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Erratum: Elements of Applied Bifurcation Theory

3rd Edition

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Preface to the Third Edition

p. VIII, l. -5:

Finally, may the constant support by my wife, Lioudmila, and my daughters, Elena and Ouliana, be acknowledged.

1

Introduction to Dynamical Systems

p. 12, l. -3:

$$S \cap f(S) = V_1 \cup V_2$$

2

Topological Equivalence, Bifurcations, and Structural Stability of Dynamical Systems

p. 63, l. -11:

which would imply that the map $(x, \alpha) \mapsto (h_{\alpha}(x), p(\alpha))$

3

One-Parameter Bifurcations of Equilibria in Continuous-Time Dynamical Systems

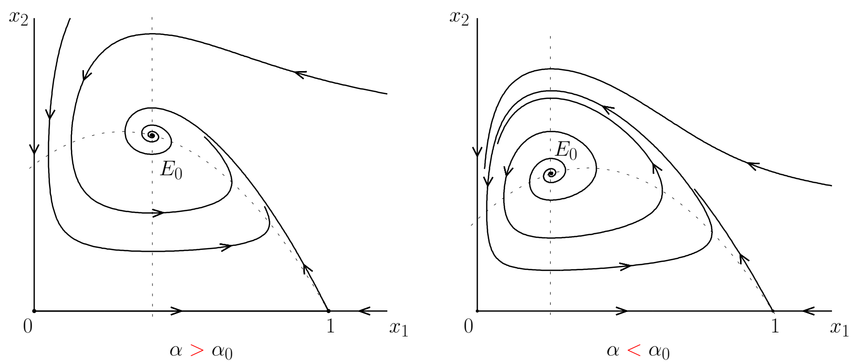

Fig. 3.11. Hopf bifurcation in the predator-prey model.

p. 105, l. -16

$$l_1 = \frac{1}{2\omega^2}\mathrm{Re}(ig_{20}g_{11} + \omega g_{21}) = -\frac{2 + A^2}{2A(1 + A^2)} < 0,$$

p. 105, l. -10
(*Hint:* Introduce $y = -\dot{x}$ and rewrite the equation as a system of two differential equations.)

p. 108, l. -21
Fix α small but positive. Both systems (A.1) and (A.2) have a limit cycle in some neighborhood of the origin. Assume that the time reparametrization resulting in the constant return time 2π is performed in system (A.1) (see the previous step). Consider a homeomorphism H that conjugates the Poincaré map of (A.1) with that of (A.2) at this parameter value.

Define a map $z \mapsto \tilde{z}$ by the following construction. Take a point $z = x_1 + ix_2$ and find values (ρ_0, τ_0), where τ_0 is the minimal time required for an orbit of system

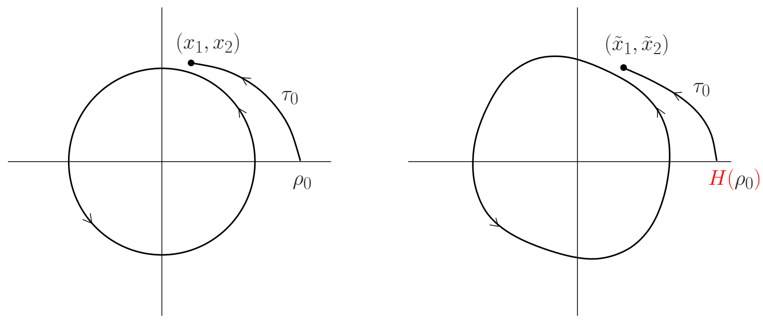

Fig. 3.14. Construction of the homeomorphism near the Hopf bifurcation.

(A.2) to approach the point z starting from the horizontal half-axis with $\rho = \rho_0$. Now, take the point on this axis with $\rho = H(\rho_0)$ and construct an orbit of system (A.1) on the time interval $[0, \tau_0]$ starting at this point. Denote the resulting point by $\tilde{z} = \tilde{x}_1 + i\tilde{x}_2$ (see **Fig. 3.14**). Set $\tilde{z} = 0$ for $z = 0$.

p. 110, bottom

$$(L_A f^{(2)})(y) = \omega_0 \begin{pmatrix} (a_2 + a_4)y_1^2 + (-2a_1 + 2a_3 + a_5)y_1 y_2 + (-a_2 + a_6)y_2^2 \\ (-a_1 + a_5)y_1^2 + (-a_2 - 2a_4 + 2a_6)y_1 y_2 + (-a_3 - a_5)y_2^2 \end{pmatrix},$$

p. 112, l. -15

$$h_j(y) = y_1^{m_1} y_2^{m_2} \cdots y_n^{m_n} \mathrm{e}_j, \tag{B.11}$$

p. 112, l. -10

$$L_A h_j = (\langle \lambda, m \rangle - \lambda_j) h_j, \tag{B.12}$$

p. 112, l. -7

The equation (B.12) means that the vector-monomial h_j defined by (B.11) is the eigenvector of L_A corresponding to the eigenvalue

$$\mu_j = \langle \lambda, m \rangle - \lambda_j.$$

Thus, the null-space of L_A is spanned in this case by vector-monomials h_j, for which $\mu_j = 0$, i.e.,

$$\lambda_j = \langle \lambda, m \rangle. \tag{B.13}$$

p. 113, l. 2

For a fixed m and each $j = 1, 2, \ldots, n$, (B.13) implies a condition on the eigenvalues of A (called the *resonance condition* or *resonance*).

