INLDS Practicum 2

For each planar system below, construct its phase portrait numerically (using the `MATLAB` tool `pplane9`) and then try to prove its essential features analytically.

Exercises

Ex.1 System with a degenerate equilibrium

$$\begin{cases} \dot{x} &= x^2 - y^2, \\ \dot{y} &= 2xy. \end{cases}$$

Hint: To find an integral of motion $H = H(x, y)$, write a differential equation for the complex variable $z = x + iy$ and take the imaginary part of its general solution. Then study the level curves of H.

Ex.2 System with a neutral focus

Find and classify all equilibria of the system

$$\begin{cases} \dot{x} &= -y - xy + 2y^2, \\ \dot{y} &= x - x^2 y. \end{cases}$$

Hint: To determine local stability of the equilibrium $(0, 0)$, compute its 1st Lyapunov coefficient l_1.

Ex.3 System with a non-degenerate saddle homoclinic orbit

$$\begin{cases} \dot{x} &= -x + 2y + x^2, \\ \dot{y} &= 2x - y - 3x^2 + \frac{3}{2}xy. \end{cases}$$

Hint: The curve $x^2(1 - x) - y^2 = 0$ is invariant, i.e. consists of orbits.

Homework

Hand-in exercise is number 3. A solution should contain a phase portrait with three equilibria and a homoclinic orbit. Classify the equilibria and prove rigorously the existence of a unique homoclinic orbit. Determine stability of the homoclinic orbit analytically.