# Particle production in pQCD
# How do the models work?

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# What is QCD?

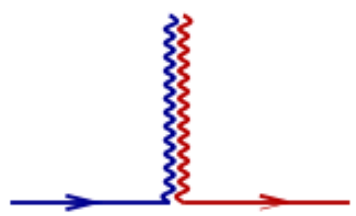
## What is QCD (Quantum Chromo Dynamics)?

Elementary fields: Quarks Gluons

$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1,\ldots,3 \\ \text{spin} & \alpha = 1,2 \\ \text{flavor} & f = u,d,s,c,b,t \end{cases} \qquad A^a_\mu \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

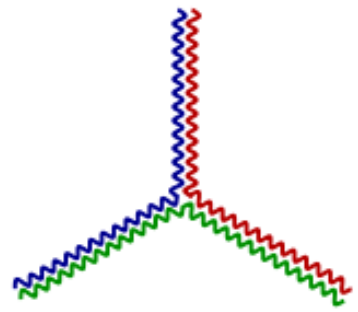
Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

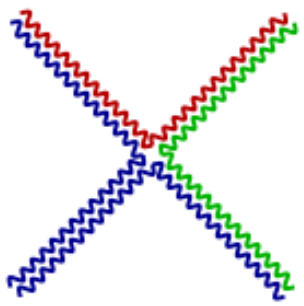
$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$



$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$i\not{D} q = \gamma^\mu \left( i\partial_\mu + g A^a_\mu t^a \right) q$$

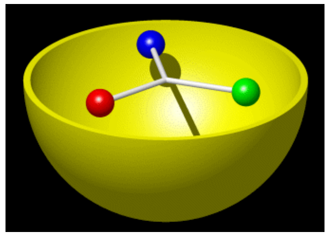




From: T. Schaefer, QM08 student talk

# QCD and hadrons

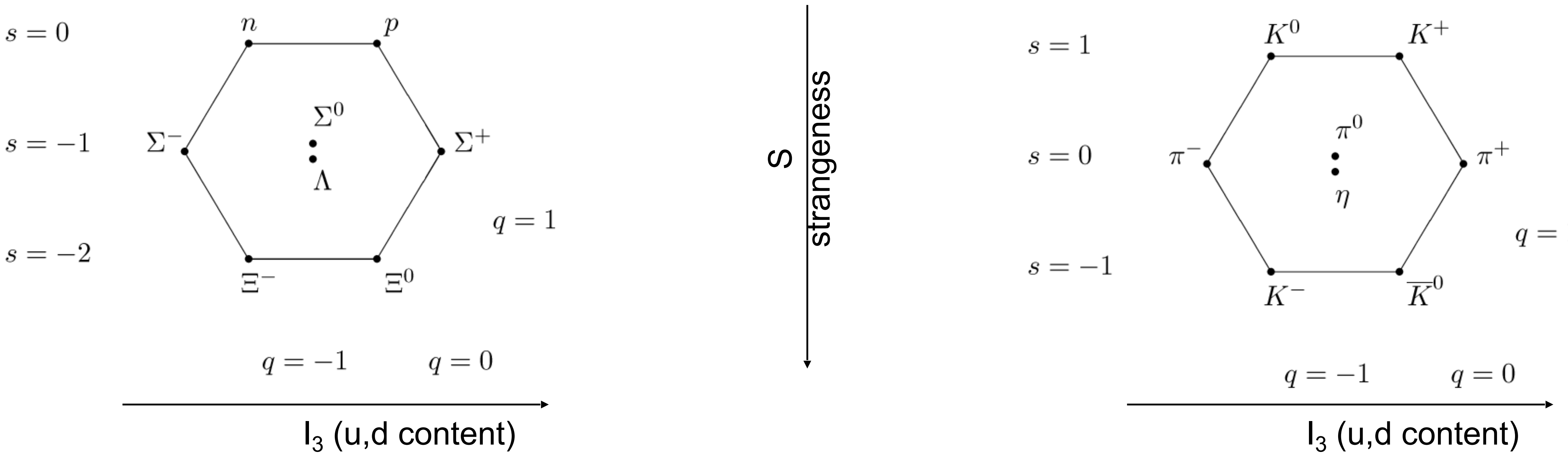
Quarks and gluons are the fundamental particles of QCD
(feature in the Lagrangian)



However, in nature, we observe hadrons:
Color-neutral combinations of quarks, anti-quarks

Baryon multiplet

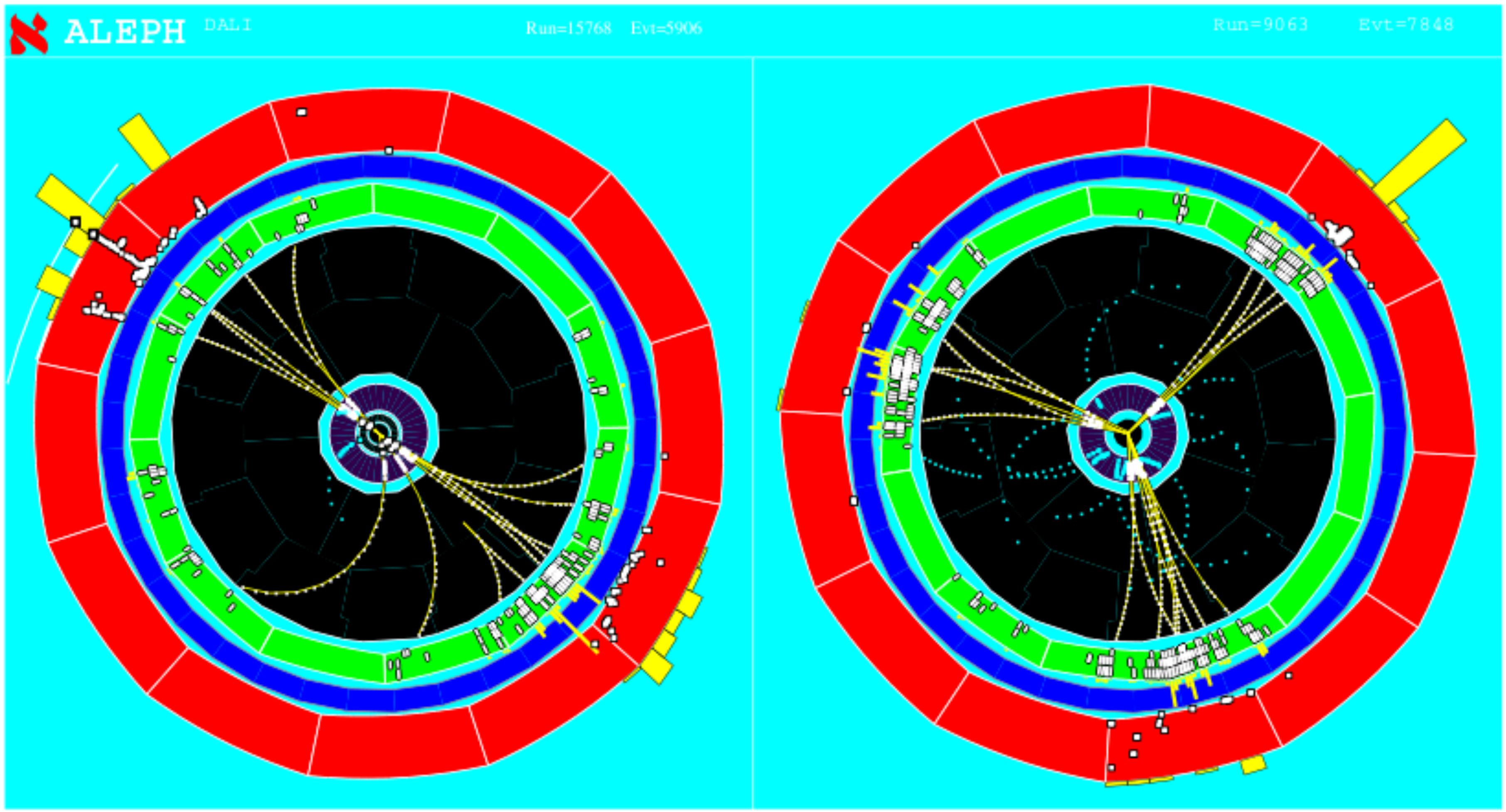
Meson multiplet

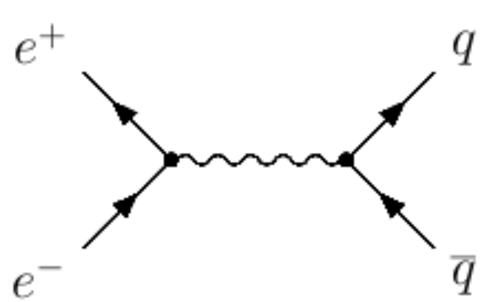


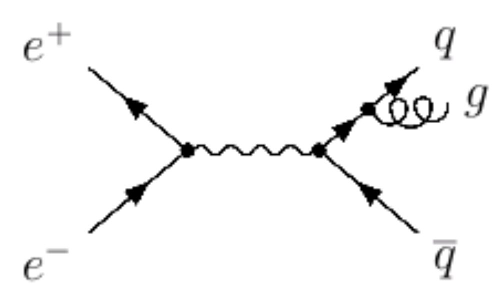

Baryons: 3 quarks

Mesons: quark-anti-quark

# Seeing quarks and gluons

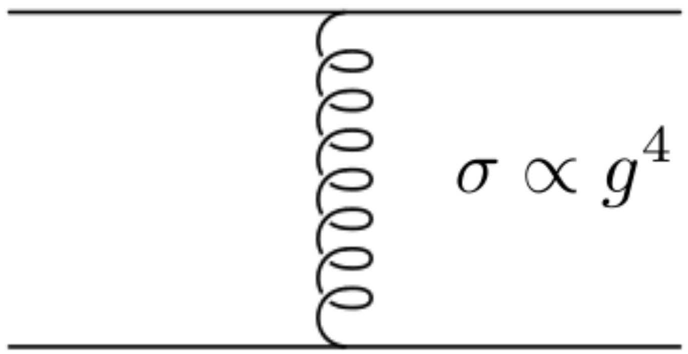


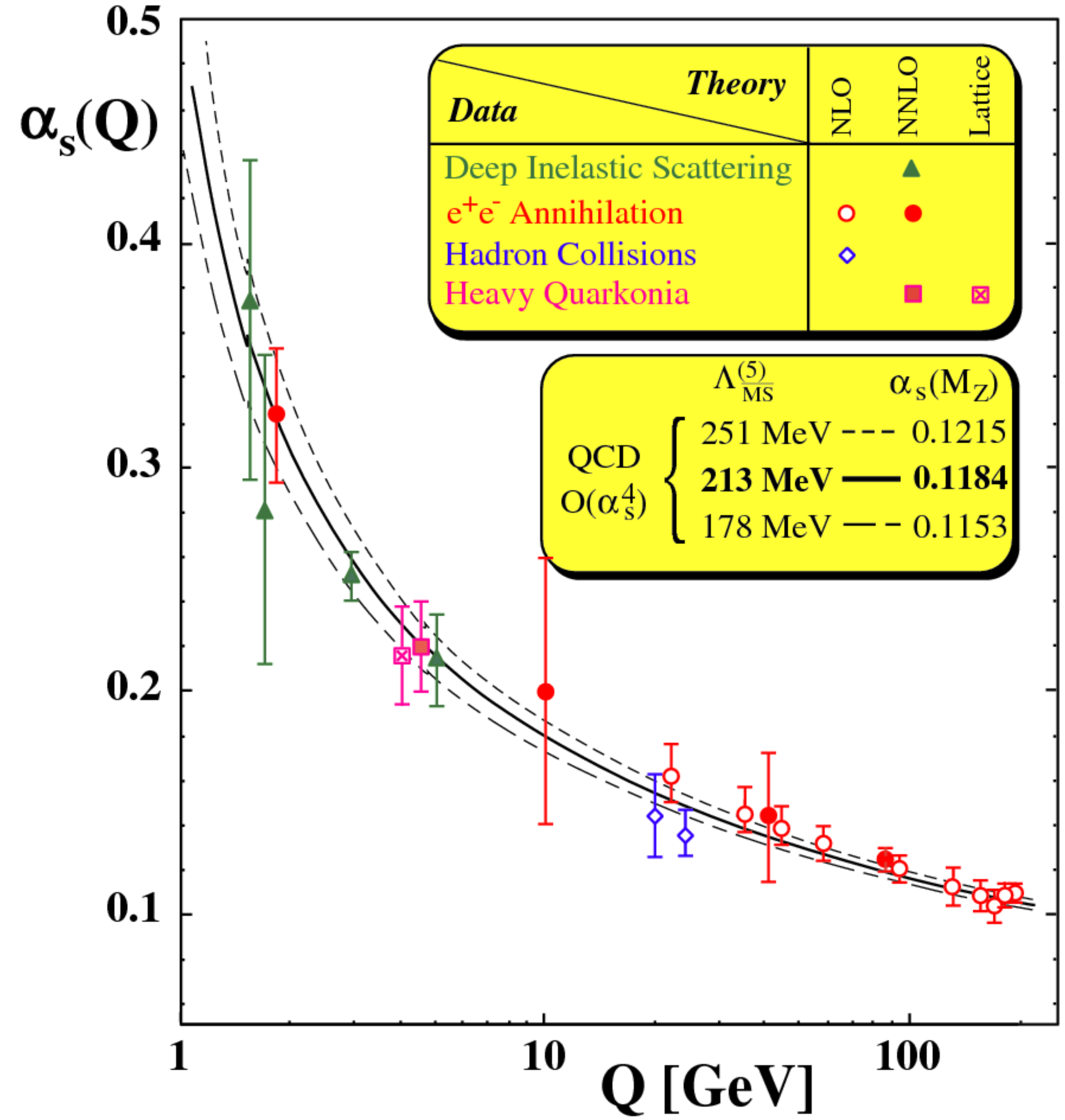

In high-energy collisions, observe traces of quarks, gluons ('jets')

# How does it fit together?



$$\sigma \propto g^4 = 16\pi^2\alpha_s^2$$

S. Bethke, J Phys G 26, R27




Running coupling:
$\alpha_s$ decreases with $Q^2$

$$\beta_1 = (11N_c - 2n_f)/3$$

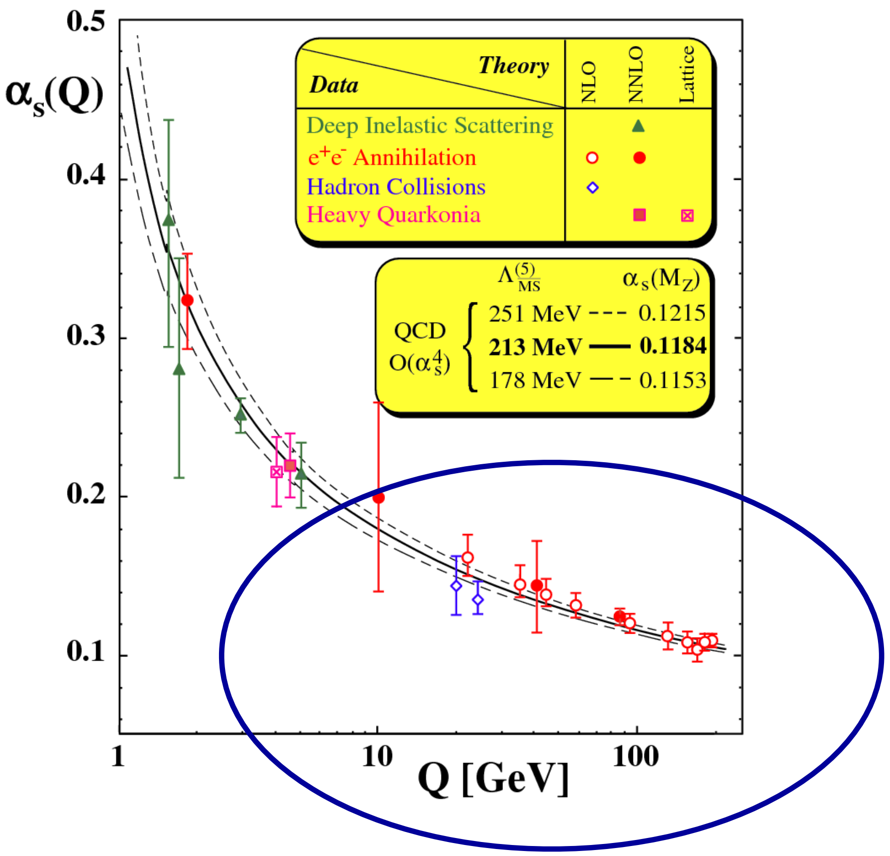
$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_1 \ln(\mu^2/\Lambda^2_{QCD})}$$

Pole at $\mu = \Lambda$

$\Lambda_{QCD} \sim 200$ MeV $\sim 1$ fm$^{-1}$

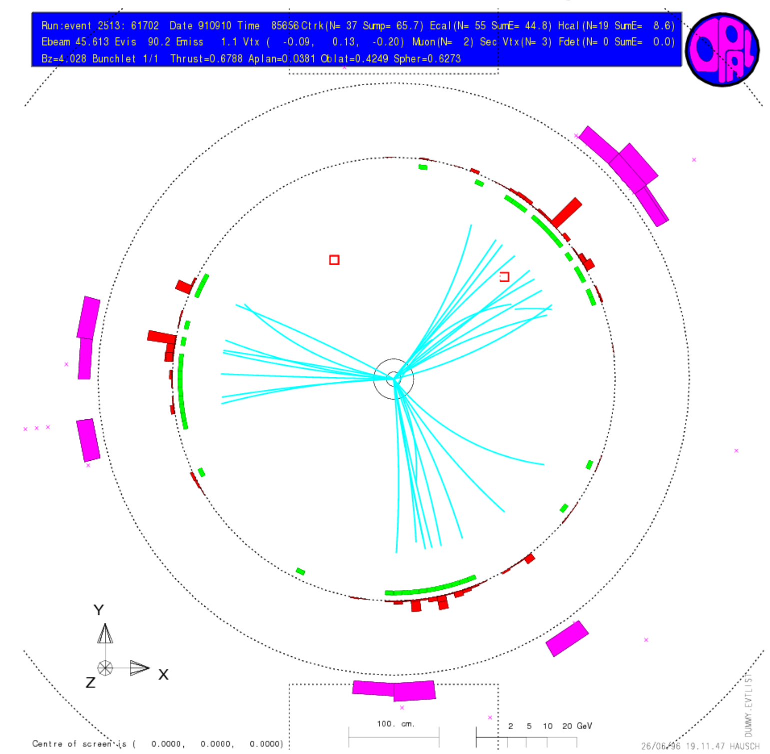
Hadronic scale

# Asymptotic freedom and pQCD

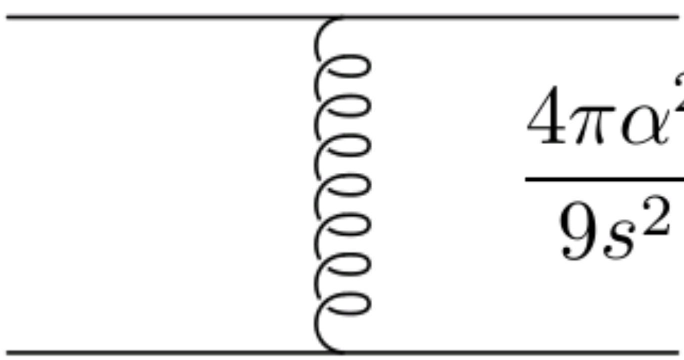



At large $Q^2$, hard processes: calculate 'free parton scattering'

$e^+ e^- \rightarrow q\,\bar{q}\,g$




At high energies, quarks and gluons are manifest



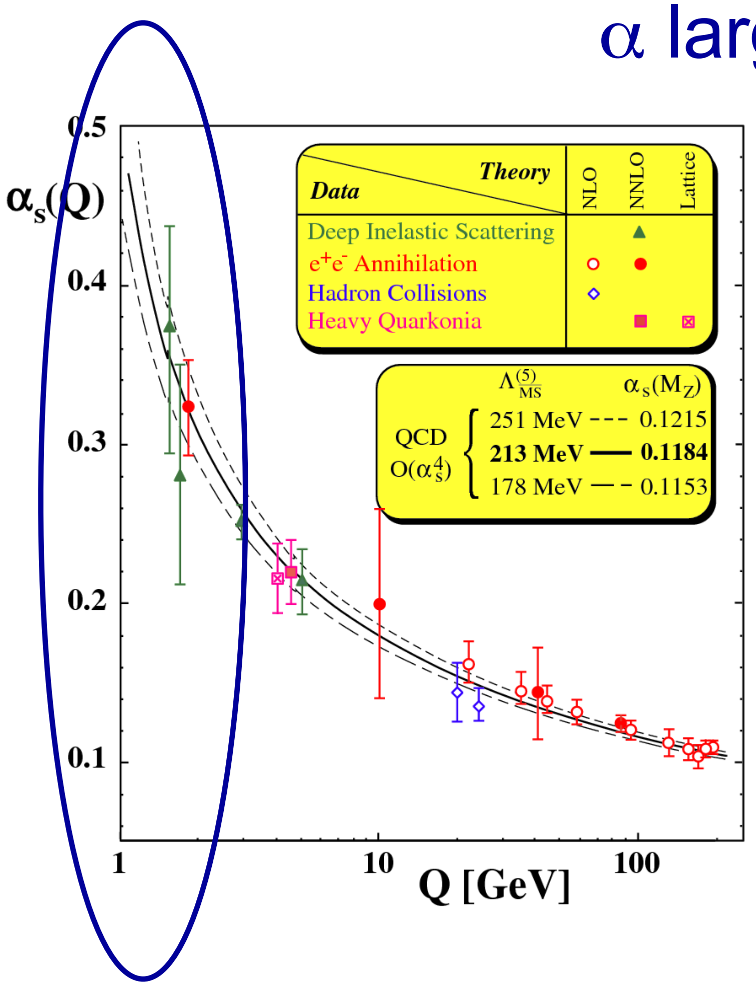
$$\frac{4\pi\alpha^2}{9s^2}\,\frac{s^2+u^2}{t^2}$$

+ more subprocesses

But need to add hadronisation (+initial state PDFs)

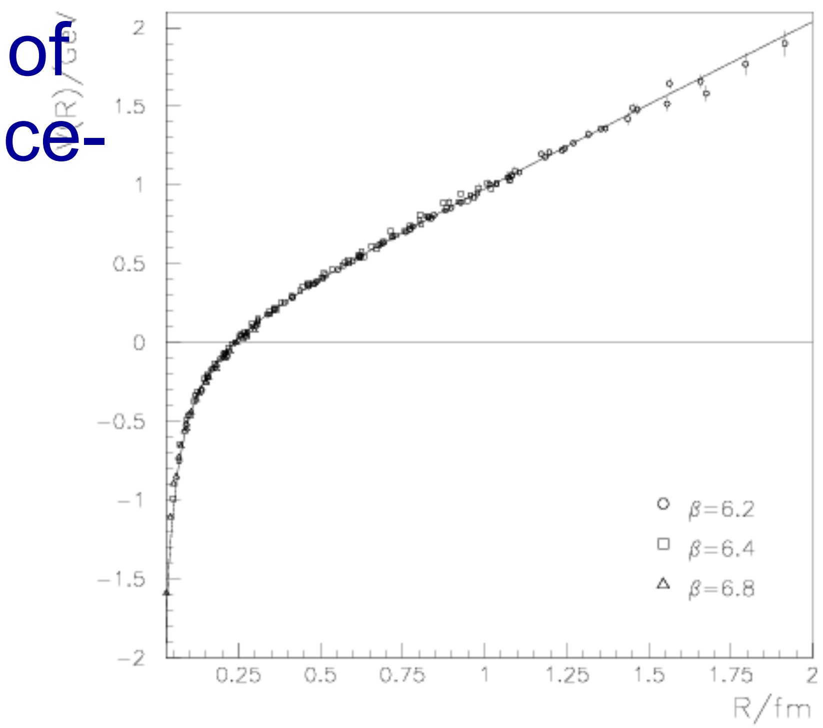
# Low Q²: confinement

$\alpha$ large, perturbative techniques not suitable

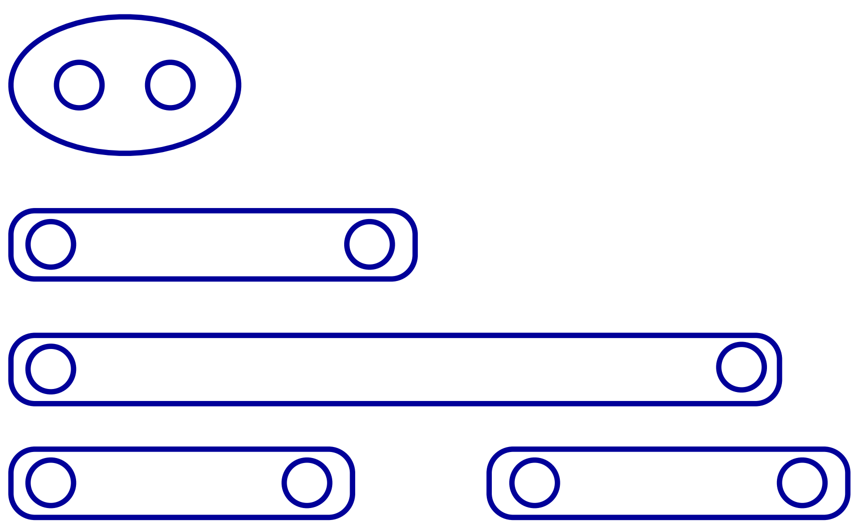



Lattice QCD: solve equations of motion (of the fields) on a space-time lattice by MC

Bali, hep-lat/9311009




Lattice QCD potential



String breaks, generate qq pair to reduce field energy

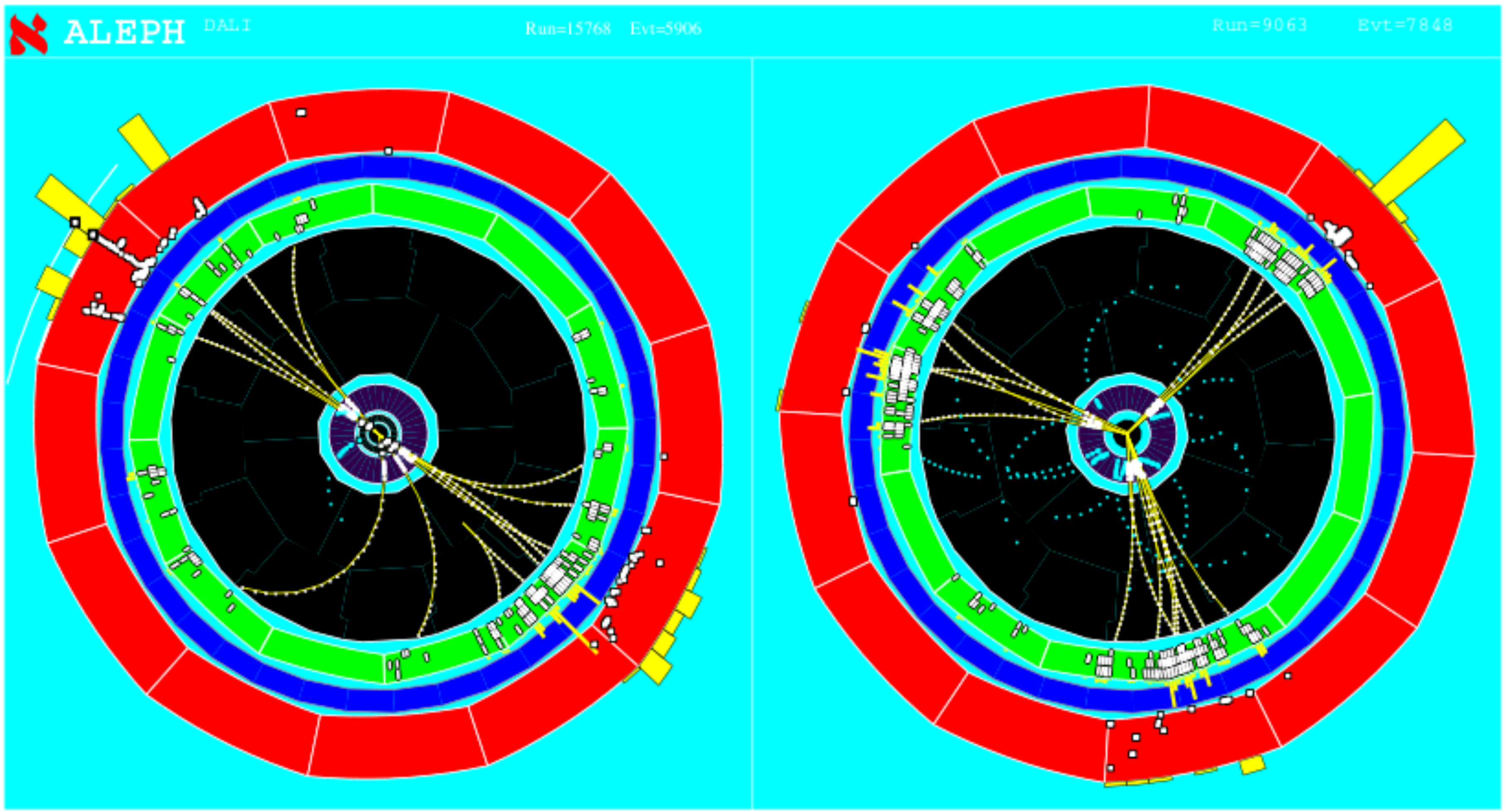
# Part I: Hard processes in fundamental collisions

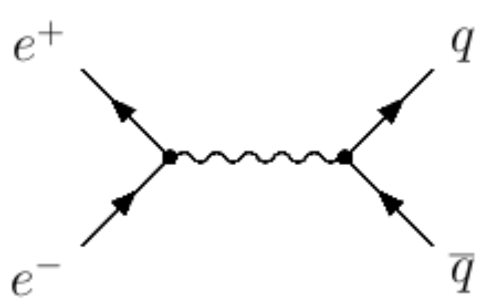
# Accelerators and colliders

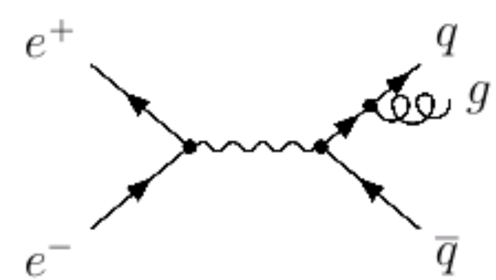
- p+p colliders (fixed target+ISR, SPPS, TevaTron, LHC)
  - Low-density QCD
  - Broad set of production mechanisms
- Electron-positron colliders (SLC, LEP)
  - Electroweak physics
  - Clean, exclusive processes
  - Measure fragmentation functions
- ep, μp accelerators (SLC, SPS, HERA)
  - Deeply Inelastic Scattering, proton structure
  - Parton density functions
- Heavy ion accelerators/colliders (AGS, SPS, RHIC, LHC)
  - Bulk QCD and Quark Gluon Plasma

Many decisive QCD measurements done

# Seeing quarks and gluons



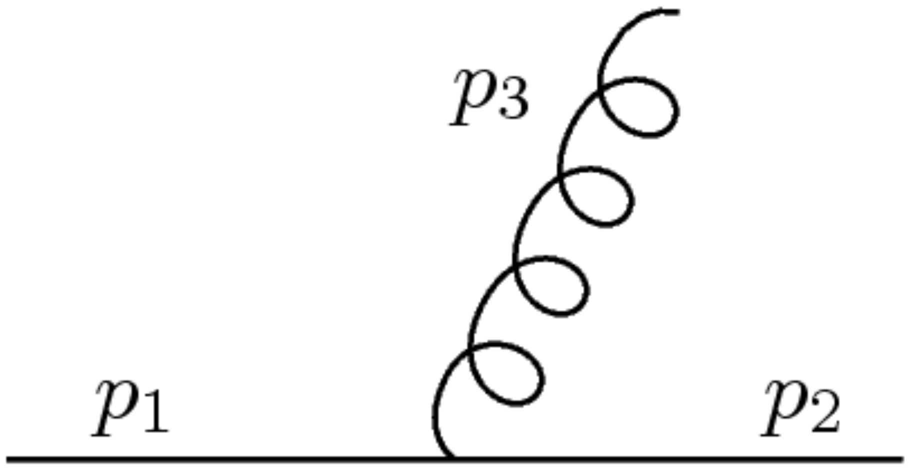




In high-energy collisions, observe traces of quarks, gluons ('jets')

# Singularities in pQCD

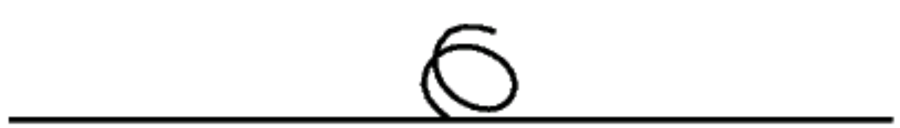
$$\frac{d^2\sigma}{dx_1 dx_2} \propto \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$




$$x_1 = 1 - \frac{2p_2 \cdot p_3}{Q^2}$$

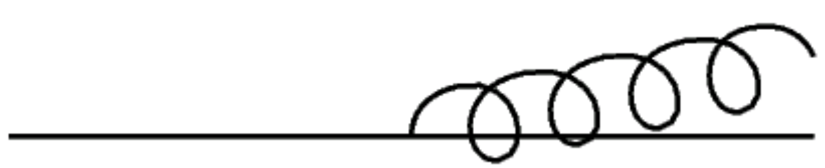
(massless case)

Soft divergence



$$p_3 \longrightarrow 0$$

Collinear divergence



$$p_2 \cdot p_3 \longrightarrow 0$$

Closely related to hadronisation effects

# Hard processes in QCD

**Factorization**

Cross section calculation can be split into
- Hard part: perturbative matrix element
- Soft part: parton density (PDF), fragmentation (FF)

$$\frac{d\sigma_{pp}^{h}}{dy d^2 p_T} = K \sum_{abcd} \int dx_a dx_b \underbrace{f_a(x_a,Q^2) f_b(x_b,Q^2)}_{\text{parton density}} \underbrace{\frac{d\sigma}{d\hat{t}}(ab \rightarrow cd)}_{\text{matrix element}} \underbrace{\frac{D_{h/c}^0}{\pi z_c}}_{\text{FF}}$$

- Hard process: scale Q >> $\Lambda_{QCD}$
- Hard scattering High-$p_T$ parton(photon) Q ~ $p_T$
- Heavy flavour production m >> $\Lambda_{QCD}$

QM interference between hard and soft suppressed (by $Q^2/\Lambda^2$ 'Higher Twist')

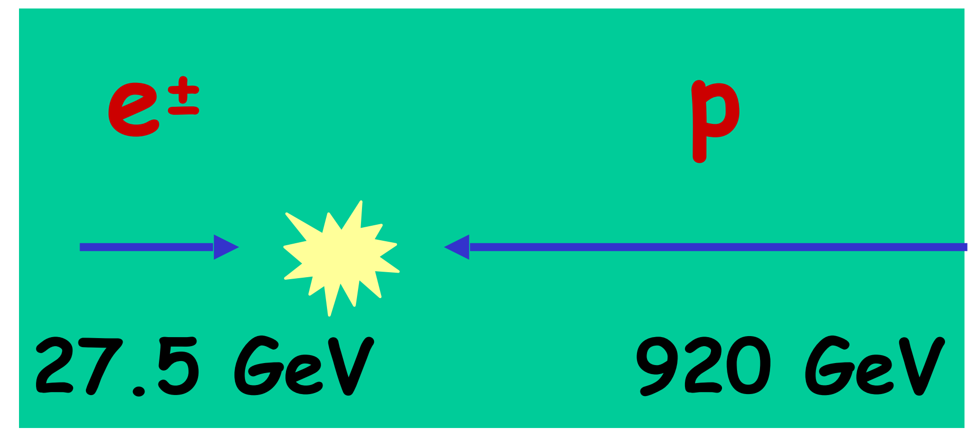
Soft parts, PDF, FF are *universal*: independent of hard process

This is likely to break down in a QGP, but still a good starting point

# The HERA Collider

## The first and only ep collider in the world

Hera at DESY near Hamburg

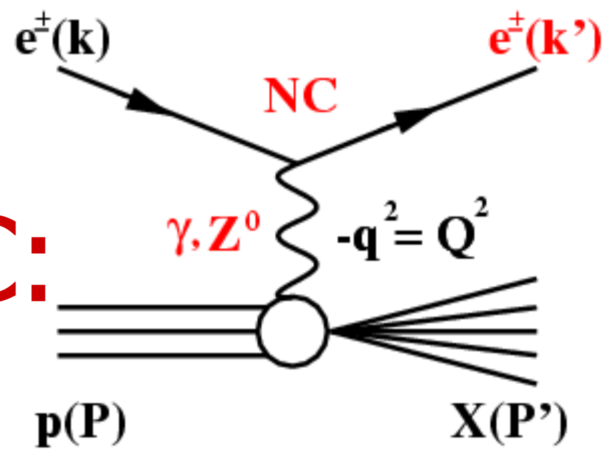

$\sqrt{s}$ = 318 GeV

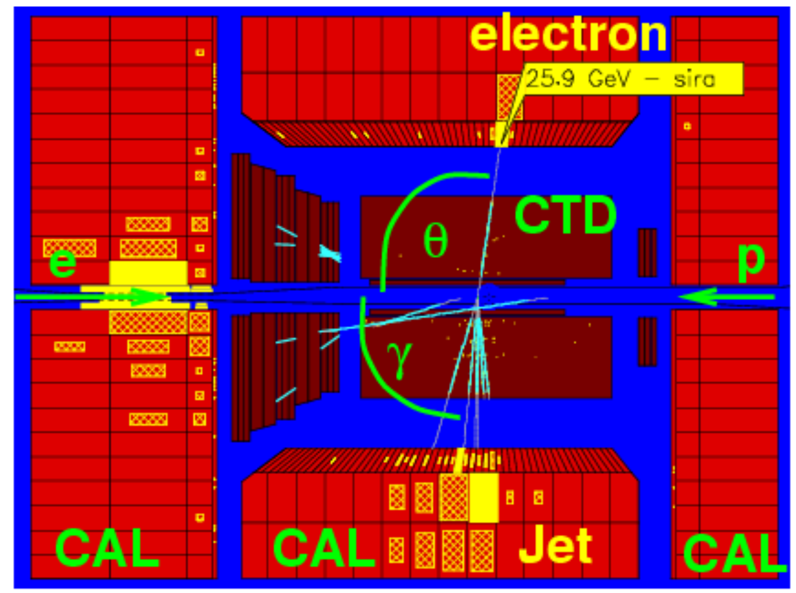
Equivalent to fixed target experiment with 50 TeV e±

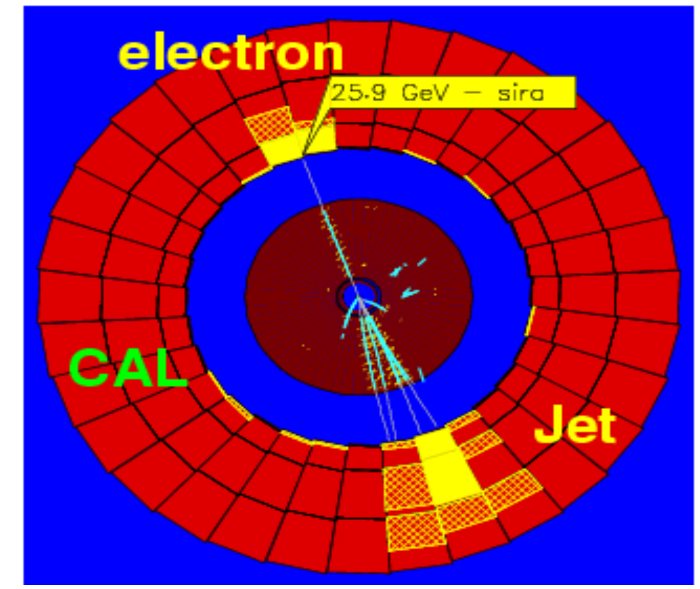
# Example DIS events

$$\text{NC}: \quad e^{\pm} + p \rightarrow e^{\pm} + X, \qquad \text{CC}: \quad e^{\pm} + p \rightarrow \bar{\nu}_e(\nu_e) + X$$

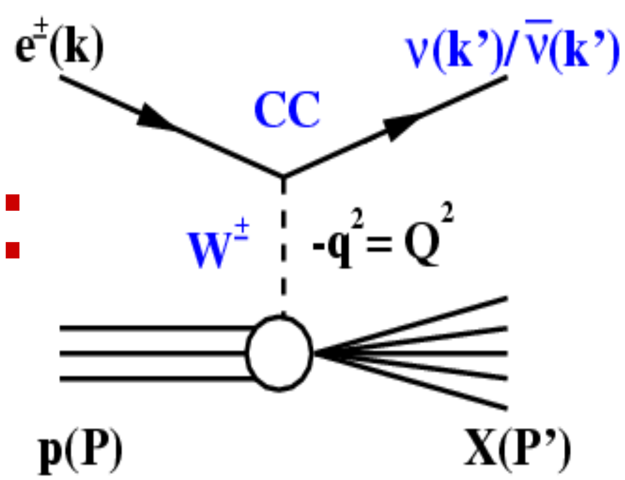
NC:



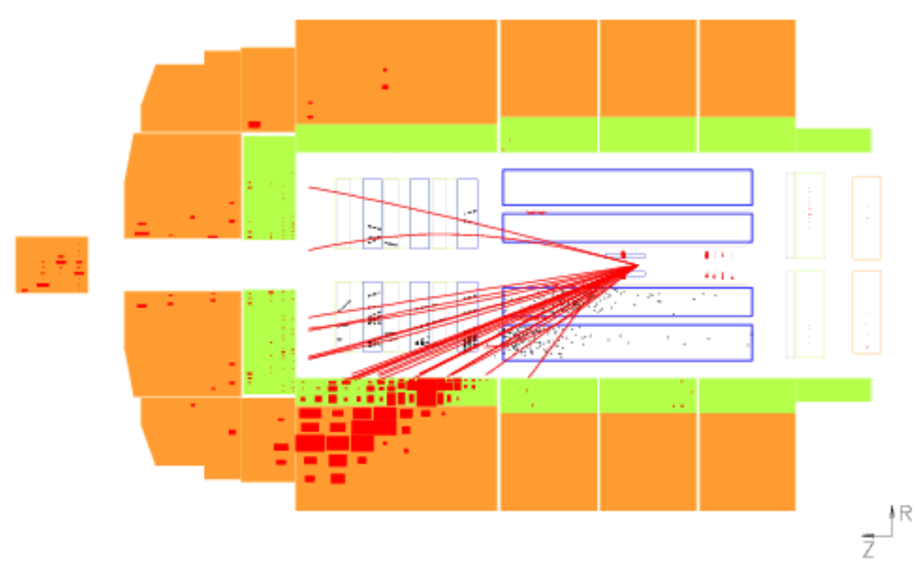



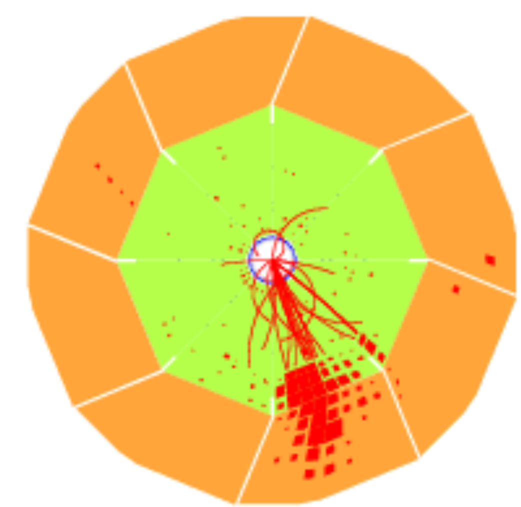




CC:



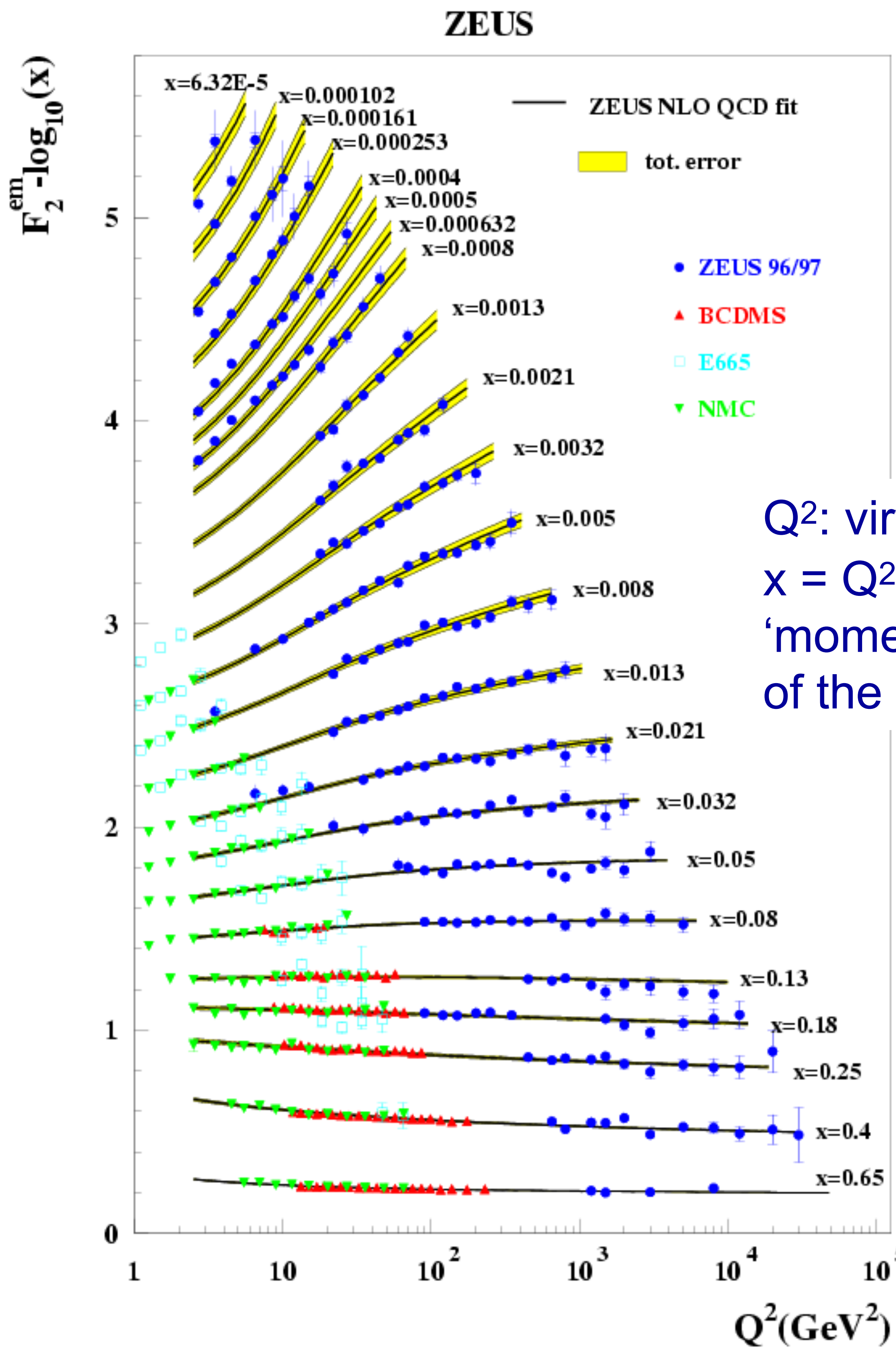

Q**2 = 21475 y = 0.55 M = 198



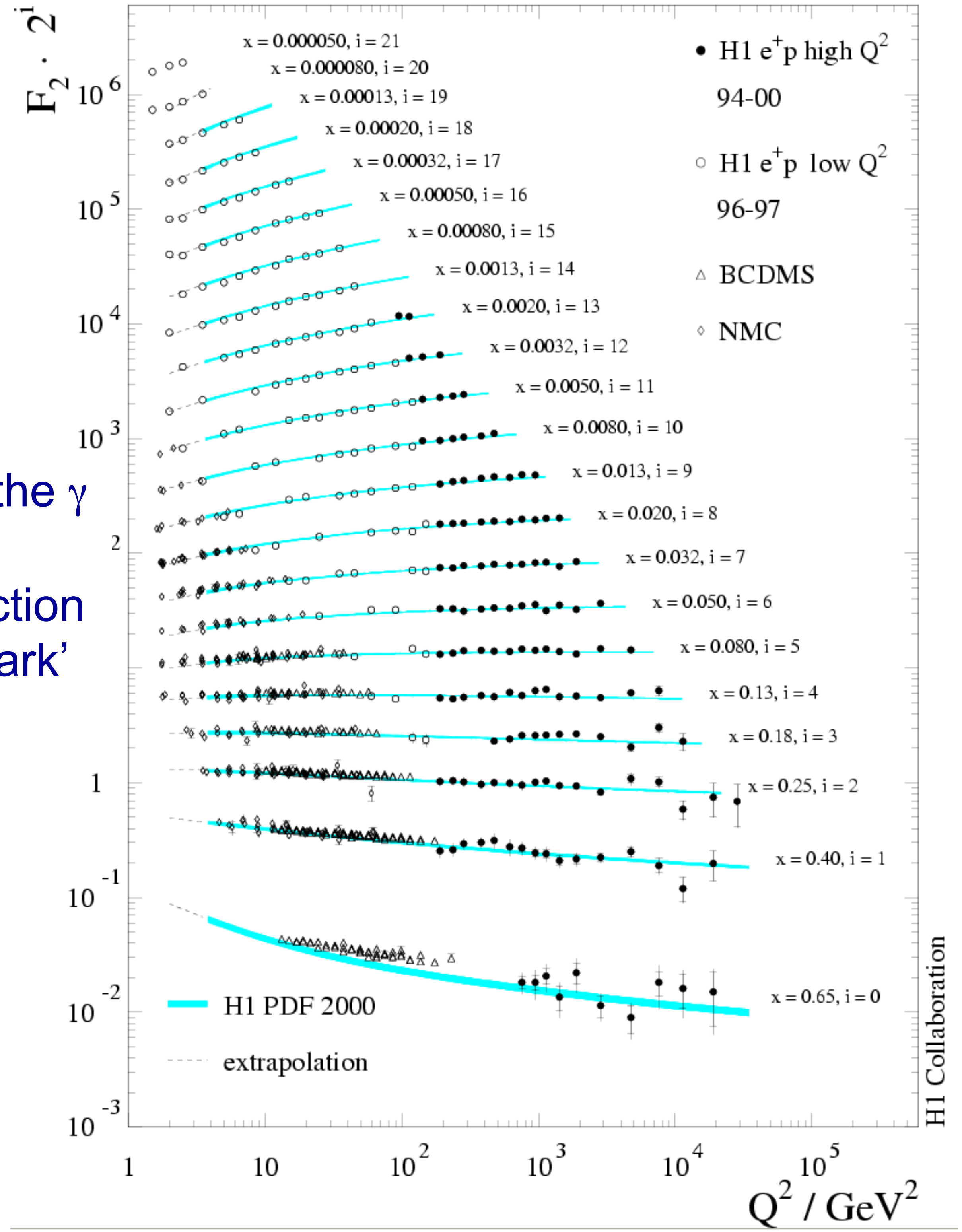



DIS: Measured electron/jet momentum fixes kinematics: $x$, $Q^2$

# Proton structure $F_2$




$Q^2$: virtuality of the $\gamma$
$x = Q^2 / 2\, p\, q$
'momentum fraction of the struck quark'




$F_2$: essentially a cross section/scattering probability

# Factorisation in DIS

the physical structure fct. is independent of $\mu_f$
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on $\mu_f$
(choice of $\mu_f$: shifting terms between long- and short-distance parts)

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]$$

yet another scale: $\mu_r$
due to the renormalization
of ultraviolet divergencies

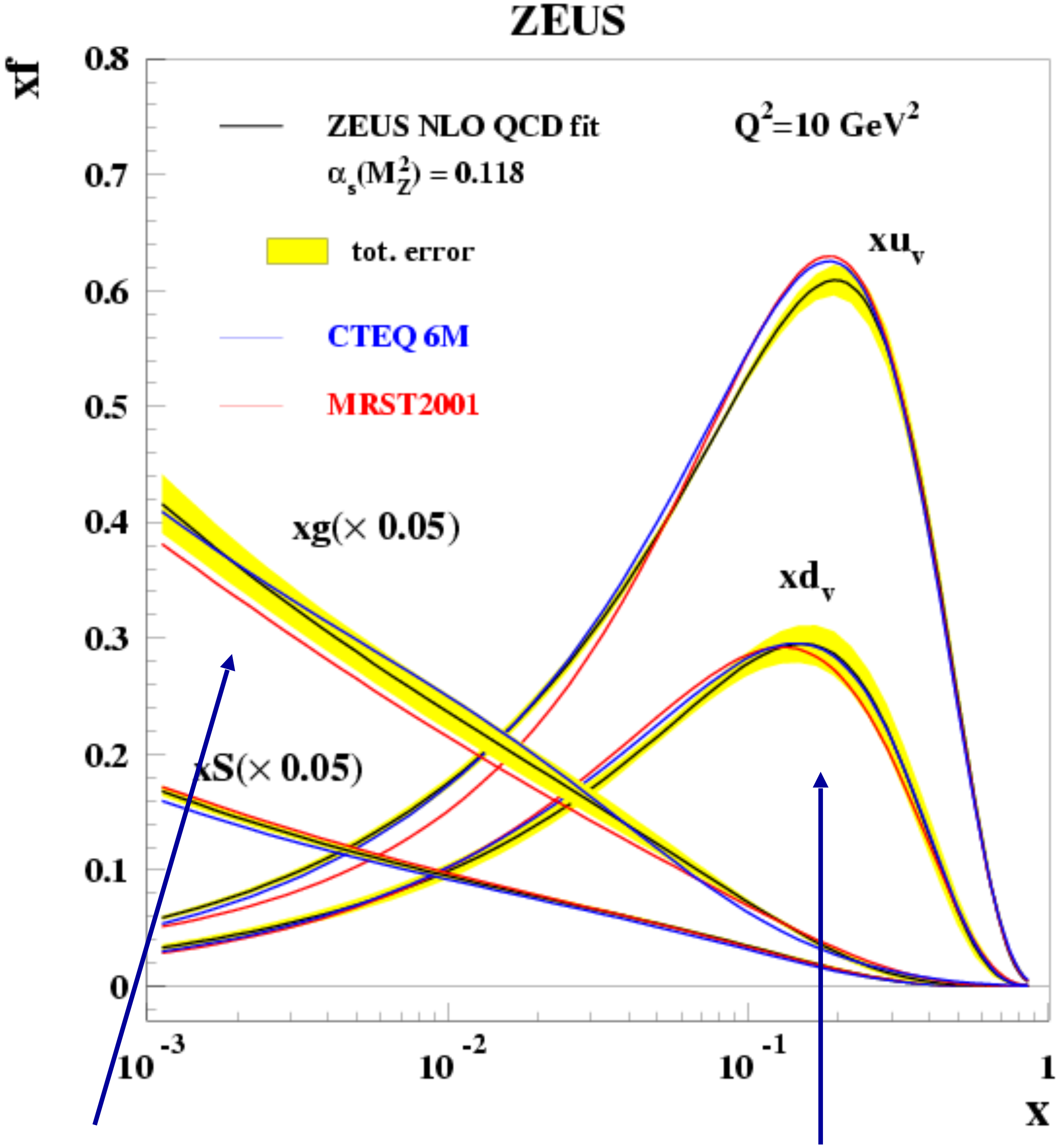
short-distance "Wilson coefficient"

choice of the factorization scheme

Integral over x is DGLAP evolution with splitting kernel $P_{qq}$

# Parton density functions

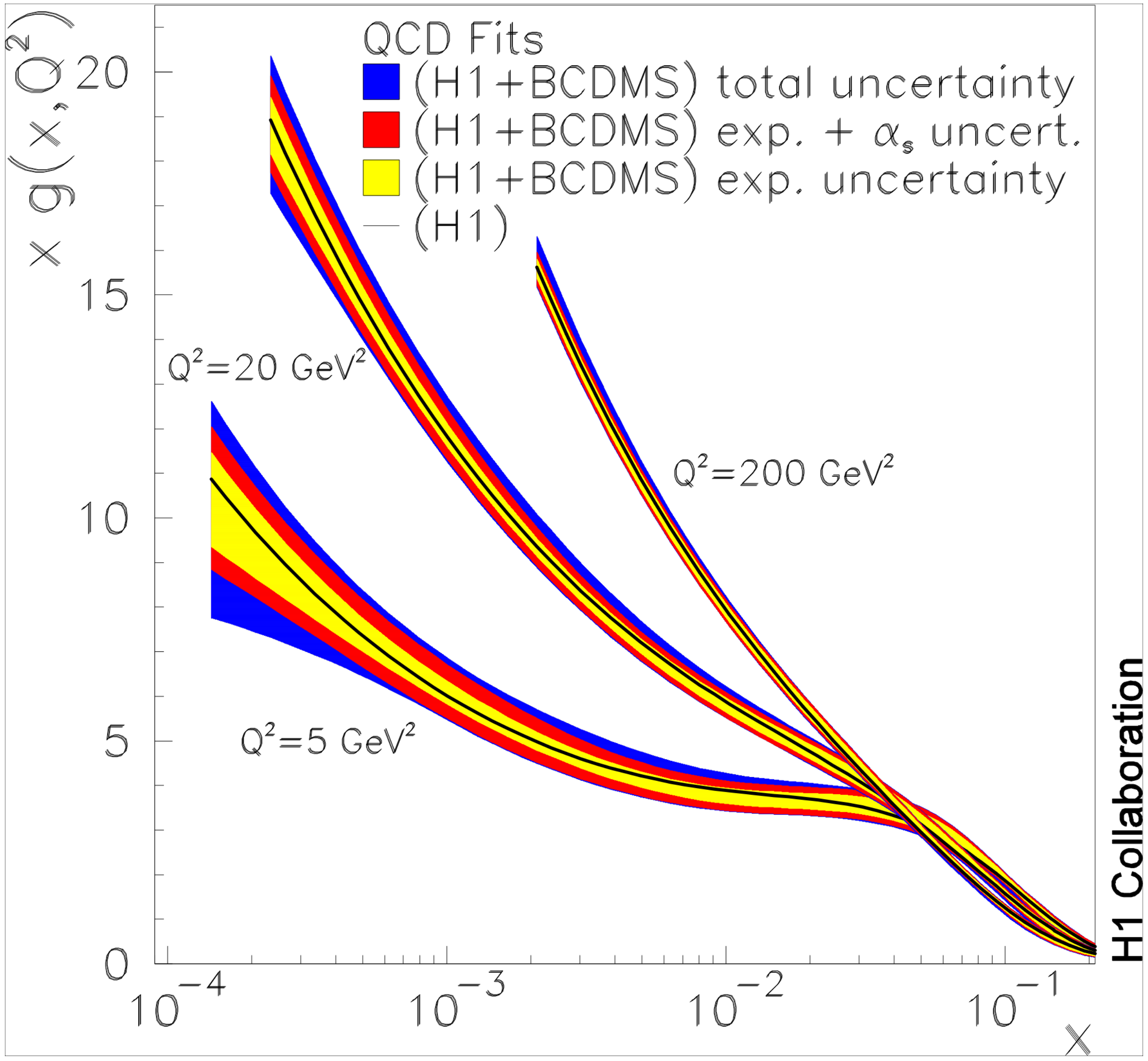
Low $Q^2$: valence structure




Soft gluons

Valence quarks (p = uud)
x ~ 1/3

$Q^2$ evolution (gluons)




Gluon content of proton rises quickly with $Q^2$

# Intermezzo I: PDFs and splitting functions

QCDNUM manual: https://www.nikhef.nl/~h24/qcdnum-files/doc/qcdnum170115.pdf

## 2.2 The DGLAP Evolution Equations

The DGLAP evolution equations can be written as

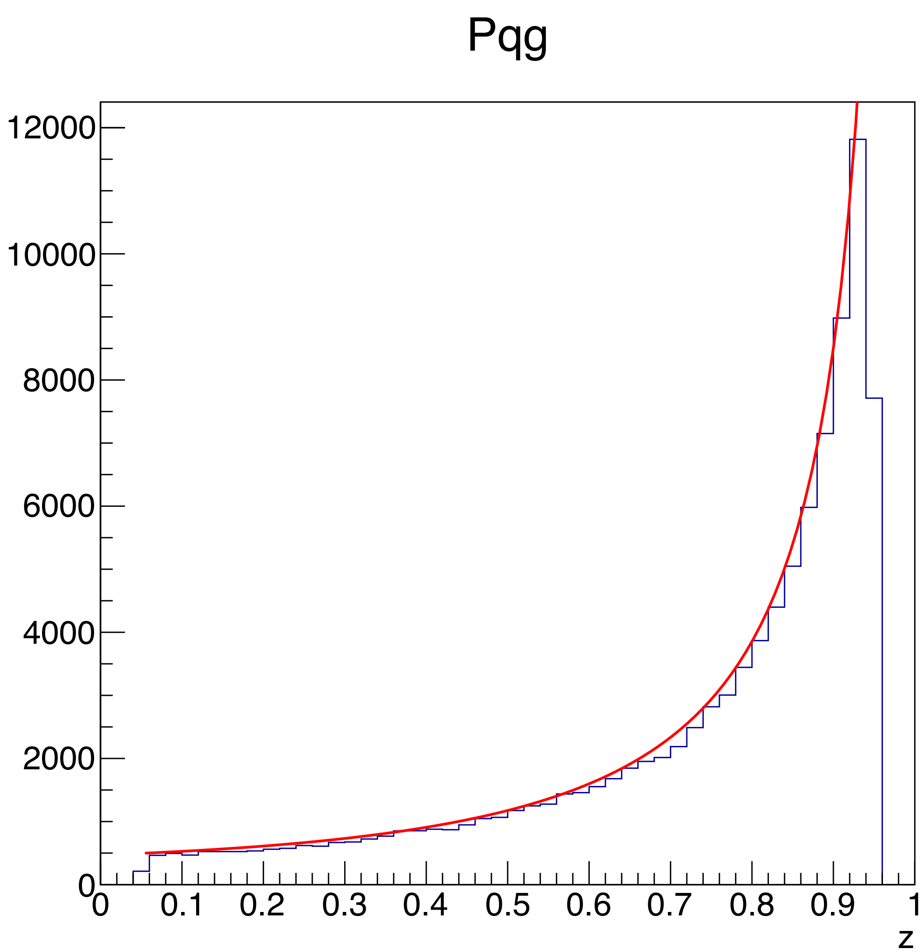
$$\frac{\partial f_i(x,\mu^2)}{\partial \ln \mu^2} = \sum_{j=q,\bar{q},g} \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(\frac{x}{z},\mu^2\right) f_j(z,\mu^2) \tag{2.4}$$

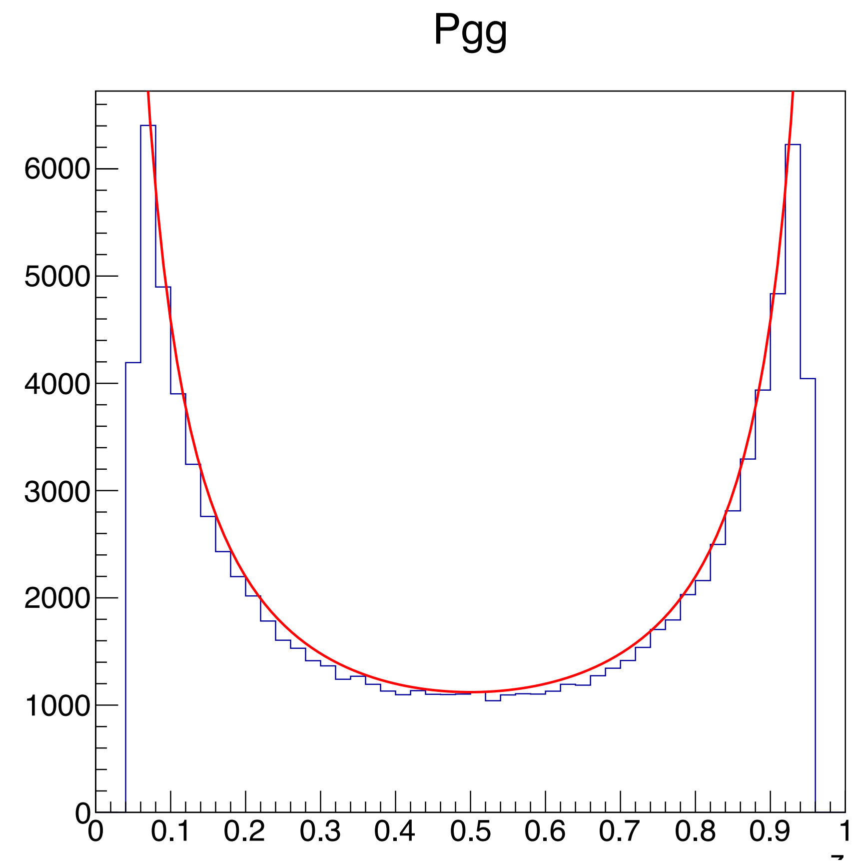
where $f_i$ denotes an un-polarised parton number density, $P_{ij}$ are the QCD splitting functions, $x$ is the Bjorken scaling variable and $\mu^2 = \mu_{\mathrm{F}}^2$ is the mass factorisation scale which we assume here to be equal to the renormalisation scale $\mu_{\mathrm{R}}^2$. The indices $i$ and $j$ in (2.4) run over the parton species *i.e.*, the gluon and $n_{\mathrm{f}}$ active flavours of quarks and anti-quarks. In the quark parton model, and also in LO pQCD, the parton densities

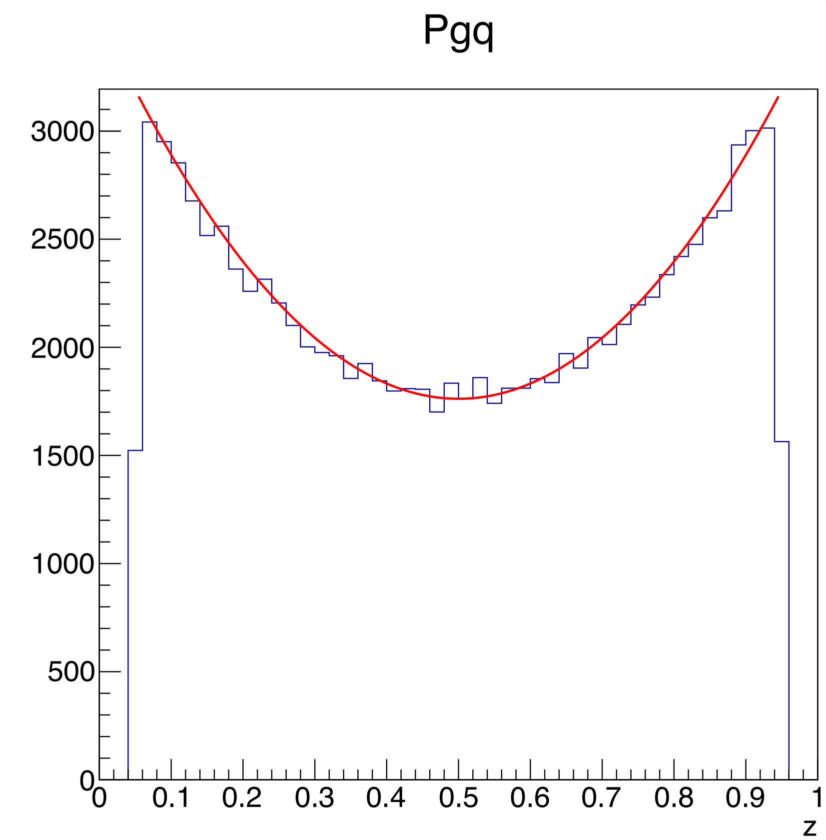
The splitting functions of the LO splitting matrix $P_{ij}^{(0)}$ in (2.14) can be written as

$$\begin{aligned}
P_{\mathrm{qq}}^{(0)}(x) &= \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] \\
P_{\mathrm{qg}}^{(0)}(x) &= 2n_{\mathrm{f}}\,\frac{1}{2}\left[x^2+(1-x)^2\right] \\
P_{\mathrm{gq}}^{(0)}(x) &= \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right] \\
P_{\mathrm{gg}}^{(0)}(x) &= 6\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \left(\frac{11}{12} - \frac{n_{\mathrm{f}}}{18}\right)\delta(1-x)\right].
\end{aligned} \tag{B.2}$$

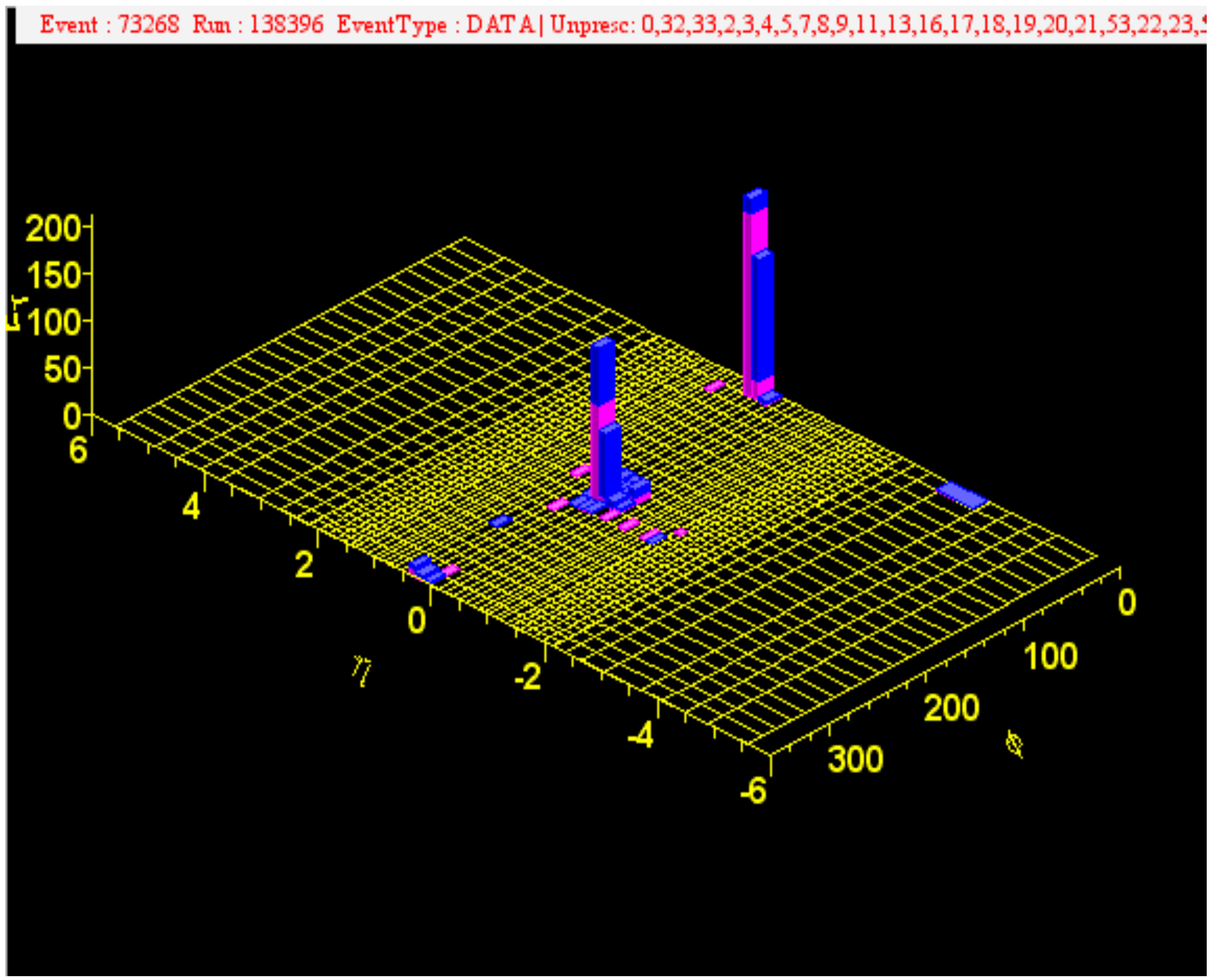
# Splitting functions illustrated

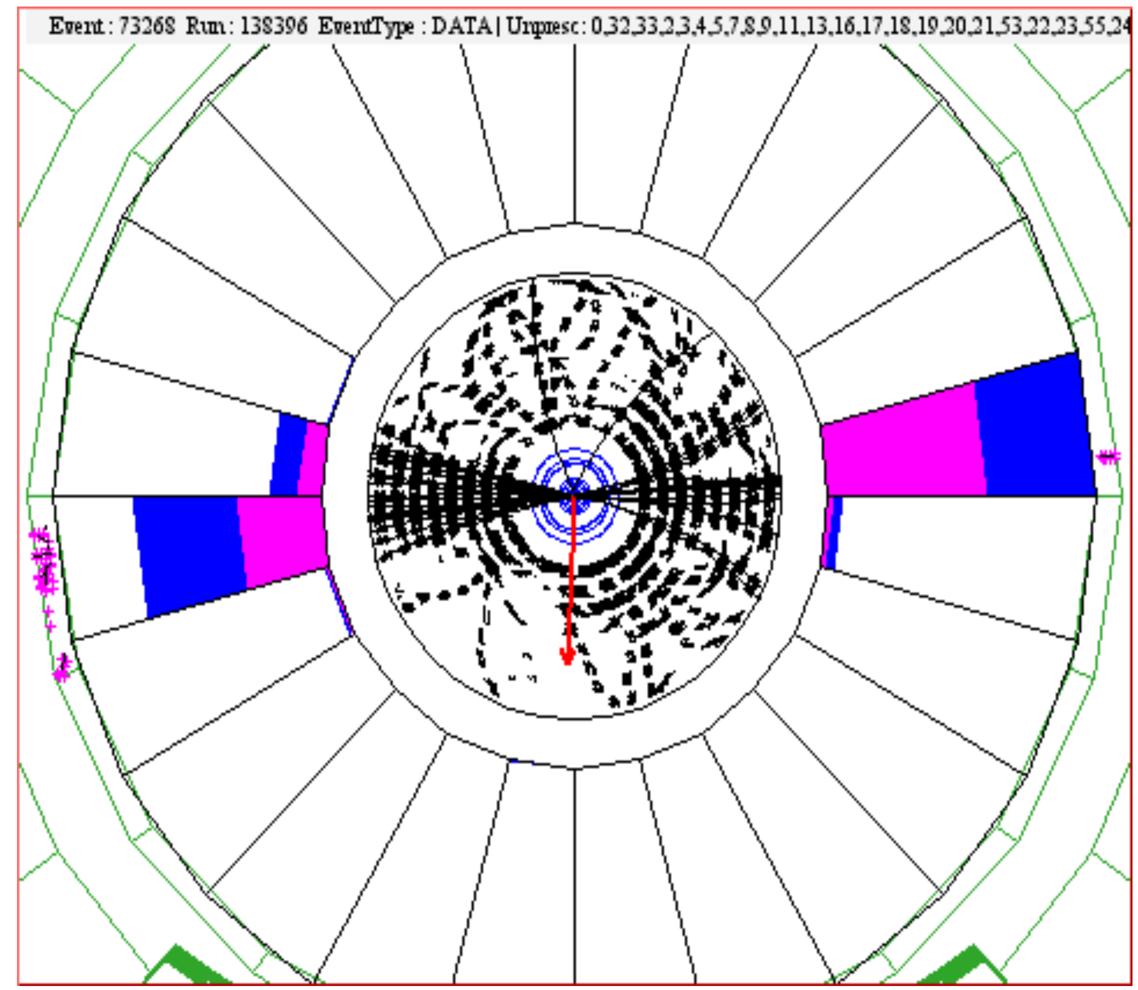


# p+$\overline{\text{p}}$ → dijet at Tevatron

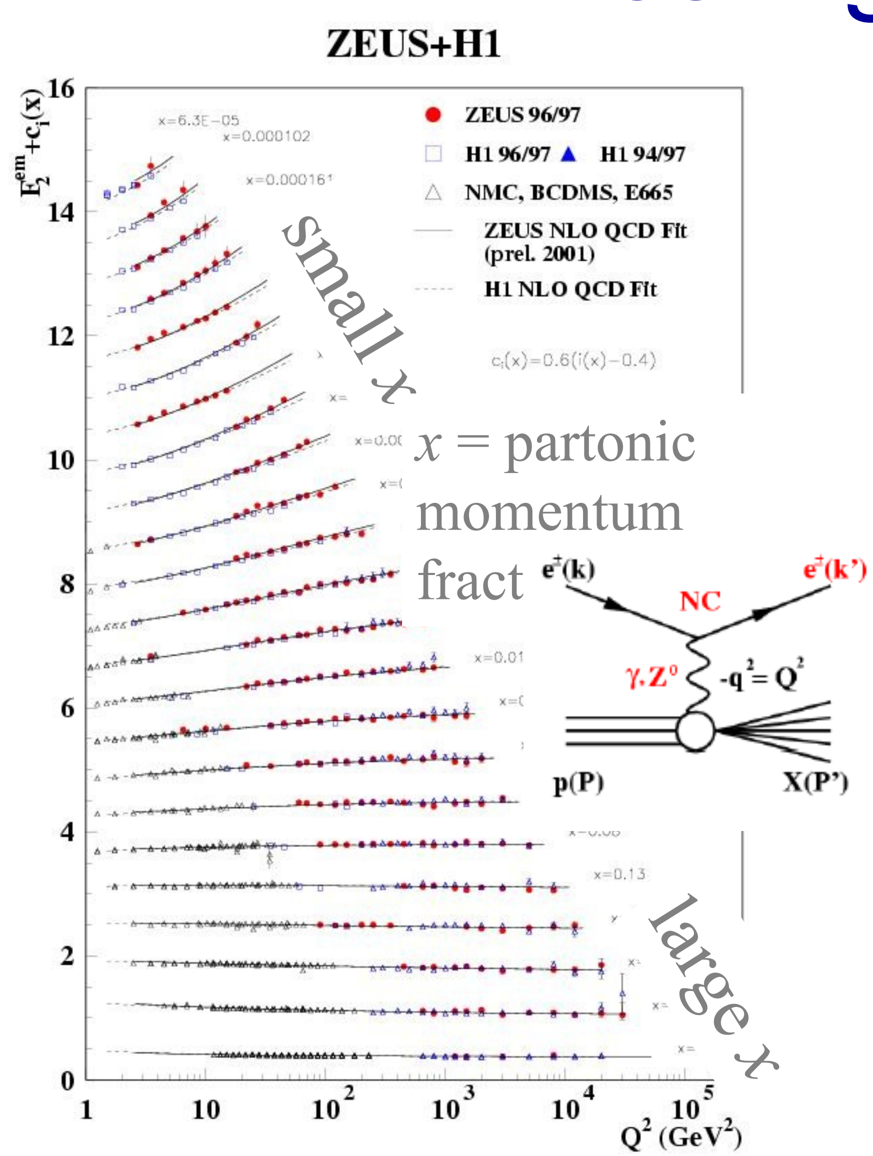


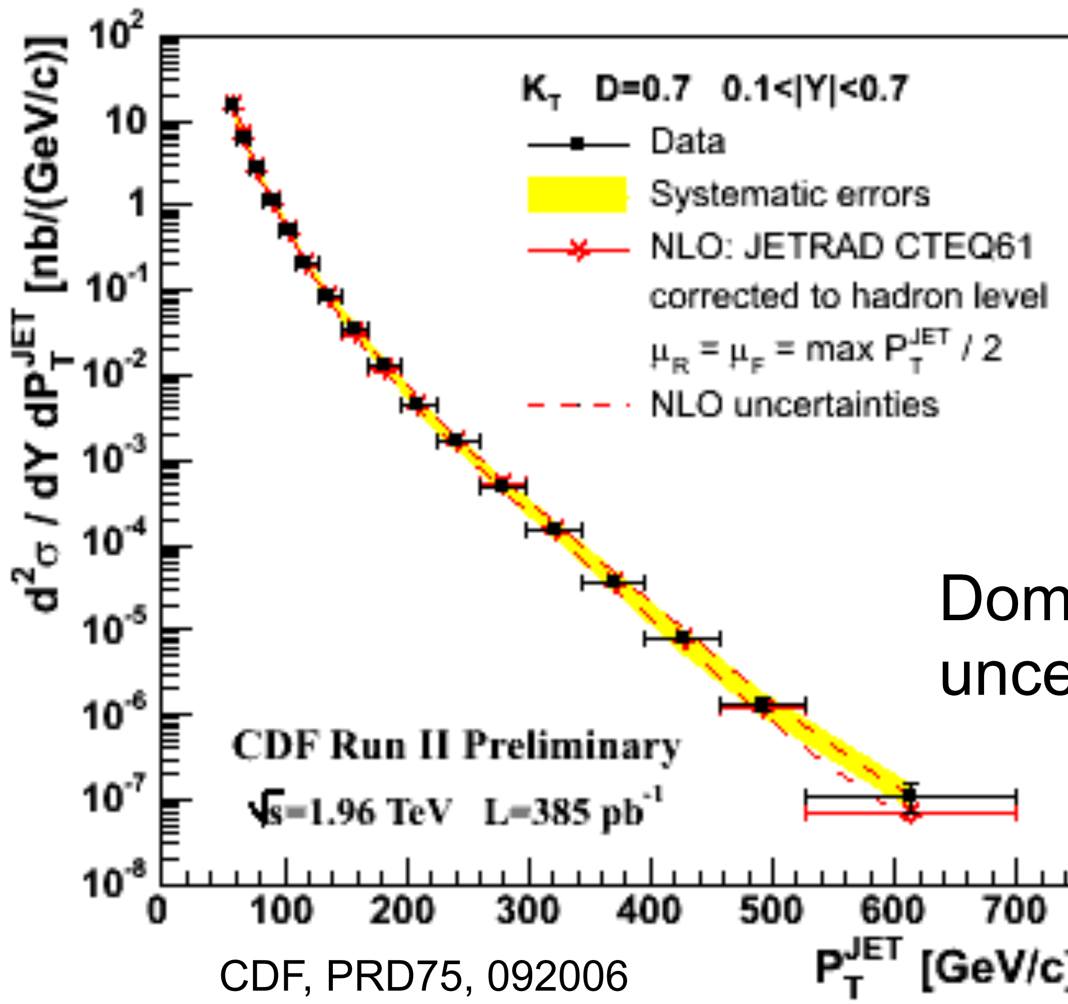

Tevatron: p + $\overline{\text{p}}$ at $\sqrt{s}$ = 1.9 TeV

Jets produced with several 100 GeV

# Testing QCD at high energy

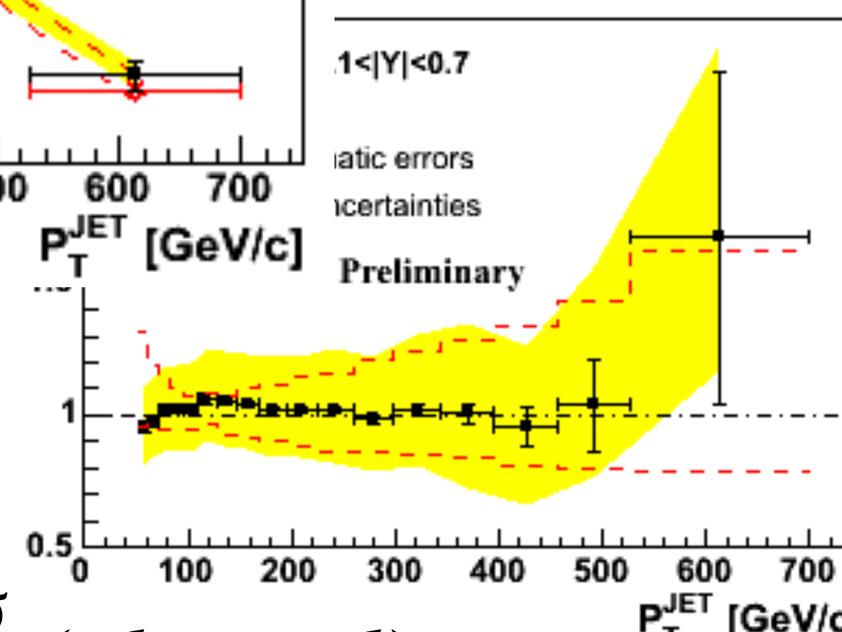



x = partonic momentum fract




CDF, PRD75, 092006

Dominant 'theory' uncertainty: PDFs




**DIS to measure PDFs**

$$\frac{d\sigma^{h}_{pp}}{dy\,d^2p_T} = K\sum_{abcd}\int dx_a dx_b \underbrace{f_a(x_a,Q^2) f_b(x_b,Q^2)}_{\text{parton density}} \underbrace{\frac{d\sigma}{d\hat{t}}(ab\rightarrow cd)}_{\text{matrix element}}$$

Theory matches data over many orders of magnitude

Universality: PDFs from DIS used to calculate jet-production in pp

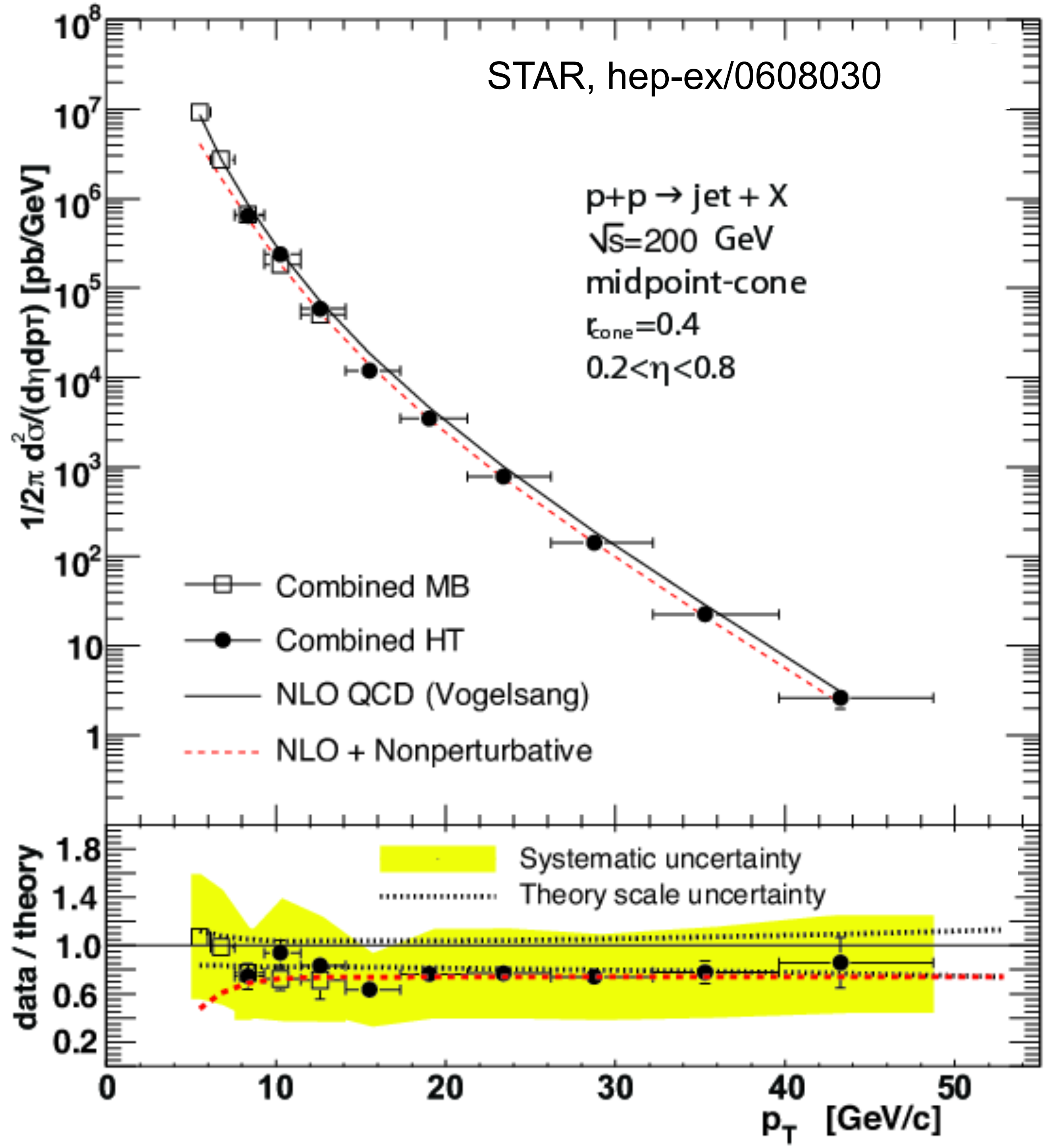
# Testing QCD at RHIC with jets

RHIC: p+p at √s = 200 GeV
(p+p at 500 GeV also available)

Jets also measured at RHIC

NLO pQCD also works at RHIC

However: significant uncertainties in energy scale, both 'theory' and experiment



# Intermezzo II: calculating the spectra

Sarcevic, Ellis and Carruthers, Phys.Rev. D40 (1989) 1446

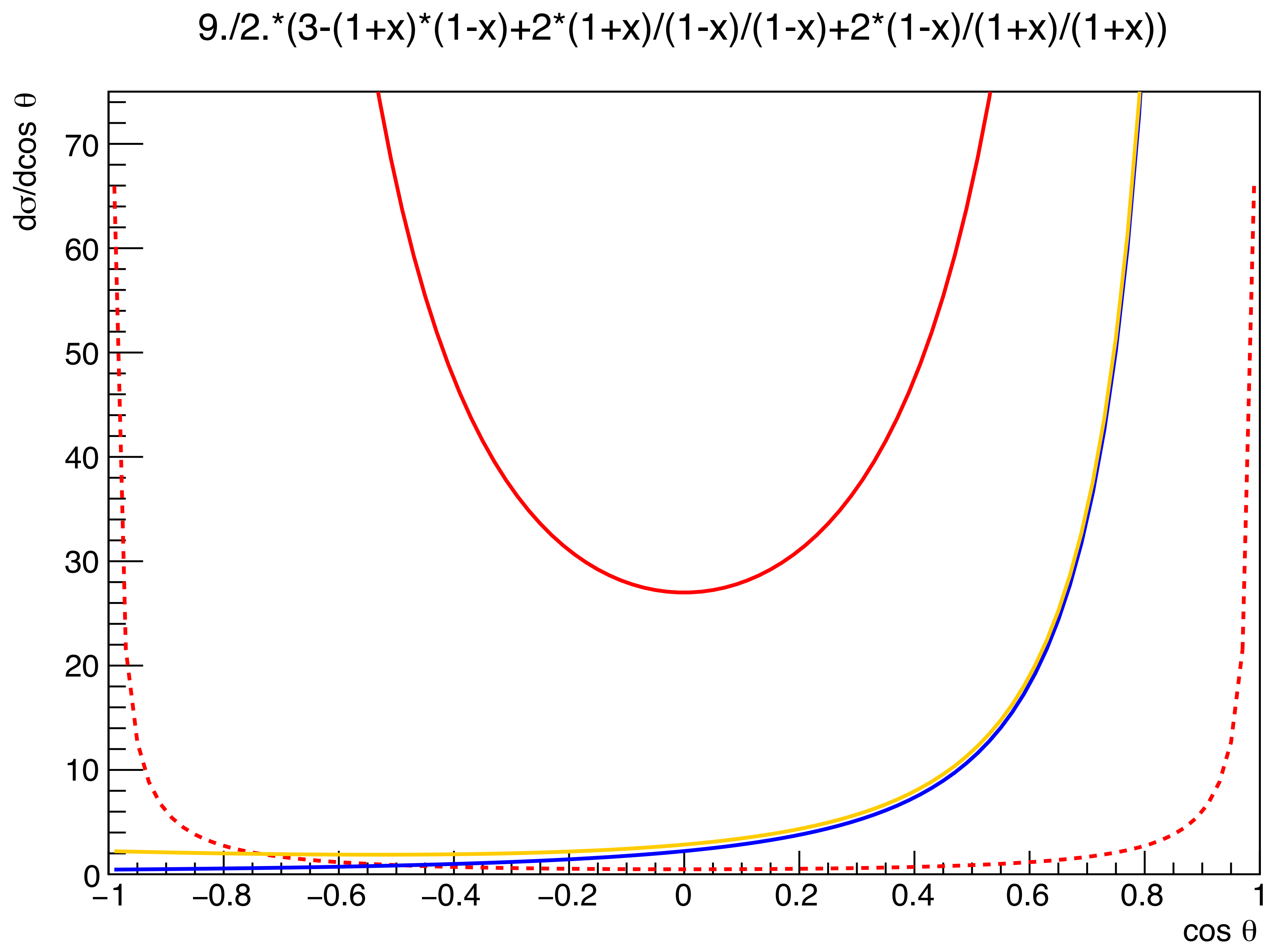
Here $y_1$ and $y_2$ refer to the rapidities of the scattered partons and the symbols with carets refer to the parton-parton c.m. system. The summation is over all flavors $i$ and $j$ and the factor $\frac{1}{2}$ in front of the integral in Eq. (1b) is due to the fact that there are assumed to be 2 jets in the final state (in general this factor should account for the actual average multiplicity of jets per event). The relations between the variables $\hat{t}, \hat{u}, \hat{s}, p_T, y_1, y_2$, and $\cos\hat{\theta}, x_a, x_b$ are given by

$$\hat{s}=x_a x_b s=4p_T^2\cosh^2\left[\frac{y_1-y_2}{2}\right], \quad \cos\hat{\theta}=\left[1-\frac{4p_T^2}{\hat{s}}\right]^{1/2}=\tanh\left[\frac{y_1-y_2}{2}\right],$$
$$x_b^a=\left[\frac{\hat{s}}{s}e^{\pm(y_1+y_2)}\right]^{1/2}, \quad \hat{t}=-\frac{\hat{s}}{2}(1-\cos\hat{\theta}), \quad \hat{u}=-\frac{\hat{s}}{2}(1+\cos\hat{\theta}). \tag{2}$$

For the total cross section the range for $\cos\hat{\theta}$ is between $-\sqrt{1-4(p_T^{\min})^2/\hat{s}}$ and $+\sqrt{1-4(p_T^{\min})^2/\hat{s}}$ for a given $\hat{s}$ (i.e., given $x_a$ and $x_b$). In Eq. (1) the $G(x,Q^2)$'s are parton distribution functions and the $\hat{\sigma}_{ij}$ are parton cross sections. In particular the required $\hat{\sigma}_{ij}$ are given by

$$\hat{\sigma}(q_iq_j'\to q_iq_j')=\frac{4\alpha_s^2}{9\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right], \quad \hat{\sigma}(q_i\bar{q}_i\to q_j\bar{q}_j)=\frac{4\alpha_s^2}{9\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2}\right],$$
$$\hat{\sigma}(q_iq_i\to q_iq_i)=\frac{4\alpha_s^2}{9\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}-\frac{2\hat{s}^2}{3\hat{t}\hat{u}}\right], \quad \hat{\sigma}(q_i\bar{q}_i\to q_i\bar{q}_i)=\frac{4\alpha_s^2}{9\hat{s}}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}-\frac{2\hat{u}^2}{3\hat{s}\hat{t}}\right],$$
$$\hat{\sigma}(q_i\bar{q}_i\to gg)=\frac{8\alpha_s^2}{3\hat{s}}(\hat{t}^2+\hat{u}^2)\left[\frac{4}{9\hat{t}\hat{u}}-\frac{1}{\hat{s}^2}\right], \quad \hat{\sigma}(gg\to q_i\bar{q}_i)=\frac{3\alpha_s^2}{8\hat{s}}(\hat{t}^2+\hat{u}^2)\left[\frac{4}{9\hat{t}\hat{u}}-\frac{1}{\hat{s}^2}\right],$$
$$\hat{\sigma}(gq\to gq)=\frac{\alpha_s^2}{\hat{s}}(\hat{s}^2+\hat{u}^2)\left[\frac{1}{\hat{t}^2}-\frac{4}{9\hat{s}\hat{u}}\right], \quad \hat{\sigma}(gg\to gg)=\frac{9\alpha_s^2}{2\hat{s}}\left[3-\frac{\hat{u}\hat{t}}{\hat{s}^2}-\frac{\hat{u}\hat{s}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right]. \tag{3}$$

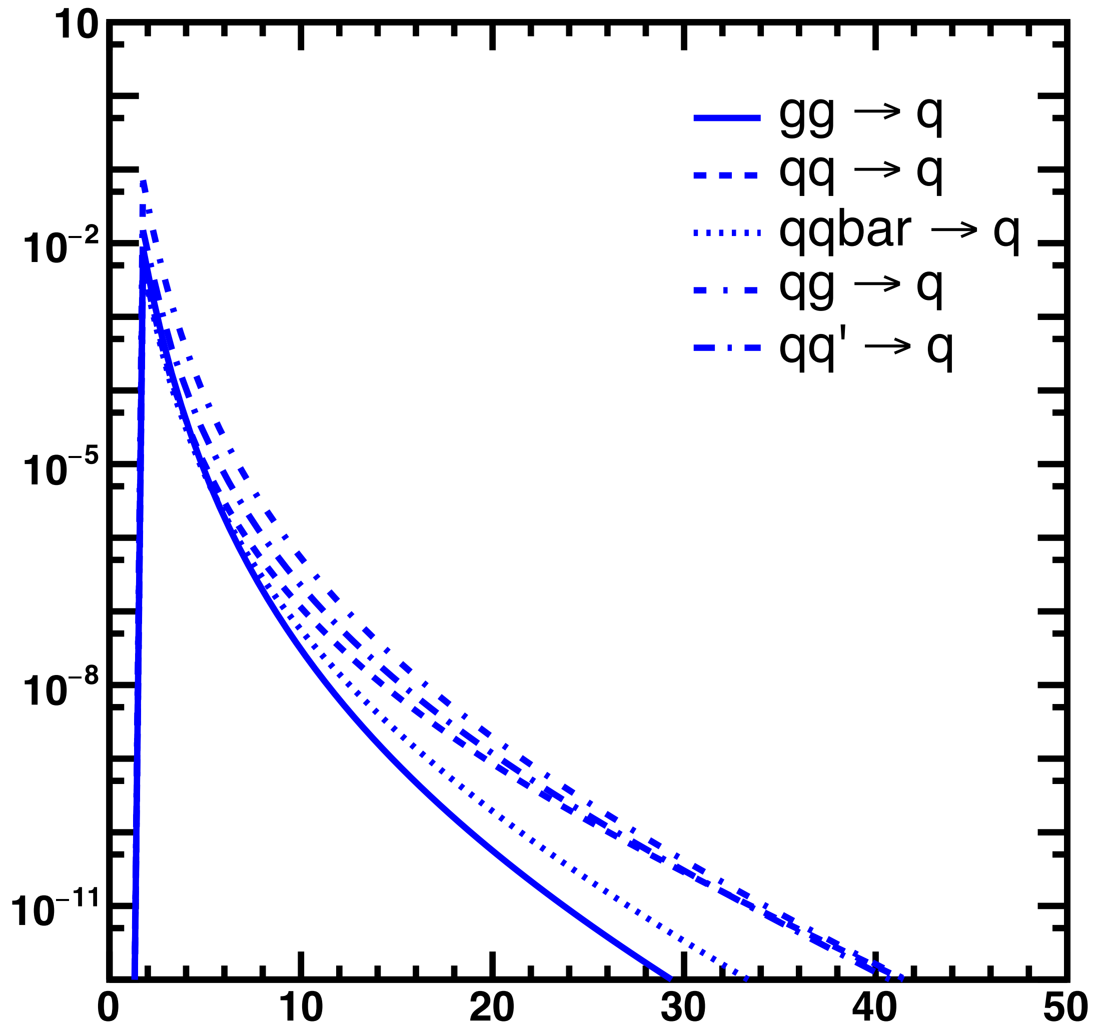
# Parton scattering cross section



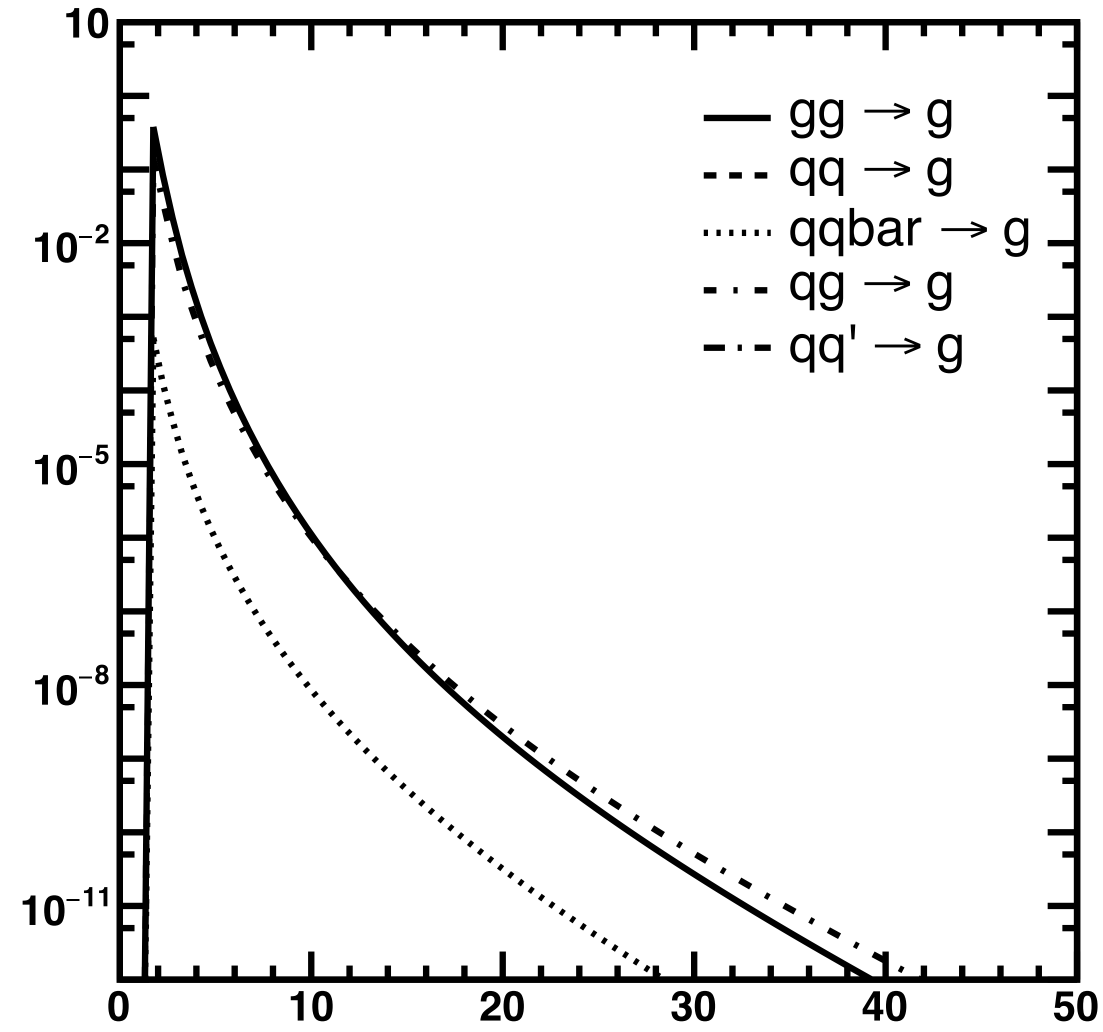

Four example processes; can you guess which ones they are?

# Subprocesses

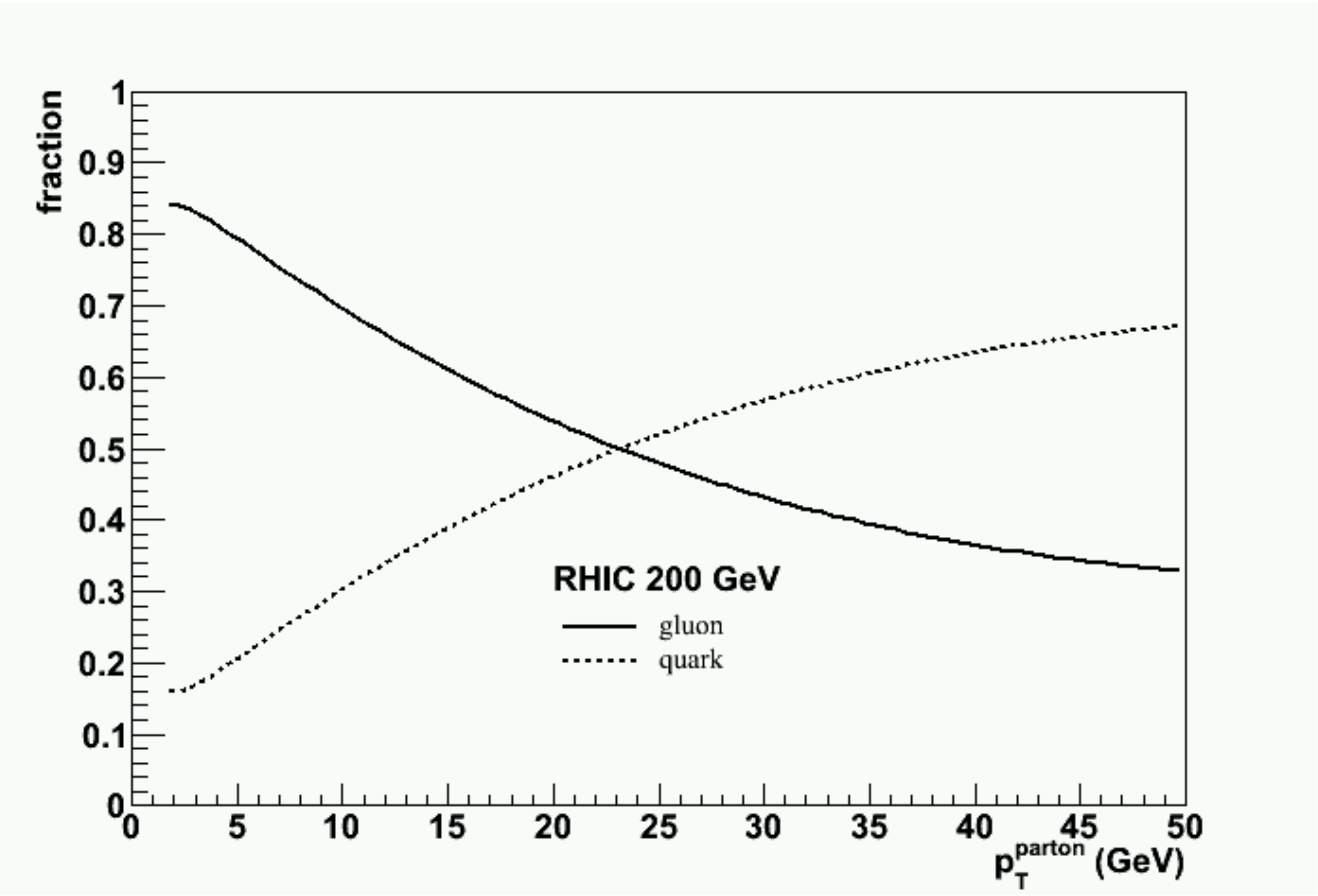
quark production

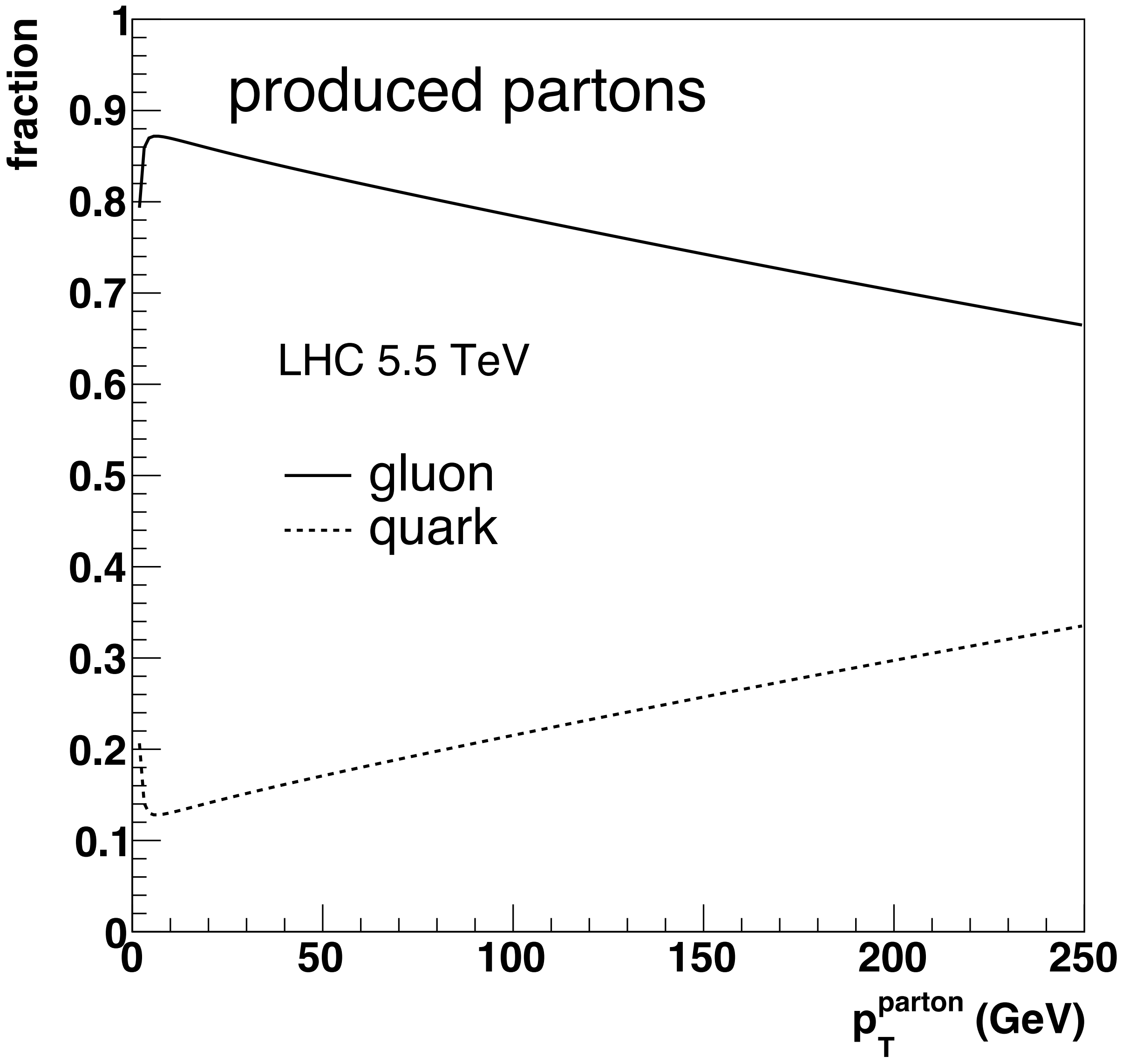



gluon production



# Quark fractions



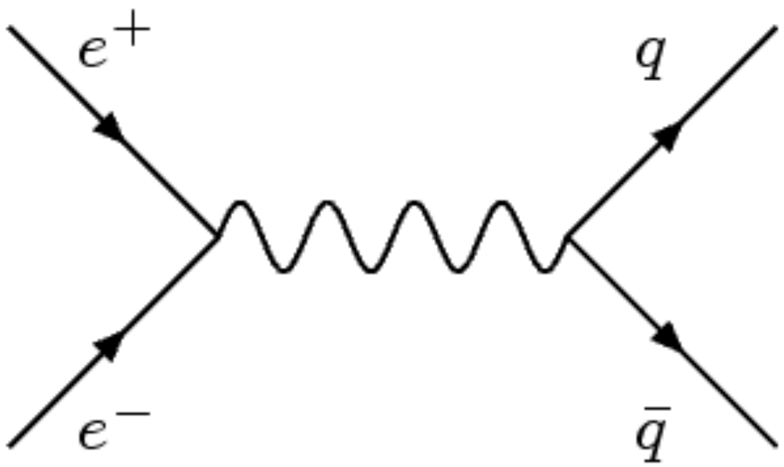




Kinematics, quark fraction very different at LHC, RHIC

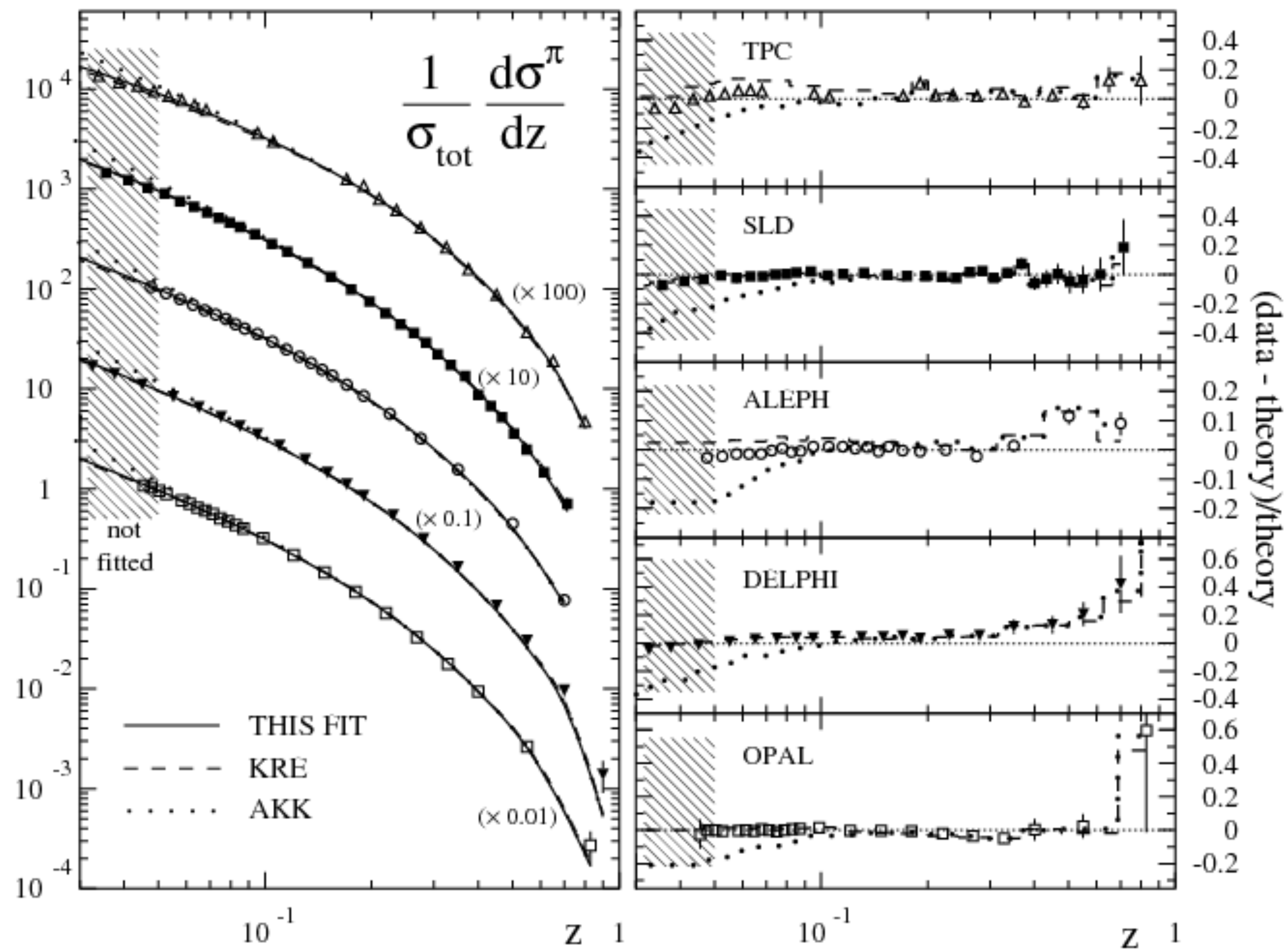
Discussion: why?

# Towards hadron production: Fragmentation Functions

# $e^+e^- \rightarrow qq \rightarrow$ jets



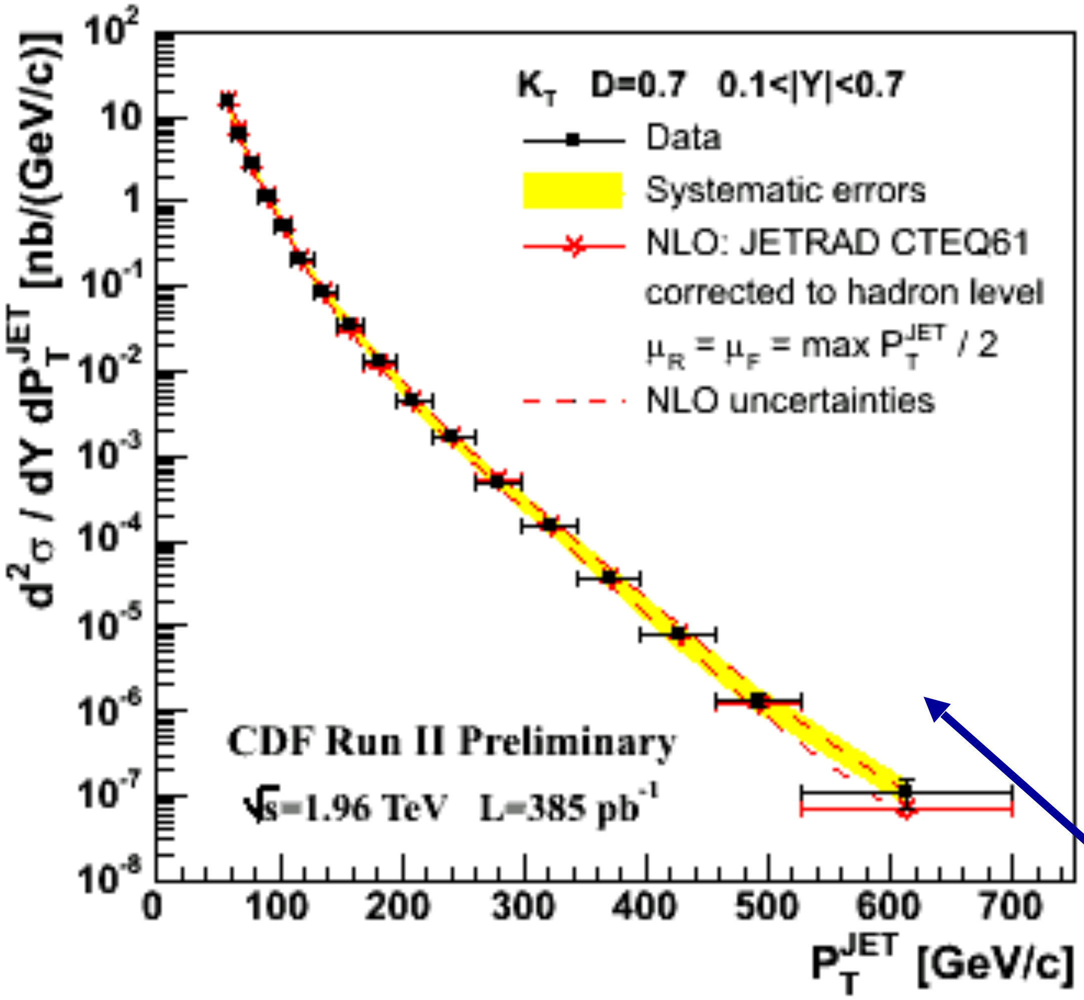

$$\vec{p}_{e^+} = -\vec{p}_{e^-}$$
$$\vec{p}_q = -\vec{p}_{\bar{q}}$$
$$p_q = p_e$$




Direct measurement of fragmentation functions

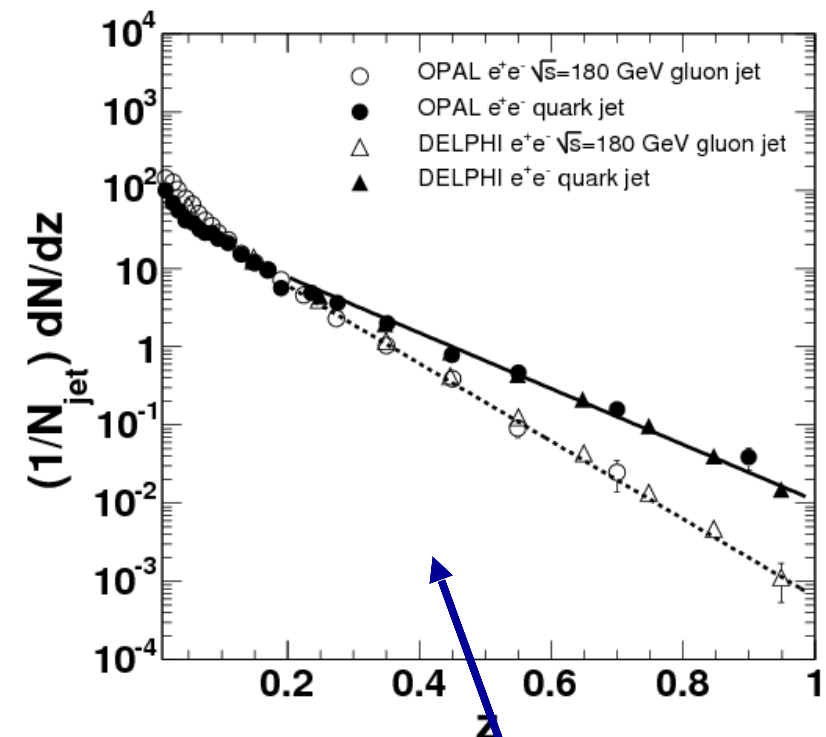
# pQCD illustrated

Jet spectrum ~ parton spectrum




CDF, PRD75, 092006

Fragmentation

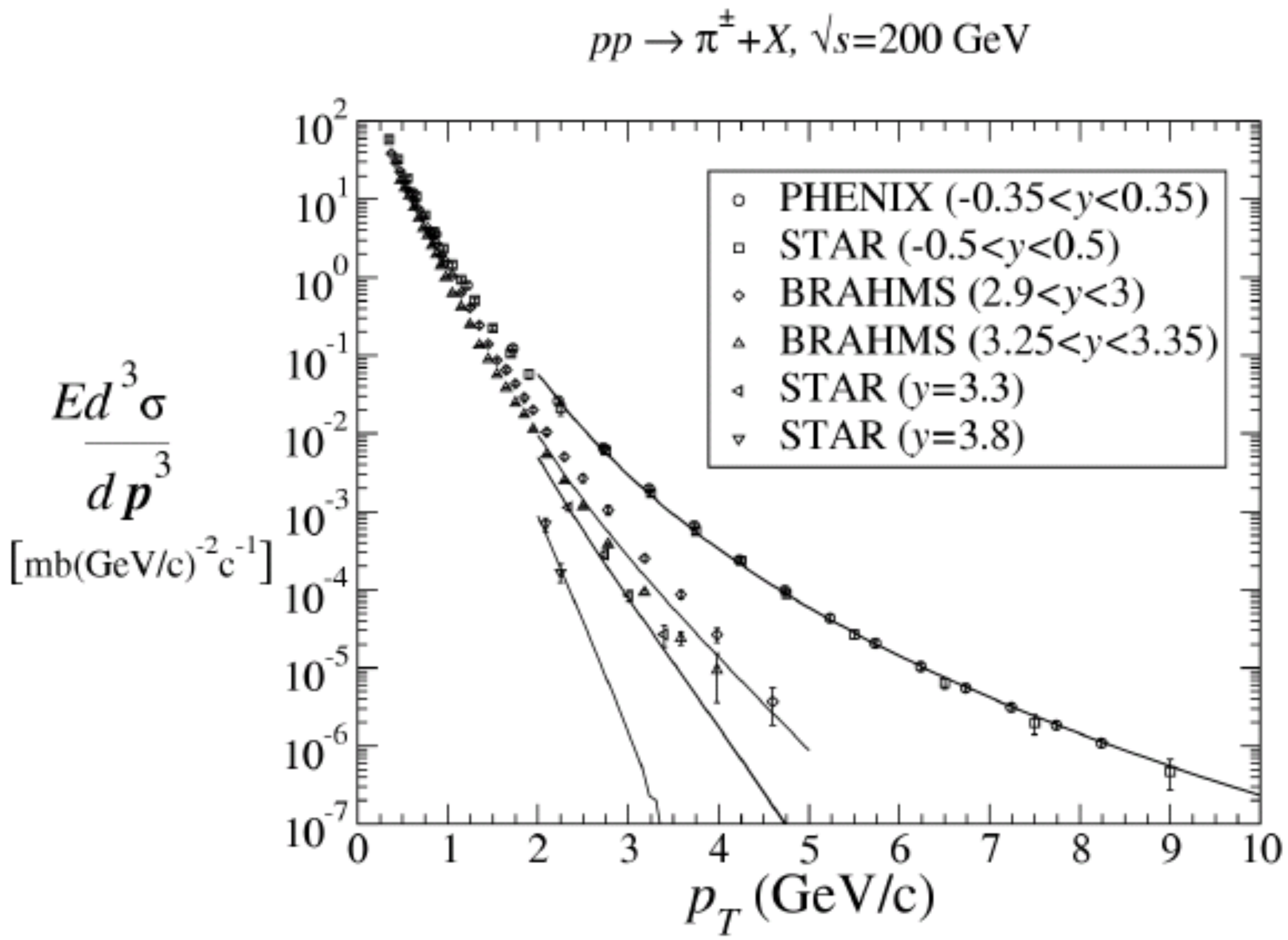


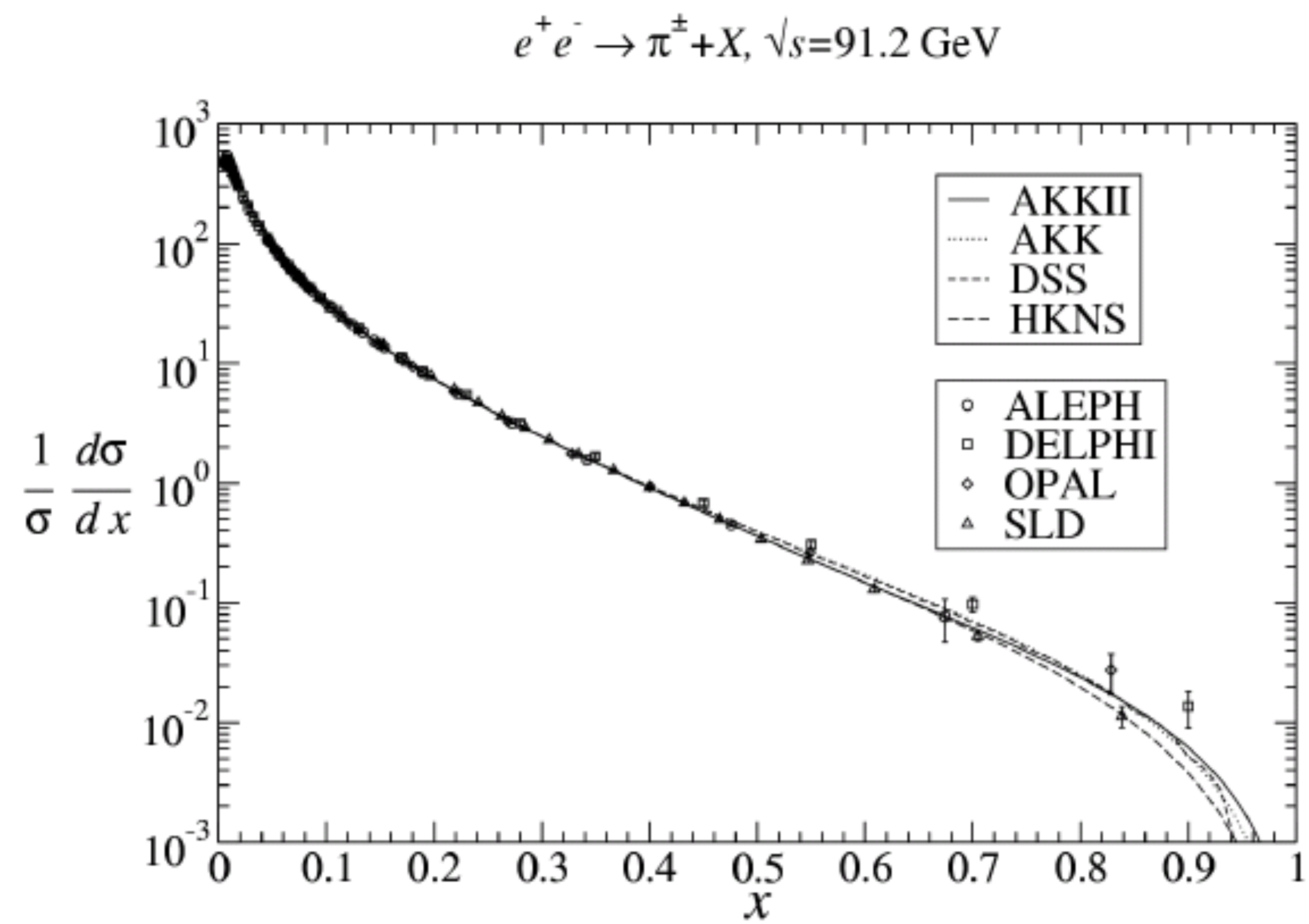

$$\frac{dN}{\hat{p}_T d\hat{p}_T} \propto \frac{1}{\hat{p}_T^n}$$

$$z = \frac{p_{T,hadron}}{P_{jet}}$$

$$\frac{d\sigma_{pp}^{h}}{dy d^2 p_T} = K \sum_{abcd} \int dx_a dx_b f_a(x_a, Q^2) f_b(x_b, Q^2) \frac{d\sigma}{d\hat{t}}(ab \to cd) \frac{D_{h/c}^0}{\pi z_c}$$

# Note: difference p+p, $e^+$+$e^-$

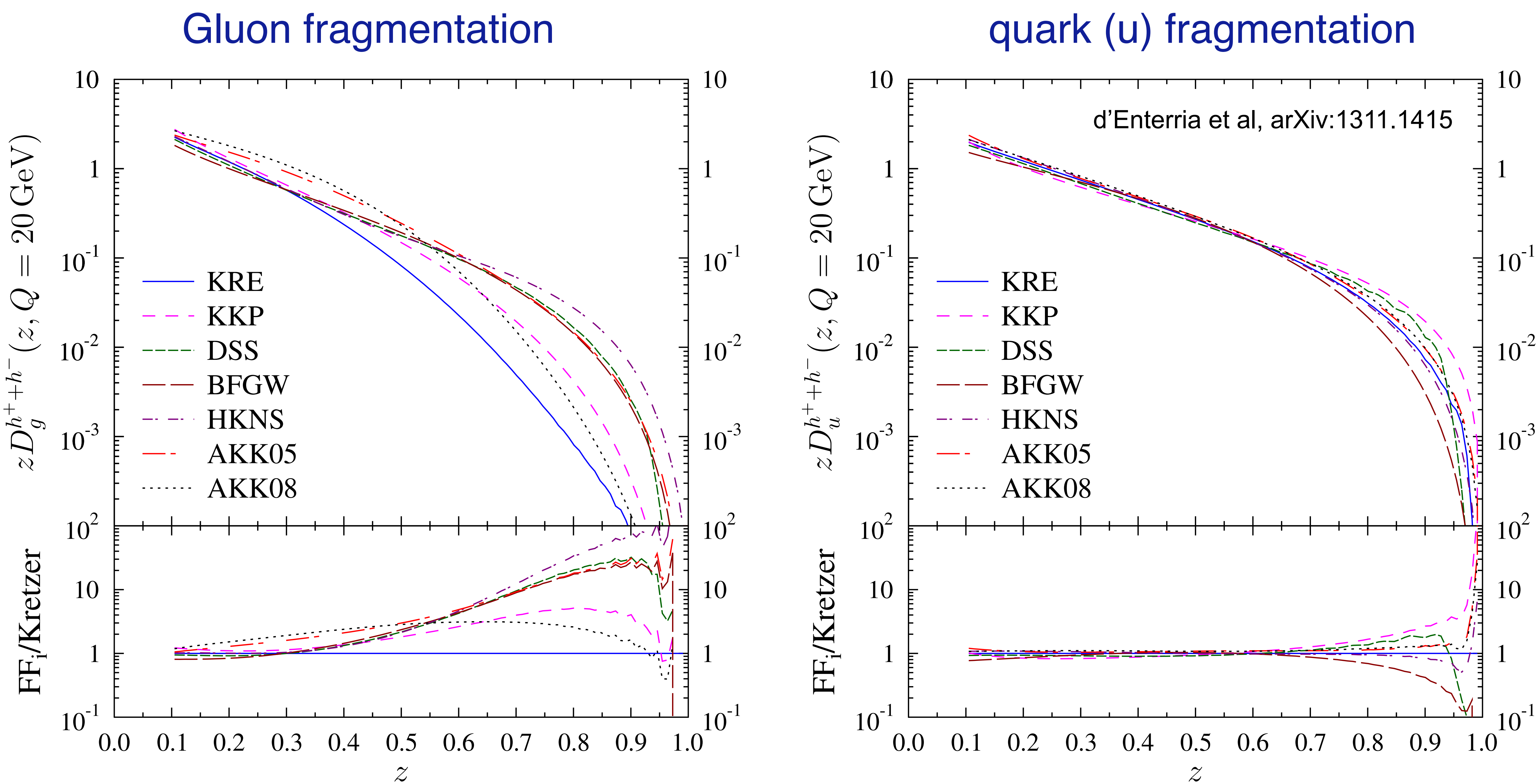



p+p: steeply falling jet spectrum
Hadron spectrum convolution
of jet spectrum with fragmentation

$e^+$ + $e^-$ QCD events: jets
have p=1/2 √s
Directly measure frag function

# Fragmentation function fits

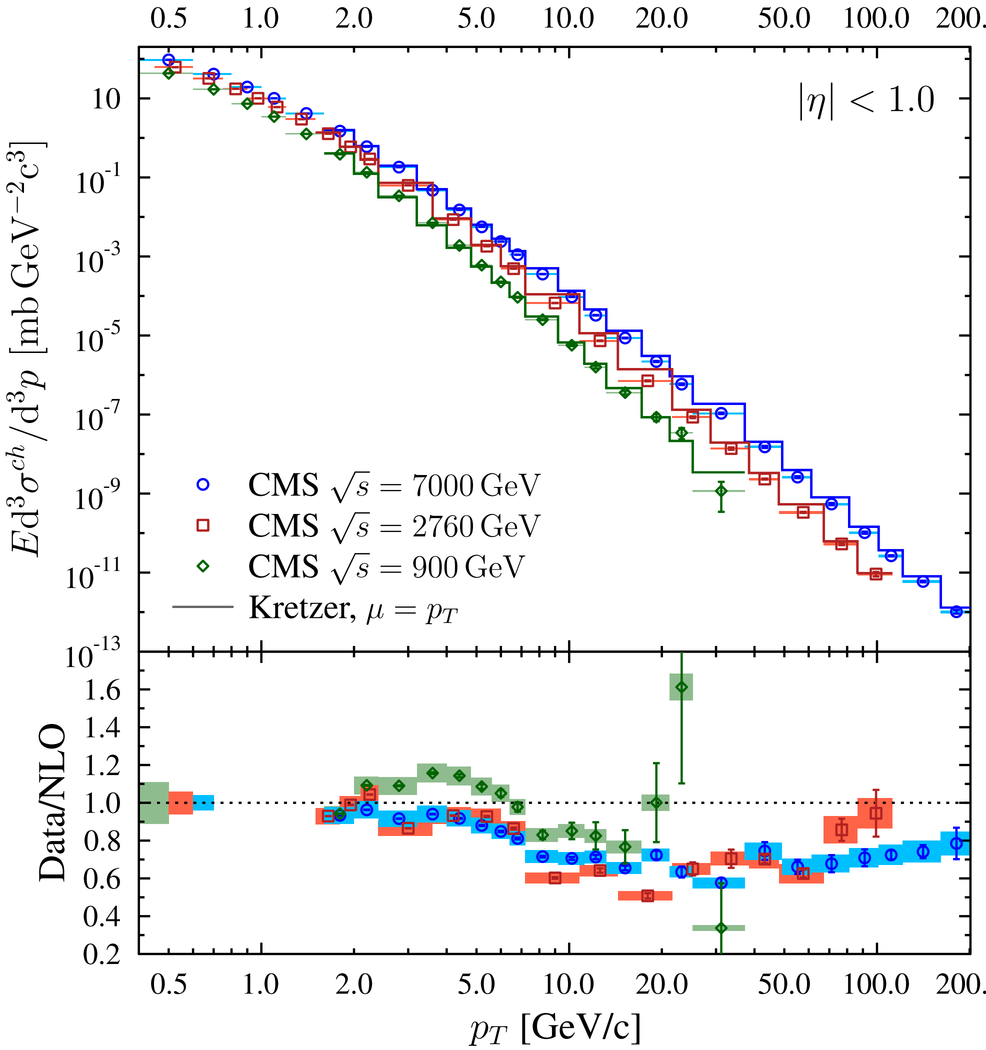



Fragmentation function fits based on $e^+e^-$ have large uncertainty in gluon fragmentation

Some groups use hadron production to further constrain FFs

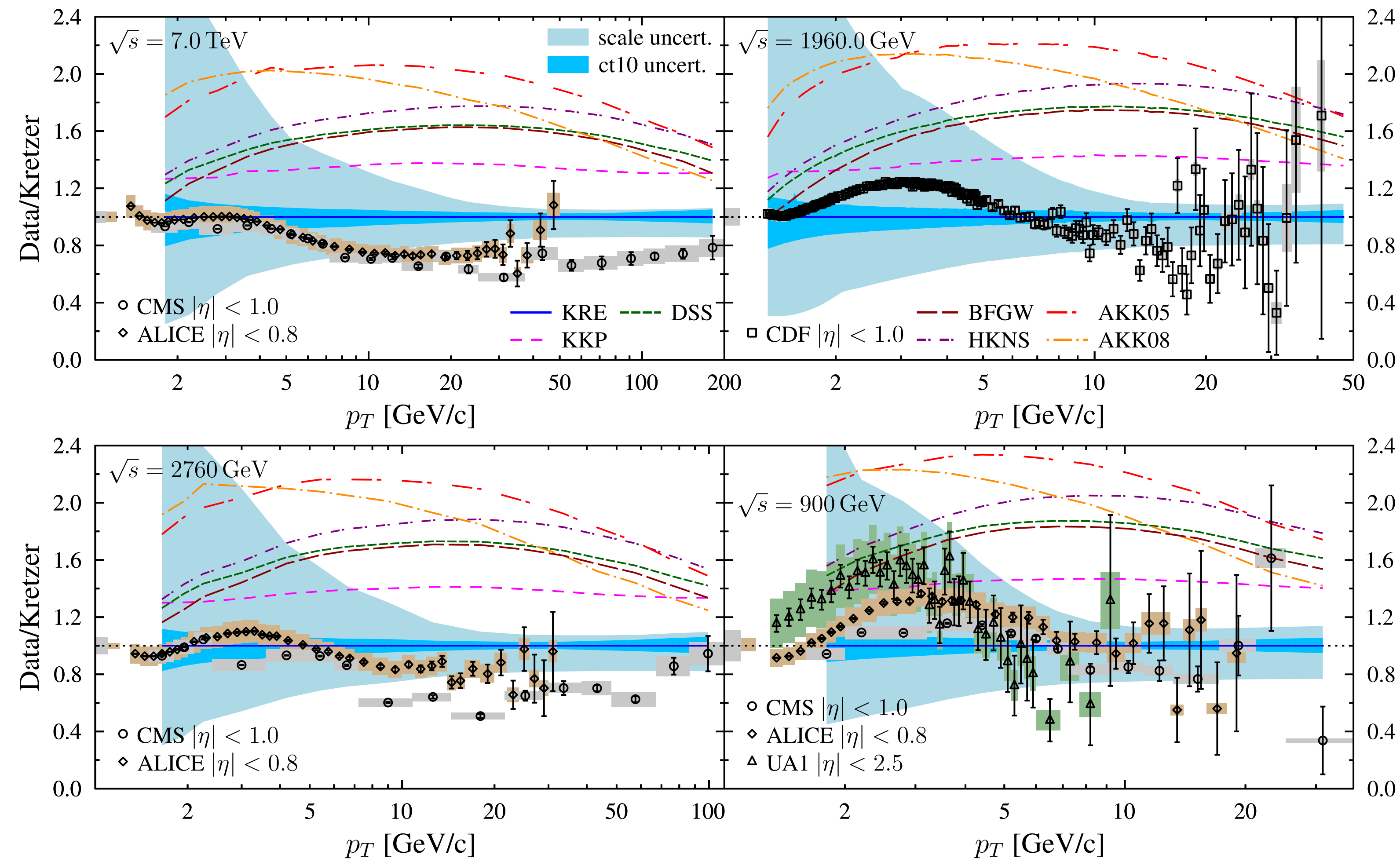
# Adding the LHC data in the game

Kretzer fragmentation




d'Enterria et al, arXiv:1311.1415

Ratios data/theory with uncertainties



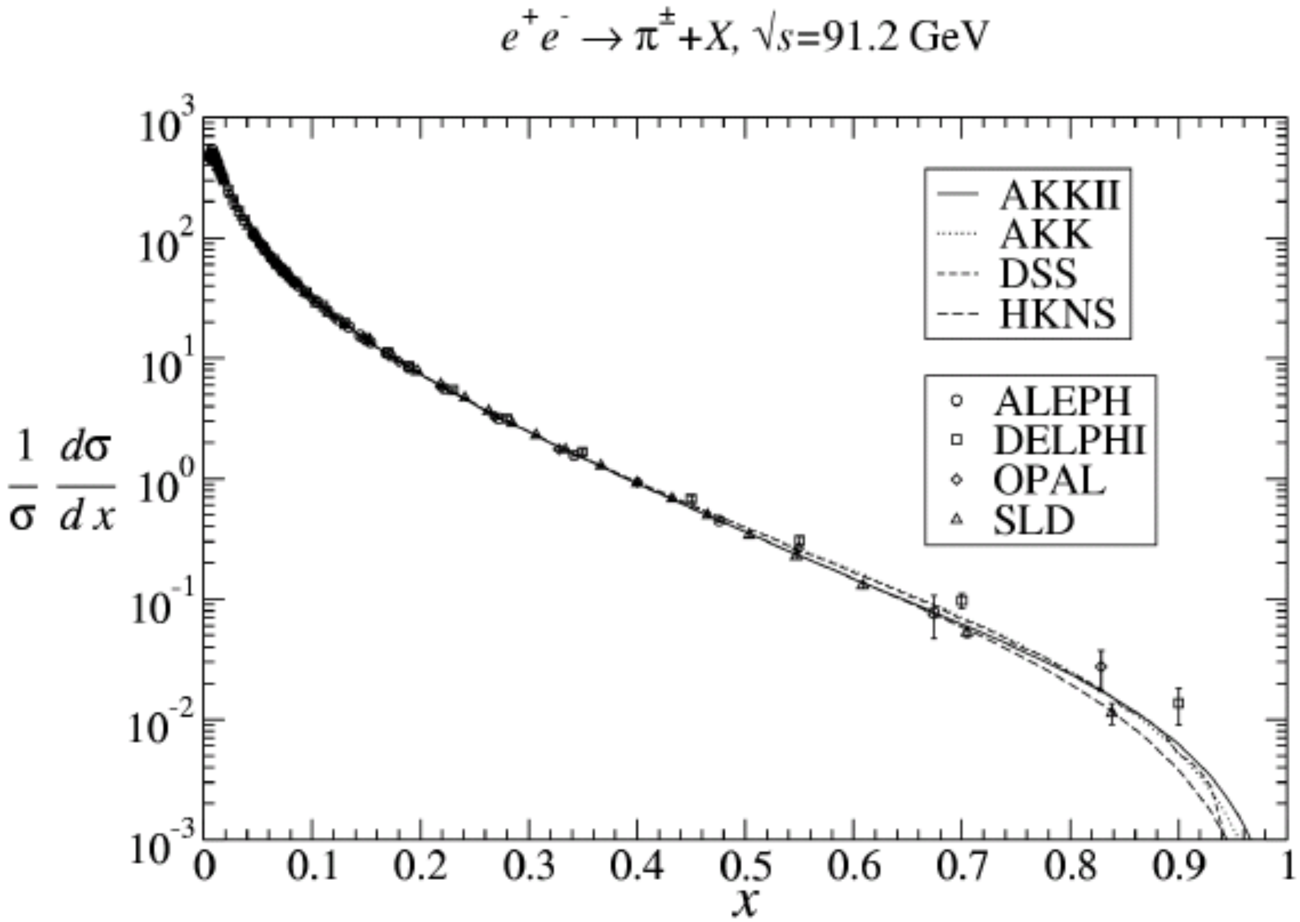

Factor ~2 spread of results due to FF parameterisations

Mostly due to uncertainty in gluons: next step: use data to constrain gluon FF

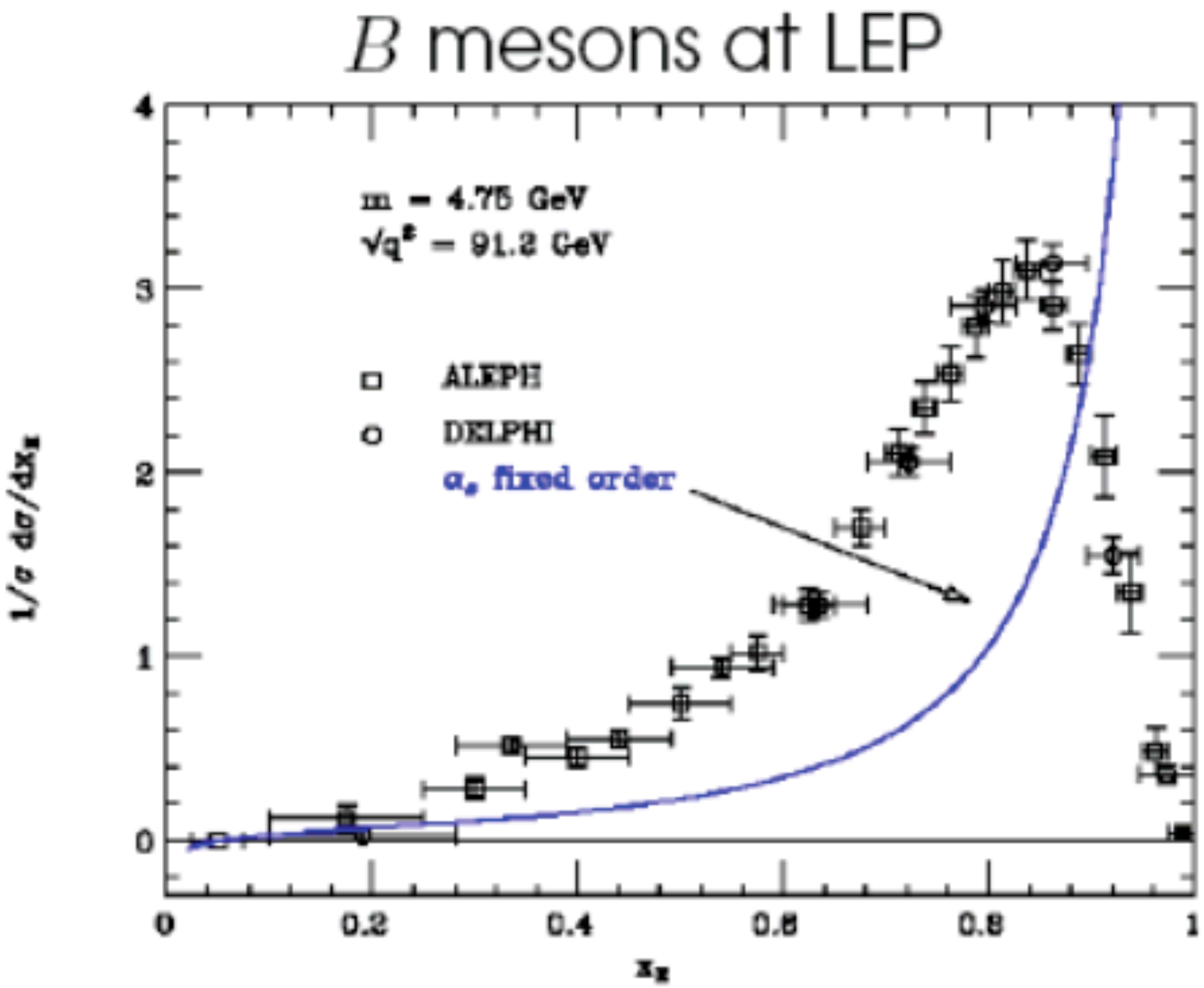
Also note: large scale uncertainties at $p_T < 5$ GeV

# Heavy quark fragmentation

Light quarks




Heavy quarks

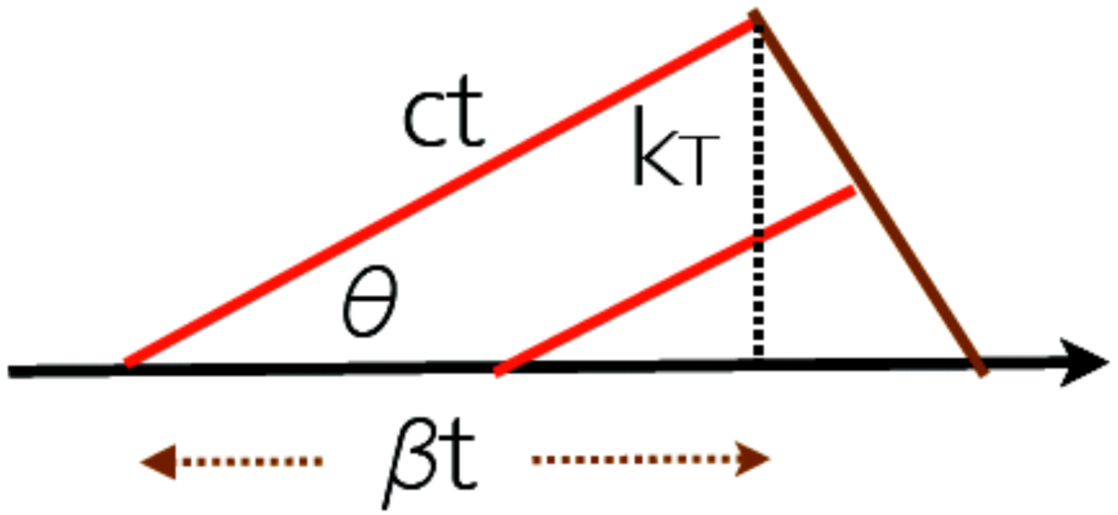



Heavy quark fragmentation: leading heavy meson carries large momentum fraction

Less gluon radiation than for light quarks, due to ‘dead cone’

# Dead cone effect

Radiated wave front cannot out-run source quark




$$\sin\theta_{\mathrm{DC}} = 1 - \beta^2 = \left(\frac{M}{E}\right)^2$$

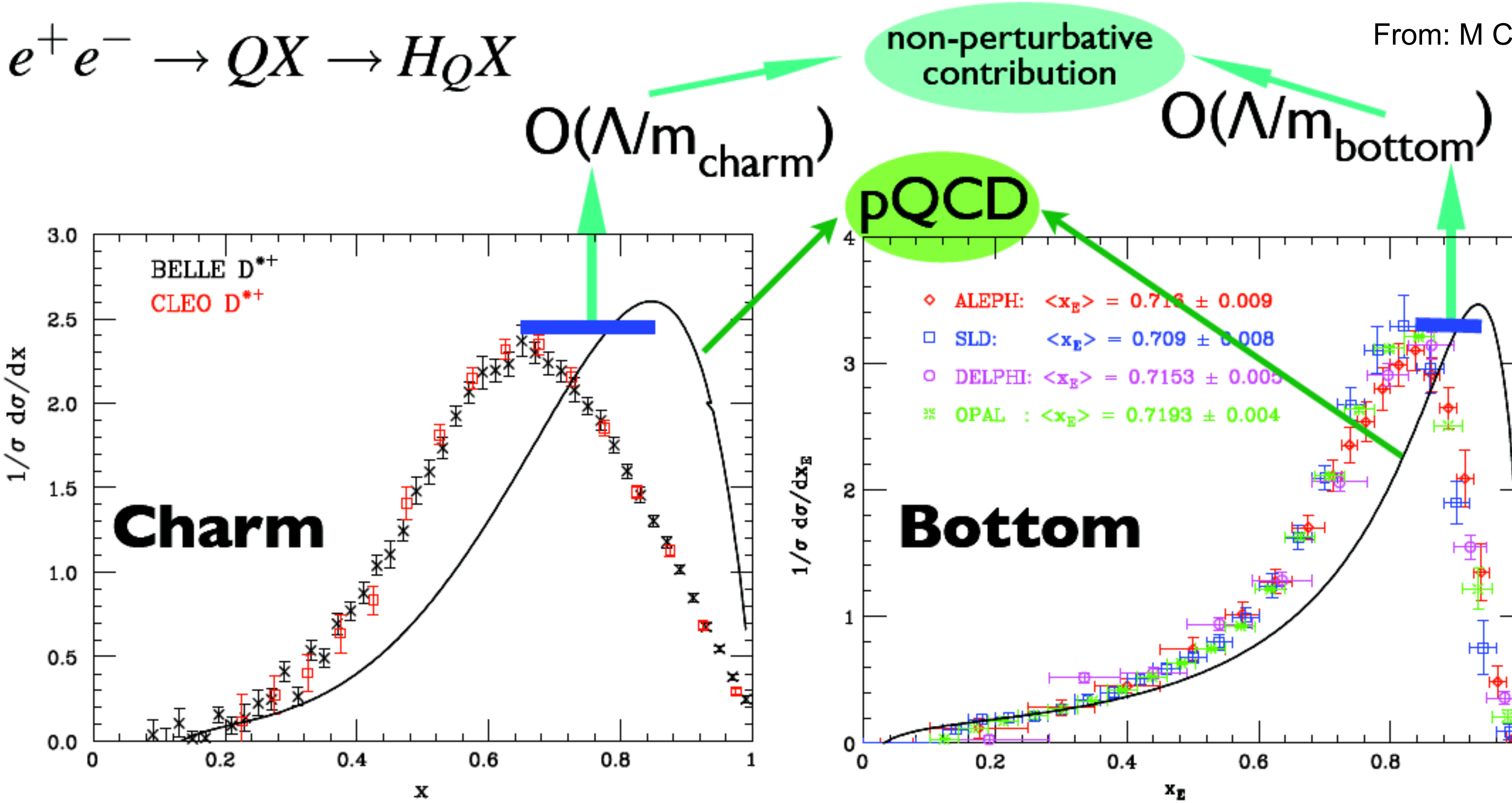
Heavy quark: $\beta < 1$

Result: minimum angle for radiation
⇒ Mass regulates collinear divergence

# Heavy Quark Fragmentation II

From: M Cacciari

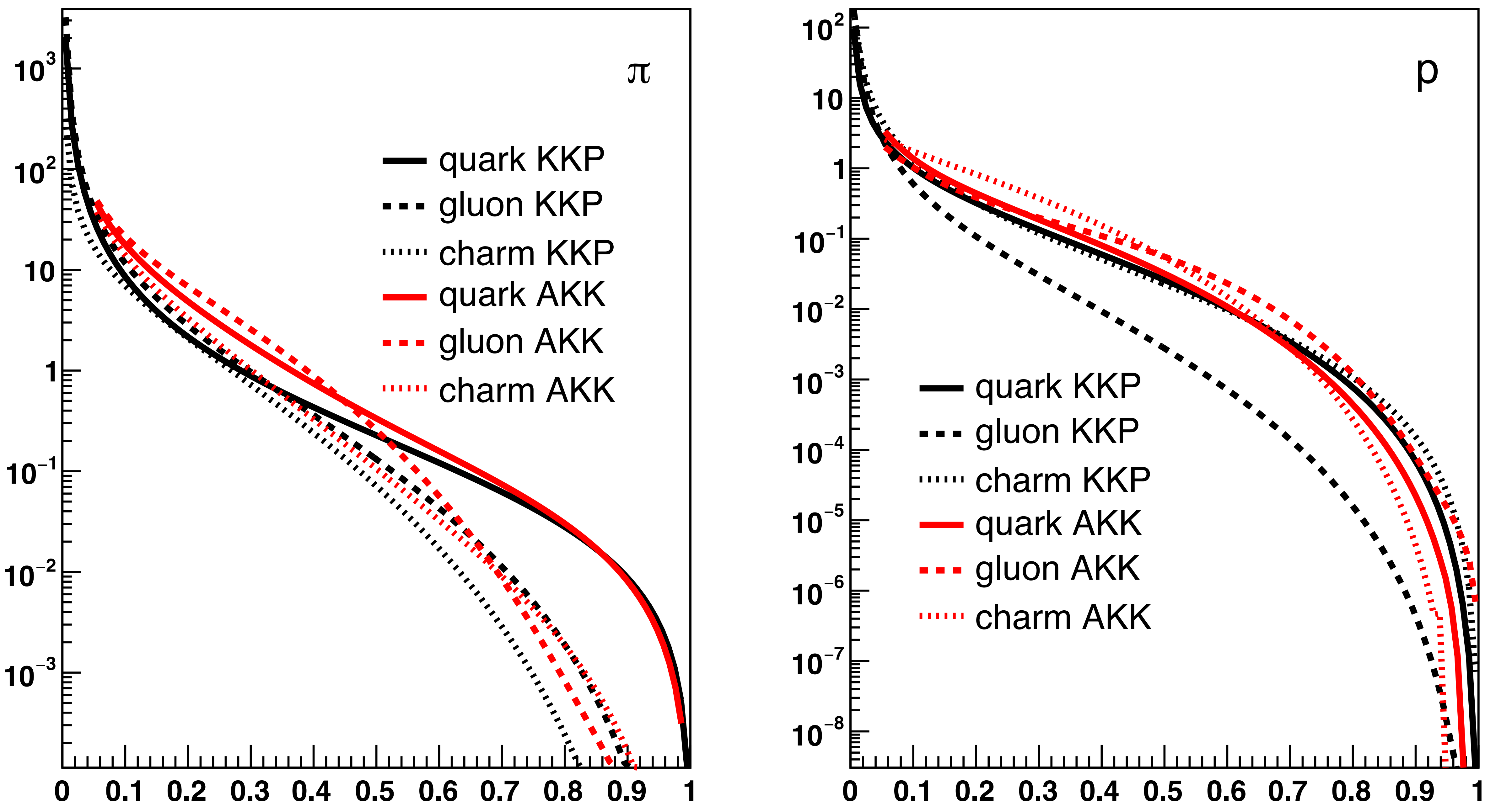
$$e^+e^- \to QX \to H_Q X$$




Significant non-perturbative effects
seen even
in heavy quark fragmentation

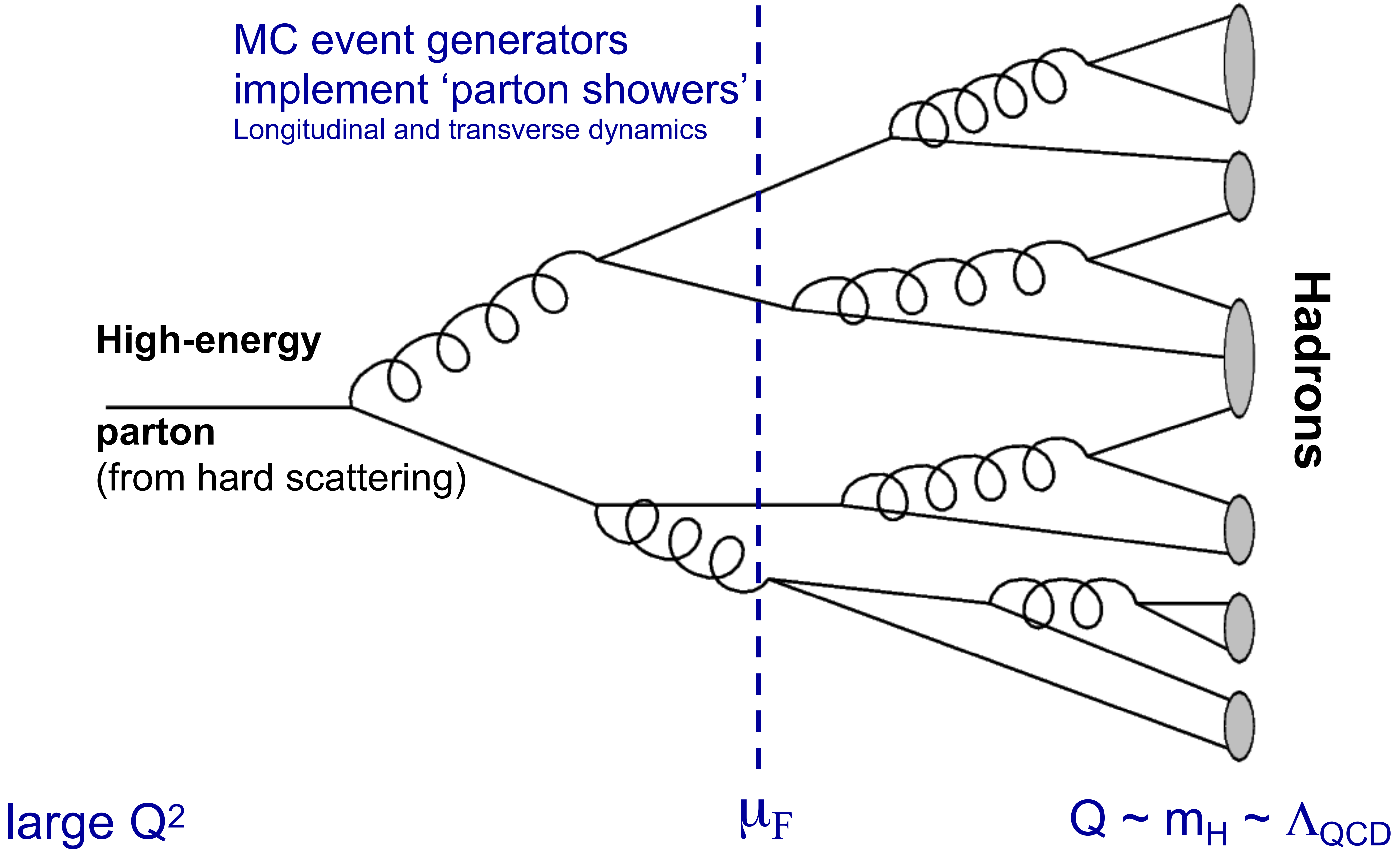
$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{dx} &= \delta(1-x)+\frac{\alpha_s(Q^2)}{2\pi}\left\{C_F+C_F\left[\ln\frac{Q^2}{m^2}\left(\frac{1+x^2}{1-x}\right)_+\right.\right.\\ &+ 2\frac{1+x^2}{1-x}\log x-\left(\frac{\ln(1-x)}{1-x}\right)_+(1+x^2)+\frac{1}{2}\left(\frac{1}{1-x}\right)_+(x^2-6x-2)\\ &+ \left.\left.\left(\frac{2}{3}\pi^2-\frac{5}{2}\right)\delta(1-x)\right]\right\} + \mathcal{O}\left(\frac{m}{Q}\right)\end{aligned}$$

# Heavy and light flavor cross talk




As expected: charm —> light is softer than quark —> light fragmentation
Fragmentation of charm to light hadrons also contributes to light flavor yields

# Fragmentation and parton showers




Analytical calculations: Fragmentation Function $D(z, \mu)$ $z=p_h/E_{jet}$

Only longitudinal dynamics

Thank you for your attention