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Distributions. Theory and applications. Transl. from the Dutch by J. P. van Braam Houckgeest. (English)

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This is a very useful, well-written, self contained, motivating book presenting the essentials of the theory of distributions of Schwartz, together with many applications to different areas of mathematics, like linear partial differential equations, Fourier analysis, quantum mechanics and signal analysis. Although there are many good books that present an introduction to the theory of distribution of Schwartz, assuming different levels of knowledge of linear functional analysis, the text of Duistermaat and Kolk is unique in that it presents distributions as a natural method of analysis from a mathematical and a physical point of view, and in that it emphasizes applications to physical phenomena like optics, quantum field theory, signal reconstruction or computer tomography. The exposition stresses in applications and interactions with other parts of mathematics.

The importance of the theory of distributions in mathematical analysis and in partial differential equations is well known. Lars Hörmander, in the preface from 1983 of his monumental treatise The Aanlysis of Linear Partial Differential Operators, Springer, Berlin, 1983, wrote that "the progress in the theory of linear partial differential equations during the last 30 years owes much to the theory of distributions created by Laurent Schwartz at the end of the 1940's. It summed up a great deal of the experience accumulated in the study of partial differential equations up to that time, and it provided an ideal framework for later developments." Despite of being such a powerful tool, it has been sometimes misunderstood and undervalued. Sometimes, the presentations of the theory required much familiarity of the reader with measure theory and abstract functional analysis. The present authors have tried hard to present a self-contained text. The reader is only assumed to have a knowledge of linear algebra, analysis of several variables and one-variable complex analysis. At several places precise references to the authors' book [Multidimensional Real Analysis. I: and II. Cambridge Studies in Advanced Mathematics 86. Cambridge: Cambridge University Press. (2004; Zbl 1077.26001), Cambridge Studies in Advanced Mathematics 87. (2004; Zbl 1077.26002)] are given.

One of the main features of this book is that many clarifying examples, presented with full detail, are included in the text. Moreover, a large number of problems is included at the end of each chapter. Their level of difficulty varies. Working through the exercises the reader would re-examine many aspects of analysis of several variables at the light of the theory of distributions. Complete solutions of 146 of the 281 problems are provided and hints are given of many of the others.

After a first, very interesting Chapter 1, that presents several motivating examples of the theory of distribution, coming from different areas of mathematics and physics, the next ten chapters cover the basic theory of distributions. The following topics are discussed: test functions, definition of distributions, differentiation and convergence of distributions, localization, distributions with compact support, multiplication by functions and convolution of distributions. One of the important characteristics of the present treatment of the theory is the systematic use of the operations of pullback and pushforward, presented in Chapter 10 and used in the rest of the book. It enables the authors a very clean and concise notation. Convergence of distributions is defined for sequences. However, seminorms and elementary theory of locally convex spaces is introduced in Chapter 8. In any case, the amount of functional analysis that is needed is reduced to a minimum; it includes the uniform boundedness principle and the theorem of Hahn-Banach.

Fundamental solutions of a linear partial differential operator with constant coefficients, their relevance and a few preliminary examples are given in Chapter 12. Parametrices are also defined and results about the singular support to be utilized in Chapter 17 are established here. Chapter 13 is very interesting; it deals with the complex powers of the differentiation operator, a concept elaborated by M. Riesz in the 1940's. Riesz's treatment of the wave equation and a fundamental solution of the wave equation are presented, too. Chapter 14 about the Fourier transform, the space $S(\mathbb{R}^n)$ of rapidly decreasing functions of Schwartz and tempered distributions is one of the longest of the book, and it has 61 problems at the end, the last one about the Radon transform. A proof of the kernel theorem of Schwartz using the Fourier inversion formula is given in Chapter 15. The theory of Fourier series is reconstructed in Chapter 16 from the theory of Fourier transform.

The first part of Chapter 17 studies elliptic partial differential operators with constant coefficients. An extremely efficient and explicit proof, due to N. Ortner and P. Wagner in 1996, of the theorem of Malgrange and Ehrenpreis that every linear partial differential operator $P(D)$ with constant coefficients has a fundamental solution is presented in Theorem 17.13. We recommend the reader the recent, even simpler proof by *P. Wagner* in [Amer. Math. Monthly 116, 457-462 (2009)]. Another proof of Malgrange-Ehrenpreis theorem, based in the Paley-Wiener-Schwartz theorem for distributions with compact support, is given in the next Chapter. Explicit formulas of other fundamental solutions are also included in Chapter 17. The main features of Chapter 18 are a study of hyperbolic operators and, especially, the sampling theorem of Whittacker, Kotelnikov and Shanon and an introduction to Paley Wiener spaces. A short introduction to Sobolev spaces $H^s(\mathbb{R}^n)$ is presented in Chapter 19. In the Appendix, Chapter 20, the authors survey the theory of Lebesgue integration with respect to a measure with the point of view of Daniell, which emphasizes linear forms and is very appropriate in the present context. A full proof of Riesz representation theorem is included.

This reviewer found the book under review very stimulating, informative and inviting to further study. It is a very useful text for students who want to learn the theory of distributions, for experts in the abstract theory who want to read an application-oriented presentation with many examples and, last but not least, for mathematicians in related areas or theoretical physicists who need a self-contained, reasonable short presentation of the theory. I would strongly recommend it to all my colleagues.

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Keywords : Theory of Distributions; Fourier transform; Linear partial differential operators; fundamental solutions; Sobolev spaces
Classification :
 ∗46-02 Research monographs (functional analysis)
 46Fxx Generalized functions, etc.