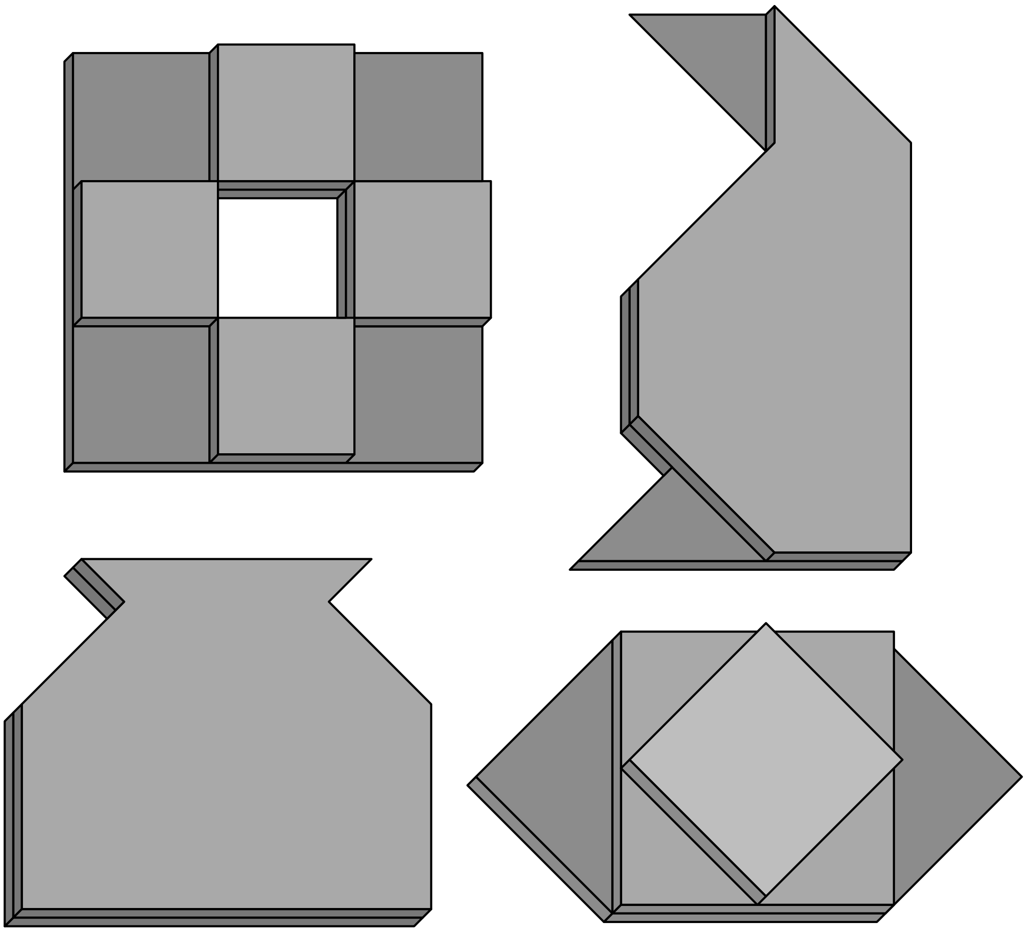
Pilogram

tangram with a new dimension



Marc van Kreveld

Puzzle idea, pieces, assignments, solutions, text, and layout by Marc van Kreveld

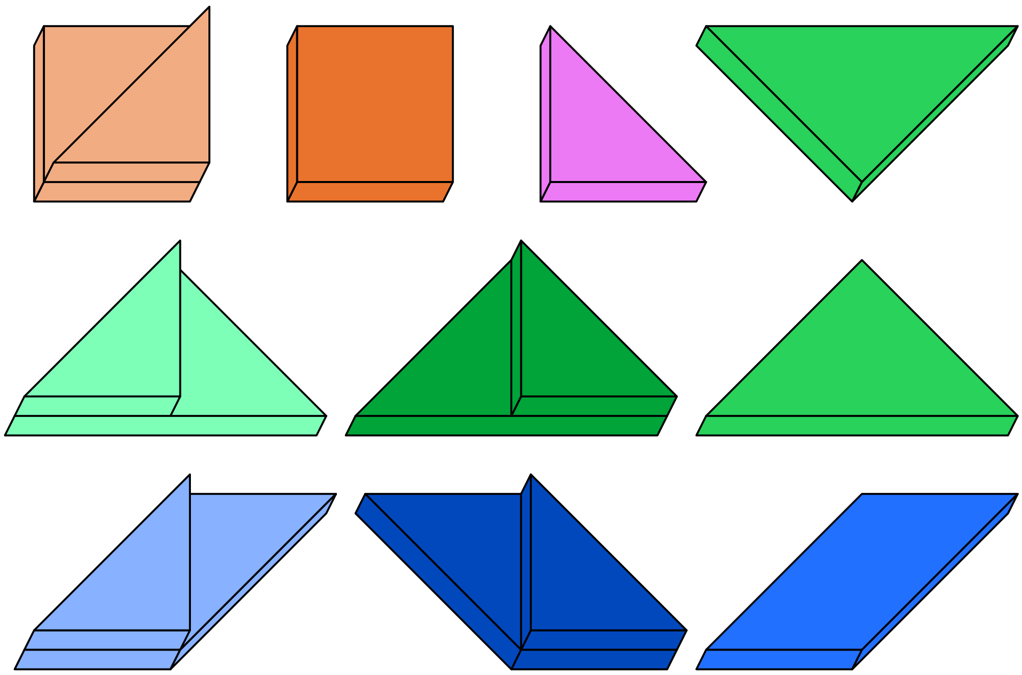
This document is a *draft*. Assignments, solutions and/or their visualizations may not be correct, and no responsibility is taken. Please send any corrections or other comments to the author by e-mail: `m.j.vankreveld@uu.nl`

Chapter 1

Introduction

Pilogram is a tangram-inspired puzzle with an extra dimension: an assignment shape extends over more than one layer. There are ten base pieces, shown in figure. With these pieces, endlessly many shapes can be built, just like with tangram. However, it is no longer possible to think 'in the plane': all assignments will use two or more layers, just because some of the pieces already occupy two layers.

When we look at the pieces, we see two squares, five triangles, and three parallelograms. Five of these shapes have a triangle on the second layer attached. One shape occurs twice. The small triangle is the same size as the five attached triangles on the second layer. Each of the base shapes could have been made from up to three copies of the small triangle.

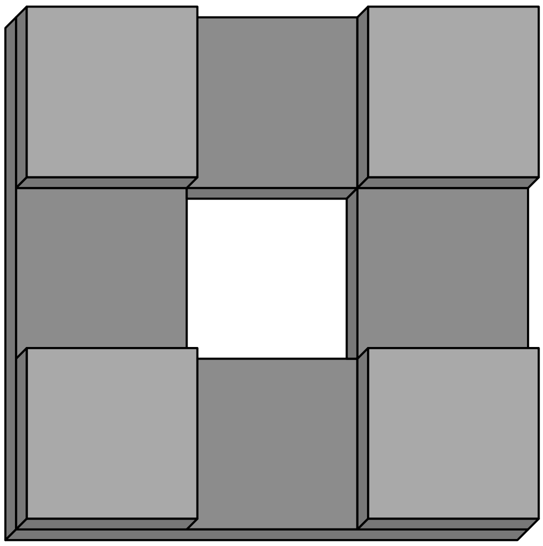


Valid assignments and solutions may not have overhangs. Any part in the second layer must be fully supported in the first layer. This also holds true when assignments use more than two layers: any part on the third layer must be fully supported on the second layer (and hence also on the first layer), any part on the fourth layer must be fully supported on the third layer (and hence also on the second and first layers), and so on.

We should note that all pieces can be used upside-down. This holds in particular for the shapes that extend over two layers. This shape by itself would create an overhang, so a different shape must then provide the support on the lower layer.

A second rule of pilograms is that all pieces must be used. If the assignment has a triangular shape, then you cannot simply choose a single triangular piece.

An assignment will always show how many layers are used everywhere. What is not shown is where the seams are, and therefore which piece is used where in the solution.

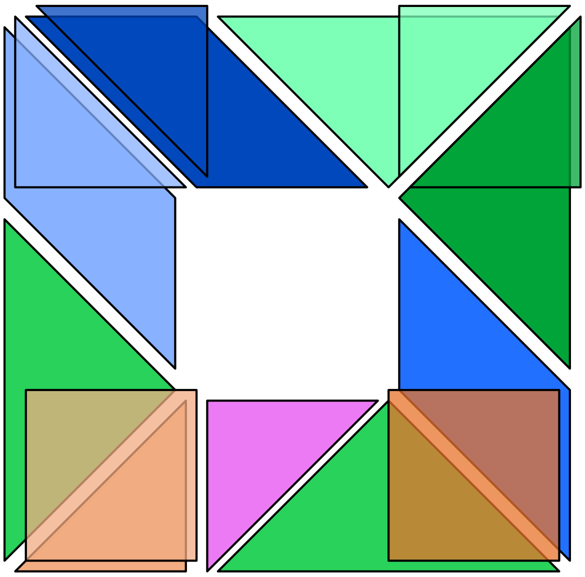


The figure above is an example of an assignment. The assignment is shown as a top view with a bit of perspective added. The perspective allows us to see how many layers are used everywhere.

We can see a large square shape where the middle is empty. This middle has a square shape as well. The four corners of the large square are also squares, they extend over two layers. In between those four corners are squares that extend over one layer only.

With some imagination we see a fortress: a square wall with raised towers in the corners.

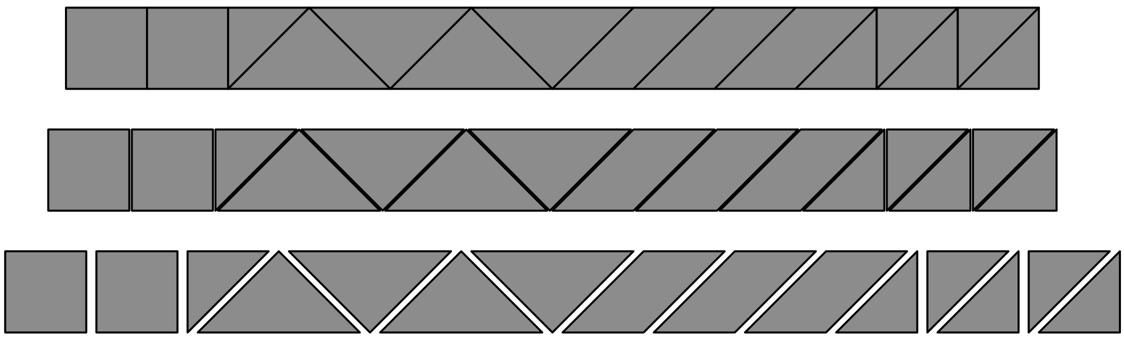
This book contains the solutions to all assignments. To show these well, we will use the colors of the pieces from the first figure. We use a simplified version of every shape, we pull all of the pieces slightly apart, and also use some degree of transparency. All of this is done to show the solution most clearly, and without much clutter. The next page shows an example. If you want to try solving the fortress assignment first, do that first, before turning the page.



Chapter 2

Making the pieces from wood

The ten pieces of pilogram are easy to make yourself from wood. You can also use thick cardboard, but wood is really nicer. You need a slat or plank of wood, roughly four times as wide as thick, for example 36×9 mm or 60×15 mm. Then use a pattern as shown in the figure to saw the wood into pieces. Use a mitre saw with mitre box for the diagonal cuts, or an electric mitre saw if you have one. Work as precisely as possible. Take into account that the saw blade has a thickness, which means that the patterns shown in the figure does not work exactly as you would think. All of the cuts need to move over sideways a little to get shapes that are exactly right.



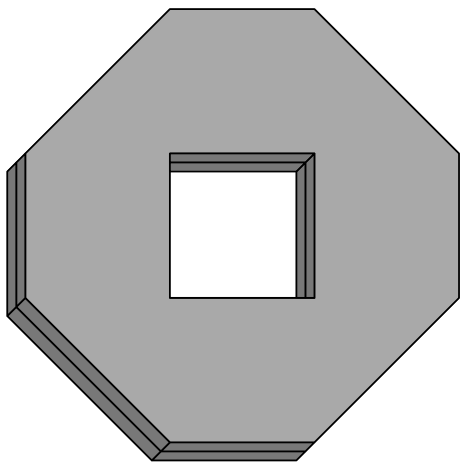
If you are going to paint the wood or finish it in a different manner, the pieces get slightly bigger too. It seems that all pieces get bigger with the same amount so this should not matter, but this is not quite true. You want the six small triangles, five of which are attached to another shape, to be a tiny bit smaller than what you would think. In the end, this will let the solutions to the assignments fit better and give a more pleasing result.

In principle you need to use a precision better than 1 mm. But larger pieces need less precision, so use a slat or plank of width at least 60 mm if your precision will be around 1 mm. It is also a good idea to saw a few more copies of every basic shape, then measure them, and discard the pieces that are off by a millimeter or more.

