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Multidimensional real analysis II: Integration. Transl. from the Dutch by J. P. van Braam Houckgeest. (English)

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The book under review is the second volume of a two volume set of an introduction to the theory of real functions on $\mathbb{R}^n$. (For a review of Part I see Zbl 1077.26001.) The present volume II is devoted to the theory of n-dimensional integration in $\mathbb{R}^n$ and the lower-dimensional integration over submanifolds of $\mathbb{R}^n$. One aim of the authors is to develop a "working knowledge" about integration of functions of several variables, in particular by discussing numerous examples. For this the theory of Lebesgue measure and integration is not needed. The authors therefore develop the integration in the sense of Riemann and the measurability of a subset of $\mathbb{R}^n$ in the sense of Jordan.

Volume II uses some results from the first volume, but it should be accessible to those readers who learned the basics of differentiable mappings and submanifolds of $\mathbb{R}^n$.

The material is organized as follows: 6. Integration, 7. Integration over submanifolds, 8. Oriented integration, Exercises.

In Chap. 6 the theory of the Riemann integral from the calculus of one real variable is extended to $\mathbb{R}^n$. Principal results are a theorem on the reduction of n-dimensional integration to successive one-dimensional integration, and the Change of Variables Theorem. For this fundamental theorem the authors present three proofs: the first proof which is given in the text, follows the standard reasoning "reduction to $\mathbb{R}$", while two other proofs are presented in the appendix to this chapter. As applications the authors treat Fourier transform and Arzelà's dominated convergence theorem (which forms an effective alternative for Lebesgue's dominated convergence theorem).

Chap. 7 begins with a discussion of densities and integration with respect to a density. In particular, the d-dimensional volume (length, area, etc.) are defined. The authors then express the integration of a total derivative in terms of an integral over the boundary of the underlying domain, which forms the generalization to $\mathbb{R}^n$ of the Fundamental Theorem of Integral Calculus in $\mathbb{R}^n$. From this the Gauss' divergence theorem is then easily deduced.

In Chap. 8 the authors first study line integrals and some properties of vector fields, and prove then an existence result for a scalar potential for a vector field. The Stokes' integral theorem is proved in $\mathbb{R}^3$. The remainder of this chapter is devoted to an introduction to differential forms and the proof of the general Stokes' integral theorem.

Analogously as Volume I, a special feature of the present volume is a very large collection of exercises which present variations and applications of the theory, and run from routine to advanced topics. Many exercises are accompanied by hints, and a number of exercises goes beyond the scope of the book.

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Throughout the text is carefully organized, proofs are complete and rigorous and the material is completed by carefully worked examples.
Summarizing, the book is an excellent introduction to the n-dimensional integration in $\mathbb{R}^n$ and the integration over submanifolds of $\mathbb{R}^n$.

Joachim Naumann (Berlin)

Keywords : Riemann integrability; change of variables theorem; integration over d-dimensional manifolds; Stokes' integral theorem
Classification :
*26-01 Textbooks (real functions)
26B15 Integration (several real variables)
26B20 Integral formulas