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Erratum: Elements of Applied Bifurcation Theory

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1

Introduction to Dynamical Systems

p. 8, l. 6

Example 1.8 (Symbolic dynamics revisited)

p. 12, l. -4

$$S \cap f(S) = V_1 \cup V_2$$

p. 19, l. -15

$$H(\varphi, \psi) = \frac{\psi^2}{2} - k^2 \cos \varphi. \ \diamond$$

p. 21, l. 2

that is smooth in (t, x_0),

p. 30, l. 13

If no generalized eigenvectors are associated to q, then the monodromy matrix $M(T_0)$ has a one-dimensional invariant subspace spanned by q and a complementary invariant $(n-1)$-dimensional subspace $\Sigma : M(T_0)\Sigma = \Sigma$. Take the subspace Σ as a cross-section to the cycle at $x_0 = 0$. One can see that the restriction of the linear transformation defined by $M(T_0)$ to this invariant subspace Σ is the linearization of the Poincaré map P defined by system (1.14) on Σ. Therefore, their eigenvalues $\mu_1, \mu_2, \ldots, \mu_{n-1}$ coincide.

If generalized eigenvectors are associated to q, the theorem remains valid, however, the proof becomes more involved and is omitted here. $\square$

2

Topological Equivalence, Bifurcations, and Structural Stability of Dynamical Systems

p. 41, l. 9
We can write the last equation in a more compact form using the symbol of map composition:

p. 43, l. 13
in $V = h(U) \subset \mathbb{R}^n$,

p. 64, l. 19
which would imply that the map $(x, \alpha) \mapsto (h_\alpha(x), p(\alpha))$

p. 77, l. 12
images of $\mathbb{R}^{n-}$ and $\mathbb{R}^{n+}$ under injective immersion.

3

One-Parameter Bifurcations of Equilibria in Continuous-Time Dynamical Systems

p. 97, l. 11

Obviously, a composition of the transformations

p. 98, l. 3

$$\dot{z} = \dot{w} + h_{20} w \dot{w} + h_{11}(w \dot{\bar{w}} + \bar{w} \dot{w}) + h_{02} \bar{w} \dot{\bar{w}},$$

p. 102, l. -12

At $\alpha = \alpha_0$, the nontrivial equilibrium E_0 ~~at $\alpha = \alpha_0$~~ has

p.104, top

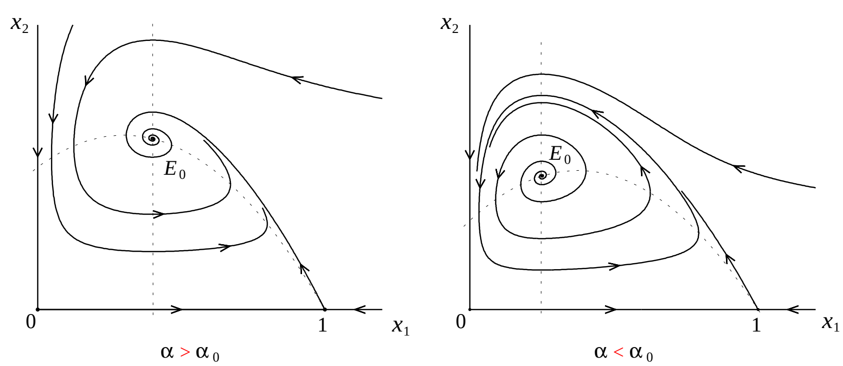

FIGURE 3.11. Hopf bifurcation in the predator-prey model.

p.106, l. -10

```
> s1:=simplify(evalc(conjugate(P[1])*q[1]+conjugate(P[2]*q[2])));
> c:=simplify(evalc(1/conjugate(s1)));
> p[1]:=simplify(evalc(c*P[1]));
> p[2]:=simplify(evalc(c*P[2]));
> simplify(evalc(conjugate(p[1])*q[1]+conjugate(p[2])*q[2]));
```

p. 107, l. 13

```
> g[2,0]:=simplify(2*evalc(coeftayl(H,[z,z1]=[0,0],[2,0])));
> g[1,1]:=simplify(evalc(coeftayl(H,[z,z1]=[0,0],[1,1])));
> g[2,1]:=simplify(2*evalc(coeftayl(H,[z,z1]=[0,0],[2,1])));
> x[2]:=evalc(X[2]+z*q[2]+z1*conjugate(q[2]));
```

p. 107, l -13

```
> l[1]:=factor(1/(2*omega^2)*Re(I*g[2,0]*g[1,1]+omega*g[2,1]));
```

p. 107, l. -11:

$$l_1 = \frac{1}{2\omega^2}\mathrm{Re}(ig_{20}g_{11} + \omega g_{21}) = -\frac{2+A^2}{2A(1+A^2)} < 0,$$

p. 107, l. -4:
(*Hint:* Introduce $y = -\dot{x}$ and rewrite the equation as a system of two differential equations.)

p. 110, l. -8:
Fix α small but positive. Both systems (A.1) and (A.2) have a limit cycle in some neighborhood of the origin. Assume that the time reparametrization resulting in the constant return time 2π is performed in system (A.1) (see the previous step). Consider a homeomorphism H that conjugates the Poincaré map of (A.1) with that of (A.2) at this parameter value.

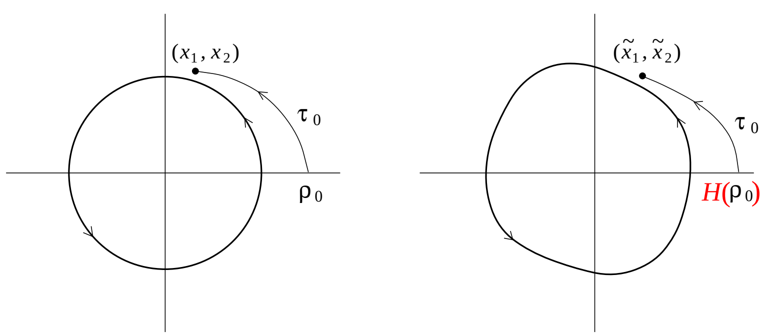

FIGURE 3.14. Construction of the homeomorphism near the Hopf bifurcation.