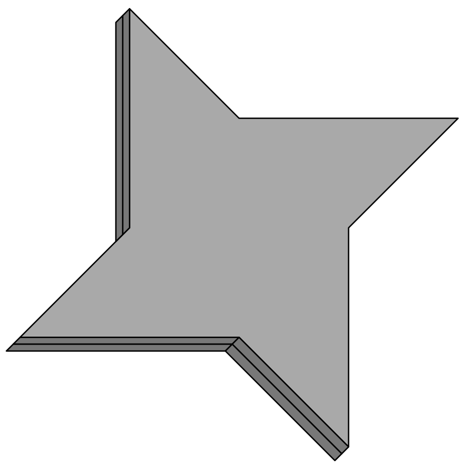
Chapter 3

Puzzles

The letter O

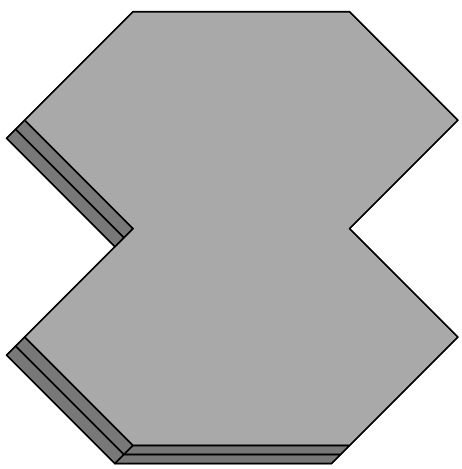


Star

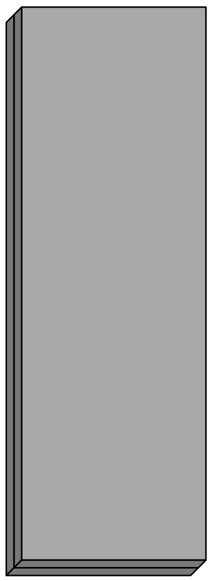


Rectangle

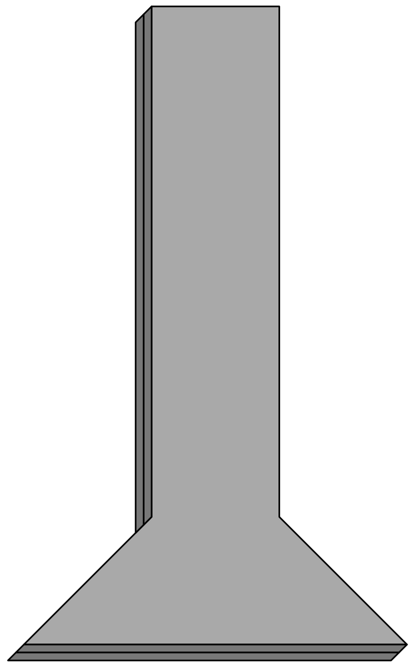
Double vase



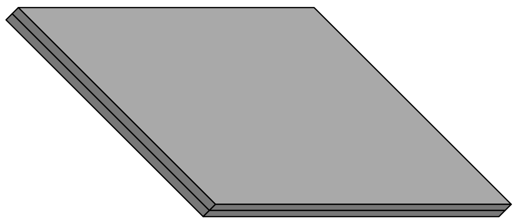
Elongated rectangle



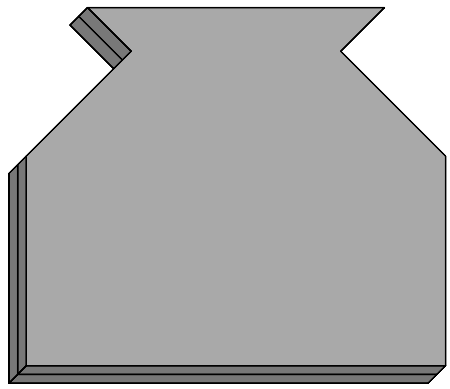
Tall vase



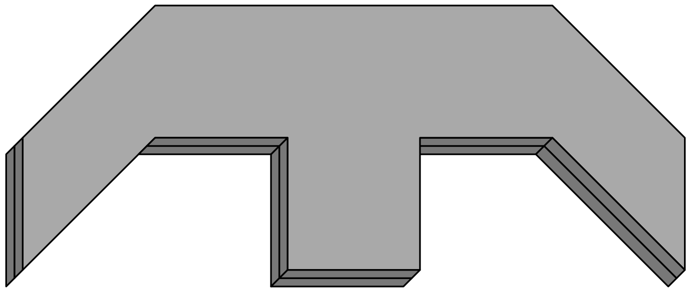
Parallelogram



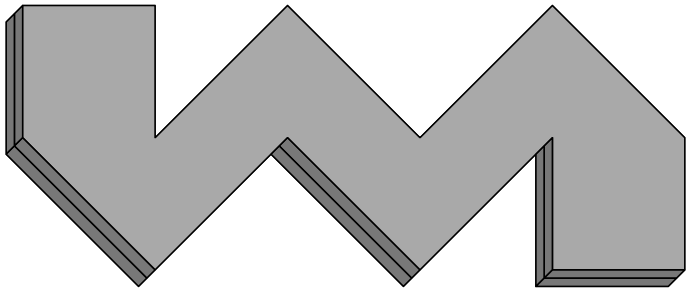
Jar



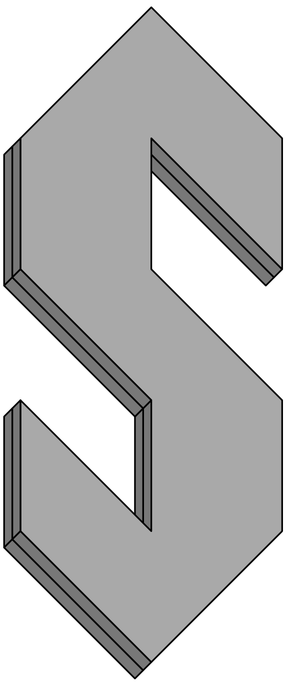
Wide shape



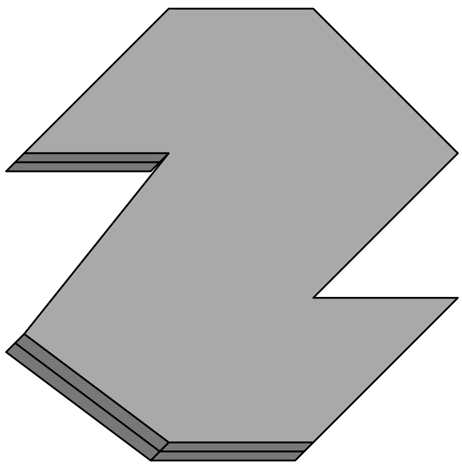
Zig-zag shape



Tall S shape

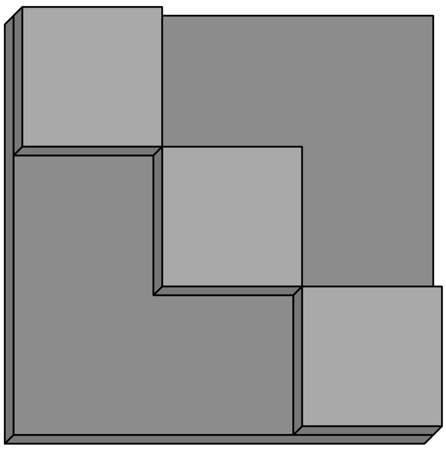


Like an S

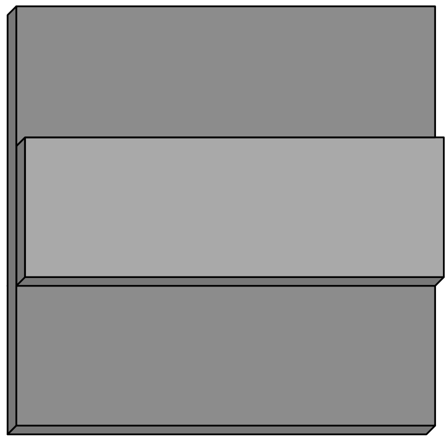


Levels 1 and 2

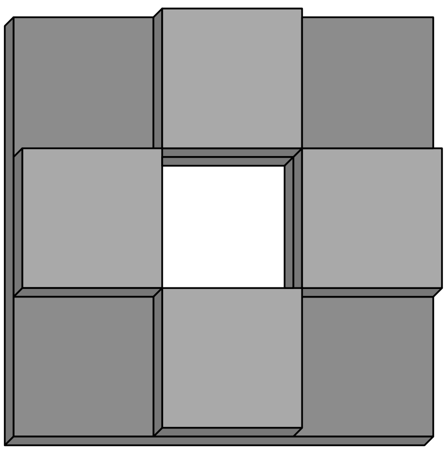
Square with diagonal



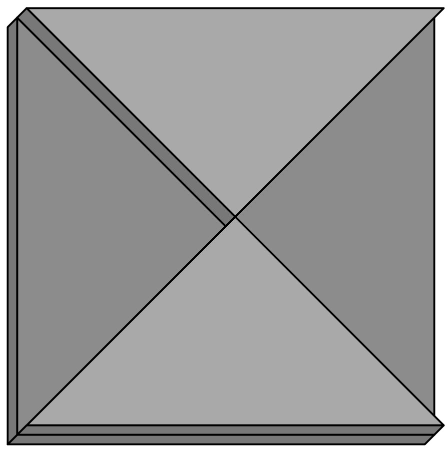
Square with line



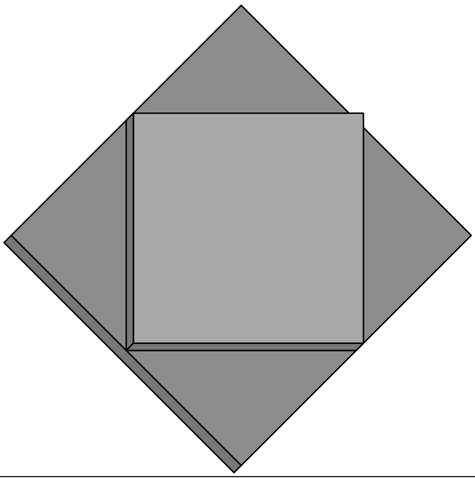
Inverted fortress



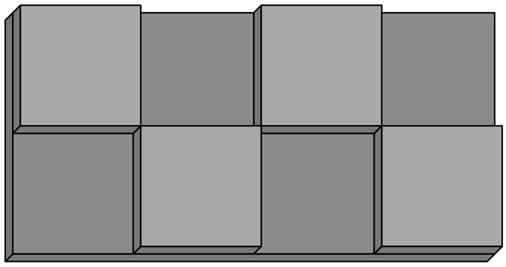
Two triangles on square



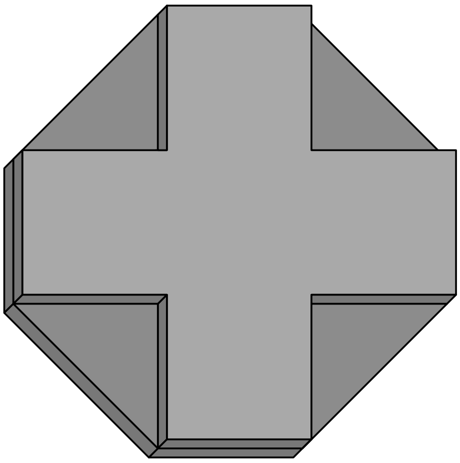
Square in square



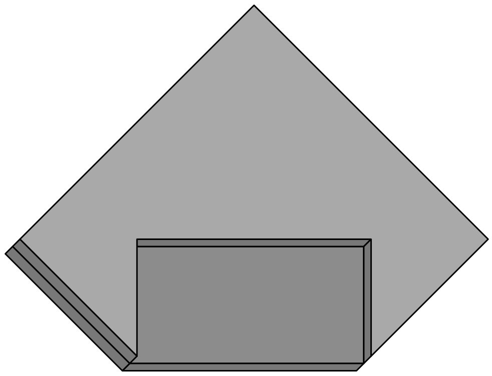
Chessboard fragment



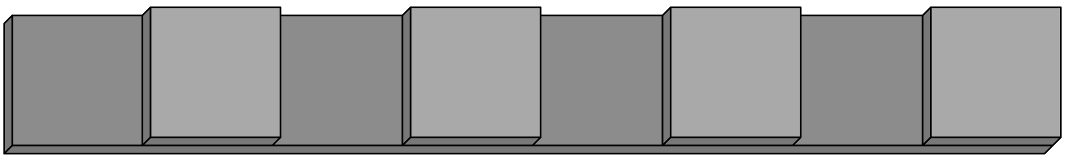
Plus on octagon



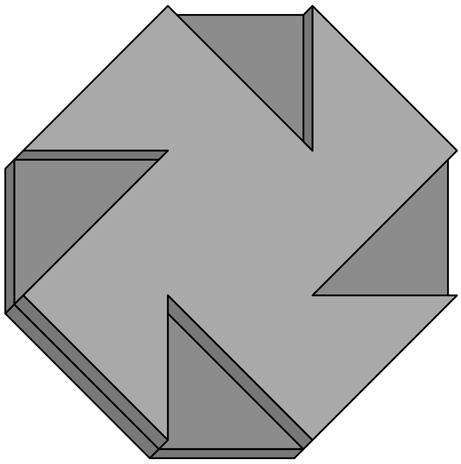
Reduced square



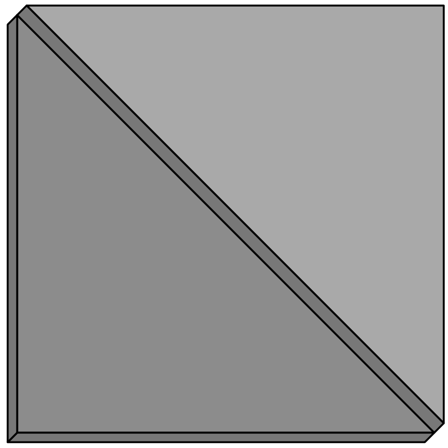
Ramparts



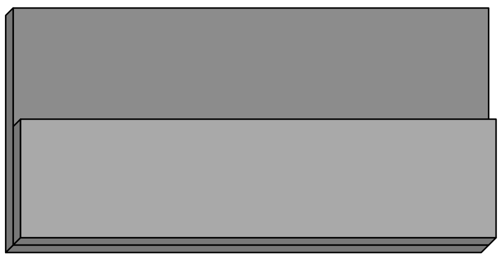
Rotation



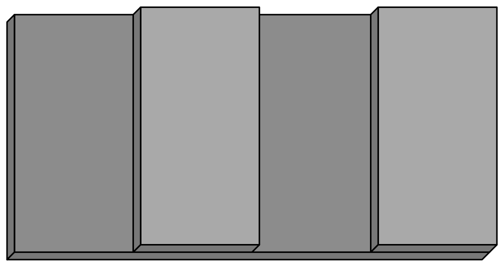
Triangle on square



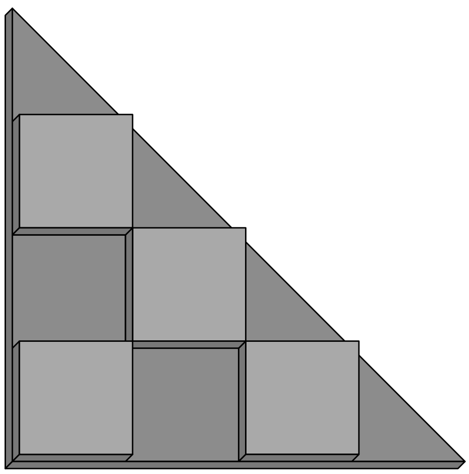
Wide step



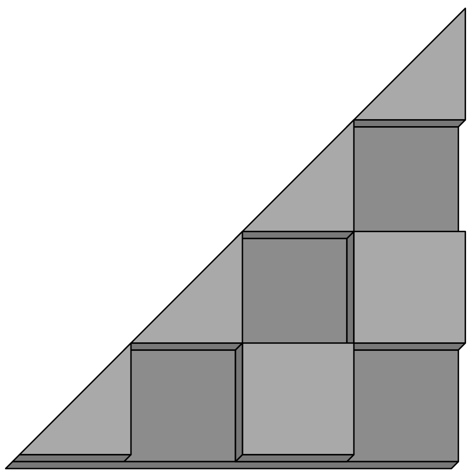
Two steps



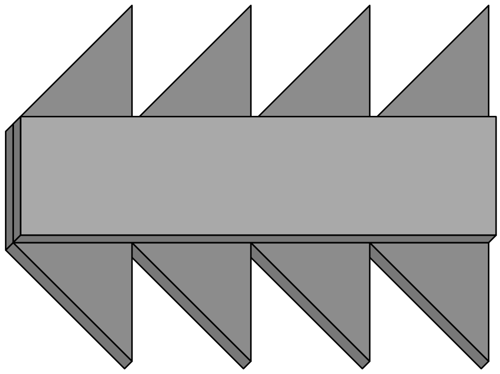
Triangular fragment of a chessboard



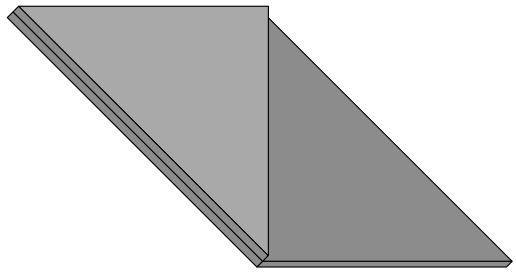
Another fragment of a chessboard



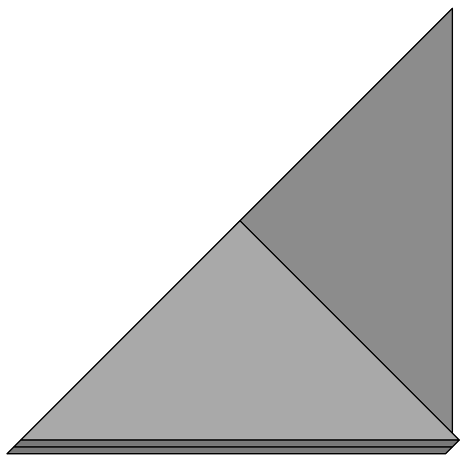
Double-sided saw



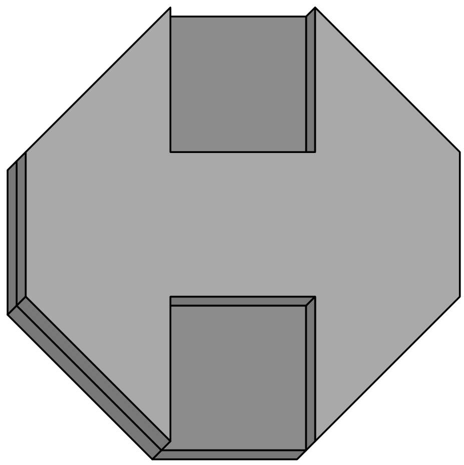
Triangle on parallelogram saw



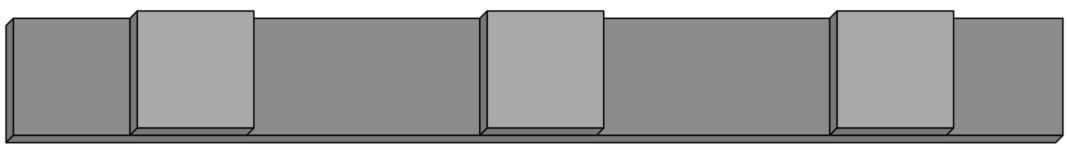
Triangle on triangle



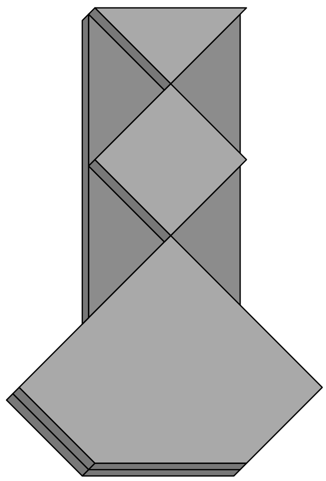
H on octagon



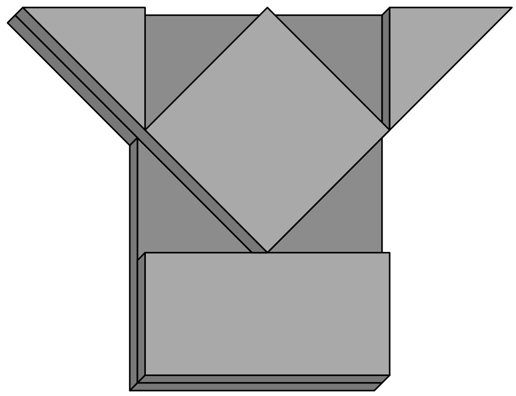
Raised squares



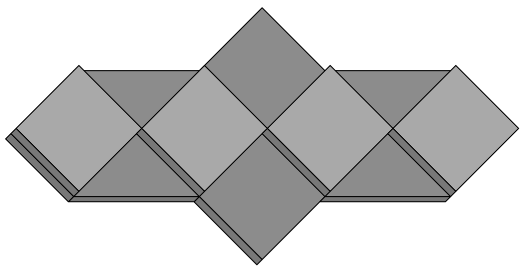
Vase



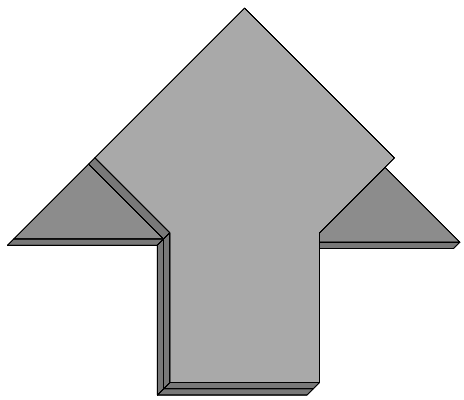
Another vase



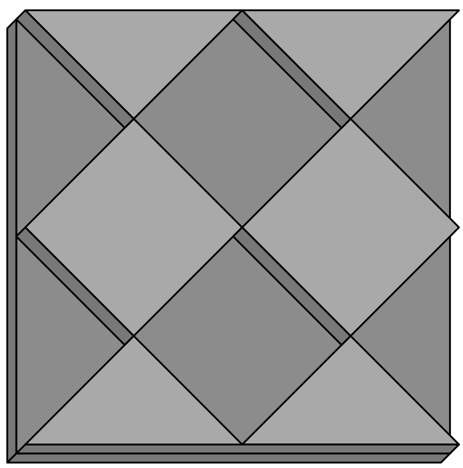
Symmetric shape



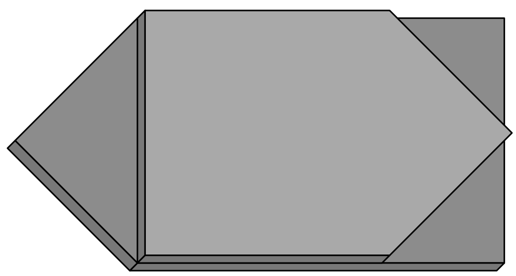
Arrow



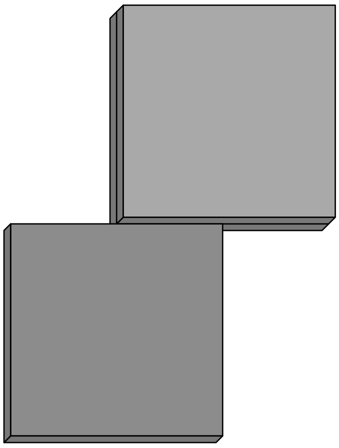
Checkered pattern



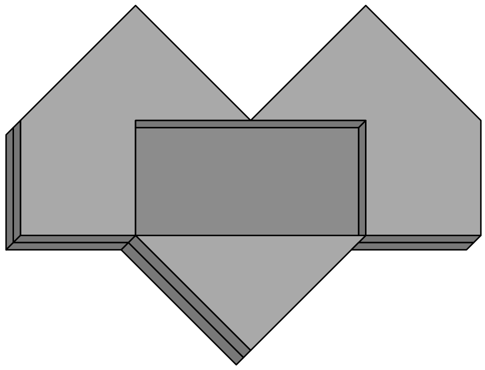
Pentagons



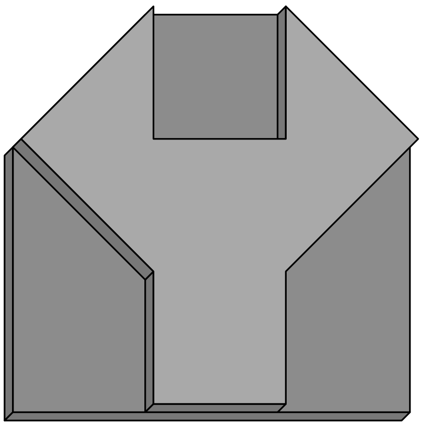
High square, low square



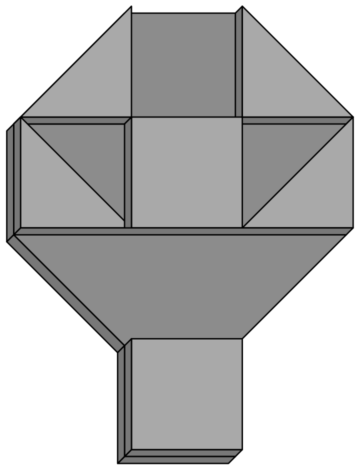
