Lecture I: pQCD and spectra

What is QCD?

What is QCD (Quantum Chromo Dynamics)?

Elementary fields: **Quarks** **Gluons**

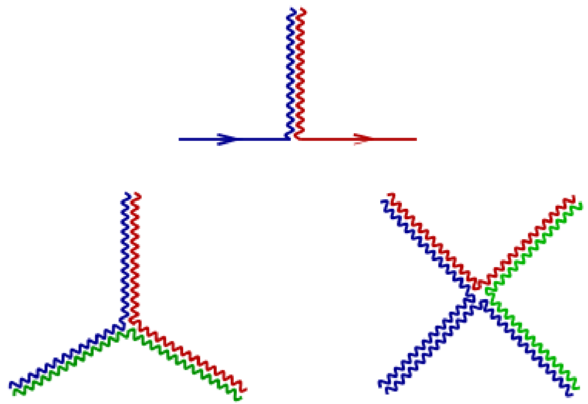
$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1,\ldots,3 \\ \text{spin} & \alpha = 1,2 \\ \text{flavor} & f = u,d,s,c,b,t \end{cases} \qquad A^a_\mu \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

$$\boxed{\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$i\not{D}q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a\right) q$$



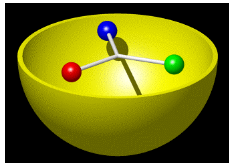
From: T. Schaefer, QM08 student talk

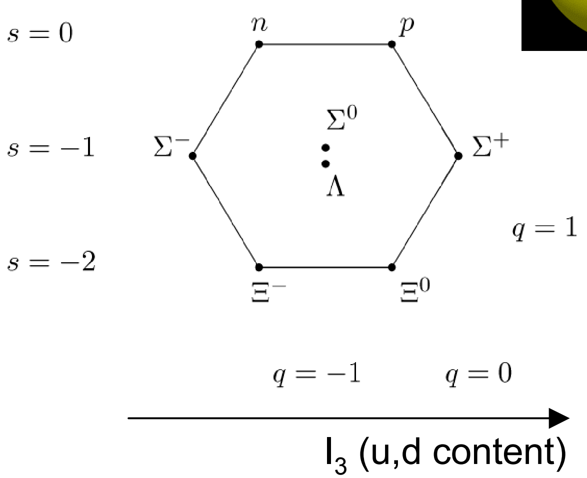
QCD and hadrons

Quarks and gluons are the fundamental particles of QCD (feature in the Lagrangian)

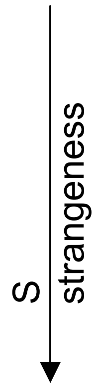
However, in nature, we observe hadrons:
Color-neutral combinations of quarks, anti-quarks

Baryon multiplet

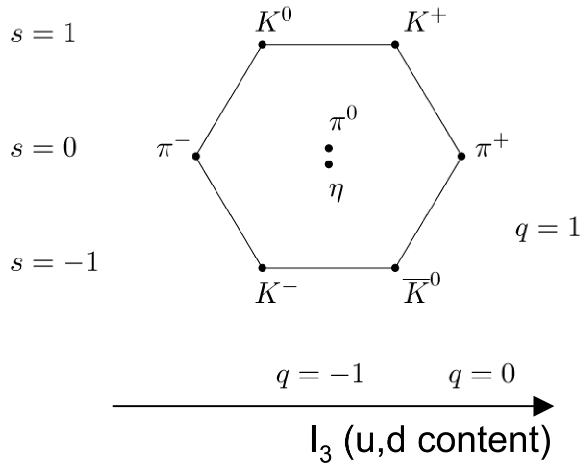




Baryons: 3 quarks

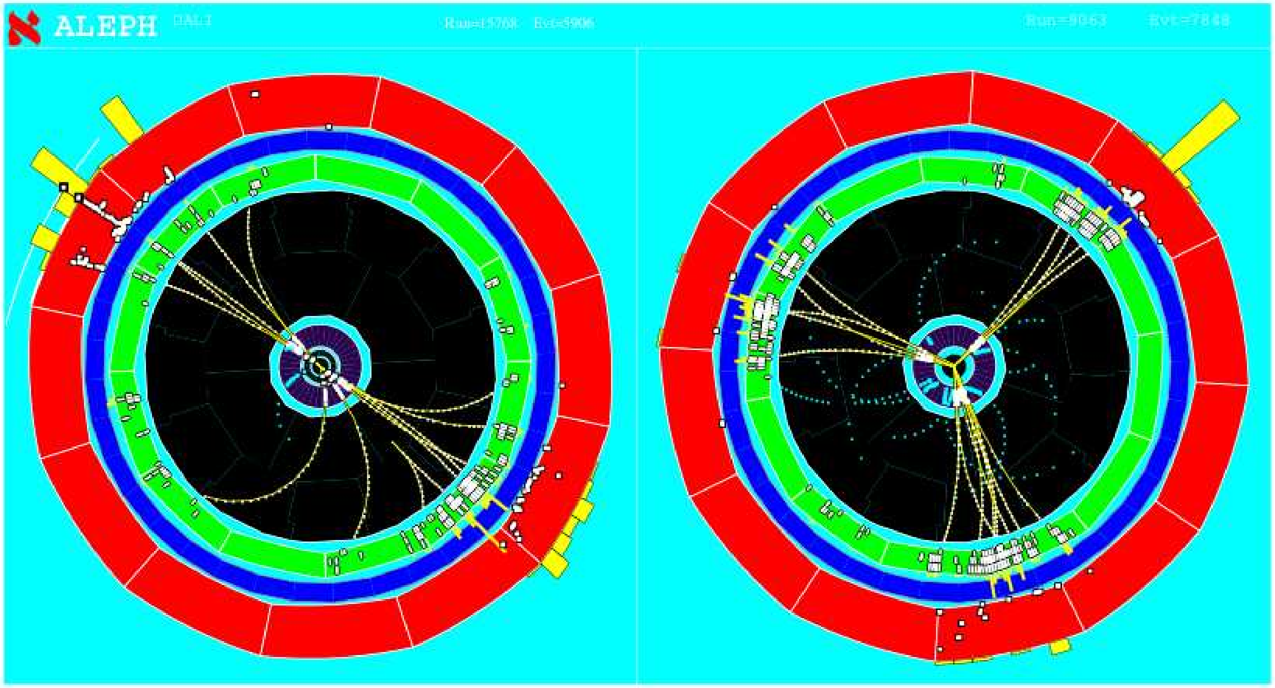


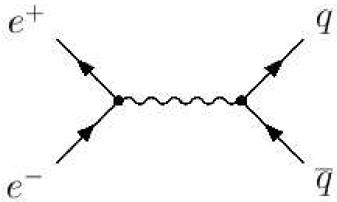
Meson multiplet

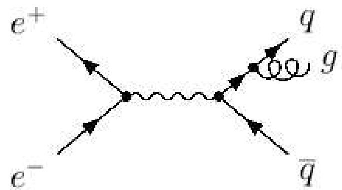


Mesons: quark-anti-quark

Seeing quarks and gluons

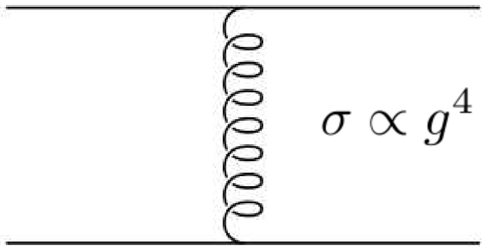






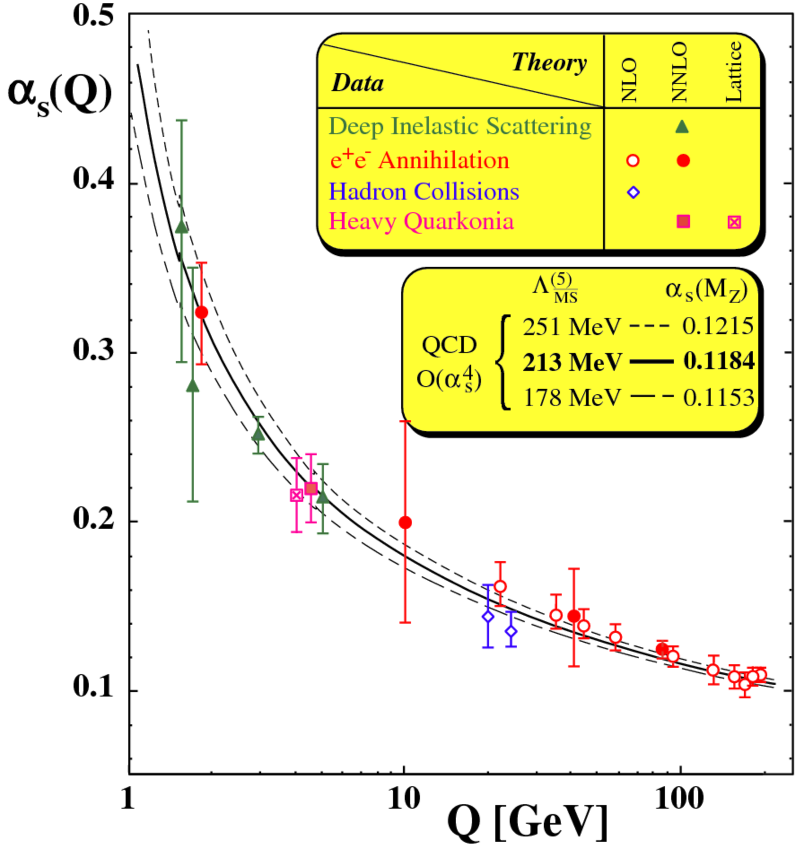
In high-energy collisions, observe traces of quarks, gluons (‘jets’)

How does it fit together?



$$\sigma \propto g^4 = 16\pi^2\alpha_s^2$$

S. Bethke, J Phys G 26, R27



Running coupling:
α_s decreases with Q^2

$$\beta_1 = (11N_c - 2n_f)/3$$

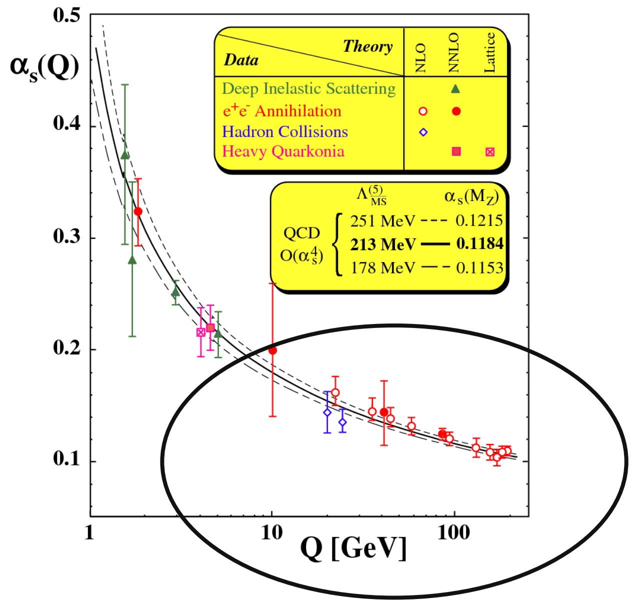
$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_1 \ln(\mu^2/\Lambda^2_{QCD})}$$

