# Radiative energy loss

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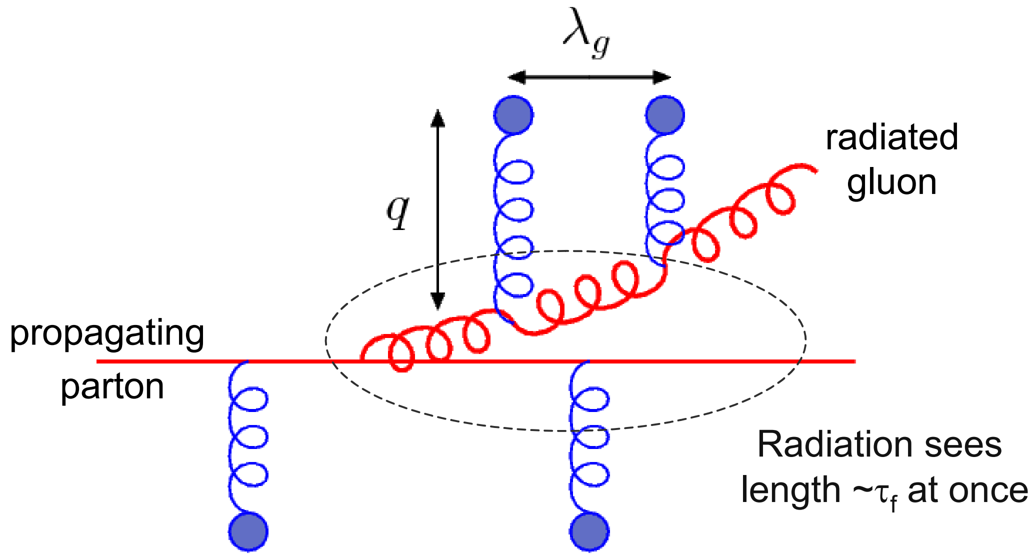
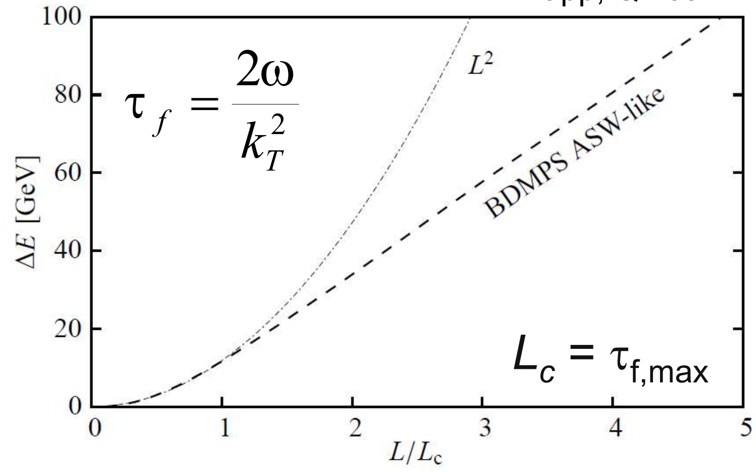
# Medium-induced radiation

Landau-Pomeranchuk-Migdal effect
Formation time important




Zapp, QM09




If $\lambda < \tau_f$, multiple scatterings add coherently

$$\Delta E_{med} \sim \alpha_S \hat{q} L^2$$

Energy loss depends on density: $\lambda \propto \frac{1}{\rho}$

and nature of scattering centers
(scattering cross section)

Transport coefficient $\hat{q} \equiv \frac{\langle q_\perp^2 \rangle}{\lambda}$

# A simple model

Parton spectrum | Energy loss distribution | Fragmentation (function)

$$\left.\frac{dN}{dp_T}\right|_{hadr} = \left.\frac{dN}{dE}\right|_{jets} \otimes P(\Delta E) \otimes D(p_{T,hadr}/E_{jet})$$

known pQCDxPDF | extract | `known' from $e^+e^-$

This is where the information about the medium is

P(ΔE) combines geometry
with the intrinsic process
– Unavoidable for many observables

Notes:

- This formula is the simplest ansatz – Independent fragmentation after E-loss assumed
- Jet, γ-jet measurements 'fix' E, removing one of the convolutions

**We will explore this model during the week; was 'state of the art' 3-5 years ago**

# Two extreme scenarios

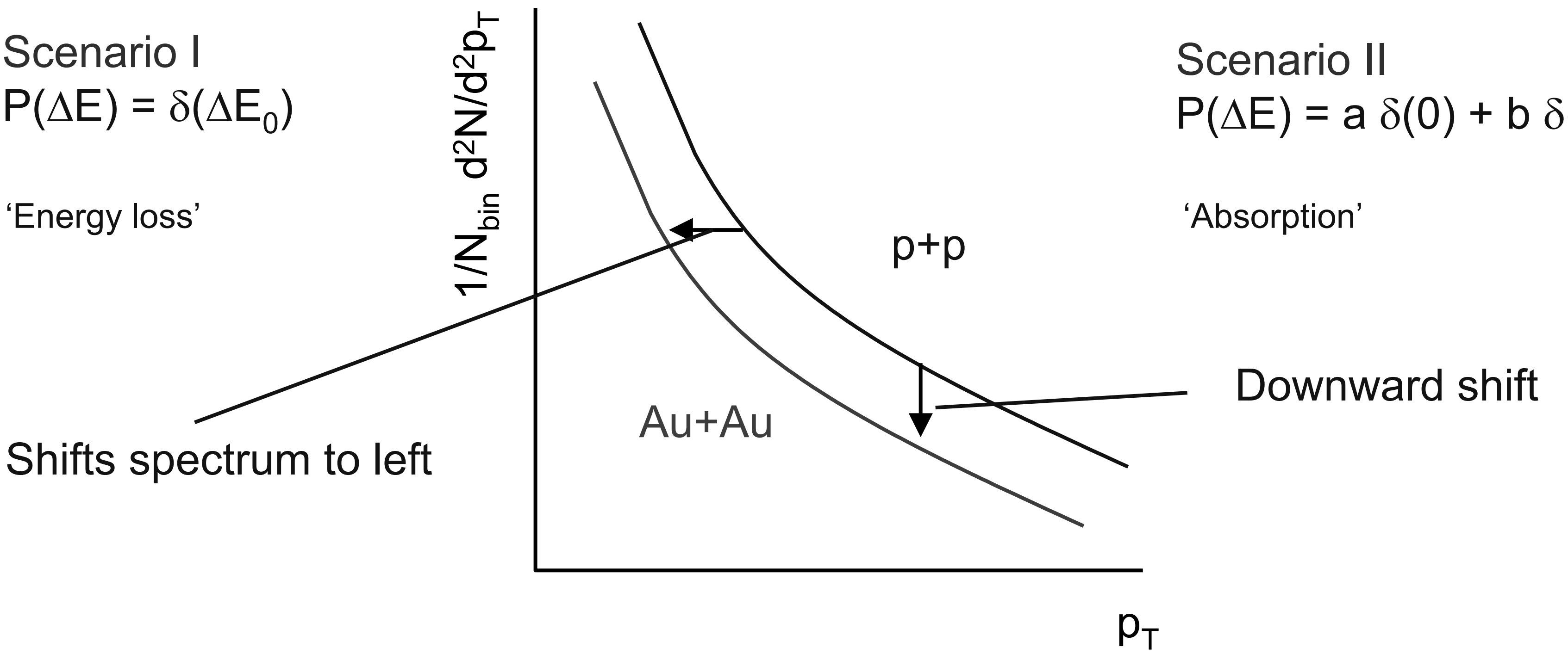
(or how $P(\Delta E)$ says it all)

Scenario I
$P(\Delta E) = \delta(\Delta E_0)$

'Energy loss'

Scenario II
$P(\Delta E) = a\,\delta(0) + b\,\delta(E)$

'Absorption'