Define a map $z \mapsto \tilde{z}$ by the following construction. Take a point $z = x_1 + ix_2$ and find values (ρ_0, τ_0), where τ_0 is the minimal time required for an orbit of system (A.2) to approach the point z starting from the horizontal half-axis with $\rho = \rho_0$. Now, take the point on this axis with $\rho = H(\rho_0)$ and construct an orbit of system (A.1) on the time interval $[0, \tau_0]$ starting at this point.

p. 111, l. -11:

Phase-portrait bifurcations in a generic one-parameter system on the plane near an equilibrium with purely imaginary eigenvalues were studied first by Andronov & Leontovich [1939].

4

One-Parameter Bifurcations of Fixed Points in Discrete-Time Dynamical Systems

p.114, l. -12:
Remark:

This bifurcation is also referred to as a *limit point, saddle-node bifurcation, turning point,* among other terms. $\diamondsuit$

p.119, l. 19:
where $x_{1,2} = \pm\sqrt{\alpha}$ (see Figure 4.4).

4.5 Generic flip bifurcation

Moved from p. 121, ll. -9 – -5:

By the Implicit Function Theorem, the system $x \mapsto f(x, \alpha)$ has a unique fixed point $x_0(\alpha)$ satisfying $x_0(0) = 0$ in some neighborhood of the origin for all sufficiently small $|\alpha|$ since $f_x(0,0) \neq 1$. We can perform a coordinate shift placing this fixed point at the origin. Therefore, we can assume without loss of generality that $x = 0$ is the fixed point of the system for $|\alpha|$ sufficiently small.

Theorem 4.3 *Consider a one-dimensional system*

$$x \mapsto f(x, \alpha), \quad x \in \mathbb{R}^1, \ \alpha \in \mathbb{R}^1,$$

with smooth f satisfying $f(0, \alpha) \equiv 0$, and let $\mu = f_x(0,0) = -1$. Assume that the following nondegeneracy conditions are satisfied:

(B.1) $\frac{1}{2}(f_{xx}(0,0))^2 + \frac{1}{3} f_{xxx}(0,0) \neq 0$;

(B.2) $f_{x\alpha}(0,0) \neq 0$.

Then there are smooth invertible coordinate and parameter changes transforming the system into

$$\eta \mapsto -(1+\beta)\eta \pm \eta^3 + O(\eta^4).$$

Proof:

The map f can be written as follows:

$$f(x,\alpha) = f_1(\alpha)x + f_2(\alpha)x^2 + f_3(\alpha)x^3 + O(x^4), \tag{4.1}$$

where $f_1(\alpha) = -[1 + g(\alpha)]$ for some smooth function g. Since $g(0) = 0$ and

$$g'(0) = -f_{x\alpha}(0,0) \neq 0,$$

p. 123, l. 13

$$x \mapsto \alpha x e^{-x} \equiv F(x,\alpha) \tag{4.17}$$

p. 123, l. -4

One can check that with $f(x,\alpha) \equiv F(x_1(\alpha_1+\alpha)+x, \alpha_1+\alpha) - x_1(\alpha_1+\alpha)$ one has

$$c(0) = \frac{1}{6} > 0, \quad f_{x\alpha}(0,0) = -\frac{1}{e^2} \neq 0.$$

p. 125, l. -6

$$\begin{aligned} |1+\alpha+d(\alpha)\rho^2| &= (1+\alpha)\left(1 + \frac{2a(\alpha)}{1+\alpha}\rho^2 + \frac{|d(\alpha)|^2}{(1+\alpha)^2}\rho^4\right)^{1/2} \\ &= 1+\alpha+a(\alpha)\rho^2 + O(\rho^4), \end{aligned}$$

p.133, l. -10

The truncated composition of the transformations

p.133, bottom

$$c_1 = \frac{g_{20}g_{11}(\bar{\mu} - 3 + 2\mu)}{2(\mu^2-\mu)(\bar{\mu}-1)} + \frac{|g_{11}|^2}{|\mu|^2 - \bar{\mu}} + \frac{|g_{02}|^2}{2(\mu^2-\bar{\mu})} + \frac{g_{21}}{2}, \tag{4.20}$$

134, l. -8

4.5, 4.6, and 4.7,

136, l. -16:

If we introduce $v_k = u_{k-1}$, the equation can be rewritten as

$$\begin{aligned} u_{k+1} &= r u_k(1 - v_k), \\ v_{k+1} &= u_k, \end{aligned}$$

p. 138. l. 2

(1) Prove that in a small neighborhood of $x = 0$ the number and stability of fixed points and periodic orbits of the map (4.2) are independent of higher-order terms, provided $|\alpha|$ is sufficiently small.

p. 145, l. 11:

A contraction map in a complete metric space has a unique fixed point $u^{(\infty)} \in U$:

p. 146, l. -13

$$\begin{aligned}\tilde{\varphi}_2 - \tilde{\varphi}_1 &= \varphi_2 - \varphi_1 + \alpha^{3/2}\left[K_\alpha(u(\varphi_2),\varphi_2) - K_\alpha(u(\varphi_1),\varphi_1)\right] \\ &\geq \varphi_2 - \varphi_1 - \alpha^{3/2}\left|K_\alpha(u(\varphi_2),\varphi_2) - K_\alpha(u(\varphi_1),\varphi_1)\right|.\end{aligned}$$

p. 146, l. -6

$$-\left|K_\alpha(u(\varphi_2),\varphi_2) - K_\alpha(u(\varphi_1),\varphi_1)\right| \geq -2\lambda(\varphi_2 - \varphi_1),$$

p. 147, l. 5:

$$\tilde{u}(\varphi) = (1-2\alpha)u(\hat{\varphi}) + \alpha^{3/2}H_\alpha(u(\hat{\varphi}),\hat{\varphi}), \tag{A2.12}$$

p. 147, l. 14:

