Introduction to General Relativity
exercises

5. Form "natural" units of mass, length and time out of the physical constants $\hbar$, G and c. Calculate their values.

6. Estimate the Bohr radius of a "gravitational" atom, e.g. two neutrons bound by their gravitational attraction in their lowest energy state.

7. Consider the star operation on a two-form in four dimensional Minkowski space given by

$$\star F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} \tag{1}$$

Show that, when it is applied on the electromagnetic field strength tensor, then $E \to B$ and $B \to -E$ for the three-vectors of the electric and magnetic field.

Show that if $F_{\mu\nu}$ is a solution of the free-space Maxwell equations, so is $\star F$ and also $e^{\star a}F \equiv Fcos\alpha + \star Fsin\alpha$ for arbitrary α. (Note that the latter transformation is called "duality rotation").