Johan van de Leur, febr. 2005

De Tsunami

Guido Schneider & C. Eugene Wayne

On the validity of 2D-surface wave models

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Tsunamis are large waves caused by earthquakes or landslides under the sea.

The waves created by these events are almost unobservable in the open sea as their height is only a few meters and their length up to 100 kilometers. In the Pacific Ocean the average depth is around 5000m which leads ... to an incredible velocity of around 700 km/h.

Pelinovsky, Talipova, Kurkin, Kharif

Nonlinear mechanism of tsunami wave generation by atmospheric disturbances

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For a tsunami to rise it is necessary that the water surface deviate from its equilibrium on a sufficient large area.

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For a tsunami to rise it is necessary that the water surface deviate from its equilibrium on a sufficient large area.

In these cases the shallow water theory or long wave theory is the good theoretical and numerical model to describe the properties of tsunami waves.

1-dim. niet-lin. ondiepe water verg.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p_{atm}}{\partial x}$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left[(\ell + u)v\right] = 0$$

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Nonlinear mechanism of tsunami wave generation by atmospheric disturbances

After leaving the generated area, the (tsunami) waves propagate as free waves. In this case Eq. (30) can be reduced to the Korteweg-de Vries equation:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + \alpha u\frac{\partial u}{\partial x} + \beta\frac{\partial^3 u}{\partial x^3} = 0$$

4 filmpjes

film1 film2 film3 film4

Guido Schneider & C. Eugene Wayne

On the validity of 2D-surface wave models

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If the earthquake happens...

On the validity of 2D-surface wave models

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On a time scale $\mathcal{O}(1/\epsilon)$ the solutions split up into the two wave packets one moving to the right and one to the left. These wave packets evolve independently as solutions of the KdV equations:

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On a time scale $\mathcal{O}(1/\epsilon)$ the solutions split up into the two wave packets one moving to the right and one to the left. These wave packets evolve independently as solutions of the KdV equations:

$$\frac{\partial u}{\partial t} = u\frac{\partial}{\partial x} + \frac{\partial^3 u}{\partial x^3}$$

Volkskrant, 11 september 1999

wetenschapsbijlage

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Eenzame golf glijdt over de Noordzee

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Eenzame golf glijdt over de Noordzee

De soliton, een geheimzinnige vloedgolf, heeft op de Noordzee heel wat consternatie veroorzaakt. Een nieuwe, snelle veerboot is vermoedelijk de oorzaak.

Stena Discovery

Guido Schneider & C. Eugene Wayne

On the validity of 2D-surface wave models

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On the validity of 2D-surface wave models

High speed ferries

On the validity of 2D-surface wave models

High speed ferries

The depth of the southern North Sea is about 30-40m, and so the velocity of the ferries of about 80 km/h is close to the approximate speed where solitary waves are created.

On the validity of 2D-surface wave models

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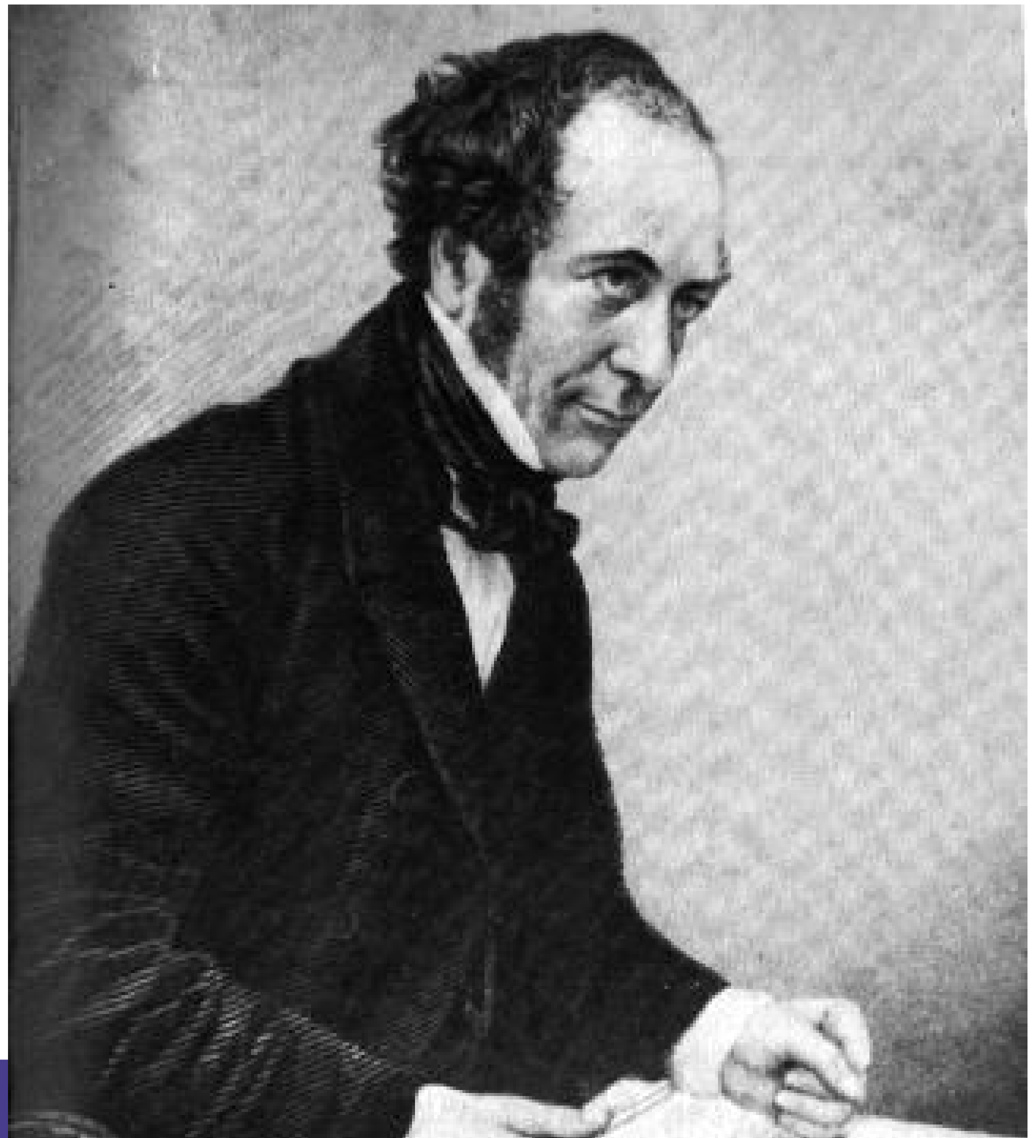
The depth of the southern North Sea is about 30-40m, and so the velocity of the ferries of about 80 km/h is close to the approximate speed where solitary waves are created.

There are now speed limits for these ferries

John Scott Russell and the solitary wave



John Scott Russell and the solitary wave

‘I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity,

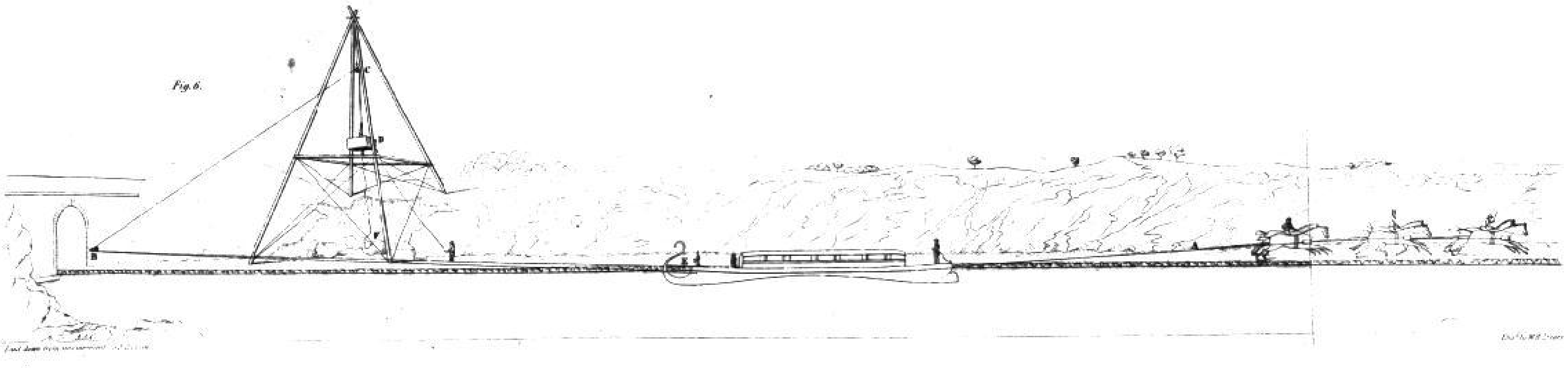
John Scott Russell and the solitary wave

assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height.

John Scott Russell and the solitary wave

Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation.

John Scott Russell and the solitary wave



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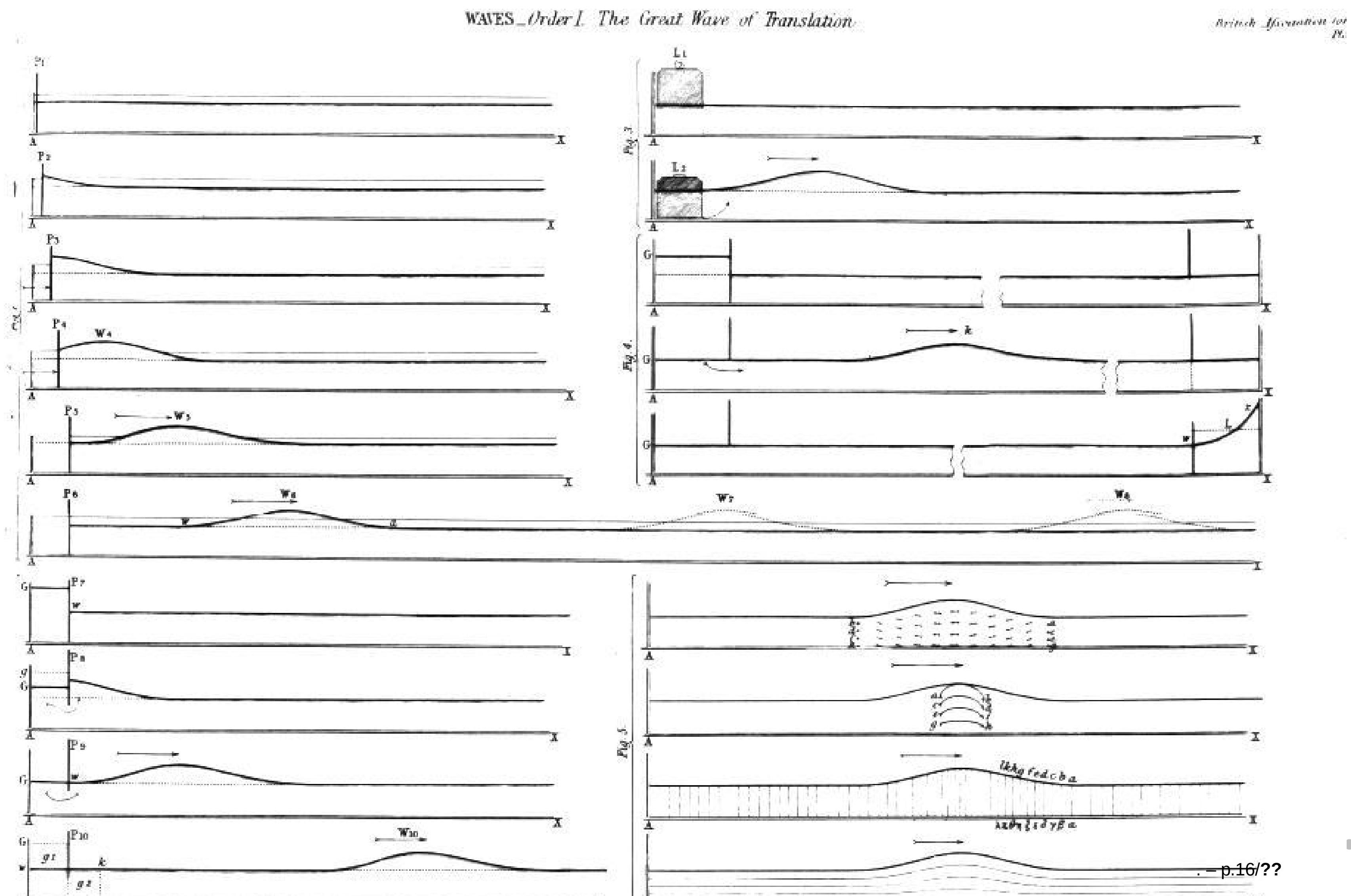

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- g gravitatieconstante

Sir George Stokes zegt in 1847

...the degradation observed by Russell is not due to the imperfect fluidity of the fluid and its adhesion to the sides and bottom of the canal but is an essential characteristic of the solitary wave.

Korteweg (en de Vries)

Korteweg en de Vries, 1895

$$\frac{du}{dt} = \frac{3}{2}\sqrt{\frac{g}{\ell}}\frac{\partial}{\partial x}\left(\frac{1}{2}u^2 + \frac{2}{3}\alpha u + \frac{1}{3}\sigma\frac{\partial^2 u}{\partial x^2}\right)$$

Korteweg en de Vries, 1895

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- ρ is de massadichtheid

KdV's oplossing

$$u = h\left(\exp\left(x\sqrt{\frac{h}{4\sigma}}\right) + \exp\left(-x\sqrt{\frac{h}{4\sigma}}\right)\right)^{-2}$$

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$$v = \sqrt{g(h+\ell)}$$ bij Scott Russell

Fermi, Pasta en Ulam, 1955

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Bestudeerden de geleiding van warmte door massieve objecten.

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Discreet model:

Gewichten (moleculen) verbonden door veren.

Kruskal en Zabusky, 1965

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Discreet model $\longrightarrow$ continu model

Kruskal en Zabusky, 1965

Discreet model $\longrightarrow$ continu model

Bestudeerden dezelfde geleiding van warmte door massieve objecten door middel van numerieke simulaties van de Korteweg de Vries vergelijking.

KdV vergelijking

$$\frac{\partial u}{\partial t} + 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

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We willen als oplossing een bewegende golf. We proberen daarom een oplossing van de vorm

$$u(x,t) = f(x - ct) = f(s)$$

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Merk op $\quad \dfrac{\partial f}{\partial t} = -c\dfrac{\partial f}{\partial s}, \qquad \dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial s}$

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Integreren geeft

$$-cf + 3f^2 + \frac{\partial^2 f}{\partial s^2} = c_1$$

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vermenigvuldig met $\frac{\partial f}{\partial s}$:

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Integreren geeft:

$$-\frac{c}{2}f^2 + f^3 + \frac{1}{2}\left(\frac{\partial f}{\partial s}\right)^2 = c_1 f + c_2$$

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Scheiden van variabelen geeft

$$\int d\,s = \int \frac{d\,f}{f\sqrt{c-2f}}$$

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Substitueer: $f = \frac{2c}{(e^w+e^{-w})^2}$

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$$-\frac{2}{\sqrt{c}} w = s + k$$

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Kies $k = 0$, dit geeft $\quad w = -\frac{\sqrt{c}}{2} s$

Wat hebben we gedaan?

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- $f = \dfrac{2c}{(e^{w} + e^{-w})^{2}}$
- $w = -\dfrac{\sqrt{c}}{2} s = \dfrac{\sqrt{c}}{2}(x - ct)$

Dit geeft de 1-soliton oplossing: $u(x,t) =$

$$2c\left(\exp\left(\frac{\sqrt{c}}{2}(x-ct)\right) + \exp\left(-\frac{\sqrt{c}}{2}(x-ct)\right)\right)^{-2}$$

Soliton 1995

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Hasegawa en Tappert, 1973

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Deze solitonen voldoen aan de Niet-Lineaire Schrödinger (NLS) vergelijking:

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- t is de tijd
- z is de afstand

NLS-soliton

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$$q = \frac{a\exp\left(-ibt + \frac{i}{2}(a^2 - b^2)z + ic\right)}{\exp\left(a(t + bz - t_0)\right) + \exp\left(-a(t + bz - t_0)\right)}$$

controleerbaarheid

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In praktijk:

$$iq_z + \frac{1}{2}q_{tt} + |q|^2 q = -i(\gamma - g(z))q$$

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In praktijk:

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- γ geeft "vezelverlies" per afstandseenheid weer
- g geeft de input van de versterkers weer

vezelverlies

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Dit wordt gemeten in decibels per kilometer.

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- eerste kabels verlies: 1000 dB/km

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- 1979: 0,2 dB/km

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- sinds 1977 zijn er commerciele systemen in gebruik
- 1993: transatlantische kabel voor 40000 telefoongesprekken = 296 mbits/seconde
- 1993: 20 Gbits/seconde foutvrij verzonden over 14000 km