$$\begin{aligned}|\tilde{u}(\varphi_1) - \tilde{u}(\varphi_2)| &\leq (1-2\alpha)|u(\hat{\varphi}_1) - u(\hat{\varphi}_2)| \\ &\quad + \alpha^{3/2}|H_\alpha(u(\hat{\varphi}_1),\hat{\varphi}_1) - H_\alpha(u(\hat{\varphi}_2),\hat{\varphi}_2)| \\ &\leq (1-2\alpha)|u(\hat{\varphi}_1) - u(\hat{\varphi}_2)| \\ &\quad + \alpha^{3/2}\lambda\Big[|u(\hat{\varphi}_1) - u(\hat{\varphi}_2)| + |\hat{\varphi}_1 - \hat{\varphi}_2|\Big] \\ &\leq (1-2\alpha+2\lambda\alpha^{3/2})|\hat{\varphi}_1 - \hat{\varphi}_2|,\end{aligned}$$

p. 147, l. -6:

$$\begin{aligned}|\tilde{u}_1(\varphi) - \tilde{u}_2(\varphi)| &\leq (1-2\alpha)|u_1(\hat{\varphi}_1) - u_2(\hat{\varphi}_2)| \\ &\quad + \alpha^{3/2}|H_\alpha(u_1(\hat{\varphi}_1),\hat{\varphi}_1) - H_\alpha(u_2(\hat{\varphi}_2),\hat{\varphi}_2)| \\ &\leq (1-2\alpha)|u_1(\hat{\varphi}_1) - u_2(\hat{\varphi}_2)| \\ &\quad + \alpha^{3/2}\lambda\Big[|u_1(\hat{\varphi}_1) - u_2(\hat{\varphi}_2)| + |\hat{\varphi}_1 - \hat{\varphi}_2|\Big],\end{aligned} \tag{A2.13}$$

pp. 148, l. 18:

Using the estimates (A2.16) and (A2.17), we can conclude from (A2.13) that

$$\|\tilde{u}_1 - \tilde{u}_2\| \leq \epsilon\|u_1 - u_2\|,$$

5

Bifurcations of Equilibria and Periodic Orbits in n-Dimensional Dynamical Systems

p.164, top

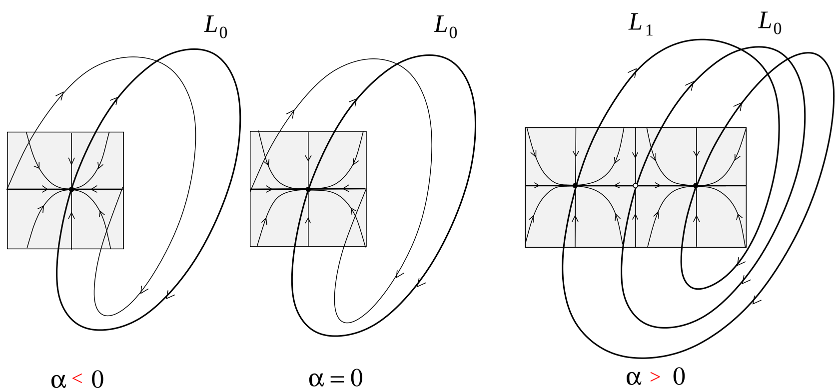

FIGURE 5.14. Flip bifurcation of limit cycles.

p.164, bottom

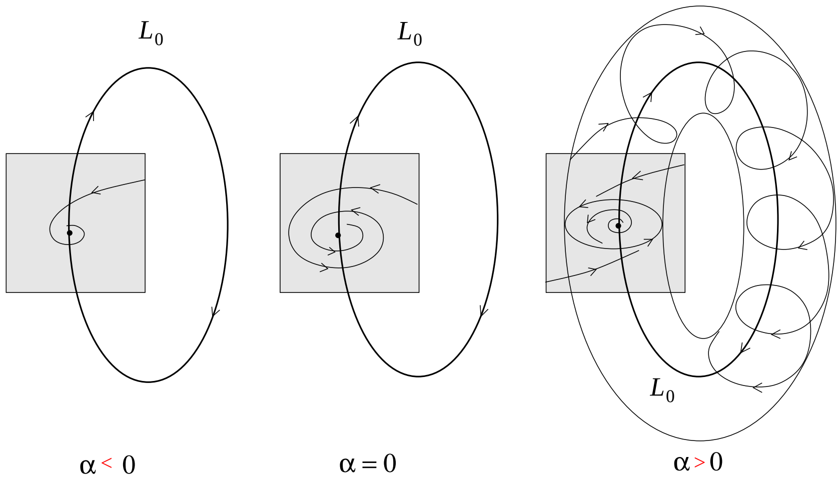

FIGURE 5.15. Neimark-Sacker bifurcation of a limit cycle.

p. 187, l. -13
giving rise to a unique saddle limit cycle for $r < r_1$ [Roschin 1978].

p. 188, l. -11
where $X = X(r,t)$, $Y = Y(r,t)$; $r \in [0, \pi]$; $t \geq 0$; $A, B, d, \theta > 0$ (see Chapter 1 and Lefever & Prigogine [1968]. Consider the case when X and Y are kept constant at their equilibrium values at the end points:

$$X(0,t) = X(\pi,t) = C,\ Y(0,t) = Y(\pi,t) = \frac{B}{C}.$$

Fix

$$C_0 = 1,\ d_0 = 2,\ \theta_0 = \frac{1}{2},$$

6

Bifurcations of Orbits Homoclinic and Heteroclinic to Hyperbolic Equilibria

p. 203, l. 1
as a composition of a near-to-saddle map

p. 206, l. 6
Step 4 (*Analysis of the complosition*)

p. 212, l. -10
(i.e., glueing points $(\xi, 1)$ and $(2, \xi)$ for $|\xi| \leq 1$).

p. 214, l. -5
The (nontrivial) multipliers of the cycle are ~~positive and~~ inside the unit circle: $|\mu_{1,2}| < 1$.

