INLDS Practicum 4

For each planar system below, construct its phase portrait for $\alpha = 0$ and for small $\alpha < 0$ and $\alpha > 0$ numerically (using the `MATLAB` tool `pplane9`). Identify the occurring bifurcation and try to support your conclusions by analytical arguments as outlined below.

Exercises

Ex.1 A system with a local bifurcation

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= \alpha - y + x^2. \end{cases} \tag{1}$$

(a) Find a multiple equilibrium at $\alpha = 0$.

(b) Introduce new coordinates (ξ, η) by the linear substitution

$$\begin{cases} x &= \xi - \eta, \\ y &= \eta. \end{cases}$$

(c) Write the principal part of the system at $\alpha = 0$ in the new coordinates. Compute the normal form coefficient a for the saddle-node bifurcation and verify that $a \neq 0$.

Ex.2 A system with a global bifurcation in which a local bifurcation is involved

$$\begin{cases} \dot{x} &= x(1 - x^2 - y^2) \; - \; y(1 + \alpha + x), \\ \dot{y} &= x(1 + \alpha + x) \; + \; y(1 - x^2 - y^2). \end{cases} \tag{2}$$

(a) Substitute $x = r\cos\varphi$, $y = r\sin\varphi$ and rewrite the system in the polar coordinates (r, φ).

(b) Prove that the unit circle $r = 1$ is invariant and study equilibria on this circle.

(c) Find a multiple equilibrium at $\alpha = 0$ on the invariant circle.

(d) Compute the normal form coefficient a for the saddle-node bifurcation of this equilibrium at $\alpha = 0$ and verify that $a \neq 0$.

Homework

Hand-in exercise is number 2.