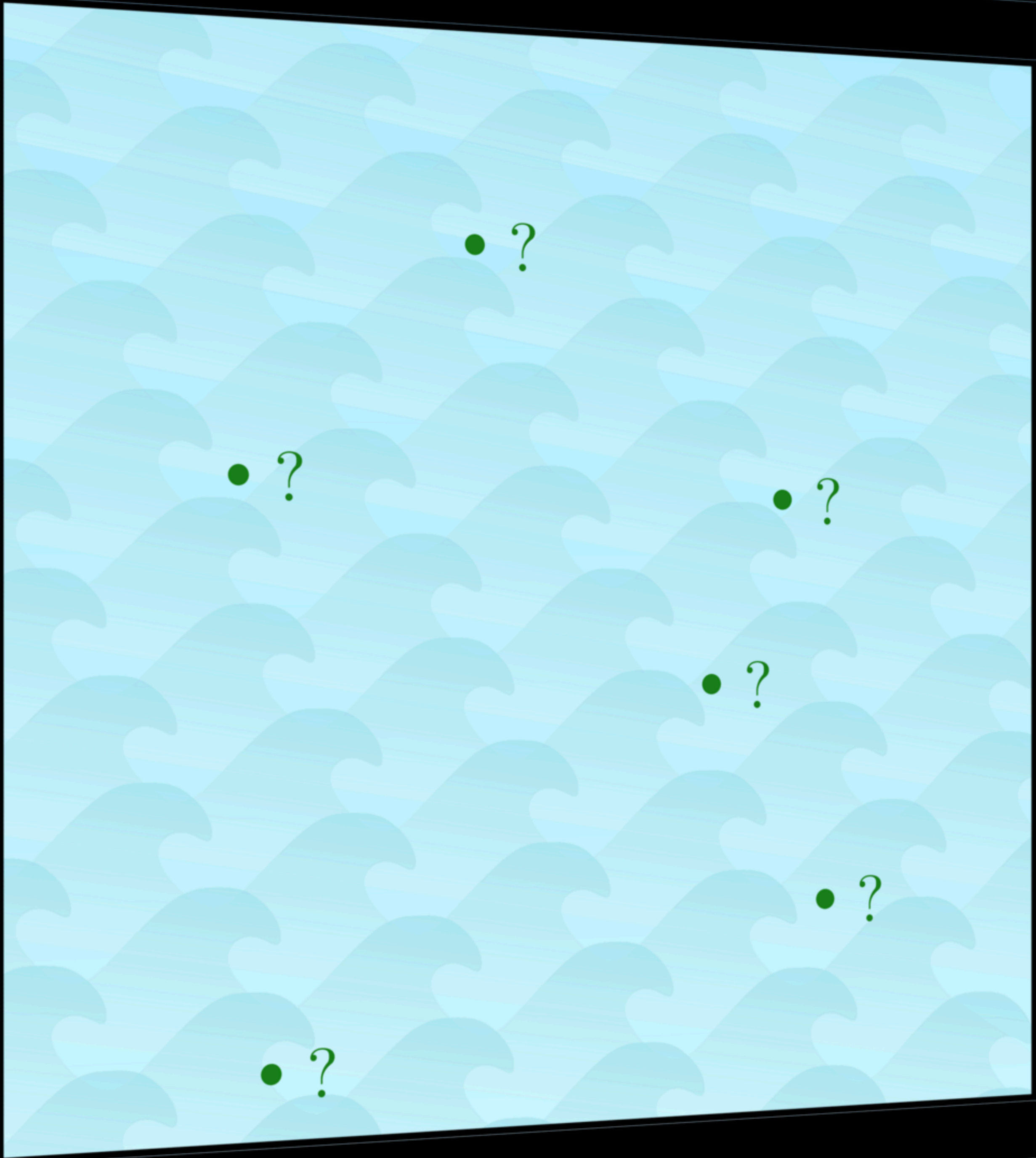
Geometry,
Uncertainty,
&
BATTLESHIPS!!!

Eva Hainzl
Maarten Löffler
Daniel Perz
Josef Tkadlec
Markus Wallinger

CHAPTER 1

UNCERTAINTY

Suppose we have a set of points, but we don’t know where they are.



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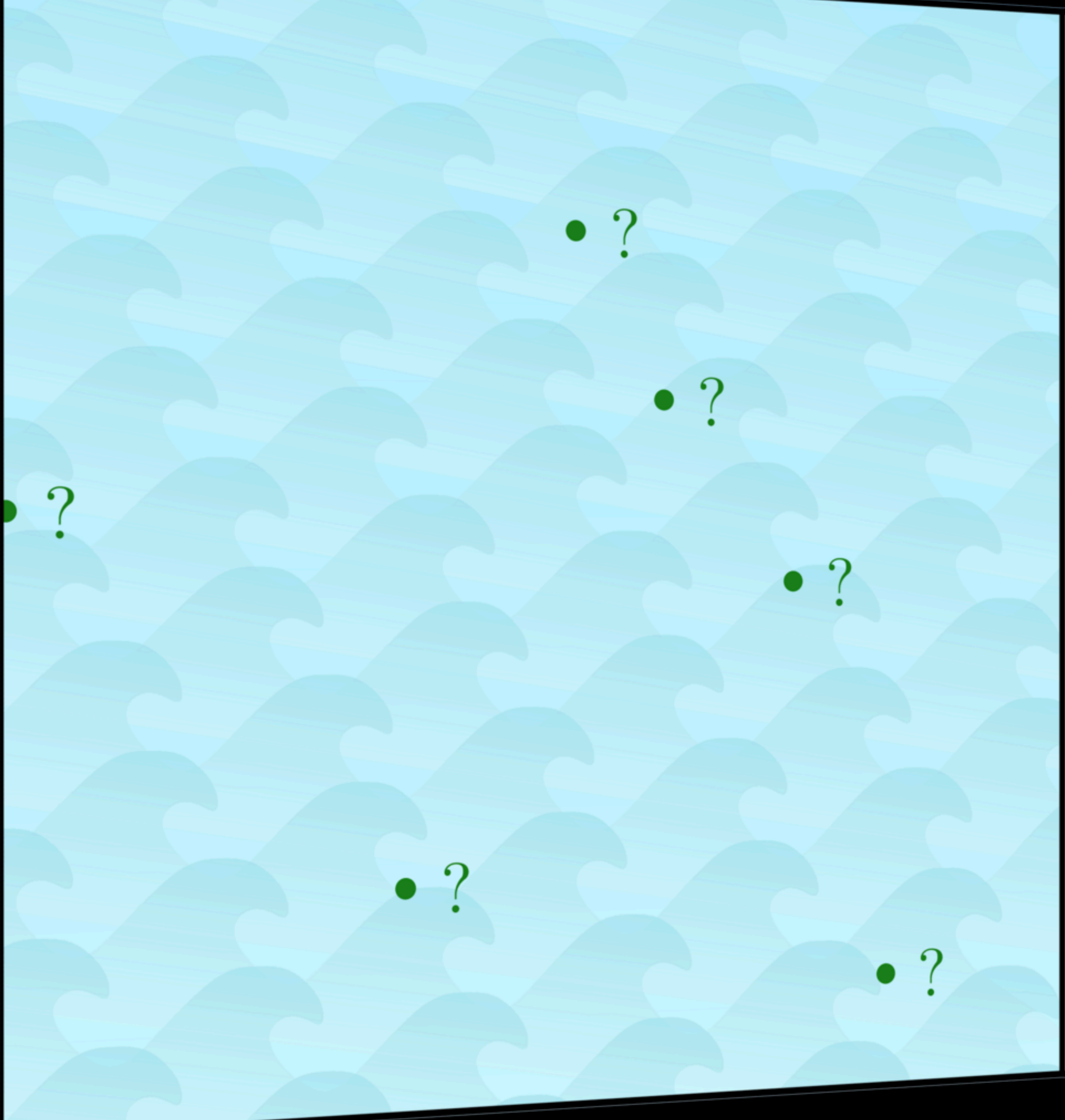

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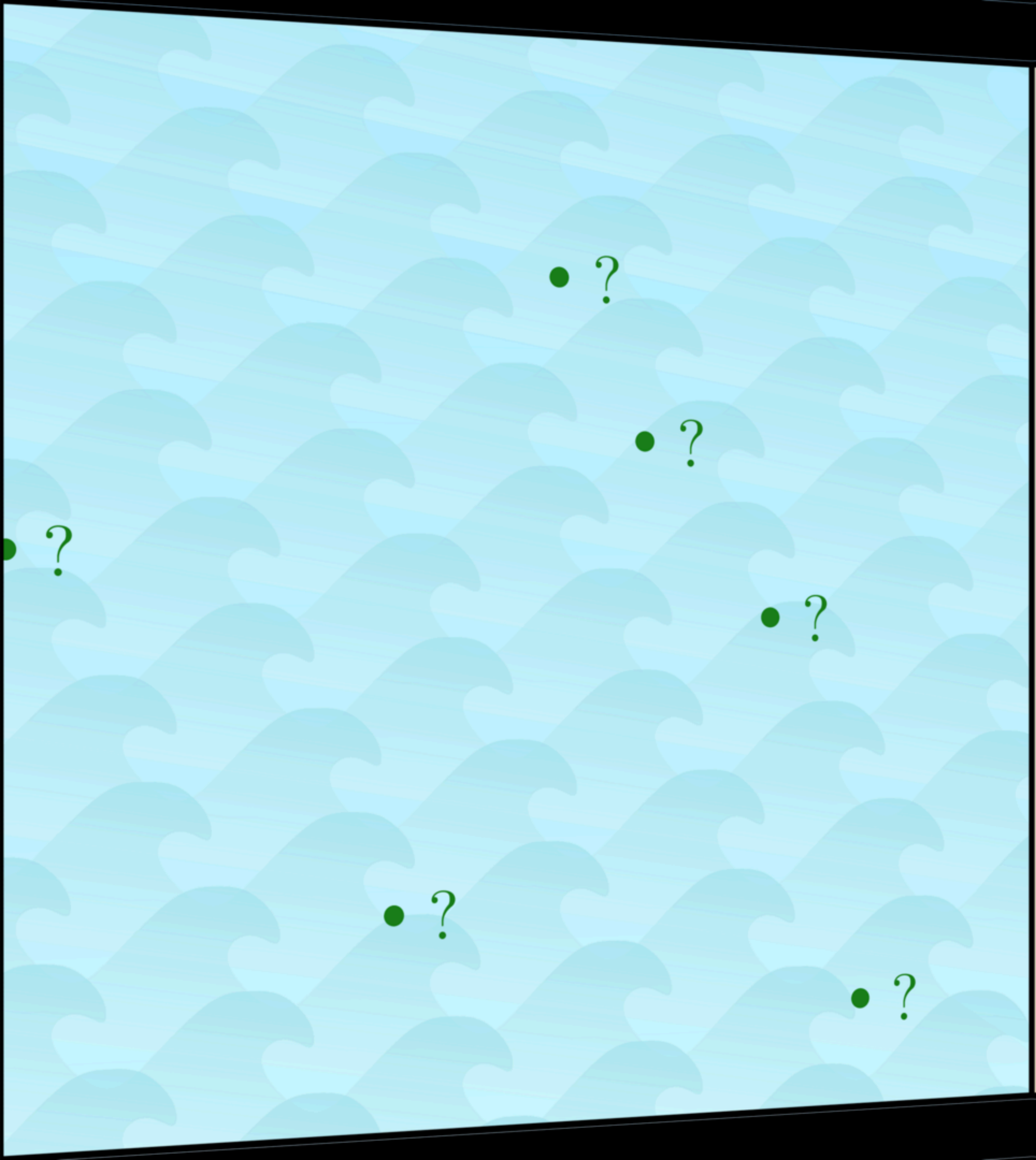

