Exercise(s) 16. 4. 2026

The last exercise is to be handed in at the beginning of the next lecture.

19. Sketch the set
$$\Gamma_{\gamma,\tau} \quad := \quad \left\{ \alpha \in \mathbb{R}^2 \;\middle|\; |2\pi\langle k \mid \alpha\rangle| \geq \frac{\gamma}{|k|^\tau} \text{ for all } k \in \mathbb{Z}^2\backslash\{0\} \right\}$$
of (γ, τ)–Diophantine frequencies.

20. Prove the Lemma of Paley–Wiener: a periodic function with Fourier series
$$f(x) \quad = \quad \sum_{k\in\mathbb{Z}} f_k \, \mathrm{e}^{2\pi \mathrm{i} kx}$$
is (real) analytic if and only if there is exponentially fast decay
$$\bigvee_{M,\eta>0} \quad \bigwedge_{k\in\mathbb{Z}} \quad |f_k| \;\leq\; M \cdot \mathrm{e}^{-|k|\cdot\eta}$$
of the coefficient functions.