

Lodder, Josje – Jeuring, Johan: Math-Bridge, bridging the math gap between high school and universities

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Abstract

In most European countries there is a gap between the actual competencies in mathematics of first year students, and the competencies needed for their studies. Within the EU-funded project Math-Bridge we develop online material to bridge the gap between secondary school and higher education. The Math-Bridge service is a learning environment in which mathematical content is available in 7 different languages. Most of the material has been used before in universities from 4 different countries. The material is encoded in an open standard (OMDoc), with metadata about for example content, difficulty, competencies and prerequisites. The metadata is used in a student model, which is updated when a student interacts with the learning environment. The student model can be used for personalized course generation. A teacher can generate a course based on the requirements of a specific programme or university. The online material consists of text, applets, animations and interactive exercises. The learning environment not only updates the student model when a student solves exercises, but it also automatically generates feedback at each step of an interactive exercise. A large scale evaluation is foreseen for autumn 2011.

1 What is Math-Bridge?

In most European countries there is a gap between the actual competencies in mathematics of first year students, and the competencies needed for their studies. The cause for this gap may differ between countries. In the Netherlands one possible cause is the introduction of the graphic calculator: students use this calculator frequently at high schools, but they lack abilities in doing algebraic manipulations themselves. Also at the Open Universiteit Nederland, the Dutch distance-teaching university, students have problems with math. The course Continue wiskunde (Calculus) is considered one of the most difficult courses. A plausible explanation is that students have forgotten their high school mathematics, which they did not use for some years. The competency gap is one of the reasons for a high drop out in sciences and technical studies. A German study (Heublein et al., 2009) on reasons for drop-out mentions 32% drop-out for sciences and 24% for technical studies. In answer to the question why these students abandon their study, half of them gives the difficulty of the subject as a reason.

Abb. 4.5 Leistungsprobleme als ausschlaggebender Grund für den Studienabbruch nach Fächergruppen
Angaben in %

ausschlaggebender Abbruchgrund	Insgesamt	Sprach-/ Kultur- wiss./Sport	Wirt- schafts-/ Sozialwiss.	Mathema- tik/Natur- wiss.	Medizin	Ingenieur- wiss.	Rechtswiss.	Lehramt
Leistungsprobleme insgesamt	20	8	18	32	27	24	14	18
Studienanforderungen zu hoch	6	2	4	14	6	10	3	7
Zweifel an persönlicher Eignung	5	2	6	5	11	3	4	5
zuviel Studien- und Prüfungsstoff	4	1	4	5	6	5	6	2
Leistungsdruck	3	2	2	5	2	3	-	1
Studieneinstieg nicht geschafft	2	1	2	3	2	3	1	3

HIS-Exmatrikuliertenbefragung 2008

Figure 1 Student motives for abandoning their study, source (Heublein et al., 2009)

Many universities try to solve problems with mathematics by offering bridging courses. However, these cost time and money (the courses don't belong to the regular curriculum) and they are not always very effective, since the deficiencies of the students attending these courses can be very diverse (Mercat, 2010).

Within the EU funded Math-Bridge project, 10 universities from 7 countries work together to develop online material to bridge the gap between secondary school and higher education.³⁰ The Math-Bridge service is a learning environment in which mathematical content is available in 7 languages (English, German, French, Finnish, Hungarian, Dutch, and Spanish). Existing material from experienced teachers from four different countries (Germany, Austria, Finland, and Netherlands) has been translated and adapted for use in this service. All content is sliced into small reusable learning objects. These objects are annotated with metadata describing amongst others content, difficulty and competencies. The system uses these metadata for a student model, which is updated when a student interacts with the learning environment. The content consists of text, applets, animations and interactive exercises.

³⁰ Participants in the project are: DFKI Saarbrücken, Universität des Saarlandes, Tampere University of Technology, Universität Kassel, Universität Paderborn, Open Universiteit Nederland, Eötvös Loránd University, Universität Wien, Université Montpellier 2, Universidad Carlos III Madrid, ERGOSIGN GmbH.