Some shape



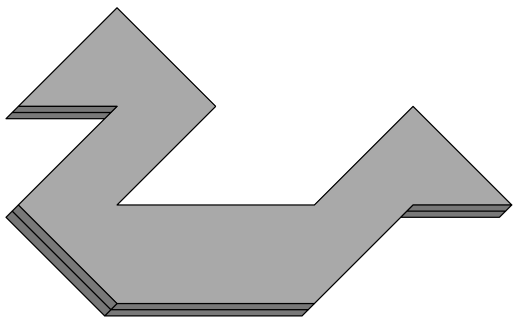
Wrench

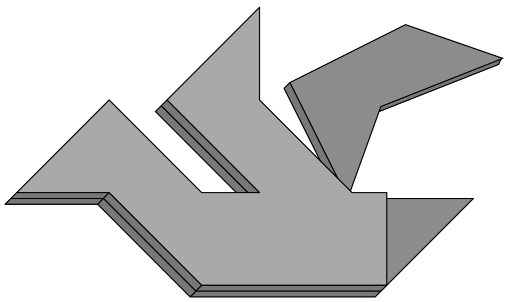


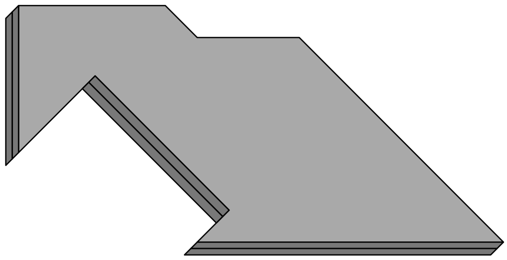
Tree

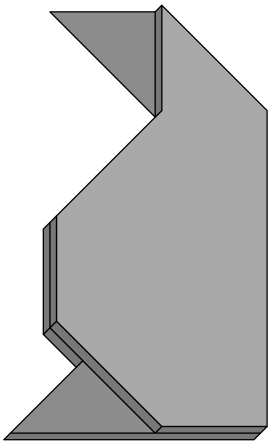


Birds

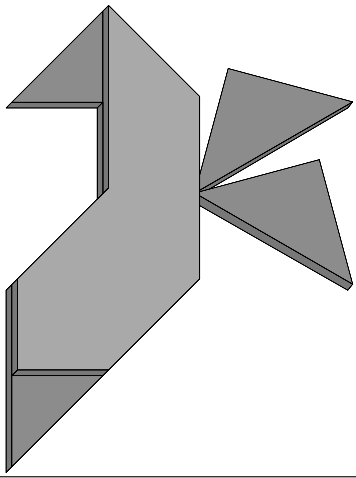

Swimming bird


Flapping bird

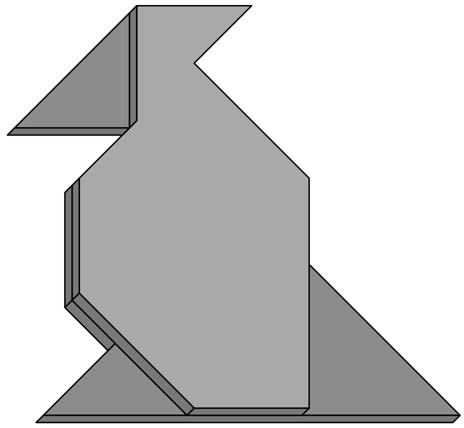

Perching bird


Penguin

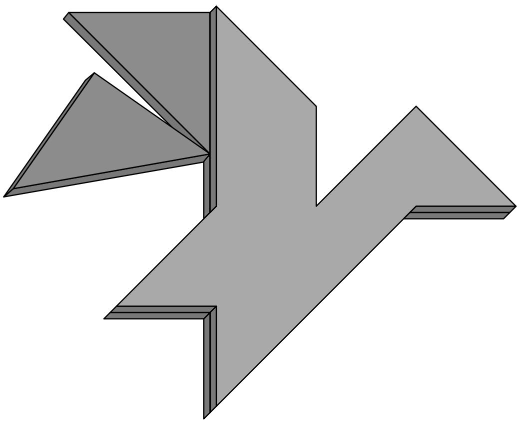
Hummingbird



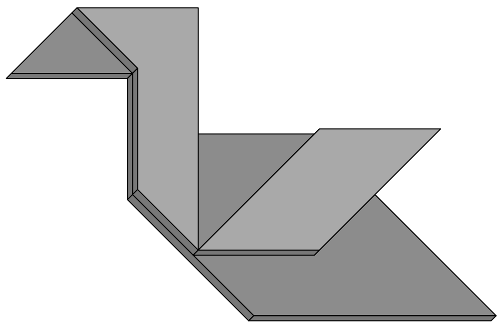
Sitting bird



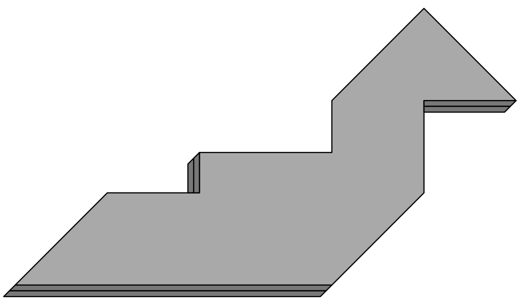
Flying bird



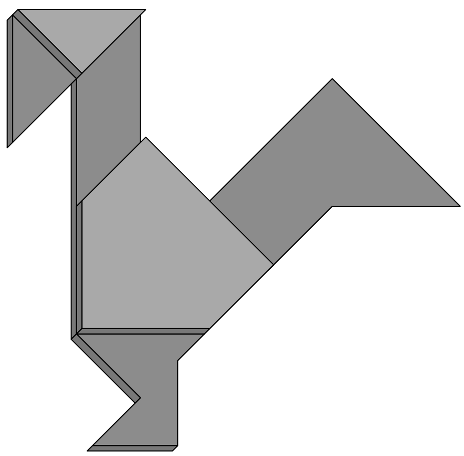
Another swimming bird



Loon

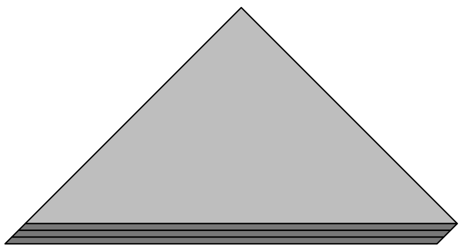


Rooster

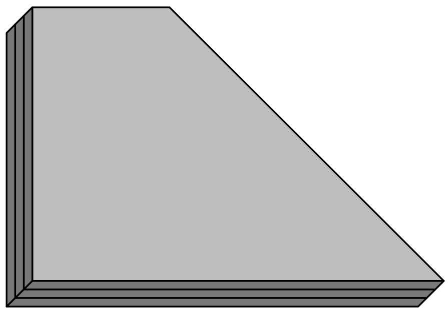


3 high

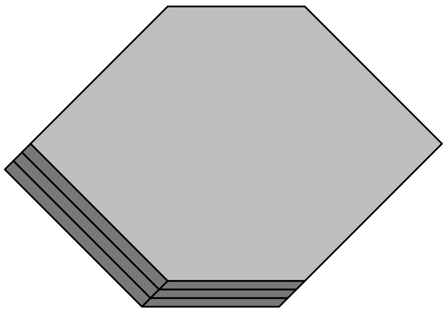
3-high triangle



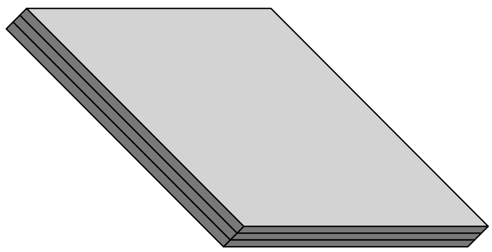
3-high quadrilateral



3-high hexagon

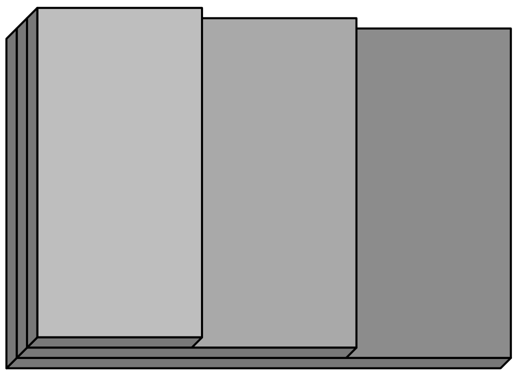


3-high parallelogram

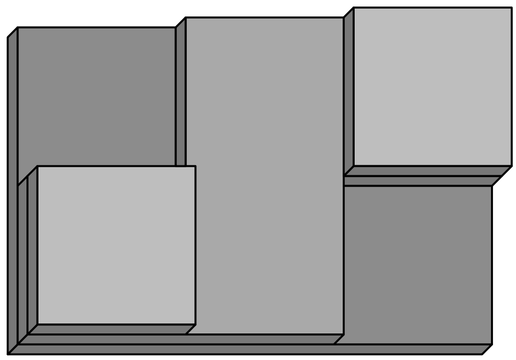


Levels 1, 2, and 3

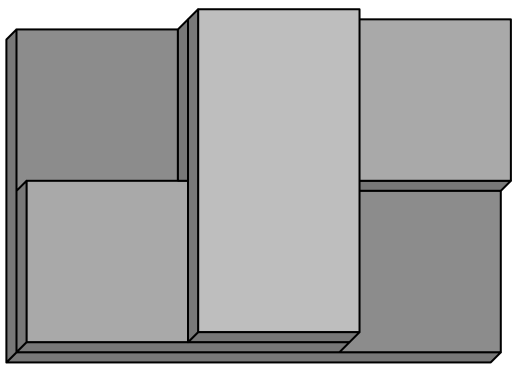
Steps



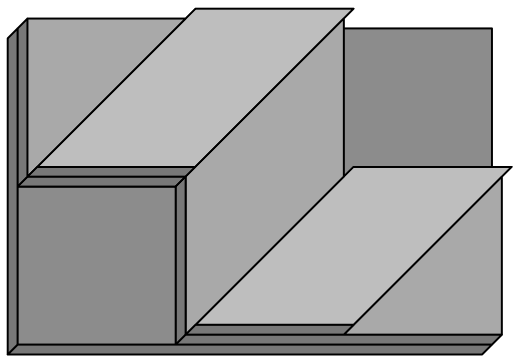
Stacking of squares, I



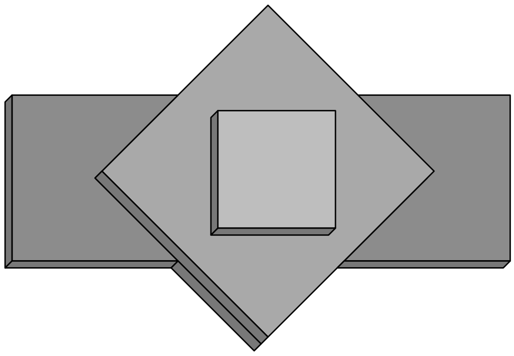
Stacking of squares, II



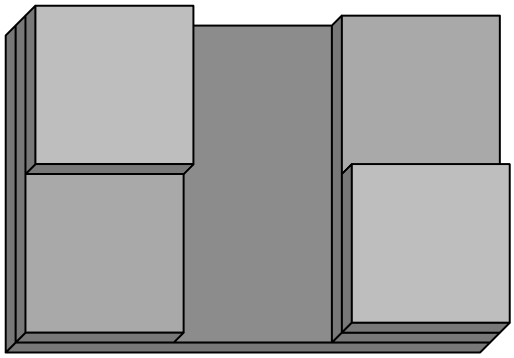
Block with parallelograms



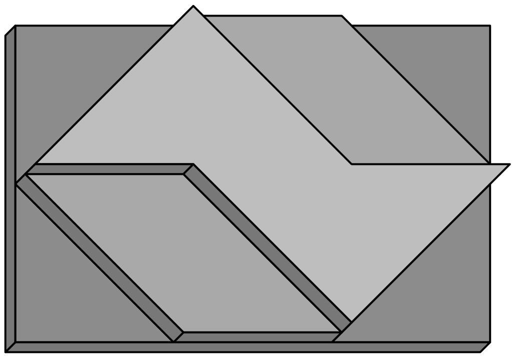
Buckle



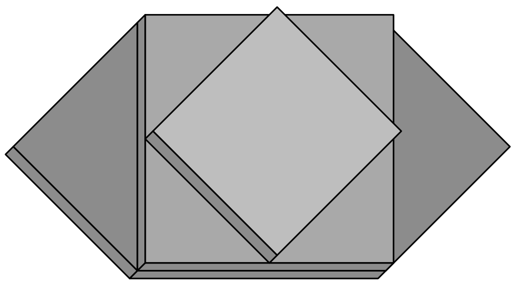
Stacking of squares, III



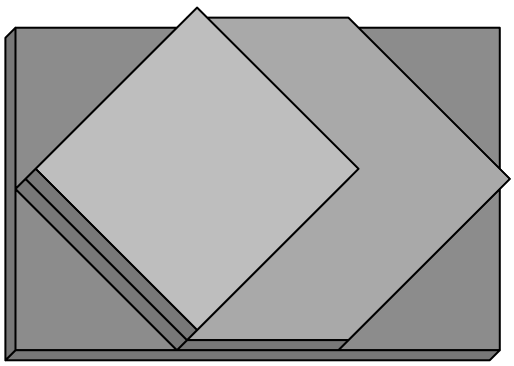
Shapes on block, I



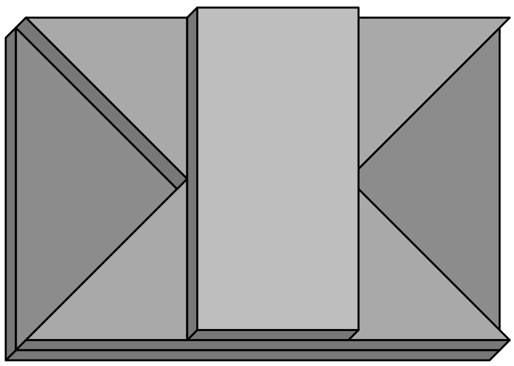
Squares on a hexagon



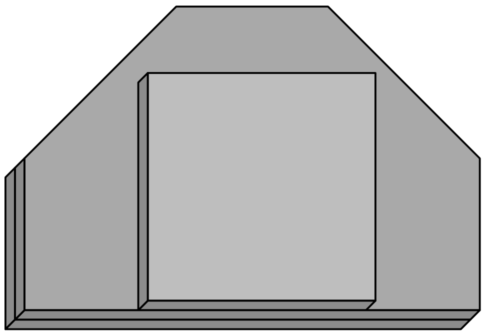
Block with hexagon and square



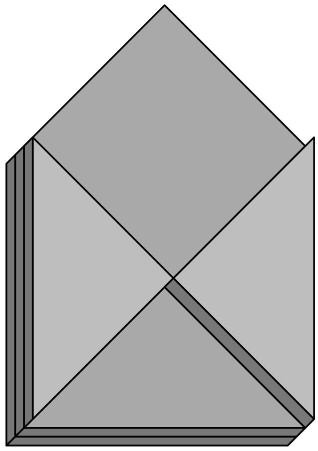
Shapes on block, II



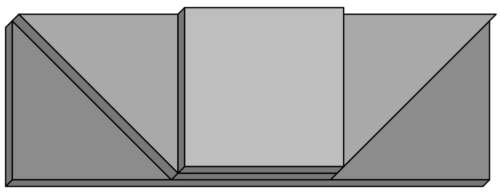
Square on a hexagon



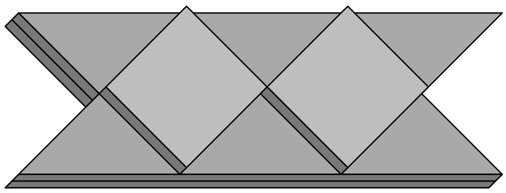
Triangles on a pentagon



Triangular steps

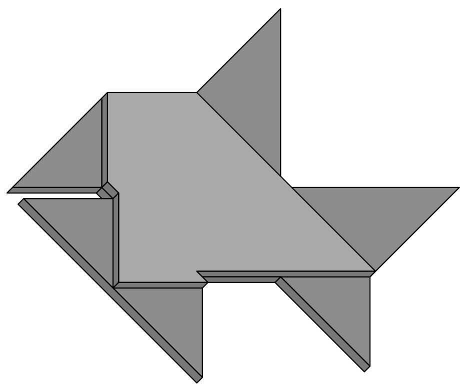


Symmetric shape, II

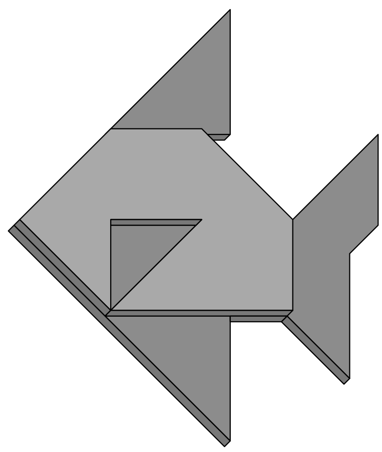


Animals

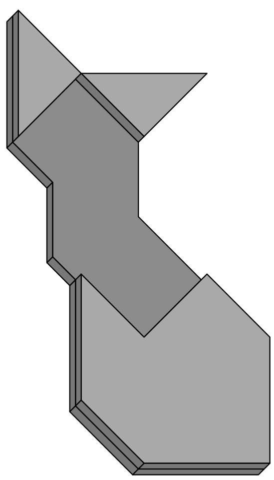
Fish, I



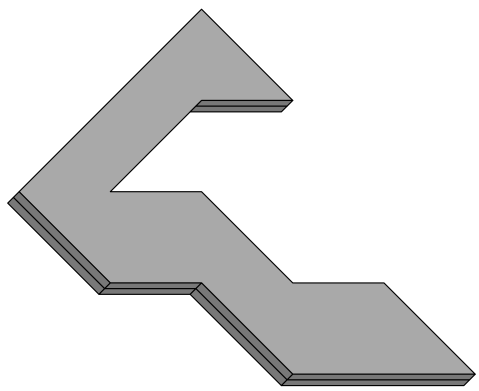
Fish, II



Cat



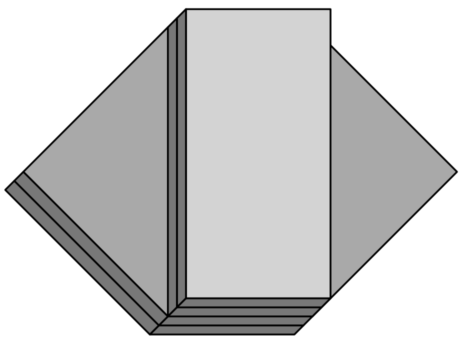
Snake



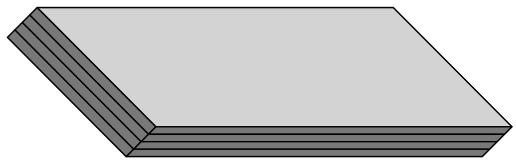
Four high and more

The puzzles in this section are at least four layers high, and sometimes five or six layers. For convenience the puzzles show which heights are used as top surfaces.

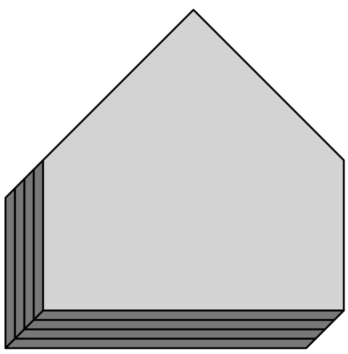
Block on hexagon (2 and 4 high)



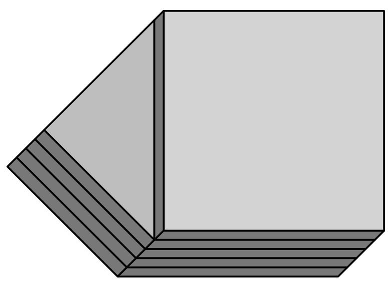
Parallelogram (4 high)



