Exercise(s) 21. 5. 2026

The last exercise is to be handed in at the beginning of the next lecture.

27. Show that the normal form of a Hamiltonian $H = H_0^0 + H_1^0 + \dots$ with $H_0^0 = H_2$ given by

$$H_2(q,p) \quad = \quad \alpha_1 \frac{p_1^2 + q_1^2}{2} \; + \; \alpha_2 \frac{p_2^2 + q_2^2}{2}$$

can in the non-resonant case be written as a function of $\sigma_1 = \frac{1}{2}(p_1^2 + q_1^2)$ and $\sigma_2 = \frac{1}{2}(p_2^2 + q_2^2)$.

28. Determine the lowest order term that can not be written as a function of $\sigma_1 = \frac{1}{2}(p_1^2 + q_1^2)$ and $\sigma_2 = \frac{1}{2}(p_2^2 + q_2^2)$ in the normal form of a Hamiltonian $H = H_0^0 + H_1^0 + \dots$ with $H_0^0 = H_2$ given by

$$H_2(q,p) \quad = \quad k \frac{p_1^2 + q_1^2}{2} \; \pm \; \ell \frac{p_2^2 + q_2^2}{2}$$

where $k, \ell \in \mathbb{N}$.