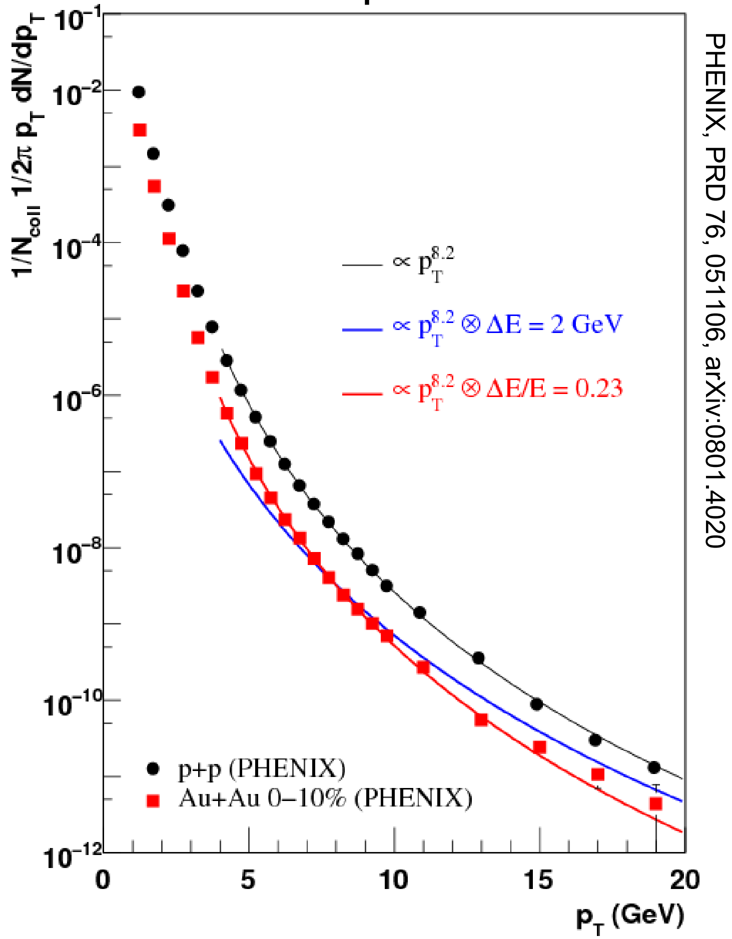



$P(\Delta E)$ encodes the full energy loss process

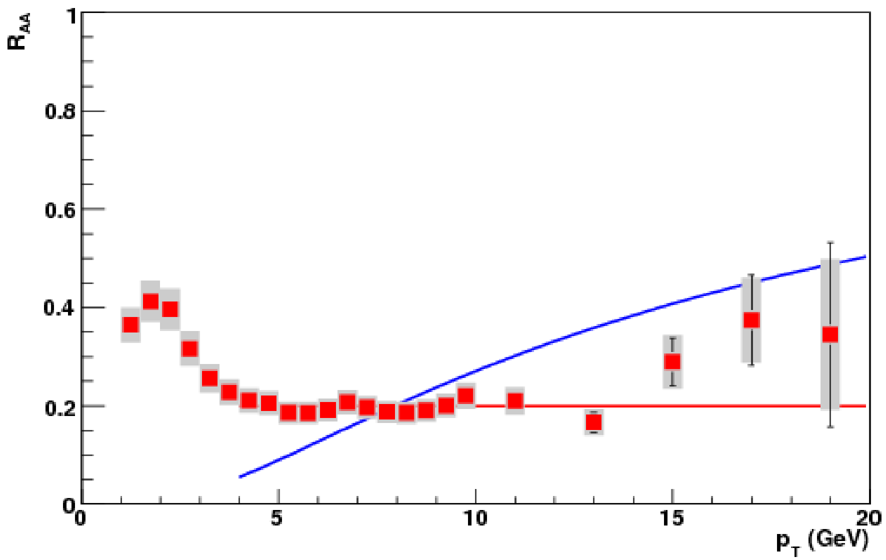
$R_{AA}$ not sensitive to energy loss distribution, details of mechanism

# What can we learn from $R_{AA}$?

$\pi^0$ spectra




Nuclear modification factor




This is a cartoon!
Hadronic, not partonic energy loss
No quark-gluon difference
Energy loss not probabilistic $P(\Delta E)$

Ball-park numbers: $\Delta E/E \approx 0.2$, or $\Delta E \approx 2$ GeV
for central collisions at RHIC

Note: slope of ‘input’ spectrum changes with $p_T$: use experimental reach to exploit this

# Four formalisms

- **Hard Thermal Loops (AMY)**
  - Dynamical (HTL) medium
  - Single gluon spectrum: BDMPS-Z like path integral
  - No vacuum radiation
- **Multiple soft scattering (BDMPS-Z, ASW-MS)**
  - Static scattering centers
  - Gaussian approximation for momentum kicks
  - Full LPM interference and vacuum radiation
- **Opacity expansion ((D)GLV, ASW-SH)**
  - Static scattering centers, Yukawa potential
  - Expansion in opacity $L/\lambda$
    (N=1, interference between two centers default)
  - Interference with vacuum radiation
- **Higher Twist (Guo, Wang, Majumder)**
  - Medium characterised by higher twist matrix elements
  - Radiation kernel similar to GLV
  - Vacuum radiation in DGLAP evolution

Multiple gluon emission

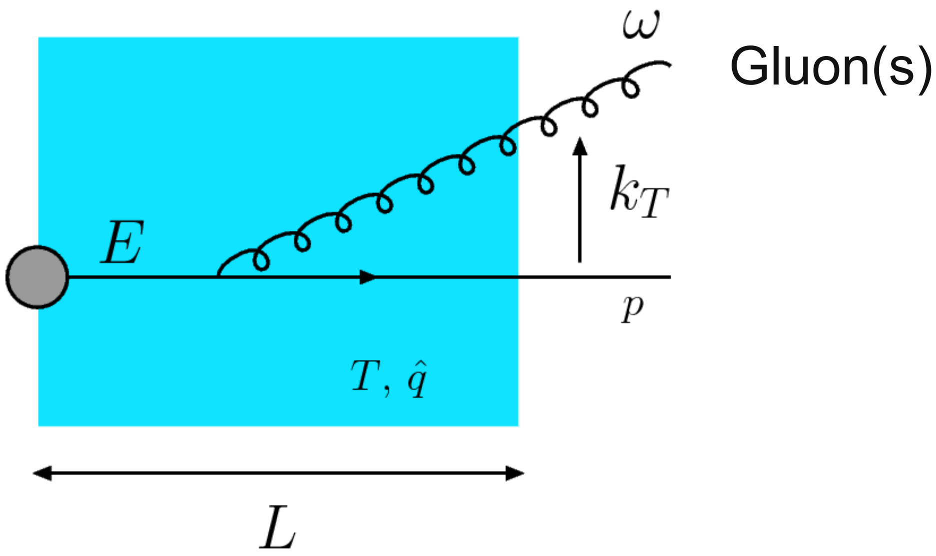
Fokker-Planck rate equations

Poisson ansatz (independent emission)