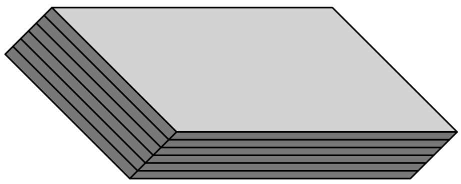
House (4 high)



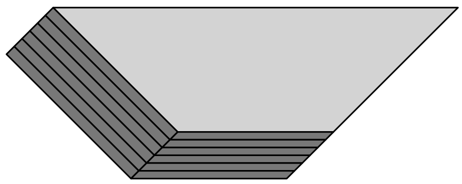
Square on pentagon (4 and 5 high)



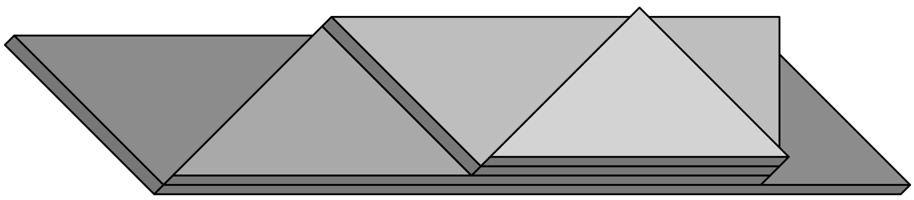
Parallelogram (6 high)



Trapezium (6 high)



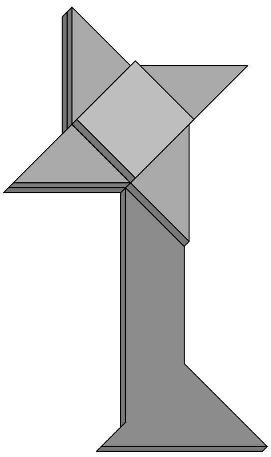
Triangle steps (1-2-3-4-3-1 high)



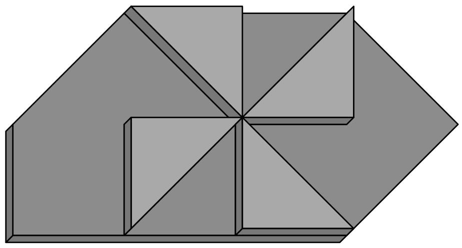
Not all pieces

In the next set of puzzles, not all pieces will be used. You may choose which pieces to exclude from the solution. On the one hand, these puzzles are easier, because the final shape is smaller. On the other hand, it is unknown which pieces are excluded, so you may try to make a solution with a pieces that does not occur in it.

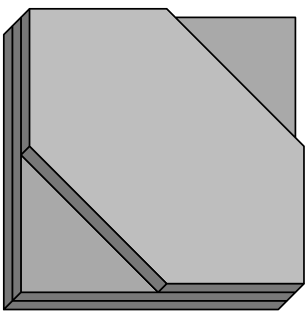
Flower



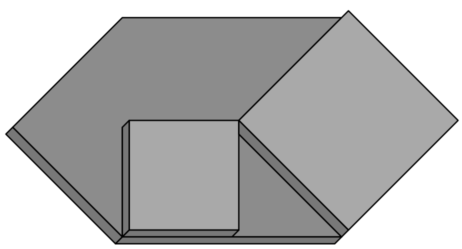
Wind spinner



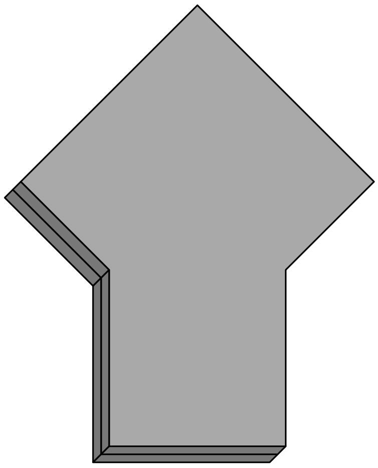
Hexagon on square



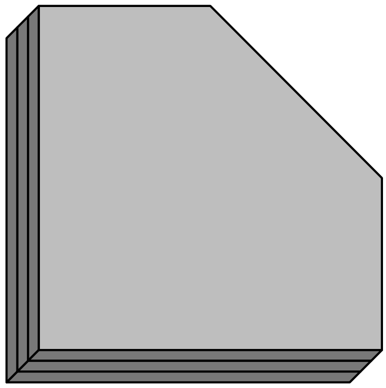
Two squares on a hexagon



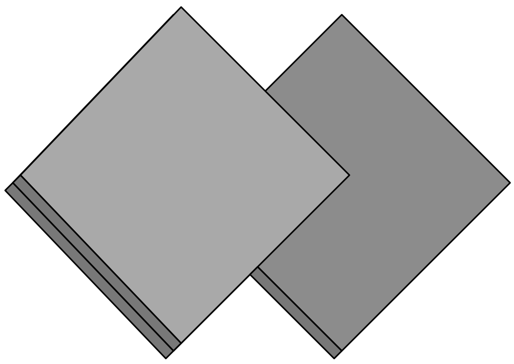
Abstract tree



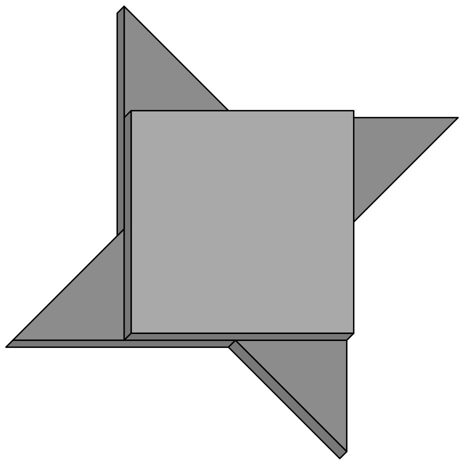
Incomplete square



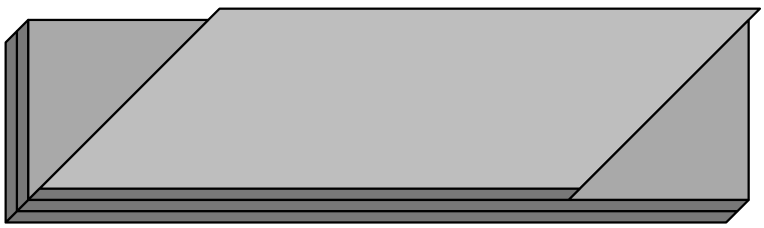
Overlapping squares



Star, II



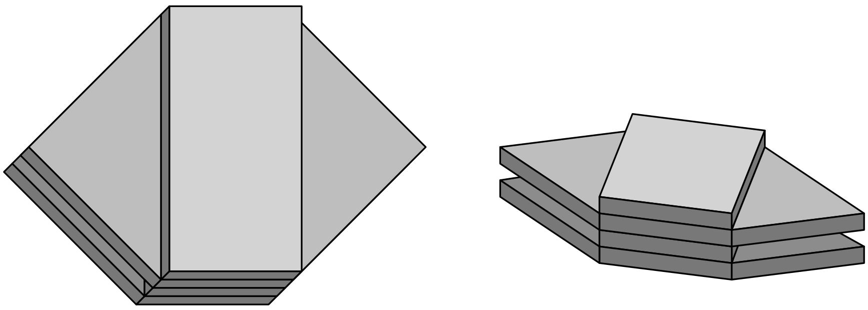
Elongated shape



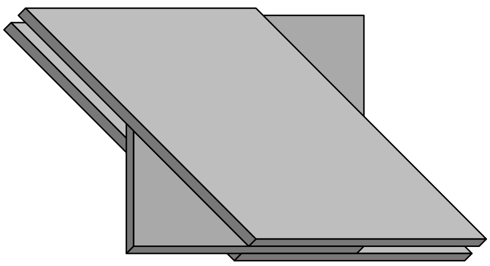
With overhang

Suppose we let go of the requirement that every part of a piece on a higher level must be supported fully on the level below. In other words, suppose we allow overhangs. Then there are possibilities for new puzzles, given in this section. Beware that we are using all pieces again.

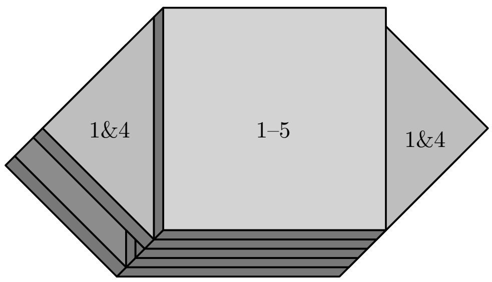
Hexagon-rectangle-hexagon-rectangle (two views of the same puzzle)



Parallelogram-square-parallelogram



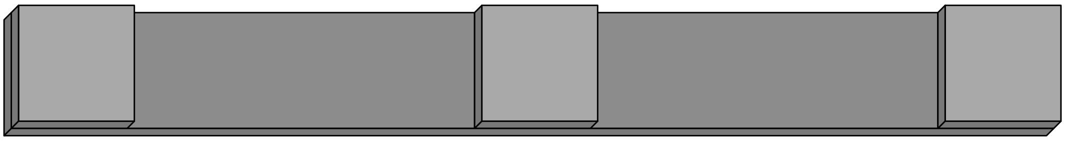
Stacked squares and hexagons



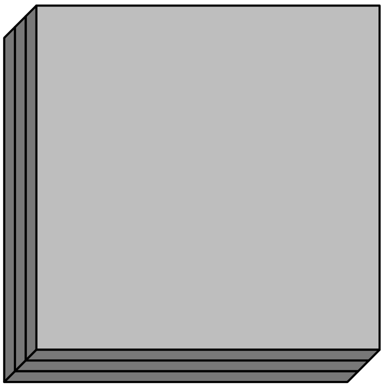
Impossible puzzles

Certain shapes cannot be made, although it seems that these might be possible. One of these is a square of three levels high. We have the right sizes of pieces in principle, but the given shapes of the pieces do not allow making the shape. The solutions section in the back obviously does not contain solutions to these “puzzles”.

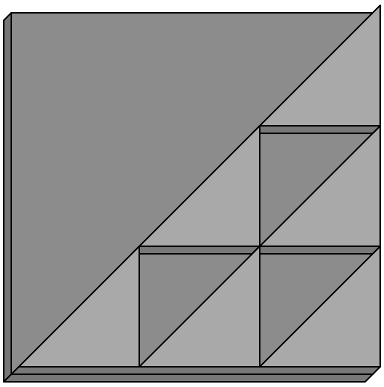
Raised squares, II



3-high square



Triangle pattern on square



Special assignments

1. Largest square perimeter

Make a shape where the outer circumference is a square of the largest possible size.