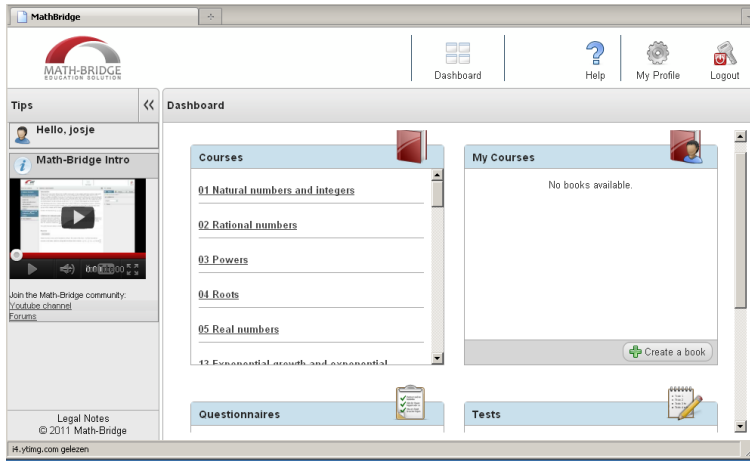


Figure 2 Homepage of the Math-Bridge learning environment

2 Multicultural and multilingual aspects

One of the distinctive features of Math-Bridge is the availability of the material in seven languages. A student who visits the Math-Bridge for the first time is asked to register and to fill in language and nationality.

Account Information

Account name:

Personal Information

I accept the [privacy policy](#).

How should MathBridge call you?
(e.g. first name or nickname)

Name

E-mail address (optional)

Language:

Country:

Roles: Author, Leamer

This information will help MathBridge choosing appropriate content items:

What is your field?

What is your educational level?

Figure 3 The login procedure

From then on, all texts are given in the language of the student. A single learning object can be rendered in different languages and the search functions allows for searching material in a specified language. Thus it is for example possible to compare the definition of a function in Dutch and English.

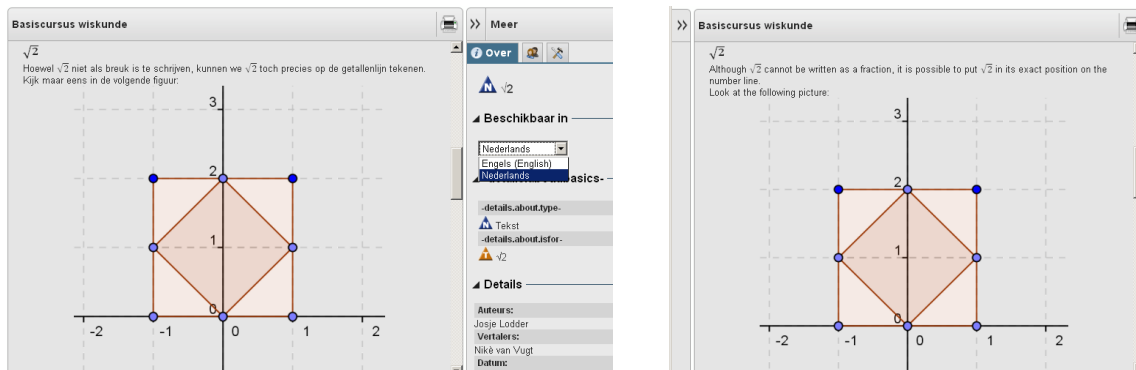


Figure 4 Translating a learning object, left the Dutch version, with the available languages, right the English version

This feature makes Math-Bridge useful for students who study abroad, and who want to compare learning materials in their own language with the foreign language, but also for students who want to prepare themselves for a study abroad, or for students studying at a distance-teaching university in a foreign language, or for immigrant students. But also for example Dutch students who study at a Dutch university, using an English textbook, can use Math-Bridge to look up terminology and definitions in Dutch (Libbrecht, 2010).

After registration, not only text is rendered in the language of the student, but also notations are rendered according to the specified language and country (Melis et al., 2009). This happens automatically: all formulae are semantically encoded, and the rendering of a formula depends on language and country. For example the tangents function is rendered in Dutch as \tan , in French as tg . Also an expression like common greatest divisor is automatically adapted: gcd in English, ggd in Dutch and $pgcd$ in French. Another example is the notation for open intervals: the semantic encoding of an open interval starting with 2 and ending with 5 is $(2,5)$; in Dutch this is automatically rendered with pointed brackets, in English with round brackets.

