

Black hole microstates

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Dedicated to the memory of Jacob D. Bekenstein.

Abstract

Usually, it is assumed that black hole microstates can only be constructed by modifying the known laws of physics. Here it is argued that basically two modifications are needed. One of these concerns the topology of the space-time continuation of the Schwarzschild metric, that must be modified such that there is only one asymptotic region rather than two. The other concerns the information contents of Fock Space for the Standard Model, for which a holographic constraint appears to be needed. A qualitative description is given of the microstates one then gets, how they cure the black hole information paradox, and the way they add to the total mass of a black hole. Thus, the quantum black hole serves as a ‘theoretical laboratory’, assisting us in our search for internal logic in quantum gravity theories. A striking observation is that (super)string theory is not among the modifications needed.

1 Introduction

Black hole physics carries a profound set of implications for the physics of the sub-atomic particles. J.D. Bekenstein[1, 2, 3] was one of the first to realise this. Even without asking questions about the quantum mechanical properties of black holes, one can speculate on the significance of the fact that the black hole parameters appear to obey relations that look exactly like the thermodynamical variables in samples of ordinary matter. Thus, black holes appear to have not only energy, charge and angular momentum, but also a temperature and an entropy.

A black hole isn't black. It is not exactly black. It couldn't be, because for something to be exactly black, one needs to say that it is surrounded by an exact vacuum, but to define an exact vacuum, one needs an infinite space. The black hole has only a finite size, and this is why it must be expected to emit some amount of light.

The first to actually compute the temperature of a black hole was Hawking [4]. Since this temperature is usually quite low, and its entropy quite big, these features tend to be hidden, so that there is no direct clash with other branches of physics, including the predictions from usual General Relativity about the way in which material is sucked in, accelerates, and disappears from view.

Our next primary observation is that black holes cannot respect additive conservation laws such as the conservation of baryon number or lepton number, because they can absorb arbitrary amounts of neutrons and proton-electron pairs, while keeping their mass to baryon number ratio much smaller than that of single particles. This became even more evident when it was realised that if indeed the black hole temperature does not vanish, black holes will radiate away energy, so that the ratio between energy and baryon or lepton number can shrink to zero.

The exception to this rule is the case where charges are coupled to Abelian fields such as electro-magnetism. In that case electric charge is manifest from the asymptotic values of the electric field strength, while any violation of electric charge conservation would clash with gauge-invariance, which would cause inconsistency with unitarity and/or locality for these gauge theories.

Thus, one might be inspired to suspect that black holes will have something to tell us about the fundamental interactions between elementary particles, normally laid down in the Standard Model¹. On the other hand, the fact that black holes may have properties in common with ordinary matter may suggest that there exist domains of physics where the fundamental distinction between black hole physics and particle physics may vanish. Thus, anyone interested in unified theories of physics should be interested in black holes; they are not just curious little puzzles of logic, they can teach us important, and highly

¹With 'Standard Model' we here refer to whatever quantum field theory that is needed to describe elementary particles and their interactions at energy scales reaching up to the Planck scale.

non-trivial, lessons.

When the physics community was striving at understanding the synthesis of quantum mechanics with *special* relativity, we had the luxury of a plethora of experimental approaches and observations that revealed to us how Nature works in this domain. Initially there were many competing theories, of which some were more severely criticised than others, just because they seemed to be logically imperfect. One theory that originally seemed to be logically deficient, *quantised field theory*, emerged victoriously, just because experimental observations pointed us the way; the logical imperfection of this approach turned out not to lead to insuperable practical barriers, while the theory did allow us to construct a complete scheme of all possibilities [5]. What is called the Standard Model now, turned out to be just one of these possibilities. Some parameters of the theory, notably a handful of masses and coupling strengths, had to be determined by careful measurements, after which many other features of the theory were accurately predicted, enabling us to scrutinise the accuracy of the model in meticulous detail.