p. 218, l. -24
Sketch of the proof of Theorems 6.3 and 6.5

p. 218, l. -13
composition

$$P = Q \circ \Delta$$

p. 220, l. -11
Sketch of the proof of Theorems 6.4 and 6.6

p. 220, l. -7
and represent the Poincaré map $P : \Sigma^+ \to \Sigma$ as a composition

p. 232, l. -9

$$M_\alpha(0) = \int_\Omega (\text{div } g)(x)\, dx_1 dx_2. \ \diamondsuit$$

p. 240, l. 13
(10) (Proofs of Theorems 6.3–6.6 revisited)

p. 240, l. 21
(c) Compute a composition of the maps

7

Other One-Parameter Bifurcations in Continuous-Time Dynamical Systems

p. 260, l. - 5

can be represented as the composition of a “local” map

p. 261, l. - 14

$Q_\beta\Sigma \cap \Pi_1$ (see Figure 7.10(a))

p. 289, l. 16

Check that $Rv = -v$, where R is the involution that leaves Lorenz system (7.15) invariant, so that case (ii) of Theorem 7.7 is applicable.

p. 290, l. 6

There are three subcases: (i) $b(0) < a(0)$; (ii) $b(0) > a(0), a(0) + b(0) < 0$; (iii) $b(0) > a(0), a(0) + b(0) > 0$.

p. 290, l. 17

$$\left\{ \begin{array}{lcl} \dot{z}_1 & = & \lambda(\alpha)z_1 + f_1(z_1, \bar{z}_1, z_2, \bar{z}_2, \alpha), \\ \dot{z}_2 & = & \lambda(\alpha)z_2 + f_2(z_1, \bar{z}_1, z_2, \bar{z}_2, \alpha), \end{array} \right. \qquad (7.23)$$

8

Two-Parameter Bifurcations of Equilibria in Continuous-Time Dynamical Systems

p. 302, l. - 15

Finally, perform a *linear scaling*

$$\eta = \xi\sqrt{|c(\mu)|},$$

and introduce new parameters:

$$\begin{aligned}\beta_1 &= \mu_1\sqrt{|c(\mu)|},\\ \beta_2 &= \mu_2.\end{aligned}$$

p. 309, l. 1

$$\frac{dw}{d\tau} = (\nu(\alpha)+i)w + d_1(\alpha)w|w|^2 + d_2(\alpha)w|w|^4 + O(|w|^6), \tag{8.21}$$

p. 309, l. 10

$$\frac{dw}{d\theta} = (\nu+i)w+((\nu+i)e_1+d_1)w|w|^2+((\nu+i)e_2+e_1d_1+d_2)w|w|^4+O(|w|^6).$$

p. 309, l. 14

$$\frac{dw}{d\theta} = (\nu(\alpha)+i)w + l_1(\alpha)w|w|^2 + l_2(\alpha)w|w|^4 + O(|w|^6),$$

p. 310, l. 2

$$\mu(0)=0,\ l_1(0) = \frac{\text{Re }c_1(0)}{\omega_0} = \frac{1}{2\omega_0}\left(\text{Re }g_{21}(0) - \frac{1}{\omega_0}\text{Im}(g_{20}(0)g_{11}(0))\right) = 0,$$

p. 311, l. 7
Then, rescaling

$$w = \frac{1}{\sqrt[4]{|L_2(\mu)|}}\, u, \quad u \in \mathbb{C}^1,$$

and defining the parameters

$$\left\{ \begin{array}{rcl} \beta_1 & = & \mu_1, \\ \beta_2 & = & \dfrac{1}{\sqrt{|L_2(\mu)|}}\, \mu_2, \end{array} \right.$$

p. 316, l. 11
where $a_{kl}(\alpha), b_{kl}(\alpha),$ and

p. 319, l. 5

$$f_{00}(\alpha) = h_{00}(\alpha), \quad f_{10}(\alpha) = h_{10}(\alpha) + 2h_{00}(\alpha)\theta(\alpha),$$

and

$$f_{20}(\alpha) \quad = \quad h_{20}(\alpha) + 4h_{10}(\alpha)\theta(\alpha) + 2h_{00}(\alpha)\theta^2(\alpha),$$

p. 319, l. -11
where

$$\mu_1(\alpha) = h_{00}(\alpha), \quad \mu_2(\alpha) = h_{10}(\alpha) - h_{00}(\alpha)h_{02}(\alpha), \tag{8.46}$$

and

$$A(\alpha) = \frac{1}{2}\left(h_{20}(\alpha) - 2h_{10}(\alpha)h_{02}(\alpha) + \frac{1}{2}h_{00}(\alpha)h_{02}^2(\alpha)\right), \quad B(\alpha) = h_{11}(\alpha). \tag{8.47}$$

p. 335, l. 10

$$\Psi - i\omega_0 \frac{G_{200}(0)}{G_{011}(0)} e_2,$$

p. 340

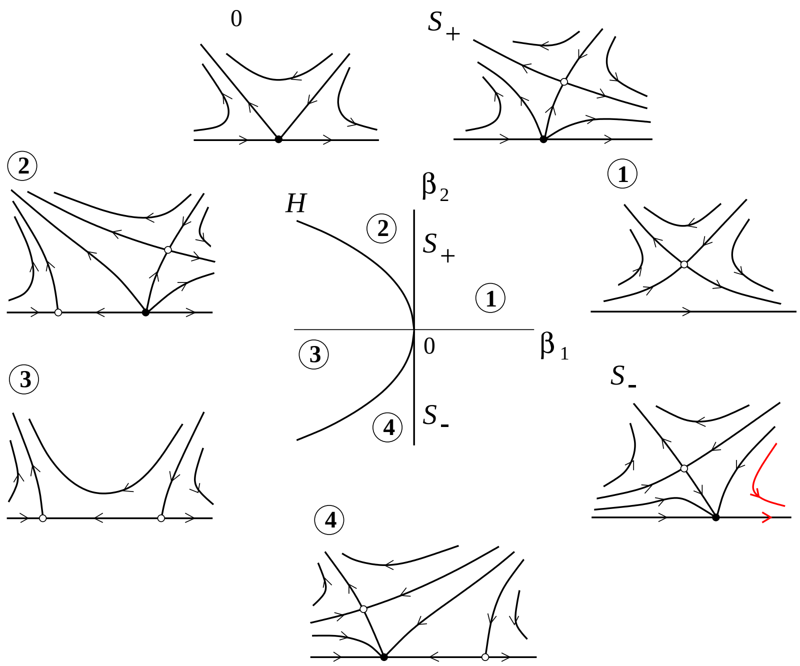

FIGURE 8.14. Bifurcation diagram of the amplitude system (8.81) ($s = -1,\ \theta < 0$).

