Introduction to General Relativity
exercises

27. In the Schwarzschild geometry a test particle of relativistic velocity v flies past a mass M at an impact parameter r_0 so great that the deflection angle Δ_{grav} is small. Calculate Δ_{grav}. (Hint: follow the lines of the 12th chapter of the lecture notes for the calculation of the bending of light rays. Note however, that the world-line of the test particle is now *timelike*. Show that the result coincide with (12.38) when $v = c$).
In flat space a test charge e flies with velocity v past a nucleus of charge Ze with impact parameter[1] r_0 so great that the deflection angle Δ_{em} is small. Calculate Δ_{em}. Why is the formula for Δ_{grav} different from that for Δ_{em} ?
Also do the exercise indicated at the end the chapter 12 of the lecture notes: calculate the "bending" of the trajectory of a nonrelativistic particle in a hyperbolic orbit around a star.

28. Show that (12.27) $\delta\phi = 2\pi\frac{3M^2}{J^2}$ for the perihelion shift in ordinary units reads as
$$\delta\phi = \frac{6\pi G m_\odot}{r_0 c^2}$$
with $m_\odot$ being the mass of the central star.

29. Show that once a rocket ship crosses the horizon $r = 2M$ in the Schwarzschild geometry, it will reach $r = 0$ in a proper time $\tau \leq \pi M$ no matter how the engines are fired. (Hint: $g_{\mu\nu}u^\mu u^\nu = -1$ for a timelike observer and use that $u^r < 0$ within the horizon after convincing yourself about its validity.

30. Find the non-rotating magnetic monopole solution by postulating a radial magnetic field.

[1] That is the smallest distance from the central mass.