Matrices depending on parameters

Valesca Peereboom
v.d.peereboom@students.uu.nl

Deadline: October15, 2018.

For diagonizable matrices the normal form can easily be written as a diagonal matrix of eigenvalues. However, not all matrices are diagonizable and to find a similar normal form smoothly depending on parameters for every matrix, we used versal deformations.

We say that two matrices commute when $SP = PS$ and the cummutator $[P, S] = SP - PS$ is in that case zero. Since the orbit of a matrix m is given by the set of matrices $G(m) = \{gmg^{-1}\}$ for all $g \in GL(n, \mathbb{C})$ which is the set of non singular $n \times n$- matrices, the tangent space of such a orbit can be written as $TG(m) = \{[g, m]\}$ for all $g \in GL(n, \mathbb{C})$.

During these exercises we take a closer look at the Sylvester family and the versal deformations with the fewest number of parameters (also called miniversal deformations) discussed in class.

Exercise 1.
Show by a direct computation of the commutators that the $n \times n$ Sylvester family:

$$A(\alpha) = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ . & . & . & . & . \\ & & & 0 & 1 \\ \alpha_1 & \alpha_2 & & \dots & \alpha_n \end{pmatrix}$$

is transversal to the orbit of it's A_0 matrices and has the minimal number of parameters.

Exercise 2.
a) Derive a miniversal deformation of the matrix:

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 11 & 6 & -4 & -4 \\ 22 & 15 & -8 & -9 \\ -3 & -2 & 1 & 2 \end{pmatrix}$$

(Hint: this matrix has eigenvalues 1 and -1, both of order 2)

b) Show that the miniversal deformation of matrix H is transversal to the orbit of its original matrix (that is $A(0) = A_0$)