p. 341

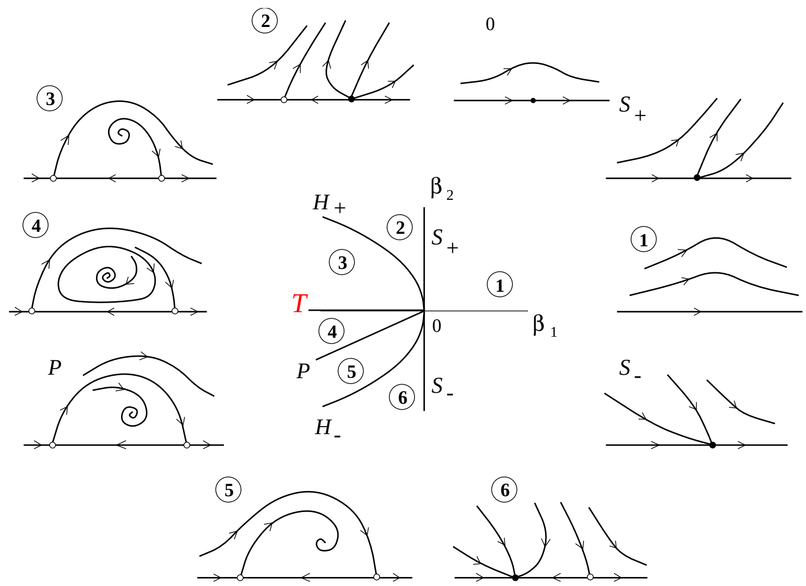

FIGURE 8.16. Bifurcation diagram of the amplitude system (8.81) ($s = 1,\ \theta < 0$).

p. 342

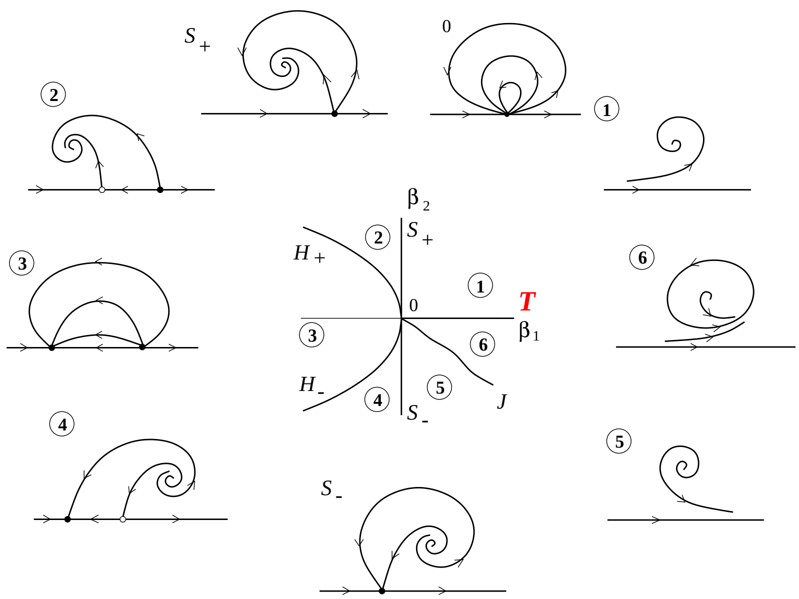

FIGURE 8.17. Bifurcation diagram of the amplitude system (8.81) ($s = -1,\ \theta > 0$).

p. 347, l. -8
with $\tilde{\Theta}_\beta,\ \rho^4\tilde{\Psi}_\beta(\xi,\rho^2) = O((\xi^2+\rho^2)^2)$.

p. 354, l. -14
Thus, $\hat{G}_{2111} = 0, \hat{H}_{1121} = 0,$

p. 358, l. -9
(otherwise, ~~reverse time and~~ exchange the subscripts in (8.112)).

p. 359

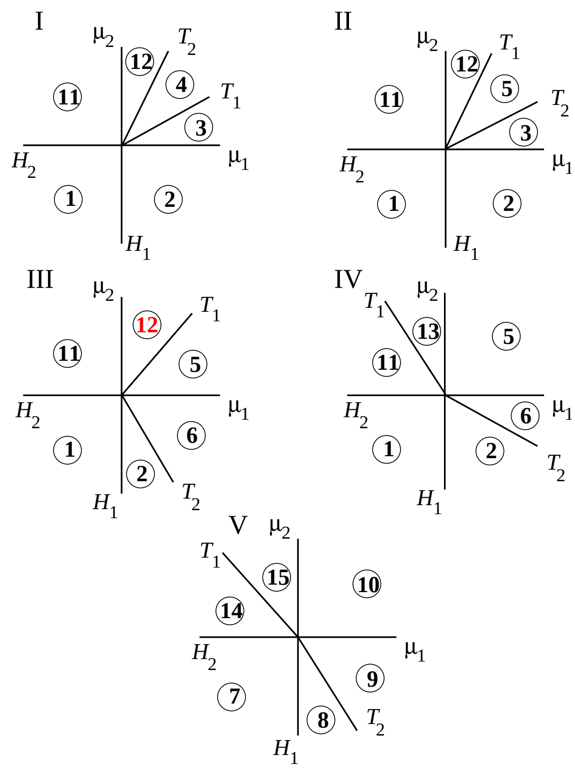

FIGURE 8.25. Parametric portraits of (8.111) (the "simple" case).

p. 362, l. 4

$$C = \left\{ \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} : \ \mu_2 = -\frac{\delta - 1}{\theta - 1}\mu_1 - \frac{(\delta - 1)\Theta + (\theta - 1)\Delta}{(\theta - 1)^3}\mu_1^2 + O(\mu_1^3) \right\}$$

that should be considered when both $\mu_1 > \theta\mu_2$ and $\delta\mu_1 > \mu_2$.

