Exercise(s) 30. 4. 2026

The last exercise is to be handed in at the beginning of the next lecture.

23. Let $A \in M_{n\times n}(\mathbb{R})$ be a matrix for which all eigenvalues are different from each other. Show that the vector space $\mathbb{R}^n$ admits the splitting

$$\operatorname{im} A \;\oplus\; \ker A \;=\; \mathbb{R}^n$$

as a direct sum of two A–invariant subspaces.

24. Compute for $H_0^0 = x_1y_2 - x_2y_1$ the eigenvalues and eigenvectors of the linear mapping

$$\begin{array}{rcccc} X_{H_0^0} & : & \mathcal{G}_{k+2} & \longrightarrow & \mathcal{G}_{k+2} \\ & & W & \mapsto & \{W, H_0^0\} \end{array}$$

and conclude that the splitting

$$\operatorname{im} X_{\mathcal{T}_4} \;\oplus\; \ker X_{\mathcal{T}_4} \;=\; \mathcal{G}_{k+2}$$

can be achieved for every $k \in \mathbb{N}$.