INLDS Practicum 7

Where helpful construct phase portraits numerically before proving the essential features analytically.

Exercises

Ex.1 Rayleigh's equation Consider the equation

$$\ddot{x} + \dot{x}^3 - 2\alpha\dot{x} + x = 0 \tag{1}$$

and rewrite it as a planar system by introducing $y = -\dot{x}$.

1. Construct the phase portrait of the resulting planar system for $\alpha = 0$ and for small $\alpha < 0$ and $\alpha > 0$ using the `MATLAB` tool `pplane9`.
2. Identify the occurring bifurcation and support your conclusions by analytical arguments as outlined below.
   (a) Introduce the complex variable $z = x + iy$ and write the planar system for $\alpha = 0$ as one complex equation $\dot{z} = i\omega z + g(z, \bar{z})$.
   (b) Compute the Taylor coefficients g_{20}, g_{11}, g_{21} and evaluate the first Lyapunov coefficient l_1.
   (c) Determine the type and direction of the Hopf bifurcation based on the sign of l_1 and the analysis of the eigenvalues of the equilibrium $x = y = 0$.

Ex.2 Brusselator Consider the planar system

$$\begin{cases} \dot{x} &=& A - (B+1)x + x^2 y, \\ \dot{y} &=& Bx - x^2 y, \end{cases} \tag{2}$$

where $A > 0$ is fixed and B is a bifurcation parameter.

(a) Find the bifurcation parameter value $B_0 = B_0(A)$ at which system (2) exhibits an Andronov-Hopf bifurcation.

(b) Determine whether this bifurcation is sub- or super-critical by computing l_1.

(c) Illustrate your analysis for $A = 1$ by constructing the phase portrait of (2) with `pplane9` at $B = B_0$ and for $B < B_0$ and $B > B_0$ with small $|B - B_0|$.

Homework

Hand-in is exercise 2.