p. 362, l. 7

(HH.7) $p_{22}(0) \neq p_{12}(0)$;
(HH.8) $p_{21}(0) \neq p_{11}(0)$.

p. 362, l. 12

$$\text{sign } l_1 = \text{sign}\left\{ -\theta\left[\theta(\theta - 1)\Delta + \delta(\delta - 1)\Theta\right]\right\}.$$

p. 364

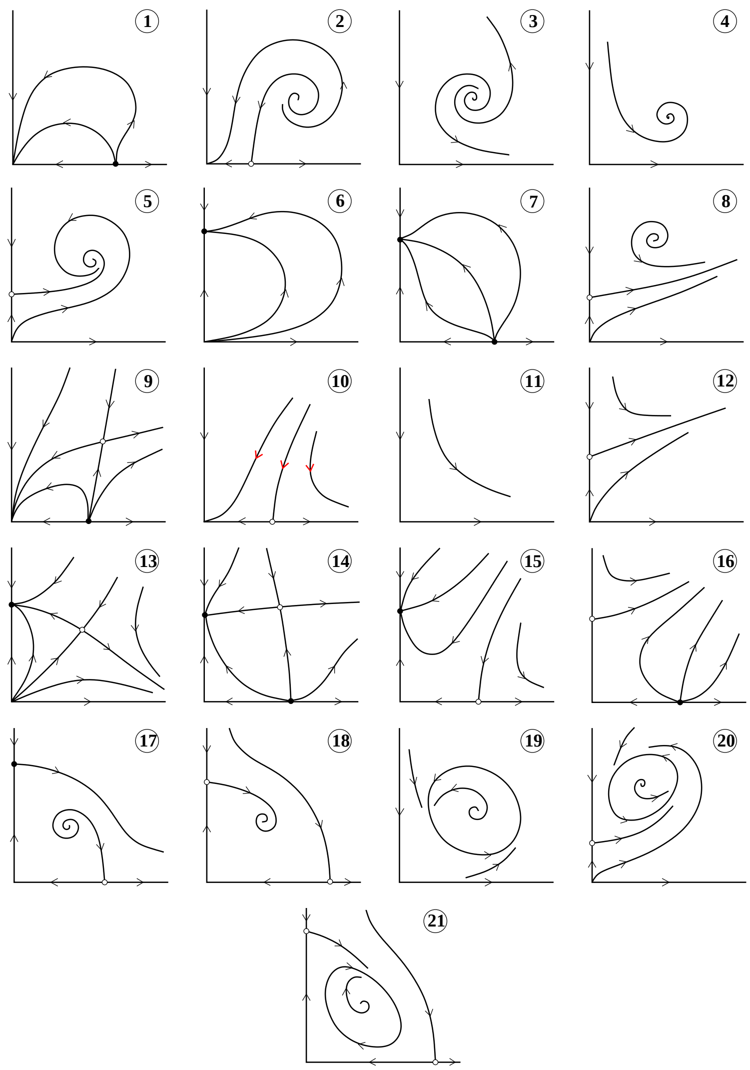

FIGURE 8.29. Generic phase portraits of (8.114).

p. 365, l. 16

$$Y = \left\{(\mu_1, \mu_2) : \ \mu_2 = -\frac{\delta - 1}{\theta - 1}\mu_1 + \frac{(\theta-1)^3\delta\Delta + (\delta-1)^3\theta\Theta}{(2\delta\theta - \delta - \theta)(\theta-1)^2}\mu_1^2 + O(\mu_1^3)\right\},$$

p. 365, l. -3

Recalling the interpretation of equilibria and cycles of the amplitude system (8.110) in the four-dimensional *truncated* normal form (8.108), we can establish a relationship between bifurcations in these two systems. The curves $H_{1,2}$ at which the trivial equilibria appear in (8.110) obviously correspond to Hopf bifurcation curves in (8.108). These are the two "independent" Hopf bifurcations caused by the two distinct pairs of eigenvalues passing through the imaginary axis. Crossing a bifurcation curve T_1 (or T_2) results in the branching of a two-dimensional torus from a cycle. Therefore, the curves $T_{1,2}$ correspond to Neimark-Sacker bifurcations in (8.108). On the curve C, system (8.108) exhibits a bifurcation that we have not yet encountered, namely, branching of a three-dimensional torus from the two-dimensional torus. The curves J describe blow-ups of three-dimensional tori, while the curve Y implies the presence of a heteroclinic coincidence of the three-dimensional stable and unstable invariant manifolds of a cycle and a three-torus.

p. 369, l. -5

(2) (Lemma 8.2) Probve that a smooth system

p. 370, l. 3

Show that this curve is well-defined near the origin and can be locally parametrized by x.

p. 371, l. -7

$$\begin{cases} \dot{x}_1 = -\alpha_1 x_2 + x_1(1 - x_1^2 - x_2^2), \\ \dot{x}_2 = \alpha_1 x_1 + x_2(1 - x_1^2 - x_2^2) - \alpha_2, \end{cases}$$