DGLAP evolution

See also: arXiv:1106.1106

# Kinematic variables

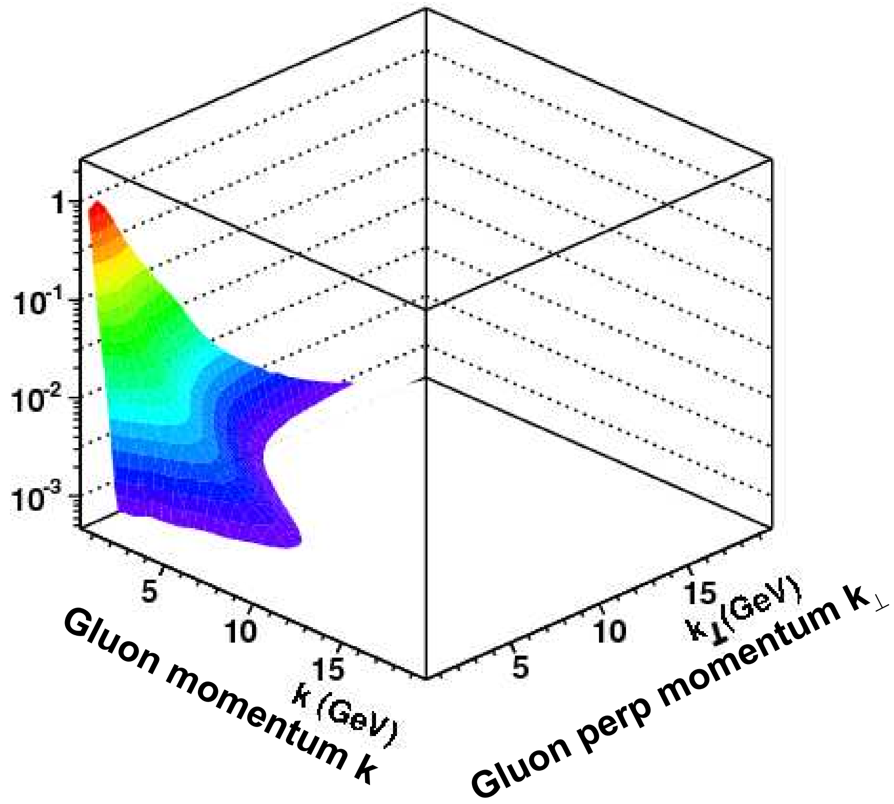


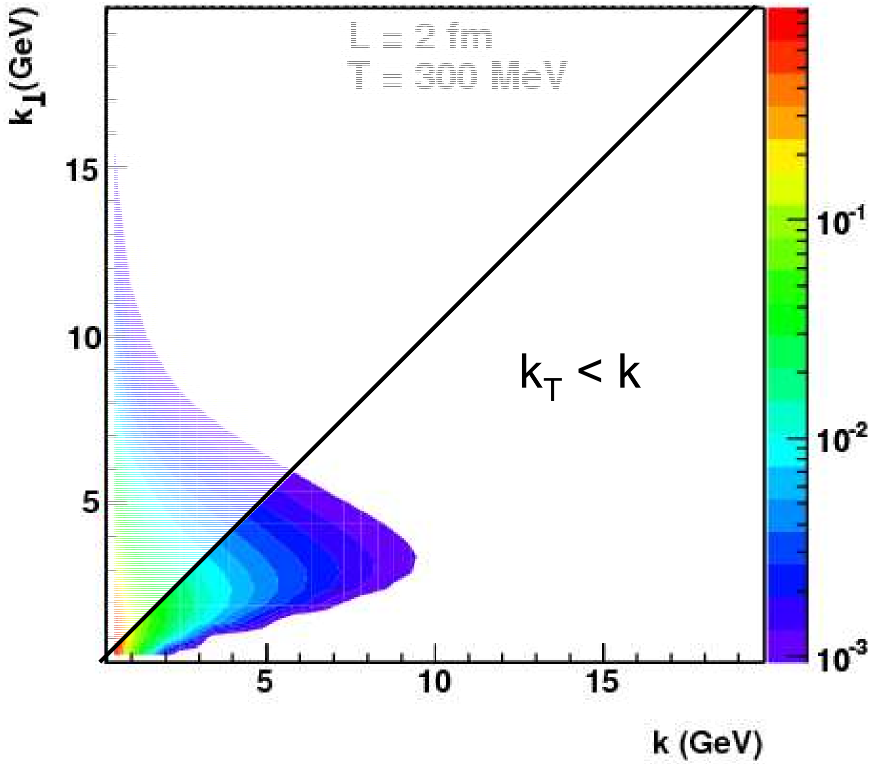

L: medium length
*T, q*: Density
*E*: incoming parton energy/momentum

ω: (total) momentum of radiated gluon (=ΔE)
$k_T$: transverse momentum of radiated gluon
*p*: outgoing parton momentum = *E*-Δ*E* = *E*-ω

# Large angle radiation

Emitted gluon distribution
Opacity expansion



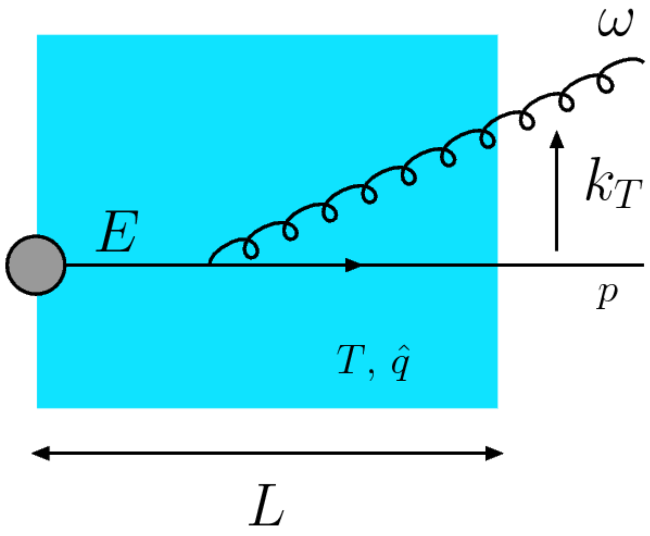




Calculated gluon spectrum extends to large $k_{\perp}$ at small k
Outside kinematic limits

GLV, ASW, HT cut this off ‘by hand’

Estimate uncertainty by varying cut; sizeable effect

# Limitations of soft collinear approach




Calculations are done in soft collinear approximation:

Soft: $\omega << E$ Collinear: $k_T << \omega$

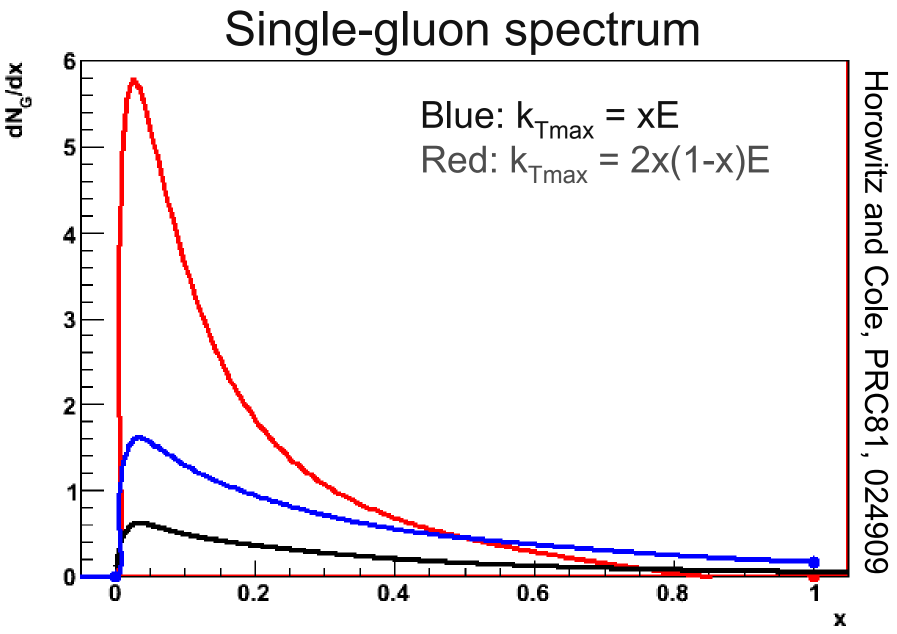
Need to extend results to full phase space to calculate observables (especially at RHIC)

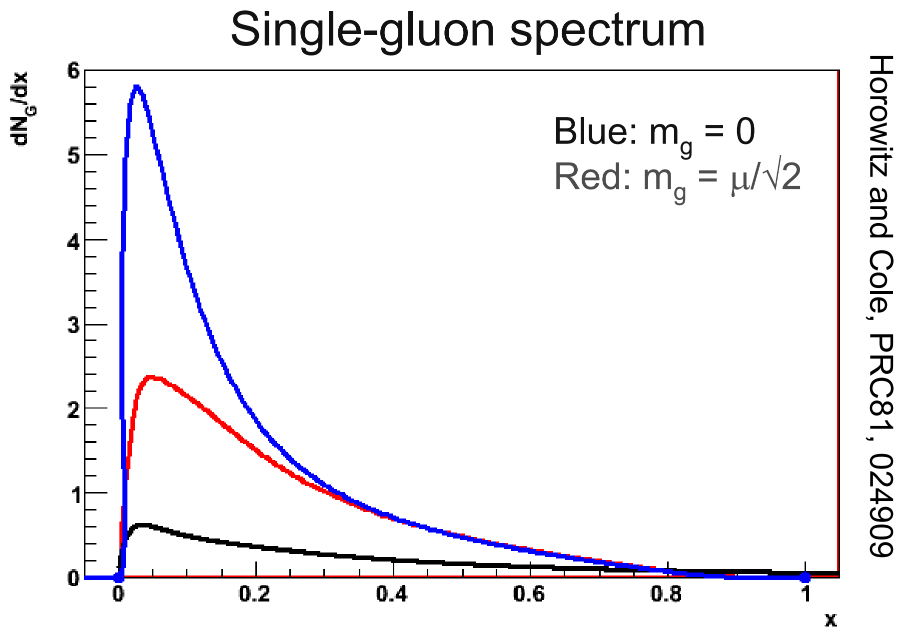
**Soft approximation not problematic:**
For large E, most radiation is soft
Also: $\omega$ > E ⇒ full absorption

