Strings from Gravity

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Abstract

The quantum properties of black holes are compared with those of elementary and composite particles. As argued before by this author, it is desirable to search for a theory of black holes in which quantum mechanical "information" is not drained away by the horizon, but such a theory requires a drastically new approach in formulating general coordinate transformations with horizons in a quantum theory. It is subsequently shown that a closed string with string tension $T = 1/8\pi G$ reproduces in a remarkable way the horizon fluctuations so that a new geometric interpretation of strings is suggested.

1. Introduction

The motivation behind the present activities in superstring theory is the desire to understand physics at the Planck length scale ($\hbar = c = G = 1$). Usually it is considered to be understood that once we have a set of dynamical variables and some functional integral, with an action and a properly defined measure, such that at the low energy limit not only matter fields but also gravitational fields are reproduced, then a "Theory Of Everything" is in sight.

Although the "superstring" seems to be on its way to do just that, one must be prepared for a complication that has not yet properly been taken care of: the gravitational collapse. Assume a large amount of matter, whatever it is made of, somewhere in space, so densely concentrated that gravitational collapse will occur. The length scale of the system may be taken to be large enough so that the details of the Planck length theory appear to be irrelevant. The formation of a "black hole" will be inevitable.

For an accurate description of this black hole, as seen by an outside observer, we need not consider its singularity at $r \to 0$ because this is well hidden behind the horizon. "Standard physics" applied to the relatively smooth space-time surrounding the horizon seems to be sufficient to formulate the black hole's properties, but if we do this we get an alarming result: black holes do not even approximately behave as either elementary or composite objects, which would contribute in the usual way to the Hamiltonian of the world. Since quantum mechanical waves disappear into the horizon there is a drainage of information, having the effect of a heat bath and this makes it appearently impossible to set up a Hilbert space with pure states. The only stationary state that is found for the black hole is a density matrix, as was first described by Hawking [1] which suggests strongly that a continuous and completely random radiation is emitted at a temperature

$$T_{\rm H} = 1/8\pi M \qquad (1.1)$$

where M is the black hole mass. This property, and the fact that the size and mass parameters may be arbitrarily small, makes the black hole quite different from any "soliton"-like object in ordinary field theories.

Quantum mechanically this solution must be seen as a run-away solution into an infinite dimensional Hilbert space, even though the finite value [2] for the black hole entropy S suggests that only a very special and much smaller linear subspace of this Hilbert space can ever be reached [3]. We shall explain this situation further in Section 2.

We conclude that a theory that generates an arbitrary curved space-time filled with matter fields with only locally defined dynamical variables will in general necessarily produce a much too large Hilbert space. Suppose that we *require* black holes to behave like particles, then this corresponds to restrictions independent of our local dynamics.

Most physicists would argue that such restrictions would therefore be irreconcilable with local dynamics. However it has always been such apparently paradoxical requirements that have led to new fundamental theories of nature. It is therefore worthwhile to stretch our imagination as far as we can before giving up this road of inquiry.

If we follow a black hole during a time interval of order $M \log M$ in Planck units we see that the horizon fluctuates with the in- and outgoing particles. We will discover in this lecture that these oscillations are surprisingly reproduced by a closed dual string amplitude. If indeed a superstring manages to commit the delicate conspiracy needed to keep quantum mechanically pure black hole radiation amplitudes then this may point us the way towards a better theory: black holes should be "unified" with the other elementary particles.

2. Run-away

For the Schwarzschild metric we take

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \qquad (2.1)$$

$$d\Omega^2 = \sin^2\theta\, d\varphi^2 + d\theta^2. \qquad (2.2)$$

If we replace the coordinates r and t by the Kruskal coordinates x and y defined by

$$xy = \left(1 - \frac{r}{2M}\right) e^{r/2M}, \qquad (2.3)$$

and

$$x/y = -e^{t/rM}, \qquad (2.4)$$

then the metric is regular at the points $r = 2M$ and $x = y = 0$. As usual, we define the regions I–IV as in Fig. 1.

It is natural to attribute to regions II and IV not more than a formal significance, representing the analytic continuation of the "physical" regions I and III. This is because if we take into account the collapsing amount of matter that produced the black hole at some instant in the past, then the metric is

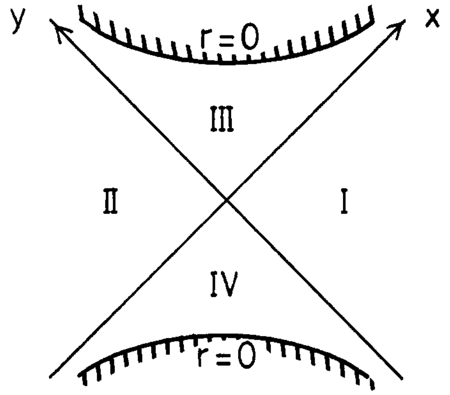

Fig. 1. Schwarzschild black hole in the Kruskal coordinate system.

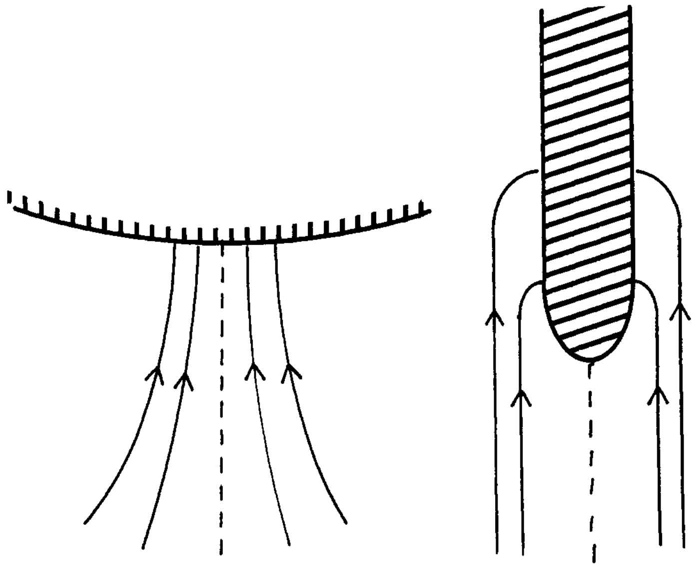

Fig. 3. Coordinate transformations of Fig. 2.

different: regions I and III are still there but curvature due to matter removes II and IV (see Fig. 2).

