Exercises 19. 9. 2012

1. Classify the linear systems on the plane. In particular, how is the behaviour under linear perturbations?

2. Consider the following vector fields. Are these Hamiltonian vector fields, or gradient systems? Where possible find Hamiltonian functions or potentials. Which systems are reversible (invariant under time reversal together with reflection at a well-chosen line) ?

$$\begin{array}{llll} \dot{x} = y & \dot{x} = \sin y & \dot{x} = y - \varepsilon x^3 & \dot{x} = -y^2 \\ \dot{y} = -x & \dot{y} = \cos x & \dot{y} = -x - \varepsilon y^3 & \dot{y} = -2xy \end{array}$$

$$\begin{array}{llll} \dot{x} = \alpha x - xy & \dot{x} = x & \dot{x} = x - x^2 y & \dot{x} = x \\ \dot{y} = x^2 - y & \dot{y} = -y & \dot{y} = -y & \dot{y} = -y + xy^2 \end{array}$$

   If you want to you can attack these examples with all the machinery of dynamical systems (equilibria and their linearization, stability analysis, Lyapunov function, etc.), leading eventually to their phase portraits.

3. Show that a system

$$\begin{aligned} \dot{x} &= x + \mathcal{O}(2) \\ \dot{y} &= -y + \mathcal{O}(2) \end{aligned}$$

   cannot be linearized by means of a smooth co-ordinate transformation. *Hint:* show in particular that cubic terms of the form

$$\begin{pmatrix} x^2 y \\ 0 \end{pmatrix} \quad \text{und} \quad \begin{pmatrix} 0 \\ xy^2 \end{pmatrix}$$

   cannot be removed.