p. 374, l. 12

$\lambda_1 = 0,\ \lambda_{2,3} = \pm i\omega_0$

p. 376, l. 6

$$\xi = \delta x,\ \rho = \delta y,\ dt = \frac{y^q}{\delta},$$

p. 380, top

```
>    [z,z1,u,u1],4);
> WW:=mtaylor(sum(sum(sum(sum(
>                W[j,k,l,m]*z^j*z1^k*u^l*u1^m,
>            j=0..3),k=0..3),l=0..3),m=0..3),
>   [z,z1,u,u1],4);
> VV1:=mtaylor(sum(sum(sum(sum(
>                 V1[j,k,l,m]*z1^j*z^k*u1^l*u^m,
>            j=0..3),k=0..3),l=0..3),m=0..3),
>    [z,z1,u,u1],4);
> WW1:=mtaylor(sum(sum(sum(sum(
>                 W1[j,k,l,m]*z1^j*z^k*u1^l*u^m,
>            j=0..3),k=0..3),l=0..3),m=0..3),
>     [z,z1,u,u1],4);
> for j from 0 to 1 do
```

```
>     for k from 0 to 1 do
>         for l from 0 to 1 do
>             for m from 0 to 1 do
>                 if j+k+l+m < 2 then
>                    V[j,k,l,m]:=0; V1[j,k,l,m]:=0;
>                    W[j,k,l,m]:=0; W1[j,k,l,m]:=0;
>                 fi;
>             od;
>         od;
>     od;
> od;
> V[1,0,0,0]:=1; V[2,1,0,0]:=0; V[1,0,1,1]:=0;
> W[0,0,1,0]:=1; W[1,1,1,0]:=0; W[0,0,2,1]:=0;
> V1[1,0,0,0]:=1; V1[2,1,0,0]:=0; V1[1,0,1,1]:=0;
> W1[0,0,1,0]:=1; W1[1,1,1,0]:=0; W1[0,0,2,1]:=0;
```

By these commands the transformation (and its conjugate) that bring the system into the normal form is defined. Its coefficients have to be found.

```
> V_z:=diff(VV,z); V_z1:=diff(VV,z1);
> V_u:=diff(VV,u); V_u1:=diff(VV,u1);
> W_z:=diff(WW,z); W_z1:=diff(WW,z1);
> W_u:=diff(WW,u); W_u1:=diff(WW,u1);
```

p. 382, l. -1

$$\zeta_1 = \frac{\eta_1}{\nu}, \quad \zeta_2 = \frac{\eta_2}{\nu^{3/2}}, \quad t = \frac{\tau}{\nu^{1/2}}. \tag{A.3}$$

p. 383, l. 1
This rescaling reduces (A.2) to

p. 389, l. -9
(i.e., all extrema are minimum points).

p. 390, l. 15
By the Inverse Function Theorem, these equations define a smooth function $\beta(\gamma)$.

p. 390, l. -11
The homoclinic curve $\mathcal{P}$ given by (A.13) is mapped by (A.20) into the curve

9

Two-Parameter Bifurcations of Fixed Points in Discrete-Time Dynamical Systems

p.399, top

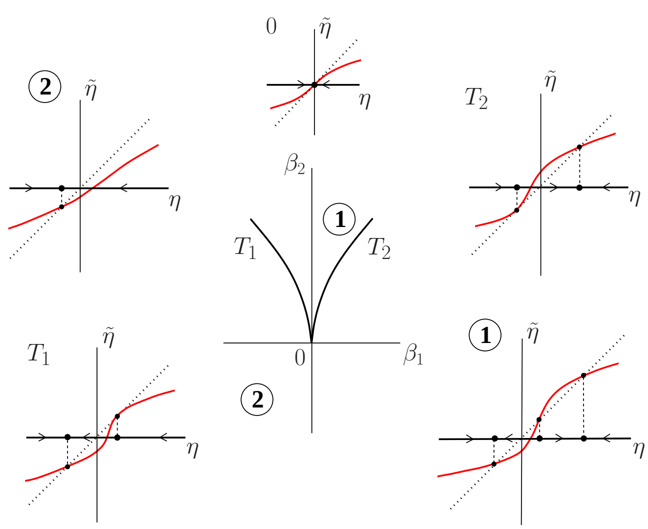

FIGURE 9.2. Bifurcation diagram of the normal form (9.11).

p. 402, l. 5

$$\eta \mapsto g_\beta(\eta) = -(1+\beta_1)\eta + \beta_2\eta^3 + \eta^5.$$

p. 402

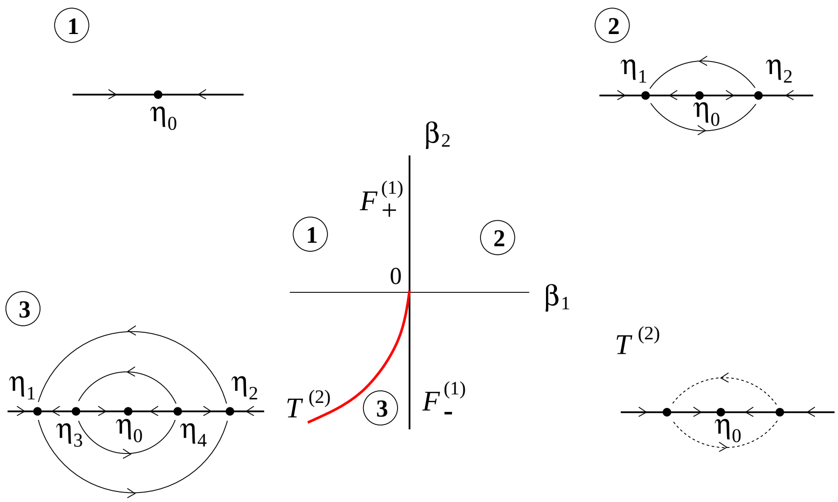

FIGURE 9.3. Bifurcation diagram of the normal form (9.17).

p. 402, l. -1
at the half-parabola:

$$T^{(2)} = \left\{(\beta_1, \beta_2) : \beta_1 = -\frac{1}{4}\beta_2^2, \beta_2 < 0\right\},$$

p. 405, l. -11

$$\begin{aligned} w \quad \mapsto \quad & e^{i\theta(\beta)}(1 + \beta_1 + (\beta_2 + iD_1(\beta))|w|^2 + (D_2(\beta) + iE_2(\beta))|w|^4)w \\ & + \Psi_\beta(w, \bar{w}), \end{aligned} \qquad (9,21)$$

p. 405, l. -4

$$w \mapsto e^{i\theta(\beta)}(1 + \beta_1 + (\beta_2 + iD_1(\beta))|w|^2 + (D_2(\beta) + iE_2(\beta))|w|^4)w. \qquad (9.22)$$

p.406, l. 7

(CH.2) $$L_2(0) = \frac{1}{2}[\mathrm{Im}\ d_1(0)]^2 + \mathrm{Re}\ d_2(0) \neq 0.$$

