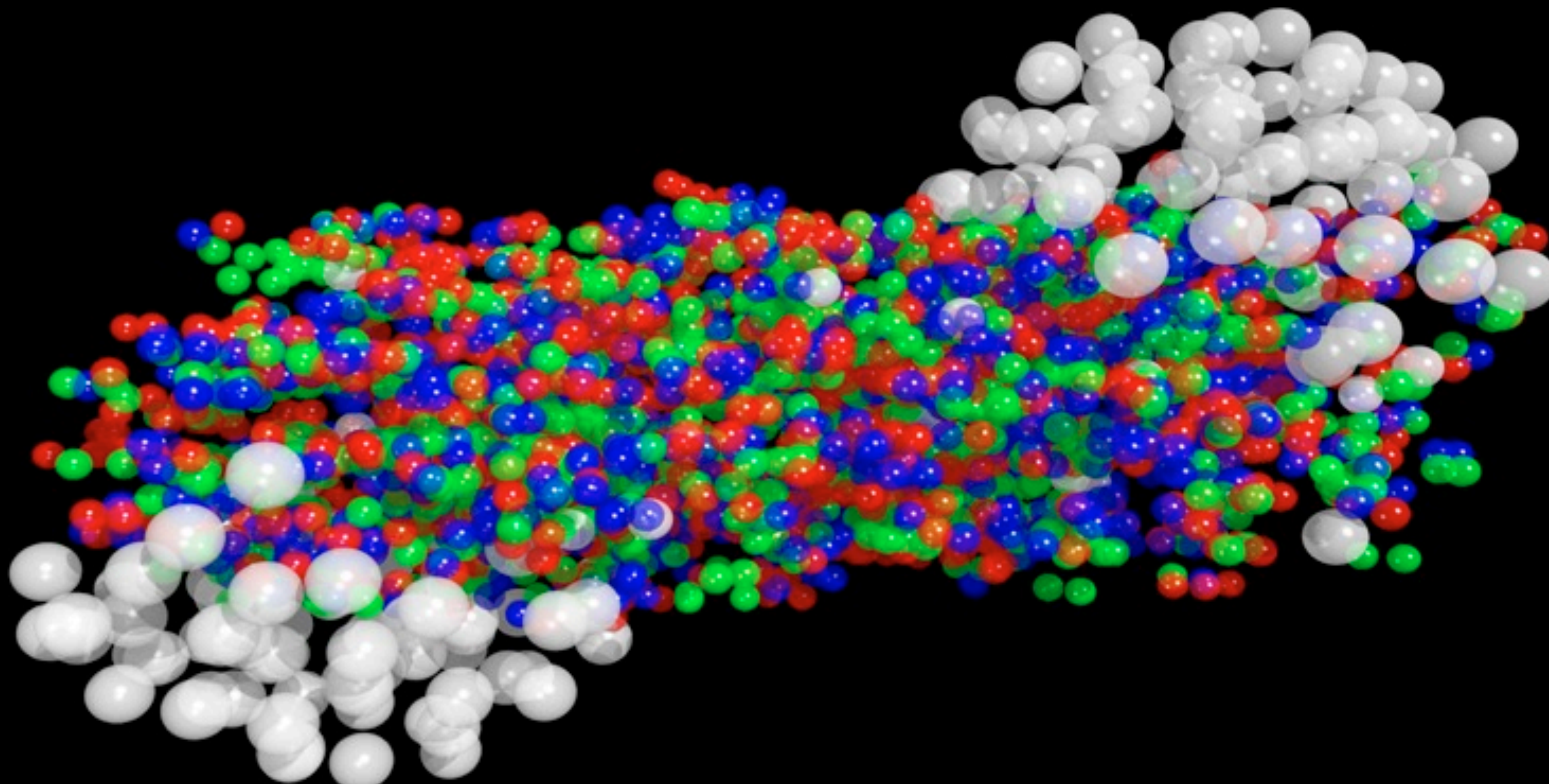
# Anisotropic Flow



Raimond Snellings

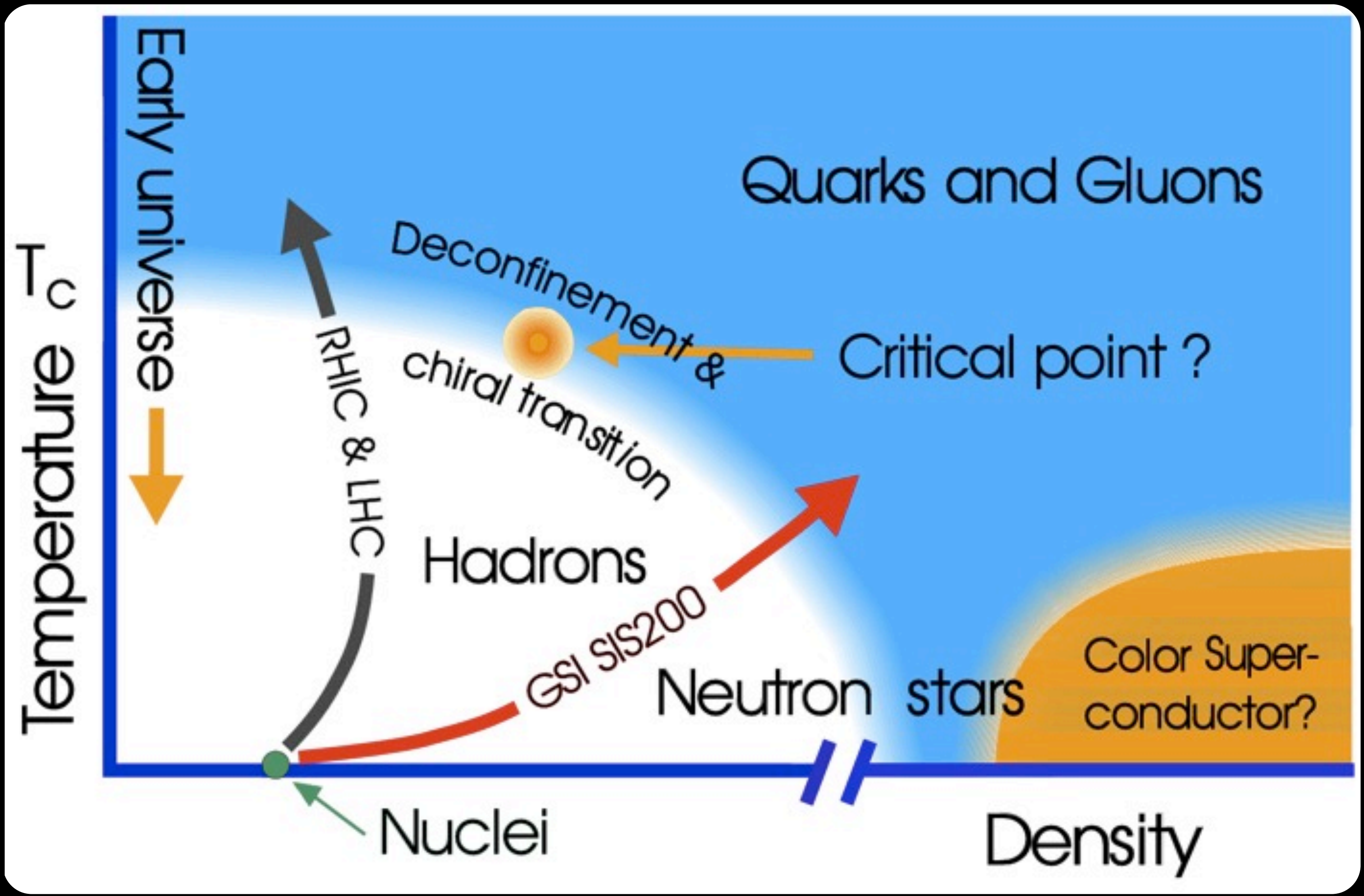
# Content

1) the QCD phase diagram, the equation of state, anisotropic flow results RHIC

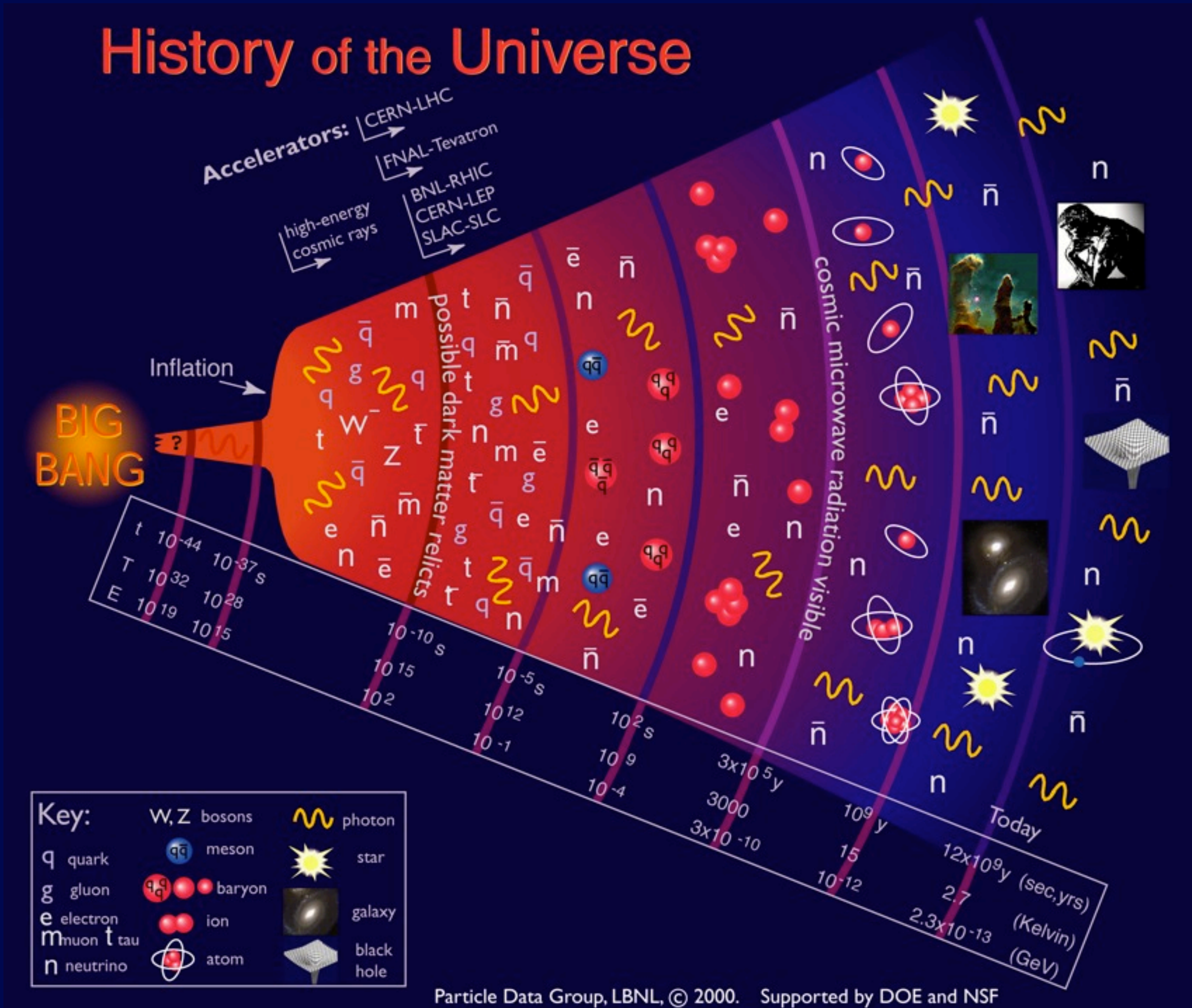
2) how do we measure flow

exercise: do flow analysis with various methods

# What happens when you heat and compress matter to very high temperatures and densities?




Based on Krishna Rajagopal and Frank Wilczek: Handbook of QCD


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 10 32 10 28
E 10 19 10 15
10 -10 s
10 15
10 2
10 -5 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
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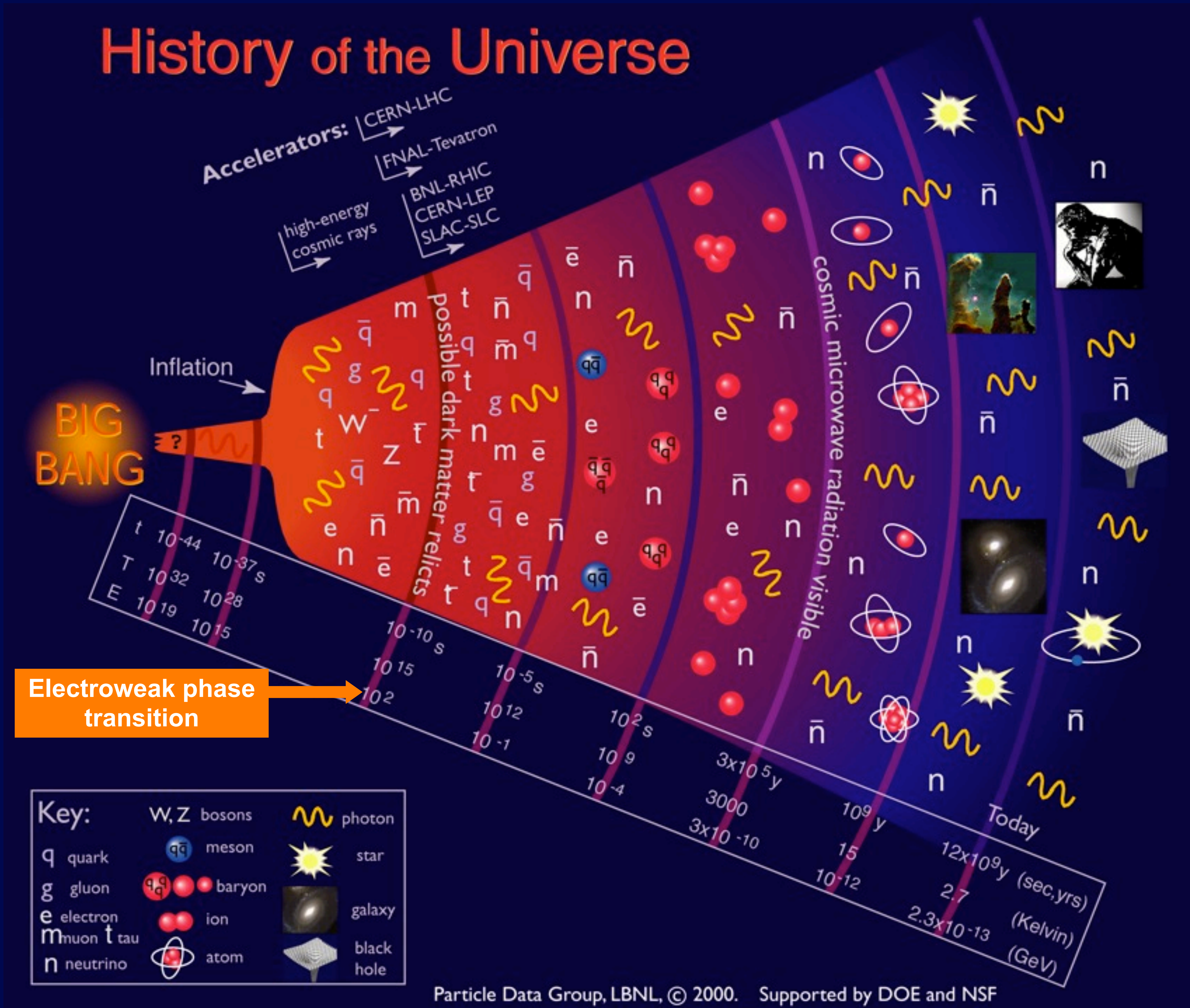
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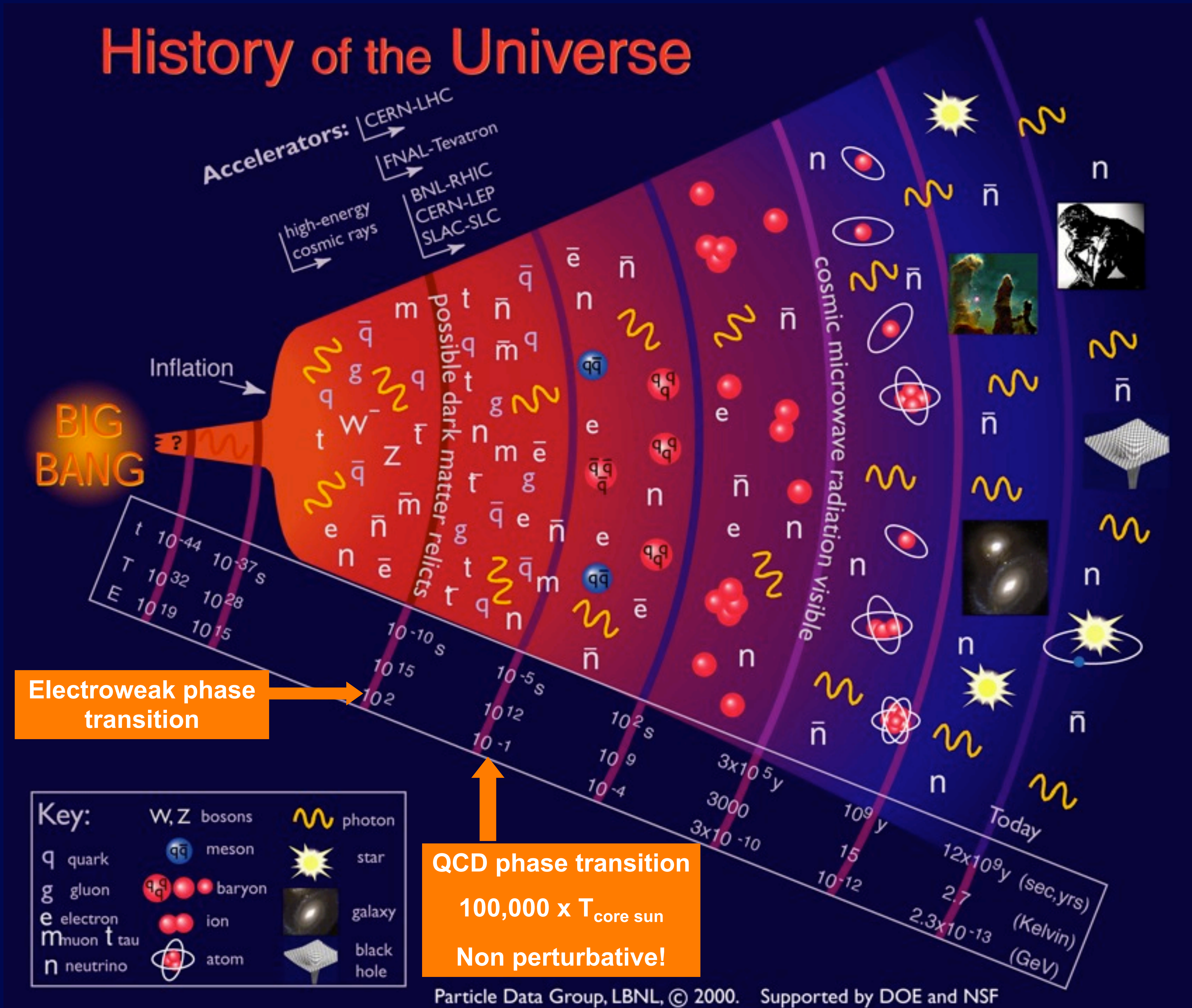
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Today
Electroweak phase transition
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100,000 x T core sun
Non perturbative!
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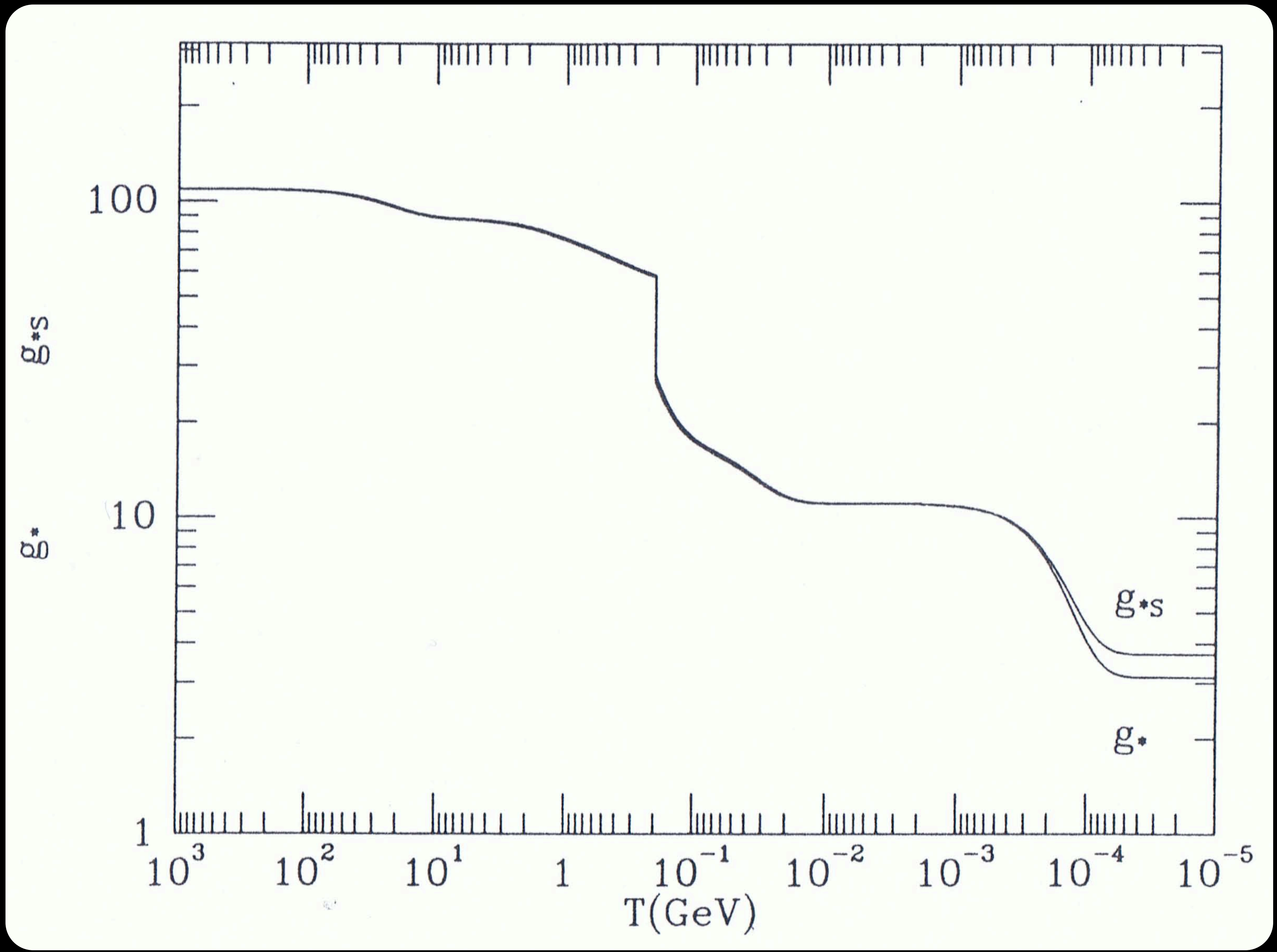
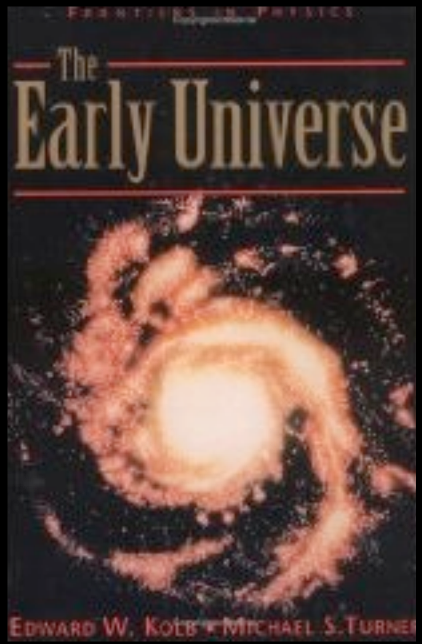
# Early Universe: degrees of freedom




E. Kolb and M. Turner: the early universe



# rough estimate: EoS and degrees of freedom

ideal gas Equation of State: $p = \frac{1}{3}\varepsilon = g\frac{\pi^2}{90}T^4$

$$\frac{\varepsilon}{T^4} = g\frac{\pi^2}{30}$$

➡ energy density of g massless degrees of freedom

$$\frac{\varepsilon}{T^4} = 3\frac{\pi^2}{30}$$

➡ hadronic matter dominated by lightest mesons ($\pi^+$, $\pi^-$, and $\pi^0$)

➡ deconfined matter, quarks and gluons

$$g = 2_{\text{spin}} \times 8_{\text{gluons}} + \frac{7}{8} \times 2_{\text{flavors}} \times 2_{\text{quark/anti-quark}} \times 2_{\text{spin}} \times 3_{\text{color}}$$

$$\frac{\varepsilon}{T^4} = 37\frac{\pi^2}{30}$$

➡ during phase transition large increase in degrees of freedom !

# rough estimate: QCD phase transition temperature

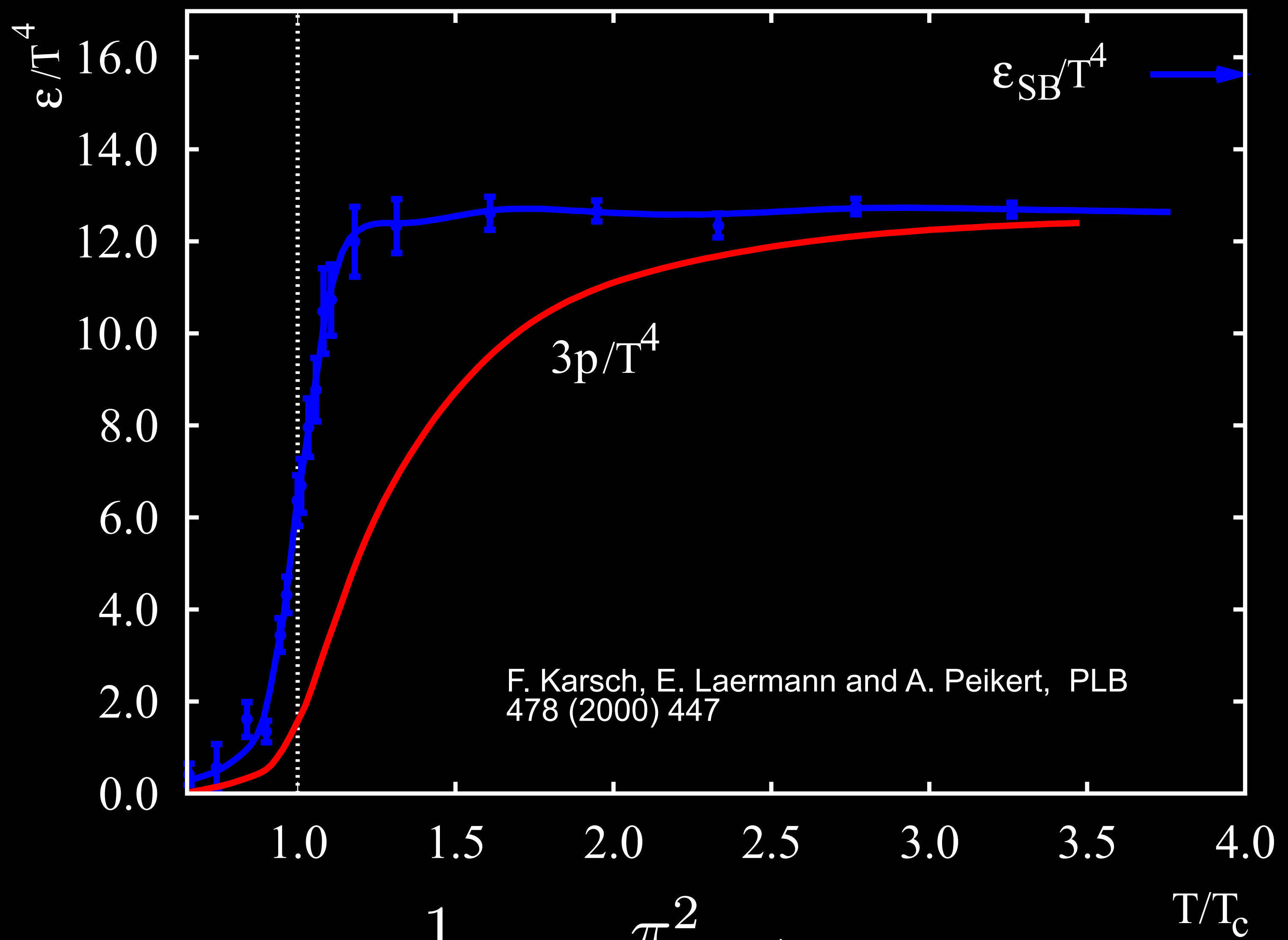
- confinement due to bag pressure B (from the QCD vacuum)
  - $B^{1/4} \sim 200$ MeV
- deconfinement when thermal pressure is larger than bag pressure

$$p = \frac{1}{3}\epsilon = g\frac{\pi^2}{90}T^4$$

$$T_c = (\frac{90B}{37\pi^2})^{1/4} = 140 \text{ MeV}$$

crude estimate!

# QCD on the Latice




$T_C \sim 170 \pm 20$ MeV, $\varepsilon_C \sim 0.6$ GeV/fm$^3$

at the critical temperature a strong increase in the degrees of freedom

✓ gluons, quarks & color!

not an ideal gas!?

✓ residual interactions

at the phase transition dp/dε decreases rapidly

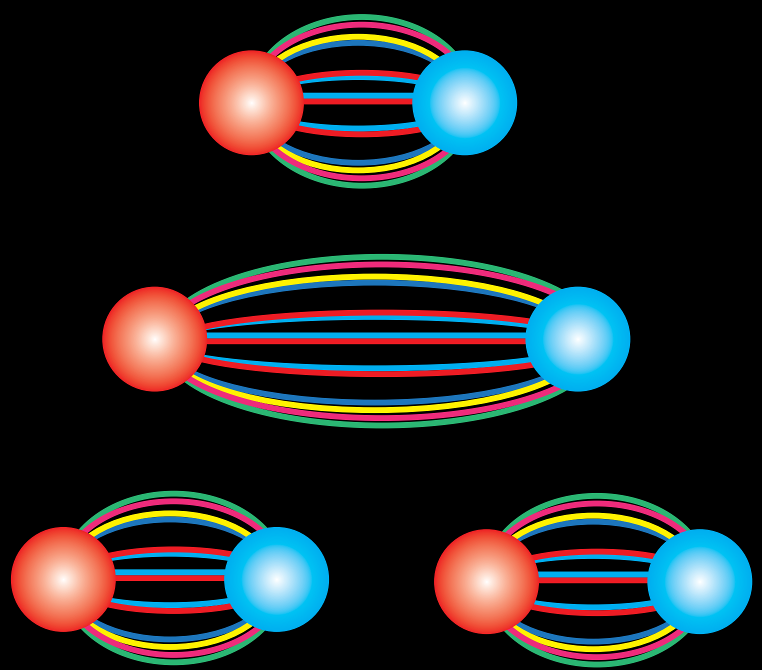
$$p = \frac{1}{3}\epsilon = g\frac{\pi^2}{90}T^4$$

$$g_{\mathrm{H}} \approx 3 \quad g_{\mathrm{QGP}} \approx 37$$

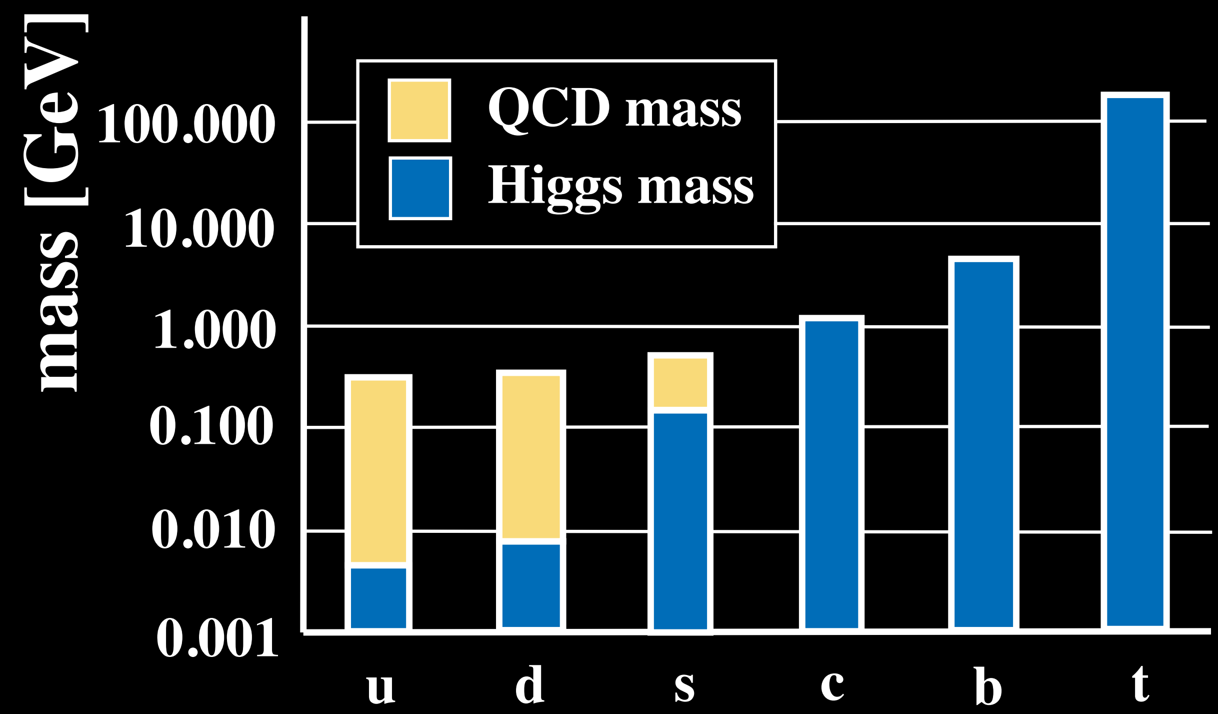
$$g = 2_{\mathrm{spin}} \times 8_{\mathrm{gluons}} + \frac{7}{8} \times 2_{\mathrm{flavors}} \times 2_{q\bar{q}} \times 2_{\mathrm{spin}} \times 3_{\mathrm{color}}$$

The macroscopic quantities of the QGP will give us better understanding of the underlying microscopic theory (QCD) in the non-perturbative regime

mechanism of confinement

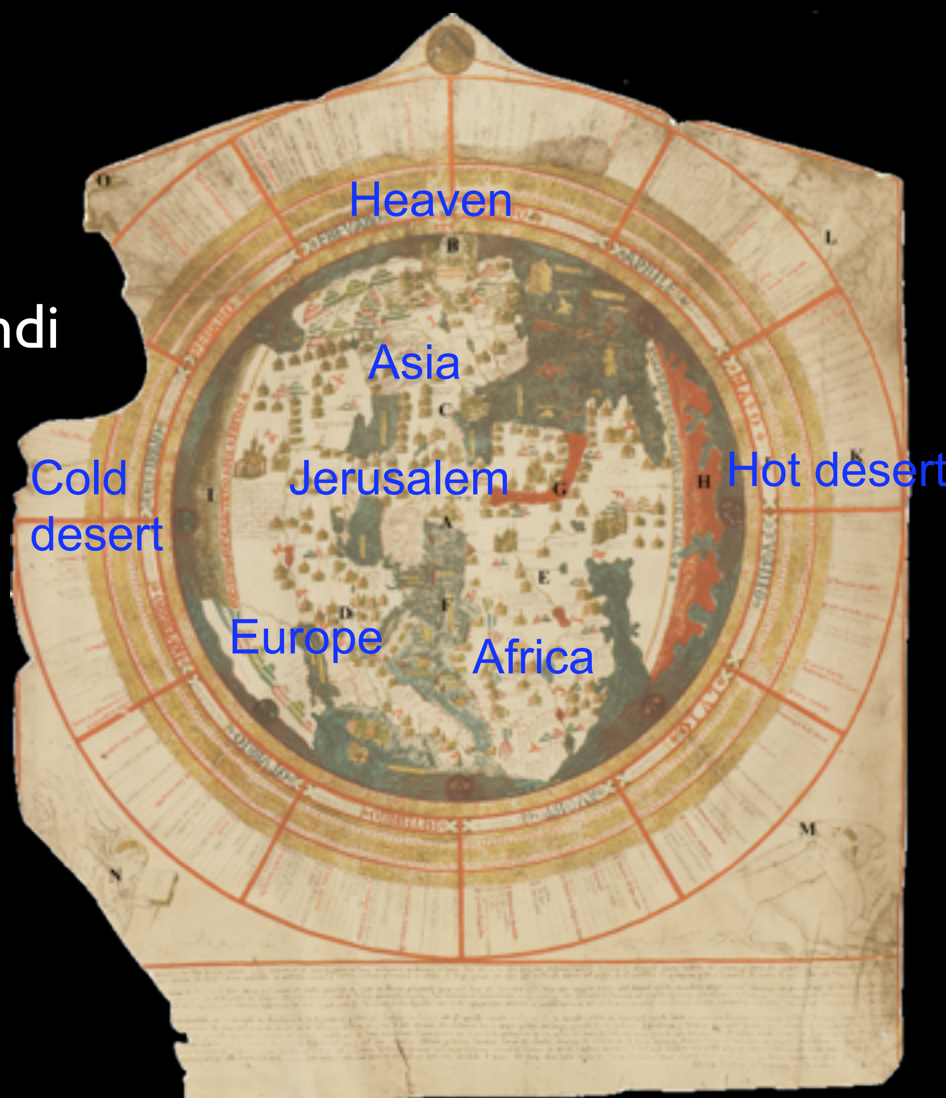


mass generation in the strong interaction

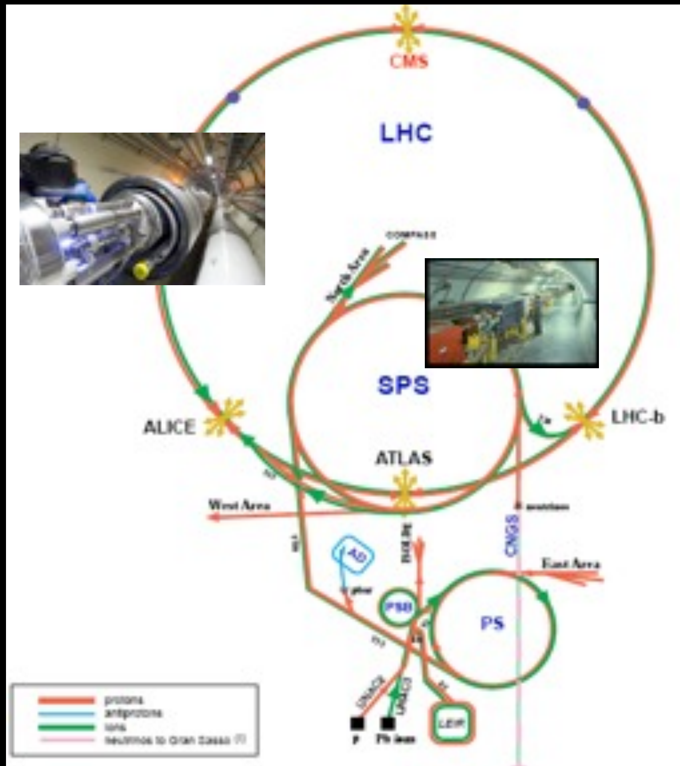


# so far only a theory view of the world!

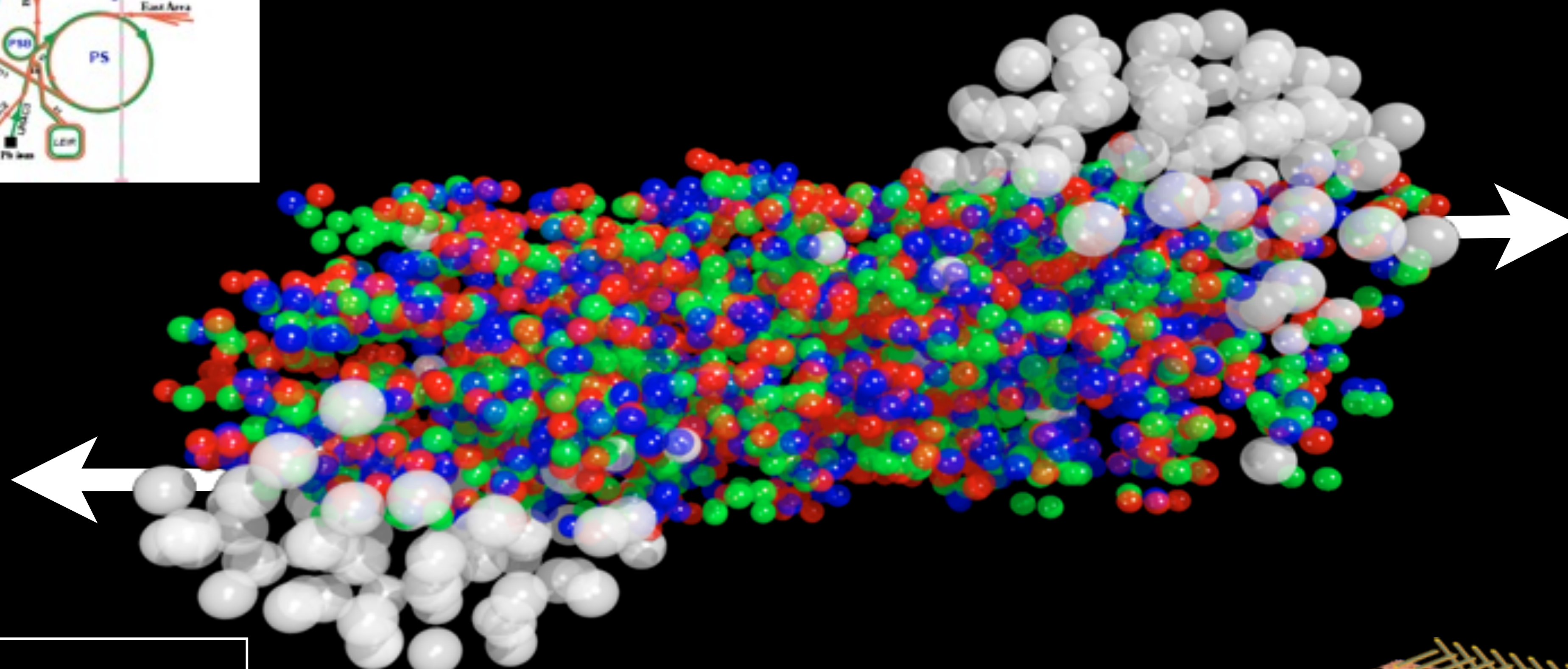
mappa mundi
1452

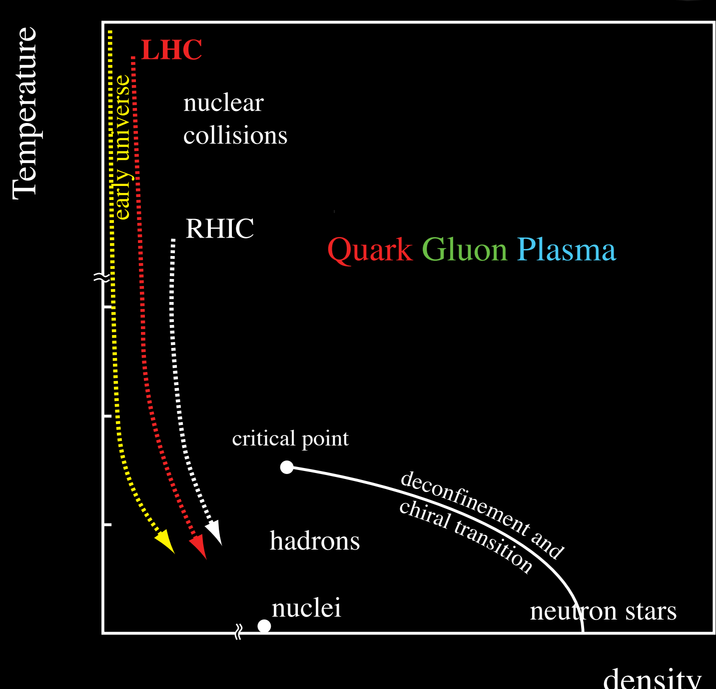



explore experimentally the properties of this Quark Gluon Plasma

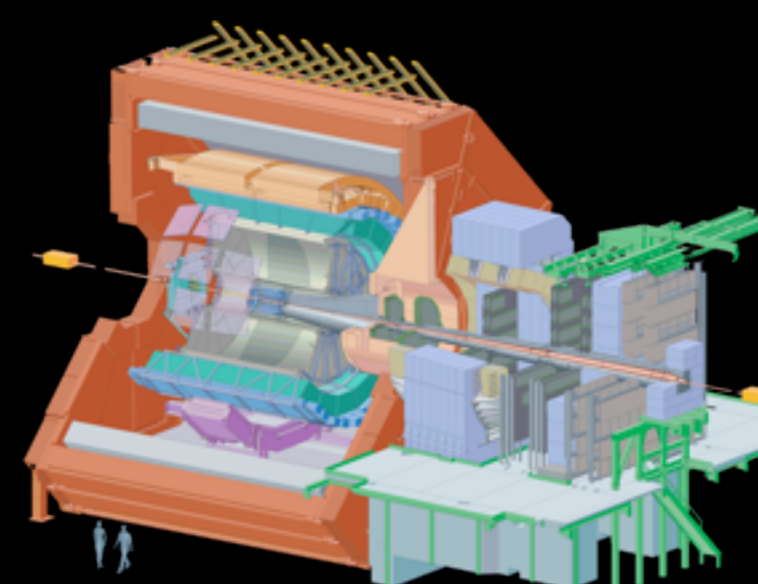



# How?




study phase transition in controlled lab conditions by colliding heavy-ions



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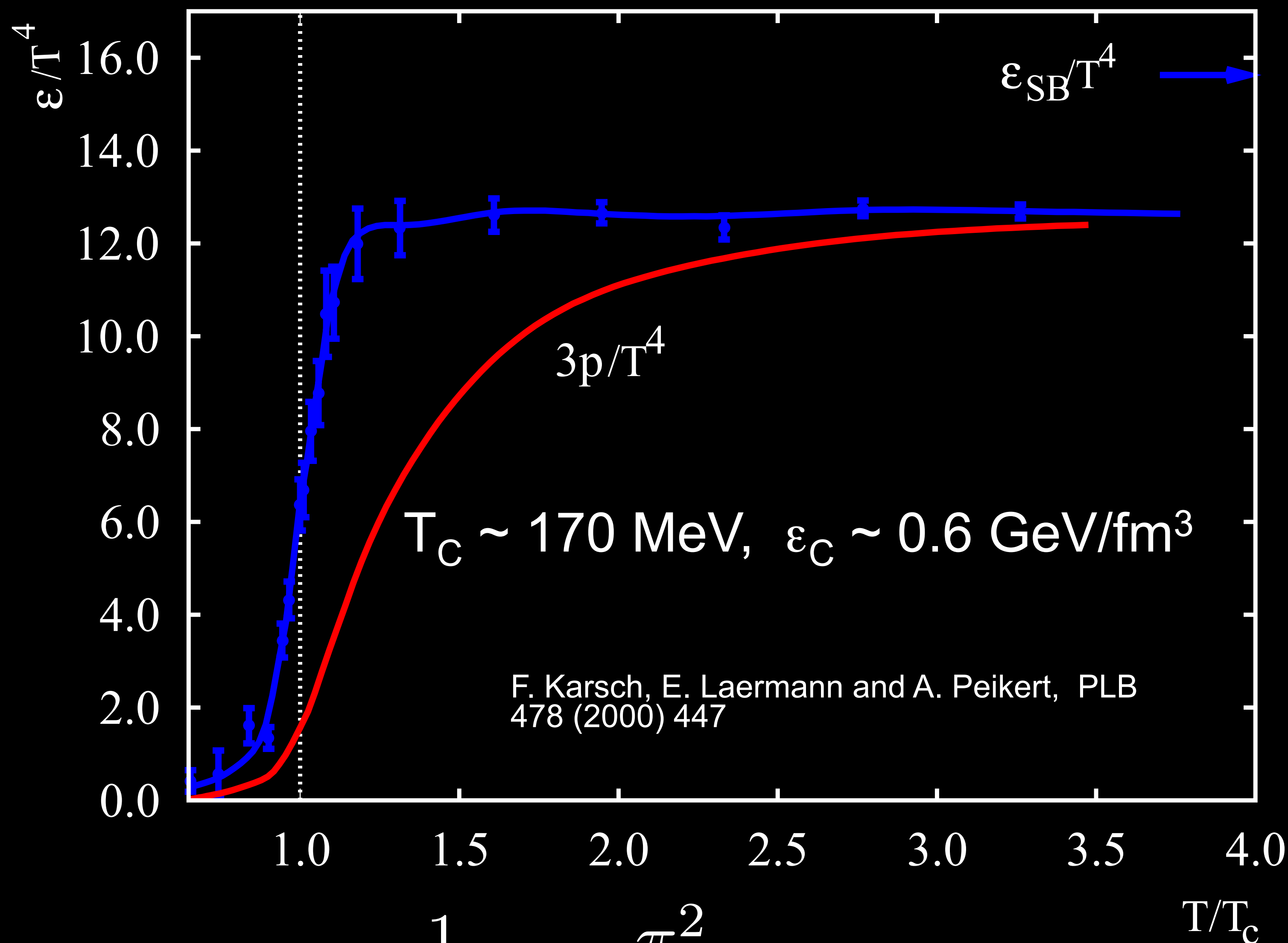


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at the critical temperature a strong increase in the degrees of freedom

✓ gluons, quarks & color!

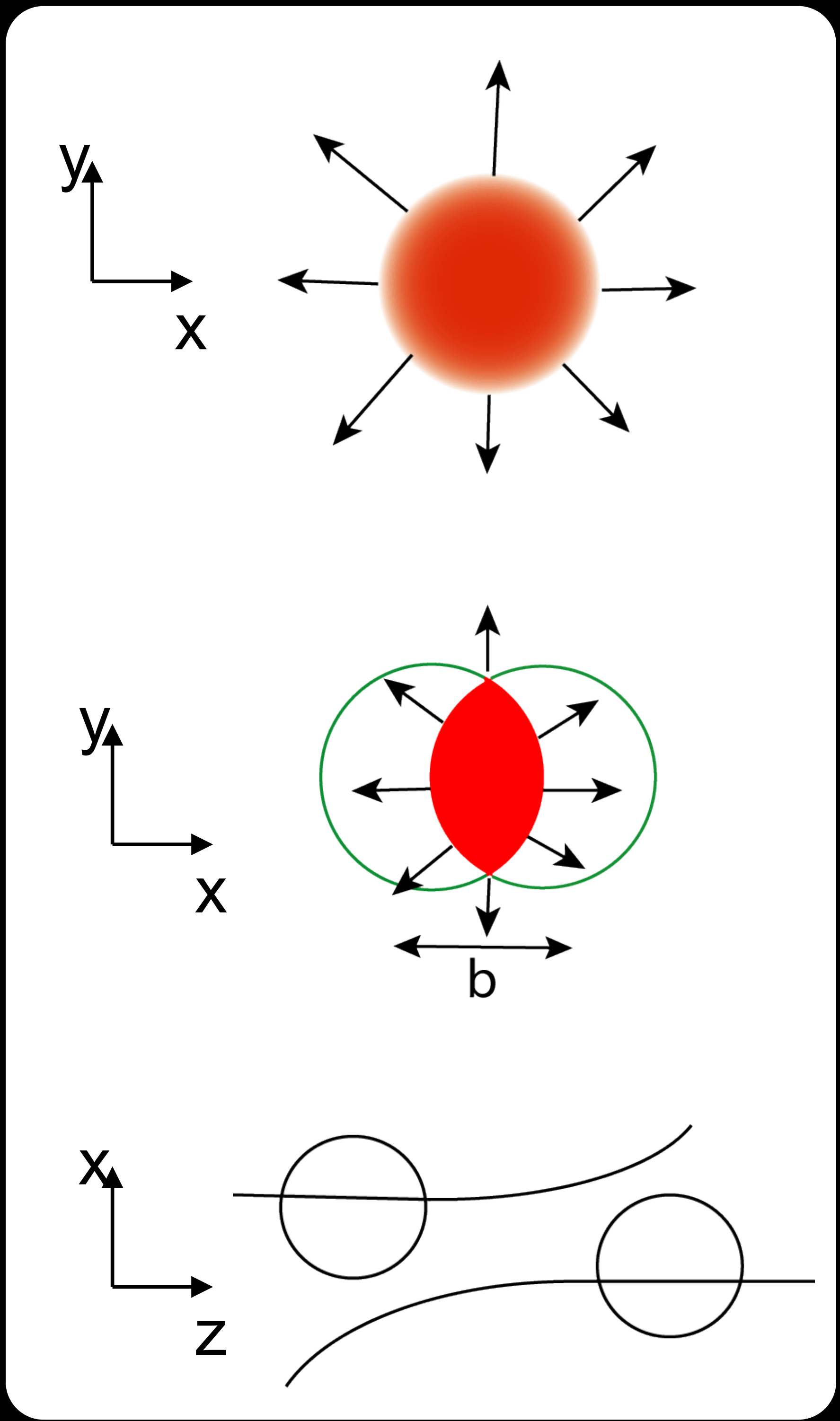
not an ideal gas!

✓ residual interactions

at the phase transition dp/dε decreases rapidly

dp/dε drives the collective expansion of the system

# Collective Motion




only type of transverse flow in central collision (b=0) is radial flow Integrates pressure history over complete expansion phase

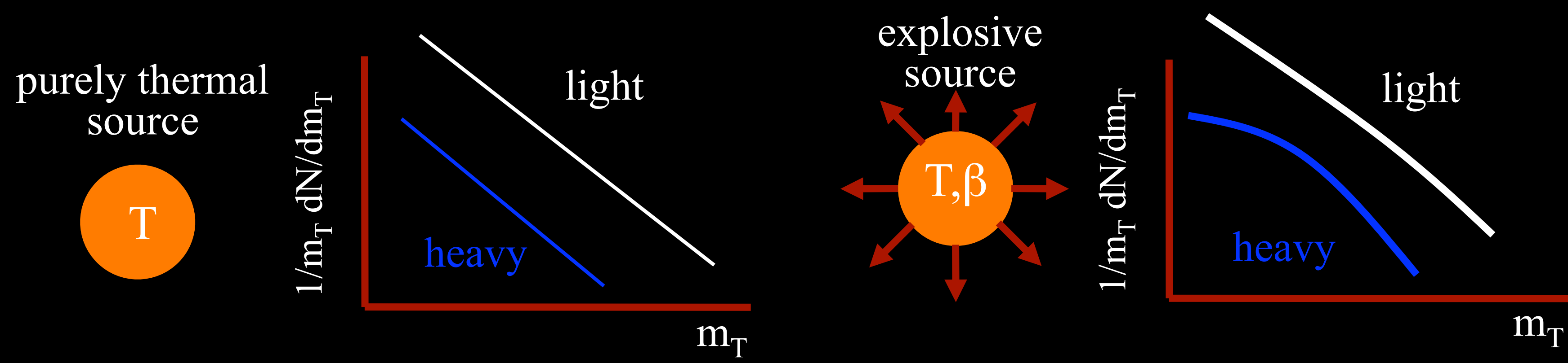
elliptic flow ($v_2$) , $v_4$ , $v_6$, … caused by anisotropic initial overlap region (b > 0) more weight towards early stage of expansion

directed flow ($v_1$) , sensitive to earliest collision stage (b > 0), pre-equilibrium at forward rapidity, at midrapidity perhaps different origin

# Collective Motion

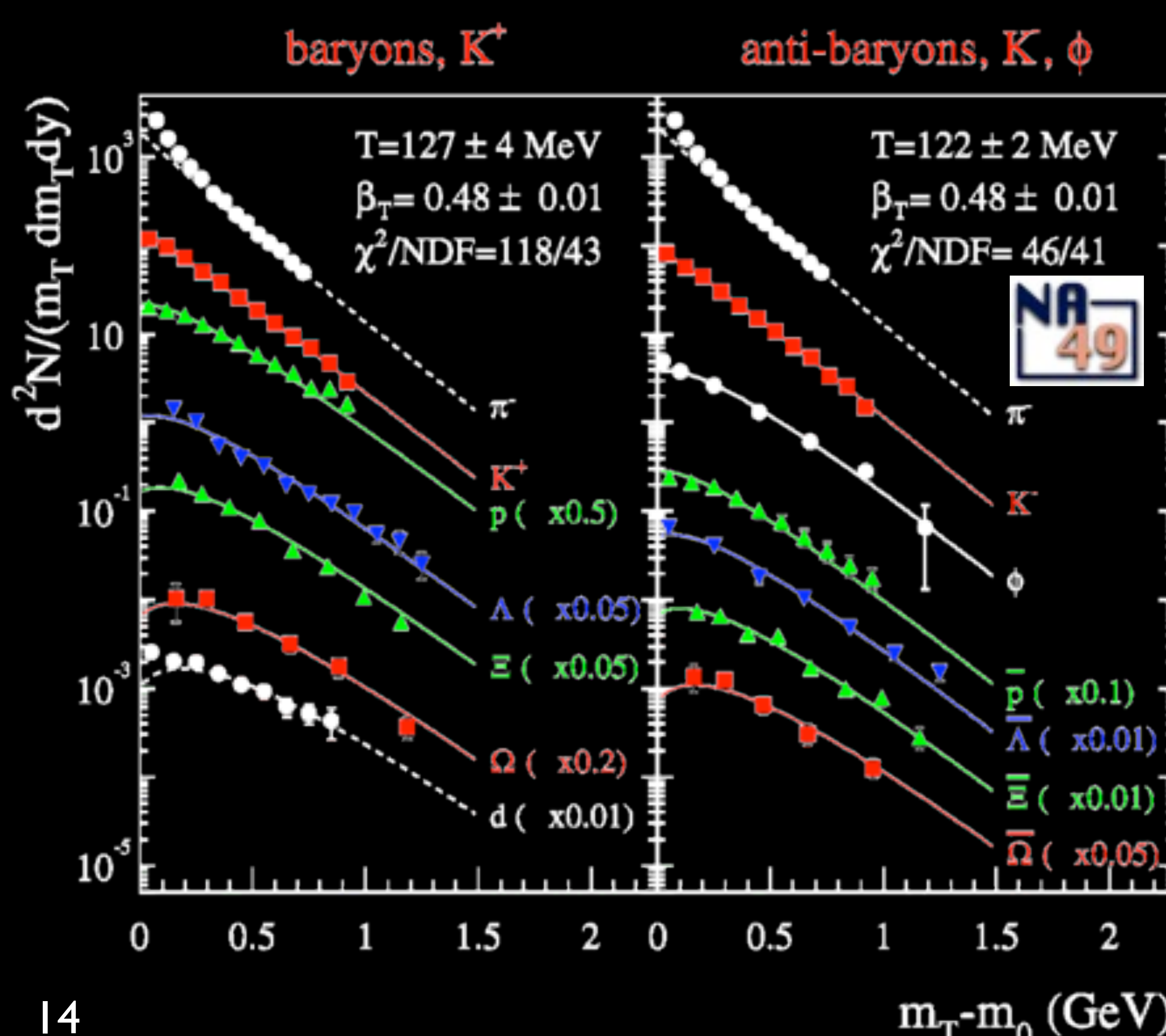
$$m_T = \sqrt{(m^2 + p_t^2)}$$

$$\frac{dN}{m_T dm_T} \propto e^{-m_T/T}$$



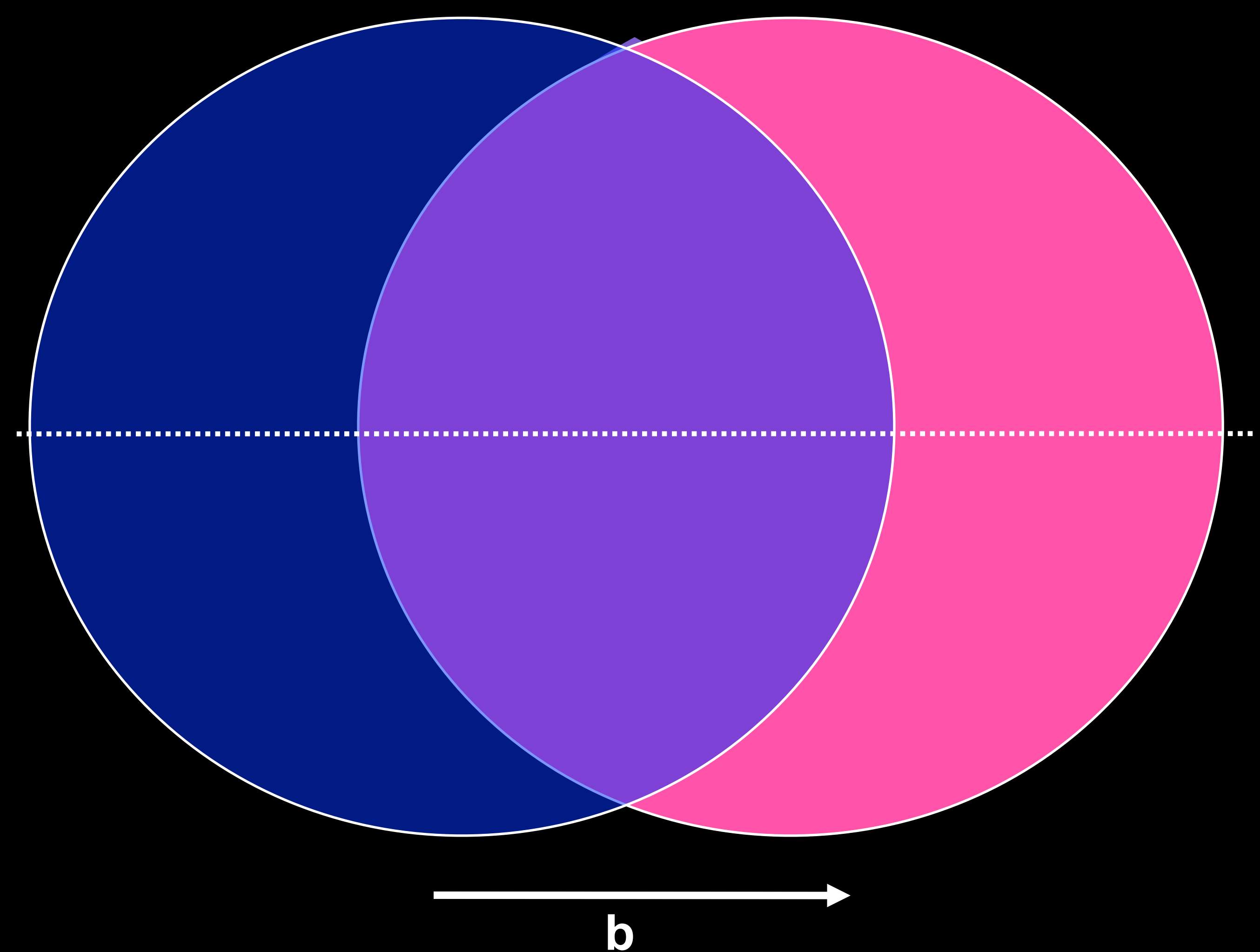

in p-p at low transverse momenta the particle yields are well described by thermal spectra ($m_T$ scaling)

boosted thermal spectra give a very good description of the particle distributions measured in heavy-ion collisions



# Elliptic Flow

Animation: Mike Lisa



# Elliptic Flow

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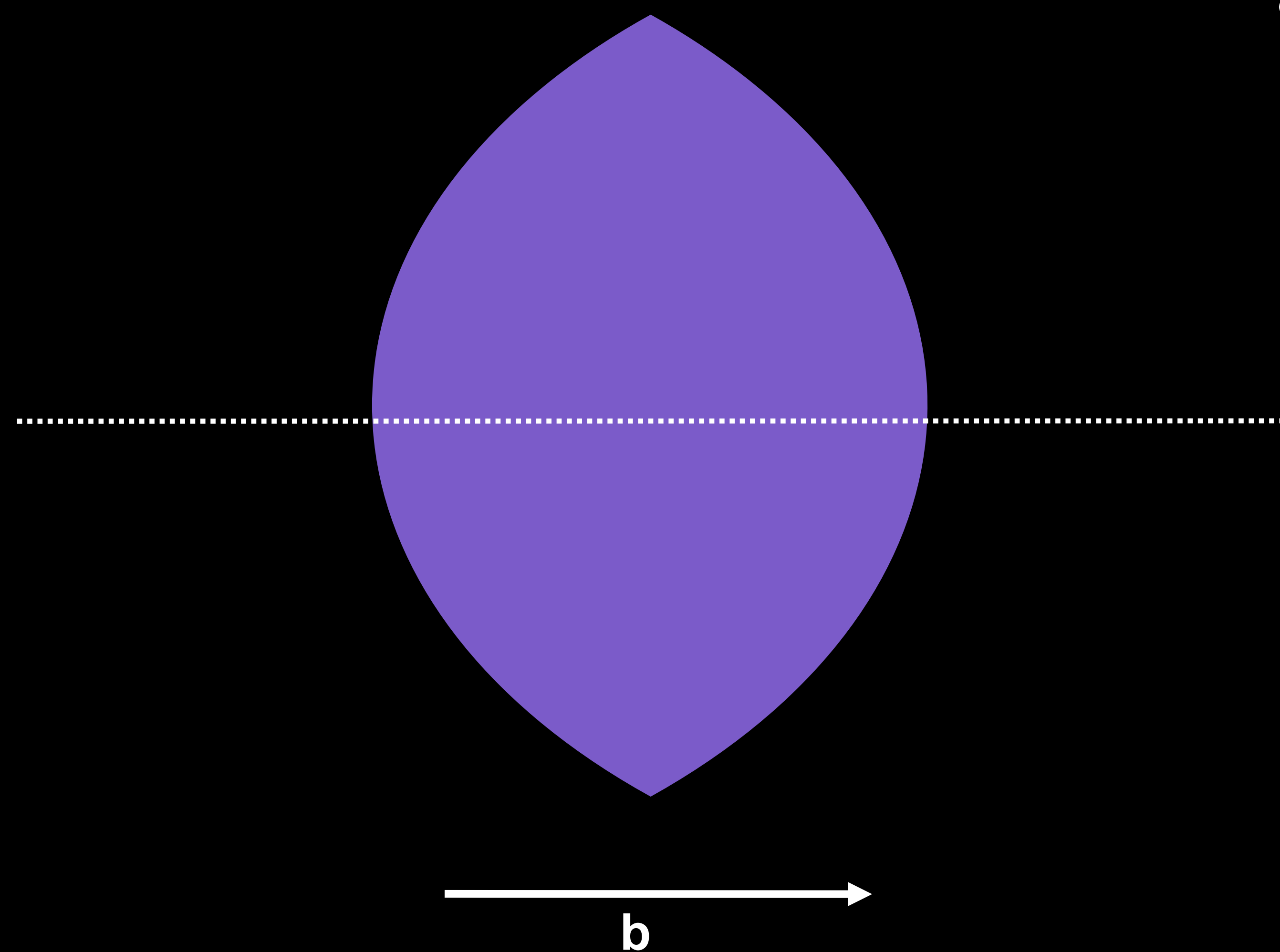
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



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1) superposition of independent p+p:

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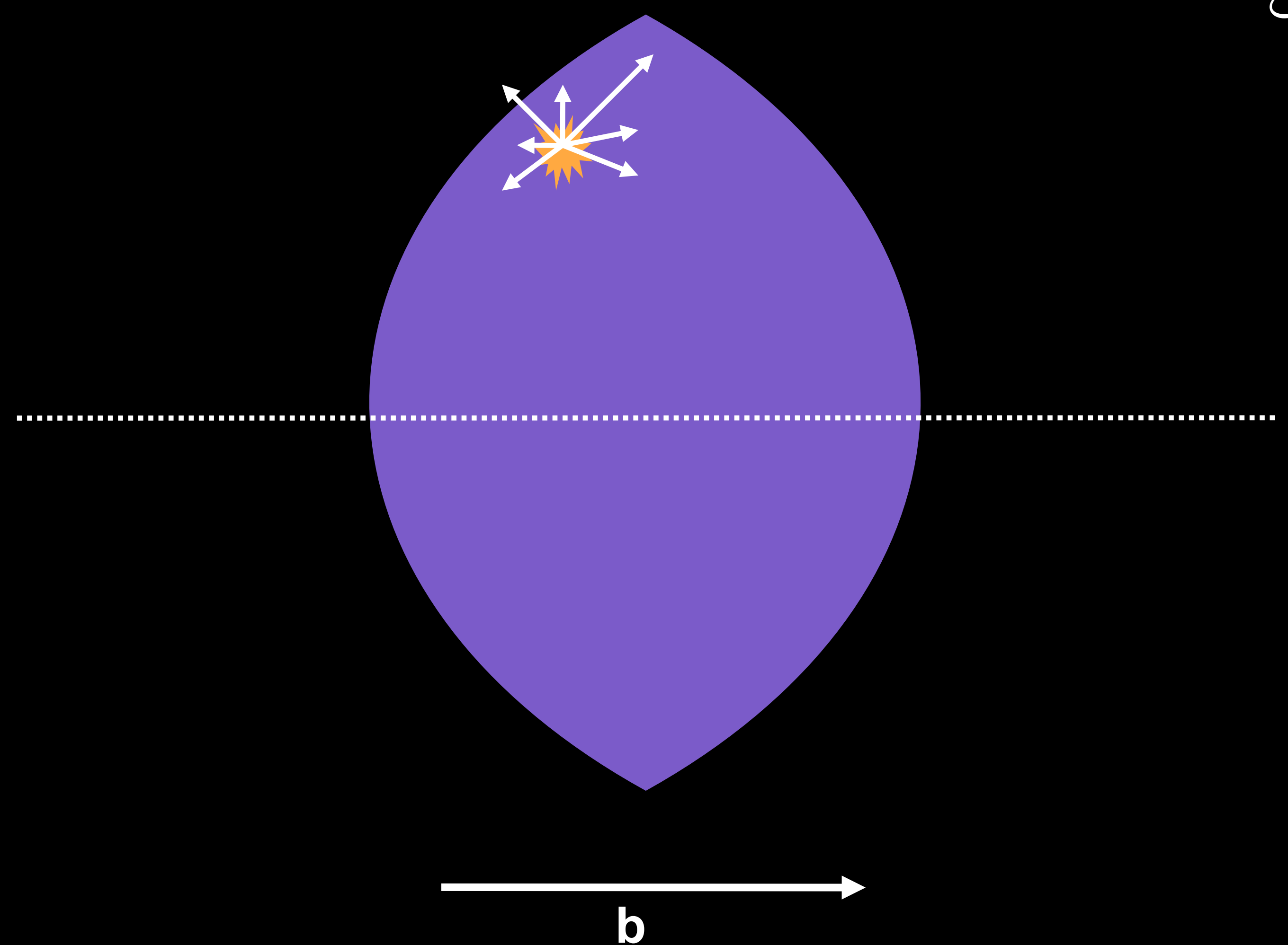

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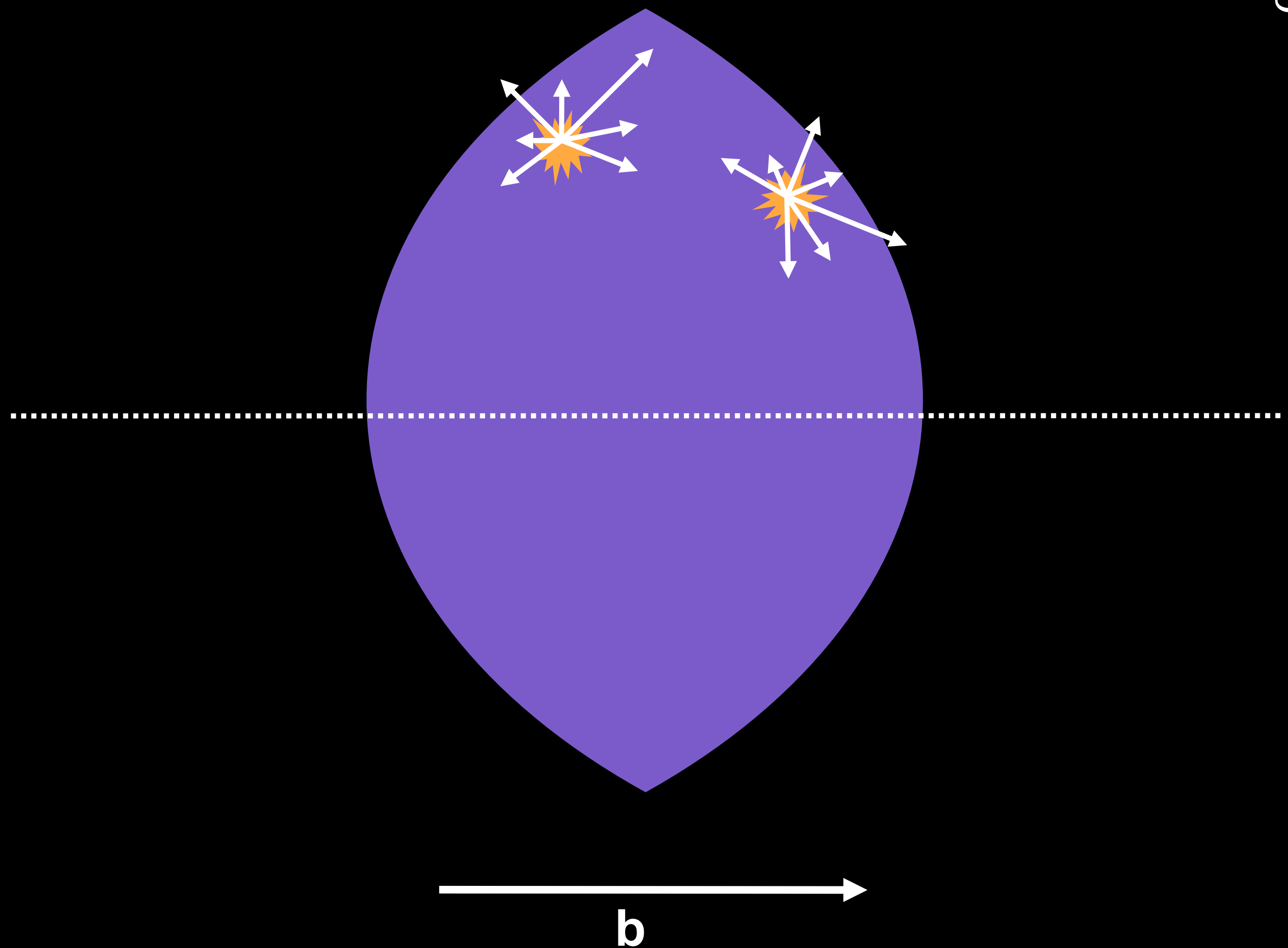

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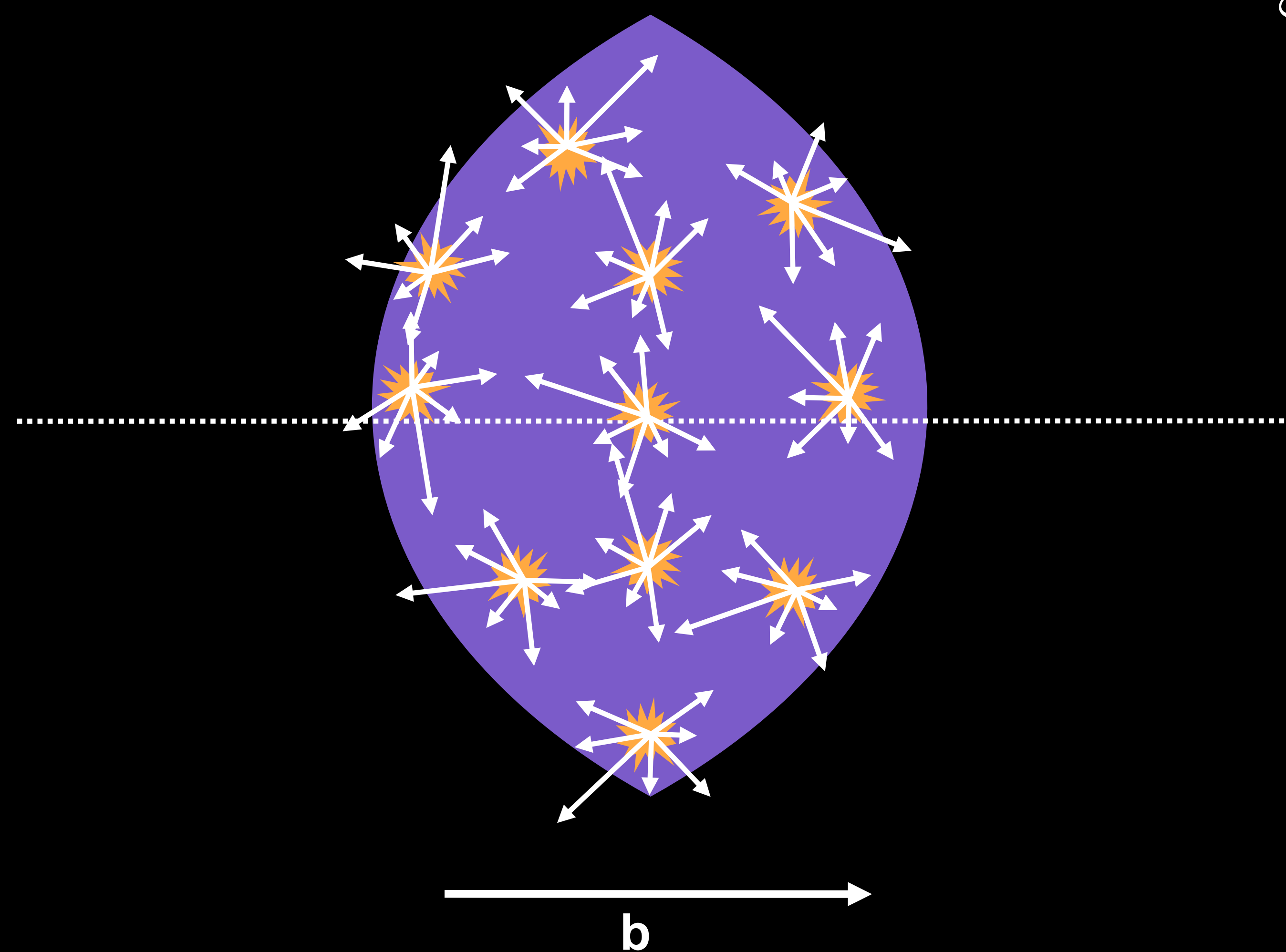

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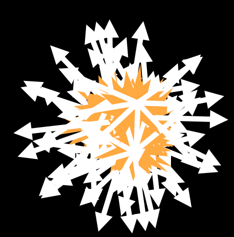
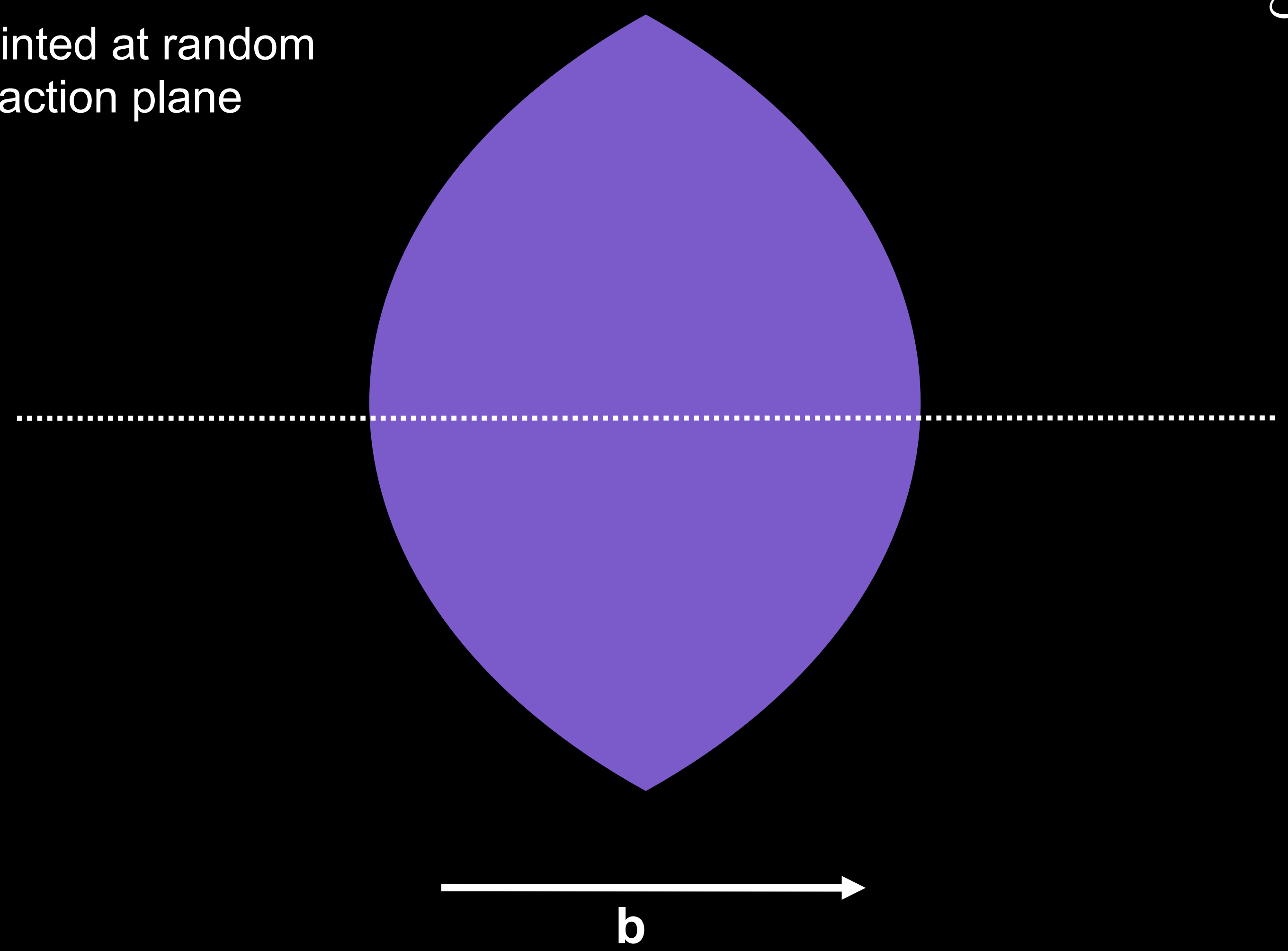
momenta pointed at random
relative to reaction plane



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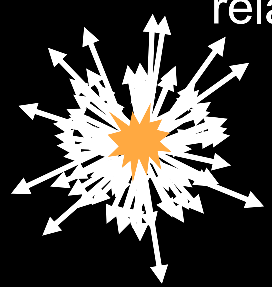

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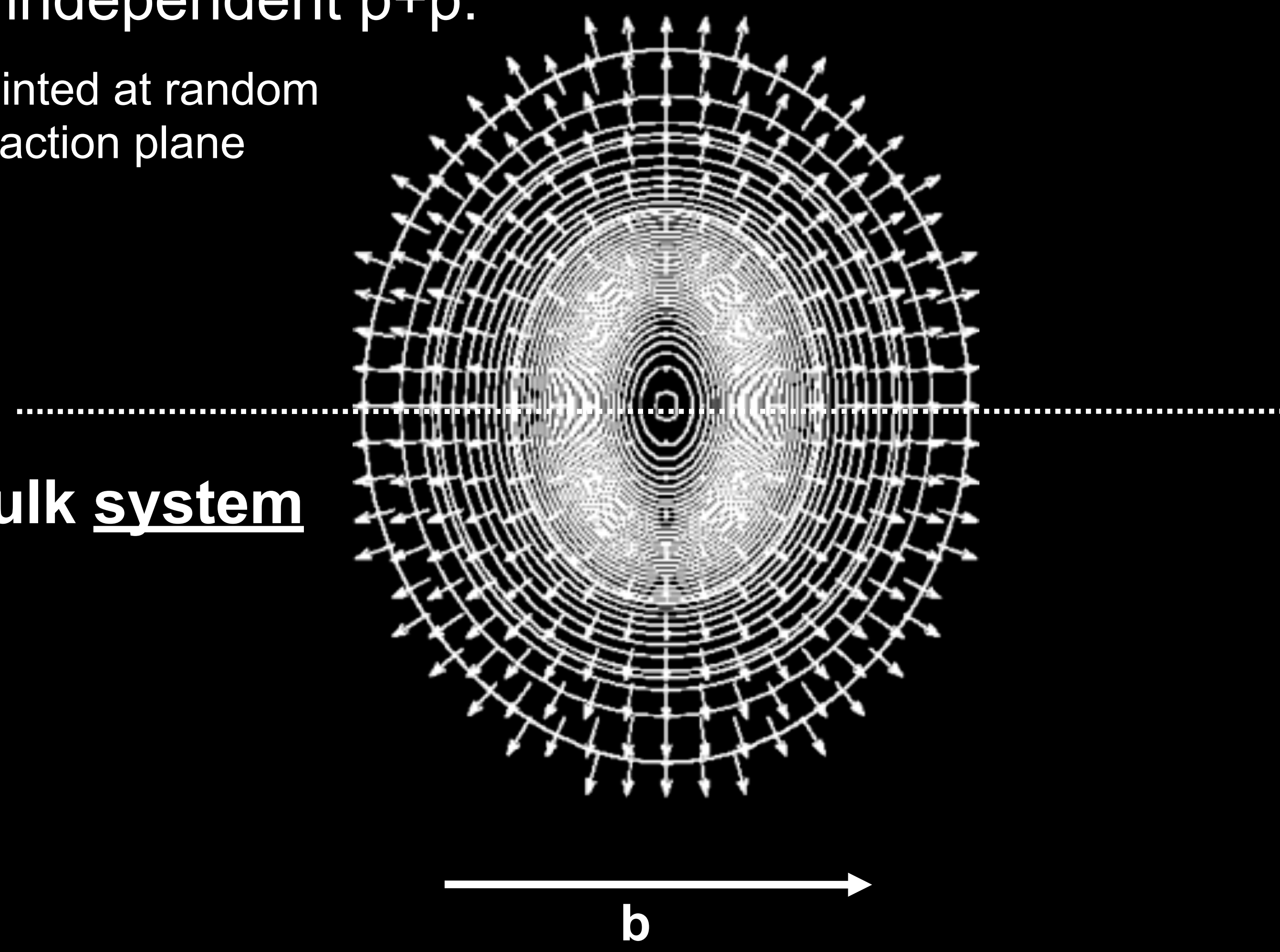
2) evolution as a **bulk <u>system</u>**



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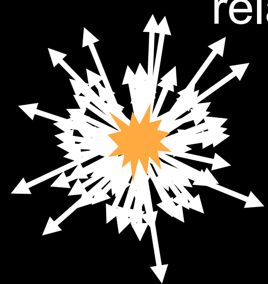

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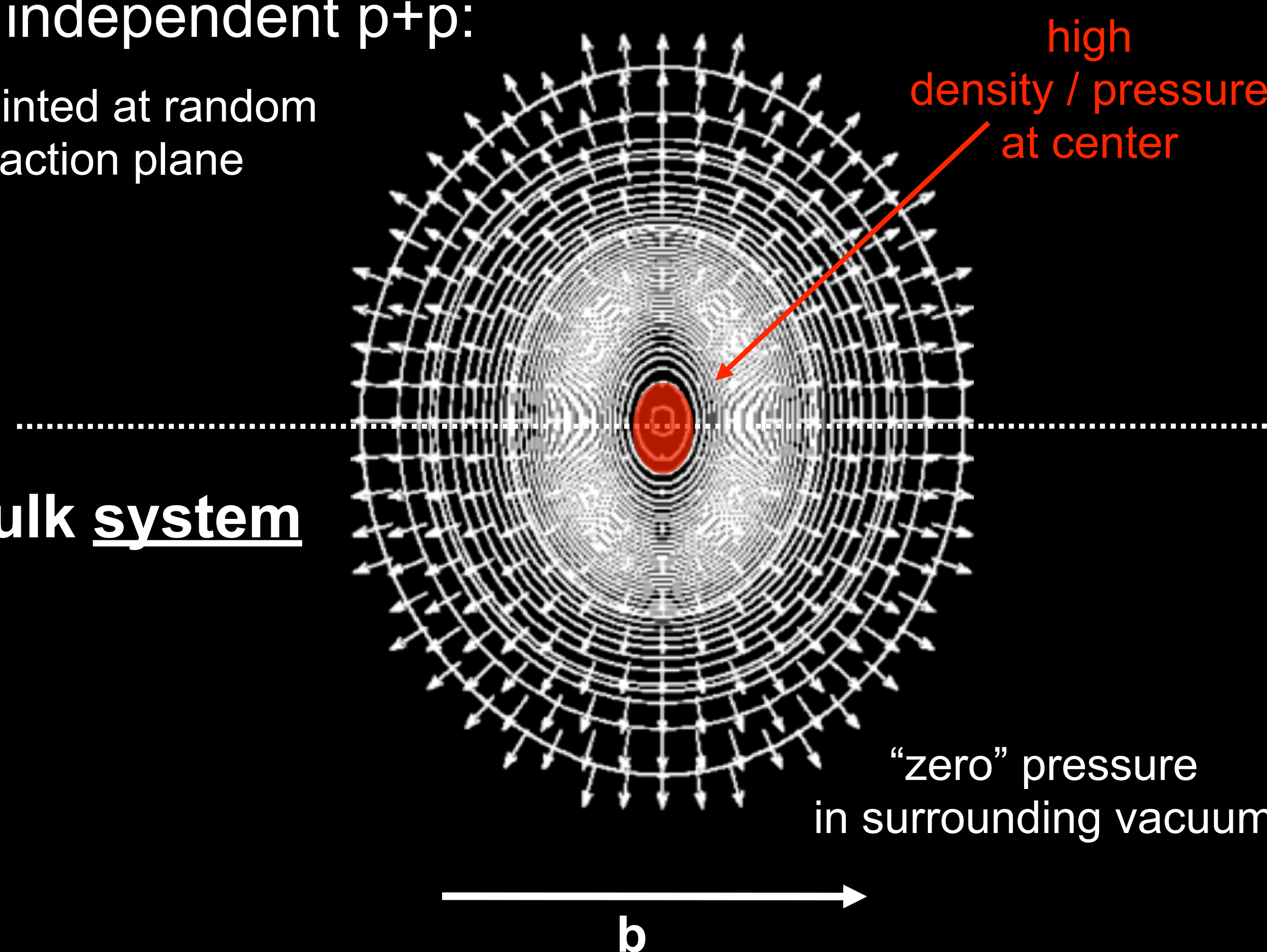

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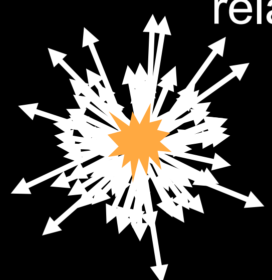

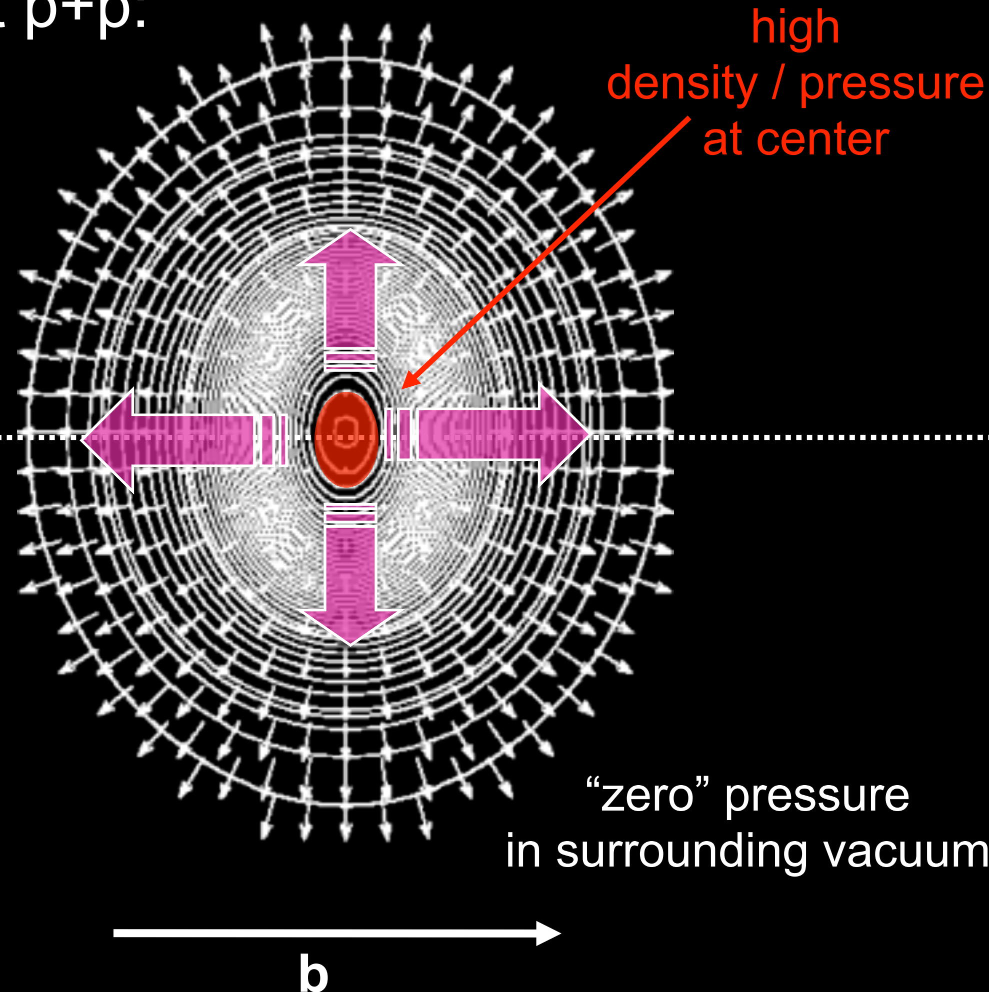


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pressure gradients (larger in-plane)
push bulk “out” → “flow”

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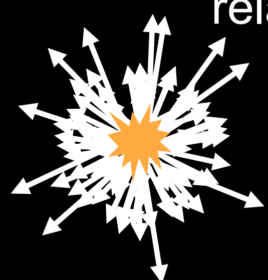

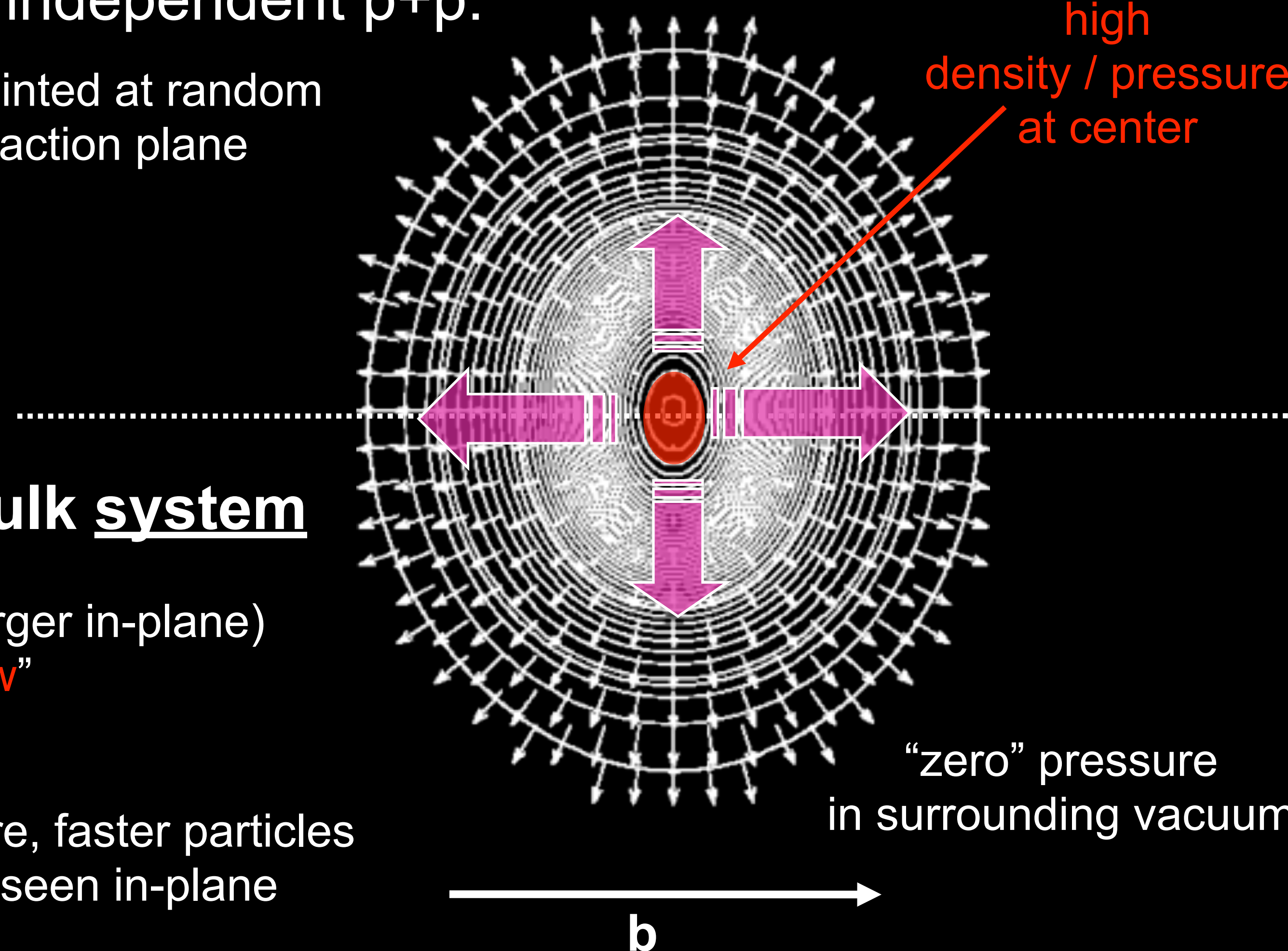


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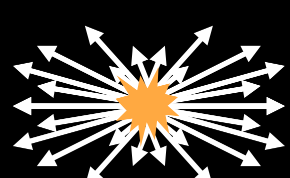

more, faster particles
seen in-plane

# Elliptic Flow

1) superposition of independent p+p:

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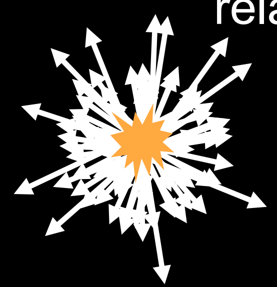

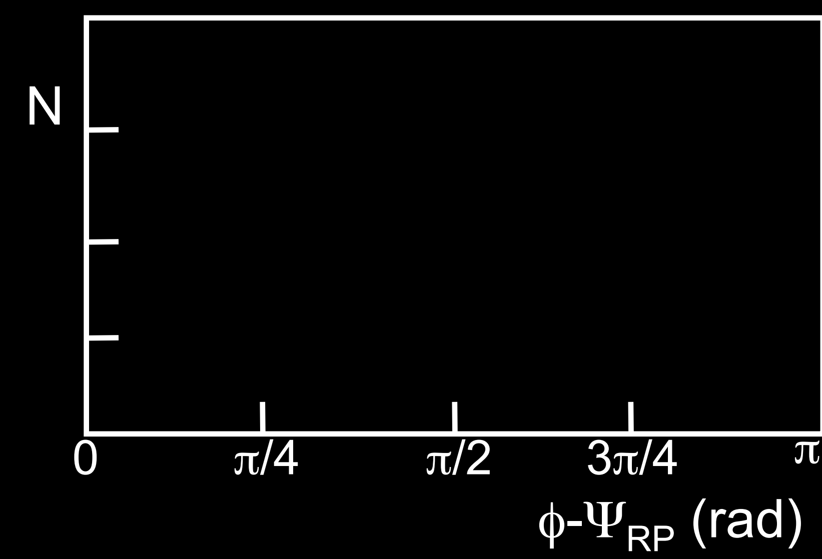

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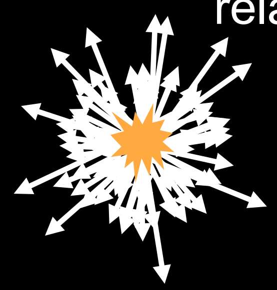

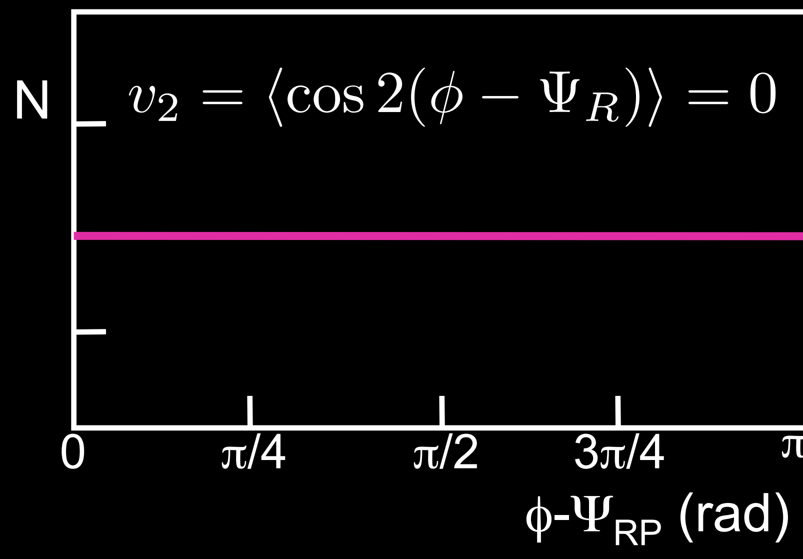

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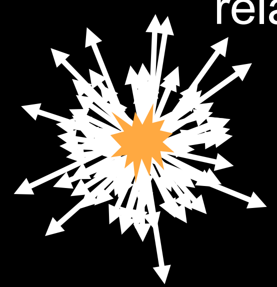

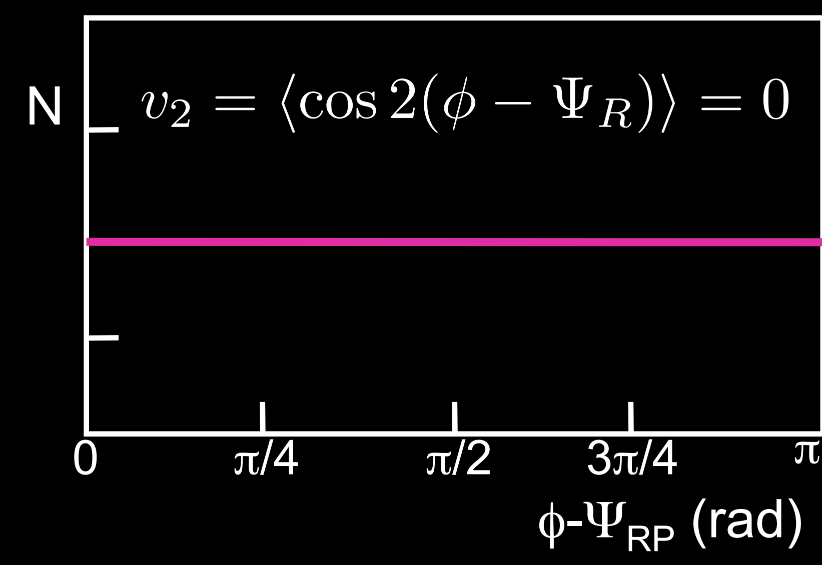


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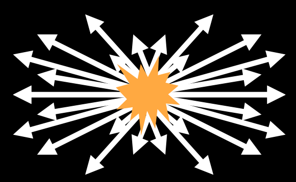
push bulk "out" → "flow"



more, faster particles
seen in-plane

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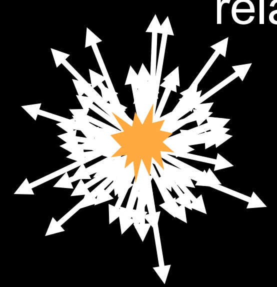

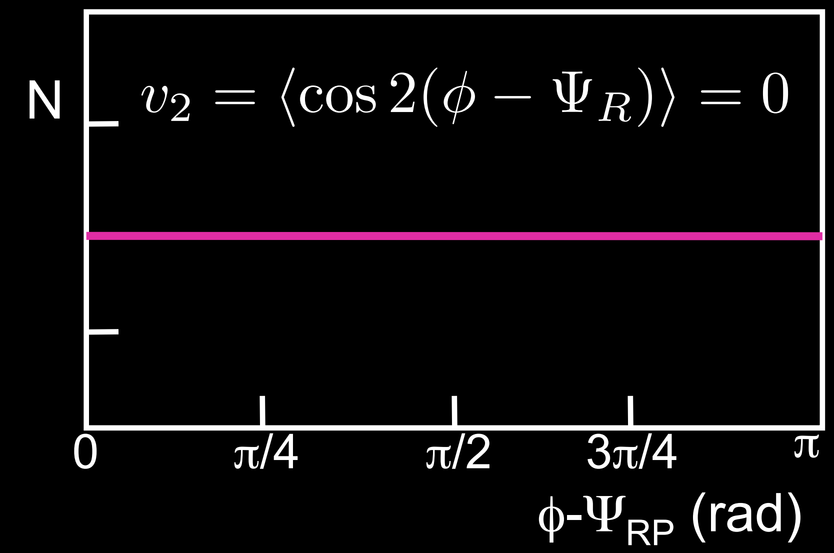


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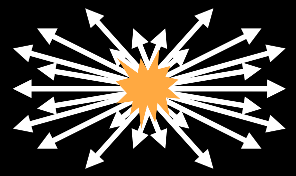
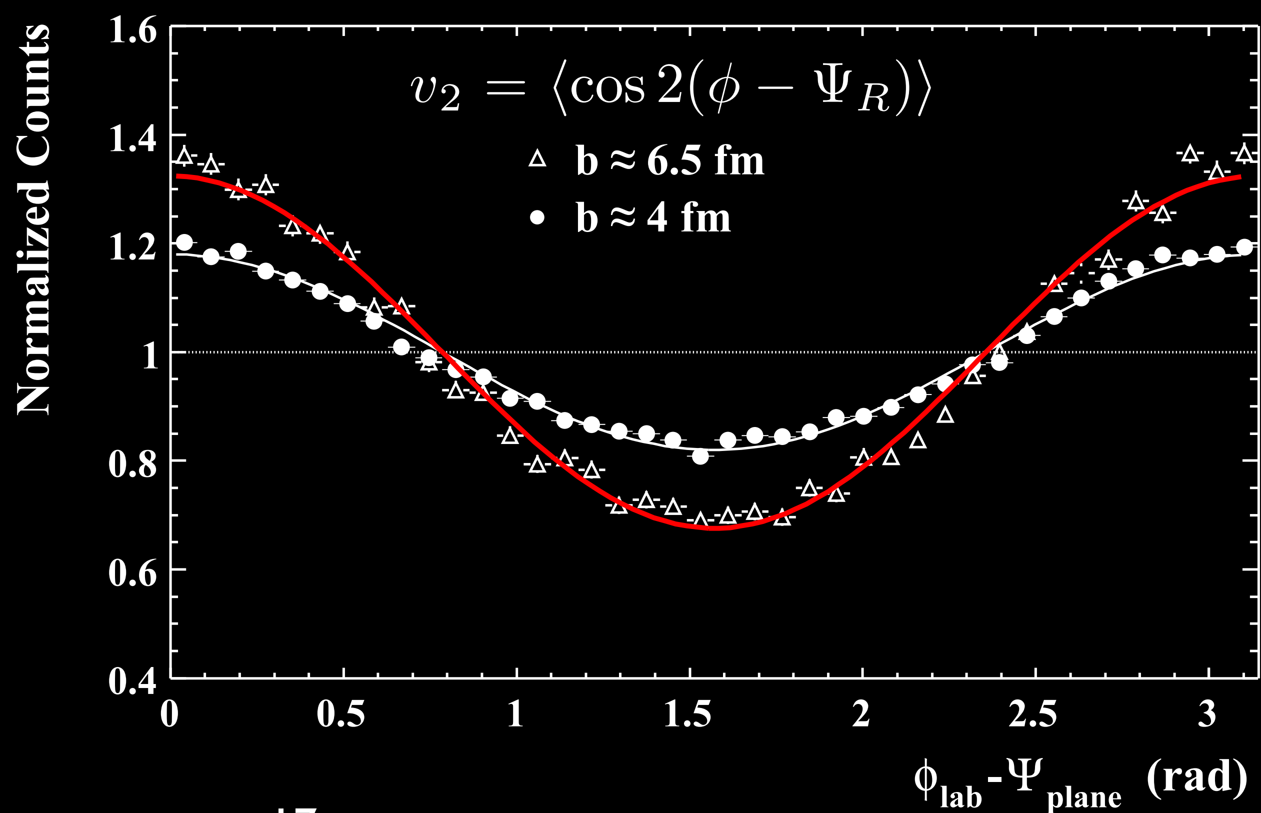
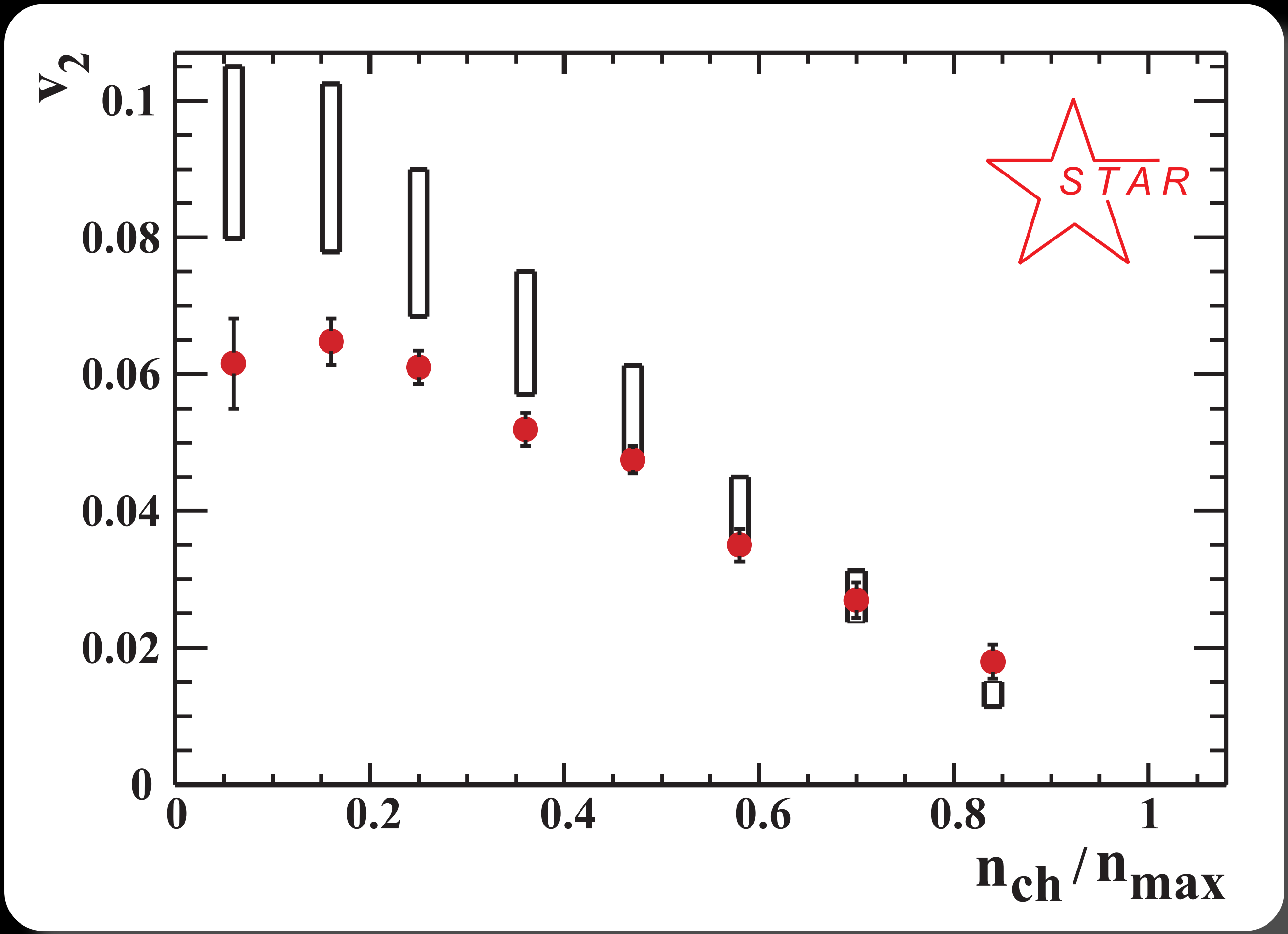


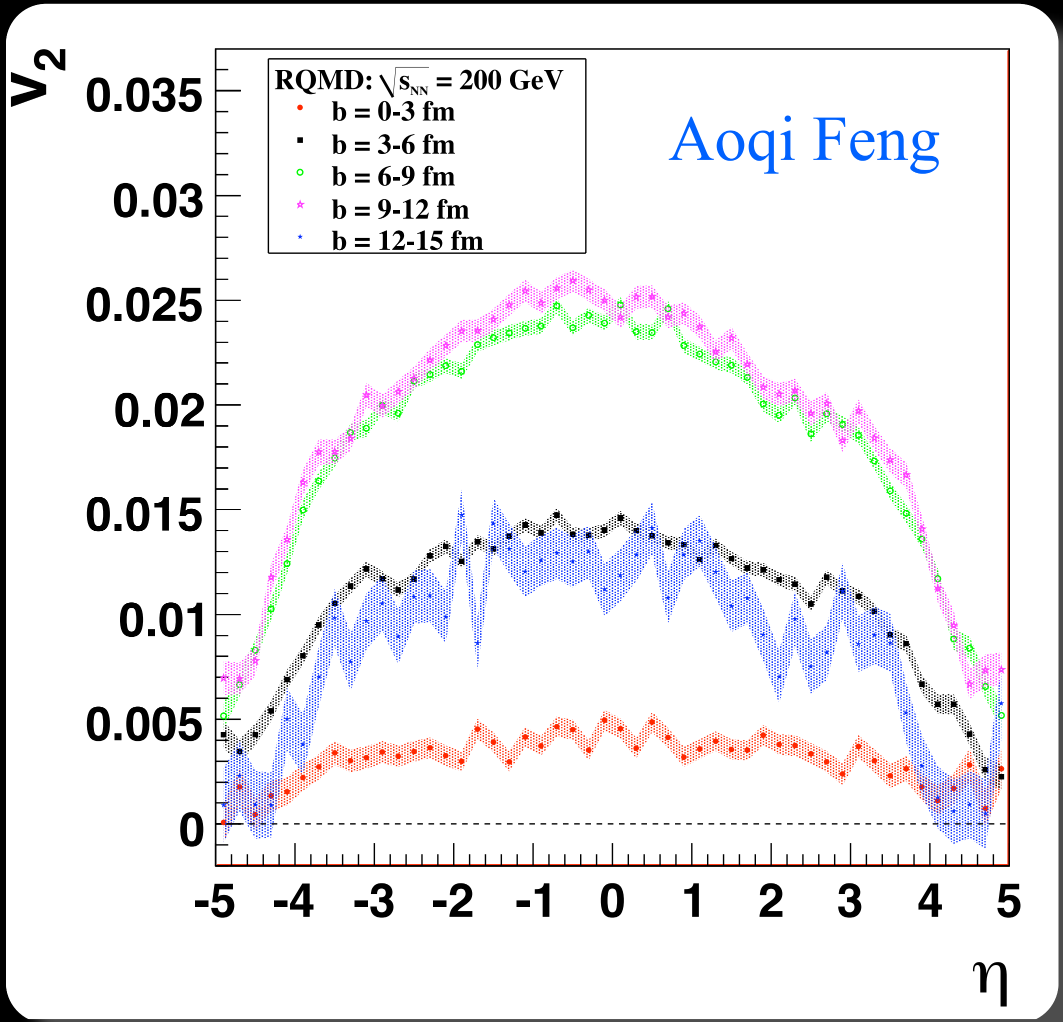
more, faster particles seen in-plane



# Flow at RHIC




STAR Phys. Rev. Lett. 86, 402–407 (2001)

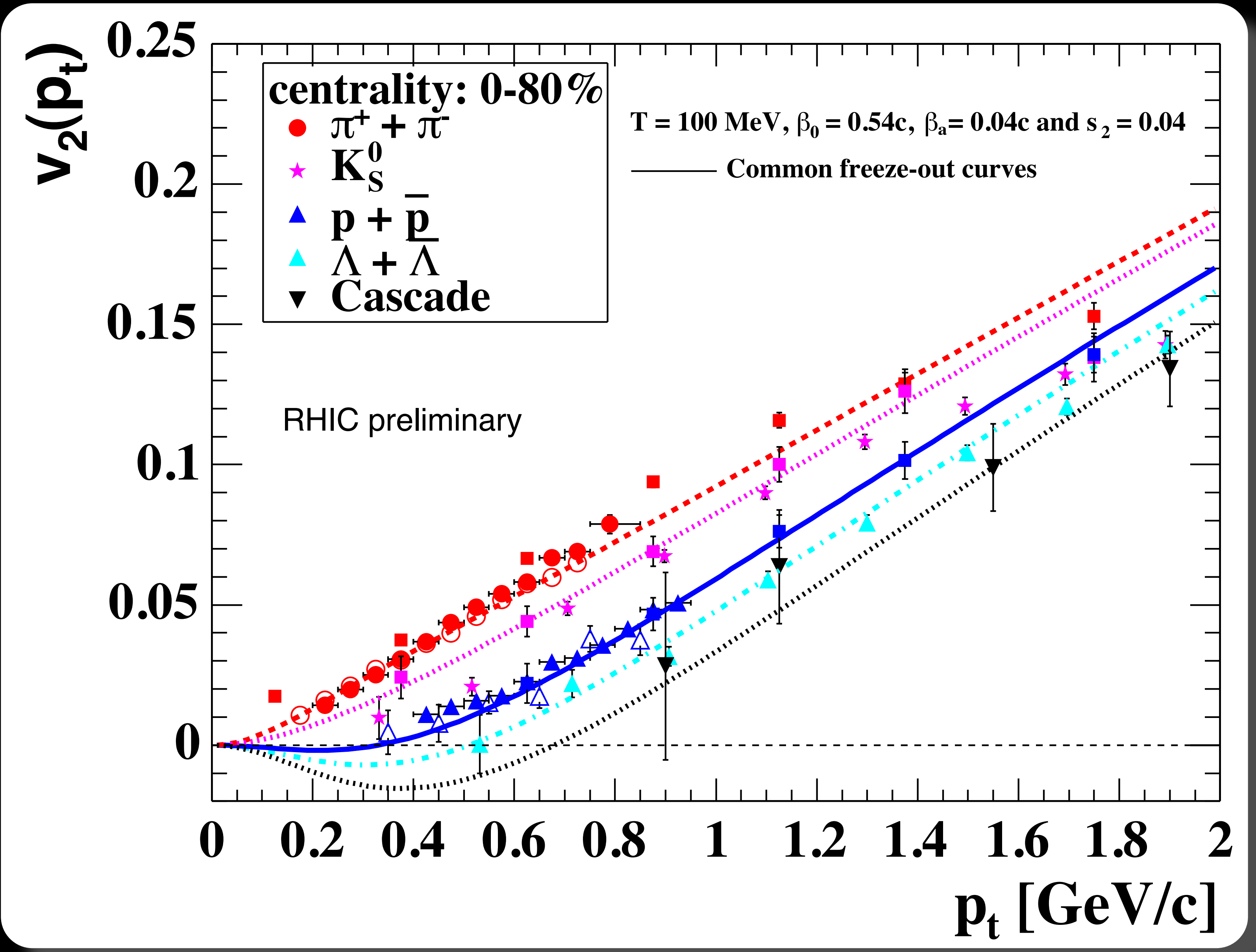



ideal hydro gets the magnitude for more central collisions

hadron transport calculations are factors 2-3 off

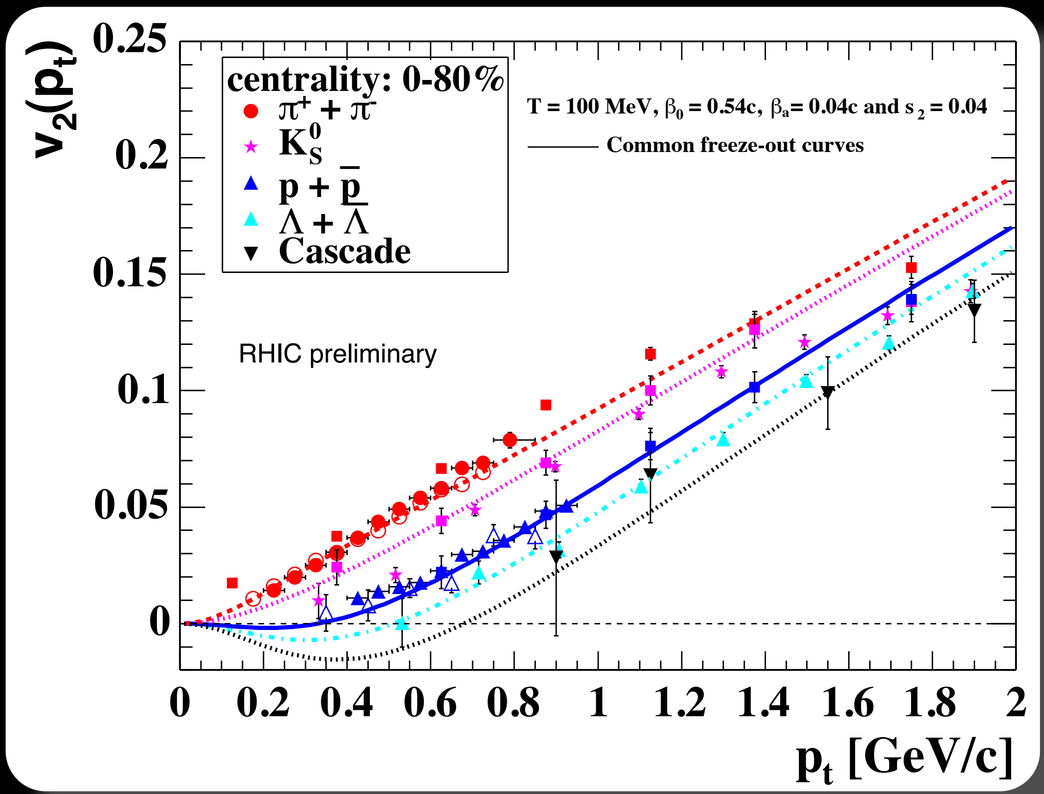
# boosted thermal spectra

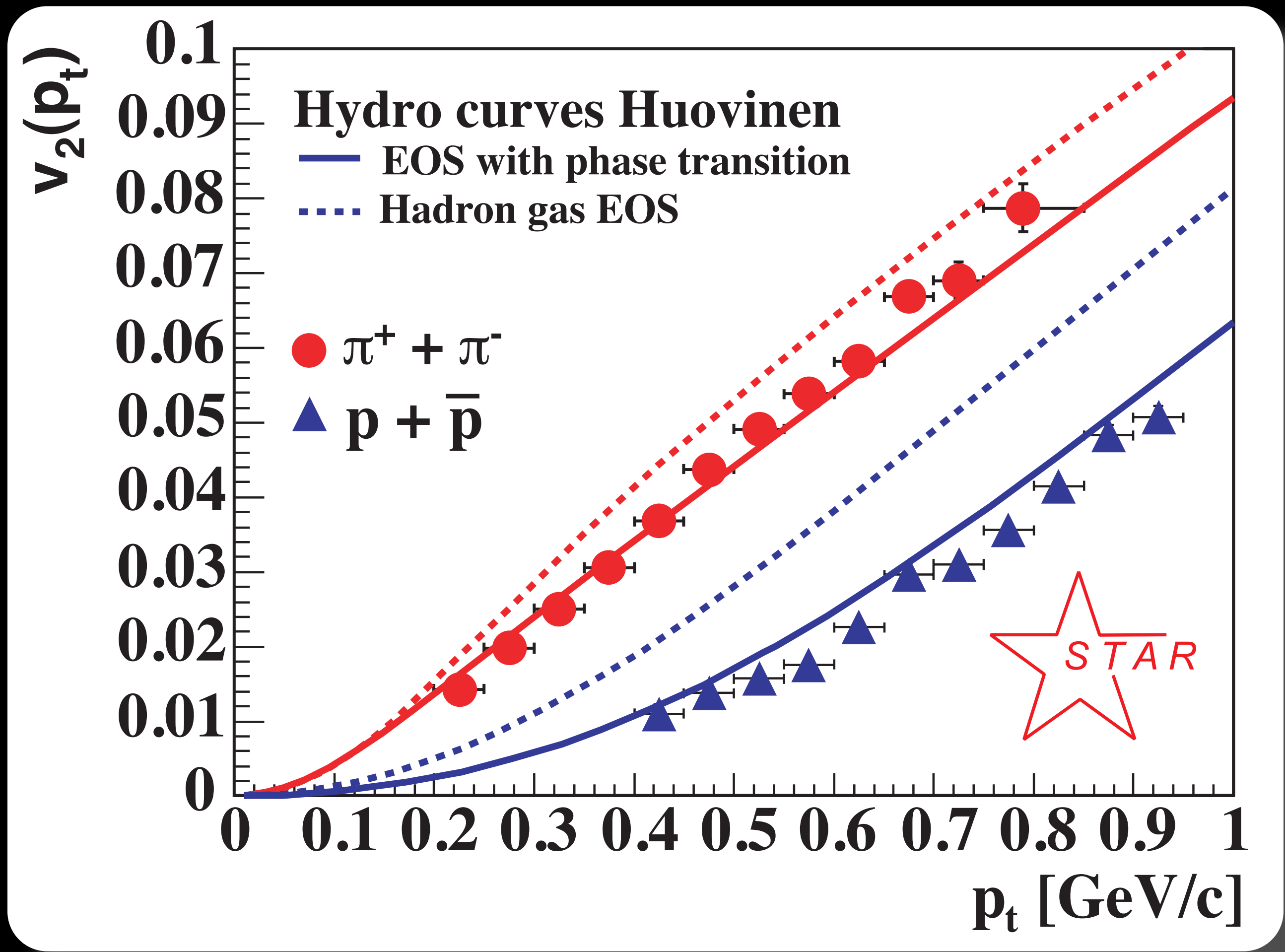
the observed particles are characterized by a single freeze-out temperature and a common azimuthal dependent boost velocity




Fits from STAR Phys. Rev. Lett. 87, 182301 (2001)

# The EoS



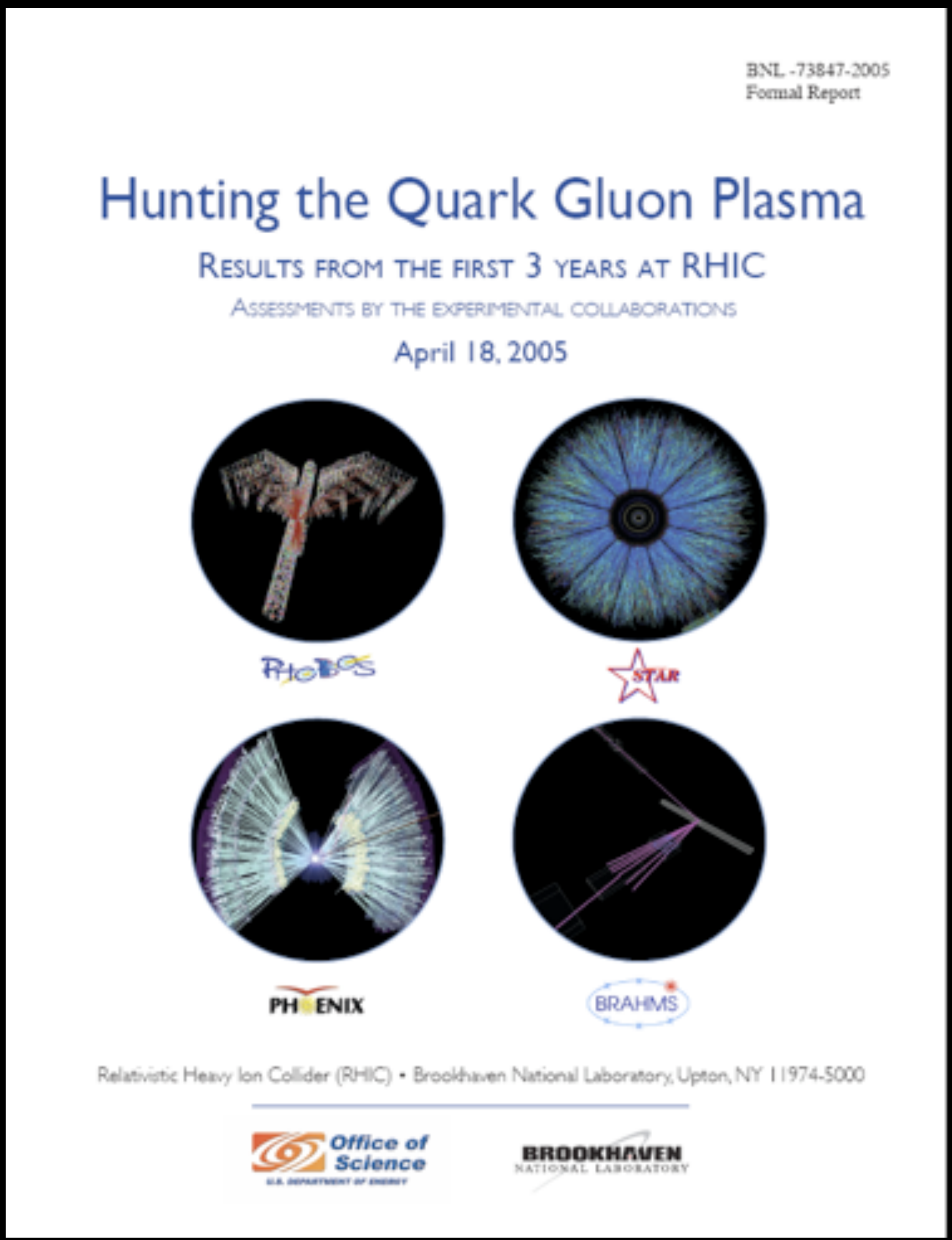

STAR Phys. Rev. Lett. 87, 182301 (2001)

The species dependence is sensitive to the EoS

# RHIC Scientists Serve Up "Perfect" Liquid

## New state of matter more remarkable than predicted -- raising many new questions

## April 18, 2005


BNL -73847-2005
Formal Report

Hunting the Quark Gluon Plasma

Results from the first 3 years at RHIC

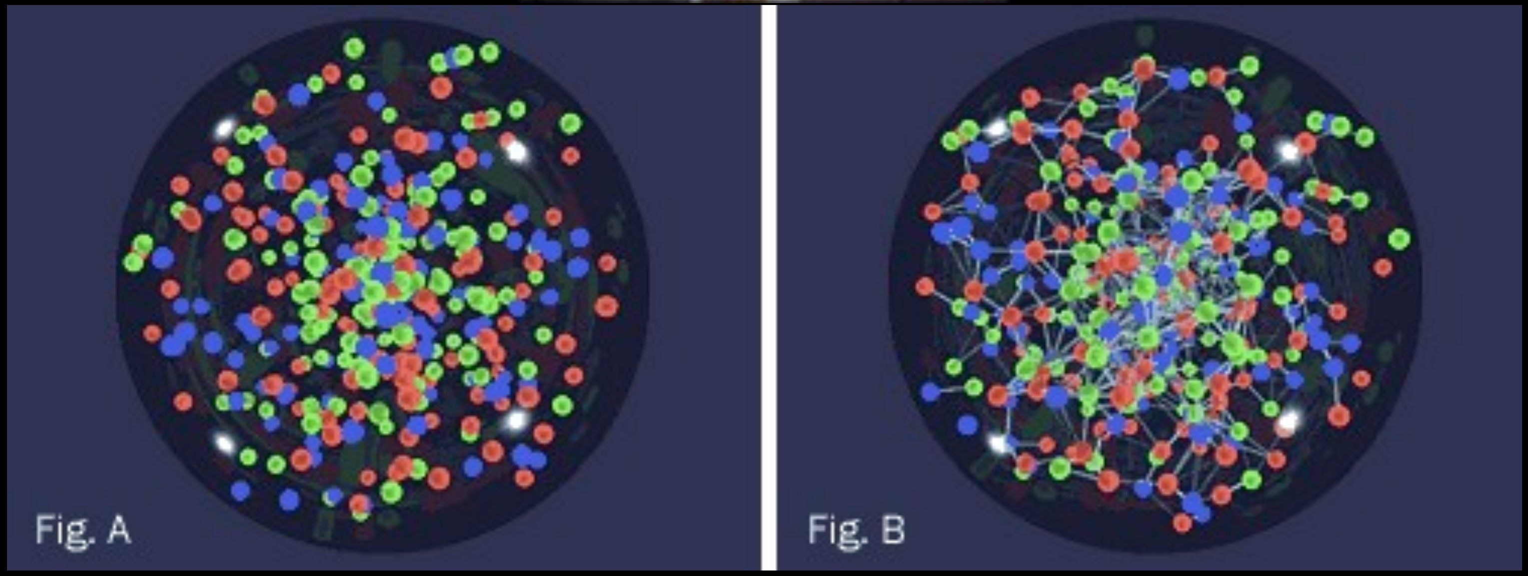
Assessments by the experimental collaborations

April 18, 2005

PHOBOS STAR PHENIX BRAHMS

Relativistic Heavy Ion Collider (RHIC) • Brookhaven National Laboratory, Upton, NY 11974-5000

Office of Science BROOKHAVEN NATIONAL LABORATORY



# RHIC Scientists Serve Up "Perfect" Liquid

## New state of matter more remarkable than predicted -- raising many new questions

## April 18, 2005

### Early Universe Went With the Flow

Posted April 18, 2005 5:57PM

**Between 2000 and 2003 the lab's Relativistic Heavy Ion Collider repeatedly smashed the nuclei of gold atoms together with such force that their energy briefly generated trillion-degree temperatures. Physicists think of the collider as a time machine, because those extreme temperature conditions last prevailed in the universe less than 100 millionths of a second after the big bang.**

### Universe May Have Begun as Liquid, Not Gas

Associated Press
Tuesday, April 19, 2005; Page A05

The Washington Post

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have pervaded the first microseconds of existence.

### Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

by Mark Peplow
news@nature.com

nature

The Universe consisted of a perfect liquid in its first moments, according to results from an atom-smashing experiment.

Scientists at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory on Long Island, New York, have spent five years searching for the quark-gluon plasma that is thought to have filled our Universe in the first microseconds of its existence. Most of them are now convinced they have found it. But, strangely, it seems to be a liquid rather than the expected hot gas.

### New State of Matter Is 'Nearly Perfect' Liquid

Physicists working at Brookhaven National Laboratory announced today that they have created what appears to be a new state of matter out of the building blocks of atomic nuclei, quarks and gluons. The researchers unveiled their findings--which could provide new insight into the composition of the universe just moments after the big bang--today in Florida at a meeting of the American Physical Society.

SCIENTIFIC AMERICAN

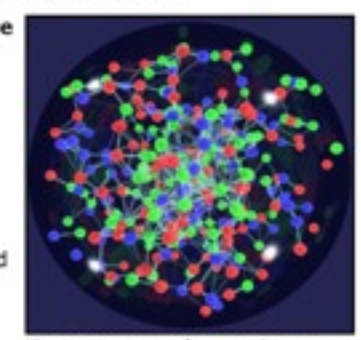
Image: BNL

There are four collaborations, dubbed BRAHMS, PHENIX, PHOBOS and STAR, working at Brookhaven's Relativistic Heavy Ion Collider (RHIC). All of them study what happens when two interacting beams of gold ions smash into one another at great velocities, resulting in thousands of subatomic collisions every second. When the researchers analyzed the patterns of the atoms' trajectories after these collisions, they found that the particles produced in the collisions tended to move collectively, much like a school of fish does. Brookhaven's associate laboratory director for high energy and nuclear physics, Sam Aronson, remarks that "the degree of collective interaction, rapid thermalization and extremely low viscosity of the matter being formed at RHIC make this the most nearly perfect liquid ever observed."

### Early Universe was 'liquid-like'

**Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.**

BBC NEWS



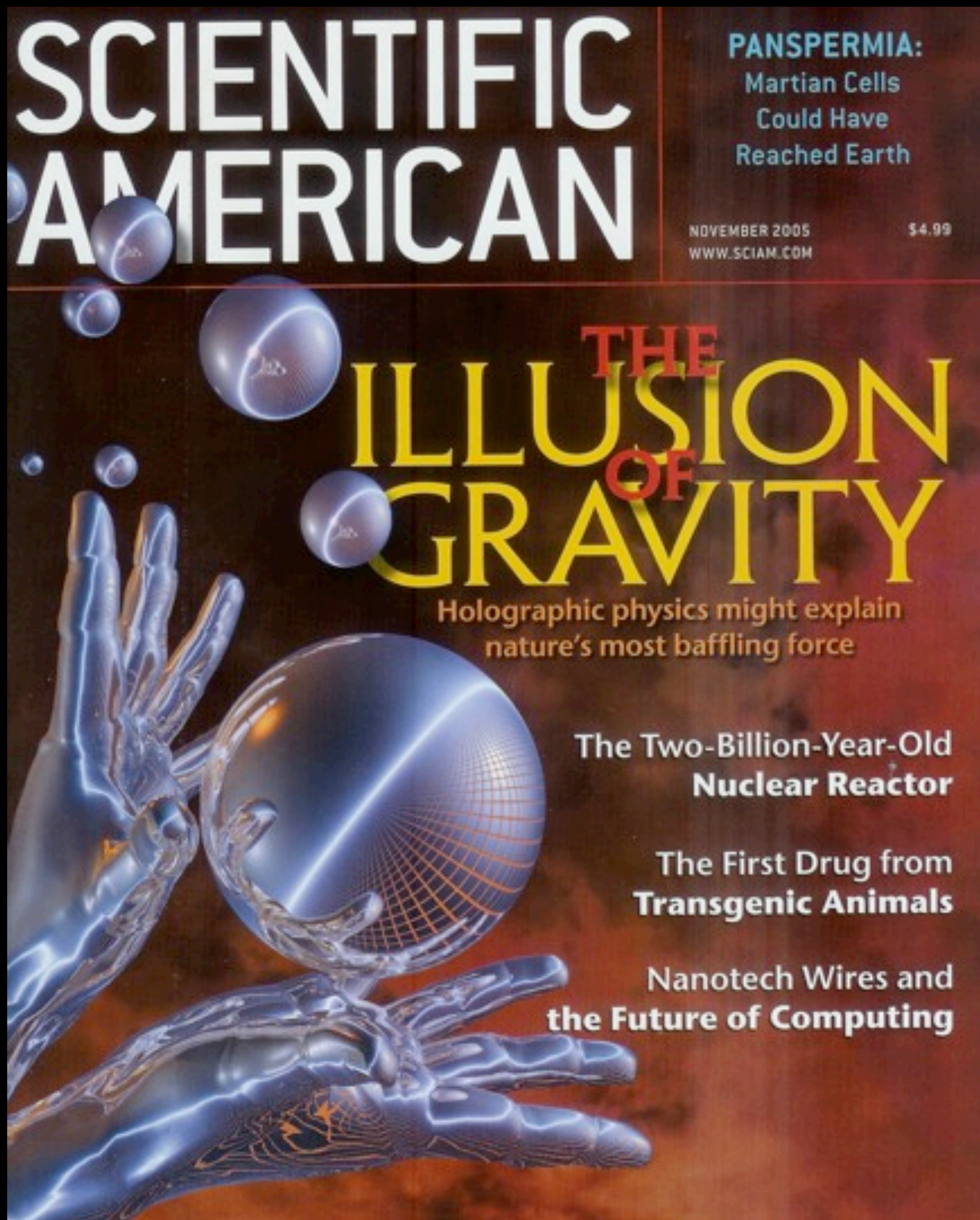
The impression is of matter that is more strongly interacting than predicted

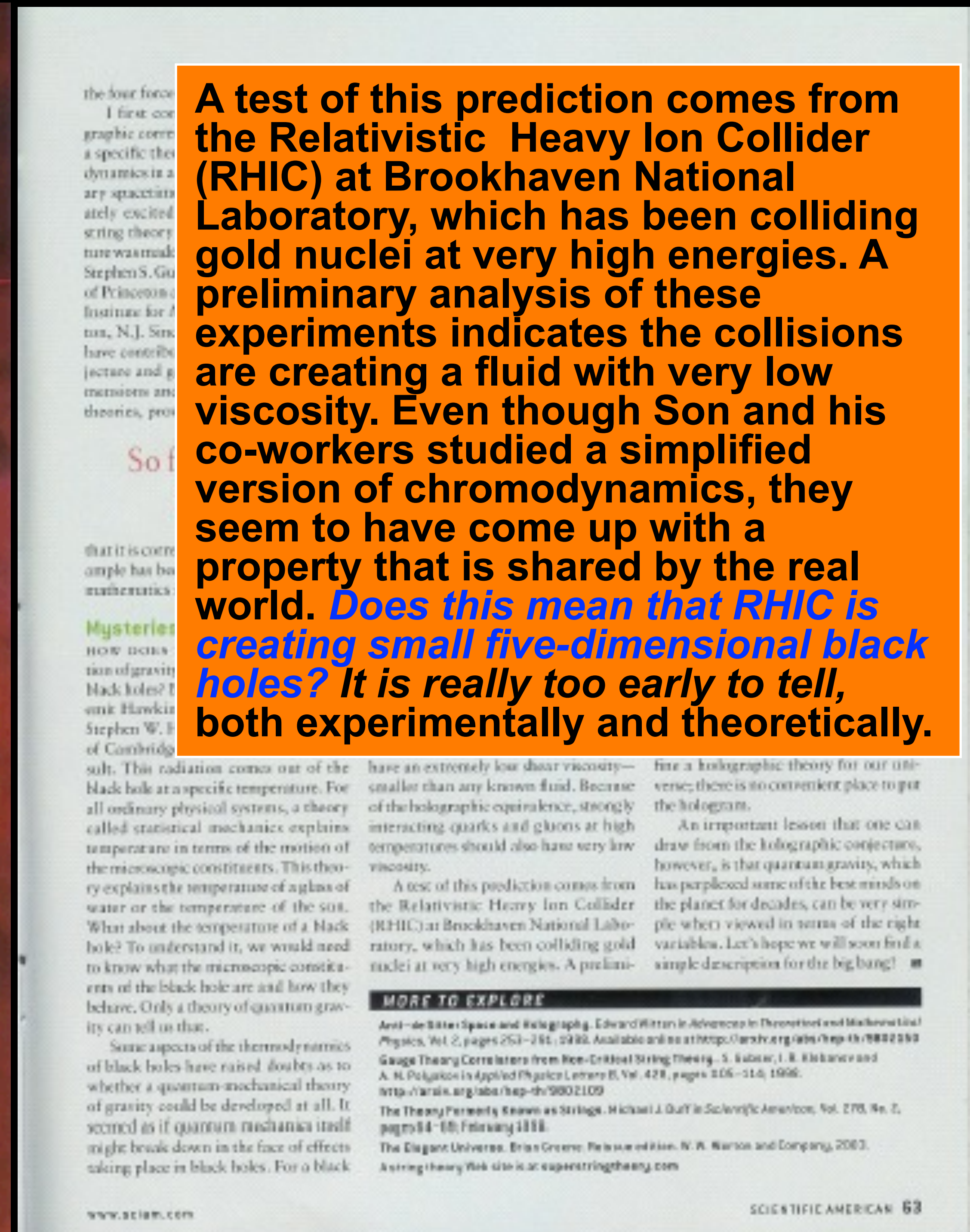
The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".

The work is expected to help scientists explain the conditions that existed just milliseconds after the Big Bang.

# AdS/CFT

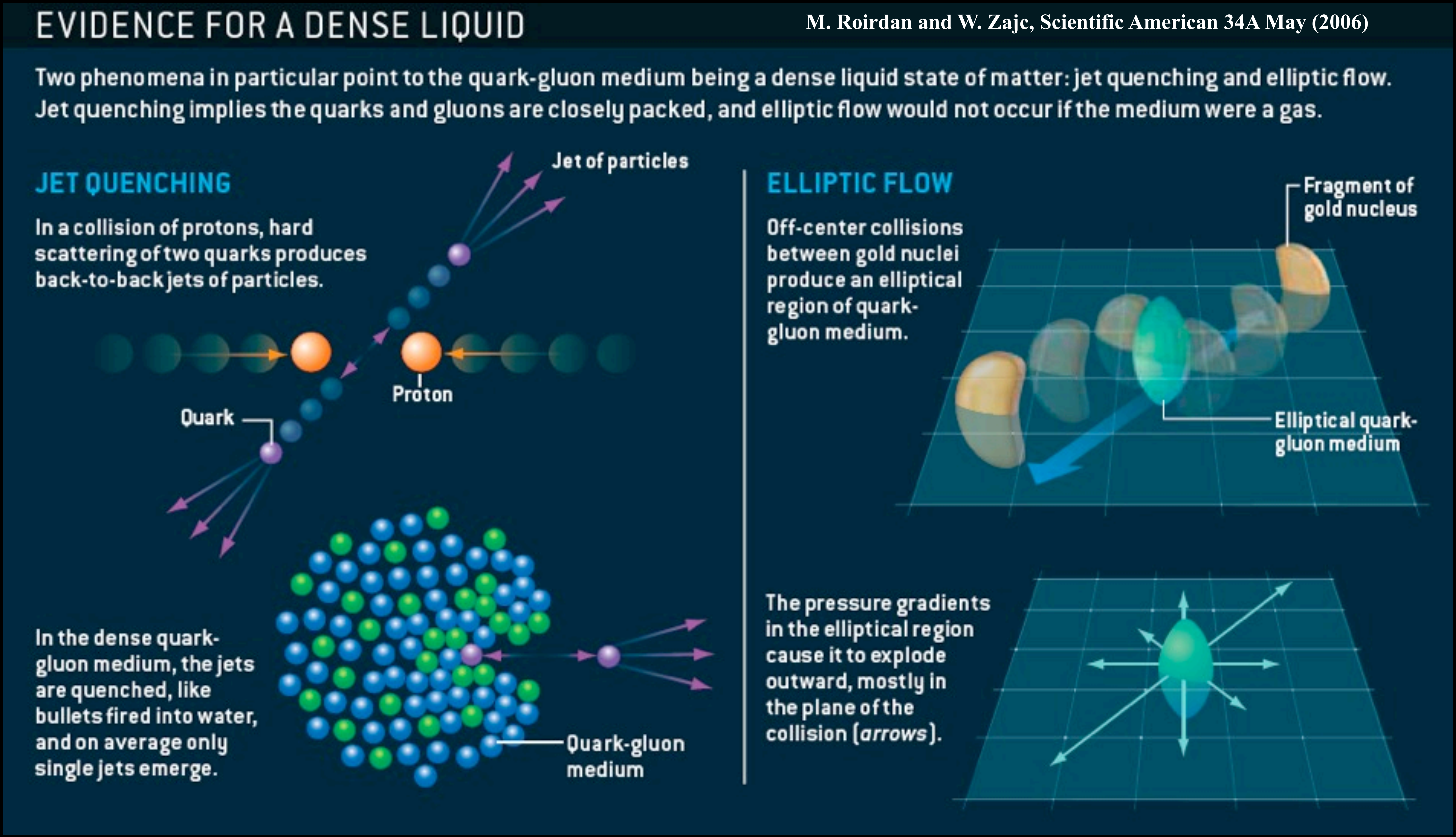


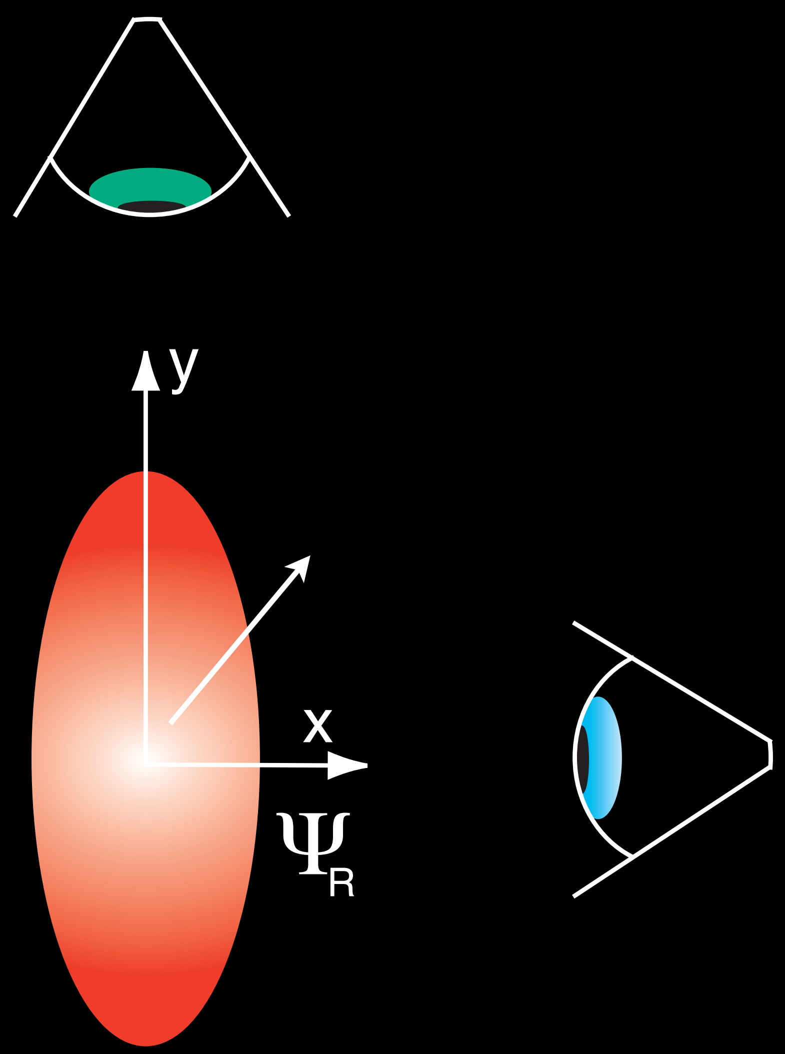
**A test of this prediction comes from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, which has been colliding gold nuclei at very high energies. A preliminary analysis of these experiments indicates the collisions are creating a fluid with very low viscosity. Even though Son and his co-workers studied a simplified version of chromodynamics, they seem to have come up with a property that is shared by the real world. *Does this mean that RHIC is creating small five-dimensional black holes? It is really too early to tell,* both experimentally and theoretically.**

November, 2005 Scientific American “The Illusion of Gravity” J. Maldacena

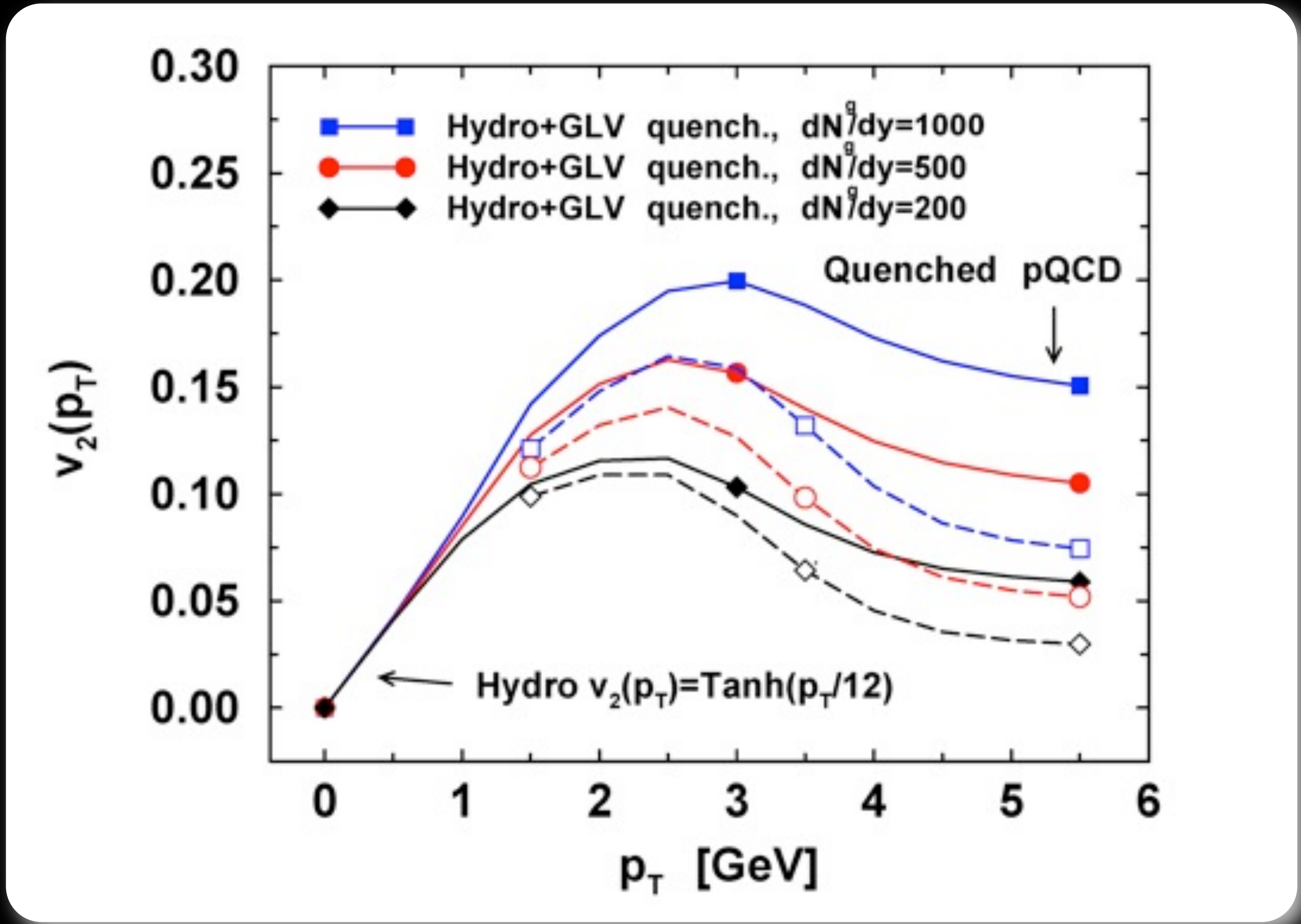
# highlights at RHIC



# parton energy loss



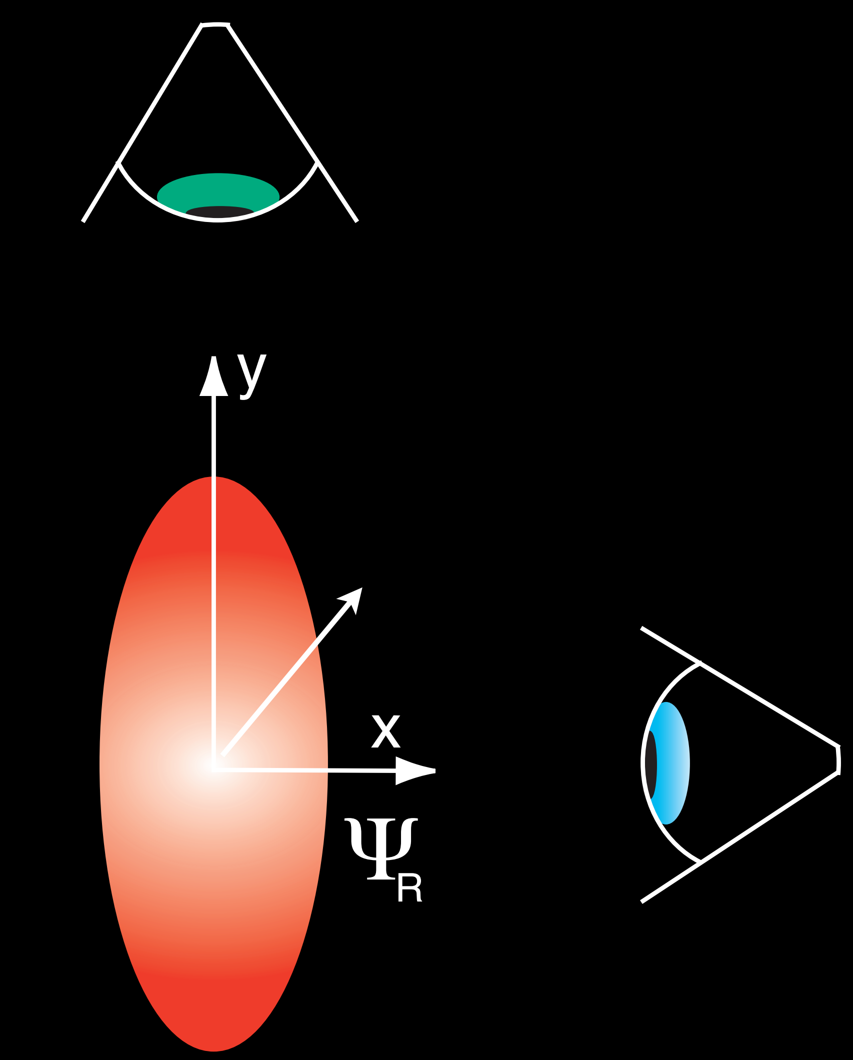

$$v_2 = \langle \cos 2(\phi - \Psi_R) \rangle$$

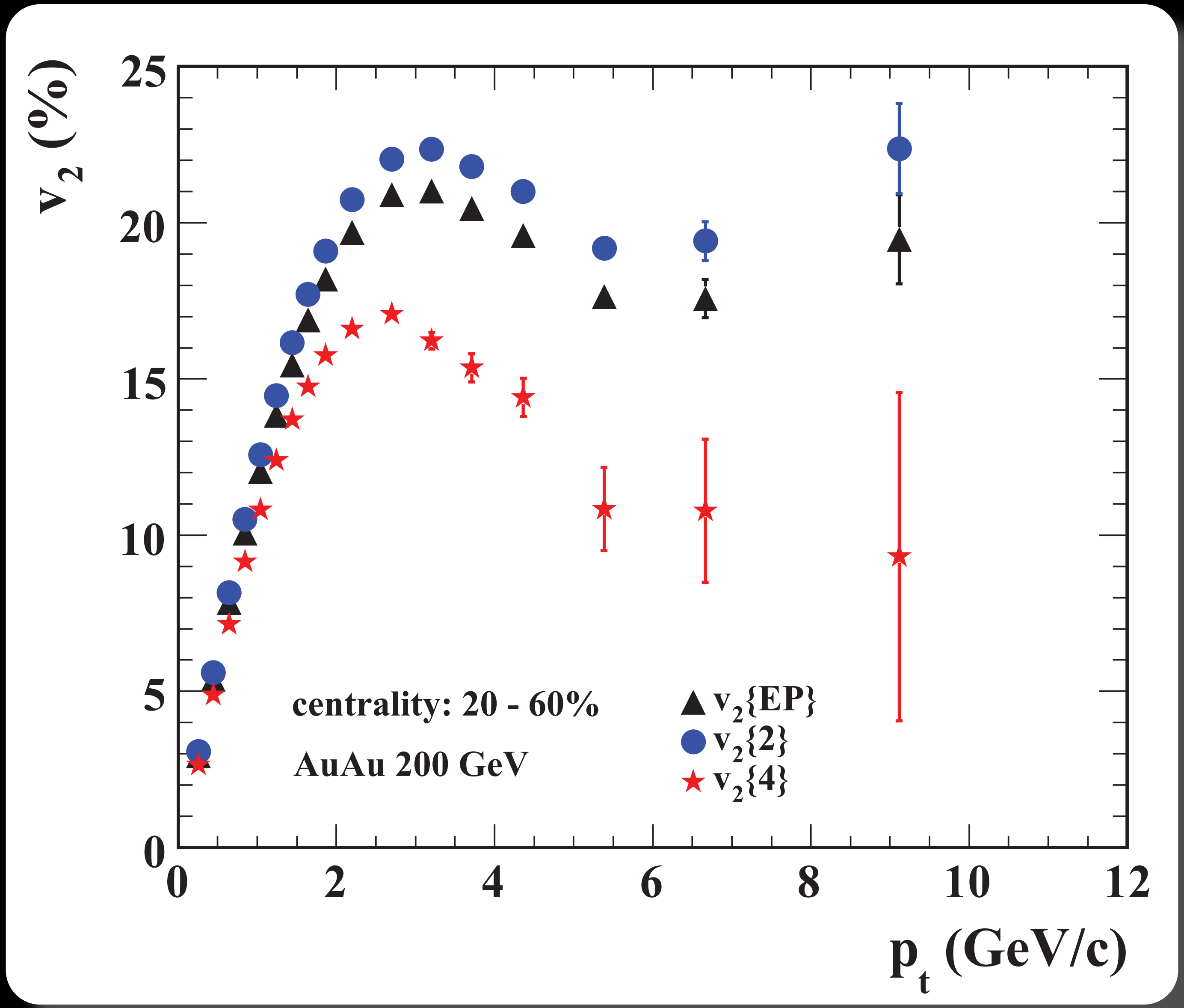



M. Gyulassy, I. Vitev and X.N. Wang
PRL 86 (2001) 2537

R.S, A.M. Poskanzer, S.A.Voloshin,
nucl-ex/9904003

# parton energy loss




$$v_2 = \langle \cos 2(\phi - \Psi_R) \rangle$$

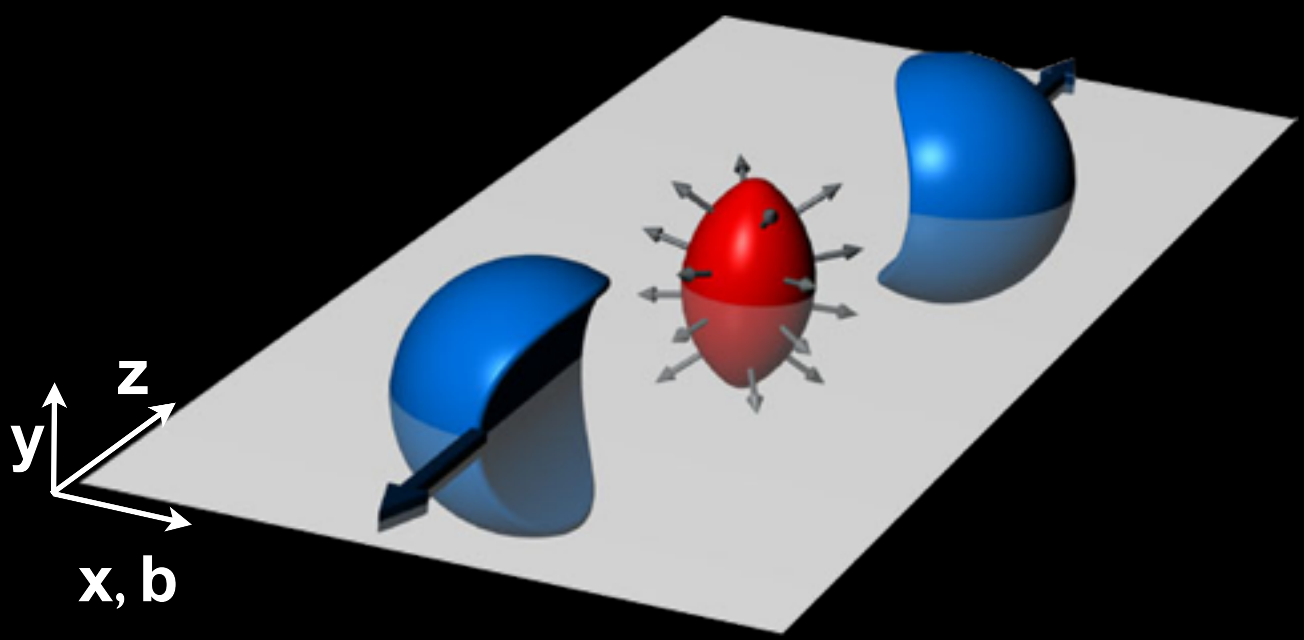
Yuting Bai, Nikhef PhD thesis

**strong path length dependence observed!**

# Summary

- event anisotropy is a powerful tool
  - provides access to equation of state of hot and dense matter
  - provides access to transport properties like viscosity and parton energy loss

# Anisotropic Flow




Azimuthal distributions of particles measured with respect to the reaction plane (spanned by impact parameter vector and beam axis) are not isotropic.

$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{\mathrm{RP}}\right)\right)\right)$$

$$v_n = \langle \cos n(\phi - \Psi_{\mathrm{RP}})\rangle \qquad v_n = \left\langle e^{in(\phi_1-\Psi_R)}\right\rangle$$

harmonics $v_n$ quantify anisotropic flow

S. Voloshin and Y. Zhang (1996)

# measure anisotropic flow

- since reaction plane cannot be measured event-by-event, consider quantities which do not depend on it's orientation: multi-particle azimuthal correlations

$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \left\langle e^{in\phi_1}\right\rangle\left\langle e^{-in\phi_2}\right\rangle + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\mathrm{corr}}$$

zero for symmetric detector when averaged over many events

$$\begin{aligned}\langle\langle 2\rangle\rangle \equiv \left\langle\!\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\!\right\rangle &= \left\langle\!\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}}-(\phi_2-\Psi_{\mathrm{RP}}))}\right\rangle\!\right\rangle \\ &= \left\langle\!\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}})}\right\rangle\left\langle e^{-in(\phi_2-\Psi_{\mathrm{RP}})}\right\rangle\!\right\rangle \\ &= \left\langle v_n^2\right\rangle\end{aligned}$$

- assuming that only correlations with the reaction plane are present

# intermezzo

- why do we define the correlations like this:

$$\Big\langle \langle x \rangle_{\text{particles in single event}} \Big\rangle_{\text{over events}}$$

  - easy to relate to $v_n$

$$\left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle$$

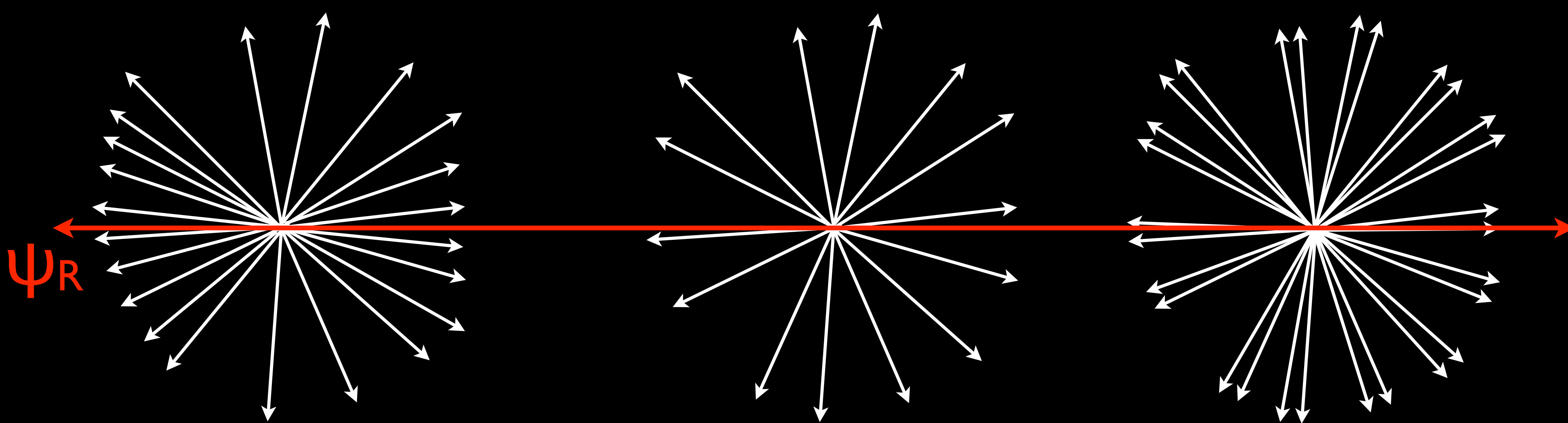
$$\left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\right\rangle$$

  - vanishes for independent particles
  - does not depend on frame $\Phi + \alpha$ (shifting all particles by fixed angle) gives same answer for the correlation

# nonflow

- however, there are other sources of correlations between the particles which are not related to the reaction plane which break the factorization, lets call those $\delta_2$ for two particle correlations

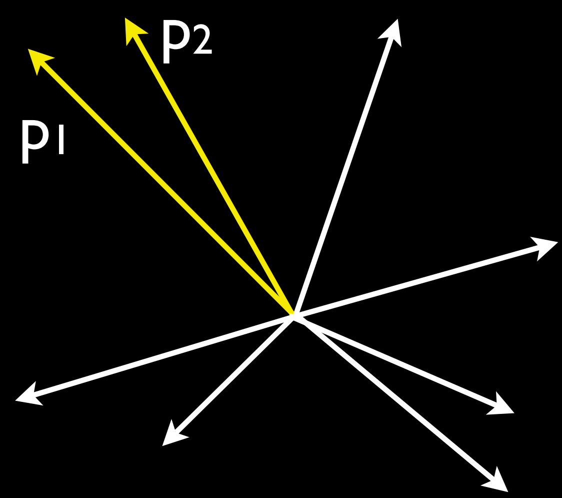
$$\left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle = \langle v_n^2 \rangle + \delta_2$$




$v_2 > 0$, $v_2\{2\} > 0$ $v_2 = 0$, $v_2\{2\} = 0$ $v_2 = 0$, $v_2\{2\} > 0$

# nonflow

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = \langle v_n^2 \rangle + \delta_2$$




particle 1 coming from the resonance. Out of remaining M-1 particles there is only one which is coming from the same resonance, particle 2. Hence a probability that out of M particles we will select two coming from the same resonance is ~ 1/(M-1). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$

- not easily satisfied: M=200 $v_n$ >> 0.07

# can we do better?


- use the fact that flow is a correlation between all particles: use multi-particle correlations

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = v_n^2 + \delta_2$$

$$\left\langle\!\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\!\right\rangle = v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 + \delta_4$$

- not so clear if we gained something

# Can we do better?


- build cumulants with the multi-particle correlations

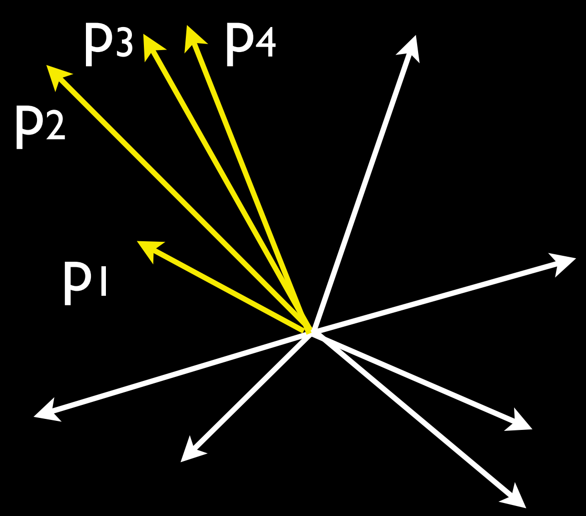
  Ollitrault and Borghini

- for detectors with uniform acceptance 2nd and 4th cumulant are given by:

$$
\begin{aligned}
c_n\{2\} &\equiv \left\langle\!\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\!\right\rangle = v_n^2 + \delta_2 \\
c_n\{4\} &\equiv \left\langle\!\!\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\!\!\right\rangle - 2\left\langle\!\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\!\right\rangle^2 \\
&= v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 - 2(v_n^2+\delta_2)^2 + \delta_4 \\
&= -v_n^4 + \delta_4
\end{aligned}
$$

- got rid of two particle non-flow correlations!

# Can we do better?


Particle 1 coming from the mini-jet. To select particle 2 we can make a choice out of remaining M-1 particles; once particle 2 is selected we can select particle 3 out of remaining M-2 particles and finally we can select particle 4 out of remaining M-3 particles. Hence the probability that we will select randomly four particles coming from the same resonance is 1/(M-1)(M-2)(M-3). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M, \qquad \delta_4 \sim 1/M^3$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$
$$v_n^4 \gg 1/M^3 \quad \Rightarrow \quad v_n \gg 1/M^{3/4}$$

# Can we do better?


- it is possible to extend this:

$$v_n^{2k} \gg 1/M^{2k-1} \Rightarrow v_n \gg 1/M^{\frac{2k-1}{2k}}$$

- for large k (or even M particle correlations e.g. Lee Yang Zeroes)

$$v_n \gg 1/M$$

- as an example: M=200 $v_n$ >> 0.005 (more than order of magnitude better than two particle correlations)
- to reliably measure small flow in presence of other correlations one needs to use multi-particle correlations!

# Calculate Correlations

## (using nested loops)

To evaluate average 2-particle correlation

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1-\phi_2)} \right\rangle = \frac{1}{\binom{M}{2}2!} \sum_{\substack{i,j=1 \\ (i\neq j)}}^{M} e^{in(\phi_i-\phi_j)}$$

in a nested loop # operations

$$\frac{1}{2!}\frac{M!}{(M-2)!}$$

- With M=1000, this approach already for 4-particle correlations gives $1.2 \times 10^{12}$ operations per event!
- calculation of average 6-particle correlation requires roughly $1.4 \times 10^{17}$ operations, and of average 8-particle correlation roughly $8.4 \times 10^{21}$ operations per event
- clearly not the way to go

# Calculate Correlations

(using Q-cumulants)

A. Bilandzic, RS, S. Voloshin (2010?)

azimuthal two particle correlations:

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1-\phi_2)} \right\rangle = \frac{1}{\binom{M}{2}2!} \sum_{\substack{i,j=1 \\ (i\neq j)}}^{M} e^{in(\phi_i-\phi_j)}$$

definition of Q vector of harmonic n

$$Q_n \equiv \sum_{i=1}^{M} e^{in\phi_i}$$

can write two particle correlation in terms of Q vector of harmonic n

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

# Calculate Correlations

(using Q-cumulants)

two particle correlations can be expressed in Q vectors

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

but also four particle correlations (and more)

note the mixed harmonics

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2 \cdot \mathbf{Re}\left[Q_{2n} Q_n^* Q_n^*\right] - 4(M-2) \cdot |Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

with this it becomes trivial to make cumulants again

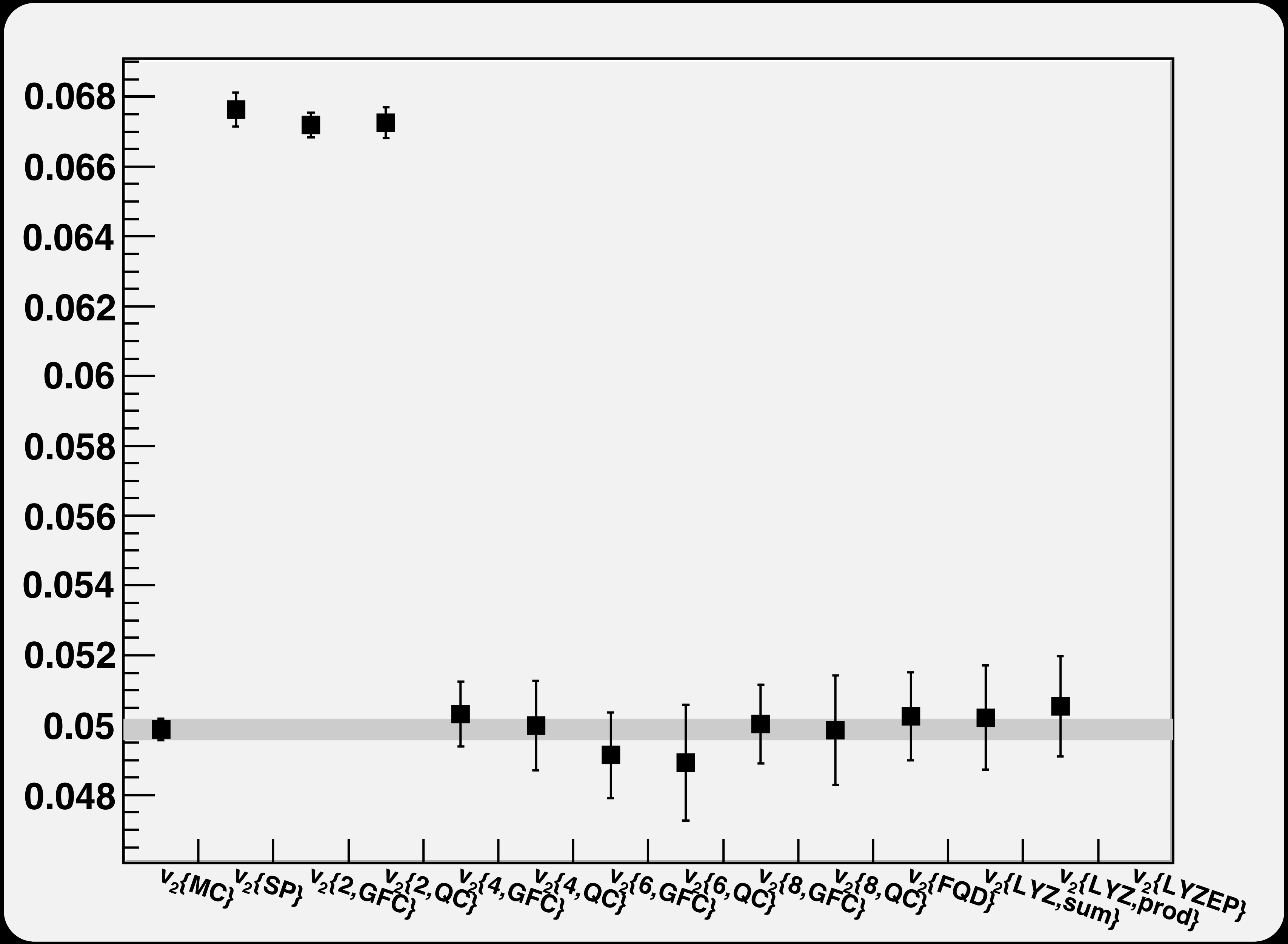
# Calculate Correlations

(using Q-cumulants)

- pros Q-cumulants
  - exact solutions, give same answer as nested loops
  - one loop over data enough to calculate all multi-particle correlations
  - number of operations to get all multi-particle correlations up to $8^{th}$ order is 4 x 2 x Multiplicity
    - for multiplicities of ~ 1000 the number of operations is reduced by a factor $10^{18}$ (helps to get your PhD degree in time)

# nonflow example

Example: input $v_2 = 0.05$, $M = 500$, $N = 5 \times 10^4$ and simulate nonflow by taking each particle twice




as expected only two particle methods are biased

# Flow Fluctuations

Both two and multi-particle correlations have an extra feature one has to keep in mind!

- By using multi-particle correlations to estimate flow we are actually estimating the averages of various powers of flow

$$\langle\langle 2\rangle\rangle = \left\langle v^2\right\rangle \,, \quad \langle\langle 6\rangle\rangle = \left\langle v^6\right\rangle$$
$$\langle\langle 4\rangle\rangle = \left\langle v^4\right\rangle \,, \quad \langle\langle 8\rangle\rangle = \left\langle v^8\right\rangle$$

- But what we are after is: $\langle v\rangle$

# Flow Fluctuations

- in general: take a random variable x with mean $\mu_x$ and spread $\sigma_x$ . The the expectation value of some function of a random variable x, E[h(x)], is to leading order given by

$$\langle h(x)\rangle \equiv E[h(x)] = h(\mu_x) + \frac{\sigma_x^2}{2} h''(\mu_x)$$

- using this for the flow results:

$$\begin{aligned} \langle v^2\rangle &= \langle v\rangle^2 + \sigma_v^2 \\ \langle v^4\rangle &= \langle v\rangle^4 + 6\sigma_v^2 \langle v\rangle^2 \\ \left\langle v^6\right\rangle &= \langle v\rangle^6 + 15\sigma_v^2 \langle v\rangle^4 \\ \langle v^8\rangle &= \langle v\rangle^8 + 28\sigma_v^2 \langle v\rangle^6 \end{aligned}$$

- remember cumulants are combinations of these quantities

# Flow Fluctuations

- flow estimates from cumulants can be written as:

$$
\begin{aligned}
v\{2\} &= \left\langle v^2 \right\rangle^{1/2} \\
v\{4\} &= \left(-\left\langle v^4 \right\rangle + 2\left\langle v^2 \right\rangle^2\right)^{1/4} \\
v\{6\} &= \left[\frac{1}{4}\left(\left\langle v^6 \right\rangle - 9\left\langle v^2 \right\rangle\left\langle v^4 \right\rangle + 12\left\langle v^2 \right\rangle^3\right)\right]^{1/6} \\
v\{8\} &= \left[-\frac{1}{33}\left[\left\langle v^8 \right\rangle - 16\left\langle v^6 \right\rangle\left\langle v^2 \right\rangle - 18\left\langle v^4 \right\rangle^2\right.\right. \\
&\qquad \left.\left. +144\left\langle v^4 \right\rangle\left\langle v^2 \right\rangle^2 - 144\left\langle v^2 \right\rangle^4\right]\right]^{1/8}
\end{aligned}
$$

- take the expression from previous slide and use:

$$\sigma_v \ll \langle v \rangle$$

- take up to order $\sigma^2$, the surprisingly simple result is:

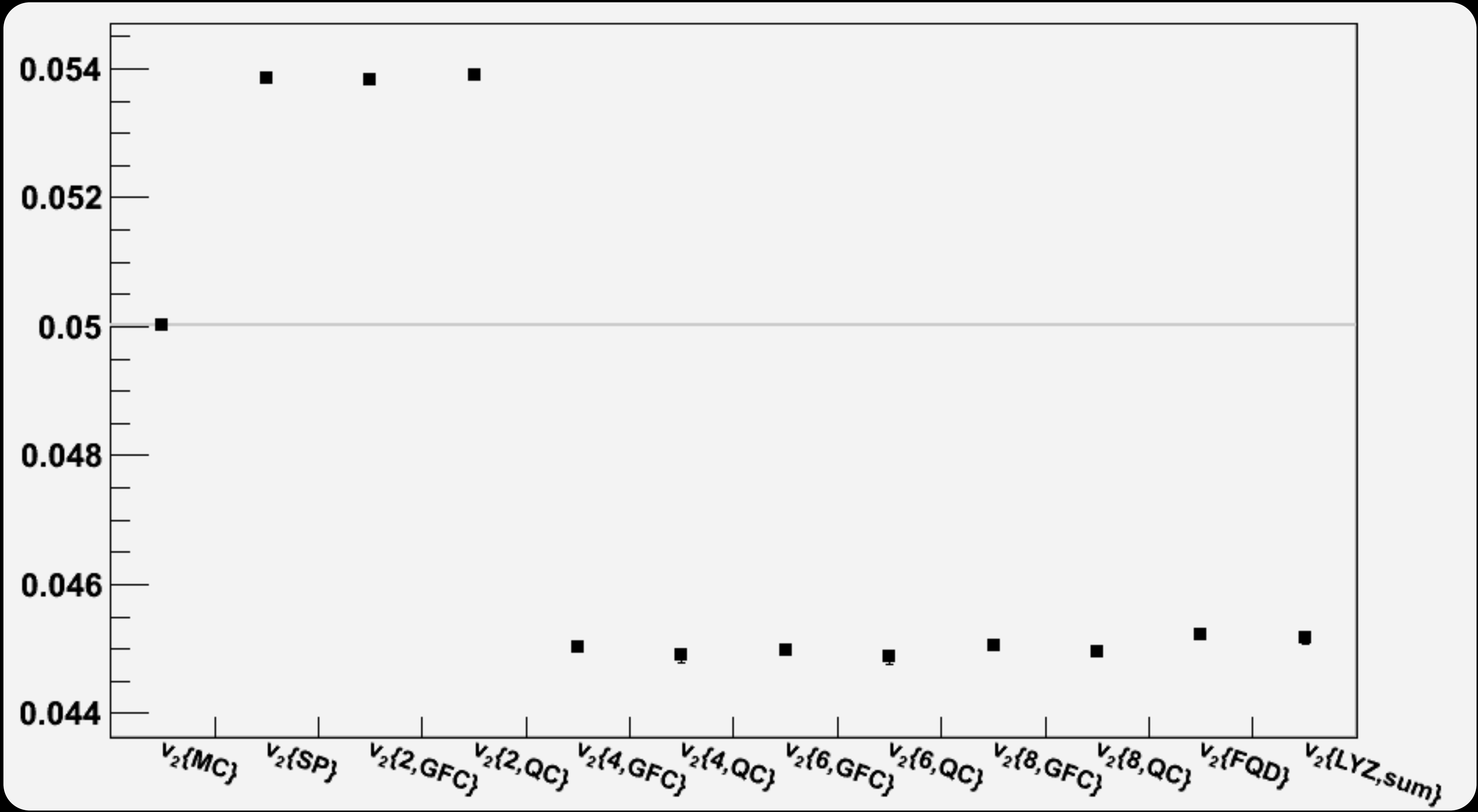
# Flow Fluctuations

$$\begin{aligned} v\{2\} &= \langle v \rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle} \\ v\{4\} &= \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle} \\ v\{6\} &= \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle} \\ v\{8\} &= \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle} \end{aligned}$$

- for $\sigma_v$ << <v> this is a general result to order $\sigma^2$

# Flow Fluctuations

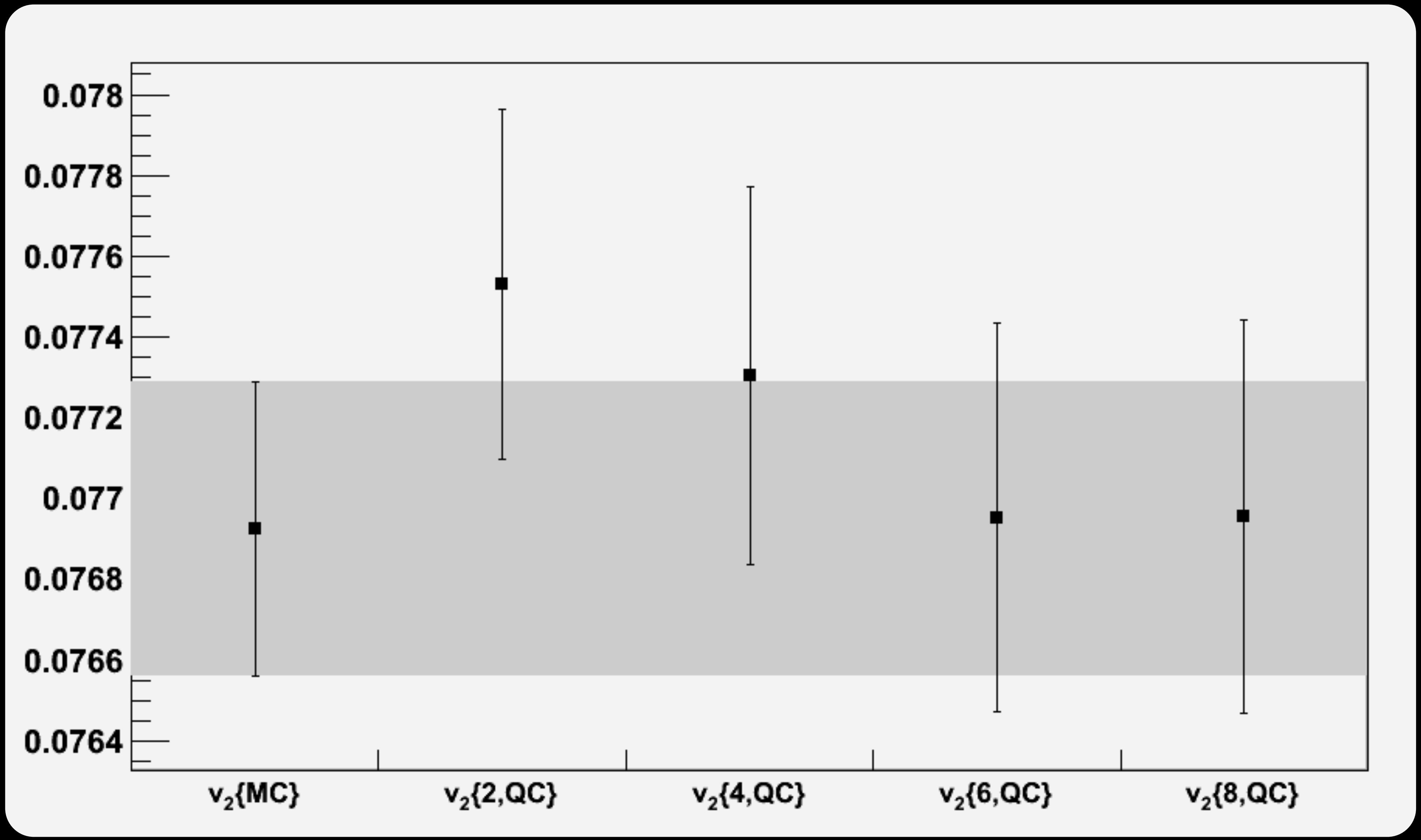
Example: input $v_2$ = 0.05 +/- 0.02 (Gausian), $M$ = 500, $N = 1 \times 10^6$




Gaussian fluctuation behave as predicted also for Lee Yang Zeroes and fitting Q distribution (more on that later)

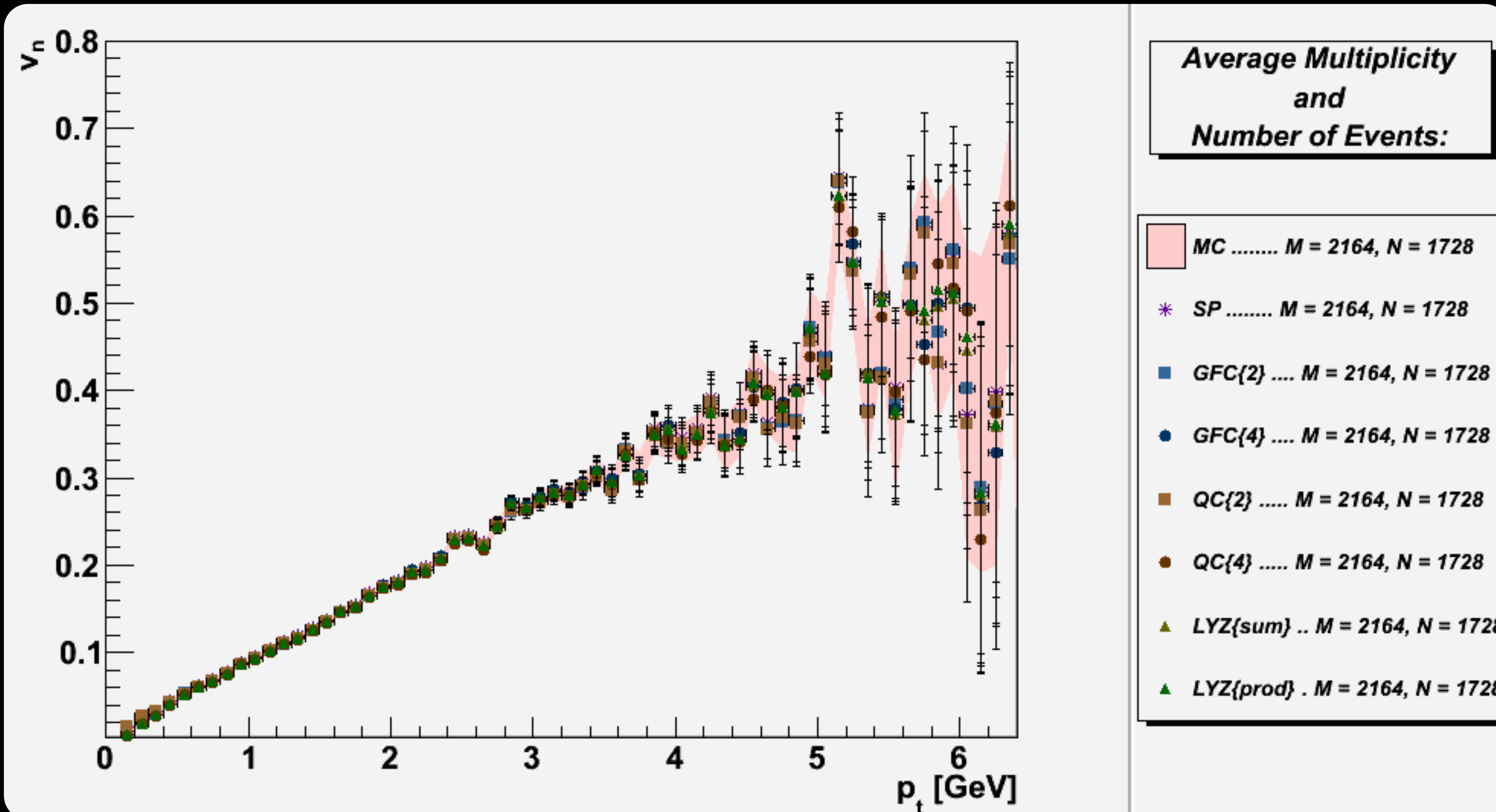
# Statistical Uncertainty

Therminator "realistic" LHC events (<M>=2000 and N = 2000 )



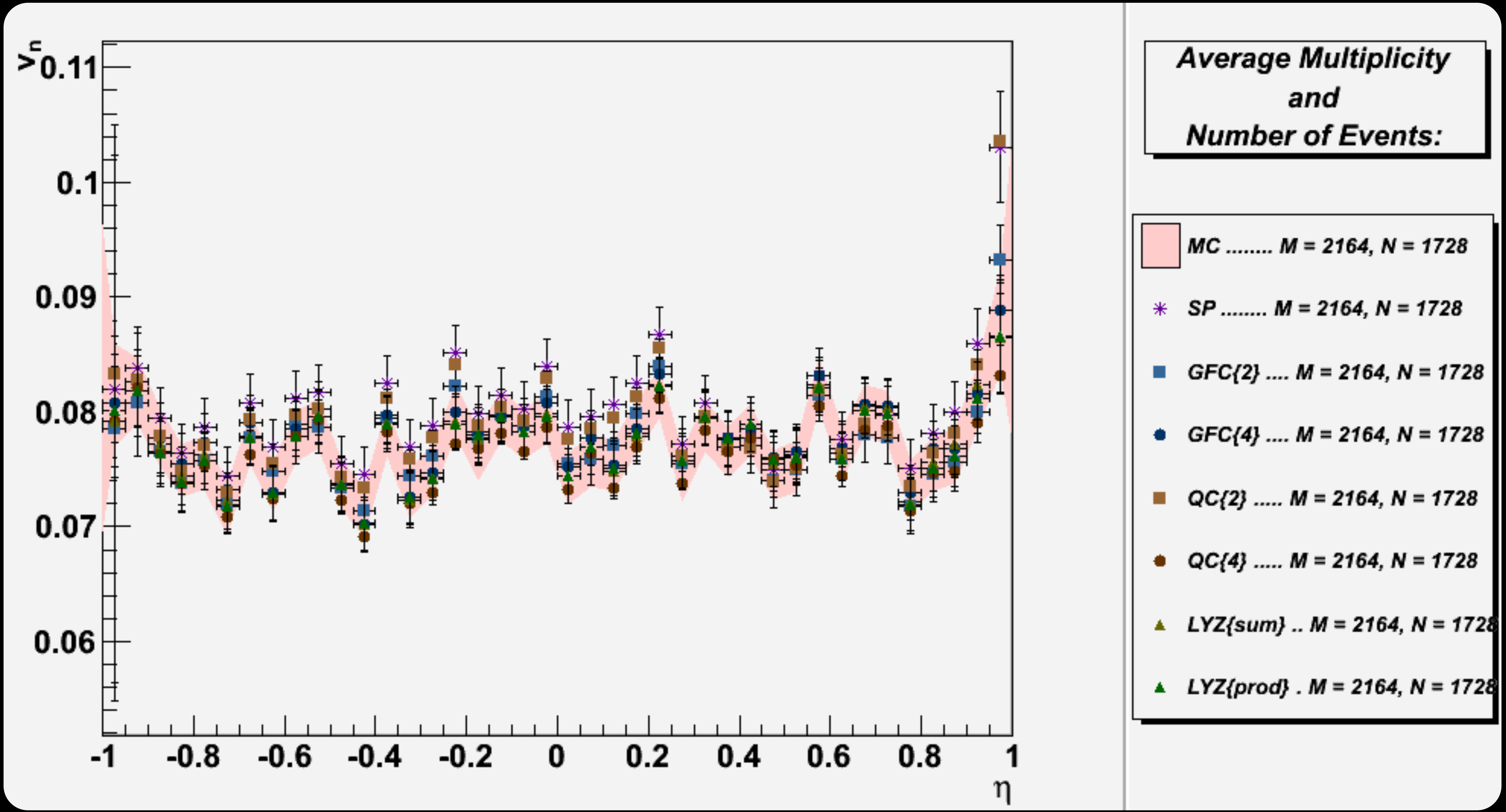

in the regime of sizable flow these multi particle estimates are a precision method!

# Precision Method




only 2000 events!

# Precision Method



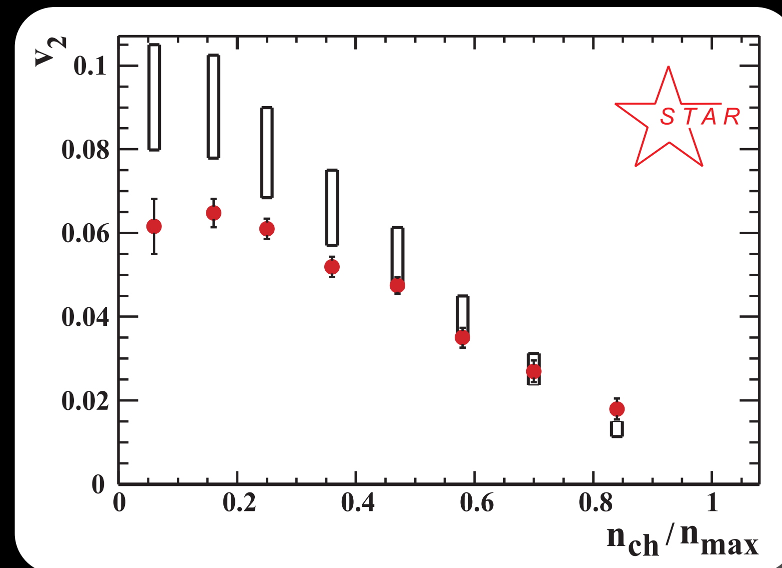

only 2000 events!

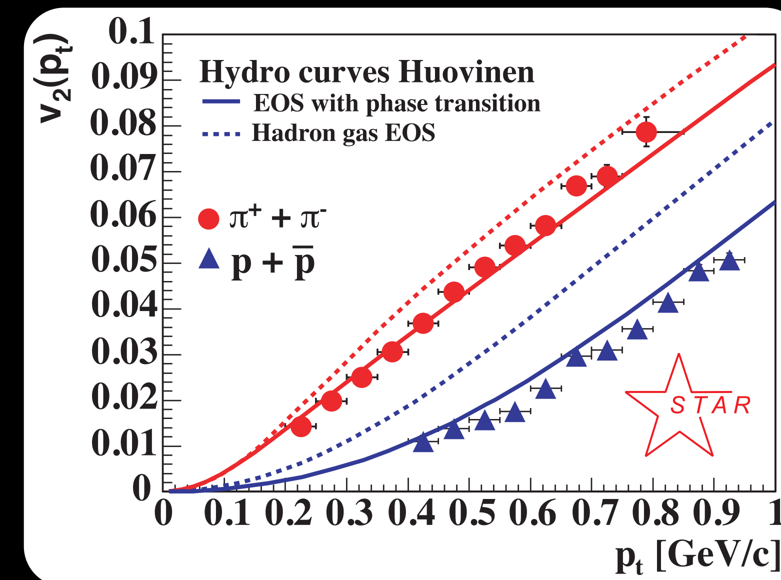
# Summary Methods

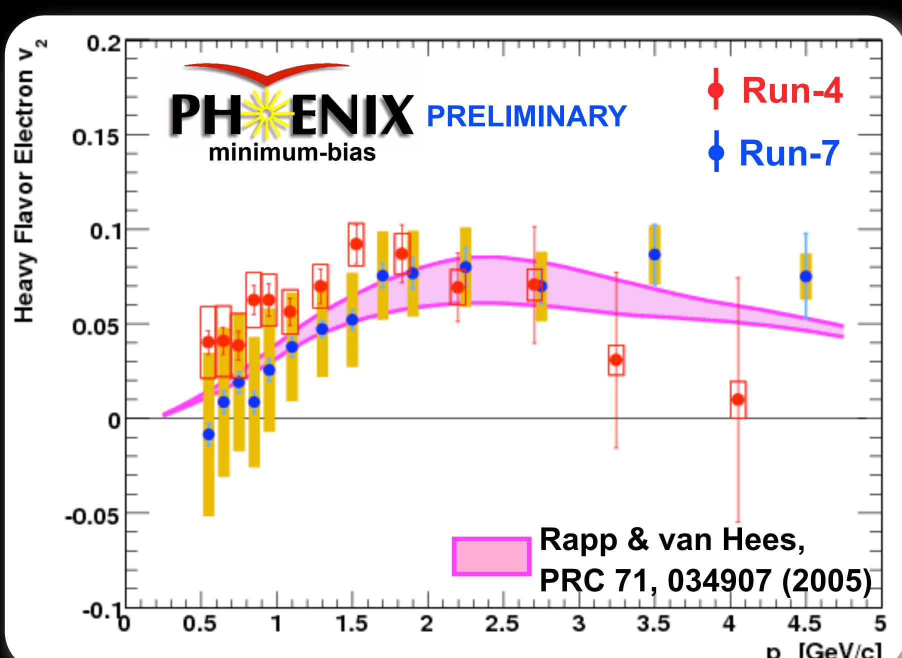
- all methods behave differently (not a bad thing!)
- two particle methods (including event plane method) are very sensitive to nonflow
- all methods are effected by event-by-event fluctuations of the flow
  - but for most cases this happens in a controlled way (although we can not disentangle nonflow and fluctuations yet)
- being able to correct for detector effects is important and the best correction is done in one pass over the data
- when other harmonics are sizable (certainly when they dominate) one should be careful with some methods

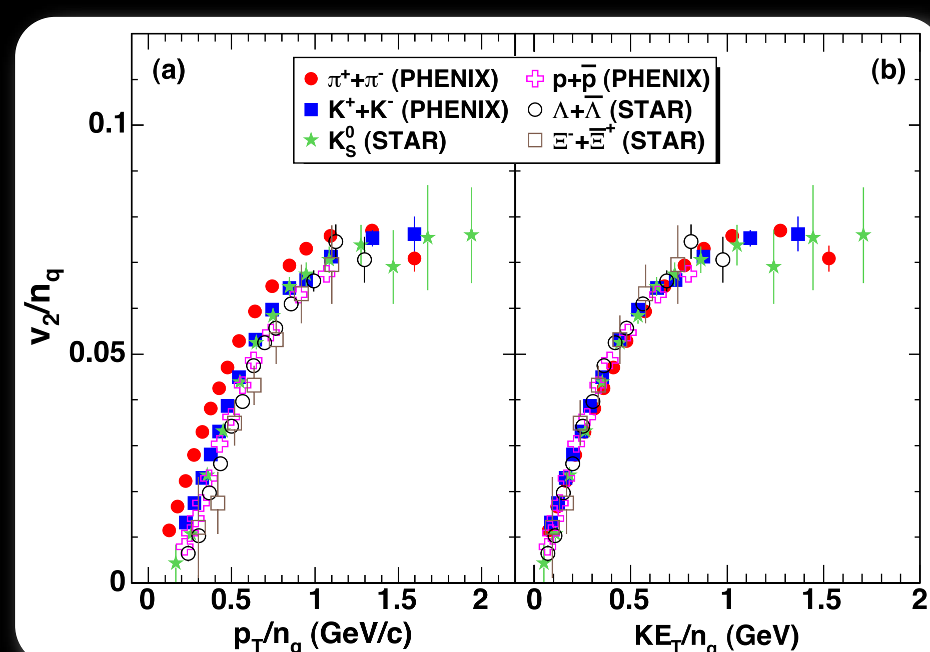
# Elliptic Flow at RHIC

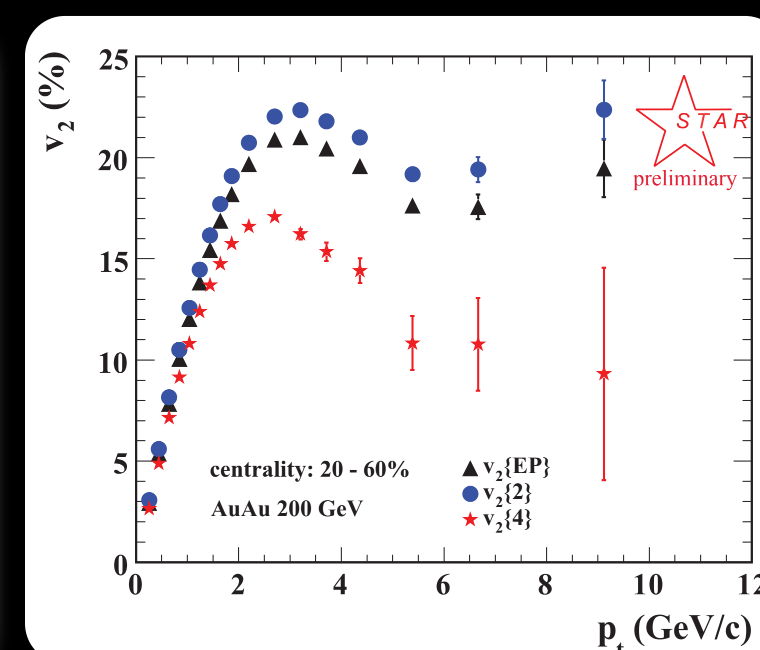
- strong elliptic flow
- constituent quark degrees of freedom
- large energy loss




+ all measurements which have flow as background

# Conclusions

- Anisotropic flow measurements have provided us with better knowledge of the properties of the created hot and dense system
- Measurements are fairly well under control and various methods are also rather well understood
  - uncertainties of ~ 10%
- At the LHC we expect to see a very rich program of correlations versus the reaction plane