What was left was the need to generalise these results such that *General* Relativity would be included. Here, we again have various contenders for the position of a unifying theory. But this time, understanding of the interplay of quantum mechanics and General Relativity, is being hampered by the unavailability of direct experimental tests. This drawback is used by some to argue that string theories are completely useless, while others, notably string theorists themselves, tend to downplay this difficulty as being merely a technical hindrance.

Yet we have seen how important the role of experiments has been in arriving at the Standard Model, and how easy it is to make severe errors of judgment when we ask for a valid theoretical scheme.

It turns out to be instructive now to regard quantised black holes as a *theoretical laboratory* [6]. The theoretical laboratory is not quite as good and impartial as the experimental one. We can only perform *thought experiments*. Our point is that, the quantum black hole behaviour depends in an essential way on the physical properties of its horizons. The black hole horizon acts as an *infinite microscope*: particles that emerge from the past horizon experience the effects of the particles that entered into the future horizon some time earlier, as if these have been magnified by almost infinite factors, and this way they fill a gap that cannot be reached by conventional particle accelerators.

Of course, the theoretical lab does not give us unequivocal results on the outcome of experiments, but it does give us something: the outcome of the theoretical experiments must, at all times, agree with some form of internal logic. This logic includes the laws for unlimited magnifications at the horizons, and it is non-trivial.

We demand this logic to cover some essential features:

- A black hole must behave exactly as the ‘classical’ or, ‘unquantised’ black hole, in

the limit where the black hole radius R_{BH} is large compared to the Planck length, or, the black hole mass, M_{BH} , is much larger than the Planck mass.

- The quantum black hole should allow a description in terms of quantum states, or energy levels, in terms of which one can describe its evolution by imposing some Schrödinger equation. It is these quantum states that are often referred to as *black hole microstates*. There exists a quantum Hamiltonian H for the black hole, that accurately dictates its evolution. Unitarity demands that this Hamiltonian be hermitian.

Strictly speaking, unitarity does not hold for systems with an asymptotic domain, but this merely implies that we have to split off the effects from particles and other features that are far separated from the black hole; imagine some boundary condition far from the black hole, in the form of a box that keeps matter particles from escaping to infinity. In particular, massless particles must be handled with care, but all of this should not lead to any difficulties of principle.

- We demand a notion of *locality*. In some treatises, it looks as if a quantum black hole has direct communication lines open with some other black hole at some far distance, or even in some other universe. This could easily lead to direct interactions between such distant partners. Locality forbids this (We return to this point in Sect. 5).

These principles may seem to be logical and inescapable, but nevertheless papers were published purportedly showing that they are mutually incompatible [7]. This is because the first and the second demand above appear to imply that an observer falling into a black hole cannot pass through the horizon. This would be contradictory, because one can also conclude from classical general relativity that the exact location of the horizon cannot be determined by Cauchy data alone; one needs to know what happens in the infinite future, thus the fate of an observer cannot depend on the position of a horizon at any finite time.

The resolution of this problem includes a postulate concerning space-time topology [8, 9], a possibility that was overlooked until recently. In such a space-time, locality does not forbid a direct interaction between a point on the horizon and its antipode.

Just because they are almost, but not quite, mutually incompatible, our logical constraints turn out to be powerful and restrictive, so that they provide important new information. As we shall see at the end, Sect. 5, one more assumption will be needed, but it is important to emphasise that we neither postulate nor exclude the validity of string theory. The point to remember here is, that it is physics at a lower energy scale where we encounter sufficient internal structure for characterising the microstates. What we do see is that the emergent mathematics concerns the horizon, which resembles a string worldsheet. For that reason, it is conceivable that a modified form of string theory might emerge [10, 11].

