Comments on "Malle's conjecture for fair counting functions"

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Conjecture 5.1 is not correct as stated. We expect this conjecture to be correct after the following two modifications:

- On page 15, the action of $G \times (\mathbb{Z}/|G|\mathbb{Z})^*$ on $\ker(\phi) - \mathrm{id}$ should send n to $gn^{-\alpha}g^{-1}$ (instead of $gn^{\alpha}g^{-1}$ as written). We thank Daniel Loughran and Tim Santens for pointing this out to us.

- The constant $c_{(H,\phi)}(G)$ may be zero. We now conjecture that $\mathrm{Epi}_{(H,\phi)}(G_{\mathbb{Q}}, G)$ is either empty, or that $c_{(H,\phi)}(G)$ is positive and the conjecture holds as stated. We thank Jiuya Wang for bringing this problem to our attention.

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