If no resonances of order m exist, Theorem B.1 implies that all terms of order m in (B.1) can be eliminated by a polynomial transformation. In the presence of resonances, the resonant monomials satisfying (B.13) cannot be removed from the j-component of the right-hand side of (B.1) by all such transformations.

p. 113, l. -8

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + f(x), \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2, \tag{B.14}$$

where $\omega_0 > 0$. If we introduce a complex variable $z = x_1 + ix_2$, then (B.14) can be written as one complex equation

p. 114, l. 1

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} i\omega_0 & 0 \\ 0 & -i\omega_0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} g_1(z_1, z_2) \\ g_2(z_1, z_2) \end{pmatrix}, \tag{B.15}$$

in which the first equation is equivalent to (B.14)), if we substitute $z_1 = z$ and $z_2 = \bar{z}$. The system (B.15)) is called the *complexification* of (B.14)). Notice that (B.15) has the diagonal linear part with $\lambda_1 = i\omega_0$ and $\lambda_2 = -i\omega_0$.

4

One-Parameter Bifurcations of Fixed Points in Discrete-Time Dynamical Systems

4.5 Generic flip bifurcation

[Moved from p. 128, ll. 2-6]

By the Implicit Function Theorem, the system $x \mapsto f(x,\alpha)$ has a unique fixed point $x_0(\alpha)$ satisfying $x_0(0)=0$ in some neighborhood of the origin for all sufficiently small $|\alpha|$ since $f_x(0,0) \neq 1$. We can perform a coordinate shift placing this fixed point at the origin. Therefore, we can assume without loss of generality that $x=0$ is the fixed point of the system for $|\alpha|$ sufficiently small.

p. 127, bottom

Theorem 4.3 *Consider a one-dimensional system*

$$x \mapsto f(x,\alpha), \quad x \in \mathbb{R}^1, \ \alpha \in \mathbb{R}^1,$$

with smooth f satisfying $f(0,\alpha) \equiv 0$, and let $\mu = f_x(0,0) = -1$. Assume that the following nondegeneracy conditions are satisfied:

(B.1) $\frac{1}{2}(f_{xx}(0,0))^2 + \frac{1}{3} f_{xxx}(0,0) \neq 0$;
(B.2) $f_{x\alpha}(0,0) \neq 0$.

Then there are smooth invertible coordinate and parameter changes transforming the system into

$$\eta \mapsto -(1+\beta)\eta \pm \eta^3 + O(\eta^4).$$

p. 128, top
Proof:

The map f can be written as follows:

$$f(x,\alpha) = f_1(\alpha)x + f_2(\alpha)x^2 + f_3(\alpha)x^3 + O(x^4), \tag{4.1}$$

where $f_1(\alpha) = -[1+g(\alpha)]$ for some smooth function g. Since $g(0)=0$ and

$$g'(0) = -f_{x\alpha}(0,0) \neq 0,$$

p. 129, l. -3

$$x \mapsto \alpha x e^{-x} \equiv F(x,\alpha) \qquad (4.17)$$

p. 130, l. -11

One can check that with $f(x,\alpha) \equiv F(x_1(\alpha_1+\alpha)+x, \alpha_1+\alpha) - x_1(\alpha_1+\alpha)$ one has

$$c(0) = \frac{1}{6} > 0, \quad f_{x\alpha}(0,0) = -\frac{1}{e^2} \neq 0.$$

p. 140, l. -13

$$c_1 = \frac{g_{20}g_{11}(\bar{\mu}-3+2\mu)}{2(\mu^2-\mu)(\bar{\mu}-1)} + \frac{|g_{11}|^2}{|\mu|^2-\bar{\mu}} + \frac{|g_{02}|^2}{2(\mu^2-\bar{\mu})} + \frac{g_{21}}{2}, \qquad (4.25)$$

p. 142, l. -10 If we introduce $v_k = u_{k-1}$, the equation can be rewritten as

$$\begin{cases} u_{k+1} = ru_k(1-v_k), \\ v_{k+1} = u_k, \end{cases}$$

p. 144, l. -14

(1) Prove that in a small neighborhood of $x=0$ the number and stability of fixed points and periodic orbits of the map (4.7) are independent of higher-order terms, provided $|\alpha|$ is sufficiently small.

p. 151, l. 2

A contraction map in a complete metric space has a unique fixed point $u^{(\infty)} \in U$:

$$\mathcal{F}(u^{(\infty)}) = u^{(\infty)}.$$

p. 151, bottom

$$\begin{aligned} \tilde{\varphi}_2 - \tilde{\varphi}_1 &= \varphi_2 - \varphi_1 + \alpha^{3/2}\left[K_\alpha(u(\varphi_2),\varphi_2) - K_\alpha(u(\varphi_1),\varphi_1)\right] \\ &\geq \varphi_2 - \varphi_1 - \alpha^{3/2}\left|K_\alpha(u(\varphi_2),\varphi_2) - K_\alpha(u(\varphi_1),\varphi_1)\right|. \end{aligned}$$

p. 152, l. 5 (after Fig. 4.17)

$$-\left|K_\alpha(u(\varphi_2),\varphi_2) - K_\alpha(u(\varphi_1),\varphi_1)\right| \geq -2\lambda(\varphi_2-\varphi_1),$$

p. 152, l. -11

$$\tilde{u}(\varphi) = (1-2\alpha)u(\hat{\varphi}) + \alpha^{3/2}H_\alpha(u(\hat{\varphi}),\hat{\varphi}), \qquad (B.12)$$

p. 153, top

$$\begin{aligned} |\tilde{u}(\varphi_1) - \tilde{u}(\varphi_2)| &\leq (1-2\alpha)|u(\hat{\varphi}_1) - u(\hat{\varphi}_2)| \\ &\quad + \alpha^{3/2}|H_\alpha(u(\hat{\varphi}_1),\hat{\varphi}_1) - H_\alpha(u(\hat{\varphi}_2),\hat{\varphi}_2)| \\ &\leq (1-2\alpha)|u(\hat{\varphi}_1) - u(\hat{\varphi}_2)| \\ &\quad + \alpha^{3/2}\lambda\Big[|u(\hat{\varphi}_1) - u(\hat{\varphi}_2)| + |\hat{\varphi}_1 - \hat{\varphi}_2|\Big] \\ &\leq (1-2\alpha+2\lambda\alpha^{3/2})|\hat{\varphi}_1 - \hat{\varphi}_2|, \end{aligned}$$