2 Replacing Fock space by microstates

To formulate any theory for a physical system, one must first specify what the fundamental degrees of freedom are. In most treatments addressing the quantisation of gravity, one assumes some space-time that exhibits curvature way beyond any perturbative description. The problem with this is that one then loses control over what one might expect to happen. Are there asymptotic states at all? Is there a vacuum state? To what extent may one allow for non-trivial topologies? And so on. The beautiful (and important) feature of black holes is that one might limit oneself to states that do allow for perturbative descriptions. This immediately opens the possibility of a systematic treatment of ‘all microstates’. We claim that, in describing a simple, semiclassical black hole, these microstates do form a complete set, and that this set does admit total control. It is difficult to overemphasise the importance of this. Usually investigators refer to their system as being ‘chaotic’, a euphemism for ignorance and lack of understanding.

Our discussion in this paper is mainly a qualitative one. More technical details are to be found in Refs. [12, 13]. First, we distinguish what we call *hard* particles from *soft* particles. Hard particles are particles whose mass and/or momentum exceed the Planck value, so that their effects on space-time curvature cannot be handled perturbatively. Soft particles are all other particles. We don’t ask to *ignore* the effects of soft particles on space-time curvature; rather, we assume that the effects soft particles have on space-time, can be handled perturbatively. Thus, there will be gravitons in our soft-particle spectrum, but these are handled as in non-Abelian gauge theories.

Next, the stage on which the microstates are defined will be represented as a Cauchy surface in space-time, and this space-time will be represented as a Penrose diagram for the *eternal* black hole, while all points in region *II* of this diagram are *identified* with the antipodes of their mirror images in region *I* (this will be referred to as the ‘antipodal identification’).

To many colleagues, it sounded surprising, or even objectionable, that, on this space-time metric, neither the effects of the imploding matter that was responsible for the black hole being formed, nor the effects of the evaporating particles at later times, are taken care of – the metric assumes the black hole background to be strictly static. This however is for a very good reason: we *only* include the soft particles as quantum particles on our Cauchy surface. Since the hard particles are as yet omitted, also their effects on the metric are not shown. How to include hard particles will be considered later (Sect. 4). Note that, for characterising microstates, only the Cauchy surface is needed (solid line in Fig. 1), and this does not cross parts of space-time in the future or in the past; only the present counts, so it should not matter what hard particles would be doing in the early past or the late future.

Regions *III* and *IV* in Fig. 1 are not needed. The *black hole singularity* (wavy lines

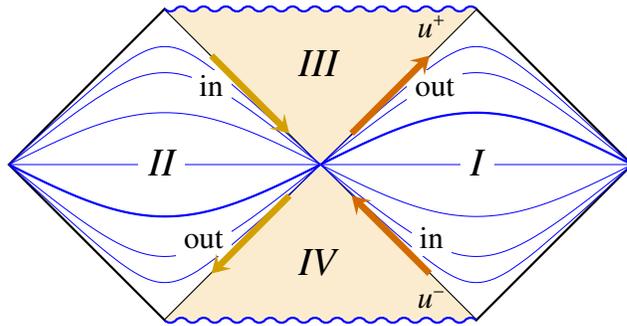


Figure 1: Penrose diagram for (Schwarzschild) black hole with the effects of in- and out going matter omitted. Curved lines are equal-time lines.

in Fig. 1) is always in the *infinite future* and the *infinite past*².

It is easy to follow the evolution of particles in this space-time during a short time-interval, where the time for the external observer stays within the range

$$|t| \ll \mathcal{O}(M_{\text{BH}} \log M_{\text{BH}}) , \quad (2.1)$$

in Planck units. During this period the black hole metric stays the same, apart from small perturbative changes due to the soft particles present. All in- and out-going particles will be soft ones, and their effects can be calculated using ‘Standard Model’ physics, combined with perturbative (quantum) gravity.

All out-going particles will be kept in the domain of soft particles. In principle, we could extend our analysis to include hard particles sent in from the outside, such as other black holes. This is known to occur in the world of black holes, but we can omit such initial states from the beginning. Also, out-going particles that are as hard as black holes, will be a rarity that is not expected to contribute significantly to the unitarity constraint. These would be genuine tunnelling events that are exponentially suppressed.