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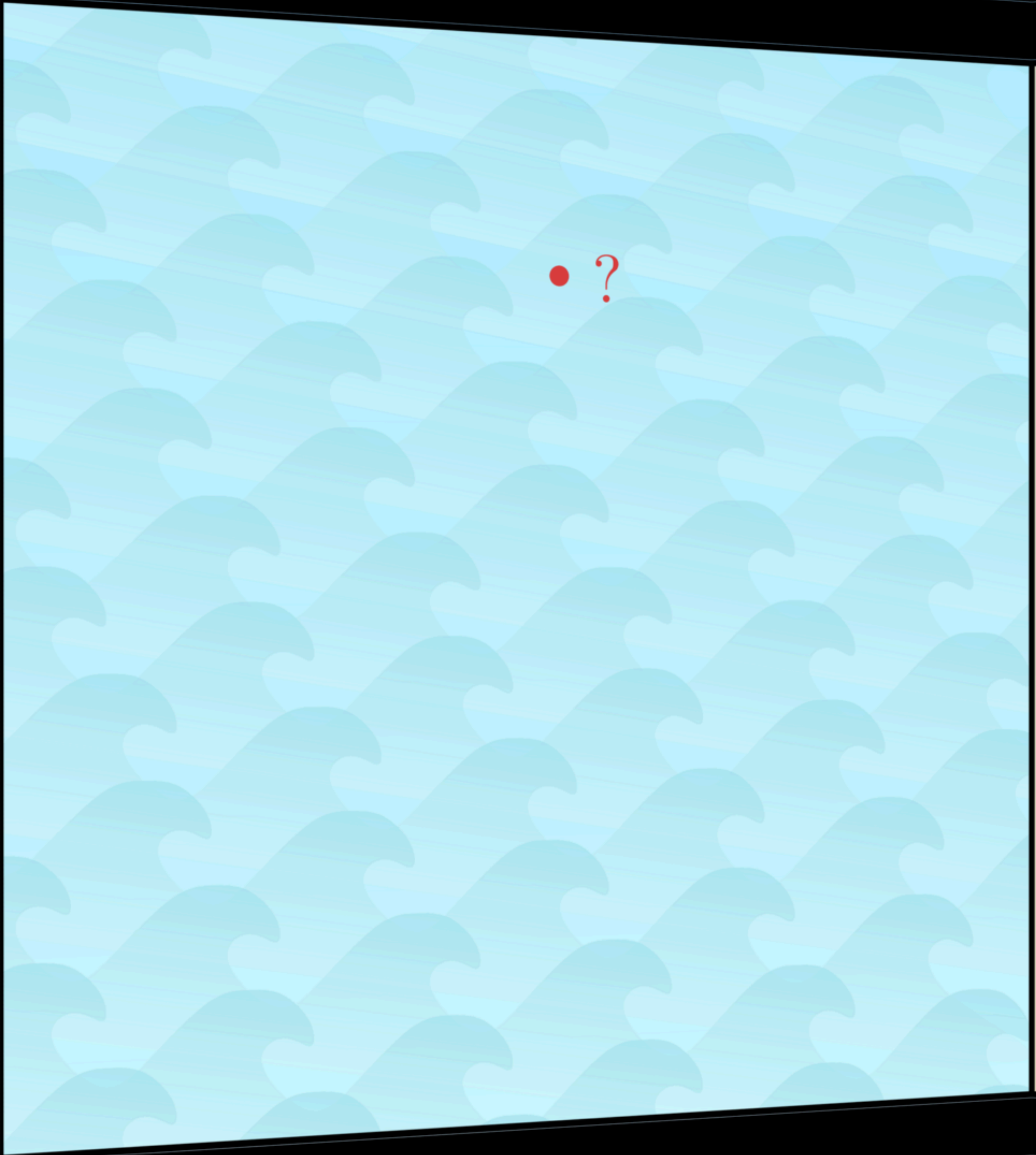

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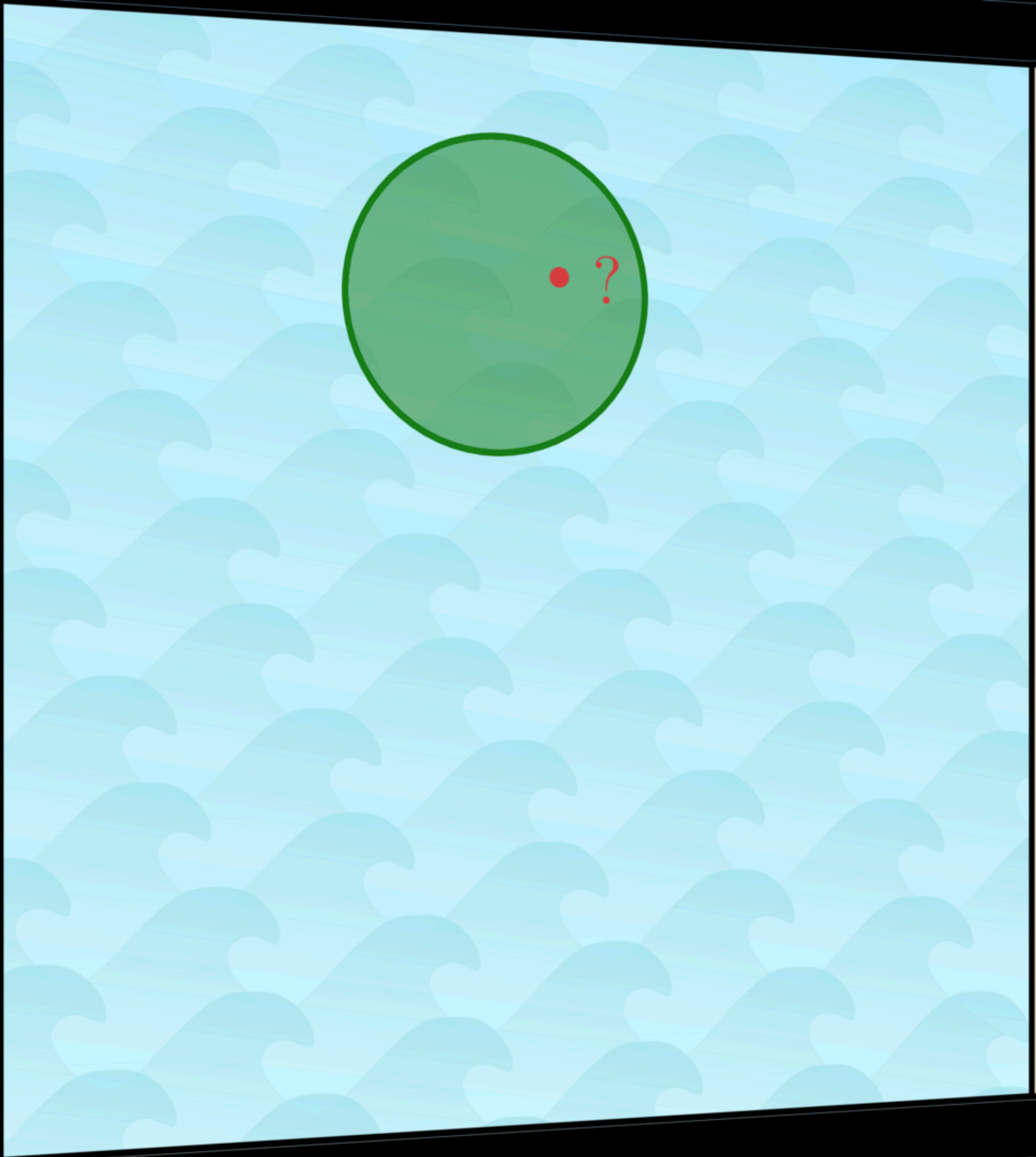

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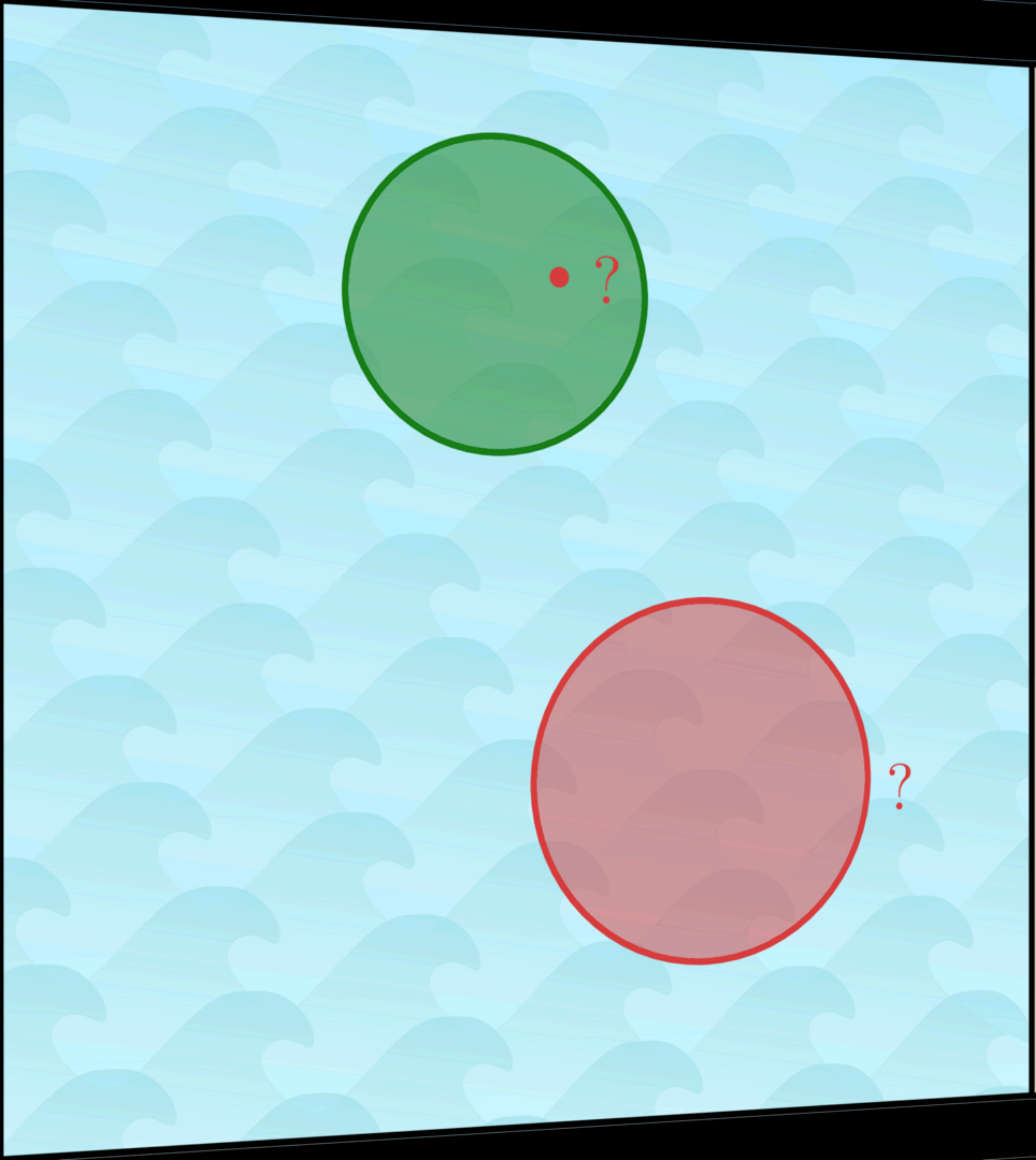