p. 153, l. 14

$$
\begin{aligned}
|\tilde{u}_1(\varphi) - \tilde{u}_2(\varphi)| &\le (1-2\alpha)|u_1(\hat{\varphi}_1) - u_2(\hat{\varphi}_2)| \\
&\quad + \alpha^{3/2}|H_\alpha(u_1(\hat{\varphi}_1),\hat{\varphi}_1) - H_\alpha(u_2(\hat{\varphi}_2),\hat{\varphi}_2)| \\
&\le (1-2\alpha)|u_1(\hat{\varphi}_1) - u_2(\hat{\varphi}_2)| \\
&\quad + \alpha^{3/2}\lambda\Big[|u_1(\hat{\varphi}_1) - u_2(\hat{\varphi}_2)| + |\hat{\varphi}_1 - \hat{\varphi}_2|\Big],
\end{aligned} \tag{B.13}
$$

pp. 153-154, bottom-top

Using the estimates (B.16) and (B.17), we can conclude from (B.13) that

$$\|\tilde{u}_1 - \tilde{u}_2\| \le \epsilon\|u_1 - u_2\|,$$

5

Bifurcations of Equilibria and Periodic Orbits in n-Dimensional Dynamical Systems

p. 172

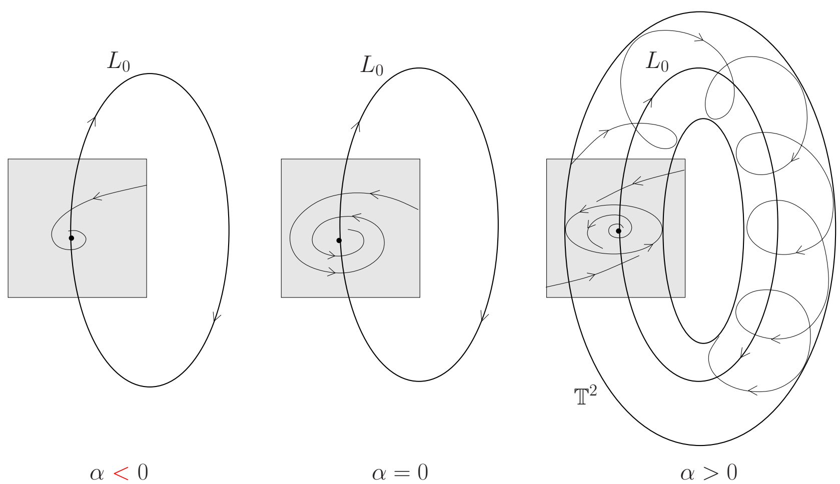

Fig. 5.17. Neimark-Sacker bifurcation of a limit cycle.

p. 189, l. 4
(b) Prove that this Hopf bifurcation is *subcritical* and, therefore, gives rise to a unique saddle limit cycle for $r < r_1$.

p. 190, l. -20
where $X = X(r,t),\ Y = Y(r,t);\ r \in [0,\pi];\ t \geq 0;\ A, B, d, \theta > 0$ (see Chapter 1 and Lefever & Prigogine [1968]. Consider the case when X and Y are kept constant at their equilibrium values at the end points:

$$X(0,t) = X(\pi,t) = C,\ Y(0,t) = Y(\pi,t) = \frac{B}{C}.$$

Fix

$$C_0 = 1,\ d_0 = 2,\ \theta_0 = \frac{1}{2},$$

and show that at

$$B_0 = 1 + C_0^2 + d_0(1 + \theta_0) = 5$$

p. 193, l. -14

$$l_1(0) = \frac{1}{2\omega_0} \mathrm{Re}\left[\langle p, C(q, q, \bar{q})\rangle + 2\langle p, B(q, w_{11})\rangle + \langle p, B(\bar{q}, w_{20})\rangle\right], \tag{A.9}$$

6

Bifurcations of Orbits Homoclinic and Heteroclinic to Hyperbolic Equilibria

p. 212, l. -17
(i.e., glueing points $(\xi, 1)$ and $(2, \xi)$ for $|\xi| \leq 1$).

p. 214, l. -7
The (nontrivial) multipliers of the cycle are ~~positive and~~ inside the unit circle: $|\mu_{1,2}| < 1$.

p. 232, l. 4

$$M_\alpha(0) = \int_\Omega (\text{div } g)(x)\, dx_1 dx_2. \ \diamondsuit$$

p. 239, l. 23
(c) Compute a composition

p. 239, l. -4
How many periodic orbits one expects near the bifurcation?

p. 240, l. 1
(13) (Melnikov integral) Prove that the Melnikov integral (6.25) is nonzero for the homoclinic orbit Γ_0 in the system (6.8) from Example 6.1. (*Hint:* Find $t_\pm = t_\pm(x)$ along the upper and lower halfs of Γ_0 by integrating the first equation of (6.8). Then transform the integral (6.25) into the sum of two integrals over $x \in [0, 1]$.)

p. 246, l. 18
In this text, however, redundant genericity conditions are often assumed

7

Other One-Parameter Bifurcations in Continuous-Time Dynamical Systems

p. 261, l. 1 (after Fig. 7.10)
$Q_\beta \Sigma \cap \Pi_1$

p. 291, l. -11
Check that $Rv = -v$, where R is the involution that leaves Lorenz system (7.21) invariant, so that case (ii) of Theorem 7.7 is applicable.

p. 292, l. 17
There are three subcases: (i) $b(0) < a(0)$; (ii) $b(0) > a(0), a(0) + b(0) < 0$; (iii) $b(0) > a(0), a(0) + b(0) > 0$.

p. 292, l. -17

$$\begin{cases} \dot{z}_1 = \lambda(\alpha) z_1 + f_1(z_1, \bar{z}_1, z_2, \bar{z}_2, \alpha), \\ \dot{z}_1 = \lambda(\alpha) z_2 + f_2(z_1, \bar{z}_1, z_2, \bar{z}_2, \alpha), \end{cases} \tag{7.29}$$

