

**Exercise.** Let  $F$  be a finite field of characteristic  $p$ .

- (1) Define  $\phi : F \rightarrow F$ ,  $\alpha \mapsto \alpha^p$ . Prove that  $\phi$  is a ring automorphism (i.e. ring isomorphism to itself). Is  $\phi$  an  $\mathbb{F}_p$ -automorphism? The map  $\phi$  is known as the Frobenius automorphism.
- (2) Suppose  $f(x) \in F[x]$  is irreducible. Prove that  $f(x)$  is separable. Conclude that all polynomials over  $F$  are separable. *Hint: use Thm. 4.8.2 and the Frobenius automorphism.*
- (3) Give an example of an inseparable polynomial.