In a quantum theory we should be more careful however. In a description where we want to make use of time-reversal invariance (PCT), expected in any theory with only pure quantum mechanical transitions, region IV has as much raison-d'être as region III.

Temporarily keeping Fig. 2 as representing our black hole, it is sometimes more convenient to picture it as in Figs. 3(a) or 3(b). The latter shows that although the $r = 0$ singularity is space-like as seen by a local observer, it looks rather time-like as seen from the outside.

The difficulty as described in the Introduction can easily be seen to occur in every theory where gravity is "quantized" more or less conventionally. We must choose some gauge condition. Suppose we took the gauge

$$g_{00} = -1;$$

$$g_{0i} = 0 \quad (i = 1, 2, 3) \tag{2.5}$$

(temporal gauge), then the metric of Figs. 2 and 3 would have the singularity at a finite time. The foliation (choice of equal-time slices) is shown in Fig. 4.

Now there is a way to choose a gauge that avoids the singularity. Take some large S_2 spheres at space-like infinity such that time is fixed to a definite value on each sphere. A foliation inside the spheres is then defined by requiring the volume of 3-space inside each sphere, as defined by the induced 3-metric, to be maximal:

$$V = \int \sqrt{\det_{ij}(g_{ij})}\, d^3x, \tag{2.6}$$

$$\delta V = 0. \tag{2.7}$$

Here, indices from the middle of the Latin alphabet run from 1 to 3. One easily convinces oneself that, in Minkowski space, V has a uniquely defined maximum, whereas in Euclidean space one would have a unique minimal 3-volume.

Equation (2.7) amounts to

$$g^{ij}\partial_0 g_{ij} = 2g^{ij}\partial_i g_{0j}. \tag{2.8}$$

If furthermore we choose the spacelike coordinates x^i to be fixed by

$$g_{0i} = g^{0i} = 0, \tag{2.9}$$

we have

$$g^{ij}\partial_0 g_{ij} = 0, \tag{2.10}$$

$$\det_{ij}(^3g_{ij})|_{x^0=t} = \det_{ij}(^3g_{ij})|_{x^0=-\infty}. \tag{2.11}$$

which we can choose to be one. So,

$$\det_{ij}(^3g_{ij}) = 1 = \det_{ij}(^3g^{ij}),$$

$$g_{0i} = 0 = g^{0i}. \tag{2.12}$$

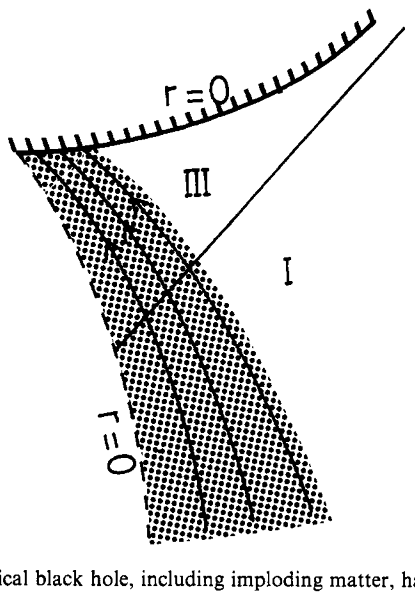

Fig. 2. A physical black hole, including imploding matter, has the Kruskal metric outside, but the regions II and IV are sealed off.

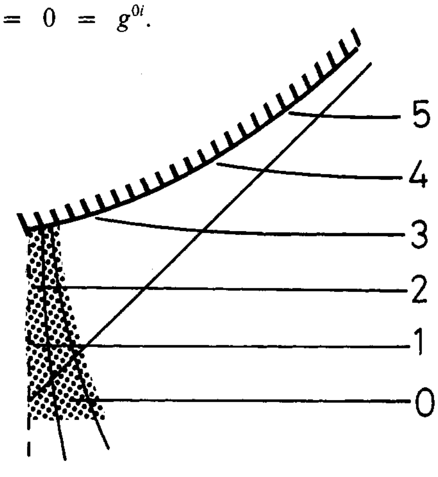

Fig. 4. Time as defined in gauge (2.5).

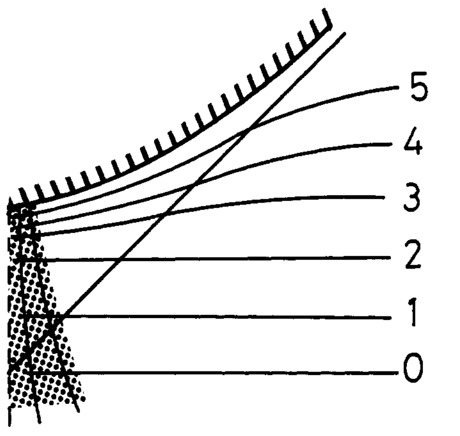

Fig. 5. Time as defined in maximal-volume gauge (2.12).

Further investigations show that this gauge is ghost-free.

In the black hole the singularity develops at

$$r \downarrow 0 \tag{2.13}$$

(indeed also charged and rotating black holes only have singularities in the subspace $r = 0$).