8

Two-Parameter Bifurcations of Equilibria in Continuous-Time Dynamical Systems

p. 304, l. 3

Finally, perform a *linear scaling*

$$\eta = \xi\sqrt{|c(\mu)|},$$

and introduce new parameters:

$$\beta_1 = \mu_1\sqrt{|c(\mu)|},$$
$$\beta_2 = \mu_2.$$

p. 310, l. -7

$$\frac{dw}{d\tau} = (\nu(\alpha)+i)w + d_1(\alpha)w|w|^2 + d_2(\alpha)w|w|^4 + O(|w|^6), \qquad (8.21)$$

p. 311, l. 3

$$\frac{dw}{d\theta} = (\nu+i)w + ((\nu+i)e_1 + d_1)w|w|^2 + ((\nu+i)e_2 + e_1d_1 + d_2)w|w|^4 + O(|w|^6).$$

p. 311, l. 7

$$\frac{dw}{d\theta} = (\nu(\alpha)+i)w + l_1(\alpha)w|w|^2 + l_2(\alpha)w|w|^4 + O(|w|^6)),$$

p. 311, l. -7

$$\mu(0)=0,\ l_1(0) = \frac{\text{Re } c_1(0)}{\omega_0} = \frac{1}{2\omega_0}\left(\text{Re } g_{21}(0) - \frac{1}{\omega_0}\ \text{Im}(g_{20}(0)g_{11}(0))\right) = 0,$$

p. 312, l. -4

Then, rescaling

$$w = \frac{1}{\sqrt[4]{|L_2(\mu)|}}\ u, \quad u \in \mathbb{C}^1,$$

and defining the parameters

$$\begin{cases} \beta_1 = \mu_1, \\ \beta_2 = \dfrac{1}{\sqrt{|L_2(\mu)|}}\, \mu_2, \end{cases}$$

p. 321, l. 1

$$f_{00}(\alpha) = h_{00}(\alpha), \quad f_{10}(\alpha) = h_{10}(\alpha) + 2h_{00}(\alpha)\theta(\alpha),$$

and

$$f_{20}(\alpha) = h_{20}(\alpha) + 4h_{10}(\alpha)\theta(\alpha) + 2h_{00}(\alpha)\theta^2(\alpha),$$

p. 321, l. 12

where

$$\mu_1(\alpha) = h_{00}(\alpha), \quad \mu_2(\alpha) = h_{10}(\alpha) - h_{00}(\alpha)h_{02}(\alpha), \tag{8.46}$$

and

$$A(\alpha) = \frac{1}{2}\left(h_{20}(\alpha) - 2h_{10}(\alpha)h_{02}(\alpha) + \frac{1}{2}h_{00}(\alpha)h_{02}^2(\alpha)\right), \quad B(\alpha) = h_{11}(\alpha).$$

p. 340

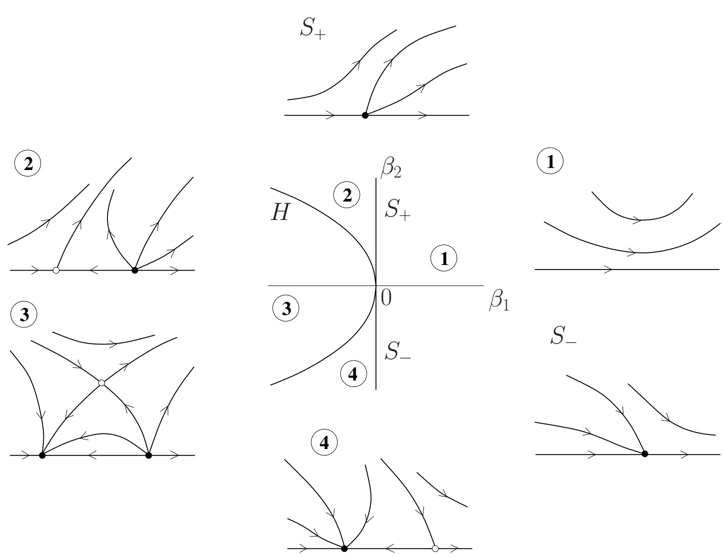

Fig. 8.13. Bifurcation diagram of the amplitude system (8.82) ($s = 1,\ \theta > 0$).

p. 341

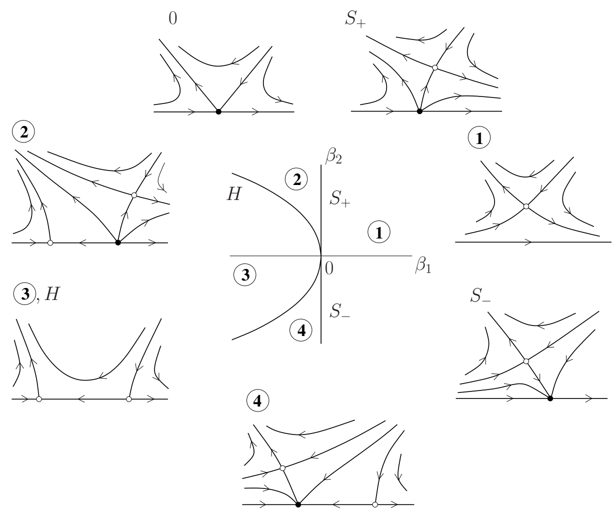

Fig. 8.14. Bifurcation diagram of the amplitude system (8.82) ($s = -1,\ \theta < 0$).