**Cannot enforce collinear limit:**
Small $\omega$, $\omega \rightarrow k_T$ always a part of phase space with large angles

# Opacity expansions

## GLV and ASW-SH




Different definitions of *x*:

ASW: $x_E = \frac{\omega}{E}$ GLV: $x_+ = \frac{\omega_+}{E_+}$

$$x_E = x_+ \left(1 + \left(\frac{k_T}{x_+ E^+}\right)^2\right)$$

$x_+ \sim x_E$ in soft collinear limit, but not at large angles

Different large angle cut-offs:

$k_T < \omega = x_E E$

$k_T < \omega = 2\, x_+ E$

Factor ~2 uncertainty from large-angle cut-off

# Opacity expansion vs multiple soft

OE and MS related via path integral formalism

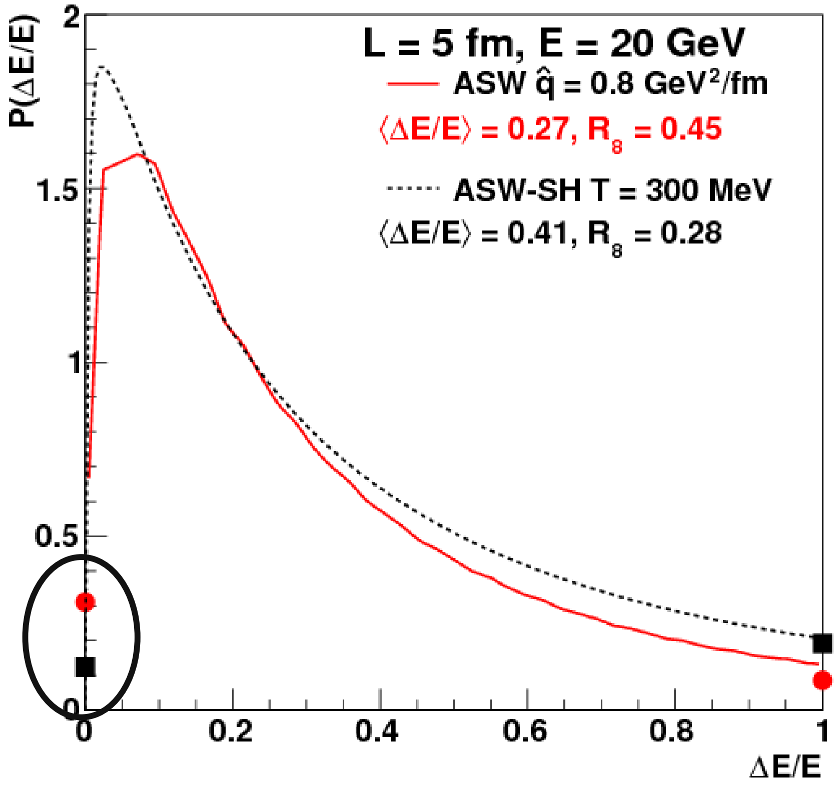
Salgado, Wiedemann, PRD68, 014008

Different limits:

SH (N=1 OE): interference between neighboring scattering centers

MS: ‘all orders in opacity’, gaussian scattering approximation

Two differences at the same time




Quantitative differences sizable

# AMY, BDMPS, and ASW-MS

Single-gluon kernel from AMY based on scattering rate:

$$\frac{d\bar{\Gamma}_{\rm el}}{d^2q_\perp} = \frac{1}{(2\pi)^2}\frac{g^2 T m_{\rm D}^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_{\rm D}^2)}$$

BMPS-Z use harmonic oscillator:

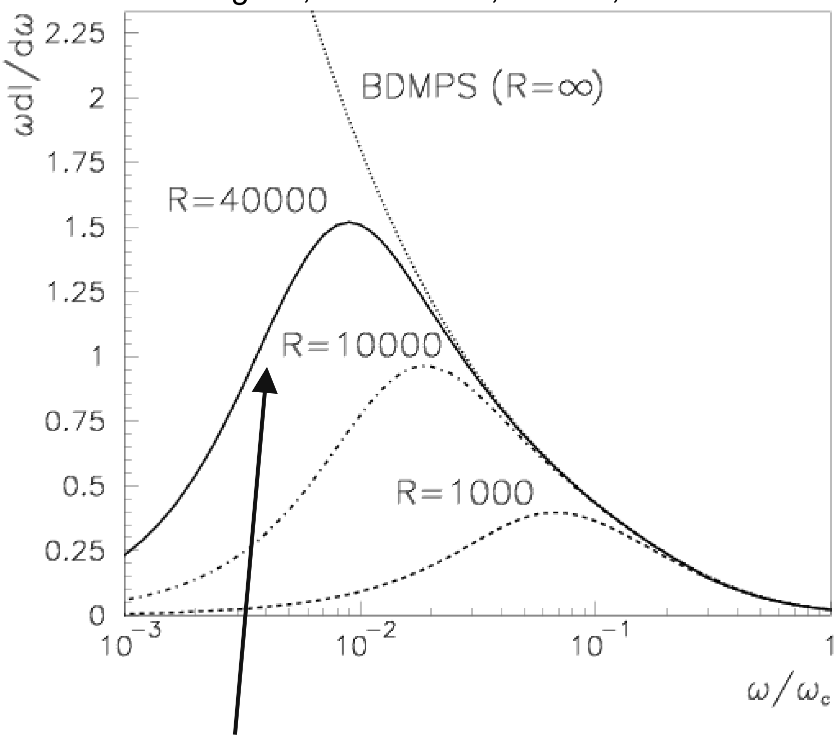
$$\bar{\Gamma}_2(\mathbf{b},t) = \int d^2q_\perp \frac{d\bar{\Gamma}_{\rm el}}{d^2q_\perp}\left(1 - e^{i\mathbf{b}\cdot\mathbf{q}_\perp}\right)$$

$$\bar{\Gamma}_2(\mathbf{b},t) = \frac{1}{4}\hat{\bar{q}}b^2$$

BDMPS-Z:

$$k\frac{dI}{dk} = \frac{\alpha x P_{s\to g}(x)}{\pi}\ln|\cos(\omega_0 L)|$$

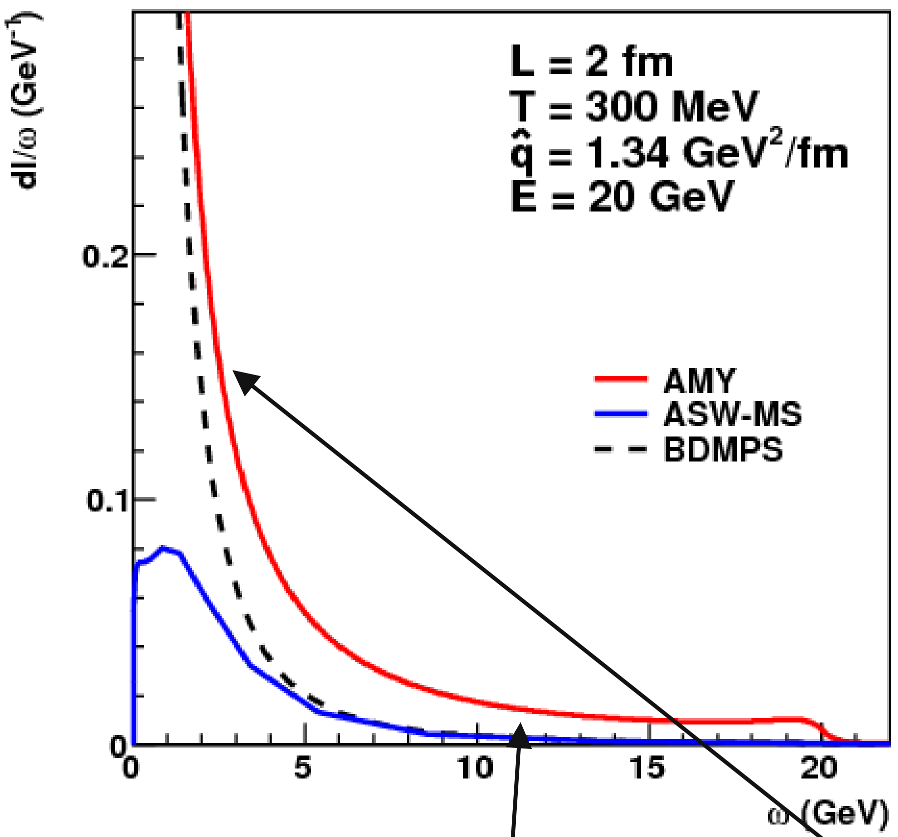
Salgado, Wiedemann, PRD68, 014008



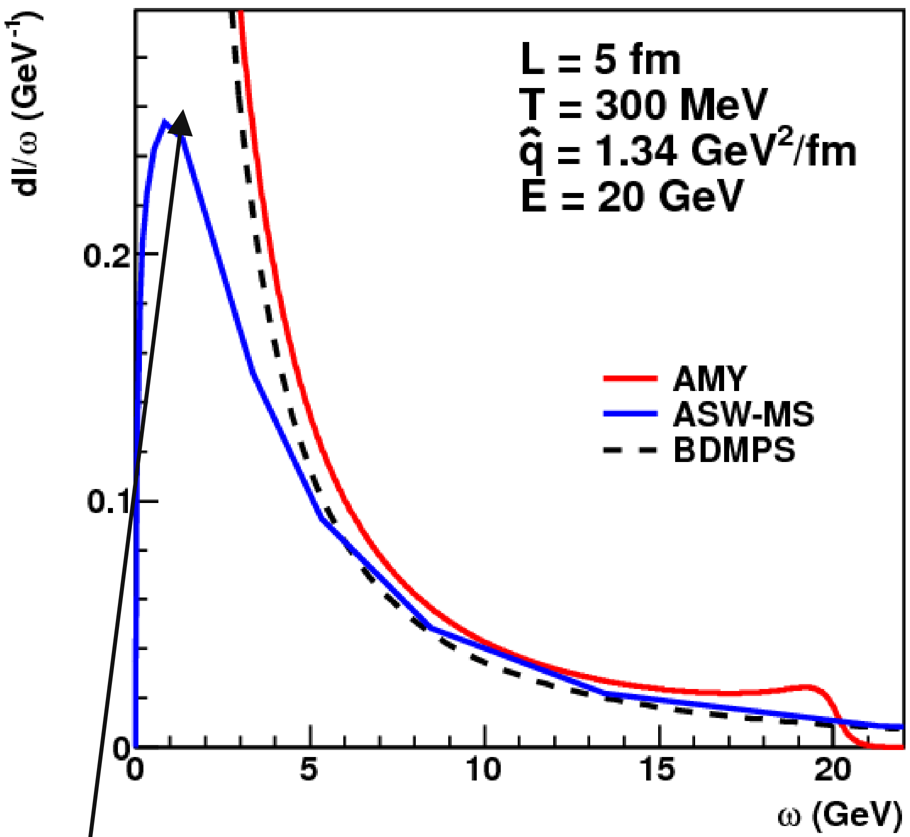

Finite-L effects:
Vacuum-medium interference
+ large-angle cut-off

# AMY and BDMPS

L=2 fm Single gluon spectra




L=5 fm Single gluon spectra



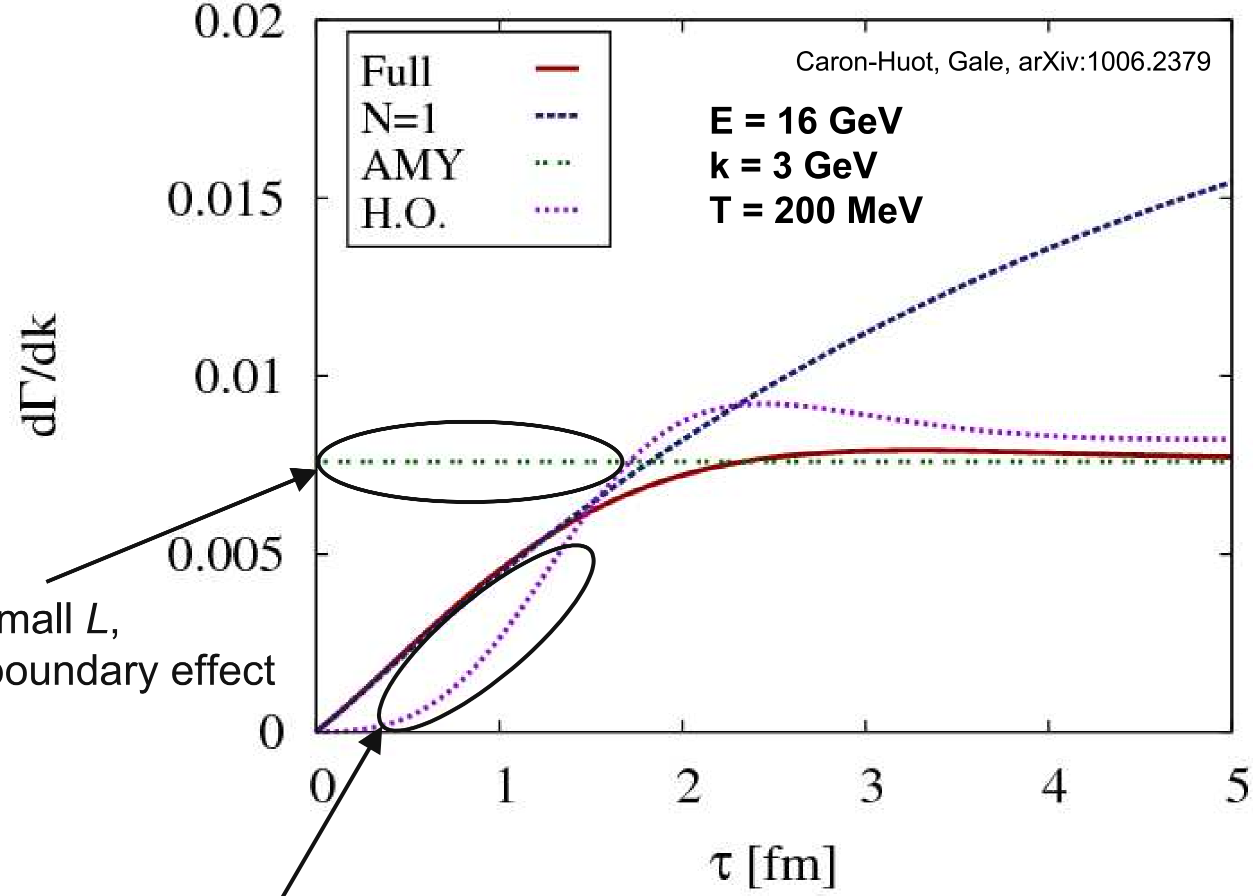

AMY: no large angle cut-off

+ sizeable difference at large ω at L=2 fm

Using $\hat{q}(T)$ based on AMY-HTL scattering potential

# *L*-dependence; regions of validity?

Emission rate vs $\tau$ (=*L*)




GLV N=1
Too much radiation
at large L
(no interference
between scatt centers)

Full =
numerical solution of
Zakharov path integral
= 'best we know'

AMY, small *L*,
no $L^2$, boundary effect

H.O = ASW/BDMPS like (harmonic oscillator)
Too little radiation at small *L*
(ignores 'hard tail' of scatt potential)

# HT and GLV

Single-gluon kernel GLV and HT similar

HT: $$\frac{dN}{dx\,dk_T^2} = \frac{\alpha_s C_F}{\pi} P_{qg}(x) F_{gg} \frac{1}{k_T^4} \Omega$$

$$\Omega = \int_0^L d\xi \left[ 1 - \cos\left( \frac{k_T^2 \xi}{2\,p\,z\,(1-z)} \right) \right]$$

