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★**Lie groups. (English summary)**
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This well-thought-out, clearly written book is an excellent addition to the literature on Lie groups and their representations. This volume covers the theory of finite-dimensional Lie groups and their representations, with an emphasis on compact groups. The authors take a differential-geometric approach. In the first, introductory, chapter many of the fundamental concepts of Lie groups are discussed. In the remaining three chapters the main focus is on compact groups and their representations. This book is intended for advanced graduate students or above. The authors assume a familiarity with differential geometry and some knowledge of ordinary differential equations. There is, however, a short appendix in which they review some of the basic concepts used from differential geometry and ordinary differential equations.

The first chapter gives a treatment of many of the usual fundamental properties of Lie groups, Lie algebras and their relations. Besides the standard results on the exponential map, Lie subgroups, Lie homomorphisms, quotients and simply connected Lie groups, this chapter also includes a discussion of Lie's fundamental theorems and results on conjugacy classes. Instead of the usual Campbell-Baker-Hausdorff formula for the Taylor expansion at the origin of the product in logarithmic coordinates, the authors give Dynkin's formula, which is much more explicit. This chapter is relatively short, but nevertheless the authors manage to give a thorough treatment of most of basic Lie group theory. Additionally, it includes many excellent examples, which illustrate the beauty of the theory and some of the intricate difficulties one can encounter.

The main focus of the remainder of this volume is on compact groups and their representations, which are discussed extensively in the remaining three chapters. First, in Chapter 2, proper actions on manifolds are discussed. This includes a discussion of the slices and fiber bundles associated with a group action. The authors also discuss smooth functions on the orbit space, orbit types, local action types, a stratification by orbit type, principal and regular orbits and finally a desingularization. The latter is a real version of a desingularization given in a complex algebraic setting.

Using the concepts developed in Chapter 2, the book gives a thorough discussion of the structure of compact groups in Chapter 3. This includes Weyl's covering theorem, from which it follows that all elements in a connected compact group are semisimple and hence contained in a maximal torus. All of these maximal tori are shown to be conjugate. This chapter also covers the orbit structure in the Lie algebra, Weyl's integration theorem, the computation of the fundamental group, unitary groups and a discussion on non-connected compact groups. One of the only things missing from this section is a classification of compact Lie groups, which relies completely on algebraic calculations. This would be out of line with the differential-geometric approach of the book.

The last chapter discusses the representations of compact groups. The main emphasis here is on complex representations, although a discussion of real representations is included as well (Section 4.8). The first few sections lay the groundwork for the famous Peter-Weyl theorem. Then the authors continue with Weyl's character formula, Cartan's highest weight theorem, the Borel-Weil theorem and a discussion of the non-connected case. This chapter also includes a large number of examples and exercises which illustrate the subtleties of the relation between the representations and the weights, and the role of the fundamental group in this.

A number of results from the literature are collected for the first time in a text in this book. Each chapter includes an extensive bibliography and a Notes section that references the literature and gives many historical notes.

This book is extremely well written and gives a very clear exposition of the results. It is likely to become one of the standard reference works for a long time to come. I am looking forward to the next volume, which the authors mention briefly. While there are many excellent books about Lie groups and their representations, this is one of the best currently in print. *Aloysius Helminck*