Numerical bifurcation analysis of delay differential equations

with DDE-BIFTOOL and PDDE-CONT

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PDDE-CONT

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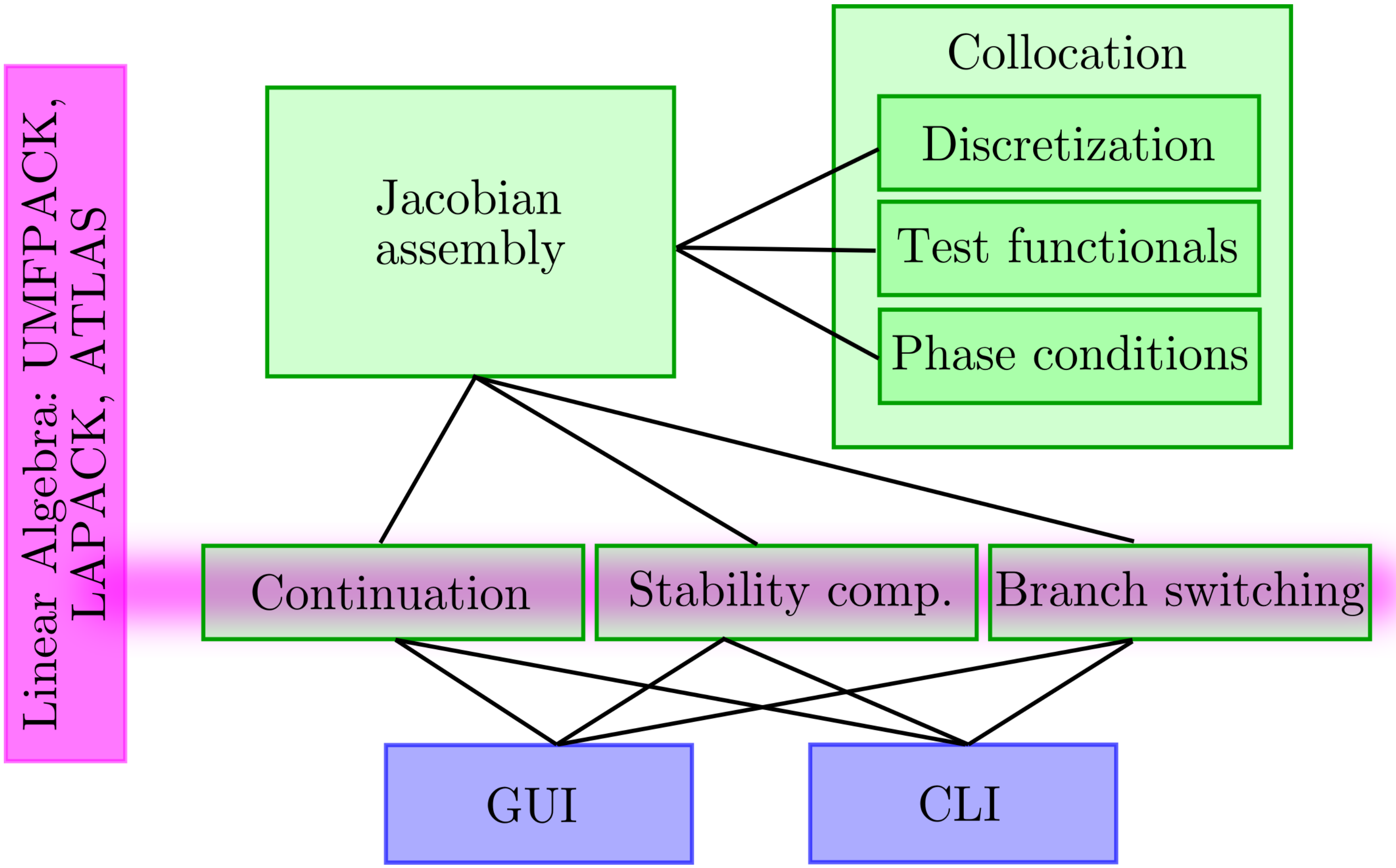
Functionality

- no explicit steady state handling
- continuation of periodic orbits, branch switching, and extension for symmetric systems
- computation of stability (see 2^{nd} lecture)
- continuation of one-codimension bifurcations (Neimark-Sacker, Period doubling, fold)
- continuation of quasi-periodic invariant tori

Structure of PDDE-CONT



Using PDDE-CONT

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An example: the Mackey-Glass equation

In order to investigate the system, two things are necessary

- The governing equation, e.g.,

$$\dot{x}(t) = ax(t) + b\frac{x(t-\tau)}{1+x^{10}(t-\tau)}$$

- starting solution (steady state or periodic)

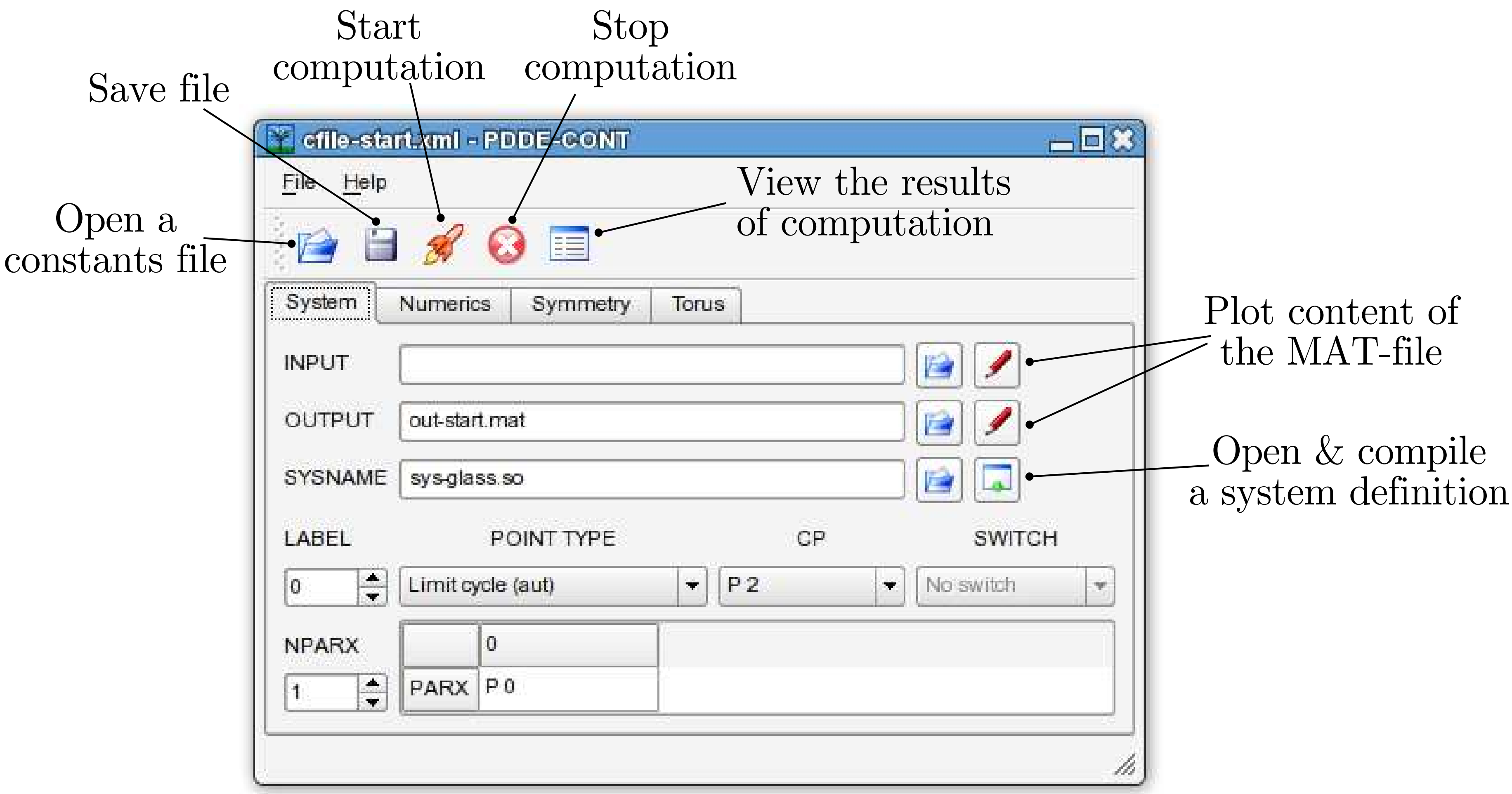
$$x = \sqrt[10]{-(a+b)/a} \quad \text{at} \quad \tau = 2, a = -1, b = 3/2, (T = 2)$$

Put these into the system definition file `sys-glass.cpp` and produce a constants file e.g. `cfile-start.xml` using the GUI.

System definition

```
#include <cmath>
#include "pddesys.h"
extern "C"
{
int  sys_ndim()  { return 1; }
int  sys_npar()  { return 4; }
int  sys_ntau()  { return 2; }
int  sys_nderi() { return 0; }
void sys_tau( Vector& y, double t, const Vector& p )
   { y(0)=0.0; y(1)=p(3); }
void sys_dtau( Vector& y, double t, const Vector& p, int vp )
   { y(0)=0.0; if (vp==3) y(1)=1.0; else y(1)=0.0; }
void sys_rhs( Vector& y, double t, const Matrix& x, const Vector& p )
   { y(0) = p(1)*x(0,0) + p(2)*x(0,1)/(1+pow(x(0,1), 10.0)); }
void sys_deri( Matrix& y, double t, const Matrix& x, const Vector& p,
   int nx, const int* vx, int np, const int* vp, const Matrix& vv ) {}
void sys_stpar( Vector& p )
   { p(0)=2.0; p(1)=-1.0; p(2)=1.5; p(3)=2.0; }
void sys_stsol( Vector& y, double t )
   { y(0) = pow((1.0-1.5)/(-1.0), 1.0/10.0); }
}
```

Editing the constants file



The output

Three branches

- Steady state (green)
- Periodic solution from Hopf (blue)
- Period-two solution (red)