Exercise

1. Let $A \in M_{n \times n}(\mathbb{R})$ be a matrix for which all eigenvalues are different from each other. Show that the vector space \mathbb{R}^n admits the splitting

 $\operatorname{im} A \ \oplus \ \ker A \ = \ \mathbb{R}^n$

as a direct sum of two A-invariant subspaces.