Een begrensde interval waarvan ondergrens en bovengrens niet meedoen, noteren we met **open** haken. Zo is $(2,5)$ het interval dat bestaat uit alle getallen tussen 2 en 5, 2 en 5 doen beide niet mee, dus:



A bounded interval in which both endpoints are not included is denoted with **open** brackets. For example, $(2,5)$ is the interval consisting of all numbers between 2 and 5, both 2 and 5 excluded, hence:



Figure 5 The rendering of an open interval in different languages

3 Adaptability and course generation

All mathematical content in Math-bridge is split up in small learning objects, and annotated with metadata. These metadata contain information about content, difficulty, competencies etc., but also about the relations between the different objects (e.g. prerequisite for ..., example for ...). These metadata are used in a student model, and for course generation. Whenever a student solves an exercise, the system updates the student model with the results of the exercise. If a student has correctly solved sufficiently many exercises on a particular subject, the system concludes that the student has mastered this subject (Gogvadze, 2009).

```
<exercise id="exc_opgave1a" for="eigenschappen-vermenigvuldiging">
  <metadata>
    <Title xml:lang="en" >Exercise 1a</Title>
    <Title xml:lang="nl" >Opgave 1a</Title>
  <extradata>
    <relation type="for" >
      <ref xref="mbase://mb_concepts/mb_numbers_and_computation/_01_02_01_03_Multiplication" type="include" />
    </relation>
    <relation type="for" >
      <ref xref="mbase://mb_concepts/mb_numbers_and_computation/_01_01_01_Natural" type="include" />
    </relation>
    <competency value="technical" level="1" />
    <competency value="solving" level="2" />
    <competency value="modeling" level="0" />
    <competency value="reasoning" level="1" />
    <exercisetype value="fill_in_blank" />
    <difficulty value="medium" />
    <learningcontext value="higher_education" />
  </extradata>
</exercise>
```

Figure 6 Metadata for an exercise

Every student can create a course consisting of existing learning objects. To create such a course, the student selects the type of the course (for example 'remedial course' or 'rehearse'), and the topics. A course is then generated automatically, with content and level adapted to the student based on the student model. Because course generation takes the dependencies between learning objects into account this results in a coherent course.

A teacher can create a course using an assembly tool. With this tool one can create a course by creating a content table, and adding content via drag and drop. A teacher composes a course according to the needs of his or her students, using his or her preferred style. It is for example easy to add extra exercises in between pieces of text. Since collections contain overlapping content, the teacher can choose between different approaches to study a particular subject, depending on the teacher's requirements about formality, depth, applications, etc. For example, Math-Bridge contains at least four different introductions to differential calculus. A special collection gathers all application oriented exercises and examples, so it is also possible to adapt a course to the field of study. A gap detection tool may be used to discover gaps in the assembled course.

5_Differenzieren collection

Finally, it is beneficial of having heard about **limit processes** and **limits**. However, we will rarely (and mainly intuitively) use these terms in this chapter.

The derivative - intuitively

We know what is the slope of a *line* - but is the slope of a curve a reasonable concept? For the moment we will be less rigorous and let intuition guide us: A curve can change its "direction", its "steepness" can vary from point to point. Thus it is reasonable to talk about direction and slope of a curve in a point? Yes it is, provided that the curve has a **tangent** at this point (i.e. it does not kink there). Then we define the *direction of the curve* to be the *direction of the tangent* and the *slope of the curve* to be the *slope of the tangent*. Let us apply this to the graph of a function: We declare: Let f be a (real) function. The **derivative** of f at x is the slope of the tangent at the graph of f at the point $(x, f(x))$. The derivative is denoted by $f'(x)$ (read " f -prime of x " or " f -prime at x ").

Remark: Of course it is assumed that the graph of f possesses a tangent at the point $(x, f(x))$. Thus this declaration is not yet a rigorous definition of the derivative but a basic idea which we use as a starting point. Thus, in principle we know what a derivative is. Given a function f with graph sketched, we want to find the derivative at x_0 . Up to now we cannot solve this **problem of the tangent** by a *calculation*, but we can *read off* the derivative *approximately* by putting a tangent to the graph at the point $(x_0, f(x_0))$ and then *determine* its slope with the help of a slope triangle. You find a corresponding example in the figure to the right. The size of the slope triangle does not matter. In the example it is chosen such that $\Delta x = 5$, then Δy is seen to be 3. The derivative is the quotient, $\frac{\Delta y}{\Delta x}$, hence $\frac{3}{5}$ or 0.6. Alternatively we can draw a slope triangle with $\Delta x = 1$, implying the slope directly to be Δy .

KasselContent collection

by $x - x_0$. Thus, the average speed for the separate sections of the path can be determined by $\bar{v} = \frac{f(x) - f(x_0)}{x - x_0}$. If we depict those considerations graphically in a coordinate system, then the quotient $\frac{f(x) - f(x_0)}{x - x_0}$ can be interpreted as slope of a straight line, which goes through the points $A(x_0, f(x_0))$ and $B(x, f(x))$. Such a straight line is called **secant**, because it intersects the function in two points.

