Some Tetraform Puzzles

by Marc van Kreveld

In this article I will describe some of my new puzzle designs. Most of them have pieces that are made of four units of some sort. First I will describe some two-layer puzzles using 8 pieces whose base is an L-tetromino or I-tetromino, and whose solution is a solid cube. Then I will describe a tetrasector puzzle, which consists of the 13 different cylindrical tetrominoes. Third and last, I will describe a puzzle based on weighted tetrominoes, where balance plays a role.

Two-layer cube puzzles

Based on wood of thickness 15 x 15 mm, saw 16 pieces of 60 mm (plus a little for the thickness of a saw cut). Saw these 16 rods into two equal pieces using a slanted cut, for example under 45 degrees. Take groups of four, glue them together so that the square bases are next to each other and form a 15 x 60 mm rectangle. The slanted tops stick out in the same direction; they can be in one of four rotations. With the proper design, you get 8 pieces that are a cube puzzle. If you glue them differently, so that the square bases form an L-tetromino instead of an I-tetromino, you can also make such a puzzle. An example of the cube and the pieces is shown in Figure 1.

Figure 1: The solution and the eight pieces of a two-layer L-shape slanted puzzle (tetraroofs)

To solve the puzzle you must make two layers of four pieces that form a square, but each with 16 slanted tops. If you take the right groups of 4 and arrange them suitably in a square, the slanted tops of the two squares will match, and a cube appears. Hence the name two-layered puzzle. The puzzle shown in the figure is not easy. Theo Geerinck suggested that this puzzle should be called tetraroofs, a nice name.

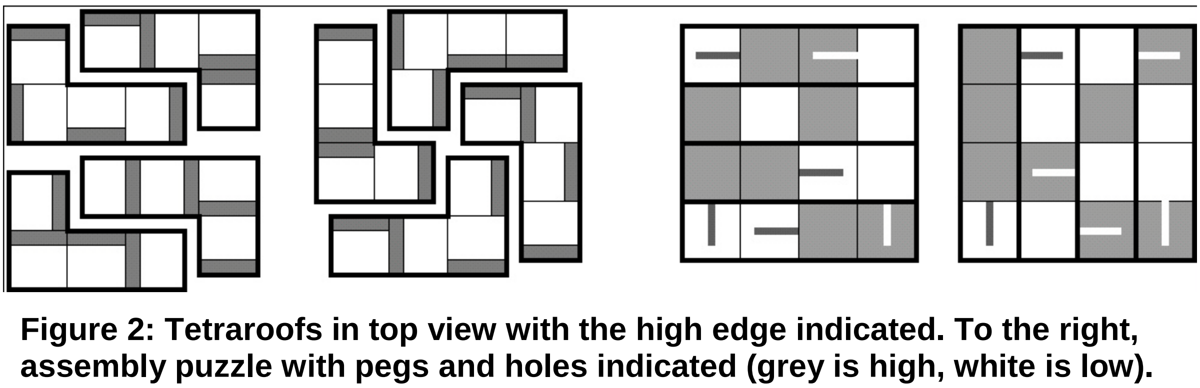

Figure 2: Tetraroofs in top view with the high edge indicated. To the right, assembly puzzle with pegs and holes indicated (grey is high, white is low).

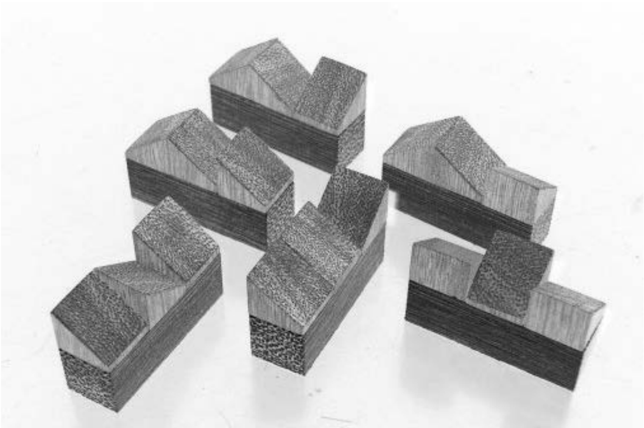

Figure 3: Triroof puzzle with houses

Even the much smaller version made from 9 rods, giving 6 I-shaped pieces, is not trivial. Figure 3 shows an example (triroofs), where the idea was to make the pieces look like houses with slanted roofs (the roofs are made of purple wood).

One can also replace the slanted tops by straight ones, but using rods with different lengths, for example 15 mm, 30 mm, and 45 mm. A short rod with length 15 mm in the bottom layer should them match with a long rod of 45 mm, and rods of length 30 mm in the top and bottom layers match with each other. Figure 4 shows this puzzle.