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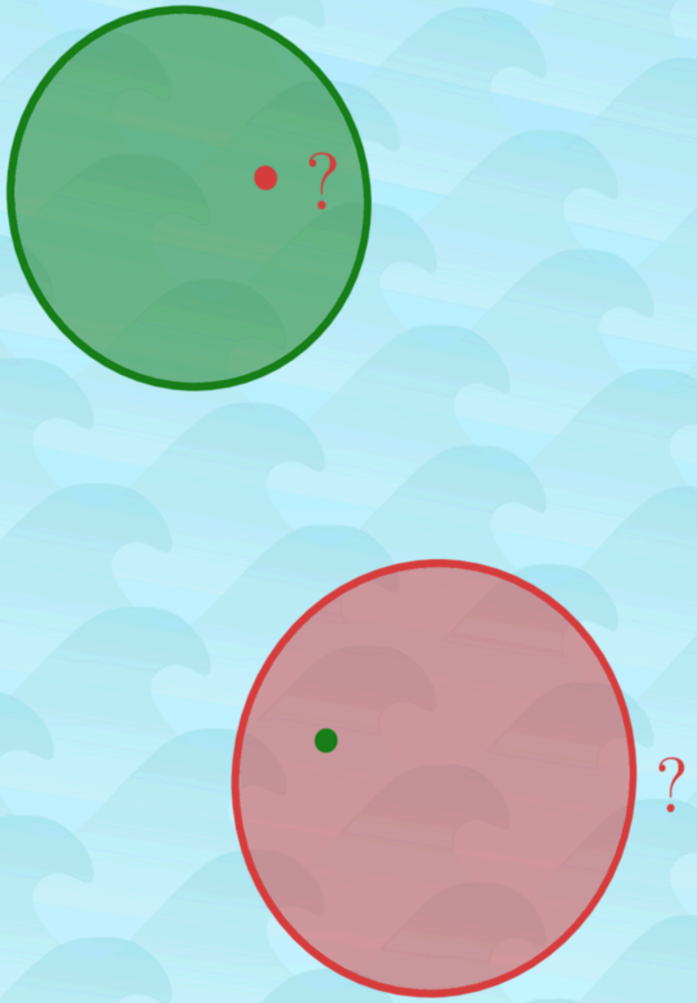

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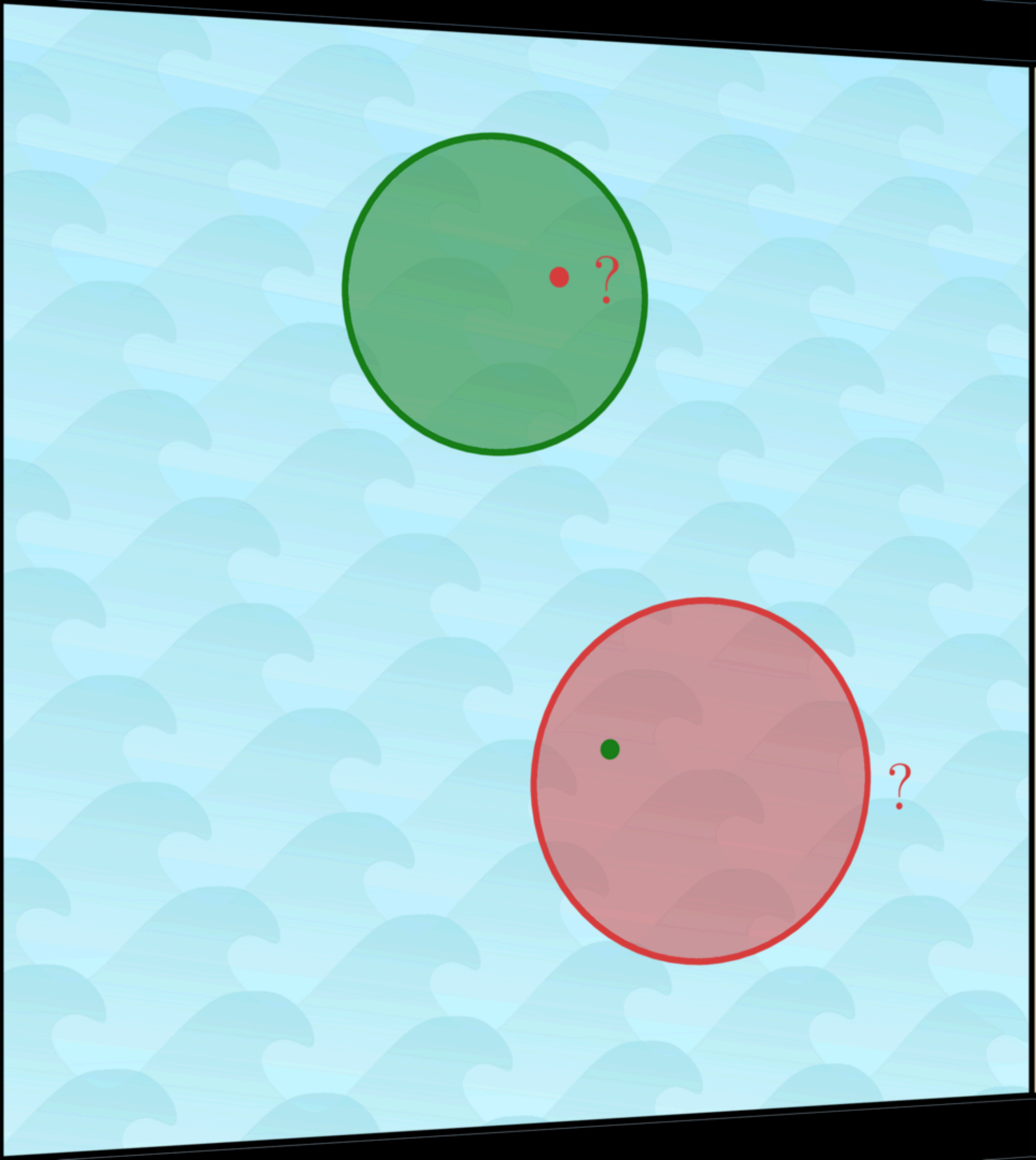

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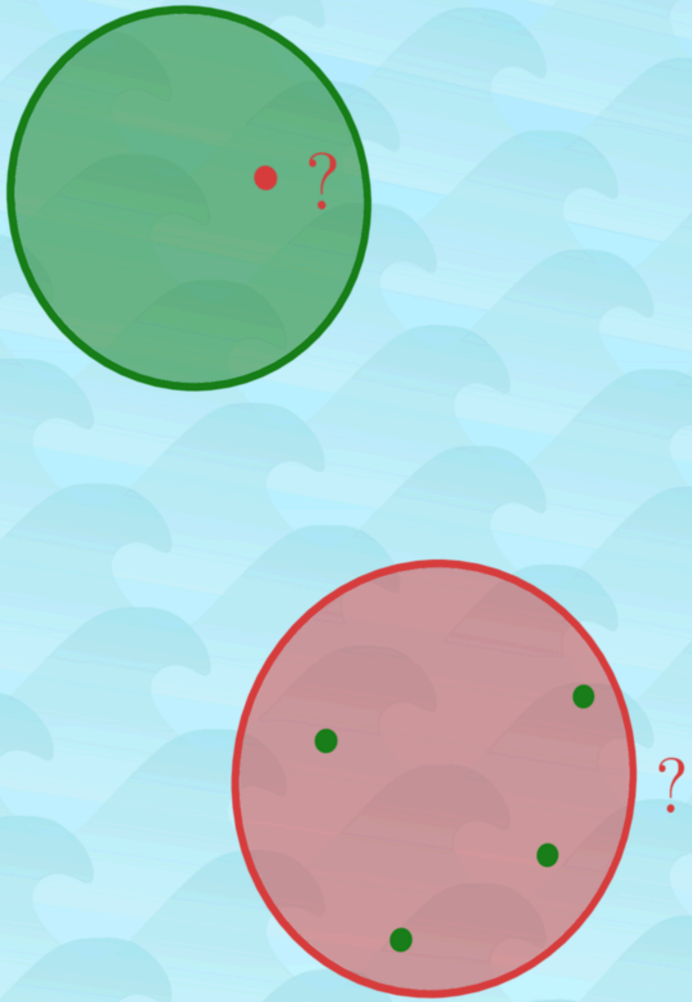

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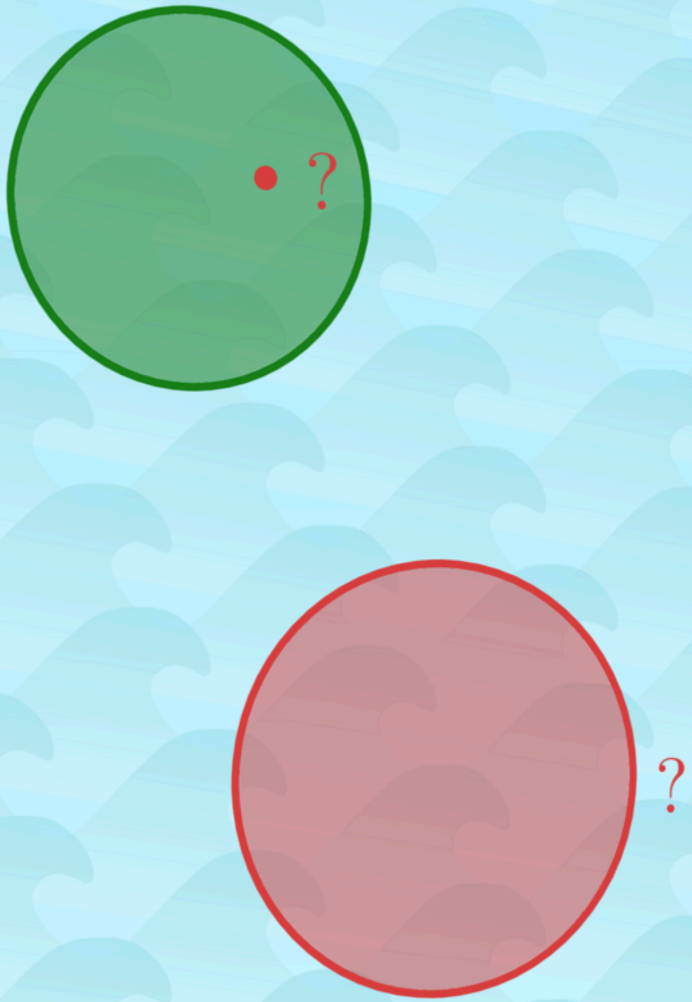