However, the slope of the secant only describes the average speed between the two points A and B . We are, however, searching for the momentary speed at the point (of time) x_0 . Now, if x tends more and more towards x_0 , then the interval of time becomes smaller and smaller and the corresponding speed approximates more and more to the momentary speed. This process can mathematically be described by $\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$. Graphically, we can imagine that the secant becomes a tangent by approximating the point B to the point A . This tangent has the slope of the function at the point x_0 . Thus, the momentary speed at the point x_0 is equivalent to the slope of the tangent at the point x_0 .


Figure 7 Two different introductions to the derivative (more informal, more formal)

The system uses open standards (OMDoc for the encoding of the learning objects), which makes the content also usable and reusable outside the Math-Bridge service in other e-learning tools.

4 Ideas

Interactive exercises are essential for learning environments, both for students to practice their knowledge, and for the learning environment to assess the student's mastery of a subject. Math-Bridge contains many interactive exercises, and uses external services to check the correctness of answers or intermediate steps from students, give hints, show worked-out examples, detect applications of common errors, etc. One of the external services used by Math-Bridge are the Ideas domain reasoners, which support step-wise solving various mathematical exercises (Heeren et al., 2010). For example, here the domain reasoner for solving quadratic equations has been used to show a worked-out example to a student:

Figure 8 An example of an interactive exercise in Math-Bridge



The ideas framework contains 29 domain reasoners for solving exercises about solving equations: linear, quadratic, higher-order, exponential, logarithmic, and with powers, solving inequations, calculating derivatives, and manipulating formulae. Using a domain reasoner, a teacher only defines an exercise, and gets all the services for hints, worked-out examples, applications of common errors, etc. for free.


5 *Math-Bridge at the Open Universiteit Nederland*

At the Open Universiteit Nederland we will use Math-Bridge for two categories of students. The first category consists of our own Computer Science students. These students have to take two math-courses in their first year. These courses contain subjects from discrete mathematics, and they do not require much knowledge of high-school mathematics. Hence, although students have to work hard for these courses, they like them, and the success rate for the exams is not lower than for other subjects. In the second year, students have to take a calculus course and here the situation is completely different. Students do need their high-school mathematics, but for most of our students high-school is too long ago (computer science students at the OUNL combine their study with a job), so they miss the skills to manipulate formulae and they forgot the definitions of e.g. logarithms and trigonometry. We offer students a pretest. After completing this test the student gets an advise; this advice might be that the student is ready to start with the course, but also that there are several deficiencies. When the level of the student is too low, we advise the student to take a preparation course, but when there are only deficiencies in some topics, a student can use Math-Bridge to master these topics.

A second group consists of students who want to start at a regular university, but who didn't take the right version of mathematics for their high school graduation. In the Netherlands, high school pupils choose between different profiles for their graduation. Each profile has its own type of mathematics. However, universities admit only students with the right type of mathematics for example sciences or medicine. Students can take a special examination to be admitted anyway. The OUNL organizes these exams, and also offers courses to prepare for these examinations. For the first part of these courses we composed a special course, combining our own material with translated material from our partners.

6 *Evaluations*

In spring 2011 we performed some small scale interviews and evaluations. In the interviews representatives from different universities in different countries were asked to evaluate the system. Leading questions were: what do you think of the system and what do you thing of the content. Of course the interviewees had some critical remarks, but overall they appreciated the intuitive design, the easy navigation and the possibility for stepwise exercises. They concluded that Math-Bridge can be used as supplementary material for bridging courses. We also had a few students who tried Math-Bridge. They could use Math-Bridge in addition to a textbook, and although one of them complained



that Math-Bridge didn't offer much extra (at that moment we had only made the first part of our course available) they seemed to like to work with Math-Bridge.

This autumn Math-Bridge will be evaluated at 6 different universities all over Europe. We prepared a pre- and posttest to measure knowledge gain, and a list of evaluation questions. We will use the results to improve the system.

7 Conclusion

The results of Math-Bridge project consists of a rich system, containing lots of material in different languages. The content can be adapted by teachers or students according to their needs. Although only one distance-teaching university (Open Universiteit Nederland) is participating in the project, the system is also interesting for other distance-teaching universities. They can use existing material, add their own translations if needed, or even add new content.

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