p. 342

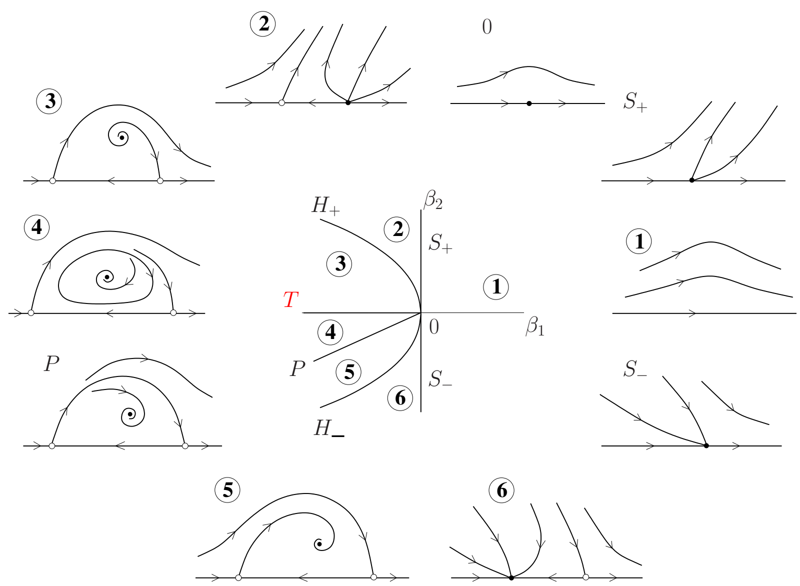

Fig. 8.16. Bifurcation diagram of the amplitude system (8.82) ($s = 1,\ \theta < 0$).

p.343

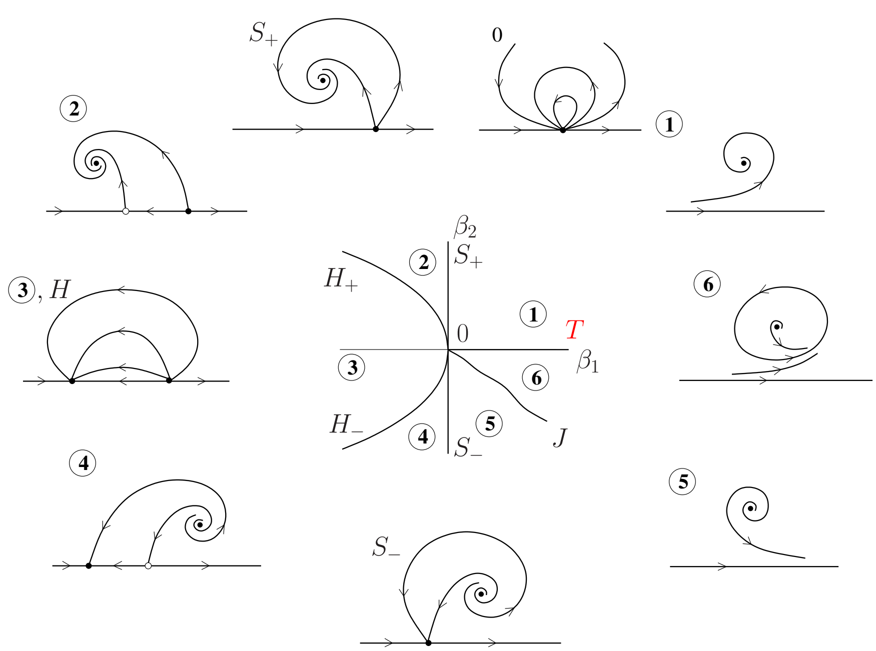

Fig. 8.17. Bifurcation diagram of the amplitude system (8.82) ($s = -1,\ \theta > 0$).

p. 349, l. -12
with $\tilde{\Theta}_\beta,\ \rho^4\tilde{\Psi}_\beta(\xi, \rho^2) = O((\xi^2 + \rho^2)^2)$.

p. 356, l. -16
Thus, $\hat{G}_{2111} = 0, \hat{H}_{1121} = 0,$

p. 360, l. -13
(otherwise, ~~reverse time and~~ exchange the subscripts in (8.112)).

p. 361

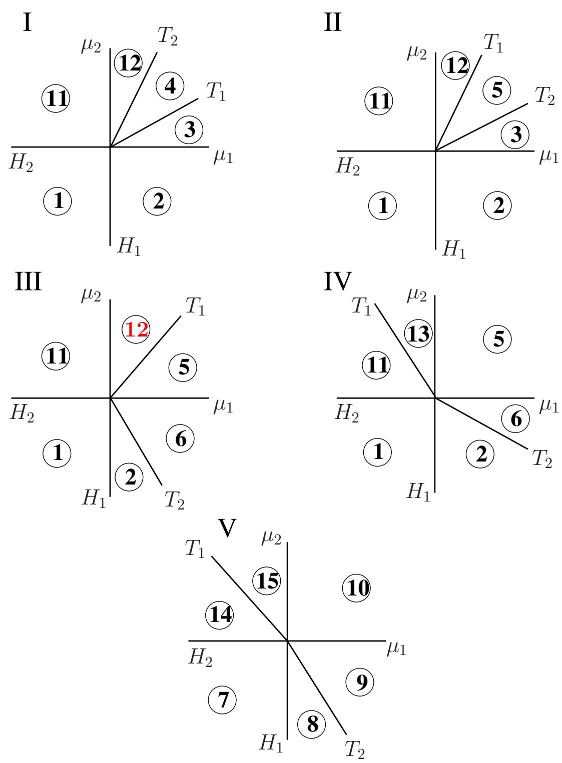

Fig. 8.25. Parametric portraits of (8.112) (the "simple" case).

p. 363, l. 11

$$C = \left\{ \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} : \ \mu_2 = -\frac{\delta - 1}{\theta - 1}\mu_1 - \frac{(\delta-1)\Theta + (\theta-1)\Delta}{(\theta-1)^3}\mu_1^2 + O(\mu_1^3) \right\}$$

that should be considered when both $\mu_1 > \theta\mu_2$ and $\delta\mu_1 > \mu_2$.

p. 363, l. -18

(HH.7) $p_{22}(0) \neq p_{12}(0)$;
(HH.8) $p_{21}(0) \neq p_{11}(0)$.