3 Time dependence

The two significant coordinates u^+ and u^- in the Penrose diagram are light-cone coordinates. The two angular coordinates are less significant for our main arguments and are

²The “black hole singularity” is often thought to reside in “the black hole center”, but this is an illusion: there is nothing in the black hole center. We explain this issue in the Appendix.

usually suppressed. Let t be the time coordinate for the external observer, then it is convenient to use a scaled time coordinate τ defined as

$$\tau = t/(4GM_{\text{BH}}) . \quad (3.1)$$

If we perform a time translation, then, near the origin, the Penrose coordinates at time τ are related to the ones at time 0 as follows:

$$u^\pm(\tau) = u^\pm(0) e^{\mp\tau} . \quad (3.2)$$

This means that, as time t flows, the positions of objects are continuously Lorentz-transformed. This also holds for the momenta of in- and out-going particles: if p^- is the in-going momentum and p^+ the out-going one, then, near the origin,

$$p^\pm(\tau) = p^\pm(0) e^{\mp\tau} . \quad (3.3)$$

We observe that the in-momentum p^- at late times inflates exponentially with time, while the point u^+ where an in-particle crosses the future event horizon, shrinks exponentially with time. An out-going (Hawking) particle leaves the past horizon at a point u^- with momentum p^+ . This momentum explodes exponentially at large negative time, while u^- expands rapidly for large positive times.

At any time, all in-going particles enter through the future event horizon and all out-going particles emerge from the past horizon. It is important to emphasise this, since it is often wrongly stated that late Hawking particles, somehow, manage to enter the scene through the future event horizon. The correct thing to say is that *black hole dynamics* describes how out-going particles emerge at the past horizon in quantum states that depend on how in-going particles enter in the future event horizon. The ensuing ‘violation of causality’ here is only apparent; in reality there will be a time delay that is large, typically

$$t_{\text{out}} = t_{\text{in}} + \mathcal{O}(4GM_{\text{BH}} \log(M_{\text{BH}}/M_{\text{Pl}})) , \quad (3.4)$$

which means that the link between out-going and in-going material is always made at values of the product u^+u^- near the Planck scale.

4 Evolution over longer time periods

In the previous section, we limited ourselves to evolution periods small compared to the intervals (2.1), for a very good reason. Following (3.3) we see that the momenta p^- of the in-going particles rapidly increase in time, while the momenta p^+ of the out-going particles rapidly increase when we follow them backwards in time. Thus, soft particles

become hard very quickly, and, as soon as we encounter hard particles, we can no longer ignore their effects on the metric.

It is not hard to suggest a flaw in our treatment so-far: we ignored the effects of the earliest in-going particles and the late out-going ones; both will severely affect the metric of space-time. Should we not replace the Penrose diagram of Fig. 1 by one where these modifications have been taken into account? Vaidya [14] and Hawking [15, 16] proposed such modifications.

This, however, would not concur with our philosophy. If we allow for these hard particles, we will very quickly be confronted with particles whose local momenta will be inflated way beyond the energy contents of the entire universe; they cannot possibly all represent existing microstates. A cut-off will be needed in any case. A much better strategy can be set up:

The only hard particles that we should expect to encounter are the ones that are squeezed either against the past horizon (the in-particles), or the future horizon (the out-particles). We can calculate the effect these will have on the metric [17, 9]. *They cause a shift $\delta u^\pm(\theta, \varphi)$ in the coordinates $u^\pm(\theta, \varphi)$ that is proportional to the momentum distribution $p^\pm(\theta', \varphi')$, and this can be calculated precisely.* We refer to this shift as the *gravitational footprint* of the in-going particles, left behind on the out-going ones.

We can adjust this result somewhat: since the earliest in-going particles give the biggest effects, and these have been long forgotten anyway, we can safely replace $\delta u^\pm(\theta, \varphi)$ by the values $u^\pm(\theta, \varphi)$ themselves. Thus, the gravitational footprint is now taken to represent the entire spectrum of the out-going particles. The out-particles *are* the gravitational footprints of the in-particles.