2. Largest triangular perimeter

Make a shape where the outer circumference is a triangle of the largest possible size.

3. Highest tower

Try to make the highest possible tower, using all pieces. How high can you get? With overhang, you can make the tower 15 layers high, but without overhang?

4. Using a 3-unit base: highest tower

Make a tower that uses a base consisting of three units (for example, the square and the small triangle, or the parallelogram and the small triangle). You do not have to use all pieces, but you cannot use overhang. How high can you get?

5. Smiley face

Using all pieces, what is the best example of a smiley face that you can make?

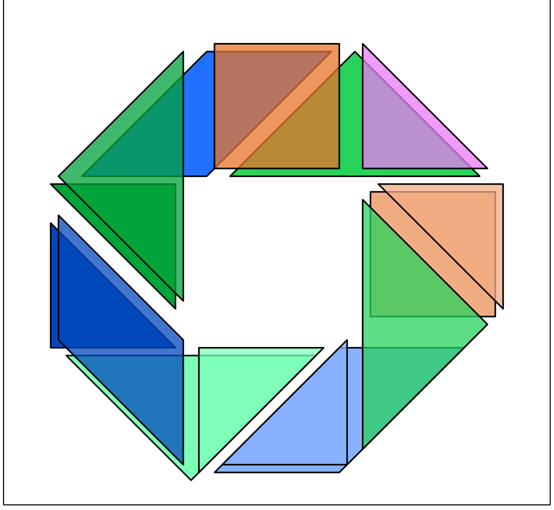
6. Alphabet

We have seen shapes that look like the letters H, O, or S. Using all pieces, can you make each letter of the alphabet in a recognizable and nice way?