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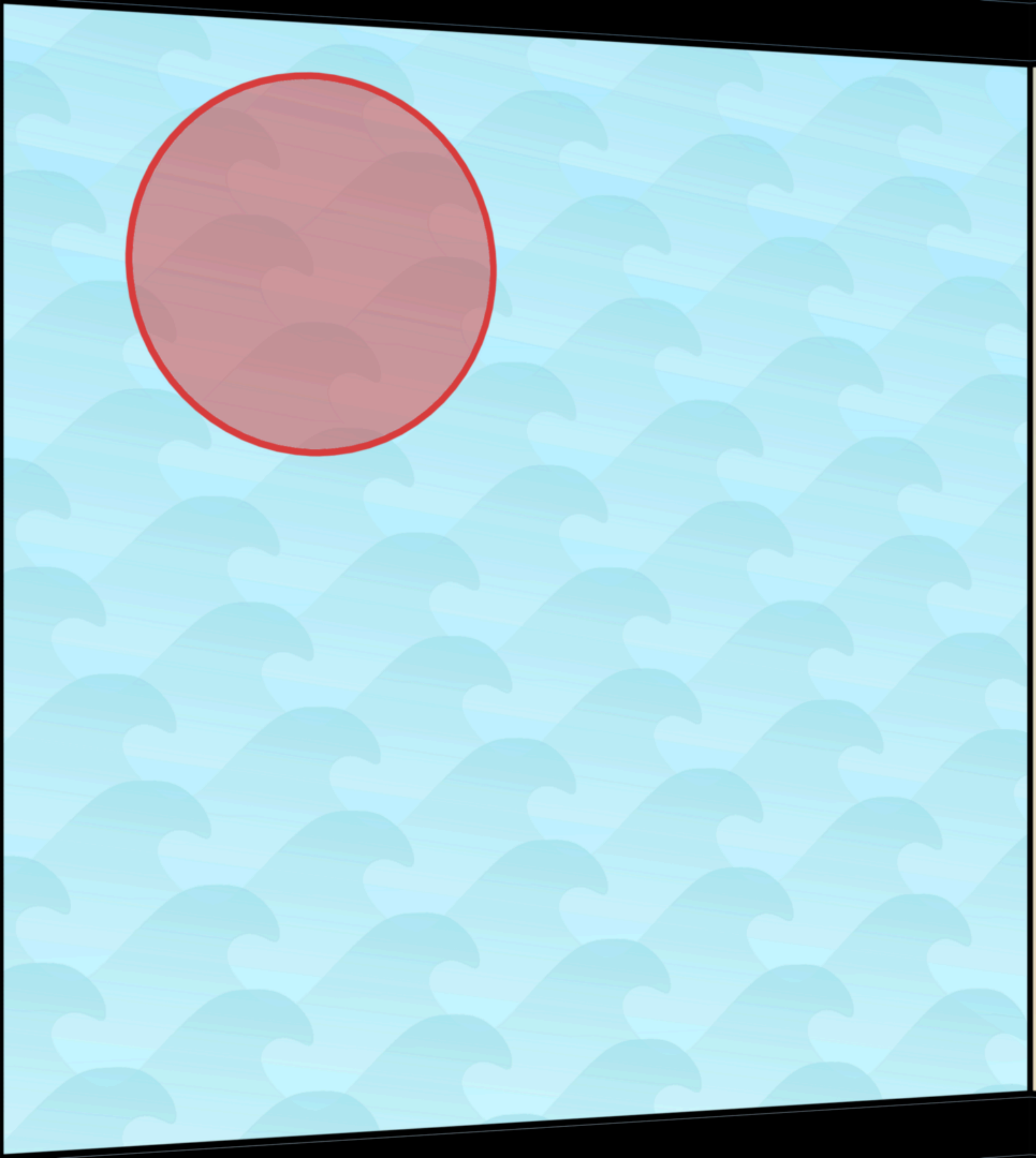

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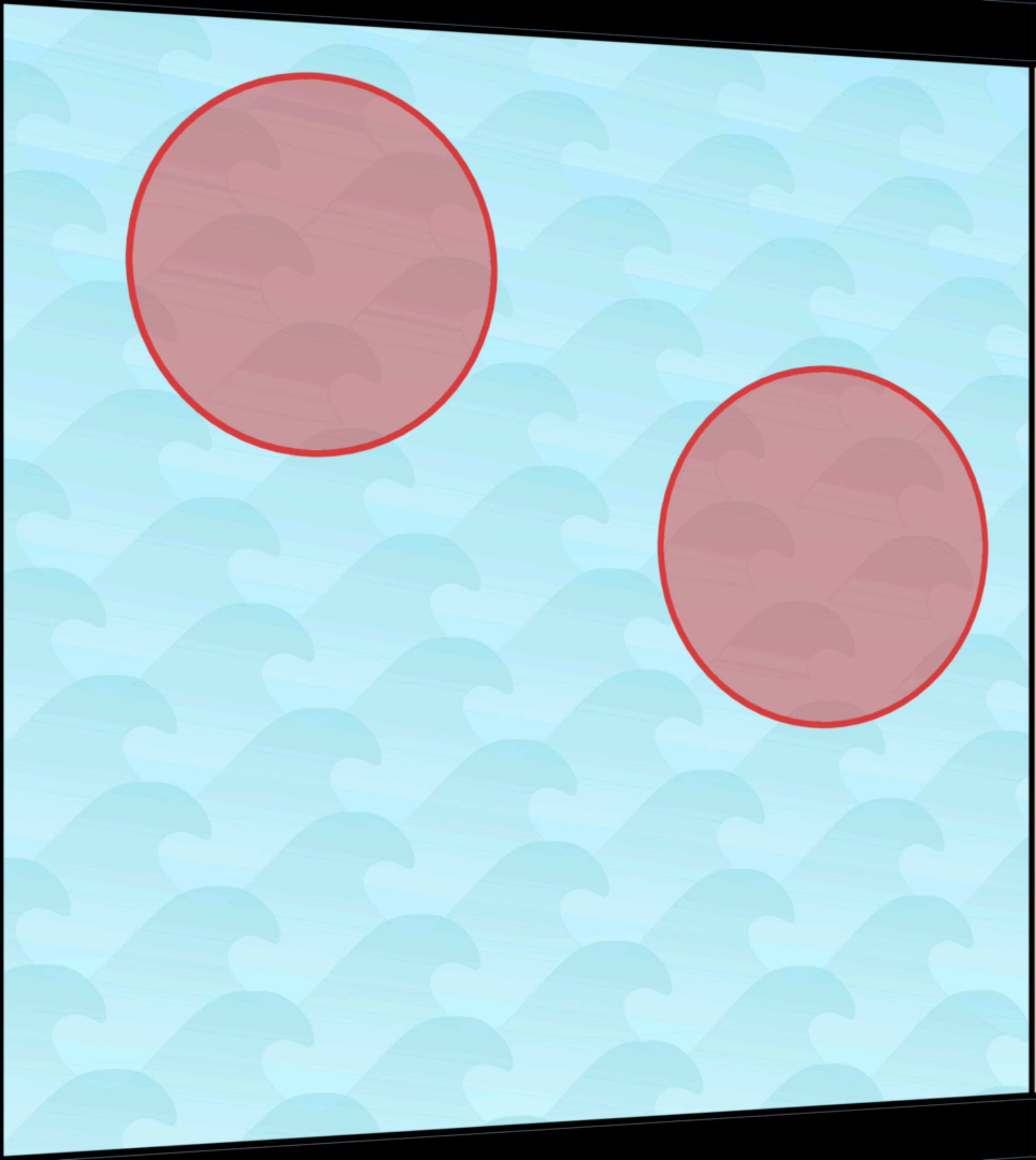

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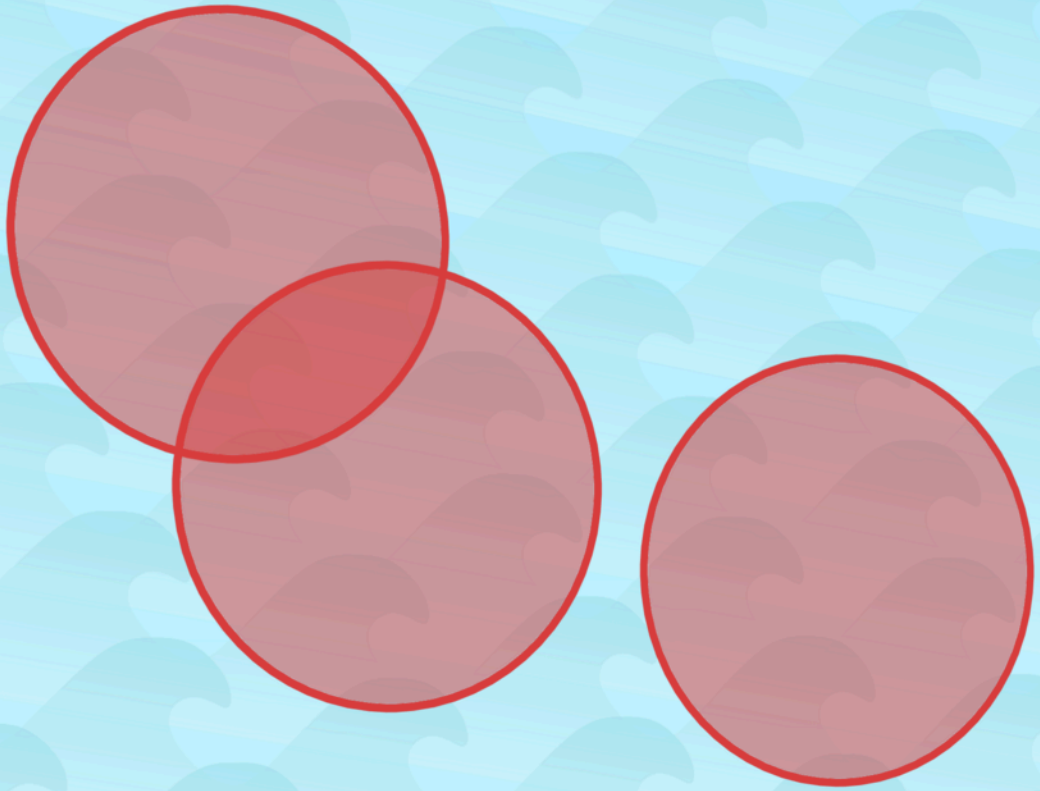

