Person Polygons — why not?

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Abstract

A new class of polygons is defined, a class that has been neglected in the past. Some properties of this new class are given, and an inefficient and impractical algorithm is given for the recognition.

1 Introduction

It is a well-known fact that simple polygons come in many different shapes and types. Some classes of polygons are named after a geometric property that defines them, such as convex polygons, weakly externally visible polygons, and x-monotone polygons. Other classes of polygons are named after their shape, such as star-shaped polygons, palm polygons, and clam-shell polygons. A notable exception are art galleries, which are in fact used for all types of simple polygons, so they hardly ever resemble what the name suggests. A good overview of types of simple polygons is given by Toussaint [1].

Although stars, palms and clams are important things in the world around us, it seems that one important subclass of the simple polygons has not received the attention it deserves: *person polygons*. Before we define person polygons, we first need the following:

Definition 1 *A simple polygon is a* limb polygon *if there is a diagonal that partitions it into two convex polygons.*

According to the definition, all convex polygons but triangles are limb polygons. We could add triangles to the class as well, being degenerate limb polygons. Now we are ready to define person polygons:

Definition 2 *A simple polygon is a* person polygon *if it can be partitioned by five non-intersecting diagonals into four limb polygons, a head polygon (which is convex) and a body polygon (which is also convex), such that the body polygon is incident to the five chosen diagonals. (The diagonals may intersect at the endpoints.)*

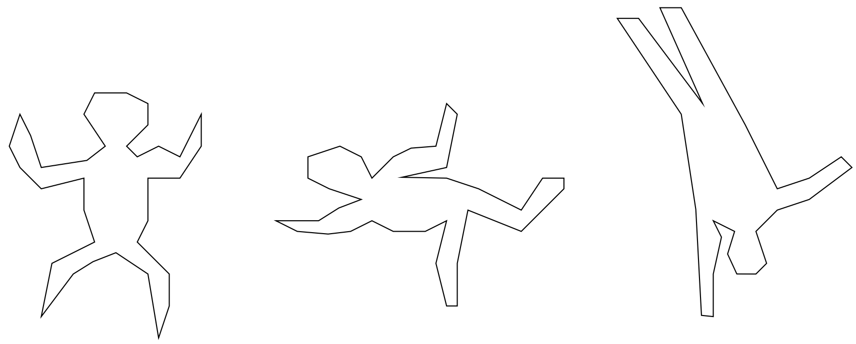

Figure 1: Examples of person polygons.

It is interesting to note that there are several interesting cases of degenerate person polygons. For instance, if only three limb polygons and a head polygon can be formed, we have such a degenerate case. The essential part of a person polygon is the body, which should not be empty. We call degenerate person polygons *incomplete person polygons*. The person polygons that were defined above are *complete person polygons*. The ultimate example of an incomplete person polygon is a triangle. Not only is it missing four limbs, it also doesn't have a head. We call it a *dead person polygon*, therefore. There are other examples of dead person polygons, which can easily be verified. It is interesting to note that the class of non-degenerate limb polygons is a subclass of the class of alive person polygons. Extending the person polygons with incomplete person polygons is important: as any other respectable class of simple polygons, person polygons should at least contain the convex polygons as a subclass.

The rest of this paper is organized as follows. In Section 2 we show some properties of person polygons and we give an algorithm to recognize them. Section 3 gives some open problems and conjectures.

2 Some properties and an algorithm

A brute-force algorithm to recognize person polygons is the following. For every five pairs of vertices of a polygon P, determine if they form five non-intersecting diagonals. If so, test if there is a body polygon incident to all five diagonals. If so, test if it is convex. If so, test if the five other polygons are four limb polygons and a head polygon. If so, the polygon is a person polygon.

This algorithm requires $O(n^{10})$ loops, each taking linear time. (Recognizing limb polygons can be done in linear time; the proof is left as an exercise to the reader.) So the overall algorithm requires $O(n^{11})$ time. Next we will show some properties which allow us to obtain a faster algorithm in some cases.

The following lemma is useful to discard a large subset of simple polygons immediately, when the goal is to determine whether a polygon is a person polygon or not. The lemma is a consequence of the observation that any person polygon can be partitioned into convex polygons using at most nine diagonals.

Lemma 1 *A person polygon has at most 18 reflex vertices.*

From the above lemma we obtain as a corollary that the link diameter of person polygons is at most 19. We can prove something stronger, however. The proof is left to the reader.

Lemma 2 *A person polygon has link diameter at most 5.*

We continue with a linear time algorithm to recognize person polygons with at least 18 reflex vertices, which could be considered the most difficult type of person polygons. However, it seems to be the most easy type. The algorithm is based on Lemma 1, and takes as the input a simple polygon P.

1. Count the number of reflex vertices of P. If there are more than 18, then stop and answer to the negative. If there are less than 18, then stop and answer "don't know".

2. Let R be the set of 18 reflex vertices of P.

3. For every 5 pairs of vertices of P, do the following steps.

4. Test if the pairs are non-intersecting. Also test if the pairs are 5 diagonals of P. If not, try the next 5 pairs.

5. Test if the 6 polygons arising from the 5 diagonals form a convex body polygon incident to all 5 diagonals, a head polygon and 4 limb polygons. If so, then stop and answer to the affirmative.

The above algorithm yields the first main result of this paper:

Theorem 1 *One can determine in linear time whether a simple polygon with at least 18 reflex vertices is a person polygon or not.*

Proof: The first two steps of the algorithm clearly take linear time. Steps 4 and 5 inside the loop take linear time, obviously. It remains to determine how often the loop is taken. One can easily see that this is only

$$\binom{18}{10}\frac{\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}}{5!} = 33,081,048$$

which is constant except in all practical situations. The theorem follows. □

Next we extend the above algorithm to run in time $O(n^{19-r})$ time, where r is the number of reflex vertices of P and n is the number of vertices. The idea is to complete the set R of reflex vertices with other vertices, until it has size 18. Then we can run the previous algorithm. There are $O(n^{17-r})$ ways to complete the set R. So this is an improvement over the brute-force algorithm for all $r \geq 9$.

3 Extensions and conjectures

We have defined an interesting class of simple polygons, the person polygons. We conclude this paper with the conjectures and open problems.

Conjecture 1 *All person polygons can be recognized in $O(n)$ time.*

Conjecture 2 *Person polygons are not a subclass, nor a superclass, for any of the classes of simple polygons given by Toussaint [1].*

Interesting open problems are the following.

1. Extend the definition of person polygons to include ears, hands, fingers, a nose, and so on.
2. Generalize the results to three dimensions. It clearly is more natural to study person polyhedra than person polygons.

The most challenging open problem in this area is to partition the class of person polygons into the *male person polygons* and the *female person polygons*. This open problem is one of the very few which may be easier to solve in 3-dimensional space than in the plane.

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References

[1] G.T. Toussaint, *A Hierarchy of Simple Polygons,* manuscript in preparation.

[2] G.T. Toussaint, Antropomorphic Polygons, *American Mathematical Monthly,* 1991, pages 31–35.