p. 363, l. -13

$$\text{sign}\ l_1 = \text{sign}\left\{-\theta\left[\theta(\theta-1)\Delta + \delta(\delta-1)\Theta\right]\right\}.$$

p. 364, l. -6

$$Y = \left\{ (\mu_1, \mu_2) : \ \mu_2 = -\frac{\delta-1}{\theta-1}\mu_1 + \frac{(\theta-1)^3\delta\Delta + (\delta-1)^3\theta\Theta}{(2\delta\theta - \delta - \theta)(\theta-1)^2}\mu_1^2 + O(\mu_1^3) \right\},$$

p. 365, l. -5

Recalling the interpretation of equilibria and cycles of the amplitude system (8.111) in the four-dimensional *truncated* normal form (8.109), we can establish a relationship between bifurcations in these two systems. The curves $H_{1,2}$ at which the trivial equilibria appear in (8.109) obviously correspond to Hopf bifurcation curves in (8.109). These are the two "independent" Hopf bifurcations caused by the two distinct pairs of eigenvalues passing through the imaginary axis. Crossing a bifurcation curve T_1 (or T_2) results in the branching of a two-dimensional torus from a cycle. Therefore, the curves $T_{1,2}$ correspond to Neimark-Sacker bifurcations in (8.109). On the curve C, system (8.109) exhibits a bifurcation that we have not yet encountered, namely, branching of a three-dimensional torus from the two-dimensional torus. The curves J describe blow-ups of three-dimensional tori, while the curve Y implies the presence of a heteroclinic coincidence of the three-dimensional stable and unstable invariant manifolds of a cycle and a three-torus.

p. 366

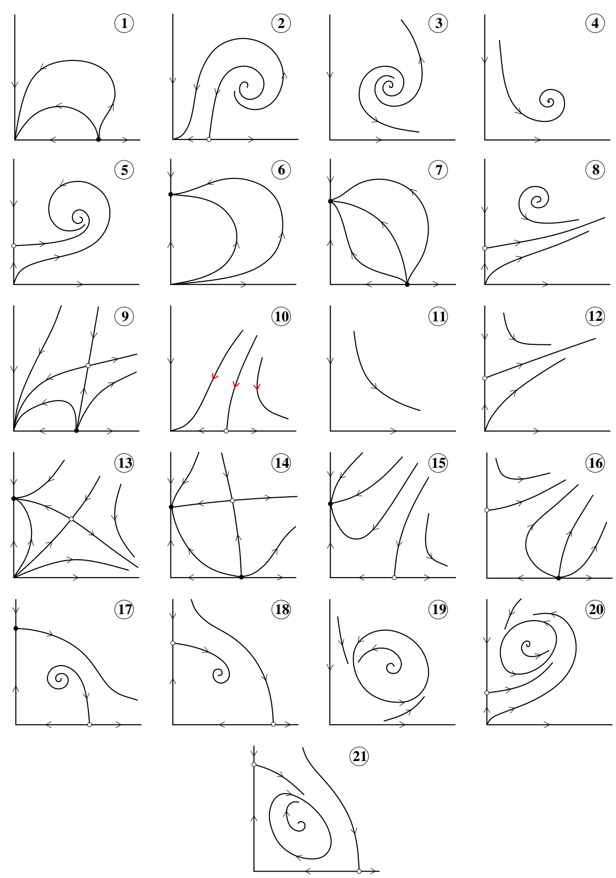

Fig. 8.29. Generic phase portraits of (8.115).

p. 380, l. -9

The vectors in $\mathbb{C}^3$

p. 384, l. 3

The resulting formulas are lengthy and can be found elsewhere (see references in Appendix ~~2~~).

p. 384, l. -19

Show that this curve is well-defined near the origin and can be locally parametrized by x.

p. 386, l. 3

$$\begin{cases} \dot{x}_1 = -\alpha_1 x_2 + x_1(1 - x_1^2 - x_2^2), \\ \dot{x}_2 = \alpha_1 x_1 + x_2(1 - x_1^2 - x_2^2) - \alpha_2, \end{cases}$$

p. 388, l. 18

$\lambda_1 = 0,\ \lambda_{2,3} = \pm i\omega_0$

p. 396, l. 4

$$\zeta_1 = \frac{\eta_1}{\nu},\quad \zeta_2 = \frac{\eta_2}{\nu^{3/2}},\quad t = \frac{\tau}{\nu^{1/2}}. \tag{A.3}$$

This rescaling reduces (A.2) to

p. 396, l. -11

This system is a *Hamiltonian system*,

$$\begin{cases} \dot{\zeta}_1 = \dfrac{\partial H(\zeta)}{\partial \zeta_2}, \\ \dot{\zeta}_2 = -\dfrac{\partial H(\zeta)}{\partial \zeta_1}, \end{cases}$$

p. 402, l. 14

(i.e., all extrema are minimum points).

p. 403, l. 1

By the Inverse Function Theorem, these equations define a smooth function $\beta(\gamma)$.

p. 403, l. 8

The homoclinic curve $\mathcal{P}$ given by (A.13) is mapped by (A.20) into the curve

p. 403, l. -19

the second Lyapunov coefficient

9

Two-Parameter Bifurcations of Fixed Points in Discrete-Time Dynamical Systems

p. 413

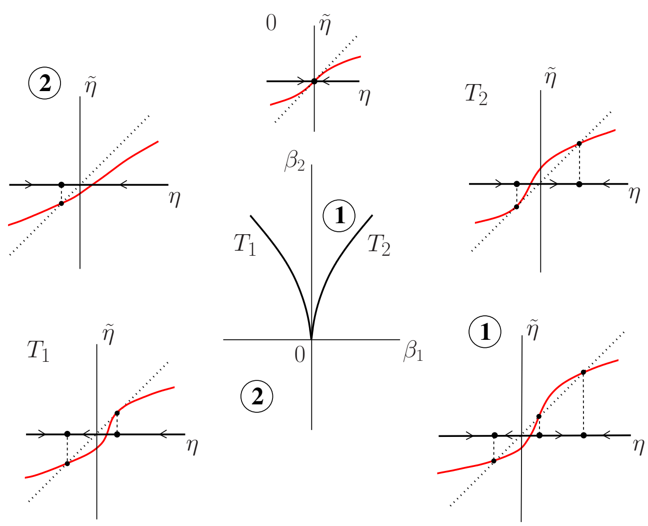

Fig. 9.2. Bifurcation diagram of the normal form (9.11).