The factor in the determinant due to the term $r^2\,d\Omega^2$ in the metric insures that the induced 3-volume tends to zero near the singularity. This is why our foliation will avoid the singularity (a fact we have checked by explicitly computing the surfaces generated by eq. (2.12)). The foliation is sketched in Fig. 5.

We do see in Fig. 5 however that, as time goes on, a larger and larger fraction of the 3-volume will occupy the region within the horizon, out of which no information can escape to infinity. Pictorially, the shape of space-time is sketched in Fig. 6. As time goes on, a larger and larger "bubble" is blown

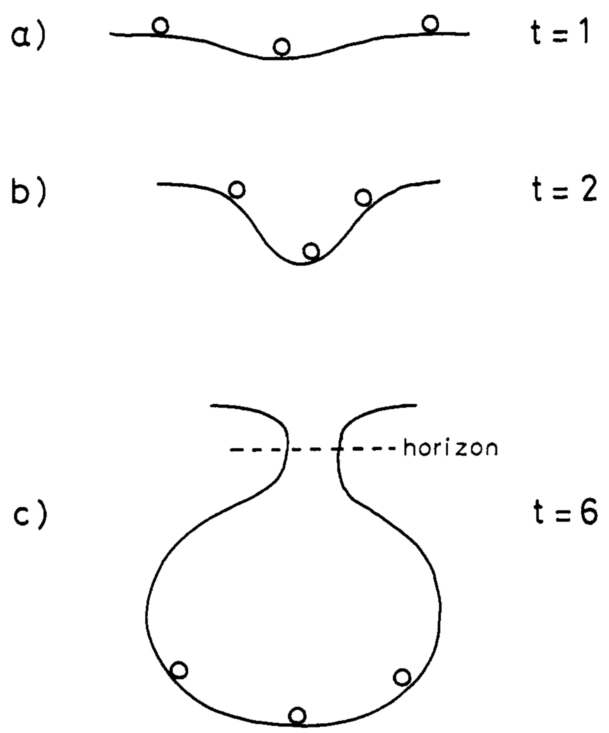

Fig. 6. Artist's impression of gravitational collapse in the maximal-volume gauge.

in 3-space: a black hole is an instability in the theory against this bubble-formation. Since essentially all of this bubble is hidden behind the horizon (dotted line), we notice nothing of it from the outside, at least according to standard general relativity.

For a quantum theory however, this situation is a disaster. Hilbert space inside the bubble is enormous, and indeed any formulation in terms of a Hilbert space would require all these nearly invisible bubbles to correspond to large sets of orthogonal basis elements, in particular if many forms of matter are allowed to live inside these bubbles.

What we actually want is a truncation of Hilbert space. Somehow, we want to characterize the states by the details at the horizon, rather than the included volume. This is also suggested by the value of the entropy of the black hole as found from its Hawking temperature applying standard thermodynamics [2].

In all respects our bubble behaves as an illegal run-away solution of the quantum theory. Very little physics was used to derive the existence of this run-away solution. Therefore we expect that any cure of this problem requires a rather drastic revision of our views on space-time.

3. The black hole scattering matrix

Hawking's result seems to imply that the quantum state of the outgoing matter does not exactly follow from the quantum state of the infalling matter: it is in a "density matrix": one single initial state results in a wide (thermal) distribution of final states, unless one keeps track of all states in an ever growing Hilbert space.

Of course one could elaborate on a theory of the world in which indeed such transitions between pure states and distributions of states (density matrices) take place [4]. However we choose to take the opposite point of view: because the entropy in a given volume seems to remain bounded we postulate the exact validity of a quantum mechanical description within a much more limited Hilbert space, and require that a quantum mechanical evolution of a black hole is determined by a well-defined Hamiltonian.

Consider now the complete history of a black hole. In the beginning there is a "star" or any other form of highly concentrated matter. An implosion (collapse) takes place and a black hole is about to be formed. In principle the black hole is only an asymptotic solution of the Einstein equations, and deviations from the metric occur close to the event horizon. But these deviations shrink exponentially with Schwarzschild time t, namely as $e^{-t/4M}$, so that at times

$$t > 4M \log M = t_1 \tag{3.1}$$

in Planck units, these deviations become essentially undetectable.

Then the well-known phenomenon of Hawking radiation sets in [1], whose temperature is presumably,* given by eq. (1.1). The energy loss per unit of time is determined by the surface of the horizon times T^4, and also a numerical factor depending on the details of the radiated particles. One easily deduces that, unless further matter is accumulated by the black hole, it will radiate away all its mass at a time scale

$$t_2 = 0(M^3). \tag{3.2}$$

* See, however, [5].

The final state is not known, but it is reasonable to assume, from time reversal invariance, that this final state consists of particles not more exotic than the ones that made the black hole in the beginning. One expects all existing particle types to occur roughly in equal amounts.

Thus we have particle states $|\{p\}\rangle_{\text{in}}$ going into the black hole, that evolve into particle states $|\{p'\}\rangle_{\text{out}}$ leaving it. Our philosophy now corresponds to assuming that there exist a unitary scattering matrix S, such that

$$|\{p'\}\rangle_{\text{out}} = S\{p\}\rangle_{\text{in}}, \tag{3.3}$$

just as in any decent quantum mechanical scattering, so that the black hole should be nothing out of the ordinary.