What is nice about this procedure are two things: one, while p^- increases exponentially with time τ , the footprint $u^-(\theta, \varphi)$ expands. It represents the wave functions of the out-particles. Since these wave functions are spreading, the out-particles rapidly turn into soft particles. This means that we successfully replaced hard particles by soft ones, while keeping all information about the spectrum intact.

Secondly, we are forced to avoid double counting. As soon as we realise that the out-particles contain all relevant information, we can discard the hard particles. Since momentum is replaced by position, the relation between in- and out-particles is a Fourier transformation. The Fourier transformation is a unitary process. This information is preserved, using soft particles only.

The procedure can now be repeated as often as we want. This way we reach all epochs in time. At all moments, we have only soft particles living in a Penrose diagram for eternal black holes³.

³from here on, there is no need to restrict ourselves on the eternal black hole, since only the data in regions *I* and *II* are considered. We may need an infinitesimal extension into regions *III* and *IV* only to check how the Cauchy surface continues from *I* into *II*.

5 Miscellaneous

It is important to note that, in the spectrum of the out-particles, their positions $u^-(\theta, \varphi)$ in the Penrose diagram can have any sign. It is an essential feature of the Fourier transform that unitarity is only maintained if both in p^\pm space and in u^\pm space, the wave functions are allowed to extend from $-\infty$ to $+\infty$. This means that both regions I and II are involved.

Now in the conventional Schwarzschild metric, all points (r, t, θ, φ) are represented twice in the Penrose diagram. Thus it would appear as if we are describing two black holes rather than one, *and these two black holes are interacting*. It was proposed to regard these two black holes as an “entangled” EPR pair [18]. This is untenable, since the two regions, I and II now interact. For a non-locally separated EPR pair it is essential that they should not interact directly. As in quantum field theory, one must impose that, when two space-time points x_1 and x_2 are space-like separated, or more generally, sufficiently far apart, then all operators $\phi(x_1)$ defined at x_1 and $\phi(x_2)$ defined at x_2 , must commute,

$$[\phi(x_1), \phi(x_2)] = 0 , \quad (5.1)$$

which is not the case [9] when x_1 is in region I and x_2 is in region II .

As we indicated in Sect. 2, the only correct way to proceed is by demanding that region II refers to the *antipodes* of region I . This was extensively discussed and motivated in Refs. [9, 13]: the simplest way to characterise Schwarzschild space-time (and its extensions to Kerr and Kerr-Newman) is by folding the Kruskal-Szekeres coordinates such that (x, y, θ, φ) is identified with $(-x, -y, \pi - \theta, \varphi + \pi)$. This is singularity-free at all points with $r > 0$. The extra singularity at $r \rightarrow 0$ is hidden far away in the unphysical region. We do see that this redefines neighbourhoods of points in I to include points in II , so that the concept of locality is affected. This is important, since often lack of locality is brought forward as an argument why there should be a “black hole information problem” [7], [19]–[25]. With antipodal identification, the Hartle-Hawking state no longer acts as an element of a density matrix (a mixed state), but it now describes Hawking particles, in terms of a pure state of the black hole. This, we claim, is a fundamental improvement of the internal logic of the system.

There are several problems still wide open. First, using the principles described in this paper, an evolution operator for the Schwarzschild black hole can be derived, but the dynamical variables that are used are not the elements of Fock space, adapted to the curved black hole background.

The fundamental parameter, instead, is the total momentum density entering the black hole at every point on the future event horizon, and it is linked to a position operator for the out-going matter at the past event horizon; since the latter is the Fourier transform of the former, and provided we use the antipodal identification constraint, the evolution operator we found is indeed unitary.