p. 423, l. 15

$$\varphi^1(x) = e^{\Lambda}x + g^{(2)}(x) + g^{(3)}(x) + \ldots + g^{(k)}(x) + O(\|x\|^{k+1}). \tag{9.27}$$

p. 426, l. 11

$$P^{-1}(\alpha)L(\alpha)P(\alpha) = \begin{pmatrix} \lambda & 1 \\ \varepsilon_1(\alpha) & \lambda + \varepsilon_2(\alpha) \end{pmatrix},$$

p. 426, l. 16

$$\varepsilon_1(\alpha) = b_{10}(\alpha) + a_{01}(\alpha)b_{10}(\alpha) - a_{10}(\alpha)b_{01}(\alpha), \quad \varepsilon_2(\alpha) = a_{10}(\alpha) + b_{01}(\alpha).$$

p. 426, l. -3

$$\begin{pmatrix} g_{00}(\alpha) \\ h_{00}(\alpha) \end{pmatrix} = P^{-1}(\alpha) \begin{pmatrix} a_{00}(\alpha) \\ b_{00}(\alpha) \end{pmatrix},$$

p. 430, l. -10

$$\xi^{(2)}(1) = \begin{pmatrix} \xi_1 + \xi_2 + \frac{1}{2}a_{20}\xi_1^2 + a_{11}\xi_1\xi_2 + \frac{1}{2}a_{02}\xi_2^2 \\ \xi_2 + \frac{1}{2}b_{20}\xi_1^2 + b_{11}\xi_1\xi_2 + \frac{1}{2}b_{02}\xi_2^2 \end{pmatrix},$$

p. 432, l. 3

$$Y_2(X) = \begin{pmatrix} A_{1010}\xi_1\nu_1 + A_{0110}\xi_2\nu_1 + A_{1001}\xi_1\nu_2 + A_{0101}\xi_2\nu_2 \\ B_{1010}\xi_1\nu_1 + B_{0110}\xi_2\nu_1 + B_{1001}\xi_1\nu_2 + B_{0101}\xi_2\nu_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}A_{2000}\xi_1^2 + A_{1100}\xi_1\xi_2 + \frac{1}{2}A_{0200}\xi_2^2 \\ \frac{1}{2}B_{2000}\xi_1^2 + B_{1100}\xi_1\xi_2 + \frac{1}{2}B_{0200}\xi_2^2 \\ 0 \\ 0 \end{pmatrix}.$$

We leave the reader to complete the proof by computing

$$X^{(2)}(1) = e^J X + \int_0^1 e^{J(1-\tau)} Y_2(X^{(1)}(\tau))d\tau$$

p. 436, l. 4

$$g(y,\alpha) = \langle p_0, F(y_1q_0 + y_2q_1, \alpha)\rangle, \quad h(y,\alpha) = \langle p_1, F(y_1q_0 + y_2q_1, \alpha)\rangle,$$

p. 436, l. 16

$$\nu_1(\alpha) = b_{10}(\alpha) + a_{01}(\alpha)b_{10}(\alpha) - a_{10}(\alpha)b_{01}(\alpha), \quad \nu_2(\alpha) = a_{10}(\alpha) + b_{01}(\alpha).$$

p. 437, l. -7

$$\begin{cases} \widetilde{\xi}_1 = -\xi_1 + \xi_2 + \sum_{2\le j+k\le 3} \gamma_{jk}\xi_1^j\xi_2^k + O(\|\xi\|^4), \\ \widetilde{\xi}_2 = -\xi_2 + \sum_{2\le j+k\le 3} \sigma_{jk}\xi_1^j\xi_2^k + O(\|\xi\|^4), \end{cases}$$

p. 438, l. -7

$$\begin{pmatrix} 3 & 0 & 0 & 0 & -1 & 0 \\ -3 & 2 & 0 & 0 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_{30} \\ \phi_{21} \\ \phi_{12} \\ \psi_{30} \\ \psi_{21} \\ \psi_{12} \end{pmatrix} = R[g,h],$$

p. 444

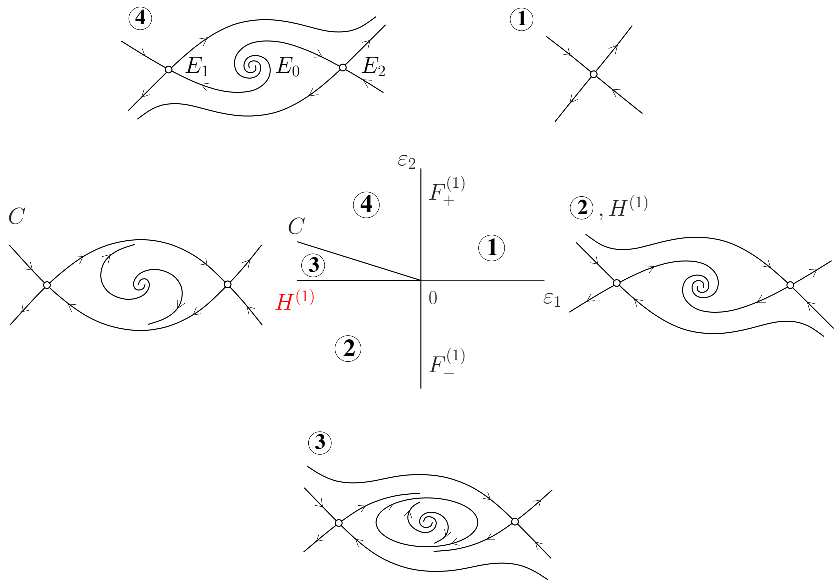

Fig. 9.9. Bifurcation diagram of the approximating system (9.75) for $s = 1$.

