INLDS Practicum 6

In principle you should work during the exercise sessions on the exercises *of that day* and only start with the homework exercise after you finished all the others. What you could not finish during an exercise session you continue at home and if questions arise you can of course ask them during the next exercise session. However, you should *not* continue working during an exercise session on a previous exercise. Since that seems not always well-understood by everybody, today's exercise session is a 'catch-up session' to bring you up to date so that next time(s) you can work during the exercise sessions on the (then) current exercise(s). If you already are up to date on the exercises: *my congratulations* and you may now proceed to today's homework exercise below.

Exercise

Ex.1 The Hopf bifurcation. After normalization the system has the form

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \beta(\alpha) & -\omega(\alpha) \\ \omega(\alpha) & \beta(\alpha) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + l_1(\alpha) \begin{pmatrix} x(x^2+y^2) \\ y(x^2+y^2) \end{pmatrix} + O(5)$$

with $\beta(0) = 0$, $\omega(0) \neq 0$ and first Liapunov coefficient l_1 for which we assume that $l_1(0) \neq 0$.

(a) Construct explicitly the equivalence that removes the $O(5)$ terms.

(b) Can you simultaneously achieve $\omega(\alpha) \equiv 1$?

(challenge) The above normal form is simplified in that there could be additional third-order terms. Show that you also can simultaneously achieve these to become 0.

Homework

Hand-in exercise is number 1.