Exercise(s) 5. 3. 2026

The last exercise is to be handed in at the beginning of the next lecture.

9. Formulate and prove Poincaré's Recurrence Theorem for Hamiltonian systems.

10. Let $H : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a given smooth function, with corresponding Hamiltonian vector field X_H. Here we use the standard symplectic structure on $\mathbb{R}^2$. Moreover, let $g : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a diffeomorphism. Consider both the function $K := H \circ g^{-1}$ — together with the associated Hamiltonian vector field X_K — and the transformed vector field $g_\star(X_H)$, defined by $g_\star(X_H)(g(p)) = D_p g X_H(p)$. Show that

$$g_\star(X_H) \quad = \quad \det(Dg) \cdot X_K \ .$$

Hint: exploit a coordinate free formulation of the fact that X_H is the Hamiltonian vector field corresponding to H. Discuss the implication for the integral curves of $g_\star(X_H)$ and X_K. Also consider the time-parametrisation of these curves. What happens in the special case that g is symplectic?