Exercise 8.7. Compute for the Hamiltonian

$$H(q,p) \;=\; \frac{p_1^2+q_1^2}{2} \;+\; 3\,\frac{p_2^2+q_2^2}{2} \;+\; \frac{q_1^3-q_2^3}{6}$$

in 1:3 resonance the normal form polynomial of degree 4.

Exercise 8.8. Let γ be a periodic orbit of the Hamiltonian system X_H with flow φ_t and Σ a hypersurface transverse to γ. Denote the unique point in $\gamma \cap \Sigma$ by x, put $h_0 := H(x)$ and consider for h close to h_0 the iso-energetic Poincaré-mapping $F_h : \Sigma_h \longrightarrow \Sigma_h$ on $\Sigma_h := \Sigma \cap \{H = h\}$. Let finally $T := T(x)$ be the return time of x, so $\varphi_T(x) = x$.

How are the eigenvalues of $D\varphi_T(x)$ related to those of $DF_{h_0}(x)$? Formulate a condition under which X_H has for each h close to h_0 a unique periodic orbit with period less than $2T$.