Now an average particle spends an amount of time of order $4M \log M = t_1$ near the horizon before it comes closer to the horizon than the Planck length. As we shall explain later in more detail, we therefore expect cross-talk between in- and outgoing particles at that time scale. But since $t_1 \ll t_2$ if $M \gg 1$ we may approximate M to be constant. Concentrating on the situation at times t with

$$t_1 \ll t \ll t_2 \tag{3.4}$$

we represent the metric there by Kruskal coordinates. The state $|\{p\}\rangle_{\text{in}}$ could be specified by giving the particle occupation numbers for all inward waves in regions I and III of the Kruskal world. Since we are mainly interested in large values of the inward momentum we will not worry for the moment about how to distinguish inward from outward particles. Regions II and IV should be left empty essentially because according to co-moving observers those regions do not exist. They are screened by matter that fell in long ago. See Fig. 7(a).

The states $|\{p'\}\rangle_{\text{out}}$ are defined on regions I and IV (see Fig. 7(b)), whereas for them regions II and III do not exist. Our scattering matrix is now supposed to link the world of Fig. 7(a) to that of 7(b). Clearly this proposal cannot be understood in terms of standard physics applied in the regular metric of Kruskal space. Why should Figs. 7(a) and 7(b) be related?

According to by now standard arguments [1] the transformation from Kruskal coordinates to Schwarzschild coordinates *does* produce outgoing particles in Fig. 7(a) and ingoing particles in Fig. 7(b). Let us write for instance a scalar field $\varphi(\boldsymbol{x}, t)$ in Schwarzschild coordinates as

$$\varphi(\boldsymbol{x}, t) = \sum_k (a(k)\,\varphi_k(\boldsymbol{x})\,\mathrm{e}^{-\mathrm{i}\omega_k t} + a^+(k)\,\varphi_k^*(\boldsymbol{x})\,\mathrm{e}^{\mathrm{i}\omega_k t}) \tag{3.5}$$

where $\varphi_k(\boldsymbol{x})$ are frequency eigen modes, and in Kruskal space

$$\varphi(x, y, \theta, \varphi) = \sum_s (a_1(s)\,\psi_s(\varrho, \theta, \varphi)\,\mathrm{e}^{-\mathrm{i}\omega_s\tau} + \text{h.c.}) \quad \text{if } \varrho > 0, \tag{3.6}$$

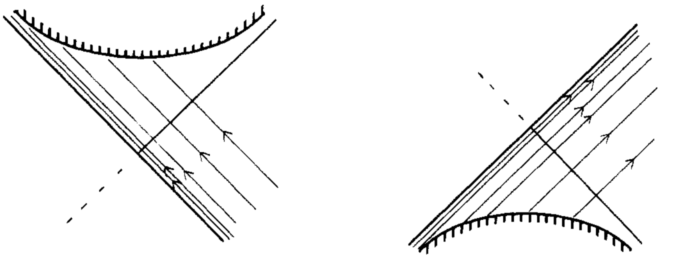

Fig. 7. (a) When all ingoing particles are included, regions II and IV disappear. (b) In the frame of outgoing particles regions II and III disappear.

and

$$\varphi(x, y, \theta, \varphi) = \sum_s (a_2(s)\,\psi_s(\varrho, \theta, \varphi)\,\mathrm{e}^{+\mathrm{i}\omega_s\tau} + \text{h.c.}) \quad \text{if } \varrho < 0, \tag{3.7}$$

where

$$\varrho = x - y$$
$$\tau = x + y \tag{3.8}$$

and ψ_s are (properly normalized) Fourier transforms of solutions to the Kruskal Klein-Gordon equation with respect to τ. The normalization is chosen such that the operators a, a_1 and a_2 satisfy the usual commutation rules.*

It is well known that one finds

$$a_1(s) = \frac{1}{\sqrt{\mathrm{e}^{\pi\omega} - \mathrm{e}^{-\pi\omega}}}(\mathrm{e}^{\pi\omega/2}A_s + \mathrm{e}^{-\pi\omega/2}B_s^+), \tag{3.9}$$

$$a_2(s) = \frac{1}{\sqrt{\mathrm{e}^{\pi\omega} - \mathrm{e}^{-\pi\omega}}}(\mathrm{e}^{\pi\omega/2}B_s + \mathrm{e}^{-\pi\omega/2}A_s^+), \tag{3.10}$$

where ω stands for ω_s, and where A_s and B_s are linear in a and satisfy the usual commutation rules for annihilation operators.

The mixing between creation operators (A, B) and annihilation operators (A^+, B^+) in the expressions for a_1 and a_2 is typical for a Bogolyubov transformation and is the reason why a state containing no outgoing particles for the Kruskal observer does have outgoing particles as seen by the Schwarzschild observer. For further details we refer to [1].

Unfortunately this does not give us the scattering matrix we want because the out state does not depend on the in state and vice versa.

As argued in earlier publications by this auther [3] there is a natural place to look for a cure to this problem. Most of the matter in Figs. 7(a) and (b) is collimated against the horizons. These particles have been boosted to such tremendous energies that their gravitational effects on the metric may no longer be ignored. In the next section we briefly resumé how these gravitational effects may be calculated.

4. Hard particles

Consider a particle at rest with a tiny rest mass m_0. At a distance $r \gg m_0$ the surrounding metric (see eq. (2.1)) can be simplified as

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \frac{2m_0}{r}(u\,\mathrm{d}x)^2 + \frac{2m_0}{r}\,\mathrm{d}r^2, \tag{4.1}$$

where we write for the 4-velocity

$$u^\mu = (1, 0, 0, 0) \tag{4.2}$$

and

$$r^2 = x^2 + (x^\mu u_\mu)^2. \tag{4.3}$$

Now that we have written the metric in a Lorentz covariant way it is easy to boost the particle to tremendous velocity [6]:

$$u^\mu \gg 1 \tag{4.4}$$

$$r \simeq |xu| \tag{4.5}$$

* Deviations from these rules are to be expected since the Hamiltonian depends explicitly on τ. These are not important for our present arguments.

$$\mathrm{d}s^2 \simeq \mathrm{d}x^2 + \frac{4m_0}{r}(u\,\mathrm{d}x)^2. \tag{4.6}$$

Let us keep

$$m_0 u^\mu = p^\mu, \quad \text{fixed,}$$

then the limit $m_0 \downarrow 0$ exists when $(xp) \neq 0$.