Pole at $\mu = \Lambda$

Λ_{QCD} ~ 200 MeV ~ 1 fm^{-1}

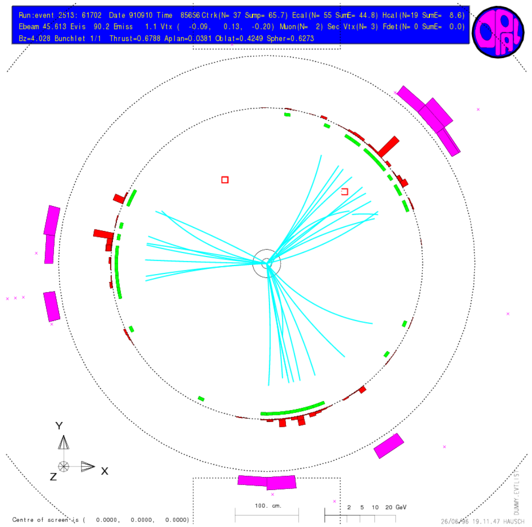
Hadronic scale

Asymptotic freedom and pQCD

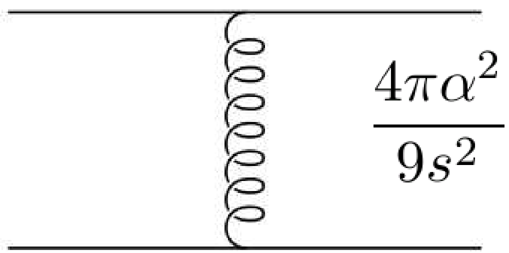


At large Q^2, hard processes: calculate 'free parton scattering'

$$e^+ e^- \rightarrow q\,\bar{q}\,g$$



At high energies, quarks and gluons are manifest



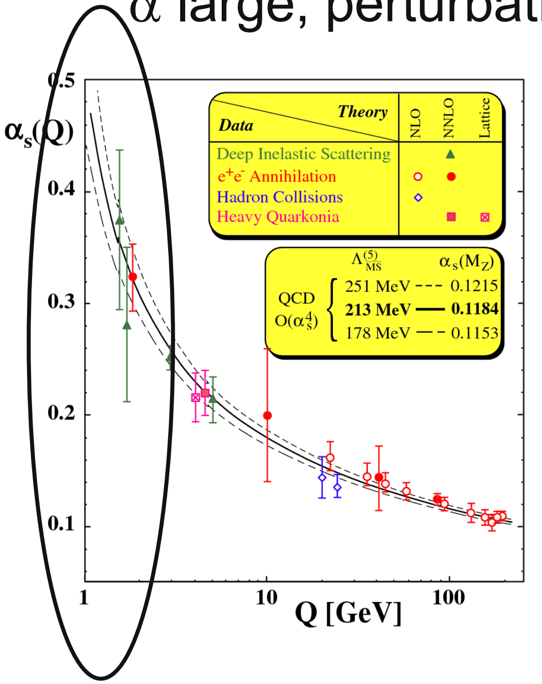
$$\frac{4\pi\alpha^2}{9s^2}\frac{s^2+u^2}{t^2}$$

+ more subprocesses

But need to add hadronisation (+initial state PDFs)

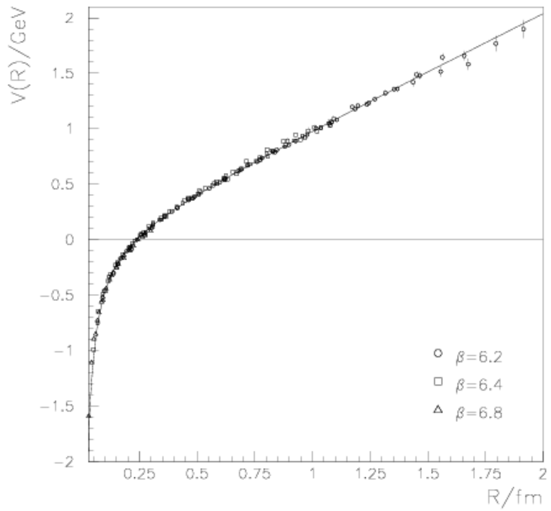
Low Q^2: confinement

α large, perturbative techniques not suitable

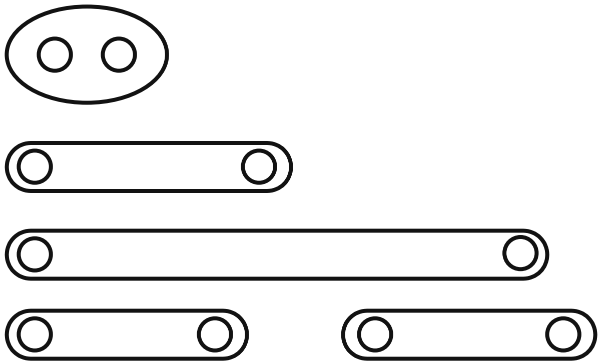


Lattice QCD: solve equations of motion (of the fields) on a space-time lattice by MC

Bali, hep-lat/9311009



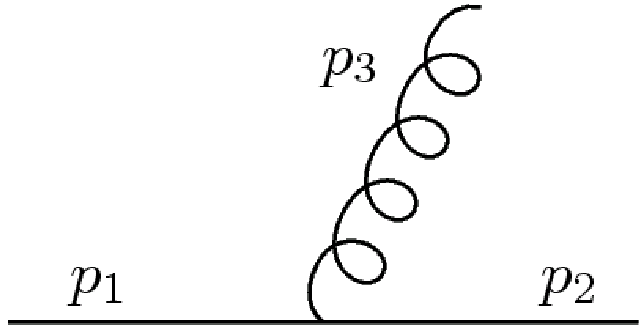
Lattice QCD potential



String breaks, generate qq pair to reduce field energy

Singularities in pQCD

$$\frac{d^2\sigma}{dx_1 dx_2} \propto \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$



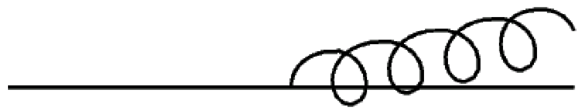
$$x_1 = 1 - \frac{2p_2 \cdot p_3}{Q^2}$$

(massless case)

Soft divergence

$p_3 \to 0$

Collinear divergence



$p_2 \cdot p_3 \to 0$

Closely related to hadronisation effects

Singularities in phase space

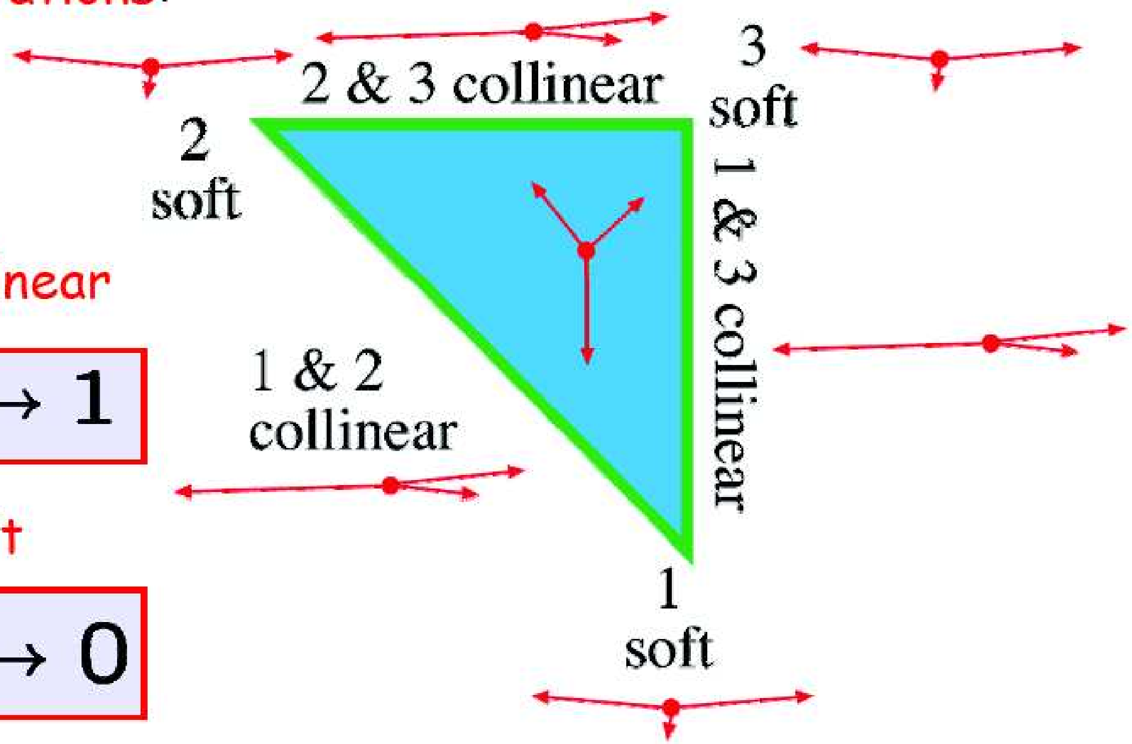
special kinematic configurations:

- edges: two partons collinear

$$\theta_{13} \to 0 \Leftrightarrow x_2 \to 1$$

- corners: one parton soft

$$p_i^\mu \to 0 \Leftrightarrow x_i \to 0$$



Hard processes in QCD

- Hard process: scale Q >> Λ_{QCD}
- Hard scattering High-p_T parton(photon) Q~p_T
- Heavy flavour production m >> Λ_{QCD}