p.406, l. -16

$$T_c = \left\{(\beta_1, \beta_2) : \beta_1 = \frac{1}{4L_2(0)}\beta_2^2 + o(\beta_2^2), \beta_2 > 0\right\} ,$$

p. 408, l. -6

$$\dot{x} = F(x) = \Lambda x + F^{(2)}(x) + F^{(2)}(x) + \cdots, \ x \in \mathbb{R}^n, \qquad (9.25)$$

p. 409, l. 16

$$\varphi^1(x) = e^{\Lambda}x + g^{(2)}(x) + g^{(3)}(x) + \ldots + g^{(k)}(x) + O(\|x\|^{k+1}). \qquad (9.27)$$

p. 411, l. 1

eigenvectors $w_{0,1} \in \mathbb{R}^2$ of

p. 413, l. -6

(R1.1) $a_{20}(0) + b_{11}(0) - b_{20}(0) \neq 0$;

p. 415, l. -5

eigenvectors $w_{0,1} \in \mathbb{R}^2$ of

p. 418, l. -7

$$\begin{aligned} \tilde{\xi}_1 &= -\xi_1 + \xi_2 + \sum_{2 \le j+k \le 3} \gamma_{jk}\xi_1^j\xi_2^k + O(\|\xi\|^4), \\ \tilde{\xi}_2 &= -\xi_2 + \sum_{2 \le j+k \le 3} \sigma_{jk}\xi_1^j\xi_2^k + O(\|\xi\|^4), \end{aligned}$$

p. 419, l. -6

$$\begin{pmatrix} 3 & 0 & 0 & 0 & -1 & 0 \\ -3 & 2 & 0 & 0 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_{30} \\ \phi_{21} \\ \phi_{12} \\ \psi_{30} \\ \psi_{21} \\ \psi_{12} \end{pmatrix} = R[g,h],$$

p. 426, l. -16

$$P = \left\{ (\varepsilon_1, \varepsilon_2) : \ \varepsilon_2 = \frac{4}{5}\varepsilon_1 + o(\varepsilon_1), \, \varepsilon_1 > 0 \right\},$$

p. 429, l. 12

$$C(\alpha) = \frac{g_{20}(\alpha)g_{11}(\alpha)(2\mu(\alpha) + \bar{\mu}(\alpha) - 3)}{2(\bar{\mu}(\alpha) - 1)(\mu^2(\alpha) - \mu(\alpha))} + \frac{|g_{11}(\alpha)|^2}{|\mu(\alpha)|^2 - \bar{\mu}(\alpha)} + \frac{g_{21}(\alpha)}{2}. \qquad (9.72)$$

p. 440, l. 2

$$-\frac{e^{i\alpha}}{\rho^2} = A(\beta) + e^{-4i\varphi},$$

p. 451, l. -9

$$D_1(0) < 0, \ C_1(0) > 0,$$

p. 457, l. 9

computed in Exercise 8 in Chapter 5.)

p. 457, l. 9

~~(notice, however, that their equation for the tangent bifurcation curve $T^{(2)}$ is only approximate)~~

10

Numerical Analysis of Bifurcations

p. 475, l. -12
method is equivalent to the representation of the Poincaré map $P : \Sigma_0 \to \Sigma_0$ by a *composition* of $N-1$ maps

p. 489, l. 8

$$\langle p, B(q,q)\rangle = \frac{1}{h^2}\left[\langle p, f(x^0+hq,\alpha^0)\rangle + \langle p, f(x^0-hq,\alpha^0)\rangle\right] + O(h^2),$$

p. 490, l. 16

$$B(v,v) = \frac{1}{h^2}\left[f(x^0+hv,\alpha^0) + f(x^0-hv,\alpha^0)\right] + O(h^2)$$

for (10.52), and

$$\begin{aligned} C(v,v,v) &= \frac{1}{8h^3}\left[f(x^0+3hv,\alpha^0) - 3f(x^0+hv,\alpha^0)\right. \\ &+ \left. 3f(x^0-hv,\alpha^0) - f(x^0-3hv,\alpha^0)\right] + O(h^2) \end{aligned}$$

p. 499, l. -3

$$\begin{pmatrix} B(q_1,q_2) \\ \ddot{y}_{(1)}^T(0)q_2 \end{pmatrix} + \begin{pmatrix} J(0) \\ q_1^T \end{pmatrix} \dot{u}(0) = \dot{\lambda}(0)q_2. \qquad (10.64)$$

p. 538, l. -10

& Wang [1997]

Appendix A

Basic Notions from Algebra, Analysis, and Geometry

p. 542, l. 8

$$\det A = \sum_{(i_1,i_2,\ldots,i_n)\in S_n} (-1)^{\delta(i_1,i_2,\ldots,i_n)} a_{i_1 1} a_{i_2 2} \cdots a_{i_n n},$$

p. 547, l. 3

their *composition* $h = f \circ g$

p. 547, l. 6

evaluated at a point $y \in \mathbb{R}^m$:

p. 547, l. 8

where $i = 1, 2, \ldots, k,\ j = 1, 2, \ldots, m$.

p. 547, l. -15

Consider a map

$$(x, y) \mapsto F(x, y),$$

where

$$F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m,$$

is a smooth map defined in a neighborhood of $(x, y) = (0, 0)$ and such that $F(0, 0) = 0$. Let $F_y(0, 0)$ denote the matrix of first partial derivatives of F with respect to y evaluated at $(0, 0)$:

$$F_y(0,0) \quad = \quad \left.\left(\frac{\partial F_i(x,y)}{\partial y_j}\right)\right|_{(x,y)=(0,0)}.$$

Theorem A.3 (Implicit Function Theorem) *If the matrix $F_y(0,0)$ is nonsingular, then there is a smooth locally defined function $y = f(x)$,*

$$f : \mathbb{R}^n \to \mathbb{R}^m,$$

such that

$$F(x, f(x)) = 0,$$

for all x in some neighborhood of the origin of $\mathbb{R}^n$. Moreover,

$$f_x(0) = -[F_y(0,0)]^{-1} F_x(0,0). \ \square$$

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