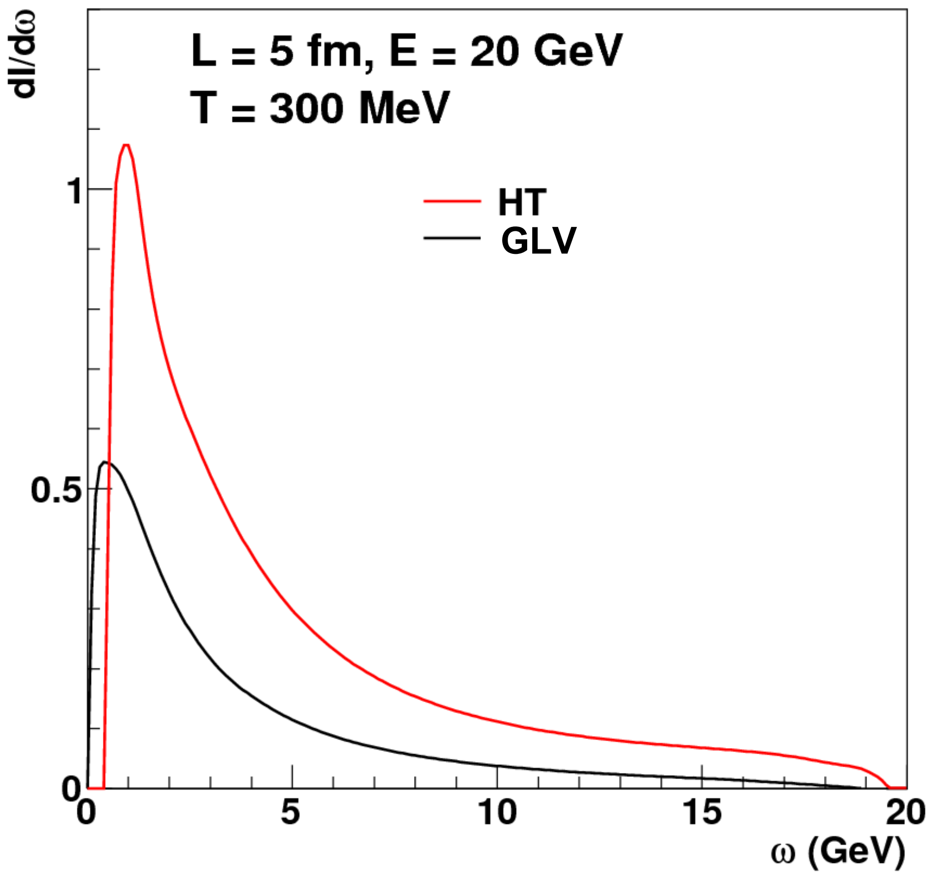
GLV similar structure, phase factor $\Omega$

However HT assumes $k_T >> q_T$, so no explicit integral over $q_T$

HT: kernel diverges for $k_T \rightarrow 0$
$\tau < L \Rightarrow k_T > \sqrt{(E/L)}$

HT: $$k_{T,\max} = \sqrt{2\,x\,(1-x)\,E\,3\,T}$$

GLV: $$k_{T,\max} = 2\,x\,(1-x)\,E \qquad q_{T,\max} = \sqrt{3\,E\,T}$$

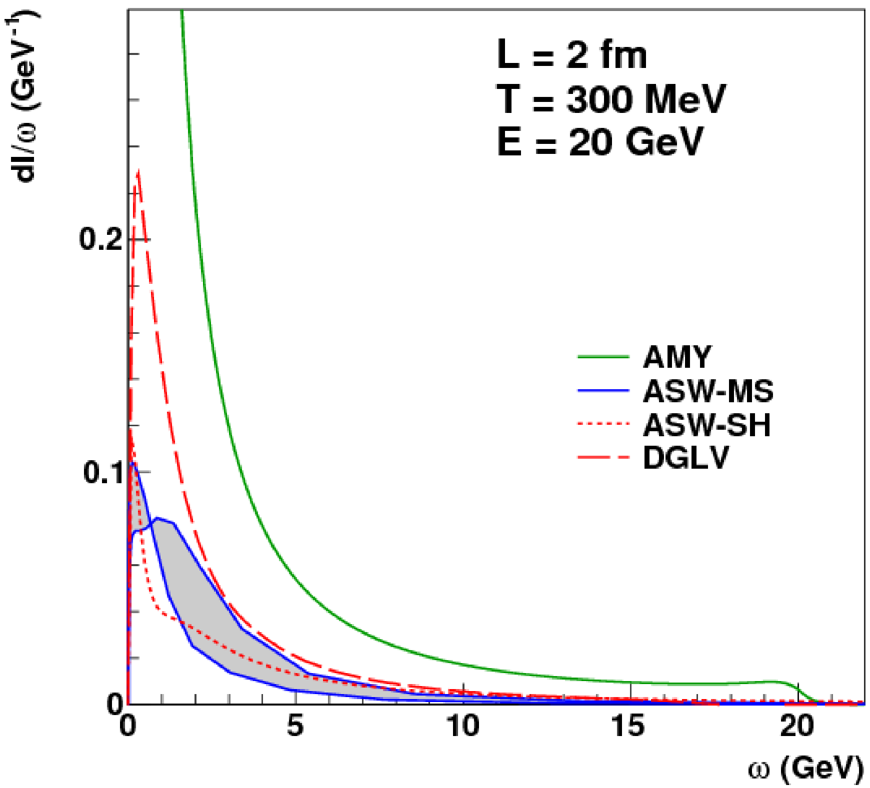



HT gives more radiation than GLV

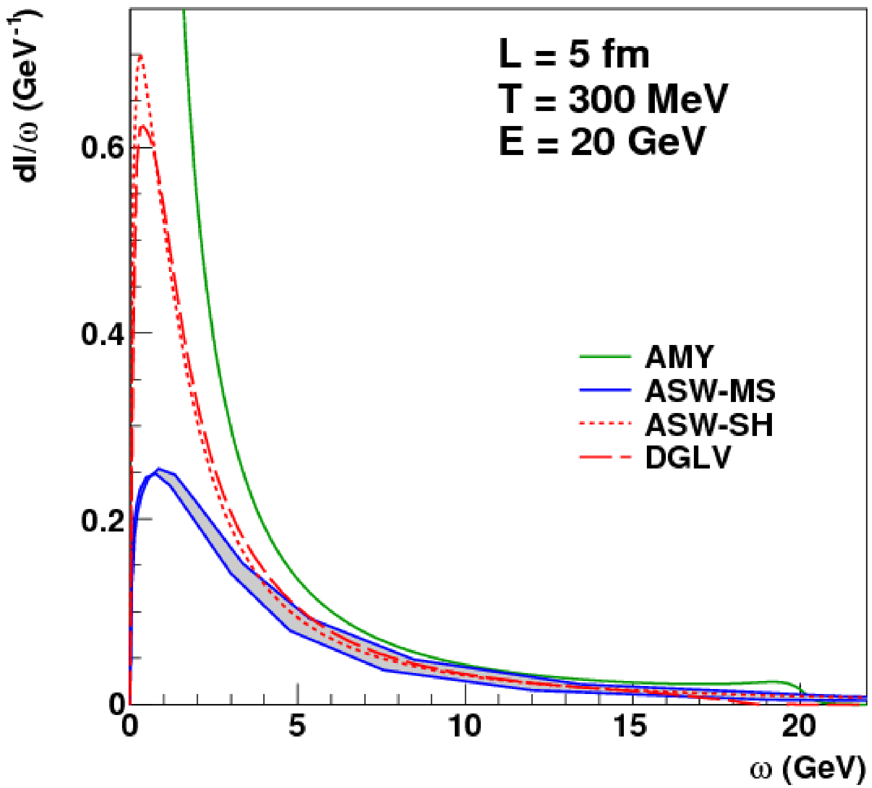
# Single gluon spectra

Same temperature

L = 2 fm




L = 5 fm




@Same temperature: AMY > OE > ASW-MS

Size of difference depends on L, but hierarchy stays

# Multiple gluon emission

Average number of gluons:

$$\langle N_{gluon} \rangle = \int \frac{dI}{d\omega} d\omega$$

Poisson fluctuations:

$$P(n) = \frac{1}{n!} \langle N_{gluon} \rangle^n e^{-\langle N_{gluon} \rangle}$$