When $(xp) = 0$ the transverse metric from eq. (4.1) is easily seen to be flat in this limit. Let us furthermore define

$$y^\mu_\pm = x^\mu \pm 2p^\mu \log r. \tag{4.8}$$

Then we see that where

$$(xp) > 0\colon \mathrm{d}s^2 \to \mathrm{d}y^2_+, \tag{4.9}$$

and

$$(xp) < 0\colon \mathrm{d}s^2 \to \mathrm{d}y^2_-. \tag{4.10}$$

On the boundary $(xp) = 0$ we have

$$y^\mu_+ - y^\mu_- \equiv -\delta y^\mu = 2p^\mu \log y^2_{\mathrm{tr}} \tag{4.11}$$

where $y_{\mathrm{tr}} = x_{\mathrm{tr}}$ are the transverse coordinates:

$$y^\mu_{\mathrm{tr}} = y^\mu_\pm - y^0 p^\mu/p_0, \quad \text{if } y^\mu \varrho_\mu = 0. \tag{4.12}$$

It is important that eq. (4.8) contains $\log r$ and not $\log|xu|$. A more precise treatment of this derivation can be found in ref. [6].

Note that the shift δy^μ in eq. (4.11) satisfies the two-dimensional Laplace equation

$$\partial^2_{\mathrm{tr}}(\delta y^\mu) = -8\pi p^\mu \delta^2(y_{\mathrm{tr}}). \tag{4.13}$$

Particles whose rest mass m_0 is negligible but Gp^μ not (G is Newton's constant, mostly put equal to one) will be referred to as "hard particles". We see that the gravitational field of a hard particle is very simple: a flat space with Minkowski coordinates y_+, at $y^\mu_+ p_\mu > 0$, is glued against another flat space with Minkowski coordinates $y^\mu_- p_\mu < 0$, shifted according to eqs. (4.11)–4.13).

Now consider a particle such as a proton in the vicinity of a black hole, about to fall in. Its world line could be expressed in terms of the Kruskal coordinates x and y of eqs. (2.3) and (2.4). After a certain amount of time Δt we could again look at its Kruskal coordinates, but now with t replaced by $t + \Delta t$ in eq. (2.4). We see that

$$x \to x\,\mathrm{e}^{\Delta t/4M}$$

$$y \to y\,\mathrm{e}^{-\Delta t/4M}, \tag{4.14}$$

so the x coordinate expands and the y coordinate shrinks, which we recognize as a Lorentz transformation in terms of the spacelike coordinate $x - y$ and the timelike coordinate $x + y$. The Lorentz boost grows exponentially with $\Delta t/4m$, so very quickly the proton becomes a hard particle, collimated against the past horizon $y = 0$. What will its gravitational field be?

This turns out to be a soluble problem [6]. One first guesses that the solution consists of two half Kruskal metrics, shifted with respect to each other at the past horizon $y = 0$. The amount of the shift is $\Delta y(\theta, \varphi)$, where θ and φ are the transverse angles on which Δy depends. By substituting this Ansatz in the Einstein equations it was found that the Ansatz is correct if Δy satisfies the equation

$$\frac{\partial^2}{\partial \Omega^2}\Delta y - \Delta y = -Cp\delta^2(\Omega - \Omega_0), \tag{4.15}$$

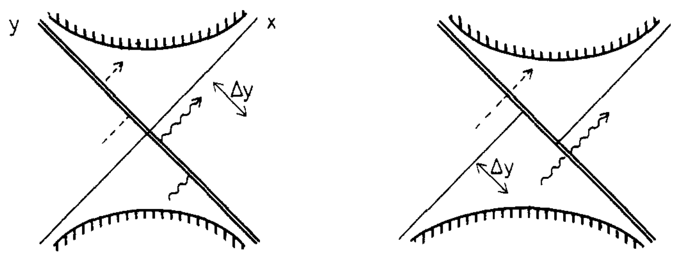

Fig. 8. Change in the metric of a black hole, induced by a single, originally soft particle falling in: (a) Two half Kruskal spaces are connected via a shift. Two geodesics are shown. (b) Coordinates can be deformed such that geodesics are continuous. Future and past horizon are seen not to coincide.

where p is the momentum of the "proton" directed inwards and the sign of Δy is defined such that $\Delta y > 0$. The constant C depends on the units used for p.

The Kruskal metric affected by the particle is pictured in Fig. 8. Earlier matter falling in (including the material that gave birth to the black hole) is here being ignored, hence the past singularity.

Equation (4.15) can be solved using spherical harmonics:

$$[l(l+1) + 1]\Delta y_{lm} = CP_{lm}. \tag{4.16}$$

Taking the case

$$\varrho(\Omega) = \delta^2(\Omega), \tag{4.17}$$

one finds:

$$\Delta y(\theta, \varphi) = \kappa \sum_l \frac{l + \frac{1}{2}}{l(l+1) + 1} P_l(\cos\theta), \tag{4.18}$$

where κ is related to C.

