Exam, due 16.1.2013 19. 12. 2012

Consider a 2–dimensional vector field with an equilibrium for which the linearization is non–hyperbolic (but has both eigenvalues non–zero) and has vanishing first Lyapunov coefficient (this is the coefficient that distinguishes between the supercritical and the subcritical case of the Hopf bifurcation). The aim is to formulate genericity conditions under which a 2–parameter unfolding is locally topologically equivalent to a standard family.

1. Use normal form theory to derive co–ordinates in which the vector field (locally) takes a particularly simple form.
2. Truncate the normal form to lowest significant order and identify the two coefficients that should serve as parameters.
3. Scale away remaining coefficients where possible. Make your own choice of the second Lyapunov coefficient to continue with.
4. Give the bifurcation diagram of the resulting standard family.
5. Show that the initial unfolding is locally topologically equivalent to the dynamics of your standard family for parameter values in the open domains of the bifurcation diagram.
6. Show that the initial unfolding is locally topologically equivalent to your standard family for 1–parameter subfamilies crossing the curves of the bifurcation diagram.
7. Show that the initial unfolding is locally topologically equivalent to your standard family.

In case you encounter problems along the way that you cannot solve, try to describe these problems as accurate as possible, and make assumptions (educated guesses) if necessary to continue.

8. Same as above, but this time for a map instead of a vector field. *Hint:* if you do not encounter problems along the way that you cannot solve, your solution is not correct!
9. How far can you get with a quasi-periodic version? Where do the problems occur?