Exercise(s) 16. 4. 2026

The last exercise is to be handed in at the beginning of the next lecture.

19. Sketch the set

$$\Gamma_{\gamma,\tau} \quad := \quad \left\{ \omega \in \mathbb{R}^2 \;\middle|\; |2\pi\langle k \mid \omega\rangle| \geq \frac{\gamma}{|k|^\tau} \text{ for all } k \in \mathbb{Z}^2 \backslash \{0\} \right\}$$

of (γ, τ)–Diophantine frequencies.

20. Prove the Lemma of Paley–Wiener: a periodic function with Fourier series

$$f(x) \quad = \quad \sum_{k \in \mathbb{Z}} f_k \, \mathrm{e}^{2\pi \mathrm{i} k x}$$

is (real) analytic if and only if there is exponentially fast decay

$$\bigvee_{M,\eta>0} \quad \bigwedge_{k \in \mathbb{Z}} \quad |f_k| \;\leq\; M \cdot \mathrm{e}^{-|k| \cdot \eta}$$

of the coefficient functions.