Using

$$\sum_{l=0}^{\infty} P_l(x)\,t^l = (1 - 2xt + t^2)^{-1/2}, \tag{4.19}$$

$$\int_0^\infty \mathrm{e}^{-s(l+1/2)} \cos\left(\frac{\sqrt{3}}{2}s\right)\mathrm{d}s = \frac{l + \frac{1}{2}}{l(l+1) + 1}, \tag{4.20}$$

we find

$$\Delta y = \kappa \int_0^\infty \frac{\cos(\sqrt{3}/2)\,\mathrm{d}s}{(\mathrm{e}^s - 2\cos\theta + \mathrm{e}^{-s})^{1/2}}. \tag{4.21}$$

Shifting the integration contour to imaginary values of s we can reexpress Δy in terms of the discontinuity of the square root. This gives:

$$\Delta y = \kappa' \int_\theta^{2\pi - \theta} \mathrm{d}z(\cos\theta - \cos z)^{-1/2}\,\mathrm{e}^{-(\sqrt{3}/2)z}, \tag{4.22}$$

in these expressions κ and κ' are certain numerical constants. We see that $\Delta y \geqslant 0$ for all θ.

Since Δy represents the horizon *shift* due to infalling matter, we conclude that this shift has everywhere the same sign: the horizon expands [6].

5. Hilbert space

The horizon shifts computed in the previous section could well turn the black hole Hilbert space into a finite one (that is, one might be able to produce a microcanonical ensemble in the vicinity of a black hole, located in a large box). We shall now show how this might work.

Figure 9 shows a black hole in the representation of Fig.

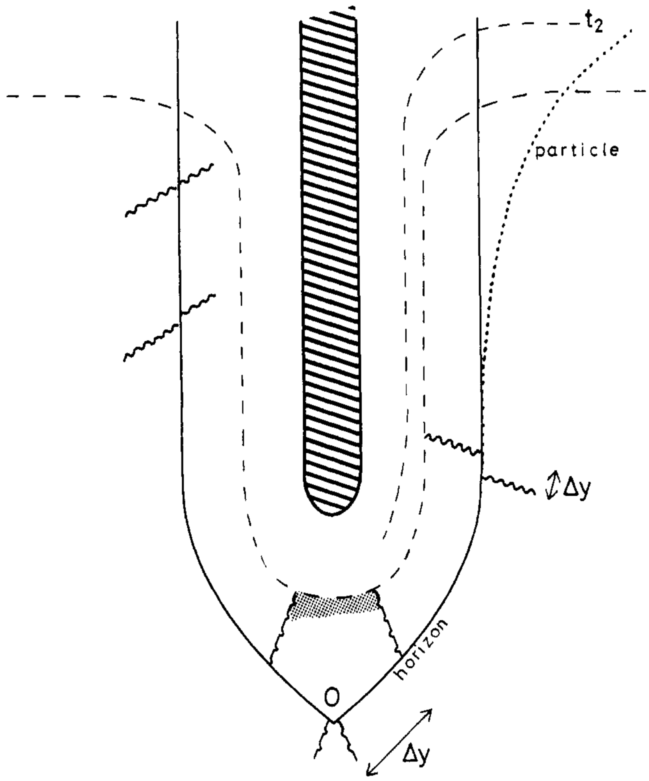

Fig. 9. Soft particles at t_1, t_2 may be hard particles at 0, producing horizon shifts. Some geodesics (wavy lines) are shown.

3(b). The broken line is a foliation at a time $t_1 \gtrsim 4M \log M$ after the black hole's formation, in a gauge that avoids the singularity as described in Section 2. Clearly it looks as if the volume of such a foliation would increase indefinitely with time. But are all possible particle and/or field configurations on the broken line allowed?

Imagine that a detector at $r = r_1 > 2M$ and time t detects either the presence or the absence of a particle. This would give us a state whose particle content is counted by the annihilation operator a_1 of eq. (3.9). If we follow the evolution of this state back to the past we see that certain particles occur at the point 0 where the horizon started. Here however it is operators of the type A and B of eqs. (3.9) and (3.10), or the operators a of eq. (3.5), which count the number of particles. Because the transformation linking these operators is of the Bogolyubov type (mixing creation with annihilation) the particle number at 0 is ill determined.

It is now not hard to convince oneself that for larger values of t_1 the particles at 0 have been boosted to tremendous energies: they are "hard" particles as discussed in Section 4. If we describe their effects on space-time in such a way that creation of these particles does not directly effect curvature of space-time everywhere, then it is natural to use coordinates such that geodesics are shifted at their respective shock waves, which all occur essentially only along the horizon: creating a particle at 0 causes a horizon shift.

The shifts in the geodesics is indicated by a few examples in Fig. 9 (wavy lines). The shift grows exponentially with negative time. Close to 0 the shift will be so large that no geodesic can enter a certain region inside the horizon, from outside. This region is indicated by shading in Fig. 9. In fact, if t_1 increases with r_1 fixed, then the boundary of the shaded region will move at a fixed distance from t_1. Thus, the "useful" part of the volume at t_1, on its broken line, will remain constant as time goes on. We may speculate that only particle states in this useful part are acceptable in a physically meaningful Hilbert space, because only these particles could have originated in a collapsing star before the black hole was formed.

The above arguments suggest that there are essentially three ways to formulate a basis Hilbert space:

(i) Specify the particle wave packets at $t = -\infty$ before the black hole was formed. Thus we prescribe exactly what goes *into* the black hole, but nothing of what comes out. This is a basis of in-states: $\{|\psi\rangle_{\text{in}}\}$.

(ii) Specify all particles that come out of the black hole, including its final explosion. This is the time-reverse of the previous picture: $\{|\psi\rangle_{\text{out}}\}$.

(iii) Specify particles that miss the black hole, and those that traverse a certain point $r \simeq 2M + \varepsilon$; $\varepsilon > 0$, where ingoing particles enter at time $t > t_{\text{in}}(\theta, \varphi)$ and outgoing particles at t leave earlier than $t_{\text{out}}(\theta, \varphi)$. The function $t_{\text{in}}(\theta, \varphi)$ depends on the outgoing particles and vice-versa.