Factorization

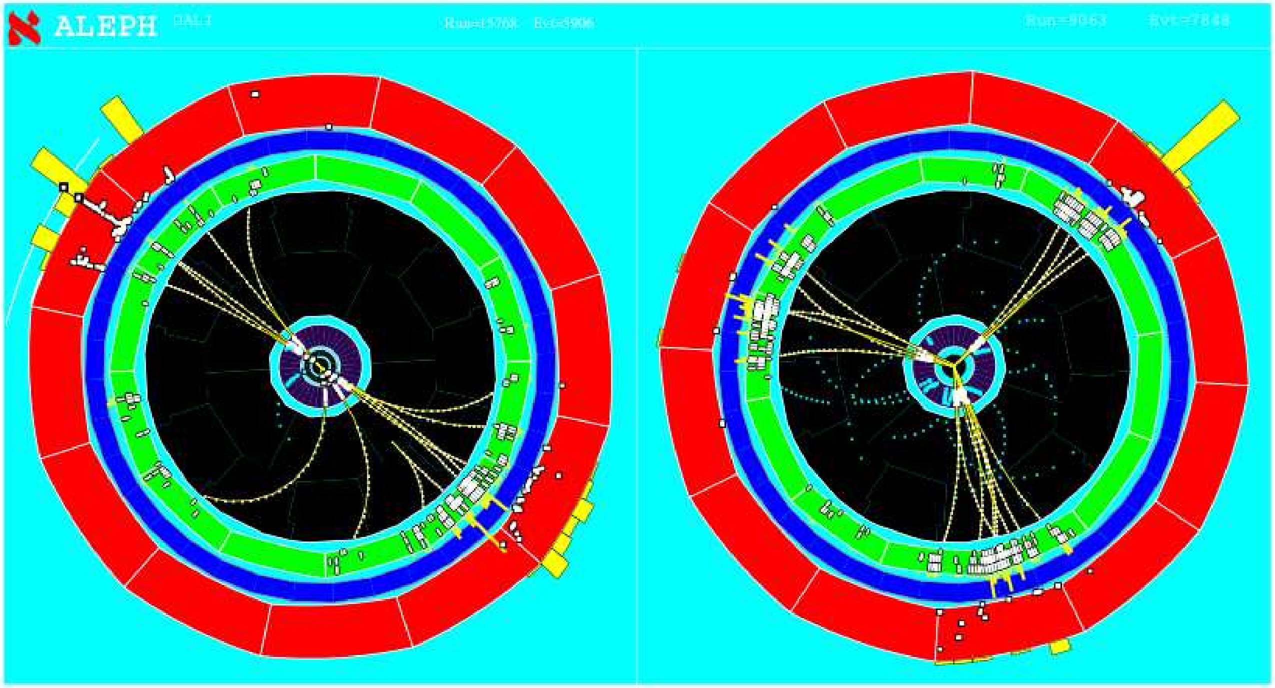
Cross section calculation can be split into
- Hard part: perturbative matrix element
- Soft part: parton density (PDF), fragmentation (FF)

$$\frac{d\sigma_{pp}^{h}}{dyd^2p_T} = K\sum_{abcd}\int \underbrace{dx_a dx_b f_a(x_a,Q^2) f_b(x_b,Q^2)}_{\text{parton density}} \underbrace{\frac{d\sigma}{d\hat{t}}(ab\rightarrow cd)}_{\text{matrix element}} \underbrace{\frac{D^0_{h/c}}{\pi z_c}}_{\text{FF}}$$

QM interference between hard and soft suppressed (by Q^2/Λ^2 ‘Higher Twist’)

Soft parts, PDF, FF are *universal*: independent of hard process

Seeing quarks and gluons



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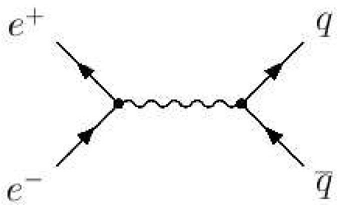
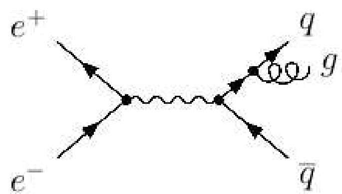




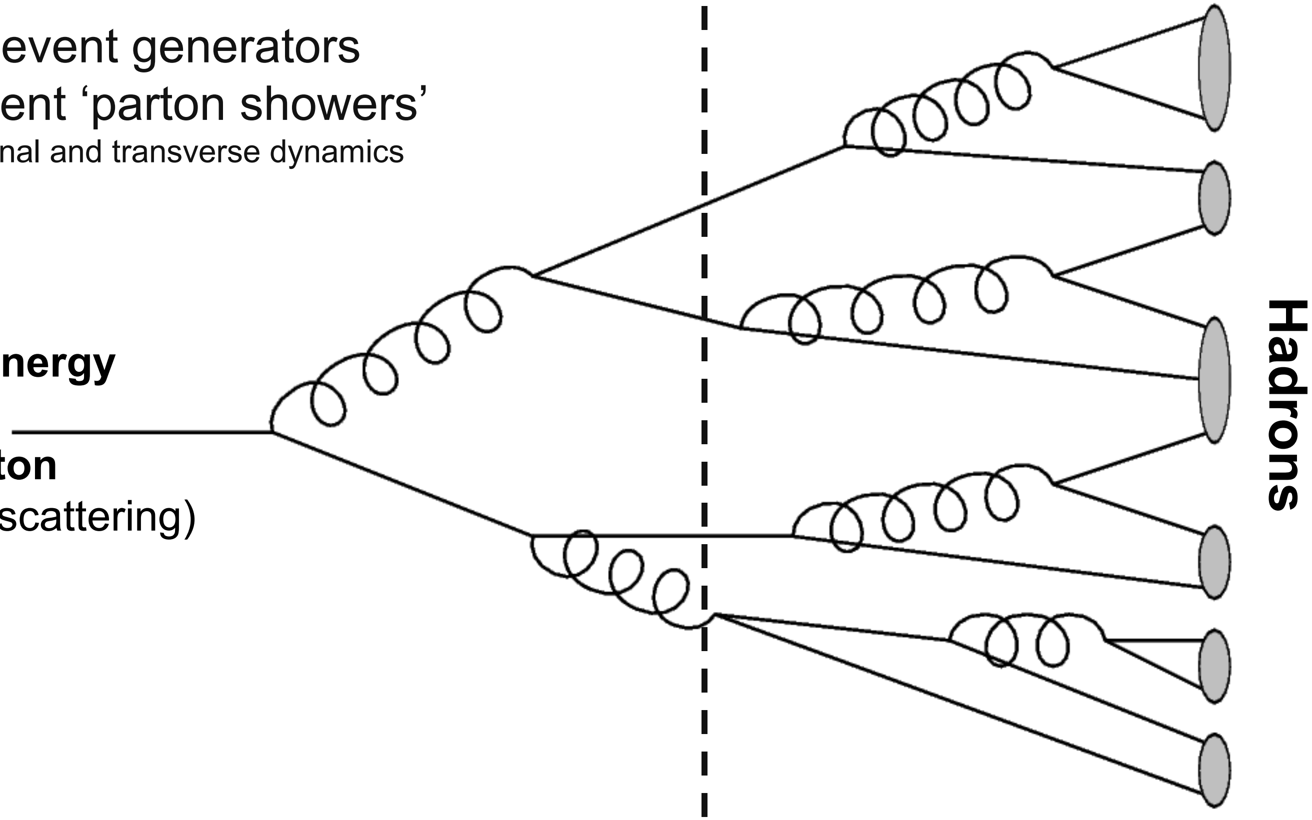
In high-energy collisions, observe traces of quarks, gluons (‘jets’)

Fragmentation and parton showers

MC event generators
implement ‘parton showers’
Longitudinal and transverse dynamics

High-energy

parton
(from hard scattering)



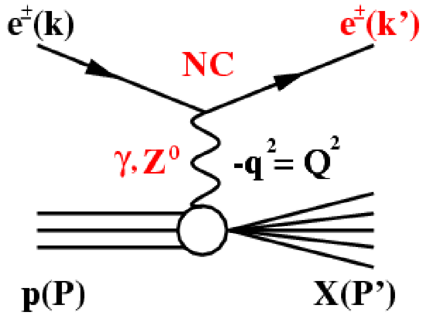
large Q^2 μ_F $Q \sim m_H \sim \Lambda_{QCD}$

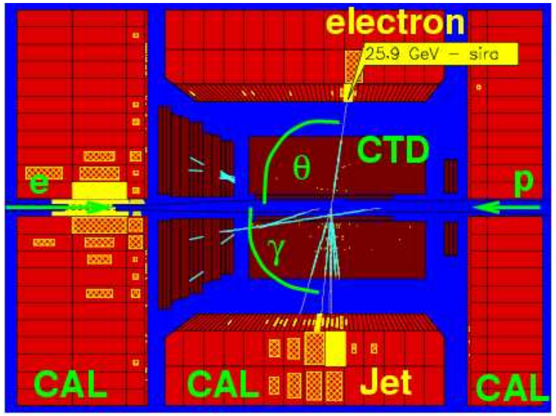
Analytical calculations: Fragmentation Function $D(z, \mu)$ $z = p_h/E_{jet}$
Only longitudinal dynamics

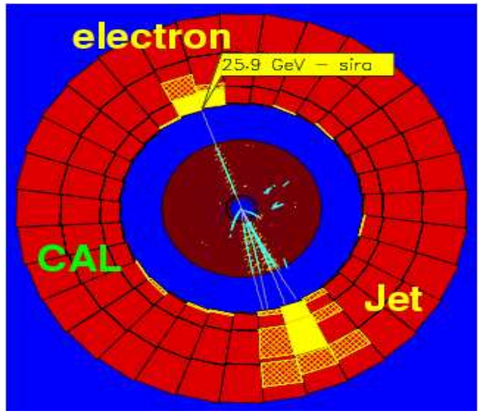
Example DIS events

$$\text{NC}: \quad e^{\pm} + p \rightarrow e^{\pm} + X, \qquad \text{CC}: \; e^{\pm} + p \rightarrow \bar{\nu}_e(\nu_e) + X$$

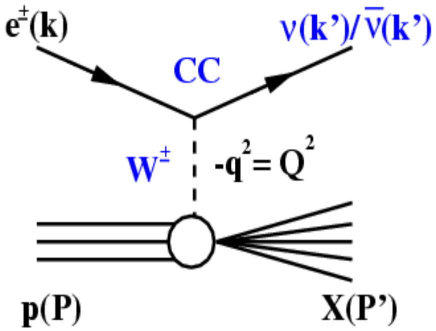
NC:

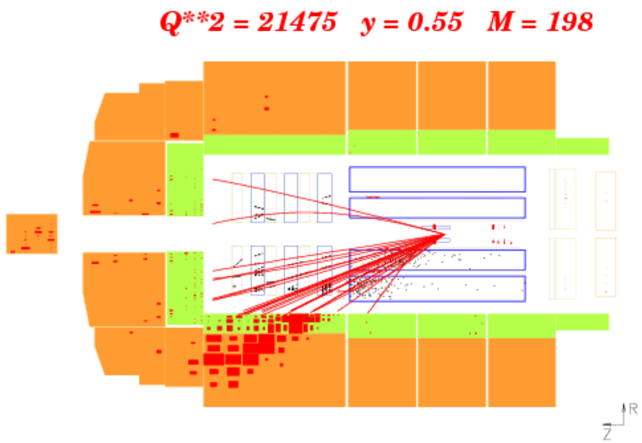


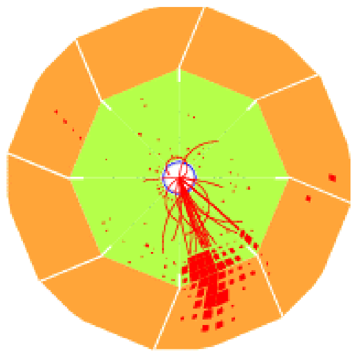




CC:

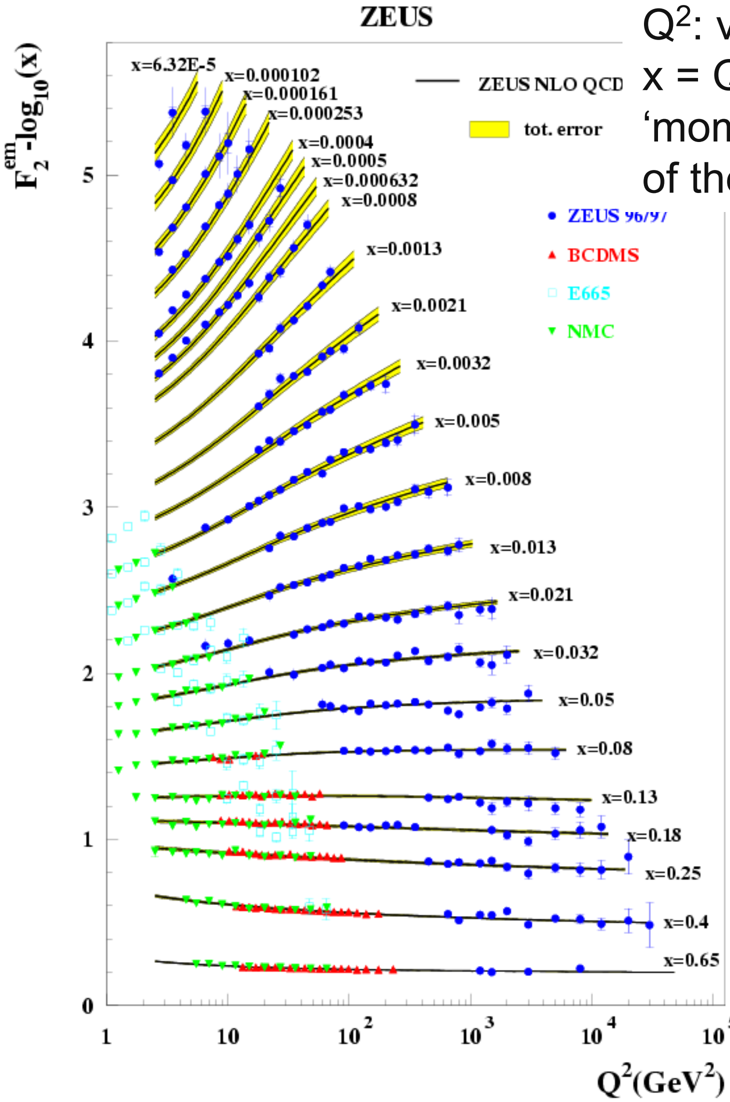






DIS: Measured electron/jet momentum fixes kinematics

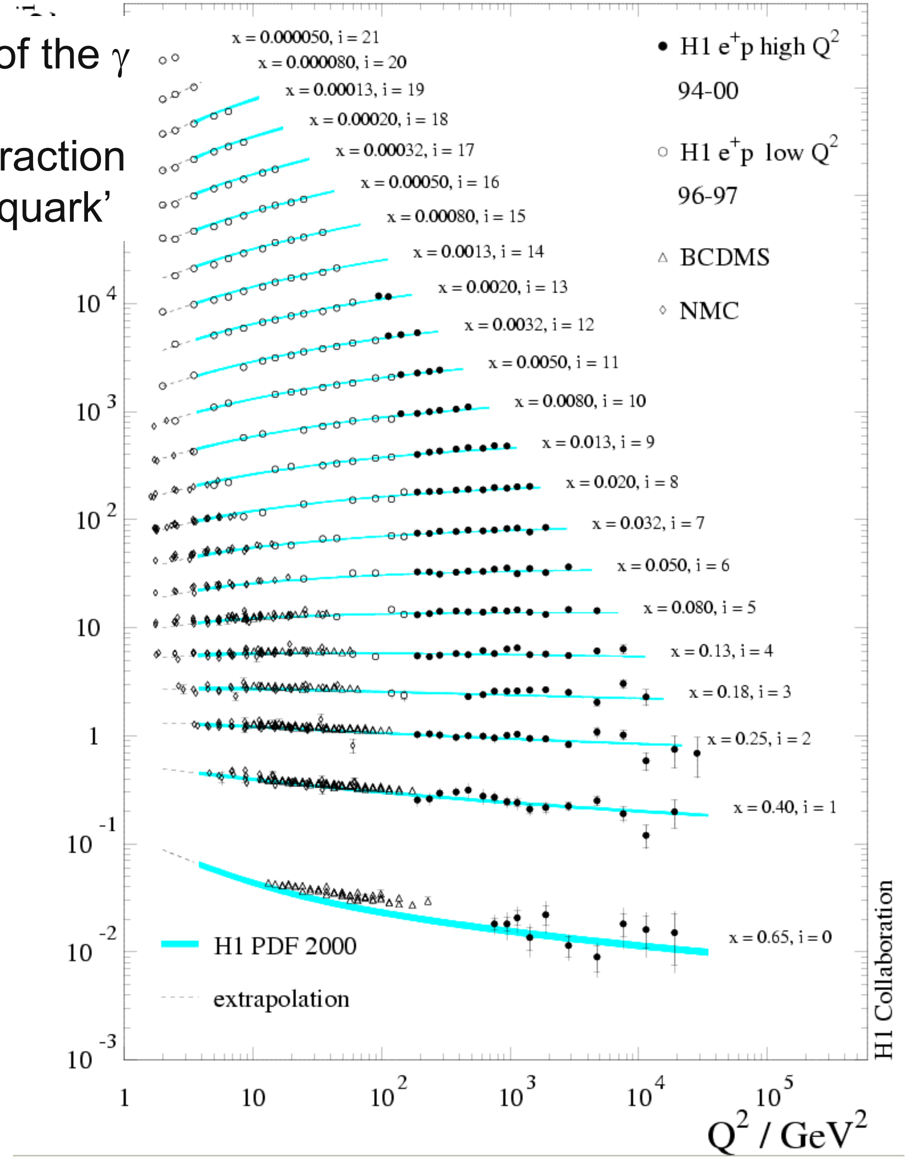
Proton structure F_2



Q^2: virtuality of the γ

$x = Q^2 / 2\,p\,q$

'momentum fraction of the struck quark'



Factorisation in DIS

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

$$F_2(x,Q^2) \;=\; x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]$$

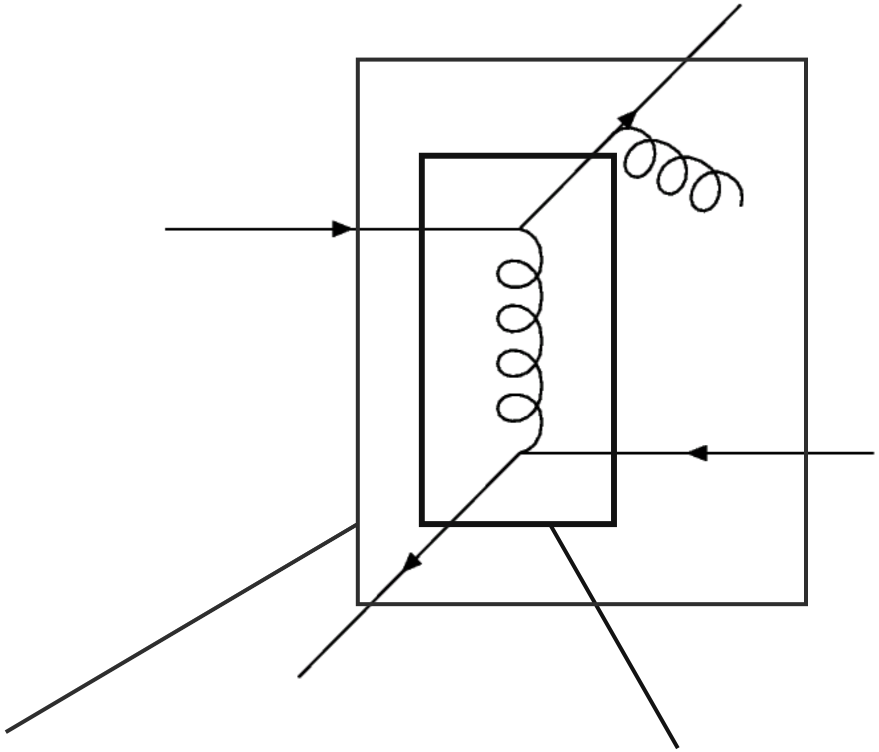
yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

short-distance "Wilson coefficient"

choice of the factorization scheme

Integral over x is DGLAP evolution with splitting kernel P_{qq}

Factorisation in pictures



Use NLO matrix element:

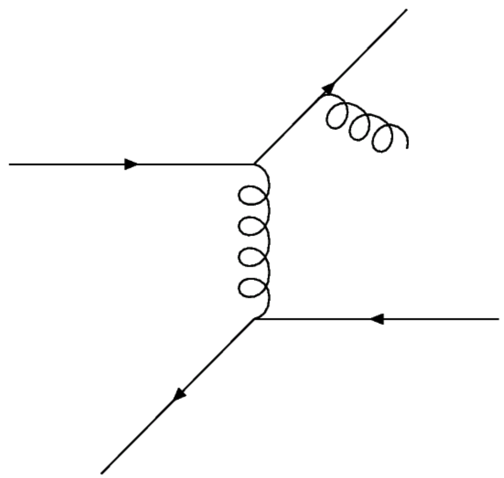
$$qq \to qqg$$

Or: LO matrix element,
include gluon radiation in fragmentation:
Evolve fragmentation