What is missing however is a unitary mapping of the Fock space elements onto the momentum density states. This may be related to another deficiency: a cut-off will be needed for the values of the ℓ parameter of the partial waves used. In principle, it should be possible to arrive at a unitary relation with a judicious choice of this cut-off, but what we are really searching for is a more complete set-up of Nature's physical degrees of freedom. This is our still open question. One might expect some version of the holographic principle to apply here: a one-to-one mapping is searched for that maps the fields describing Standard Model states on a three-dimensional Cauchy surface onto the eigenvalues of momentum operators that live on the two-dimensional horizon.

It is instructive to note how the black hole mass evolves in our procedure. In-particles arrive from far away, and out-particles leave the scenery far away. What is conserved during all stages is the total mass. As soon as an out-particle or an in-particle is sufficiently far separated from the horizon, in spite of the particle being soft, its contribution to the mass must be subtracted from the total mass, to arrive at the mass of the remaining black hole. This is a small modification in the metric of what was supposed to be the eternal black hole. Near the horizon, particles blending with the horizon continue to contribute to the total black hole mass; only when their footprints leave the black hole, their contributions to the total mass is to be subtracted, as just described. Note that the exponential inflation of p^- of a particle entering at the horizon, has no effect on the total mass at all.

Finally, we stress the importance of investigations of this nature. As our 'theoretical laboratory', we employ black holes to check the internal logic of quantum gravity theories. Our present research established that space and time possess a non-trivial topology: the Schwarzschild metric is first re-written in Kruskal-Szekeres coordinates (compressed to for Penrose diagrams), but then we observe the necessity to fold this up exactly once, such that every space-time point in this frame is identified with a point at its antipode:

$$(x, y, \theta, \varphi) \equiv (-x, -y, \pi - \theta, \varphi + \pi) . \tag{5.2}$$

This mapping, necessary for the restoration of unitarity, inverts the arrow of time.

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A The black hole singularity

In popular treatises of black holes, authors sometimes mention the black hole singularity as being something very mysterious, and “not understood” [26]. It is presented as a region of the black hole, residing at its center, where “the laws of nature become divergent”. Indeed, the Schwarzschild metric diverges at $r = 0$. The mystery is then: how could such a singularity “pop up” when a black hole is formed?

In reality, the region $r < 2GM_{\text{BH}}$ is not at the (space-like) center at all, it is at the infinite future, while what we see when we look at a black hole horizon, we see the infinite past. The r coordinate is a time coordinate there. The classical Schwarzschild black hole has one singularity at $r = 0$, and if an observer would approach this point, his time coordinate puts him beyond the infinite future for all outside observers, at all times. So we do not have to worry about singularities popping up, they emerge long after the universe ceases to exist, so that there is no clash with ordinary physics at all.

It makes more sense to compare the region behind the horizon, $r < 2GM_{\text{BH}}$, region *III* in Fig. 1, with the analytic extension of some coordinate in the complex plane. In the complex plane many models of physical systems may generate singularities such as poles and branch cuts without any alarming physical consequences. The reader is encouraged to investigate the elliptical orbit of a gravitating planet in the complex time plane, to note that the planet reaches singularities there. Astrophysicists ignore such singularities simply because the time coordinate never becomes complex. Similarly, in the case of black holes, we do not have to worry about the time-like coordinate r in the region $r < 2GM_{\text{BH}}$ just because these values for the time coordinate are unphysical.

In the present work, we also have a singularity at $r \rightarrow 0$ that corresponds to the “infinite past”. It is region *IV*, which is equally unphysical. What happened here is that we (deliberately) ignore the imploding matter that actually formed the black hole, in a distant but finite past. Ignoring that, we get an artificial singularity at the infinite past. Physically, it is of no concern. Mathematically, this allows us to regard black holes that are stationary for a sufficient amount of time in order to study its actual time dependence, as explained further in the text of this paper.

For our mathematics, we merely need the domain $r > 2GM_{\text{BH}} - \varepsilon$, for small enough $\varepsilon > 0$, in order to find the smooth extension of our metric towards the (much more important) region *II* of the Penrose diagram.

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