

**Exercise(s)**

21. 5. 2026

The last exercise is to be handed in at the beginning of the next lecture.

27. Show that the normal form of a Hamiltonian  $H = H_0^0 + H_1^0 + \dots$  with  $H_0^0 = H_2$  given by

$$H_2(q, p) = \alpha_1 \frac{p_1^2 + q_1^2}{2} + \alpha_2 \frac{p_2^2 + q_2^2}{2}$$

can in the non-resonant case be written as a function of  $\sigma_1 = \frac{1}{2}(p_1^2 + q_1^2)$  and  $\sigma_2 = \frac{1}{2}(p_2^2 + q_2^2)$ .

28. Determine the lowest order term that can not be written as a function of  $\sigma_1 = \frac{1}{2}(p_1^2 + q_1^2)$  and  $\sigma_2 = \frac{1}{2}(p_2^2 + q_2^2)$  in the normal form of a Hamiltonian  $H = H_0^0 + H_1^0 + \dots$  with  $H_0^0 = H_2$  given by

$$H_2(q, p) = k \frac{p_1^2 + q_1^2}{2} \pm \ell \frac{p_2^2 + q_2^2}{2}$$

where  $k, \ell \in \mathbb{N}$ .