Note: [DGLAP] evolution only keeps track of leading parton momentum

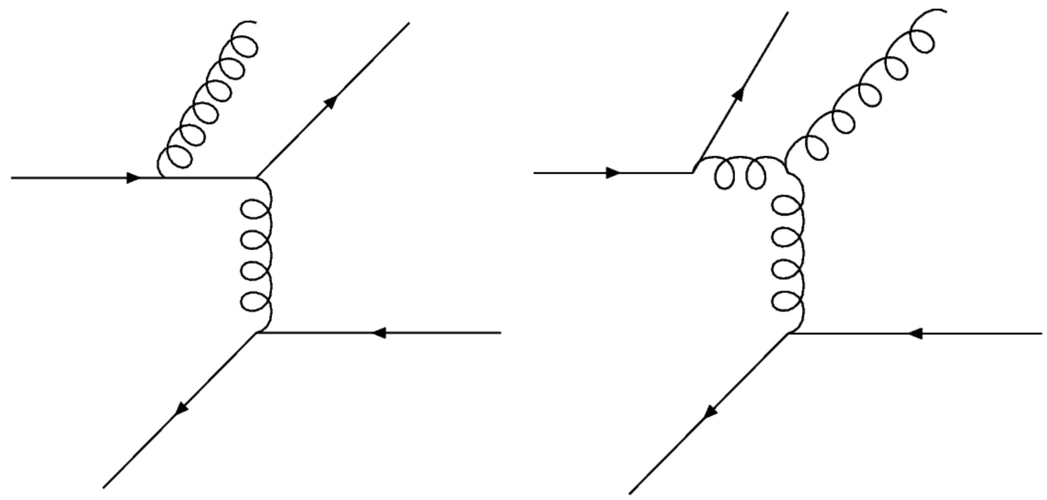
Initial state and final state radiation

Final state radiation



Fragmentation function evolves

Initial state radiation

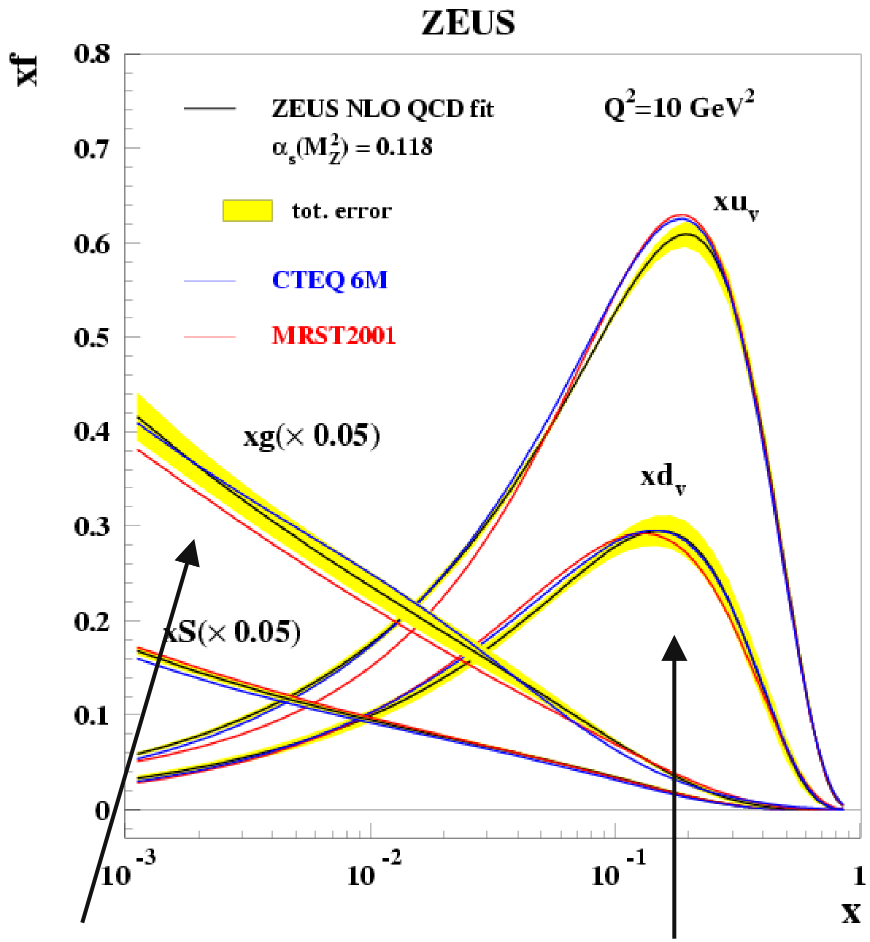


Parton density evolves

Evolution can turn (leading)
quarks into gluons
and vice versa

Parton density distribution

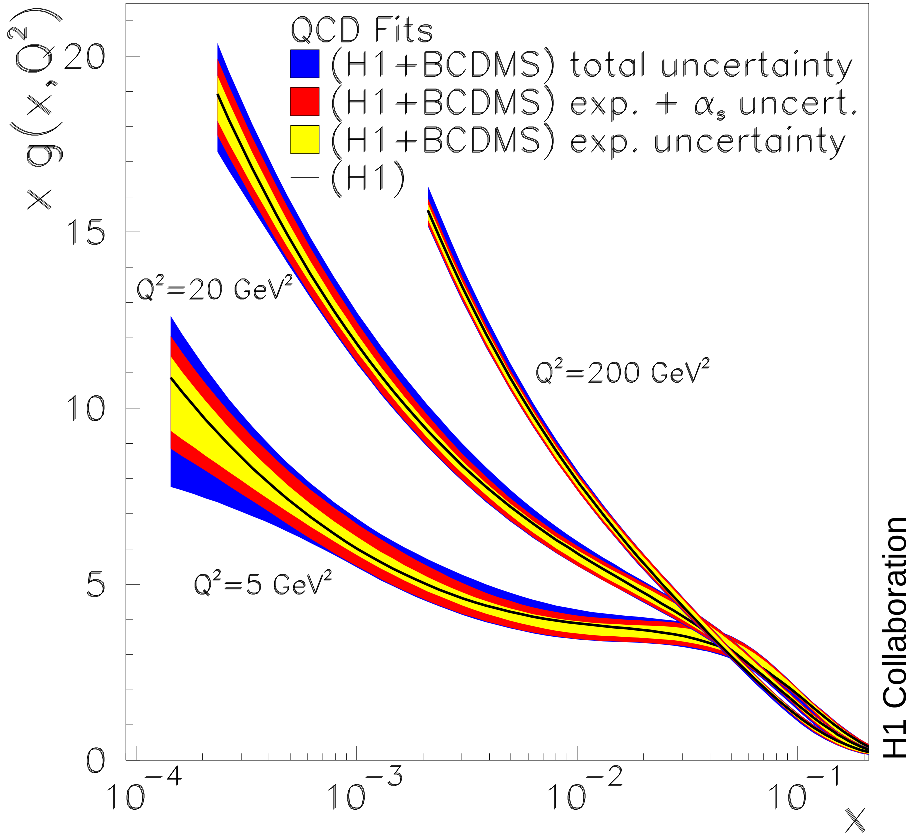
Low Q^2: valence structure



Soft gluons

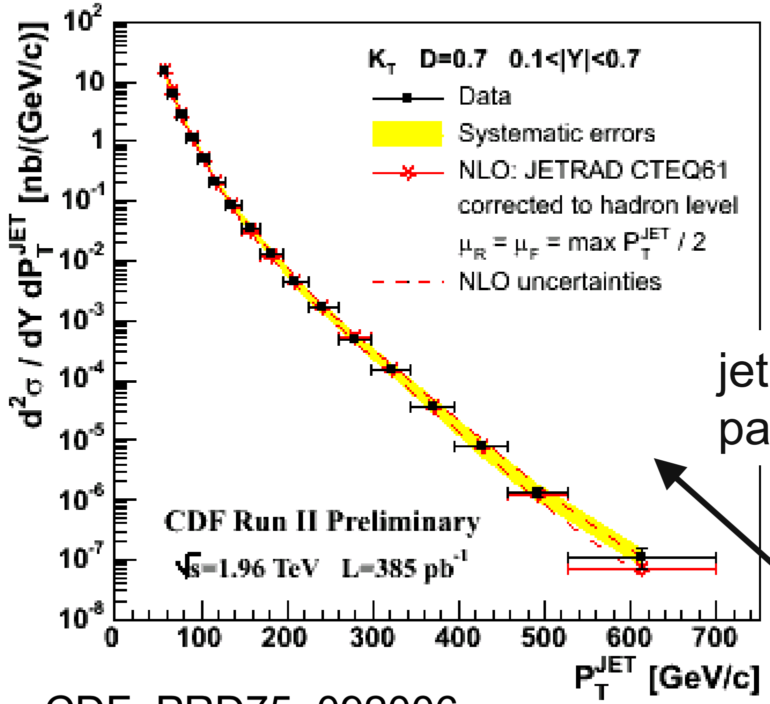
Valence quarks (p = uud)
x ~ 1/3

Q^2 evolution (gluons)



Gluon content of proton rises quickly with Q^2

pQCD illustrated

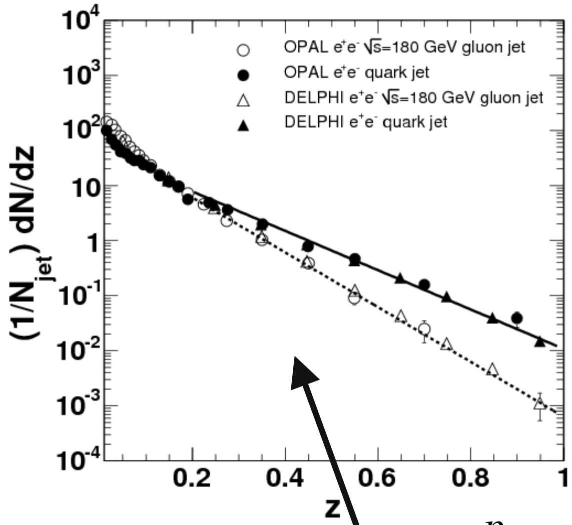


CDF, PRD75, 092006

jet spectrum ~
parton spectrum

$$\frac{dN}{\hat{p}_T d\hat{p}_T} \propto \frac{1}{\hat{p}_T^n}$$

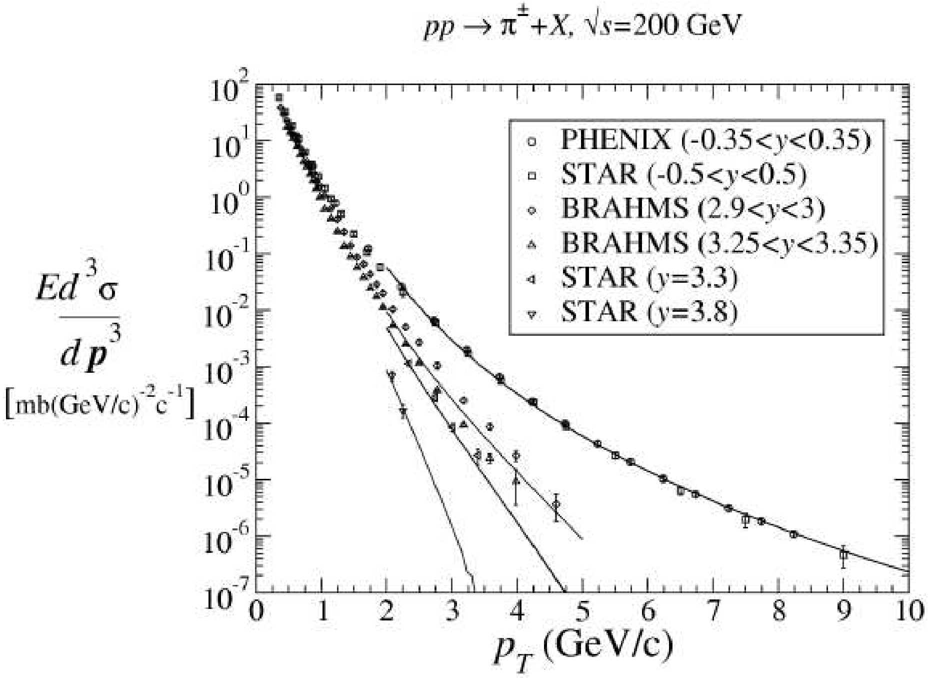
fragmentation

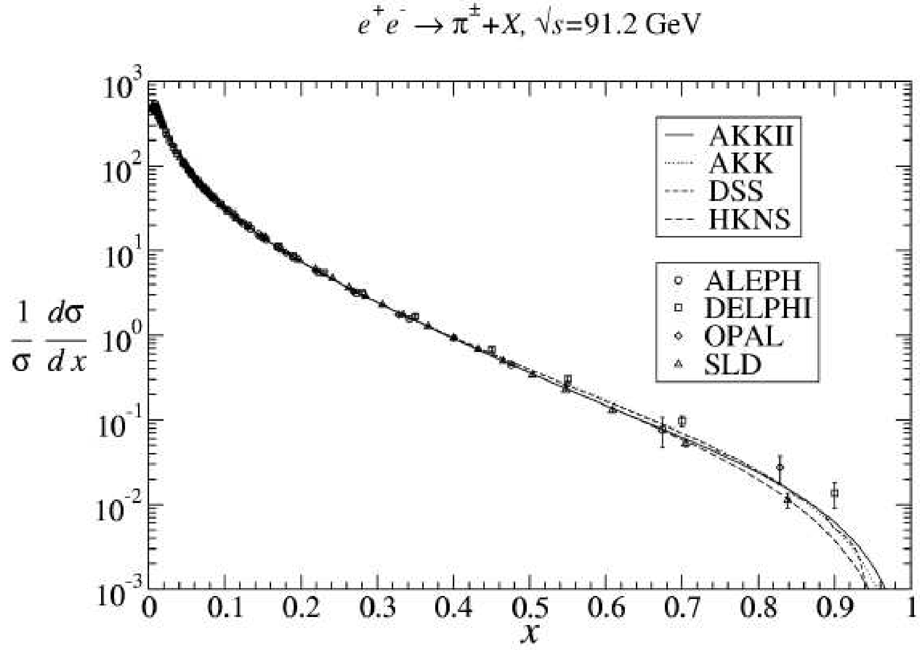


$$z = \frac{p_{T,hadron}}{P_{jet}}$$

$$\frac{d\sigma_{pp}^{h}}{dy d^2 p_T} = K \sum_{abcd} \int dx_a dx_b f_a(x_a, Q^2) f_b(x_b, Q^2) \frac{d\sigma}{d\hat{t}}(ab \to cd) \frac{D_{h/c}^0}{\pi z_c}$$

Note: difference p+p, $e^+ + e^-$



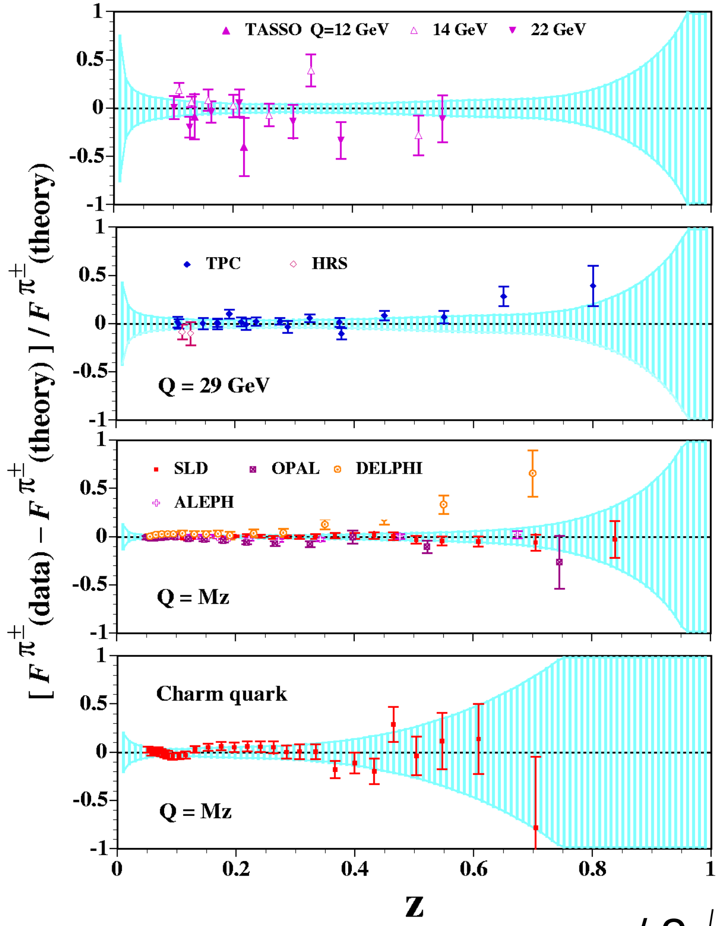


p+p: steeply falling jet spectrum
Hadron spectrum convolution
of jet spectrum with fragmentation

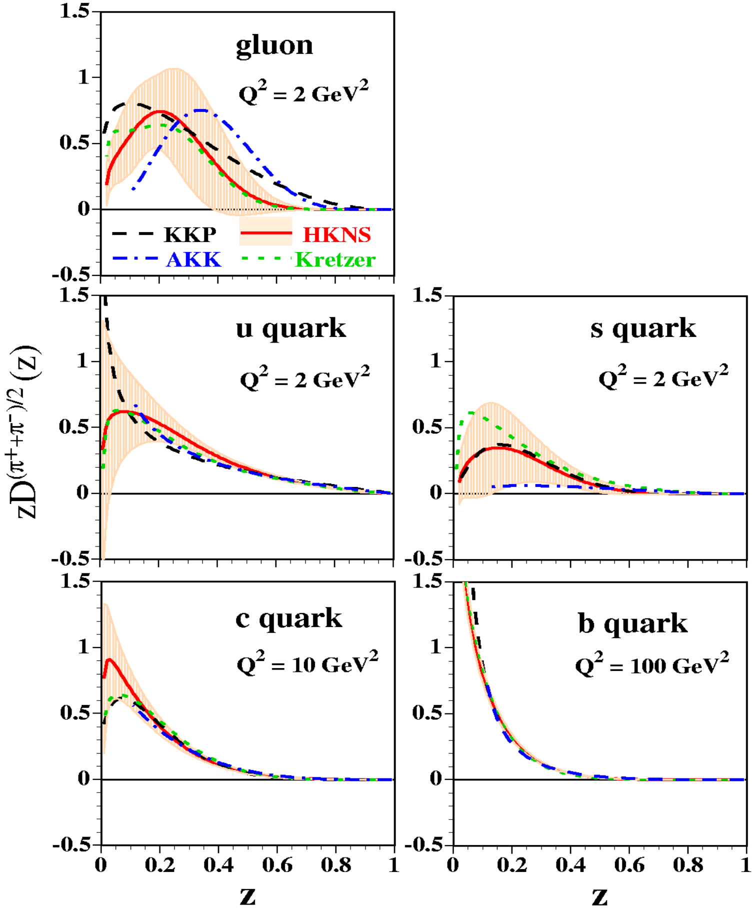
$e^+ + e^-$ QCD events: jets
have p=1/2 √s
Directly measure frag function

Fragmentation function uncertainties

Hirai, Kumano, Nagai, Sudo, PRD75:094009



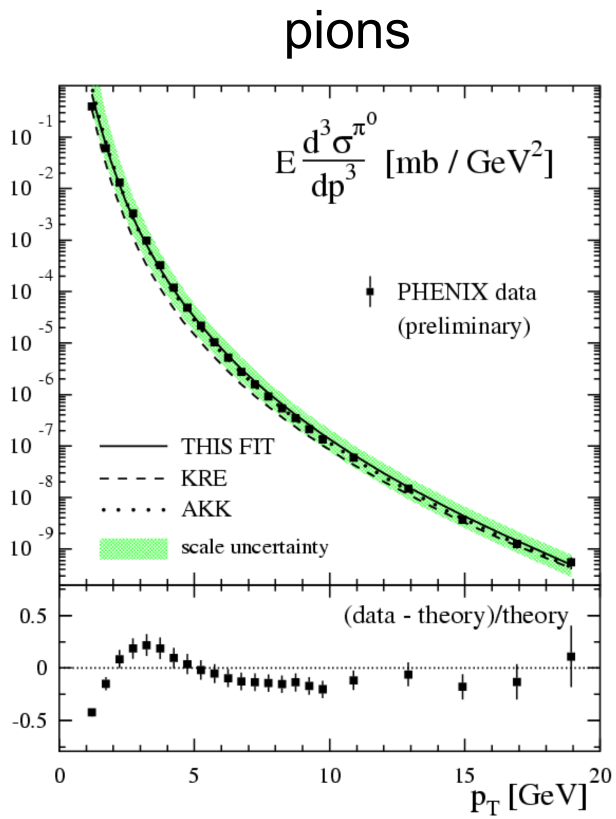
$z = p_{T,h} / 2\sqrt{s}$

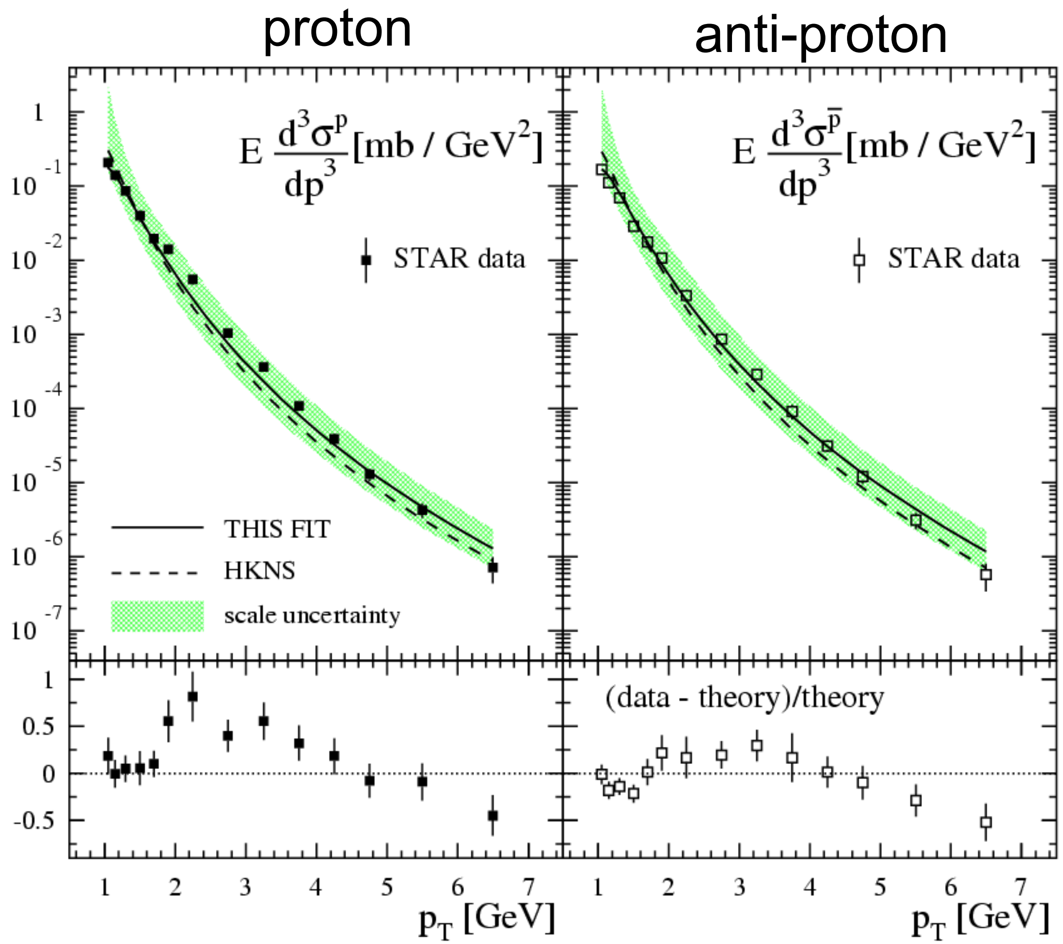


$z = p_{T,h} / E_{jet}$

Full uncertainty analysis being pursued
Uncertainties increase at small and large z