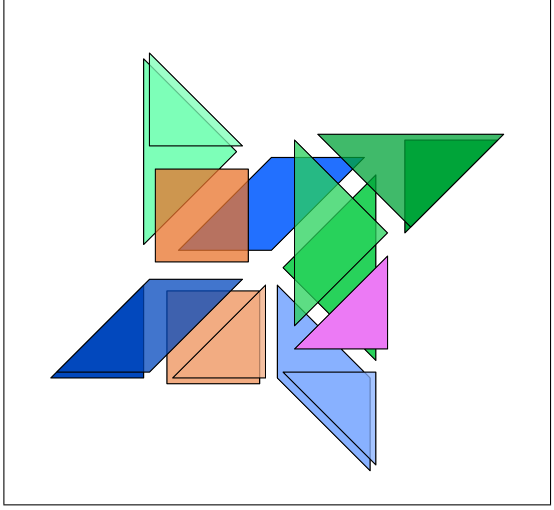
Chapter 4

Solutions

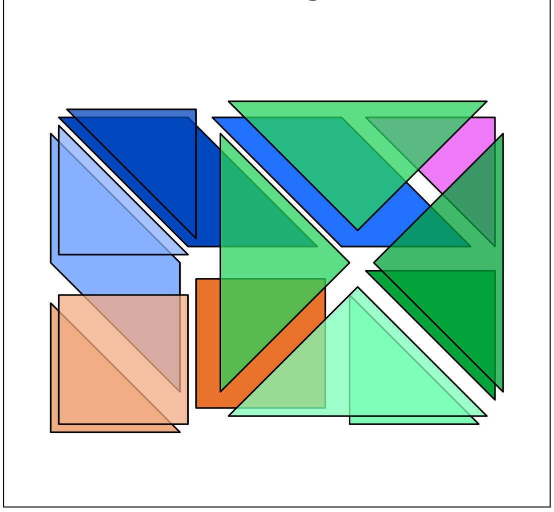
The letter O



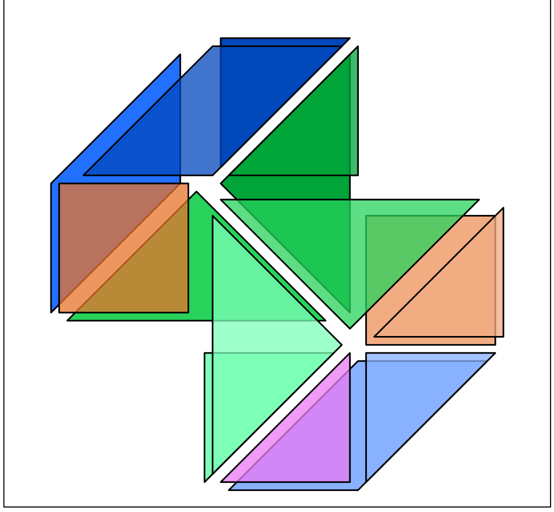
Star



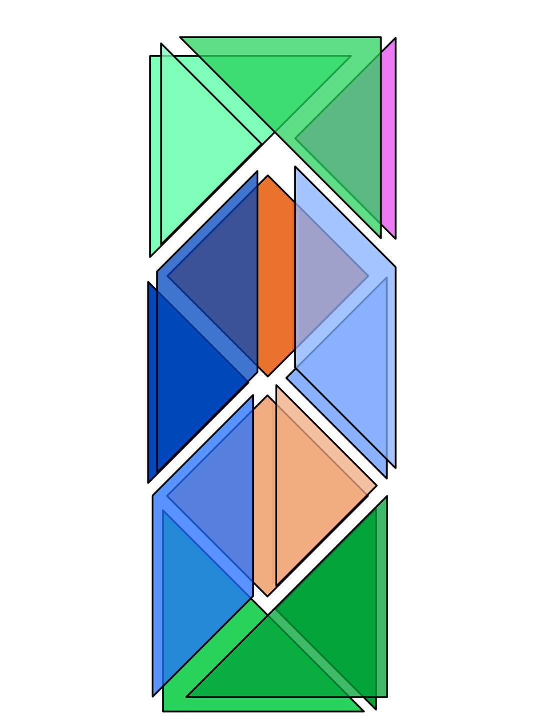
Rectangle



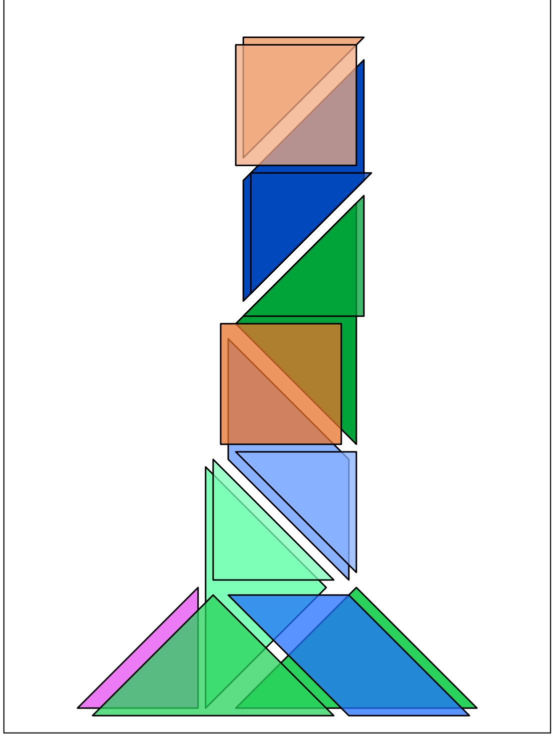
Double vase



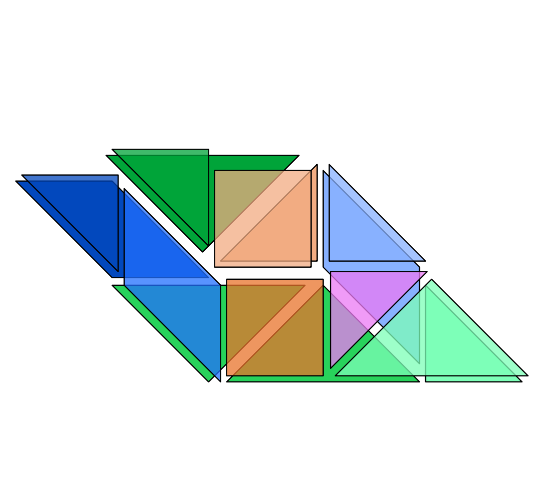
Elongated rectangle



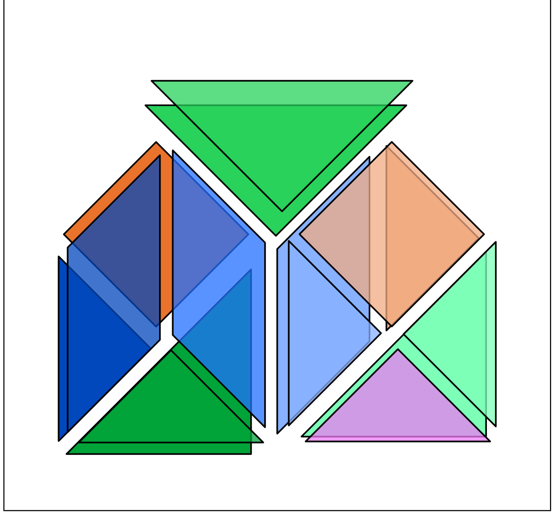
Tall vase



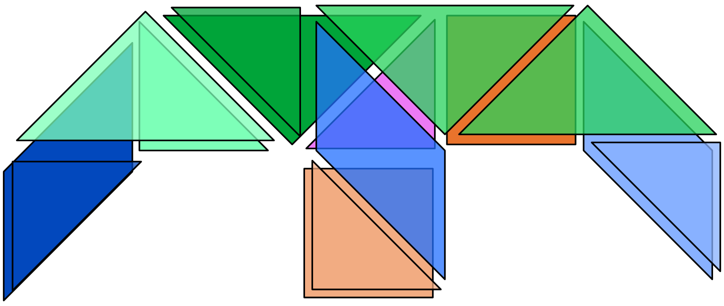
Parallelogram



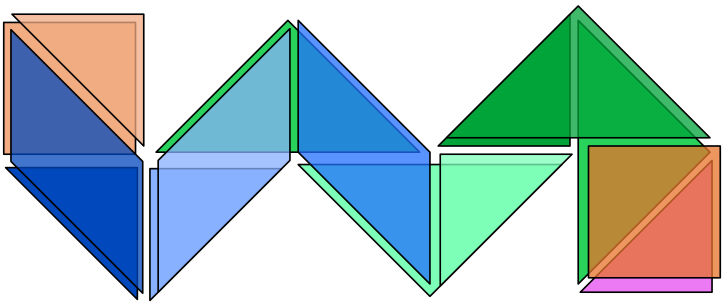
Jar



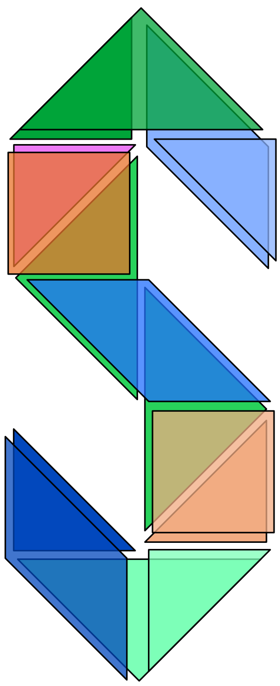
Wide shape



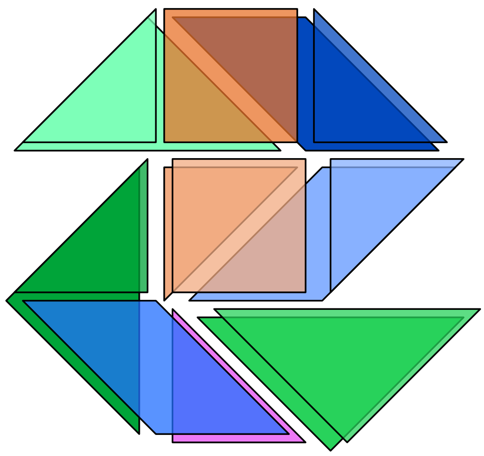
Zig-zag shape



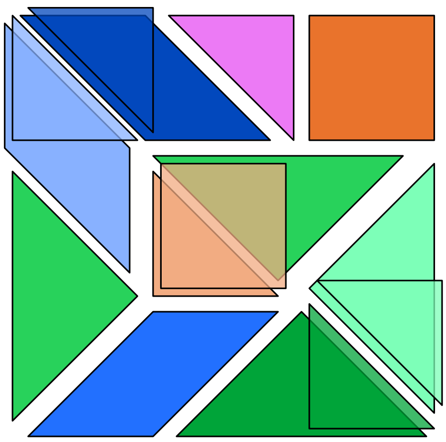
Tall S shape



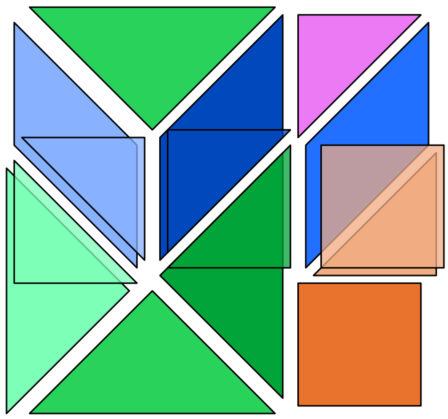
Like an S



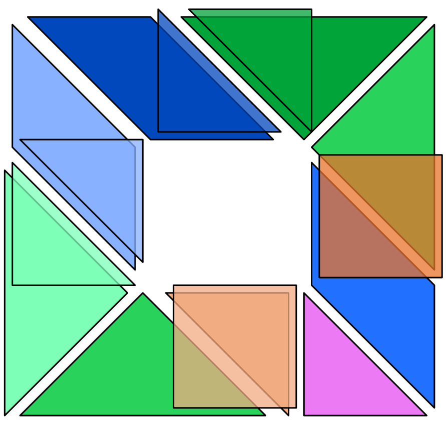
Levels 1 and 2



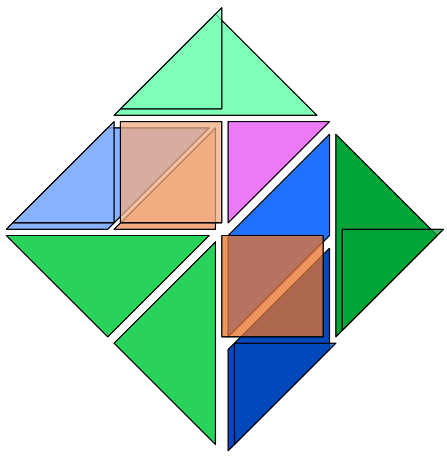
Square with diagonal



Square with line

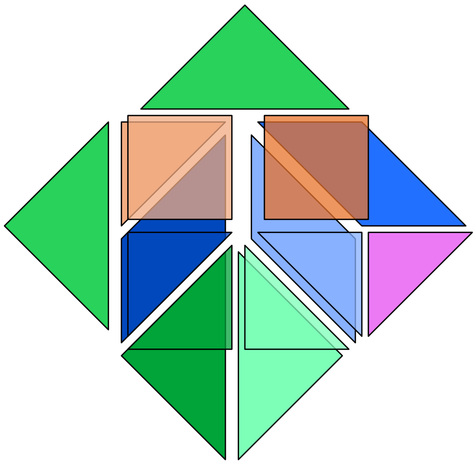


Inverted fortress

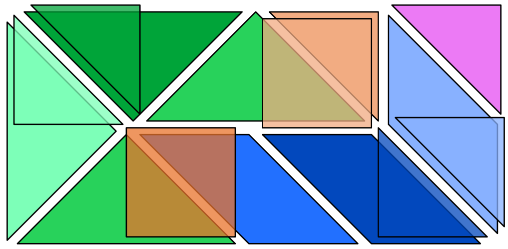


Two triangles on square

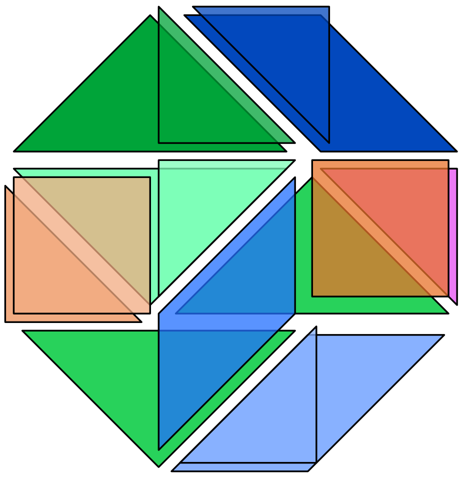
Square in square



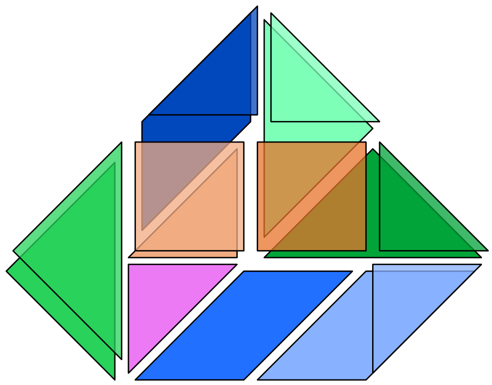
Chessboard fragment



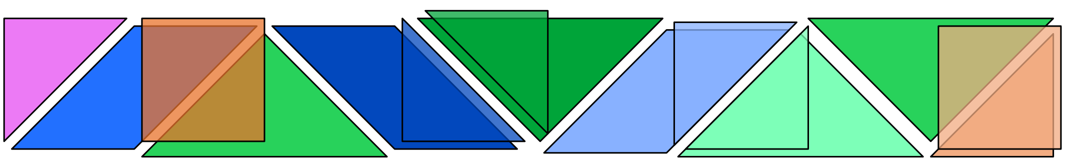
Plus on octagon



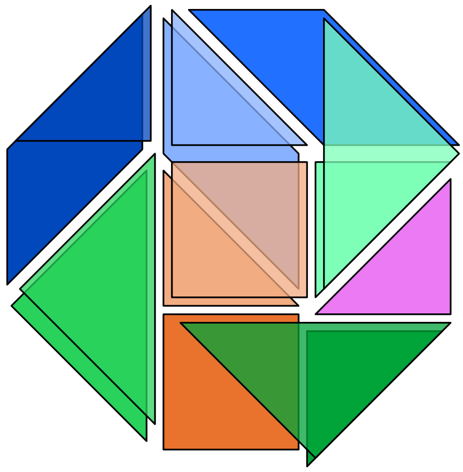
Reduced square



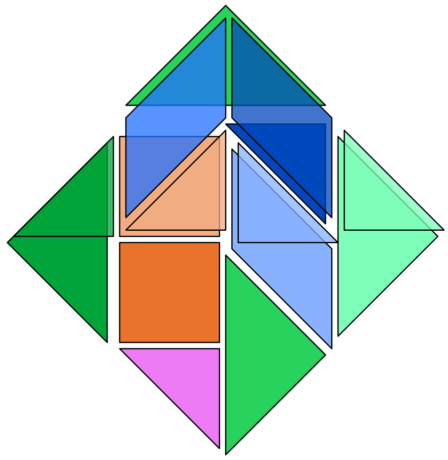
Ramparts



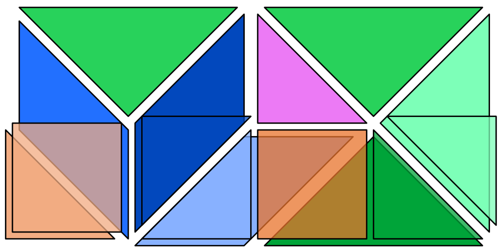
Rotation



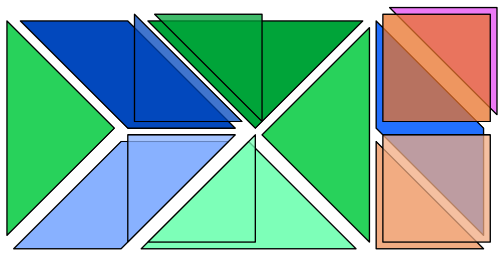
Triangle on square



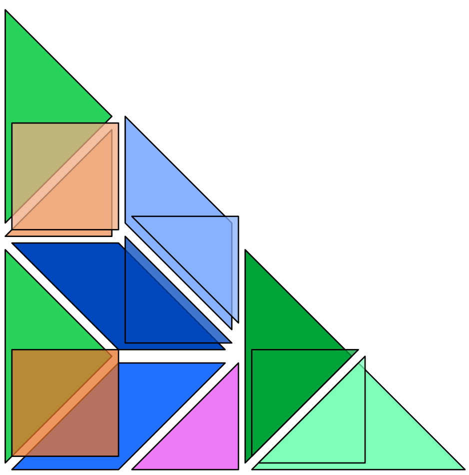
Wide step



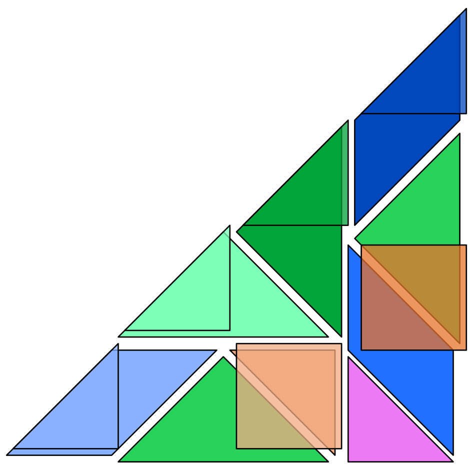
Two steps

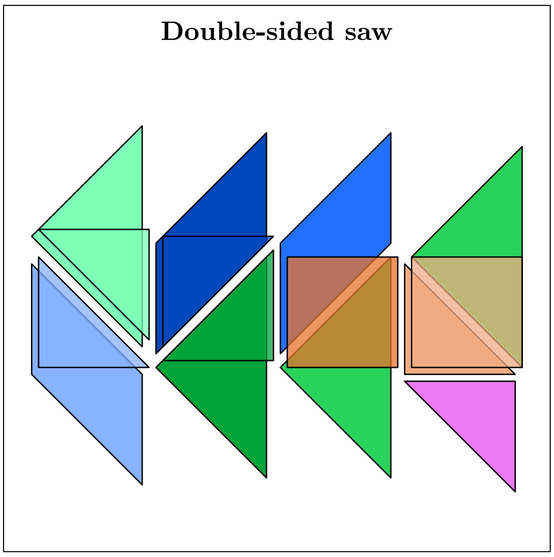


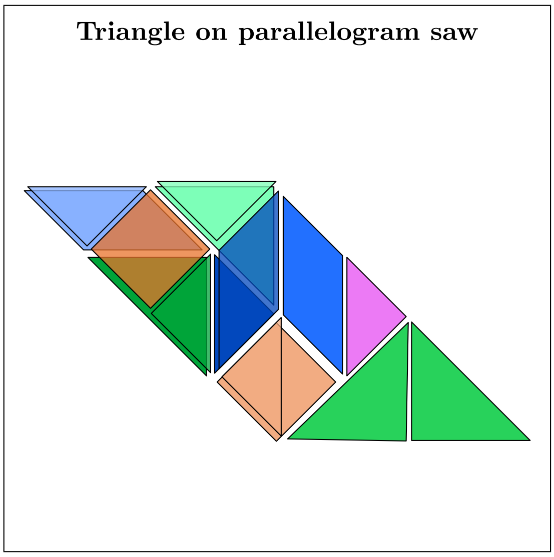
Triangular fragment of a chessboard

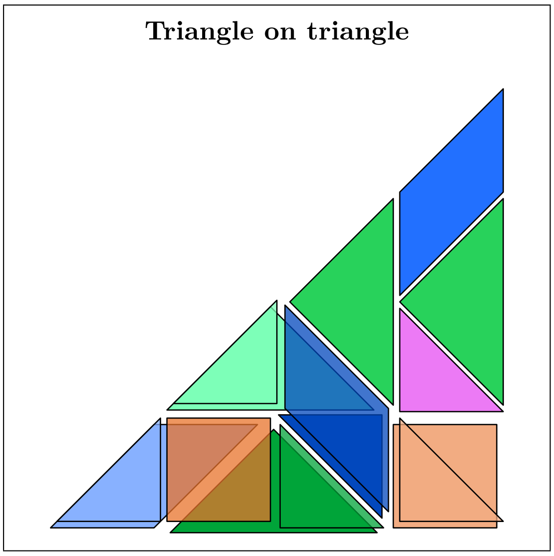


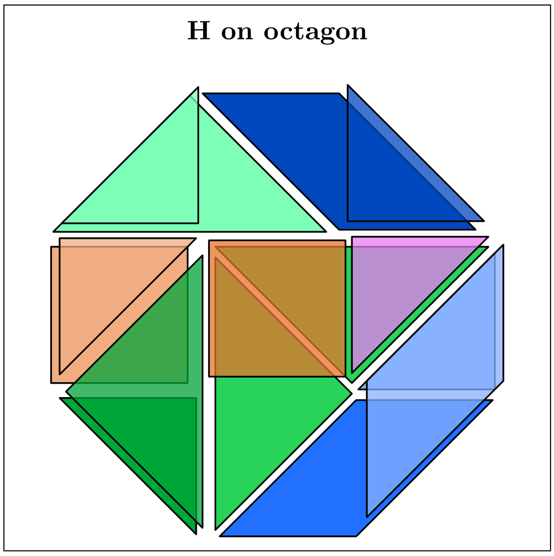
Another fragment of a chessboard

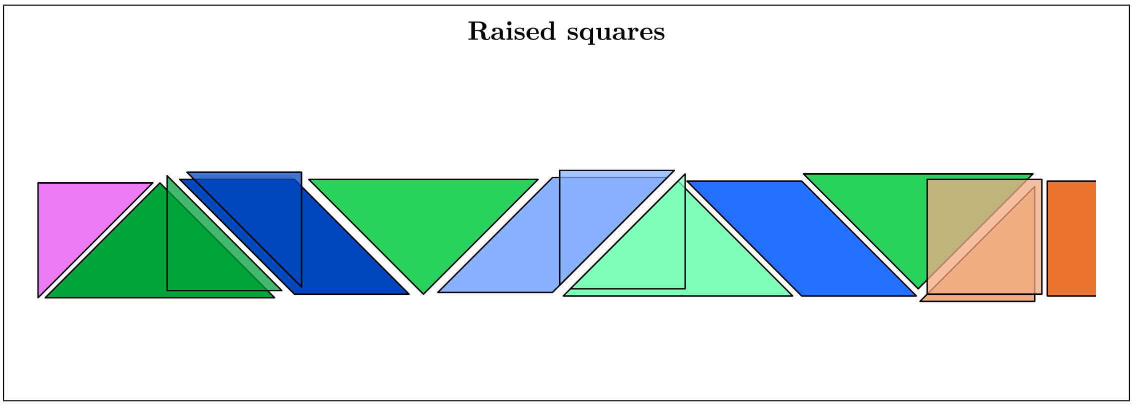



Double-sided saw

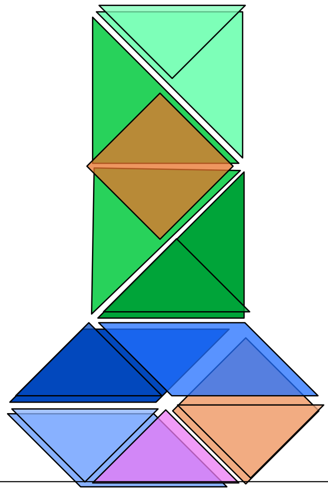

Triangle on parallelogram saw


Triangle on triangle

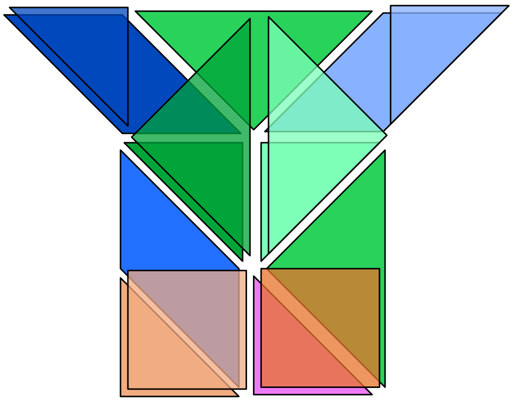

H on octagon


Raised squares

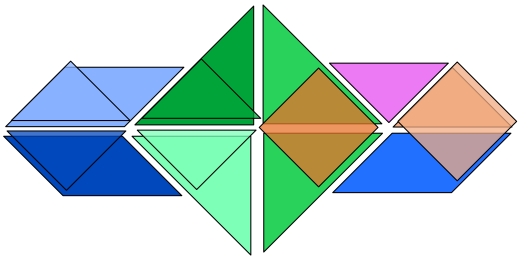
Vase



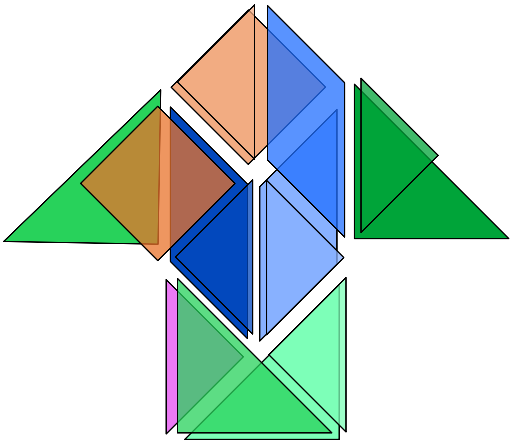
Another vase



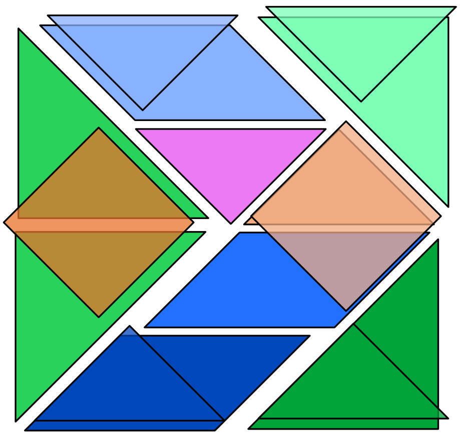
Symmetric shape



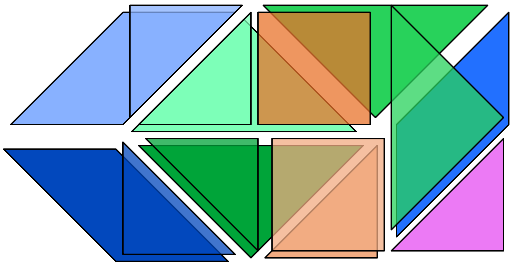
Arrow



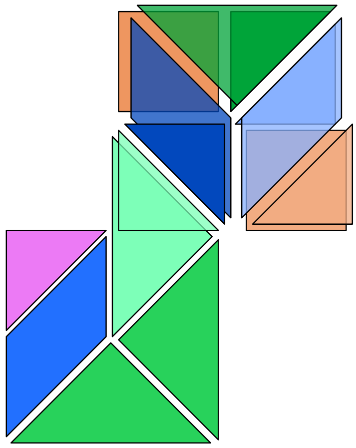
Checkered pattern



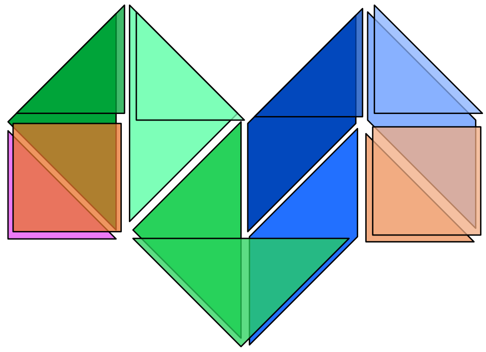
