INLDS Practicum 2

For each planar system below, construct its phase portrait numerically (using the `MATLAB` tool `pplane9`) and then try to prove its essential features analytically.

Exercises

Ex.1 Two systems without cycles

$$\left\{\begin{array}{rcl}\dot{x} & = & y, \\ \dot{y} & = & -x-y+x^2,\end{array}\right. \quad \text{and} \quad \left\{\begin{array}{rcl}\dot{x} & = & y, \\ \dot{y} & = & -x-y+x^2+y^2.\end{array}\right.$$

Ex.2 A system with infinite number of cycles

$$\left\{\begin{array}{rcl}\dot{x} & = & y, \\ \dot{y} & = & x+xy-x^3.\end{array}\right.$$

Hint: Consider the transformation $(x,y,t) \to (-x,y,-t)$.

Ex.3 A system with one stable cycle

$$\left\{\begin{array}{rcl}\dot{x} & = & y, \\ \dot{y} & = & -x+y(1-x^2-y^2).\end{array}\right.$$

Hint: Introduce polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$.

Ex.4 A system with an unstable cycle

$$\left\{\begin{array}{rcl}\dot{x} & = & x(x^2+y^2-2x-3)-y, \\ \dot{y} & = & y(x^2+y^2-2x-3)+x.\end{array}\right.$$

Ex.5 Van der Pol oscillator

$$\left\{\begin{array}{rcl}\dot{x} & = & y-x^3+x, \\ \dot{y} & = & -\varepsilon x,\end{array}\right.$$

with $\varepsilon = 1.0, 0.1$, and 0.01.

Homework

Hand-in exercise is number 4. A solution should contain a phase plane with a periodic orbit obtained with `pplane9`. Also prove rigorously the existence of at least one periodic orbit using the Poincaré-Bendixson Theorem. *Hint*: Introduce polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$ and consider an annulus $r_1 < r < r_2$ with suitable $r_{1,2} > 0$.

Challenge: Prove that the cycle is unique. *Hint*: Consider the curve $\operatorname{div} F(x,y) = 0$ as the inner border of another annulus.