Exercise(s) 19. 2. 2015

1. In this exercise we study some properties of the Poisson bracket on a symplectic manifolds.

   Show that if two functions F, G are integrals of H then so is $\{F, G\}$.

   Show that if $h : \mathcal{P} \longrightarrow \mathcal{P}$ is symplectic then $\{F, G\} \circ h = \{F \circ h, G \circ h\}$. What is the meaning of this equation?

2. In this exercise we study co-ordinate transformations in one degree of freedom.

   Let $H : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a given smooth function, with corresponding Hamiltonian vector field X_H. Here we use the standard symplectic structure on $\mathbb{R}^2$. Moreover, let $g : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a diffeomorphism. Consider both the function $K := H \circ g^{-1}$, together with the associated Hamiltonian vector field X_K, and the transformed vector field $g_\star(X_H)$, defined by $g_\star(X_H)(g(p)) := D_p g X_H(p)$. Show that

   $$g_\star(X_H) \;=\; \det(Dg) \cdot X_K \;.$$

   Hint: exploit a coordinate free formulation of the fact that X_H is the Hamiltonian vector field corresponding to H. Discuss the implication for the integral curves of $g_\star(X_H)$ and X_K. Also consider the time-parametrisation of these curves. What happens in the special case that g is canonical?