Pentagons



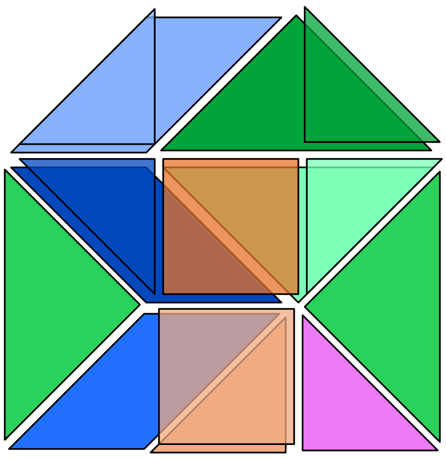
High square, low square



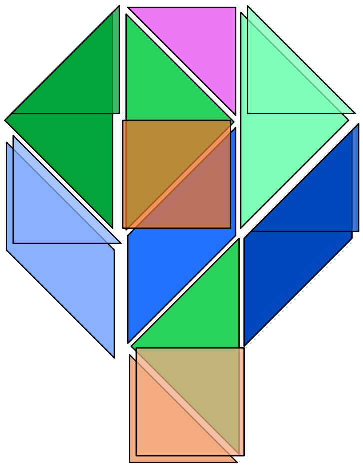
Some shape



Wrench

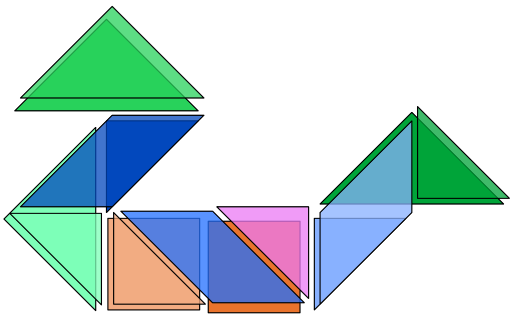


Tree

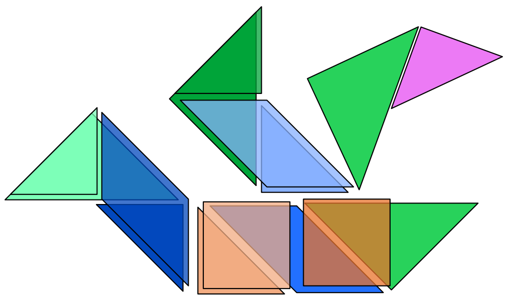


Birds

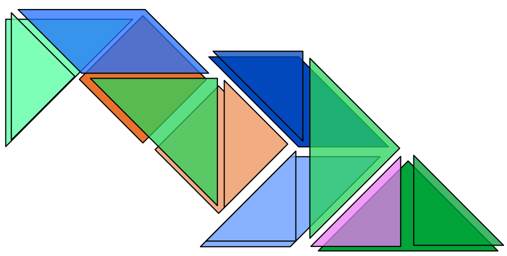
Swimming bird



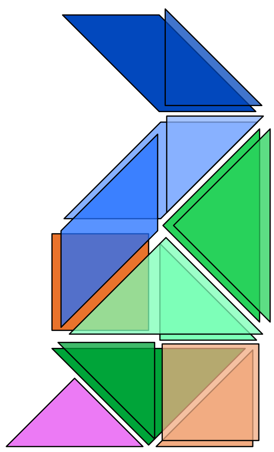
Flapping bird



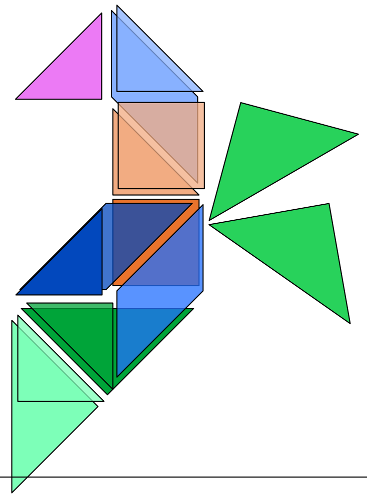
Perching bird



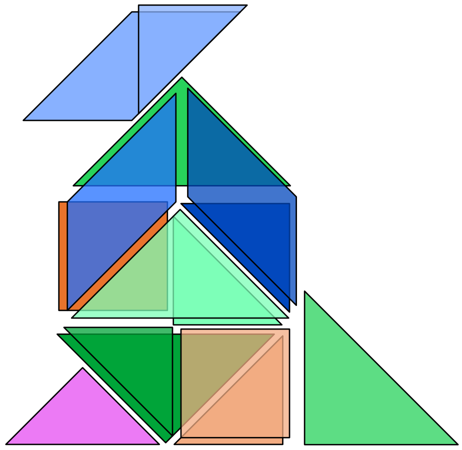
Penguin



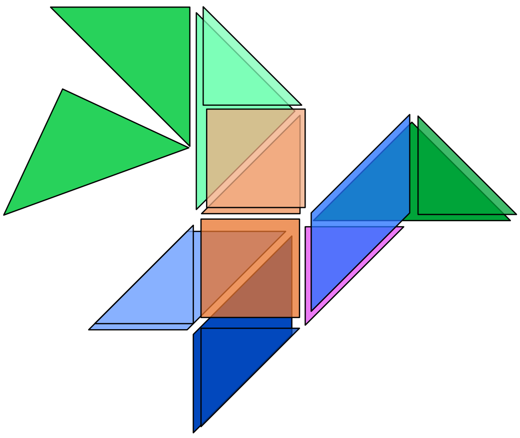
Hummingbird



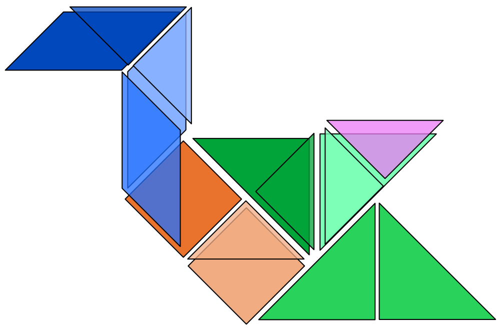
Sitting bird



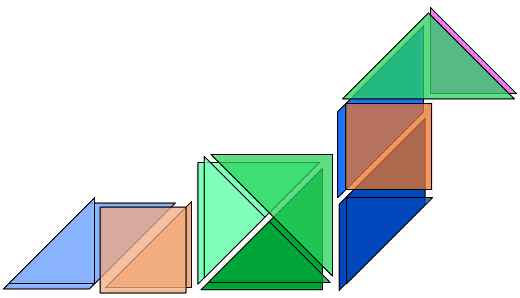
Flying bird



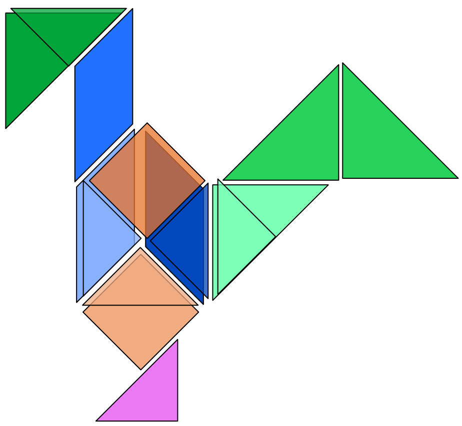
Another swimming bird



Loon

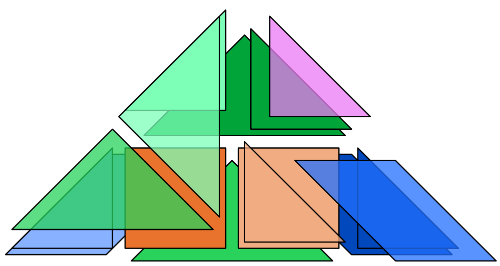


Rooster

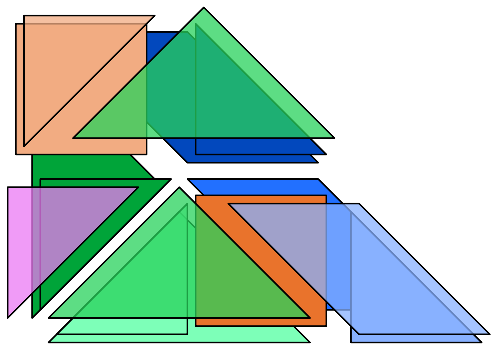


3 high

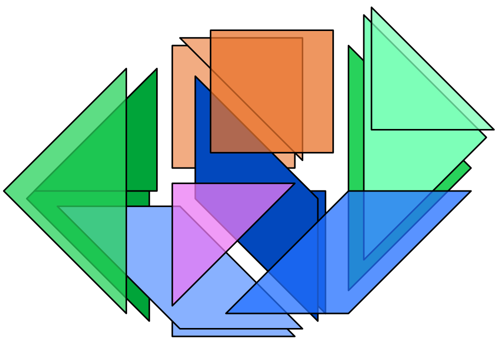
3-high triangle



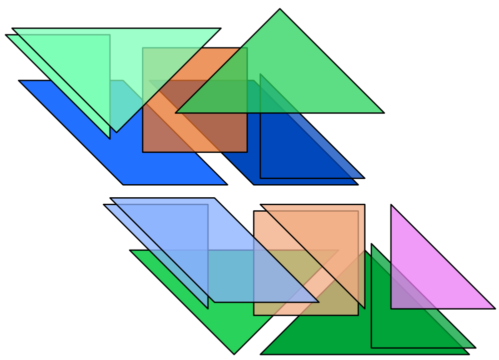
3-high quadrilateral



3-high hexagon

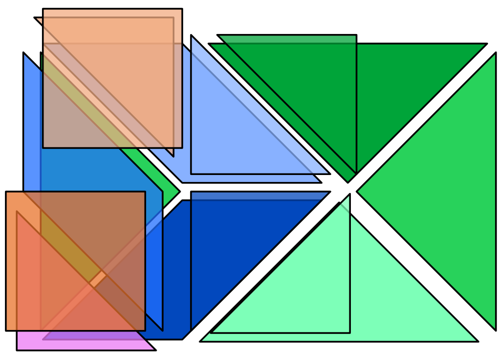


3-high parallelogram

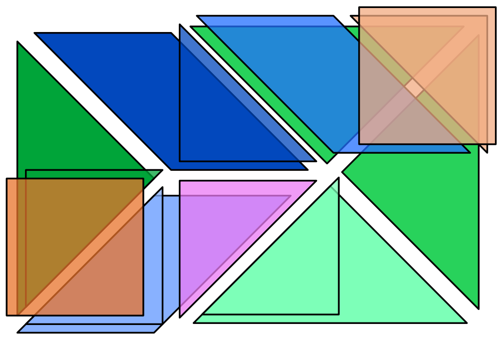


Levels 1, 2, and 3

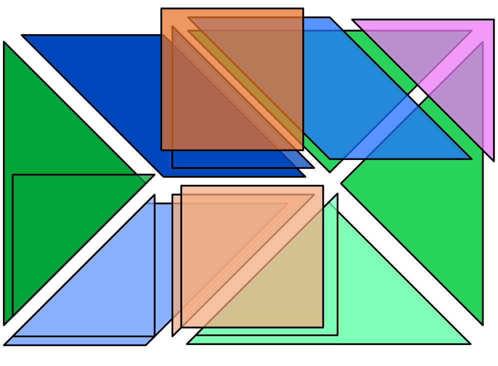
Steps



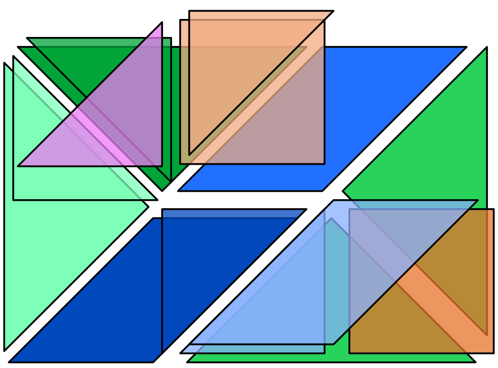
Stacking of squares, I



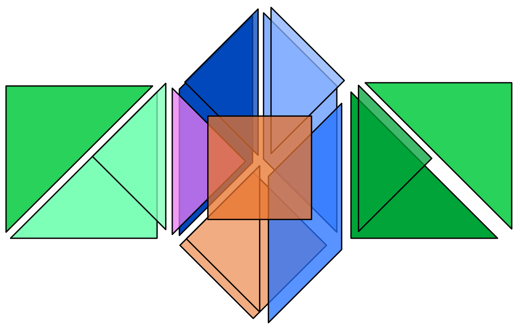
Stacking of squares, II



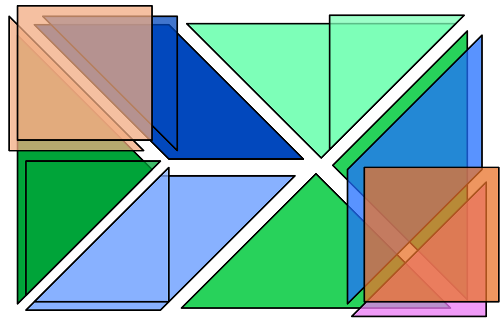
Block with parallelograms



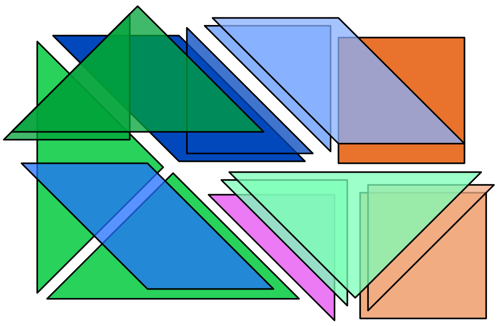
Buckle



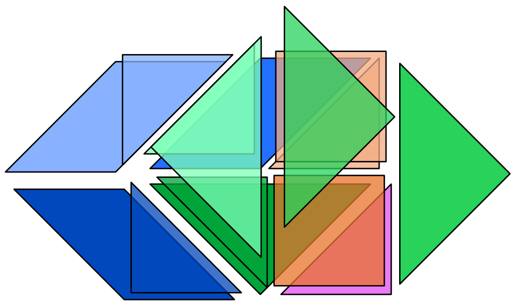
Stacking of squares, III



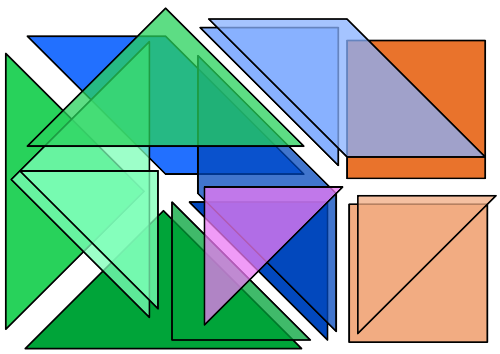
Shapes on block, I



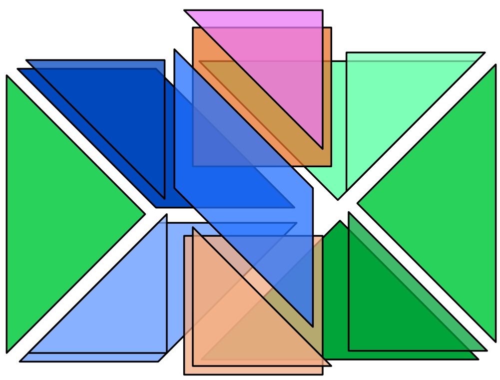
Squares on a hexagon



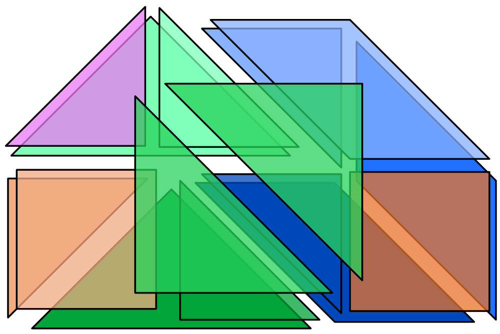
Block with hexagon and square



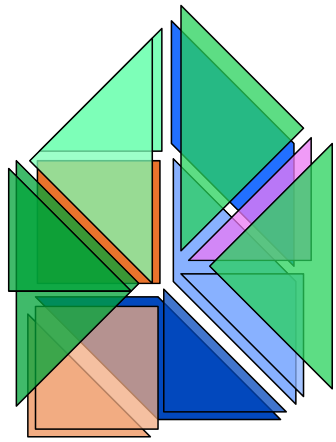
Shapes on block, II



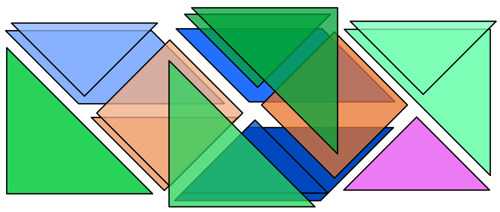
Square on a hexagon



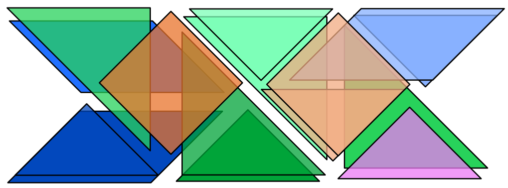
Triangles on a pentagon



Triangular steps

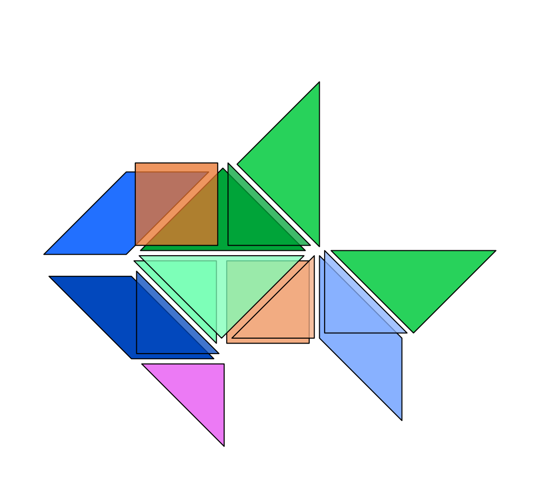


Symmetric shape, II

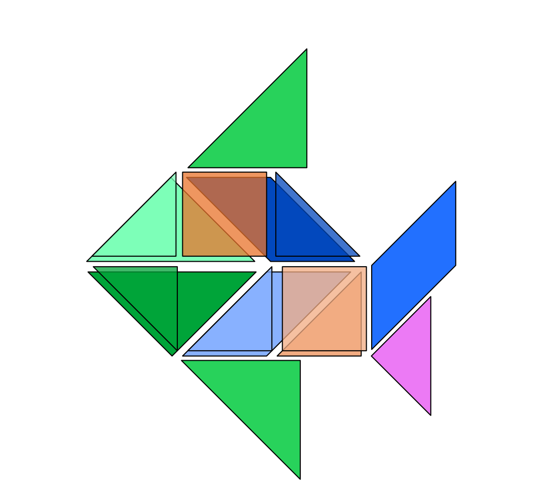


Animals

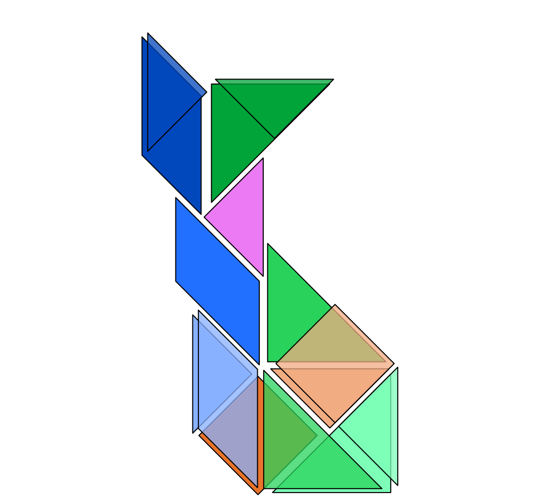
Fish, I



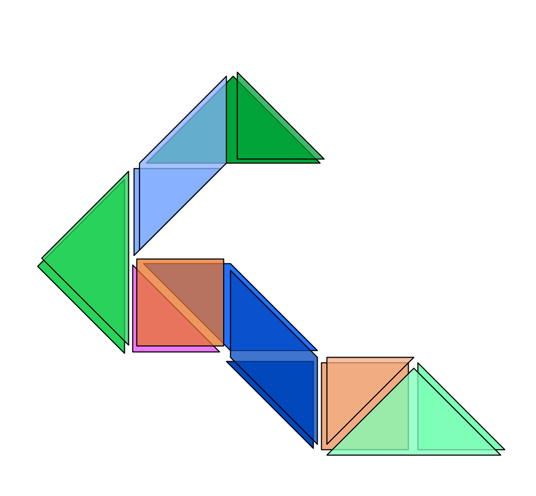
Fish, II



Cat

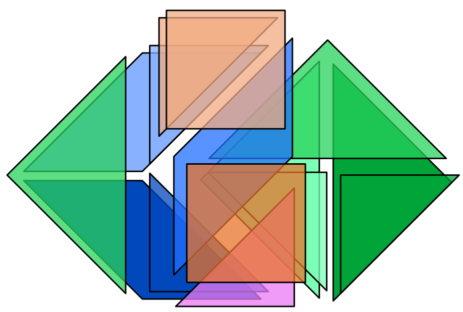


Snake

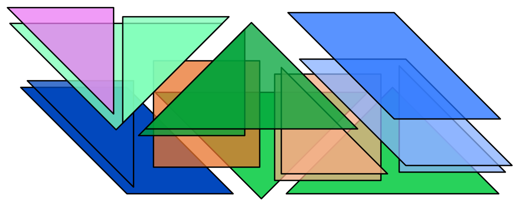


Four high and more

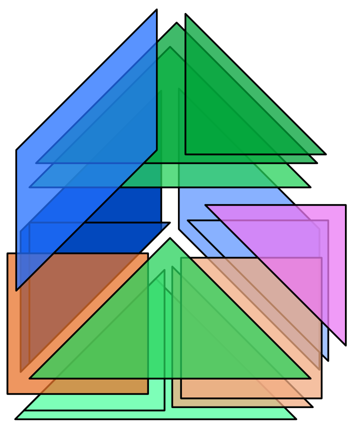
Block on hexagon (2 and 4 high)



