INLDS Practicum 1

Where helpful construct phase portraits numerically before proving the essential features analytically.

Exercises

Ex.1 Show that a hyperbolic equilibrium of $\dot{z} = X(z)$, $z \in \mathbb{R}^n$ has a neighbourhood in which all other points lie on injective orbits, i.e. does not contain another equilibrium or a periodic orbit. *Hint:* use the theorem of Hartman-Grobman.

Ex.2 For $0 \neq \lambda \in \mathbb{R}$ construct a homeomorphism $h : \mathbb{R} \longrightarrow \mathbb{R}$ that conjugates the flow φ_t of $\dot{x} = \lambda x$ with the flow $\psi_t = \mathrm{e}^{\pm t}$ of either $\dot{y} = y$ or $\dot{y} = -y$, i.e. satisfies $\psi_t \circ h = h \circ \varphi_t$ for all $t \in \mathbb{R}$.

Ex.3 Construct the phase portrait of the system

$$\begin{array}{rcl} \dot{x} & = & x^2 - xy \\ \dot{y} & = & -y + x^2 \;. \end{array}$$

Is every orbit starting in the right half plane

$$\left\{ (x, y) \in \mathbb{R}^2 \;\middle|\; x > 0 \right\}$$

converging to an equilibrium?

Ex.4 Let $A \in M_{2\times 2}(\mathbb{R})$ be a 2×2 matrix for which all eigenvalues $\lambda \in \sigma(A)$ satisfy $\mathrm{Re}(\lambda) \neq 0$. Prove that the flow φ_t of $\dot{x} = Ax$ is topologically conjugate to the flow ψ_t of

$$\text{either} \quad \dot{y} = y \quad \text{or} \quad \dot{y} = -y \quad \text{or} \quad \left\{ \begin{array}{rcl} \dot{y}_1 & = & y_1 \\ \dot{y}_2 & = & -y_2 \end{array} \right. \;.$$

Hint: in case of the third possibility use what you already proved in exercise number 2 and that linear systems simply superpose.

Homework

Hand-in exercise is number 4.

Challenge: Can you generalize (with proof) this to any dimension?