1. Let $A \in M_{n \times n}(\mathbb{R})$ be a matrix for which all eigenvalues are different from each other. Show that the vector space $\mathbb{R}^n$ admits the splitting

$$\text{im} A \oplus \text{ker} A = \mathbb{R}^n$$

as a direct sum of two $A$–invariant subspaces.