This situation of course resembles what we have in conventional field theories. Picture (iii) corresponds to the characterization using interacting fields. There however we are free to vary time t, relating $|\psi(t)\rangle$ at different t using a Hamiltonian. This in turn would enable us to compute an S-matrix that relates the bases (i) and (ii). Apparently we do not yet have a Hamiltonian or an S-matrix for a black hole.

Yet a black hole scattering matrix can be suggested. This we do in the next section.

6. The oscillating horizon and strings

In this section we propose that indeed the horizon reflects ingoing particles into outgoing ones, or rather, ingoing states into outgoing states, because no conservation of any additive particle quantum number is expected. A simplification we perform is that we replace the angles θ, φ by transverse coordinates $x_{\text{tr}} = (x, y)$, which is probably reasonable for large black holes.

Assuming, as in Section 3, that $t_2 \gg t_1$, we now may approximate space-time surrounding the horizon by Rindler space [7]. The more daring assumption we make is that we can characterize $|\psi\rangle_{\text{in}}$ entirely by giving the momentum distribution

$$p^-_{\text{in}}(x_{\text{tr}}) \tag{6.1}$$

as a function of the transverse coordinates, and possible charges $Q_{\text{in}}(x_{\text{tr}})$, which will all leave behind a gauge field configuration that is in principle measurable (we shall make little use of these charges in our present theory but they will probably be needed in a later perfection). Similarly, the outgoing states may be given by specifying

$$p^+_{\text{out}}(x_{\text{tr}}), \quad Q_{\text{out}}(x_{\text{tr}}). \tag{6.2}$$

The momenta are defined in units generated by the Minkowski (or Kruskal) coordinates, as opposed to the Rindler (or Schwarzschild) ones. This makes the present assumption rather odd: the in-momenta will increase exponentially with Schwarzschild time and the out-momenta decrease exponentially. We are *not* specifying what kind of particles contribute to $p^{\pm}_{\text{in,out}}$ and how many of them there are. Yet this attitude will not be so crazy as we will see. In any case, the curvature of the metric will only depend on these distributions (and the surrounding gauge fields depend on Q distributions).

At infinity both the future and the past horizons are well defined. They are null-surfaces that are asymptotically flat. Close to the origin however they are not flat because space-time itself is curved due to the particles present.

If we had a Minkowski vacuum the points where future and past horizons intersect would form a flat 2-surface (or an S^2 sphere in the case of a finite-size black hole). But with particles present the intersection points form a complicated curved two-dimensional subspace of space-time. Let us call this intersection surface the space-like horizon.

What is the shape of the space-like horizon? Let us for a moment assume that the effects of in and out particles can be linearly added (an assumption which is not correct but will not be needed in our later, more careful analysis).

The ingoing particles produce a shift of the coordinate x^- of the horizon. The equation is (see eq. (6.11))

$$\partial_{\rm tr}^2 \bar{x} = 8\pi G p_{\rm in}^-, \tag{6.3}$$

where G is Newton's constant and the sign is such that for the future horizon $x^- = 0$. The x^- in eq. (6.3) is the coordinate of a geodesic that still has to penetrate the stream of ingoing particles.

The outgoing particles produce a shift x^+ with

$$\partial_{\rm tr}^2 x^+ = -8\pi G p_{\rm out}^+. \tag{6.4}$$

Now let us define the external momenta of an amplitude

$$\langle \{p_{\rm out}^+(x_{\rm tr}),\, Q_{\rm out}(x_{\rm tr})\} | \{p_{\rm in}^-(x_{\rm tr}),\, Q_{\rm in}(x_{\rm tr})\}\rangle \tag{6.5}$$

as

$$p_{\rm ex}^\mu(x_{\rm tr}) = (p_{\rm in}^-,\, -p_{\rm out}^+), \tag{6.6}$$

then we have

$$\partial_{\rm tr}^2 x^\mu = 8\pi G p_{\rm ex}^\mu. \tag{6.7}$$

This fundamental equation for the oscillations of a black hole horizon can be seen to correspond to extremizing the action

$$S = C \int {\rm d}^2 x_{\rm tr} (\tfrac{1}{2}(\partial_{\rm tr} x^\mu)^2 + 8\pi G p_{\rm ex}^\mu(x_{\rm tr})\, x^\mu) \tag{6.8}$$

against variations of the variable $x^\mu(x_{\rm tr})$. Of course we have that x^μ points into 0, 3 directions only, whereas $x_{\rm tr}$ are in the 1, 2 directions.

We now make a simple but tantalizing observation. If we rewrite the transverse coordinates as

$$x = \sigma;\quad y = \tau, \tag{6.9}$$

and substitute in eq. (6.8)

$$C = -1/8\pi G \tag{6.10}$$

then, up to an innocent looking sign, our action resembles the string action to be used in describing Veneziano amplitudes:

$$S = \int {\rm d}\sigma\, {\rm d}\tau (-\tfrac{1}{2} T(\partial_\alpha x^\mu)^2 + p_{\rm ex}^\mu(\sigma, \tau)\, x^\mu). \tag{6.11}$$

The Veneziano amplitude is*

$$\int {\rm D}x^\mu \exp {\rm i}S, \tag{6.12}$$

although still a precise formalism for the measure of this functional integral is required.