p. 445, l. 4 (after Fig. 9.10)

$$P = \left\{(\varepsilon_1, \varepsilon_2) : \ \varepsilon_2 = \frac{4}{5}\varepsilon_1 + o(\varepsilon_1), \ \varepsilon_1 > 0\right\},$$

p. 448, l. 8

$$C(\alpha) = \frac{g_{20}(\alpha)g_{11}(\alpha)(2\mu(\alpha) + \bar{\mu}(\alpha) - 3)}{2(\bar{\mu}(\alpha) - 1)(\mu^2(\alpha) - \mu(\alpha))} + \frac{|g_{11}(\alpha)|^2}{|\mu(\alpha)|^2 - \bar{\mu}(\alpha)} + \frac{g_{21}(\alpha)}{2}. \qquad (9.81)$$

p. 449, l. 9
Formula (9.83) can be obtained by merely substituting $g_{02} = 0$ into (4.26) from Chapter 4.

p. 458, l. -6

$$-\frac{e^{i\alpha}}{\rho^2} = A(\beta) + e^{-4i\varphi},$$

p. 478

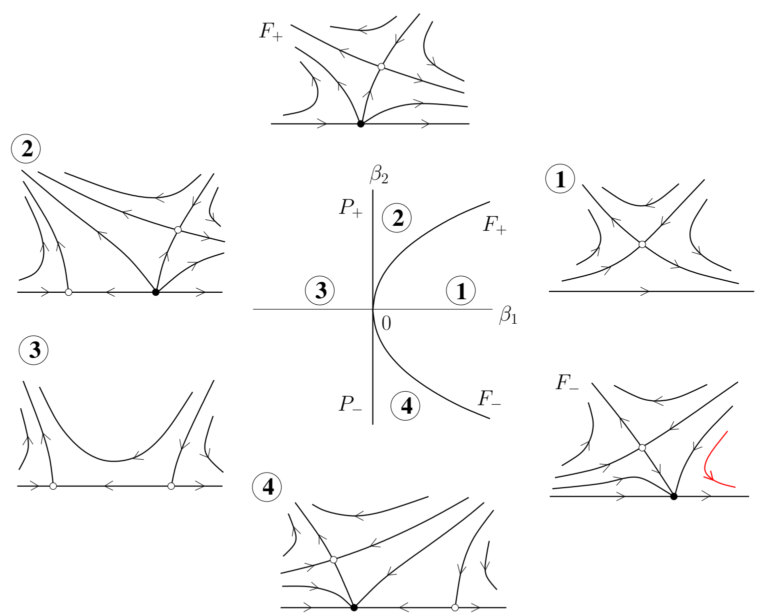

Fig. 9.27. Case 3: $a_0 > 0, b_0 < 0$.

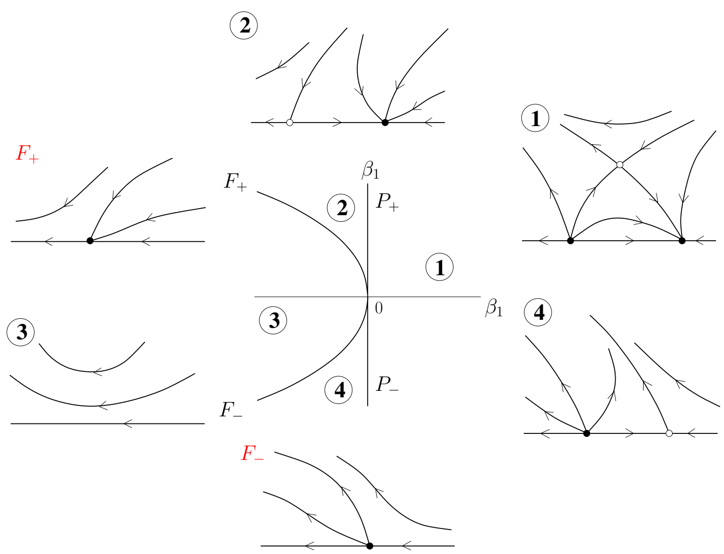

Fig. 9.28. Case 4: $a_0 < 0, b_0 < 0$.

p. 485, l. 13

As in Section 8.7.4 of Chapter 8, we make the third system solvable by selecting a proper solution h_{20} of the first system.

p. 488, l. 9

$$H(w) = w_1 q_1 + w_2 q_2 + \sum_{2 \le j+k \le 3} \frac{1}{j!k!} h_{jk} w_1^j w_2^k + O(\|w\|^4).$$

p. 489, l. 5

$$c_3 = \langle p_2, C(q_1, q_1, q_2) + B(q_2, h_{20}) + 2B(q_1, h_{11}) \rangle,$$

p. 499, l. 7

(*Hint:* Find a common point of the flip and the Neimark-Sacker bifurcation lines computed in Exercise 10 in Chapter 5.)

10

Numerical Analysis of Bifurcations

p. 541, l. -8

$$\begin{pmatrix} B(q_1, q_2) \\ \ddot{y}^T_{(1)}(0) q_2 \end{pmatrix} + \begin{pmatrix} J(0) \\ q_1^T \end{pmatrix} \dot{u}(0) = \dot{\lambda}(0) q_2. \qquad (10.64)$$

p. 580, l. -12
along the saddle-node bifurcation curve (see **Fig. 10.24**).

References

Beyn, W.-J. [1990*b*]. 'The numerical computation of connecting orbits in dynamical systems', *IMA J. Numer. Anal.* **10**, 379–405.

Gaspard, P., Kapral, R. & Nicolis, G. [1984], 'Bifurcation phenomena near homoclinic systems: a two-parameter analysis', *J. Statist. Phys.* **35**, 697–727.