10. Prove (5.5) in the lecture notes: show that two successive transformations of that form produce one of the same form.

11. Check (5.8a).

12. Derive the expression (5.27) for the curvature starting from Eq. (5.31) as a *definition* of the Riemann tensor.

13. Suppose, that we define the covariant derivative abstractly as the linear map, which obeys Leibnitz rule: (5.15), commutes with contraction: (5.16), and acts as ordinary derivative on scalar fields (5.17). Show (or argue) that the difference of two derivative operators gives rise to a tensor C^c_{ab}. (That is $(\nabla_\mu - \tilde{\nabla}_\mu)A_\nu = C^\alpha_{\mu\nu}A_\alpha$. (Hint: The difference at point P depends only on the value of the vector field A_μ at that point and it is also a linear transformation.) Show that it is symmetric in its lower indices, if we require that $\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu$ vanishes on scalar fields. (In the general case its antisymmetric part is called torsion.) Show, using (5.9) for the covariant derivative, that this is equivalent of the symmetricity of the quantity $\Gamma^\alpha_{\mu\nu}$

14. Derive the Bianchi identity (5.30) in the coordinate frame where

$$\Gamma^\mu_{\nu\alpha} = 0 \ , \tag{1}$$

following the hints in the text after (5.30).

15. Prove the following symmetry properties (5.26) and (5.29)of the Riemann curvature tensor, supposing (5.27) to be the definition. Argue that, for the latter, we may also use the special frame where $\Gamma^\mu_{\nu\alpha}$ vanishes locally.

16. Using the symmetry properties given in Chapter 5, derive the number of *mutually independent* components of the Riemann tensor.

    When we will know how to raise and lower indices we will find one more symmetry property of the Riemann tensor:

$$R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu} \tag{2}$$

    How many independent components does the Riemann tensor have then? How many does it have in d dimensions?