Figure 4: Two-layer puzzle with elements of three different heights

With the above puzzles, the only task is to make it all fit. There is no sequencing problem. But adaptations to make them into puzzles with a solution sequence are possible. To this end I used the last type of two-layer puzzle, but only used the lengths of 15 mm and 45 mm. Then I added pegs and holes at certain places, see Figures 2 and 5. The result is a nice puzzle with a solution sequence which is not hard to solve, as the assembly sequence is not unique.

Figure 5: Two-layer puzzle with assembly sequence

Tetrasectors

Normal tetrominoes lie flat in the plane. The idea of tetrasectors, or cylindrical tetrominoes, is to construct them of pie pieces. In my version, I used 1/6 of a disc as the unit. Since tetrasectors cannot be turned 90 degrees, there are two different I-shapes: one is a stack of four pie pieces, and the other is 2/3 of a pie. Similarly, there are four L-shapes, four Z-shapes, two T-shapes, and one O-shape (square). In short, there are 13 different tetrasectors, with a total of 52 units of 1/6 of a disc. Unfortunately, 52 is not a multiple of 6. A puzzle to form a cylinder out of the pieces must have at least 9 levels, in which case two gaps of 1/6 remain. For a nicely fitting puzzle, I used a full disc (grey) as the bottom with a stick of suitable length (10 times the disk thickness) going up. The elementary pie pieces all miss this center part and therefore can be placed against the center stick. The top piece is a full disc (grey) with a hole, which should be the last piece on the stick. The tetrasectors themselves are painted red to make clear that the two grey pieces are not part of the set, but rather have the role of the “box”. For the remaining 2 times 1/6 pie piece, one could use two monosectors, or one (of the two!) dosectors. We could say that one dosector is stacked and the other one flat. It appears that the flat dosector cannot be used, because the puzzle would be unsolvable. Using two monosectors or the other dosector makes the puzzle easier, so I decided to make the one dosector (stacked) to be part of the base piece. Figure 6 shows the puzzle.

Figure 6: Thirteen tetrasectors

Why can’t the flat dosector be used? The argument is a parity argument, similar to the classical argument why a chessboard of 8x8, where two opposite corners are missing, cannot be covered with 31 dominoes (each domino covers two adjacent squares, so one black and one white on a chessboard, but two opposite squares of the board have the same color, so the remaining board has 32 squares of one color and 30 of the other….). Let’s look at the 13 tetrasectors, and how many pie pieces they have on even and odd numbered levels. The Z-shaped pieces all have a neutral count (equal occupation of odd and even levels), the vertical I-shape and the O-shape as well, the horizontal I-shape gives a +4 to the even or the odd levels, and the four L-shapes all have a count of +2. One T-shape is neutral and the other gives +2. Since in the puzzle there are 9 levels, 5 odd-numbered and 4 even-numbered, there are 30 positions on odd levels and 24 on even levels. If we take away two positions from an odd level (by placing the flat dosector there), the difference between odd and even positions is 4. But the counts of the pieces does not allow a difference of 4, only of 2, 6, 10, or 14. If we take away two positions from an even level the same problem occurs. This is why the flat domino cannot be used. The stacked domino uses one position of an even level and one position of an odd level, so it preserves the count difference of 6 between the odd and even levels.

Tetrominoes with weights

The last puzzle described here involves weights. It consists of 7 tetrominoes and 2 dominoes. These 9 pieces must form two squares of 4x4 which are placed on opposite sides of a scale. If the scale is in balance, the solution is correct, otherwise it is not. So the problem is not only to form two squares, that would be easy, you also have to take care that the weight is well-distributed: several pieces have extra weights added to them. Figure 7 shows the pieces and the scale. Construction is not easy, because one has to make sure that correct weight distribution gives balance, but incorrect weight distribution must tip the scale. To this end, the scale must have a narrow, flat bottom of exactly the right position and width. My (second) attempt is reasonable at best. I used wood from a rod of 44 x 44 mm, and bolts of 12 mm thickness.

Figure 7: Pieces with holes, bolts, and nuts, two of which are on the scale

To solve the puzzle, the first idea may be to use equally many weights on both sides. Since the two T-shapes are clearly heaviest, puzzlers tend to put them on different sides. But no matter how they try, they cannot make a 4x4 square any more. The parity principle appears again, in its chessboard form. The T-shapes are the only shapes that cover 3 black and 1 white spot, or vice versa, on a chessboard. The other pieces cover 2 blacks and 2 whites (and the dominoes one of each). Therefore, the two T-pieces must be on the same side! Even one more piece with a weight must be added to form a square, which makes that one side has 8 weights and the other side only 4.

Time for some physics. A weight that is two units from the middle counts as much as two weights that are one unit from the middle. So a weight can count as 1, 2, 3, or 4, depending on where it appears. Now the total weight counts on the two sides should be the same. The 8 weights on one side should be as close as possible to the center of the scale, whereas the 4 weights on the other side should be as far away as possible. With this reasoning, it is easy to find a solution. The count is the same at a value of 13 (1+1+1+1+2+2+2+3 on one side, 2+3+4+4 on the other side).