It is tempting to conclude that black hole physics is related to string theories provided that the string constant equals to

$$T = 1/8\pi G. \tag{6.13}$$

There are some minor and some more subtle obstacles. First: usual string theories are often put in a gauge such that σ and τ correspond to the coordinates x^0 and x^3, whereas the dependent variables are the transverse x^1, x^2. In our case it is the other way around. Presumably two Wick rotations, one in the σ–τ plane and one in the x^0, x^3 plane relate our unusual gauge with the more standard one. Secondly, comparison between eq. (6.11) and eq. (6.8) at first sight suggests that a minus sign went astray. However, there is nothing wrong with identifying not x^μ but $-x^\mu$ with the string coordinates. In fact this is probably what we really want: $-x^\mu$ is what an outside observer sees when he looks at an ideal surface through the "gravitational lens" of the ingoing particles. Thus, the image of an ideal surface distorted by the fields of ingoing particles is to be identified with the presently popular "string".

We are now in a position to formulate our theory for the black hole scattering matrix more precisely: in the approximation where θ and φ are replaced by $\sigma = x^1$ and $\tau = x^2$ we have

$$\langle p_{\rm out}, Q_{\rm out} | p_{\rm in}, Q_{\rm in}\rangle = \int {\rm D}x^+ {\rm D}x^- \times \exp {\rm i} \int (-T\partial_{\rm tr} x^+ \partial_{\rm tr} x^- - p_{\rm out}^+ x^- + p_{\rm in}^- x^+)\, {\rm d}\sigma\, {\rm d}\tau, \tag{6.14}$$

where the measure is to be defined as in string theory, so that the amplitudes eventually may satisfy the unitarity requirements as derived in superstring models. The charges $Q_{\rm in}$, $Q_{\rm out}$ may perhaps correspond to compactified dimensions, although as yet we have not included those in the right hand side of eq. (6.14).

Our dependent variables being $x^\pm$ it is easy to integrate one of these, say x^+:

$$\langle\ \rangle = \int {\rm D}x^- \prod_{\sigma,\tau} \delta(T\partial^2 x^- + p_{\rm in}^-) \exp -{\rm i} \int p_{\rm out}^+ x^- = \exp -{\rm i} \int p_{\rm out}^+ x^-\{p_{\rm in}\}\, {\rm d}\sigma\, {\rm d}\tau, \tag{6.15}$$

where $x^-\{p_{\rm in}\}$ is the solution of the Laplace equation generated by the delta function. We obtain:

$$\langle\ \rangle = \exp \frac{\rm i}{T} \int p_{\rm out}^+ \Delta_{\rm tr}^{-1} p_{\rm in}^-\, {\rm d}\sigma\, {\rm d}\tau \tag{6.16}$$

which of course is the same outcome if we had first integrated over x^-.

The intermediate result (6.15) is most significant. It suggests that if we look at a state where only $p_{\rm in}^-(x_{\rm tr})$ is specified then the "standard" black hole theory gives no outgoing particles at all in the Minkowski frame. We now however suggest that the outgoing particles *are* determined, and in fact given by *probing* the shifted horizon x^- using all waves $p_{\rm out}^+$ for hypothetical outgoing objects. We see that this corresponds to eq. (6.3) only, and that it makes sense to ignore here the effects of outgoing particles directly on the metric. So our problem of non-linear interference between in- and outgoing

* Note added: eq. (6.12) is Minkowskian, whereas eq. (6.11) is Euclidean. Presumably therefore the i in the following equations is superfluous. A more precise treatment will presumably involve functional integrations in complex x-space.

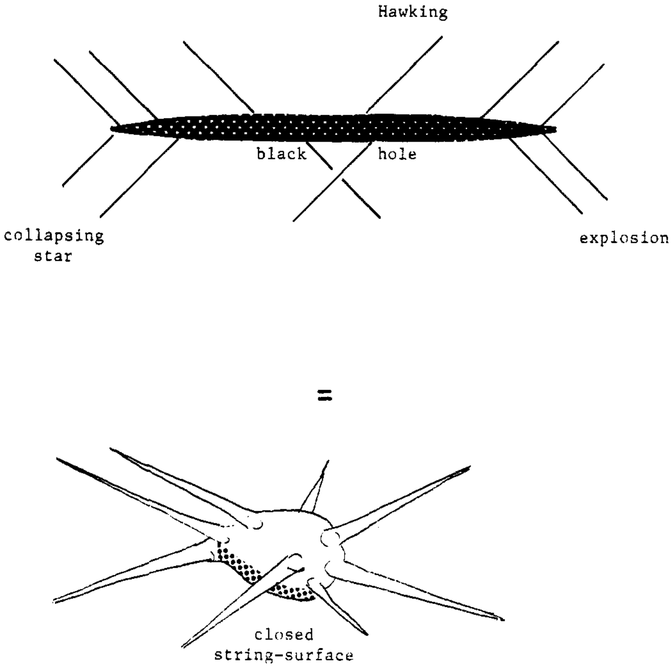

Fig. 10. Formation and destruction of a black hole. Can the amplitudes be obtained from strings theories?

particles may perhaps be irrelevant. On the other hand, time reversal invariance is built into the theory.

The new aspect of our theory is a prediction of the string constant (6.13). The curvature of space surrounding a straight string section is known to produce a conical singularity with a certain deficit angle. With our value (6.13) for the string constant this angle would be exactly one radian!

So far, we only looked at a locally flat horizon. It is easy to guess that if we look at finite size black holes with curved horizons, they must be described by a single closed string amplitude (Fig. 10). However, much care will be needed to express the transition towards asymptotically flat coordinates properly. This we have not yet understood in a satisfactory way.

Now the string amplitude preserves pure quantum states. If black holes are to be identified with closed strings then somehow their thermal properties should be reproduced. We suspect this to happen for sufficiently large black holes just because the shift of the horizon is correctly given by eq. (6.3), but the mechanism is not yet exactly understood.

Finally let us keep in mind that some of the ideas presented in these lectures are only very recent, as is often the case with young theories, their chances for survival should not be overestimated.

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