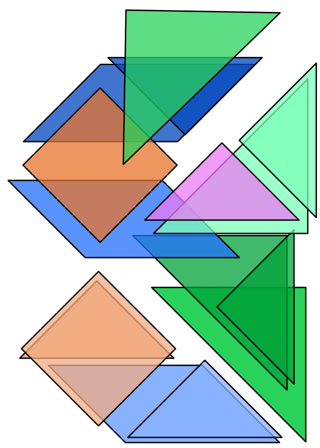
Parallelogram (4 high)



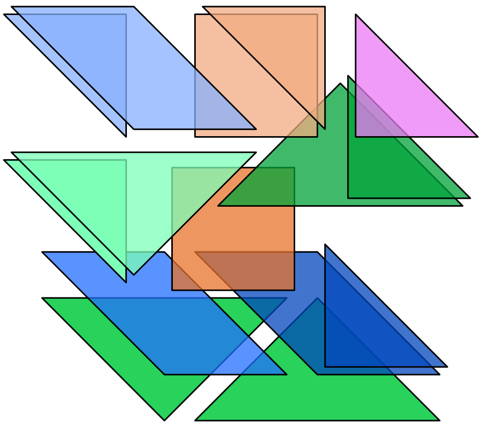
House (4 high)



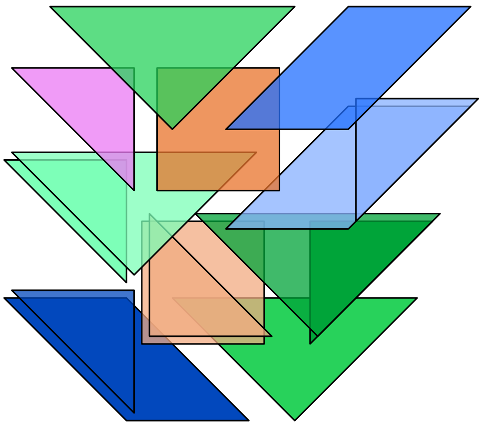
Square on pentagon (4 and 5 high)



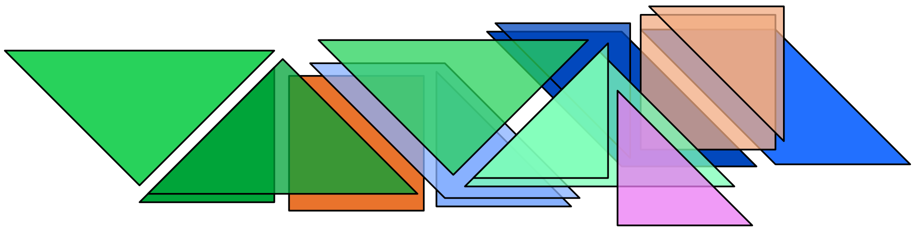
Parallelogram (6 high)



Trapezium (6 high)

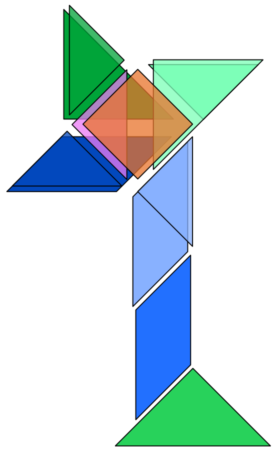


Triangle steps (1-2-3-4-3-1 high)

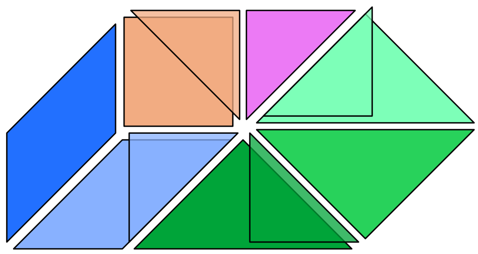


Not all pieces

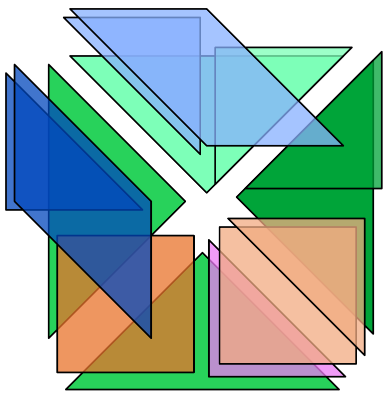
Flower



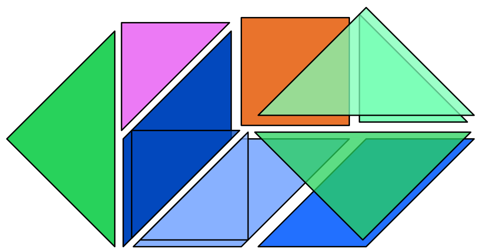
Wind spinner



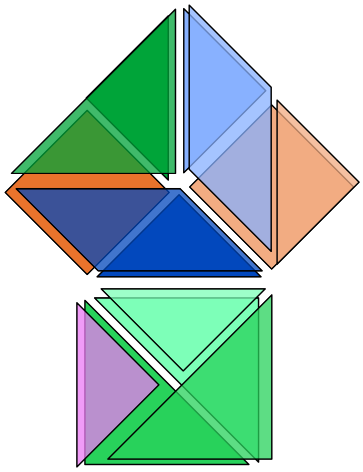
Hexagon on square



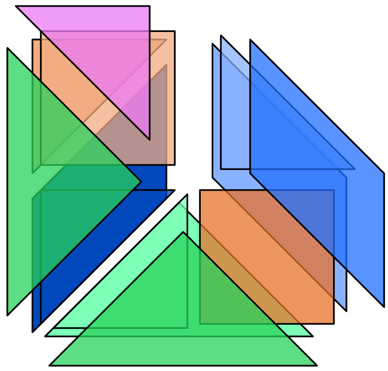
Two squares on a hexagon



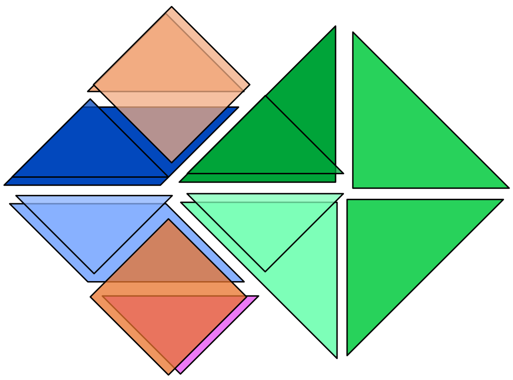
Abstract tree



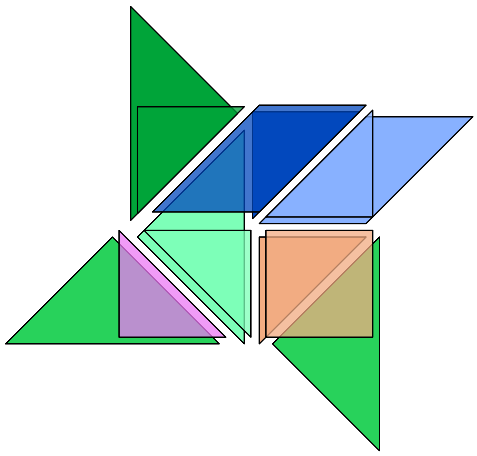
Incomplete square



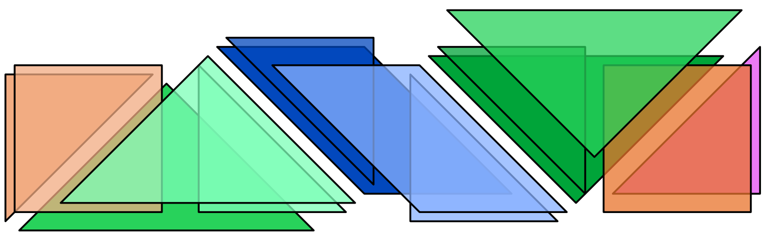
Overlapping squares



Star, II

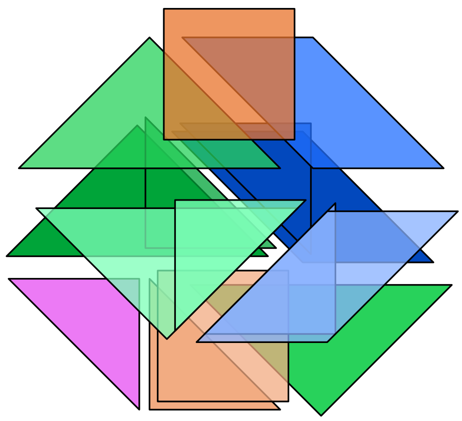


Elongated shape

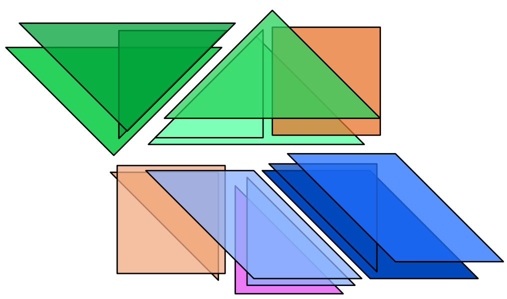


With overhang

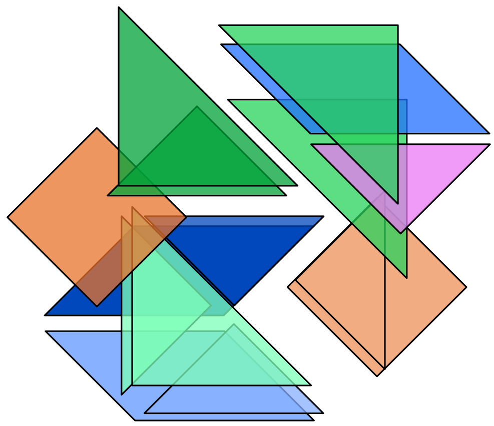
Hexagon-rectangle-hexagon-rectangle



Parallelogram-square-parallelogram

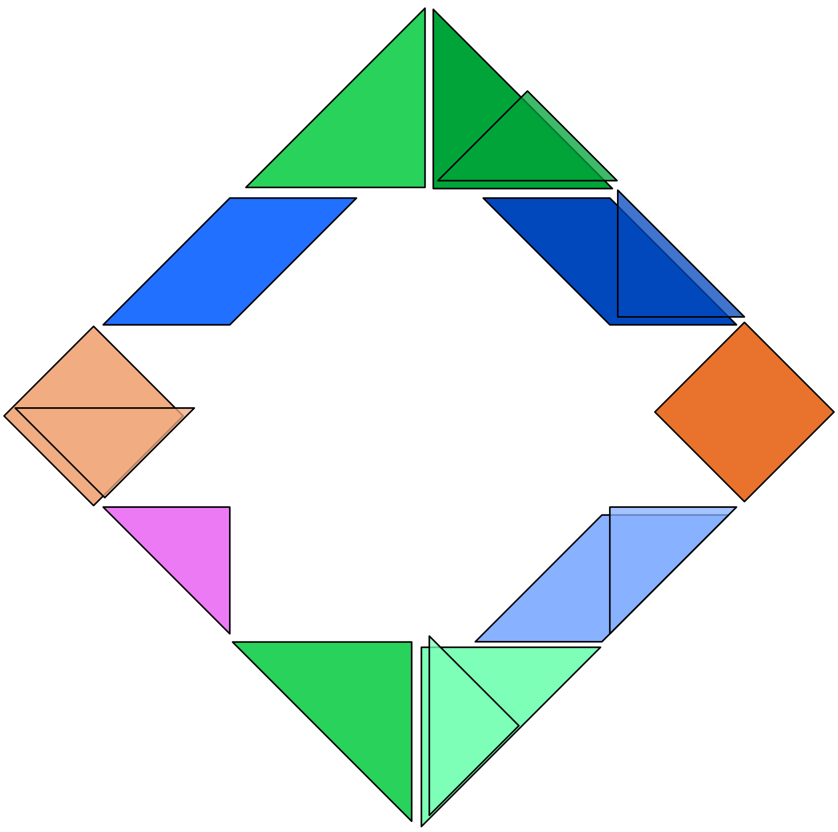


Stacked squares and hexagons

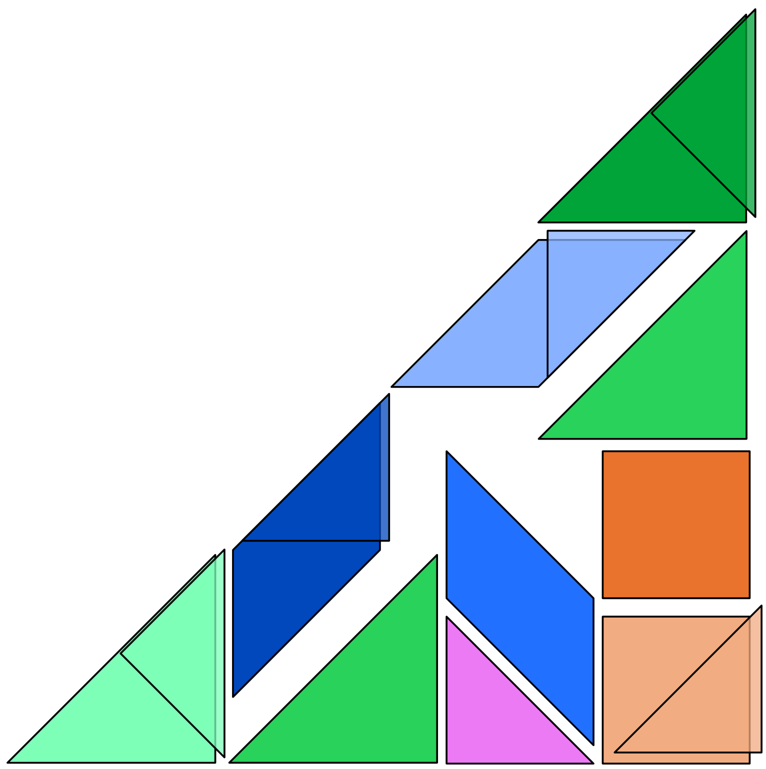


Special assignments

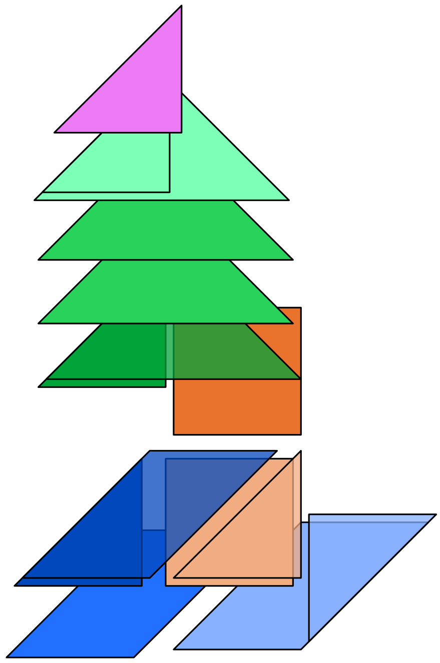
Largest square



Largest triangle



Highest tower



Highest tower base 3