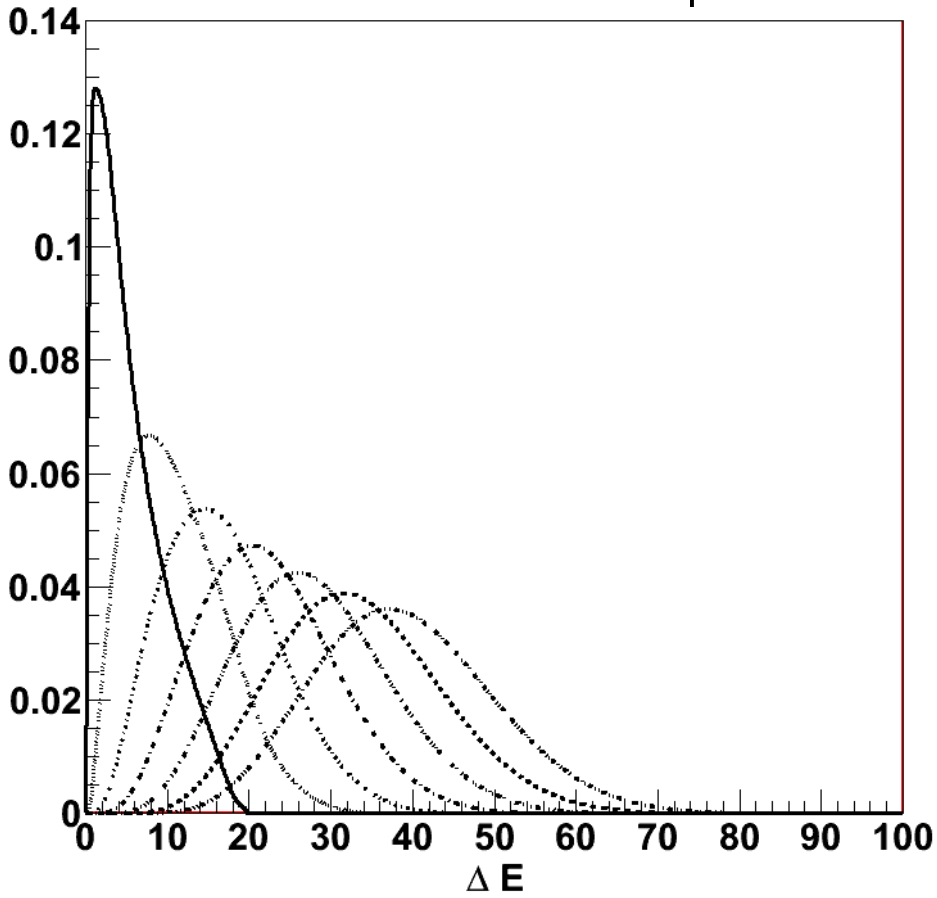
(assumed)

Total probability:

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^{n} \int d\omega_i \frac{dI(\omega_i)}{d\omega} \right]$$

$$\delta\left(\Delta E - \sum_{i=1}^{n} \omega_i\right) \exp\left[ - \int_0^{\infty} d\omega \frac{dI}{d\omega} \right]$$

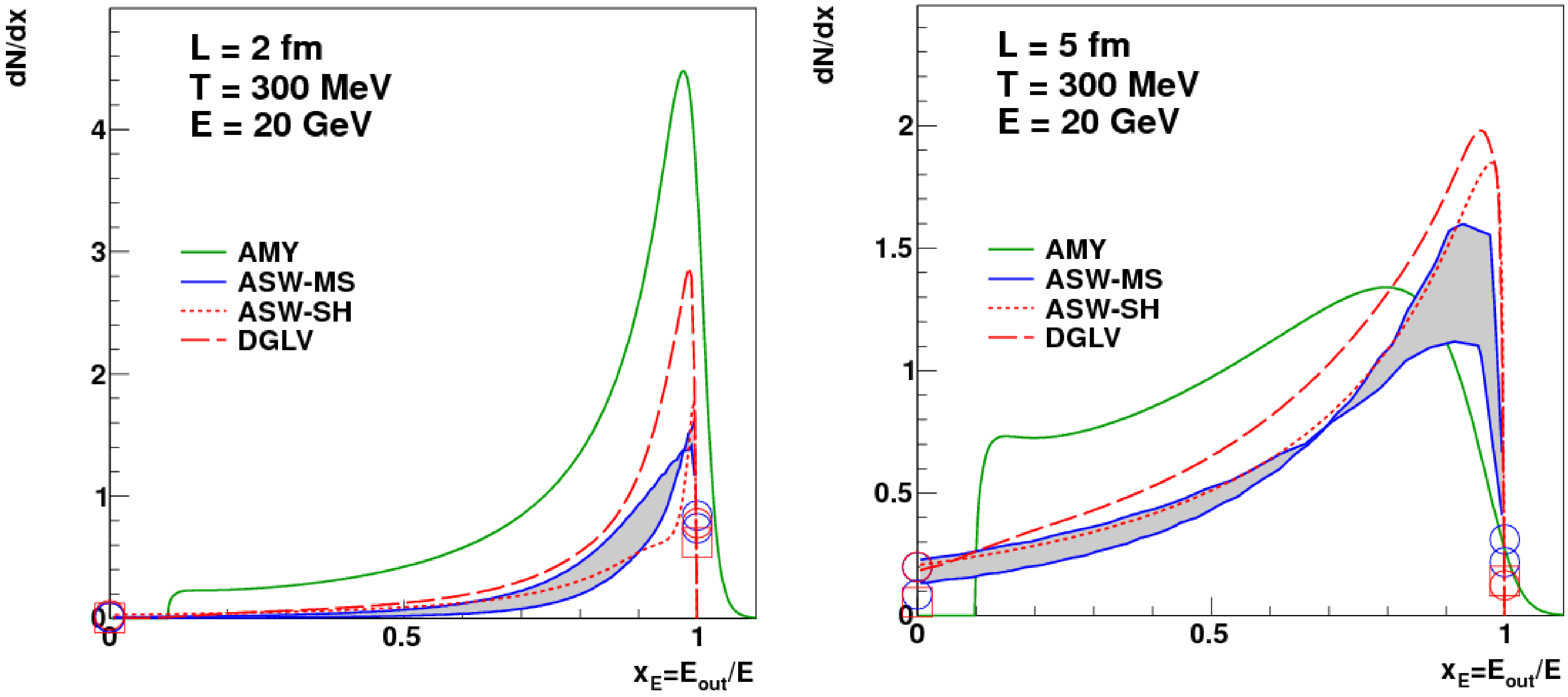
Poisson convolution example




$$P(\Delta E) = p_0 \delta(\Delta E) + p(\Delta E)$$

# Outgoing quark spectra

Same temperature: *T* = 300 MeV

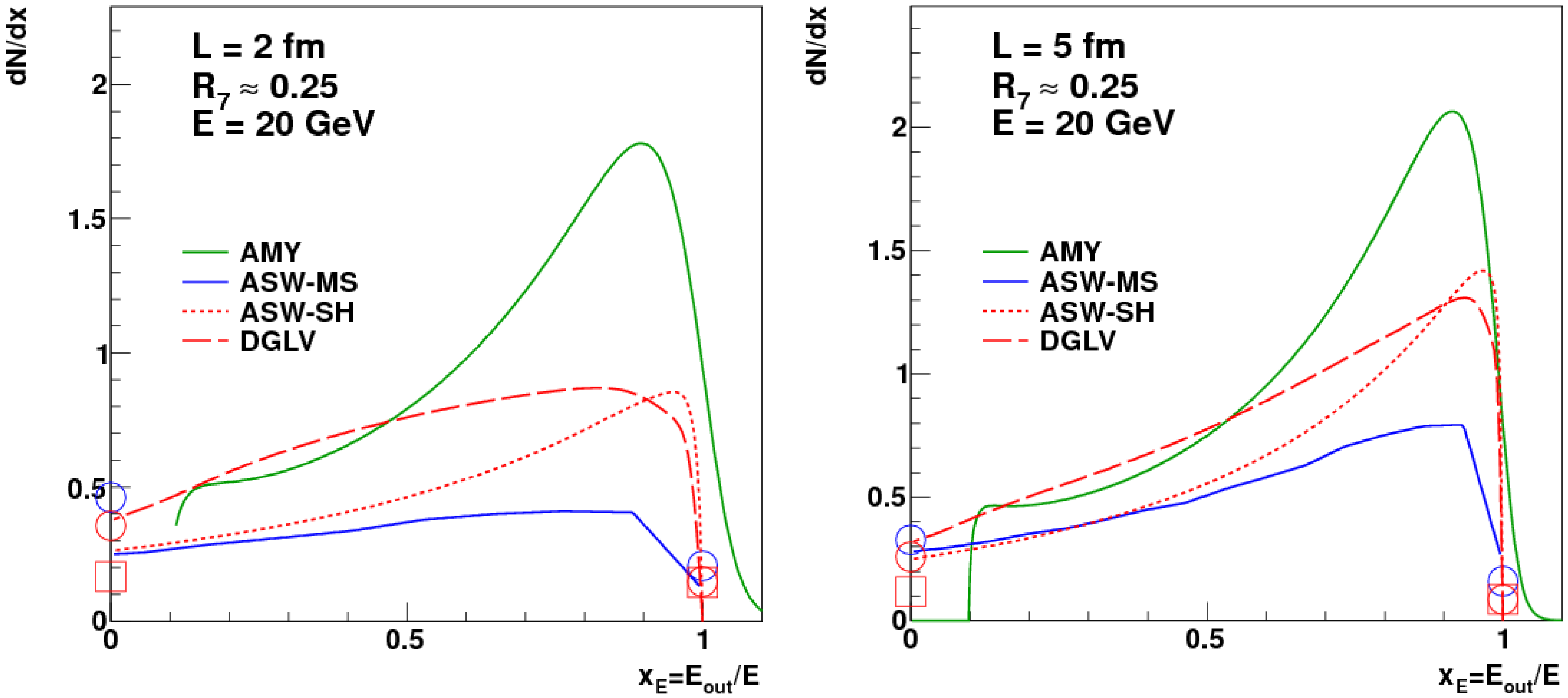



@Same *T*: suppression AMY > OE > ASW-MS

Note importance of $P_0$

# Outgoing quark spectra

Same suppression: $R_7 = 0.25$




At $R_7 = 0.25$: $P_0$ small for ASW-MS

$P_0 = 0$ for AMY by definition

# Model inputs: medium density

Multiple soft scattering (ASW-MS):