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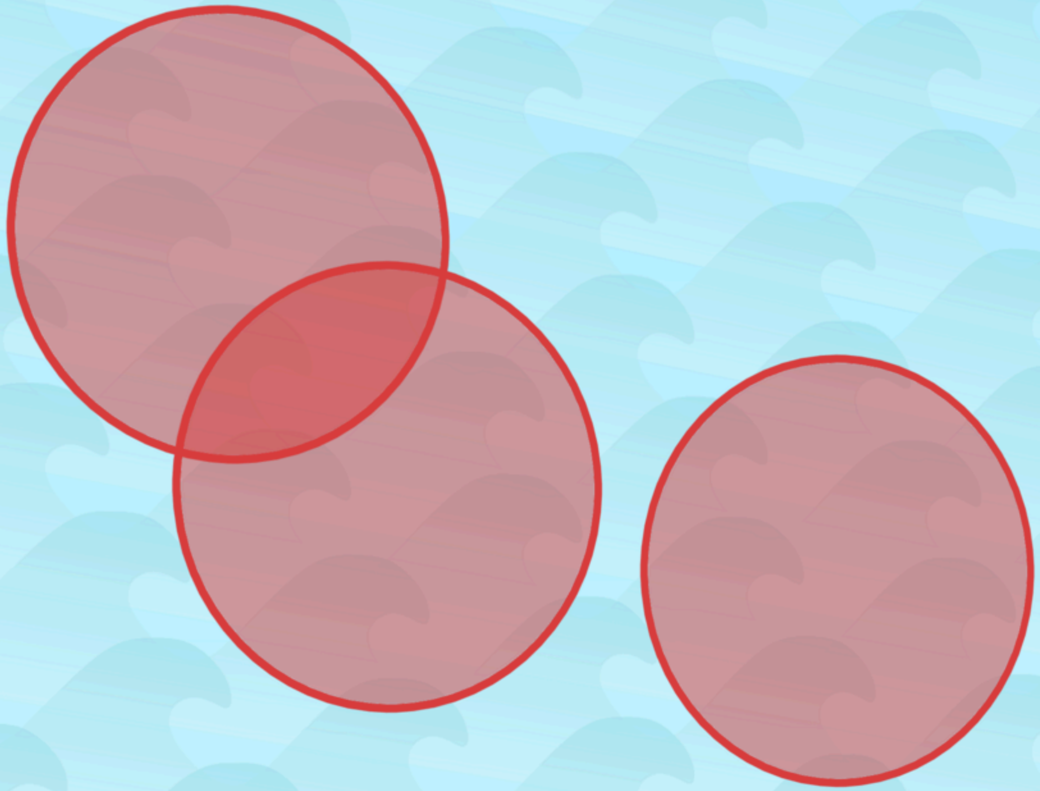

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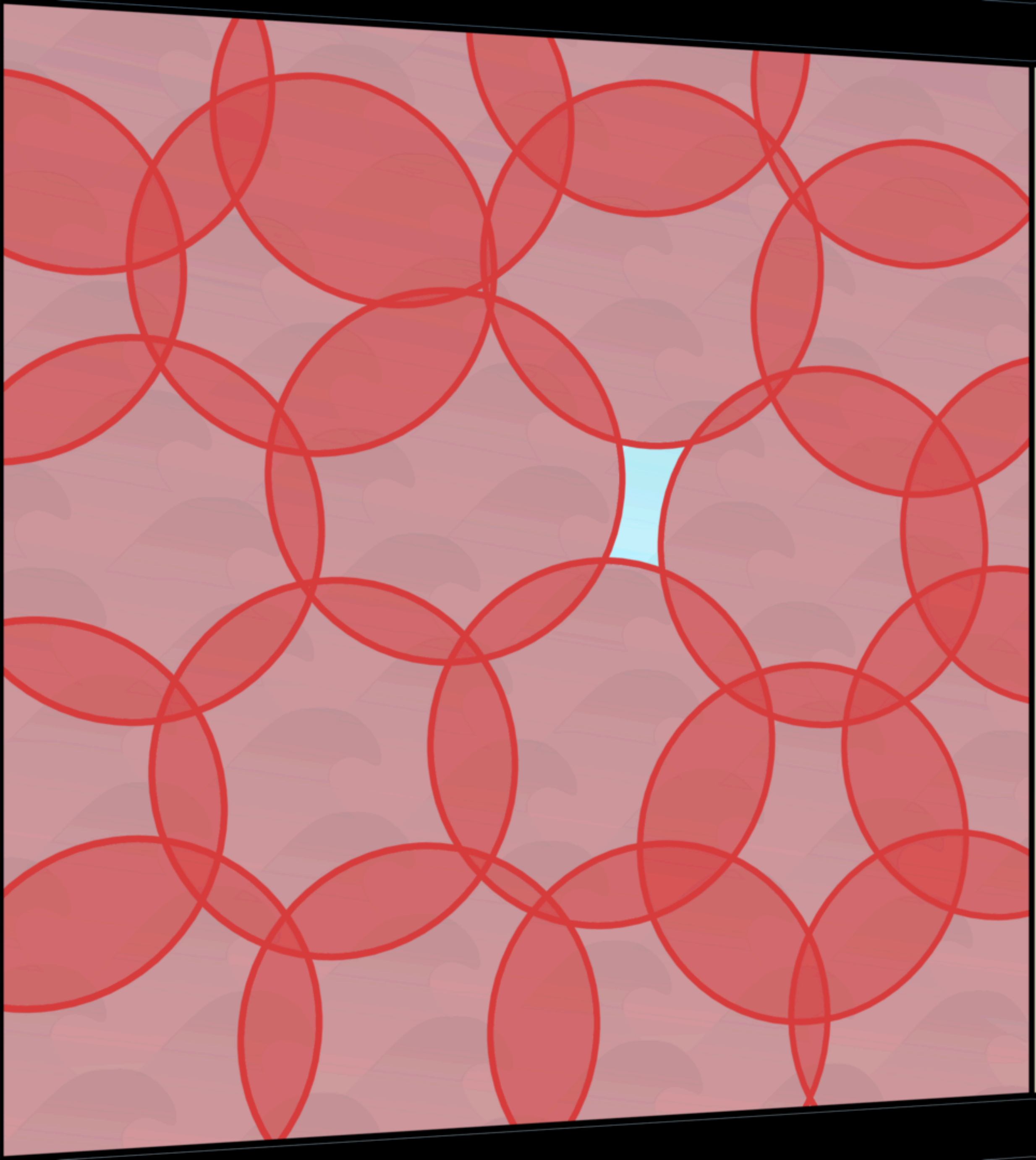

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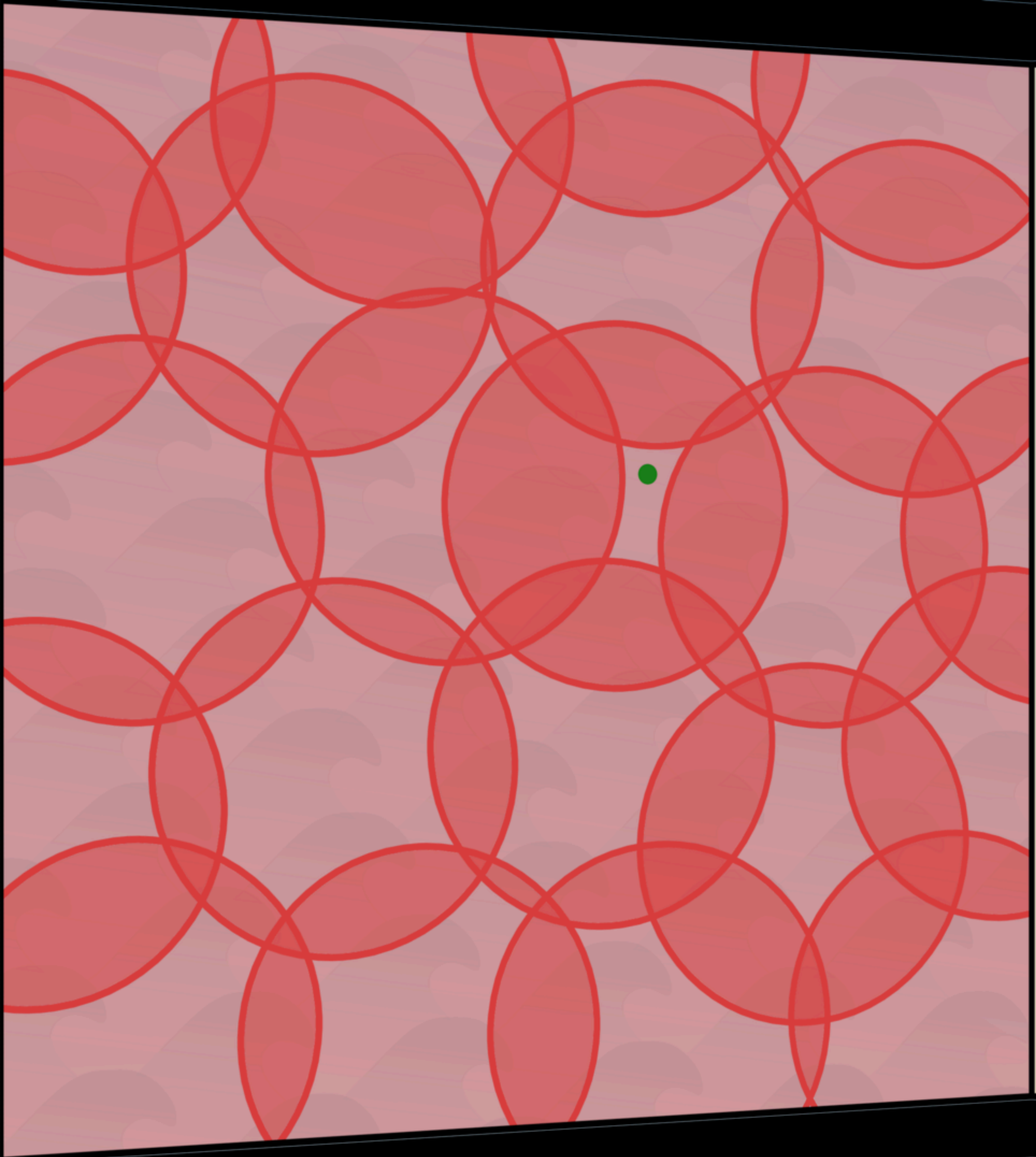

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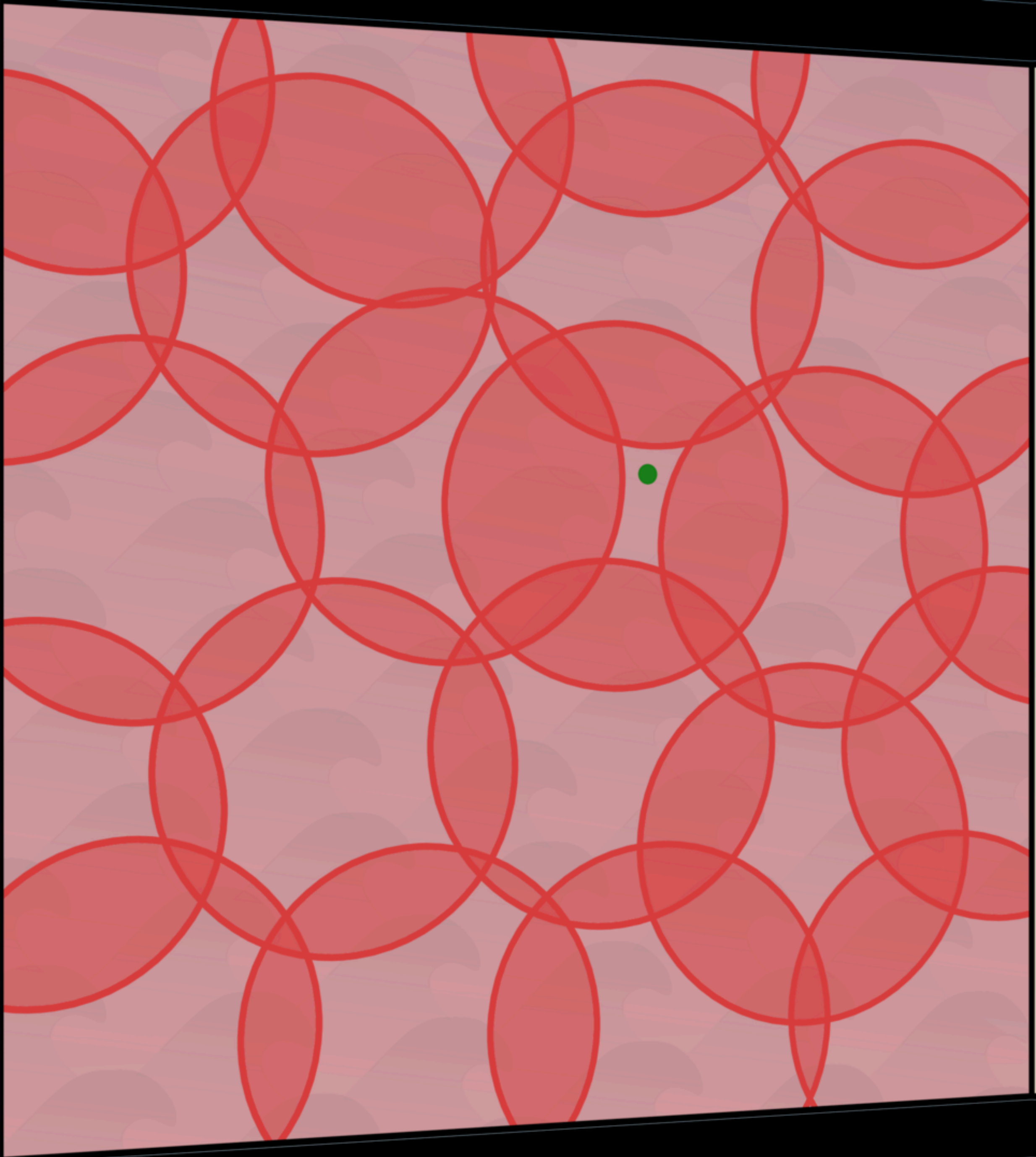

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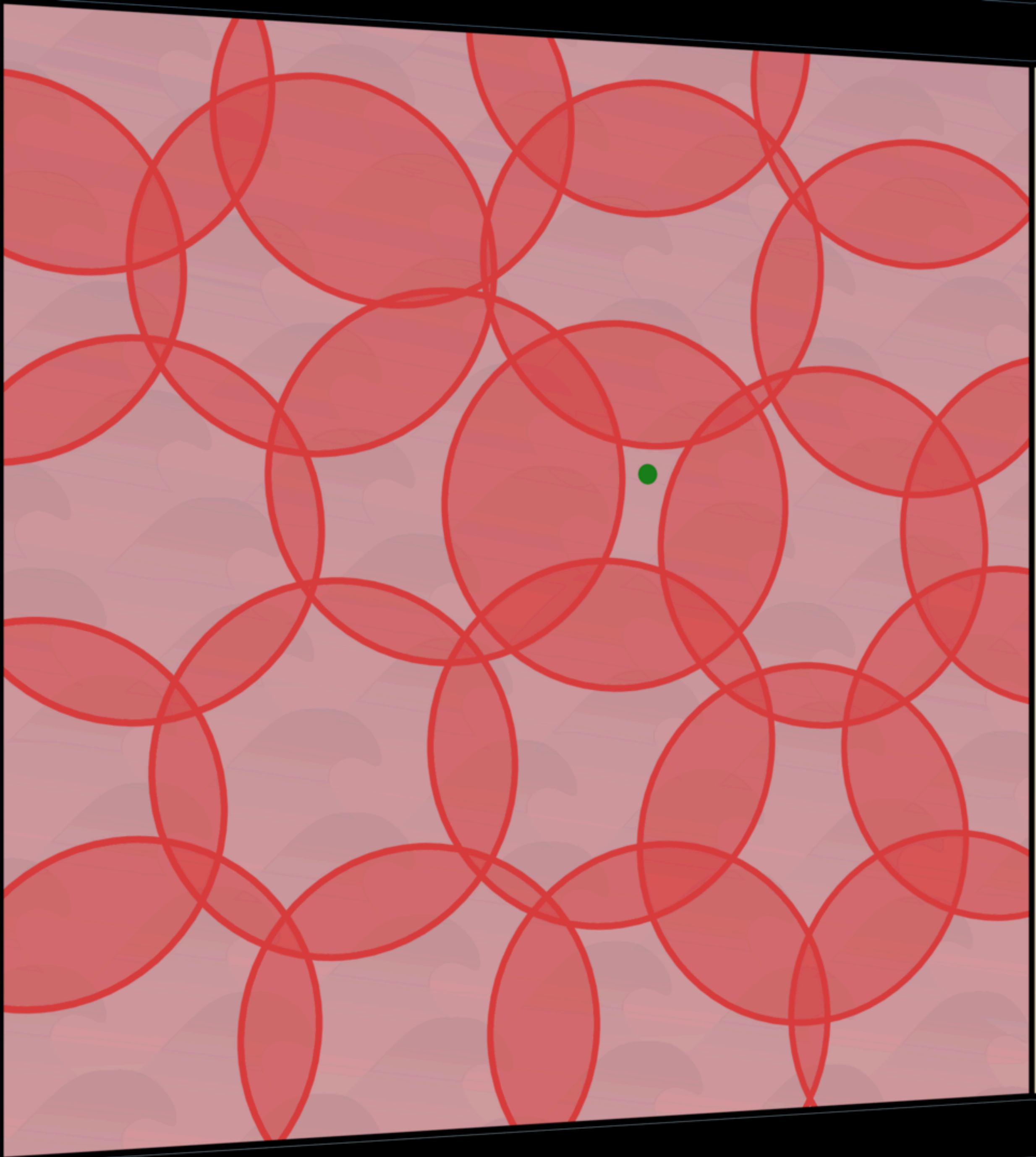

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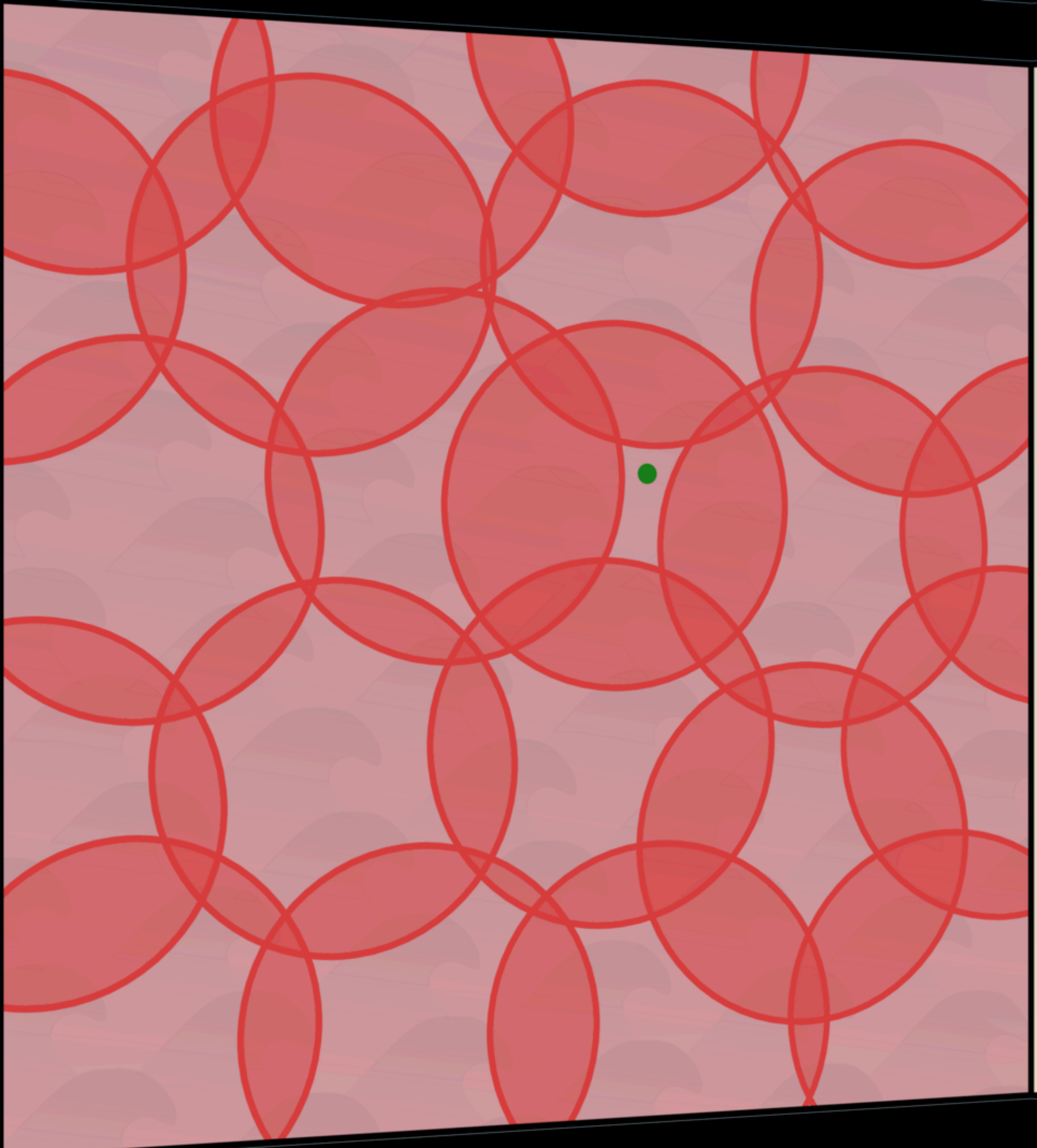

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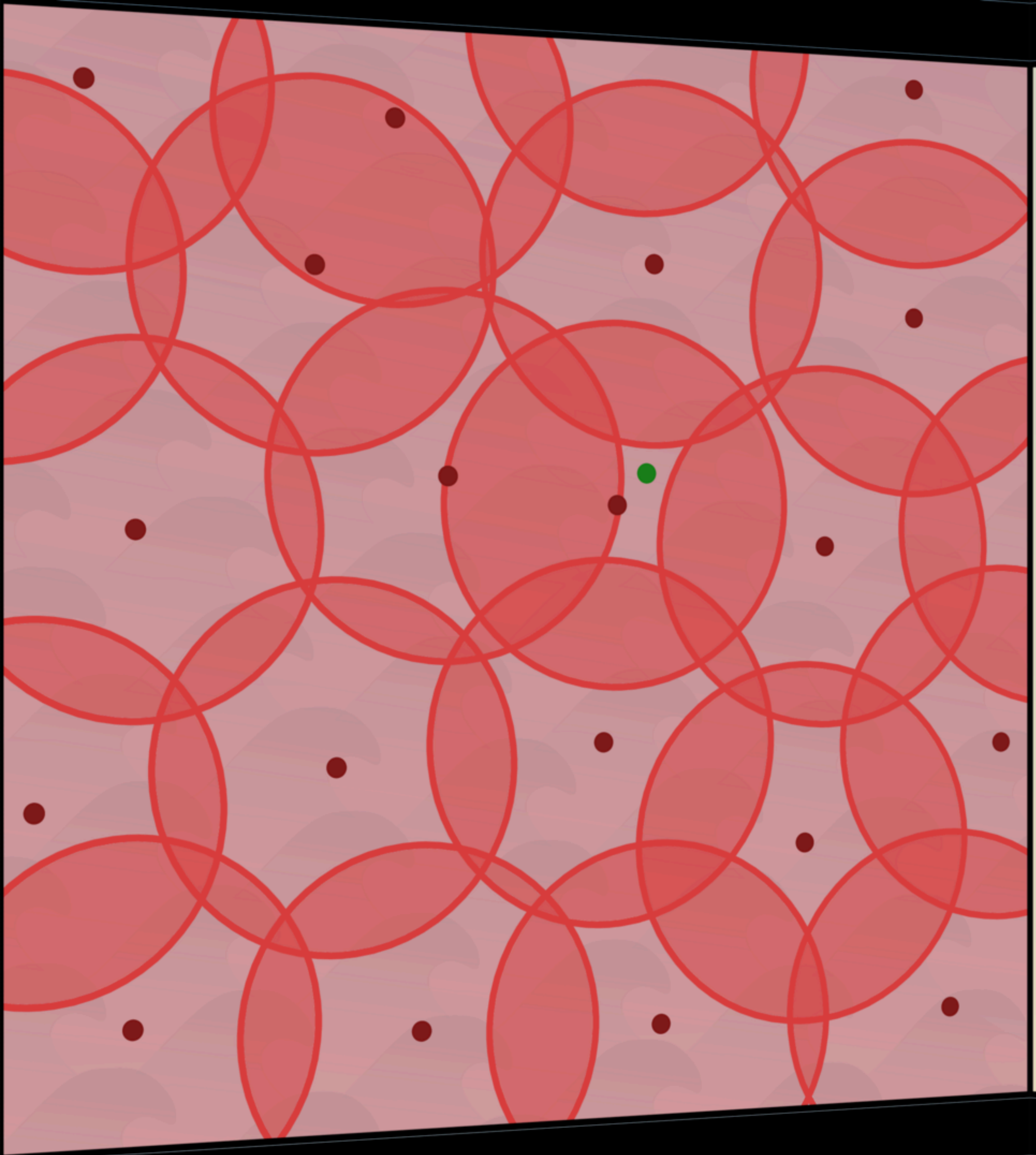

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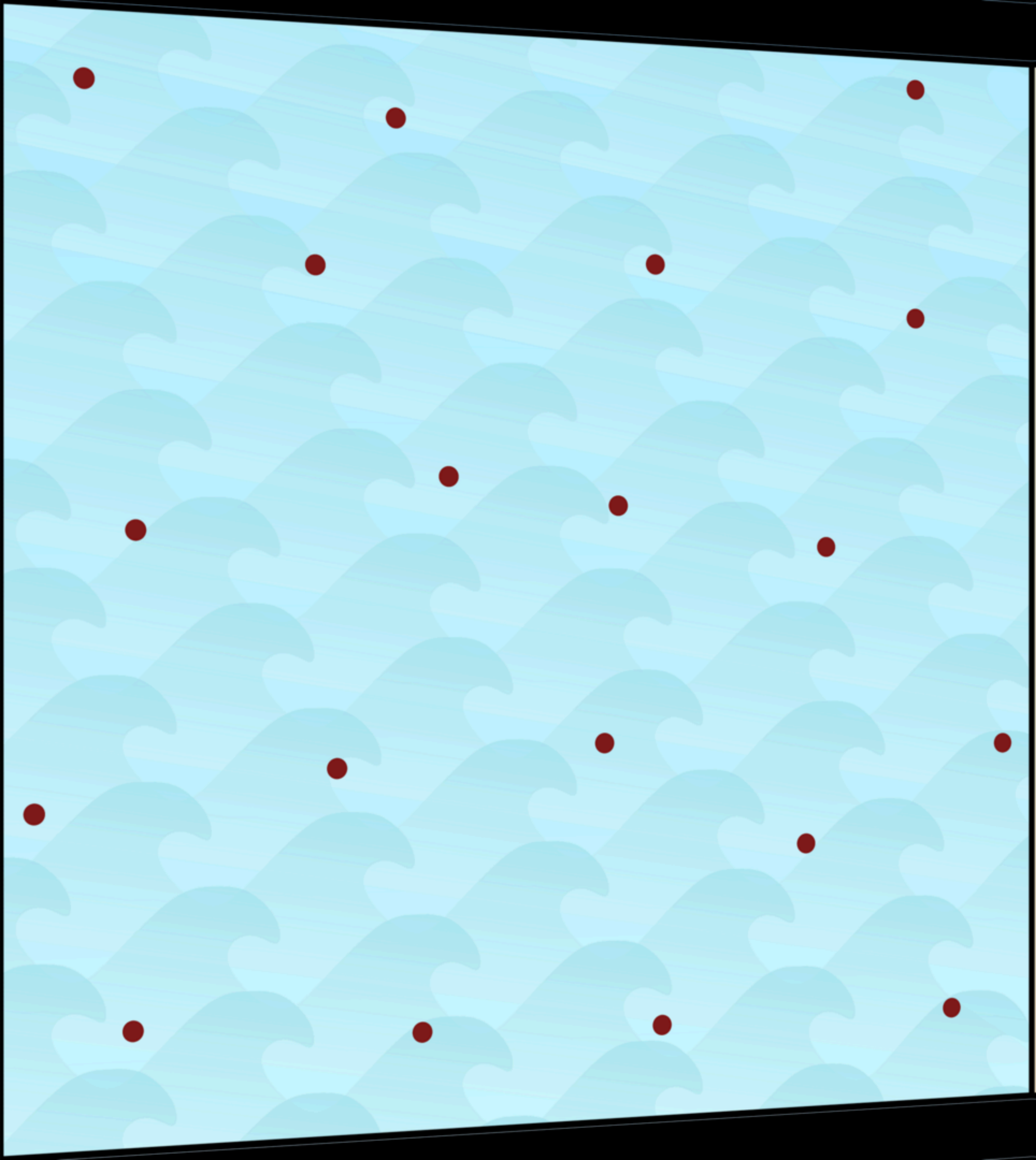

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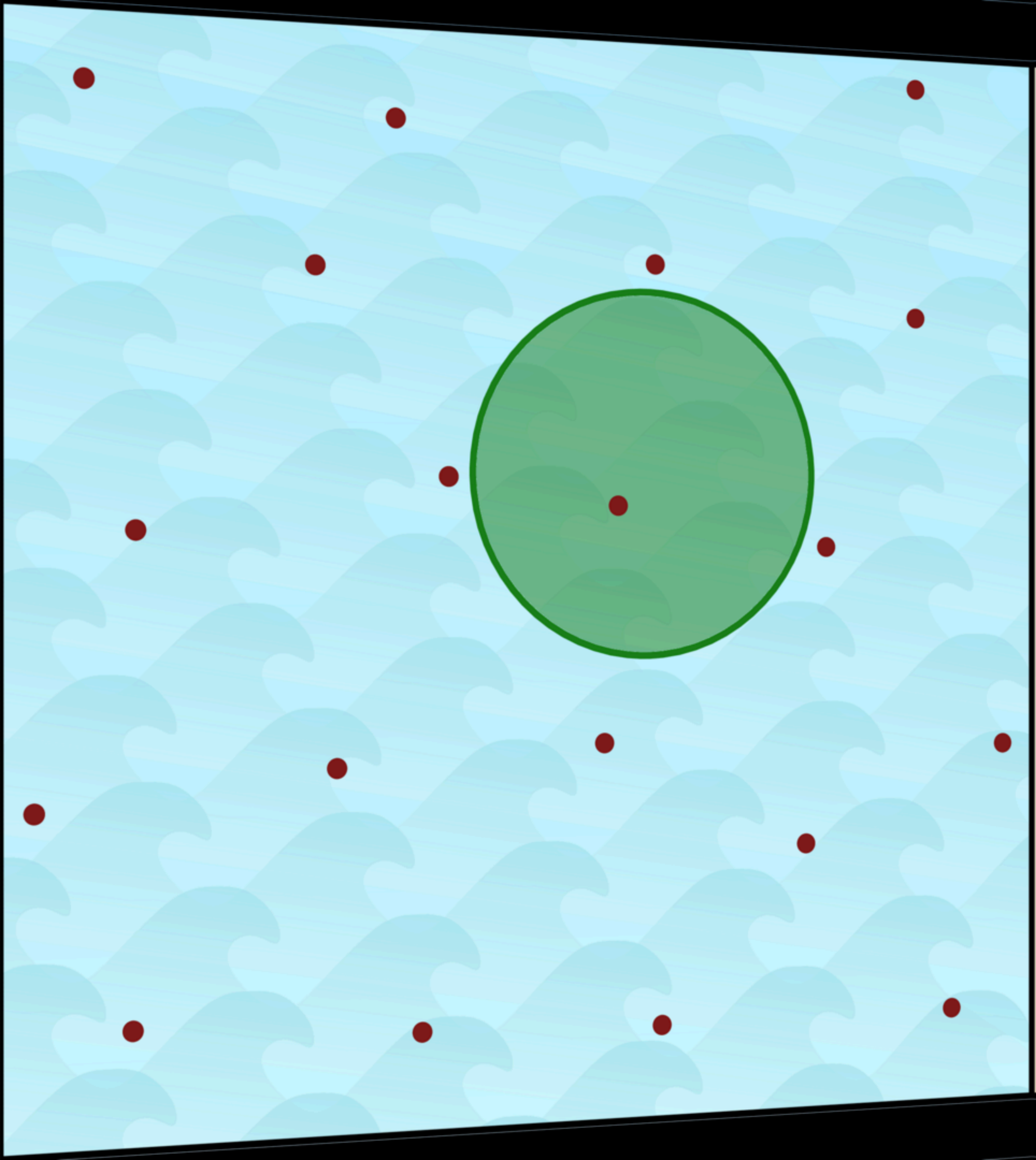

