# INLDS Practicum 1

For each planar system below, construct its phase portrait numerically (using the `MATLAB` tool `pplane9`) and then try to prove its essential features analytically.

## Exercises

### Ex.1 Two systems without cycles

$$\left\{\begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & -x - y + x^2, \end{array}\right. \qquad \text{and} \qquad \left\{\begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & -x - y + x^2 + y^2. \end{array}\right.$$

### Ex.2 A system with infinite number of cycles

$$\left\{\begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & x + xy - x^3. \end{array}\right.$$

*Hint*: Consider the transformation $(x, y, t) \to (-x, y, -t)$.

### Ex.3 A system with one stable cycle

$$\left\{\begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & -x + y(1 - x^2 - y^2). \end{array}\right.$$

*Hint*: Introduce polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$.

### Ex.4 A system with an unstable cycle

$$\left\{\begin{array}{rcl} \dot{x} & = & x(x^2 + y^2 - 2x - 3) - y, \\ \dot{y} & = & y(x^2 + y^2 - 2x - 3) + x. \end{array}\right.$$

### Ex.5 Van der Pol oscillator

$$\left\{\begin{array}{rcl} \dot{x} & = & y - x^3 + x, \\ \dot{y} & = & -\varepsilon x, \end{array}\right.$$

with $\varepsilon = 1.0, 0.1$, and $0.01$.

## Homework

Hand-in exercise is number 4. A solution should contain a phase plane with a periodic orbit obtained with `pplane9`. Also prove rigorously the existence of at least one periodic orbit using the Poincaré-Bendixson Theorem. *Hint*: Introduce polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$ and consider an annulus $r_1 < r < r_2$ with suitable $r_{1,2} > 0$.

**Challenge**: Prove that the cycle is unique. *Hint*: Consider the curve $\operatorname{div} F(x, y) = 0$ as the inner border of another annulus.