Quenching weights (AliQuenchingWeights)

Inputs: $\hat{q}, L$ $\omega_c = \frac{1}{2}\hat{q}L^2$ $R = \omega_c L$

**N.B: keep track of factors hc = 197.327 MeVfm**

Quenching weights: $P(x = \omega / \omega_c; R)$

Opacity expansion (DGLV, ASW-SH): $\mu, \frac{L}{\lambda}$

Gluon spectra $\frac{dI}{d\omega} = \frac{L}{\lambda} K_1(\omega; \mu, L)$

+Poisson Ansatz for multiple gluon radiation

# Medium properties

Some pocket formulas

gluon gas, Baier scheme:

$$\mu = gT = \sqrt{4\pi\alpha_s}\,T \qquad \lambda = \frac{1}{\rho\sigma} \qquad \rho = \frac{16\cdot 1.202}{\pi^2}T^3 \qquad \sigma = \frac{9\pi\alpha_s^2}{\mu^2}$$

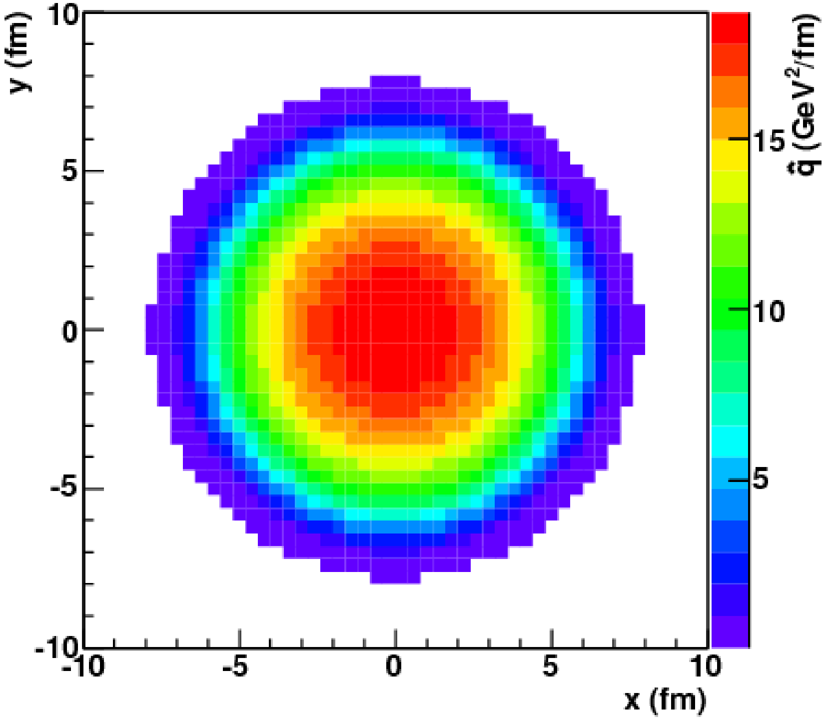
$$\hat{q} \approx \frac{\mu^2}{\lambda} \qquad \hat{q} = \frac{72\cdot 1.202\,\alpha_s^2}{\pi}T^3$$

HTL: $\hat{q} = 3\alpha_s m_D T \ln\left(\frac{\Lambda^2}{m_D^2}\right) = 1.37\ \text{Baier}\ \ln\left(\frac{\Lambda^2}{m_D^2}\right)$

See also: arXiv:1106.1106

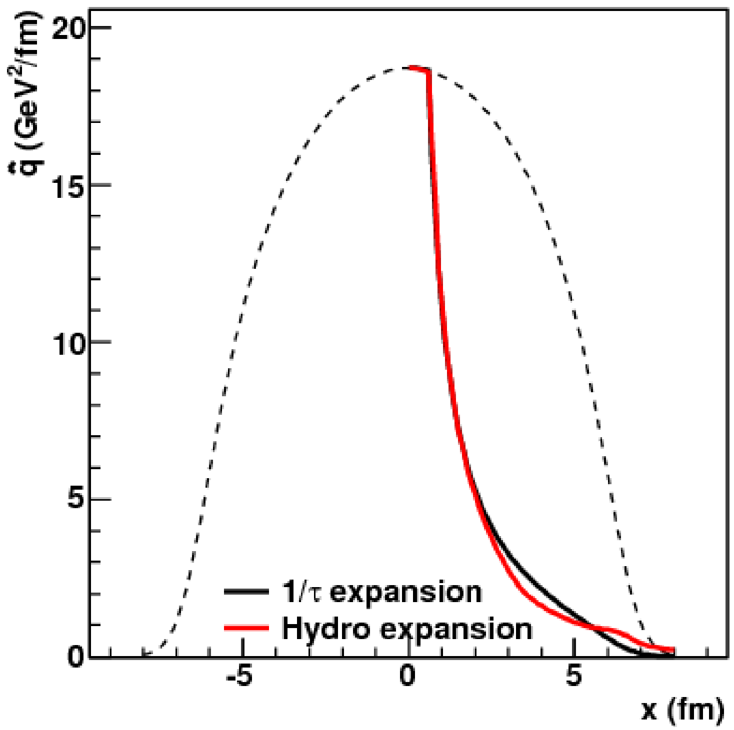
# Geometry

Density profile




Profile at $\tau \sim \tau_{form}$ known

Density along parton path




Longitudinal expansion
dilutes medium
⇒ Important effect

Space-time evolution is taken into account in modeling

# Geometry II

*L*-moments of density along path:

$$I_1 = \int dv\ \hat{q}(v)\ v = \tfrac{1}{2}\left\langle \hat{q}L^2 \right\rangle = \omega_c^{eff}$$

$$I_0 = \int dv\ \hat{q}(v) = \left\langle \hat{q}L \right\rangle$$

$$L_{eff} = \frac{2I_1}{I_0}$$

# ‘Analytic’ calculations vs MC Event Generators

- Analytic calculations
  (this lecture)
  - Easy to include interference
  - So far: only soft-collinear approx
  - Energy-momentum conservation ad-hoc
- Monte Carlo event generators
  (modified parton showers)
  - Energy-momentum conservation exact
  - May be able to introduce recoil (dynamic scatt centers)
  - More difficult to introduce interference
  - Examples: JEWEL, qPYTHIA, YaJEM

# AliQuenchingWeights

Based on Salgado, Wiedemann, hep-ph/0302184

AliQuenchingWeights::InitMult()
Initialises multiple soft scattering Quenching Weights

AliQuenchingWeights::CalcMult(ipart, R, x, cont, disc)
Multiple soft scattering Quenching Weights

**input:**

ipart 0=gluon, 1=quark

$\omega_c = \frac{1}{2}\hat{q}L^2$

$x = \omega/\omega_c$

$R = \omega_c L$

**return:**

cont: $\omega$ dI/d$\omega$

disc: P(0)

# Extra slides

# Thoughts about black-white scenario

Or: Hitting the wall with P(ΔE)

- At RHIC, we might have effectively a 'black-white scenario'
  - Large mean E-loss
  - Limited kinematic range
- Different at LHC?
  - Mean E-loss not much larger, kinematic range is?
  - Or unavoidable: steeply falling spectra

In addition: the more monochromatic the probe,
the more differential sensitivity $\gamma$-jet, jet-reco promising!

# Opacity expansion

Opacity expansion (DGLV, ASW-SH): $\mu, \frac{L}{\lambda}$

Gluon spectra $\frac{dI}{d\omega} = \frac{L}{\lambda} K_1(\omega; \mu, L)$

Poisson Ansatz: