Practicum 3, 21 August 2009, 11:00-12:30 uur

Study the following two-parameter planar systems exhibiting local codim 2 bifurcations by combining analytical and numerical methods.

- **Takens normal form**

$$\begin{cases} \dot{x} &= \beta x + y + x^2, \\ \dot{y} &= \alpha - 4x^2. \end{cases} \tag{1}$$

  1. Derive equations for the saddle-node and Andronov-Hopf bifurcations in the system.
  2. Prove that a Bogdanov-Takens (BT) bifurcation occurs in the system and find the corresponding parameter values.
  3. Compute the normal form coefficients a and b for the BT-bifurcation and verify that $ab \neq 0$.
  4. Use `pplane7` to produce all representative phase portraits of the system near the Bogdanov-Takens point. Sketch the bifurcation diagram of the system.
  5. For $\alpha = 0.25$ find numerically the value of β corresponding to the saddle homoclinic bifurcation.

- **A prey-predator model by Bazykin and Khibnik**

$$\begin{cases} \dot{x} &= \dfrac{x^2(1-x)}{n+x} - xy, \\ \dot{y} &= -y(m-x), \end{cases} \tag{2}$$

  where $x, y \geq 0$ and $0 < m < 1$.

  1. Derive an equation for the Andronov-Hopf bifurcation in the model. *Hint*: Consider the orbitally-equivalent polynomial system
  $$\begin{cases} \dot{x} &= x^2(1-x) - xy(n+x), \\ \dot{y} &= -y(m-x)(n+x). \end{cases} \tag{3}$$
  2. Using `pplane7`, produce several phase portraits of model (2) for different values of m and fixed $n = \frac{1}{4}$ and $n = \frac{1}{16}$. *Hint*: The most interesting phase portrait occurs at $(m, n) = (\frac{2}{10}, \frac{1}{16})$.
  3. Conclude that a Bautin bifurcation happens in the model. Sketch the bifurcation diagram of the model.
  4. For $m = 0.2$ find numerically the value of n corresponding to the collision and disappearance of two periodic orbits.
  5. *Challenge*: Prove that Bautin bifurcation occurs at $(m, n) = (\frac{1}{4}, \frac{1}{8})$ and verify that it is nondegenerate by computing the 2nd Lyapunov coefficient l_2.