**Solution of Exercise 2.31**.

(i) Straightforward computation.

(ii) Note that $g \circ f : \mathbf{R}^n \to \mathbf{R}$. We have

$$D(g \circ f) = ((Dg) \circ f) \circ Df = (g' \circ f)Df,$$

where $g' \circ f : \mathbf{R}^n \to \mathbf{R}$. Therefore, by means of transposition,

$$\mathrm{grad}(g \circ f) = (D(g \circ f))^t = ((g' \circ f)Df)^t = (g' \circ f)(Df)^t = (g' \circ f)\,\mathrm{grad}\, f.$$

(iii) $g : \mathbf{R}^n \to \mathbf{R}$ and $D(g \circ f) = ((Dg) \circ f) \circ Df$ imply

$$\mathrm{grad}(g \circ f) = D(g \circ f)^t = (Df)^t \circ (Dg)^t \circ f = (Df)^t (\mathrm{grad}\, g) \circ f.$$

In particular,

$$\begin{aligned}(\,\mathrm{grad}(g \circ f))_j &= \langle \mathrm{grad}(g \circ f),\, e_j \rangle = \langle (Df)^t (\mathrm{grad}\, g) \circ f,\, e_j \rangle = \langle (\mathrm{grad}\, g) \circ f,\, (Df)e_j \rangle \\ &= \langle (\mathrm{grad}\, g) \circ f,\, D_j f \rangle.\end{aligned}$$