Global analysis of FF





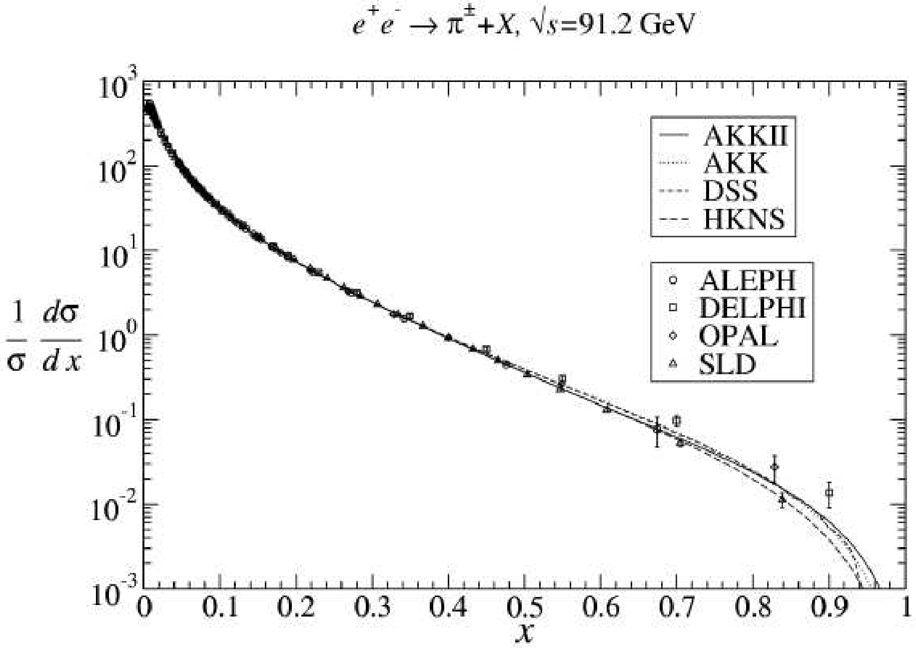
De Florian, Sassot, Stratmann, PRD 76:074033, PRD75:114010

... or do a global fit, including p+p data

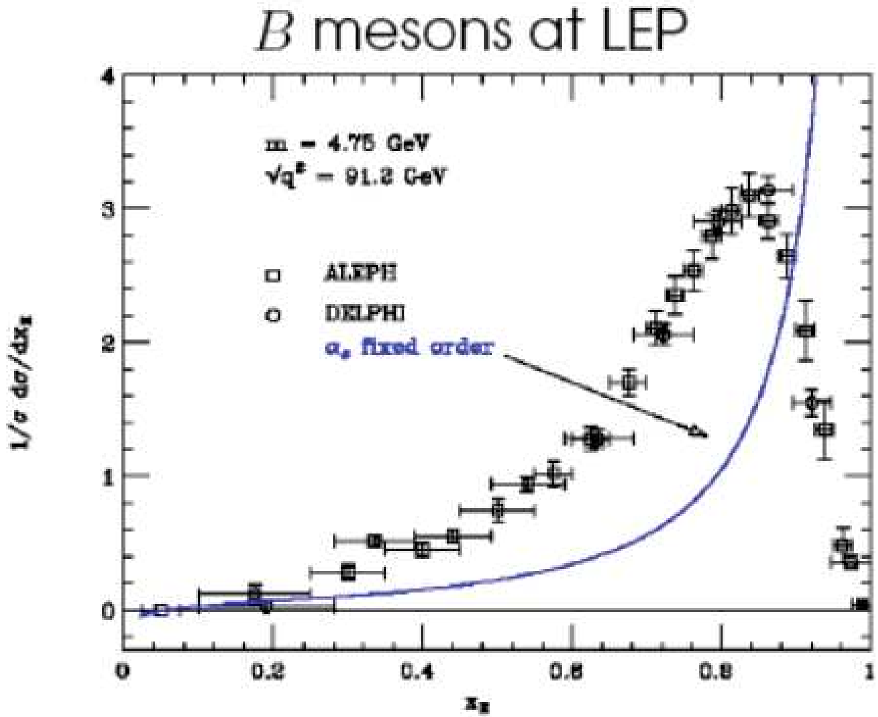
Universality still holds

Heavy quark fragmentation

Light quarks



Heavy quarks

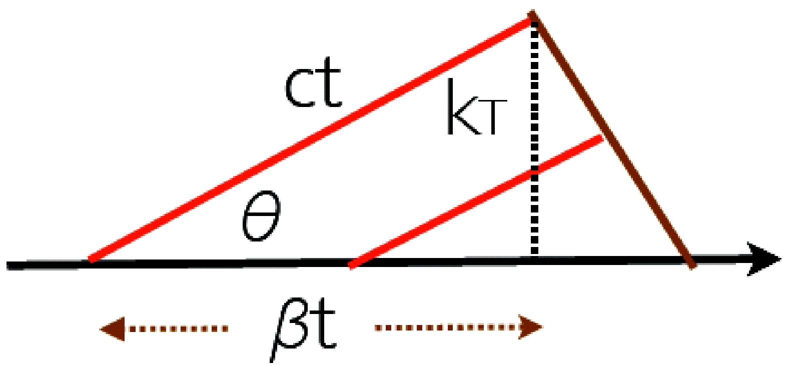


Heavy quark fragmentation: leading heavy meson carries large momentum fraction

Less gluon radiation than for light quarks, due to ‘dead cone’

Dead cone effect

Radiated wave front cannot out-run source quark



Heavy quark: $\beta < 1$

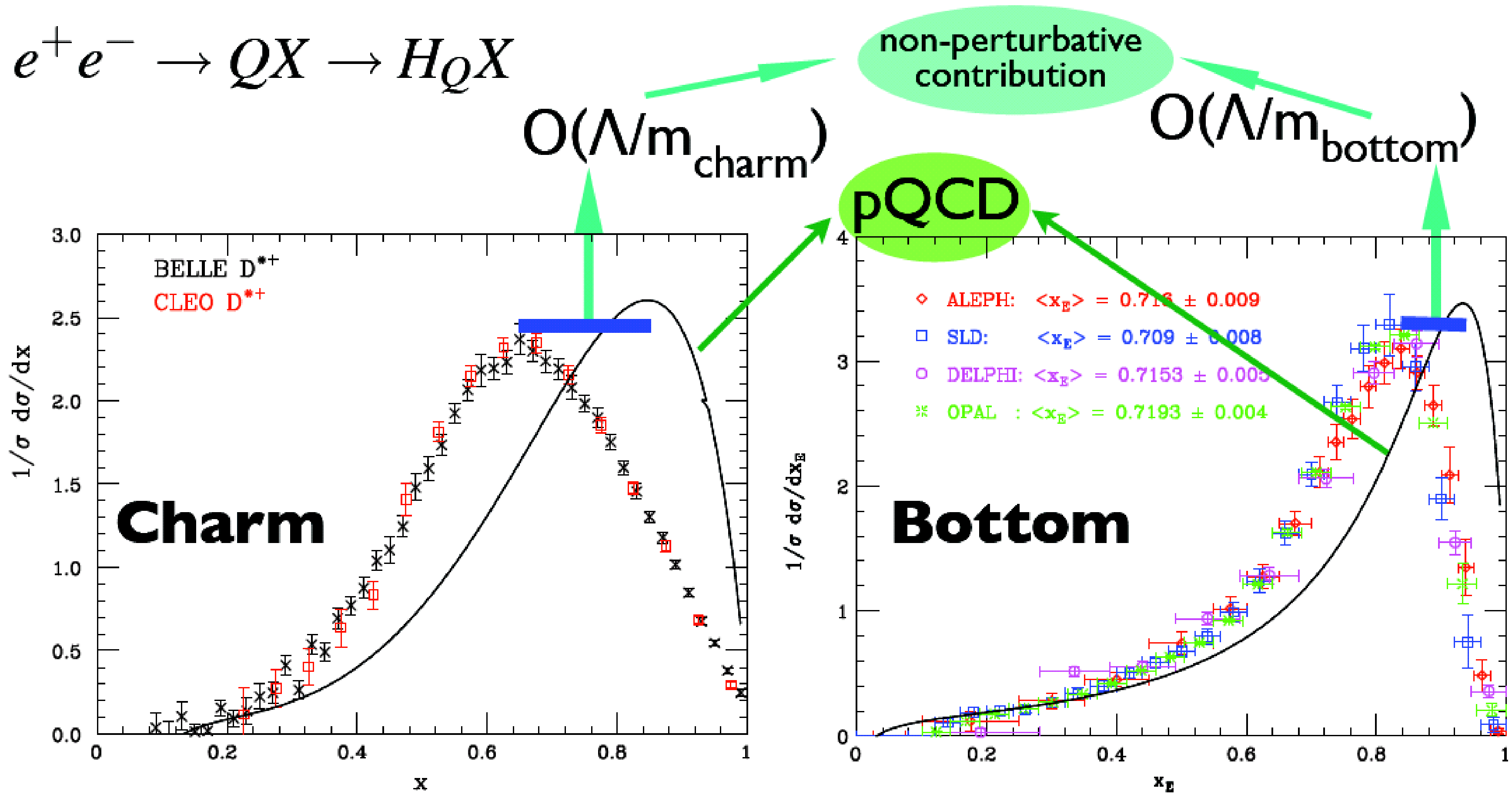
$$\sin\theta_{DC} = 1 - \beta^2 = \left(\frac{M}{E}\right)^2$$

Result: minimum angle for radiation

$\Rightarrow$ Mass regulates collinear divergence

Heavy Quark Fragmentation II

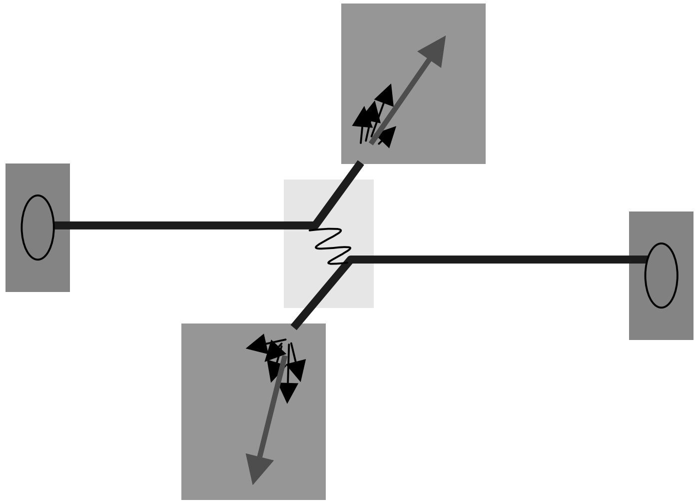
$$e^+e^- \rightarrow QX \rightarrow H_Q X$$



Significant non-perturbative effects
seen even
in heavy quark fragmentation

$$\begin{aligned}
\frac{1}{\sigma}\frac{d\sigma}{dx} &= \delta(1-x)+\frac{\alpha_s(Q^2)}{2\pi}\left\{C_F+C_F\left[\ln\frac{Q^2}{m^2}\left(\frac{1+x^2}{1-x}\right)_+\right.\right.\\
&+ \quad 2\frac{1+x^2}{1-x}\log x-\left(\frac{\ln(1-x)}{1-x}\right)_+(1+x^2)+\frac{1}{2}\left(\frac{1}{1-x}\right)_+(x^2-6x-2)\\
&+ \quad \left.\left.\left(\frac{2}{3}\pi^2-\frac{5}{2}\right)\delta(1-x)\right]\right\}+\mathcal{O}\left(\frac{m}{Q}\right)
\end{aligned}$$

Factorisation in perturbative QCD



$$\frac{d\sigma^{h}_{pp}}{dy\,d^2p_T} = K \sum_{abcd} \int dx_a dx_b f_a(x_a, Q^2) f_b(x_b, Q^2) \frac{d\sigma}{d\hat{t}}(ab \rightarrow cd) \frac{D^0_{h/c}}{\pi z_c}$$

Parton density function	Matrix element	Fragmentation function
Non-perturbative: distribution of partons in proton Extracted from fits to DIS (ep) data	Perturbative component	Non-perturbative Measured/extracted from e+e-

Factorisation: non-perturbative parts (long-distance physics) can be factored out in universal distributions (PDF, FF)

Hard processes in QCD

- Hard process: scale Q >> Λ_{QCD}
- Hard scattering High-p_T parton(photon) Q~p_T
- Heavy flavour production m >> Λ_{QCD}

Factorization

Cross section calculation can be split into
- Hard part: perturbative matrix element
- Soft part: parton density (PDF), fragmentation (FF)

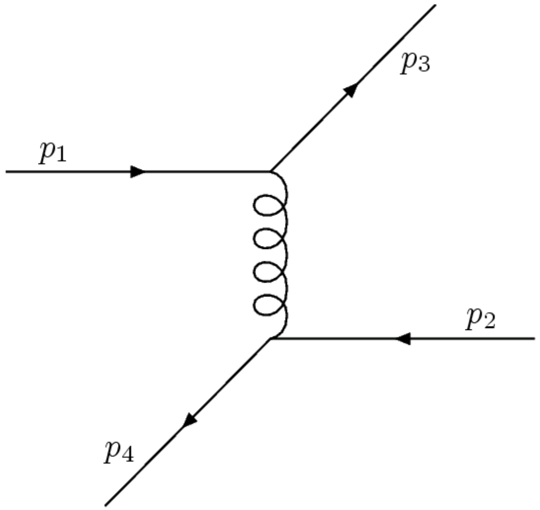
$$\frac{d\sigma^h_{pp}}{dy d^2 p_T} = K \sum_{abcd} \int dx_a dx_b \underbrace{f_a(x_a, Q^2) f_b(x_b, Q^2)}_{\text{parton density}} \underbrace{\frac{d\sigma}{d\hat{t}}(ab \to cd)}_{\text{matrix element}} \underbrace{\frac{D^0_{h/c}}{\pi z_c}}_{\text{FF}}$$

QM interference between hard and soft suppressed (by Q^2/Λ^2 'Higher Twist')

Soft parts, PDF, FF are *universal*: independent of hard process

Two-body kinematics

2 -> 2 scattering



Mandelstam variables:

$$
\begin{aligned}
s &= (p_1+p_2)^2 = (p_3+p_4)^2 \\
t &= (p_1-p_3)^2 = (p_2-p_4)^2 \\
u &= (p_1-p_4)^2 = (p_2-p_3)^2 \\
s+t+u &= m_1+m_2+m_3+m_4
\end{aligned}
$$

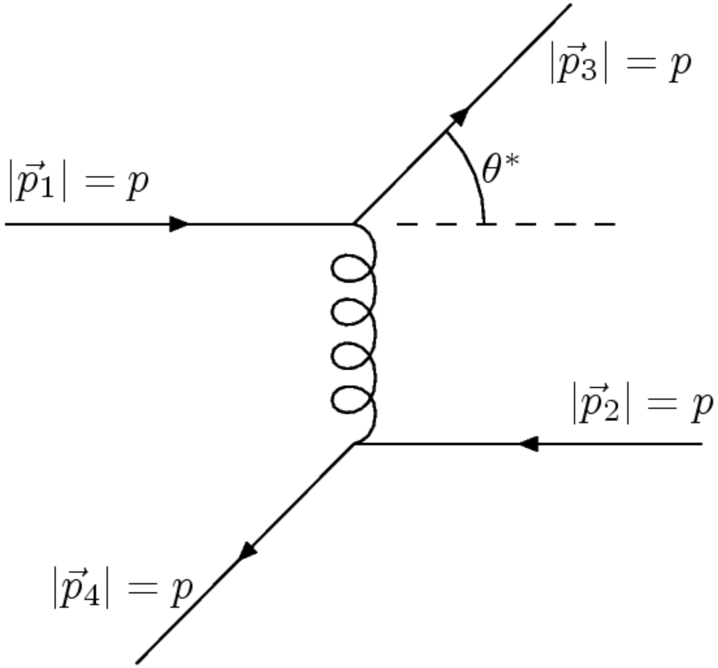
Invariants: independent of ref system
NB: *p* are four-vectors

Massless case:

$$
\begin{aligned}
s &= p_1 \cdot p_2 = p_3 \cdot p_4 \\
t &= -p_1 \cdot p_3 = -p_2 \cdot p_4 \\
u &= -p_1 \cdot p_4 = -p_2 \cdot p_3
\end{aligned}
$$

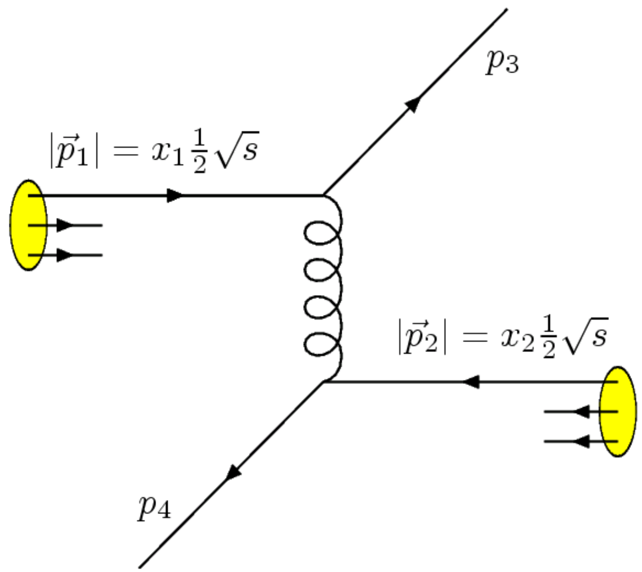
Good approximation if $\vec{p}^{\,2} >> p^2 = m$

Centre-of-mass system



$$
\begin{aligned}
s &= (2\vec{p}_1)^2 = (2\vec{p}_2)^2 = (2\vec{p}_3)^2 = (2\vec{p}_4)^2 \\
t &= -\frac{1}{2}\hat{s}(1 - \cos\theta^*) \\
u &= -\frac{1}{2}\hat{s}(1 + \cos\theta^*)
\end{aligned}
$$

p+p collision



Incoming partons carry momentum fractions x_1, x_2
Lab CMS is not parton-parton CMS

$$\hat{s} = x_1 x_2 s$$

$$\hat{y}^* = \tfrac{1}{2}(y_3 + y_4)$$

See Eskola paper for further practical details

pQCD matrix elements

$$\frac{d\sigma}{d\hat{t}} = \frac{\pi \alpha_s^2}{\hat{s}^2} \Sigma$$

Combridge, Kripfganz, Ranft, PLB70, 234

Table 1

Hard scattering subprocesses in QCD and the associated differential cross-sections in lowest order. Σ is defined by eq (1). The initial (final) colours and spins have been averaged (summed). q and g denote quark and gluon, respectively. Subscripts 1, 2 denote distinct flavours s, t, u are the Mandelstam variables of the subprocess

$q_1q_2 \to q_1q_2$, $q_1\bar{q}_2 \to q_1\bar{q}_2$	$\frac{4}{9}\frac{s^2+u^2}{t^2}$
$q_1q_1 \to q_1q_1$	$\frac{4}{9}\left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2}\right) - \frac{8}{27}\frac{s^2}{ut}$
$q_1\bar{q}_1 \to q_2\bar{q}_2$	$\frac{4}{9}\frac{t^2+u^2}{s^2}$
$q_1\bar{q}_1 \to q_1\bar{q}_1$	$\frac{4}{9}\left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2}\right) - \frac{8}{27}\frac{u^2}{st}$
$q\bar{q} \to gg$	$\frac{32}{27}\frac{u^2+t^2}{ut} - \frac{8}{3}\frac{u^2+t^2}{s^2}$
$gg \to q\bar{q}$	$\frac{1}{6}\frac{u^2+t^2}{ut} - \frac{3}{8}\frac{u^2+t^2}{s^2}$
$qg \to qg$	$-\frac{4}{9}\frac{u^2+s^2}{us} + \frac{u^2+s^2}{t^2}$
$gg \to gg$	$\frac{9}{2}\left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right)$

See also e.g. Owens, Rev Mod Phys 59,465

Key topics today

- pQCD relies on factorization
- Separation of scales
  - Parton Density Functions (soft)
  - Hard Scattering
  - Fragmentation (soft)
- PDFs from DIS
- FF from e^+e^-
- Heavy flavour fragmentation is qualitatively different from light