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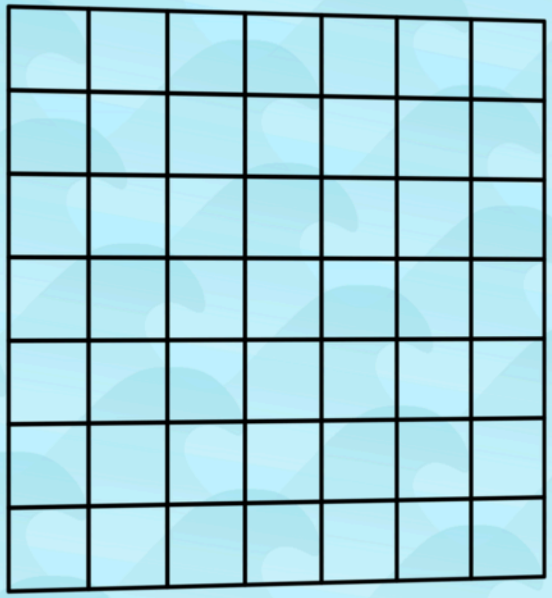
CHAPTER 2

BATTLESHIPS



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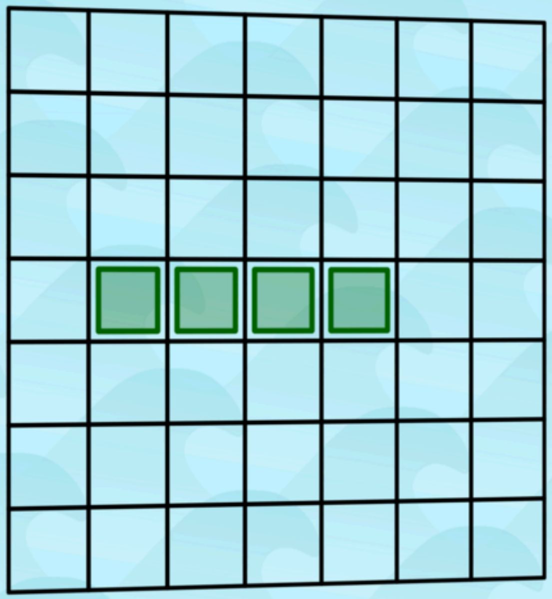

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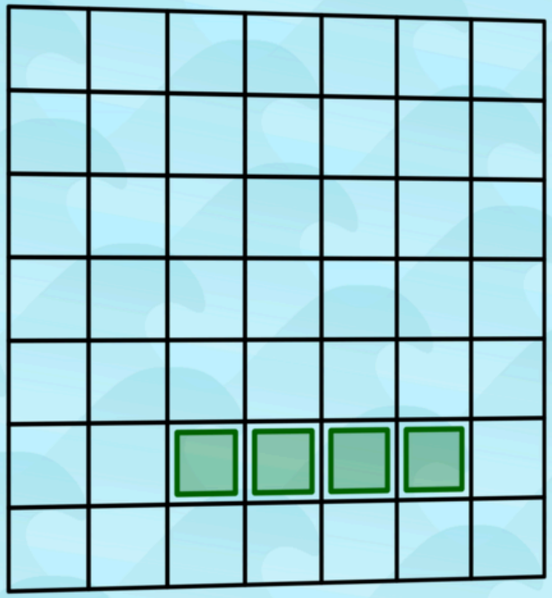

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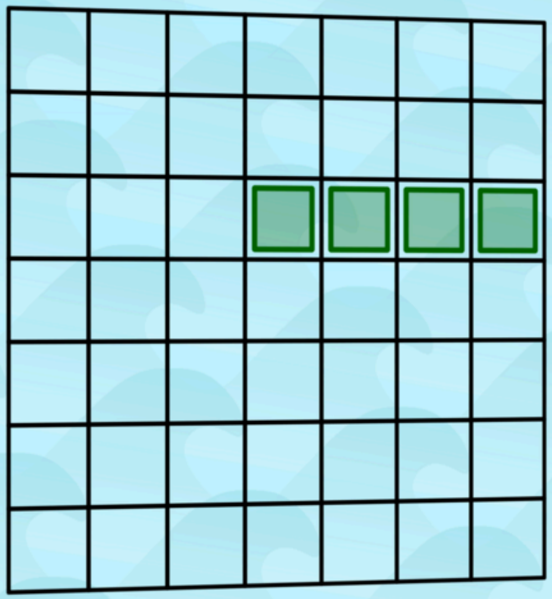

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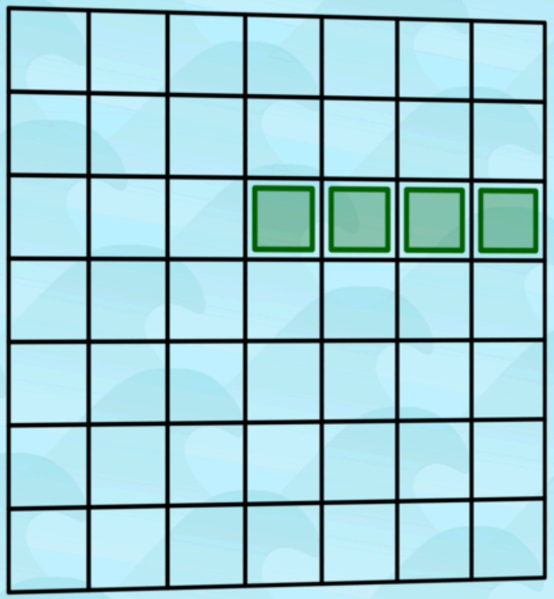

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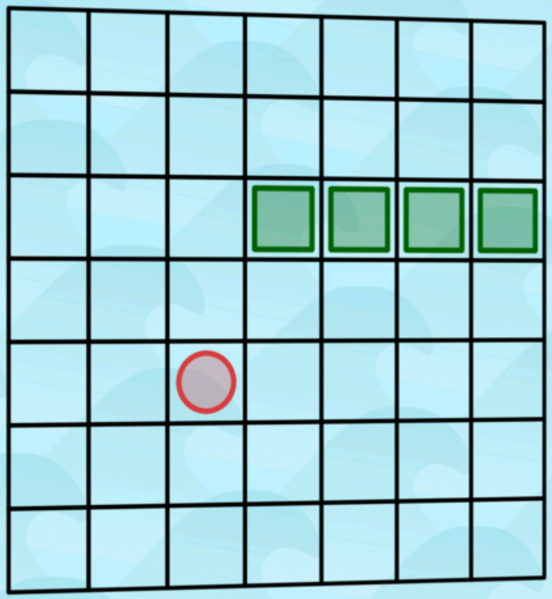

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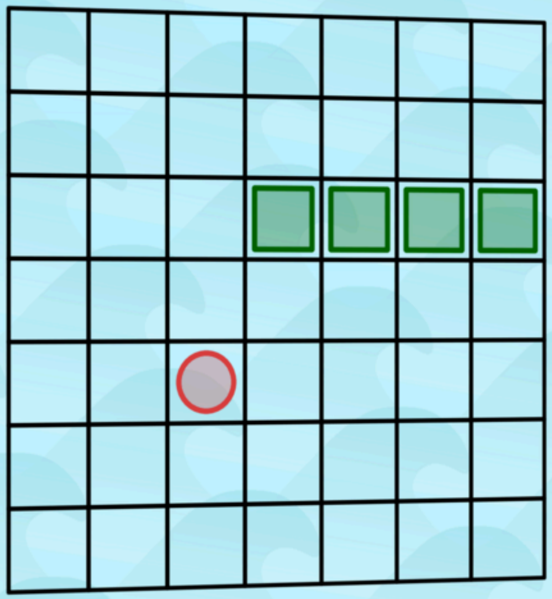

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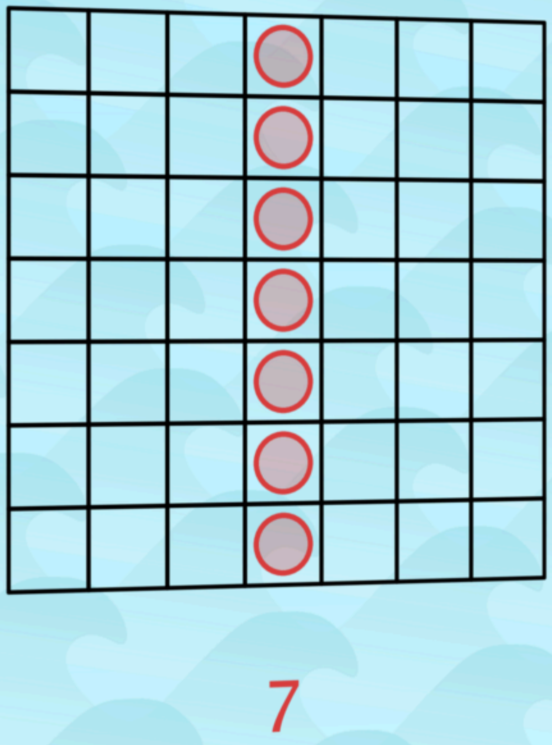

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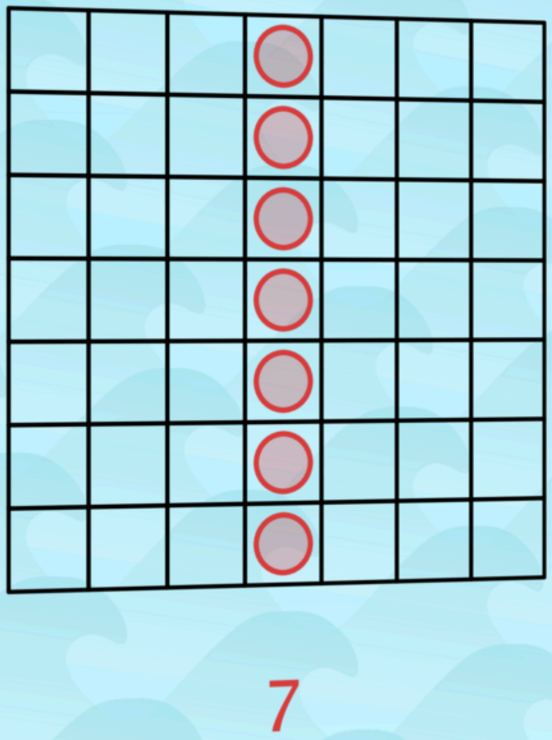

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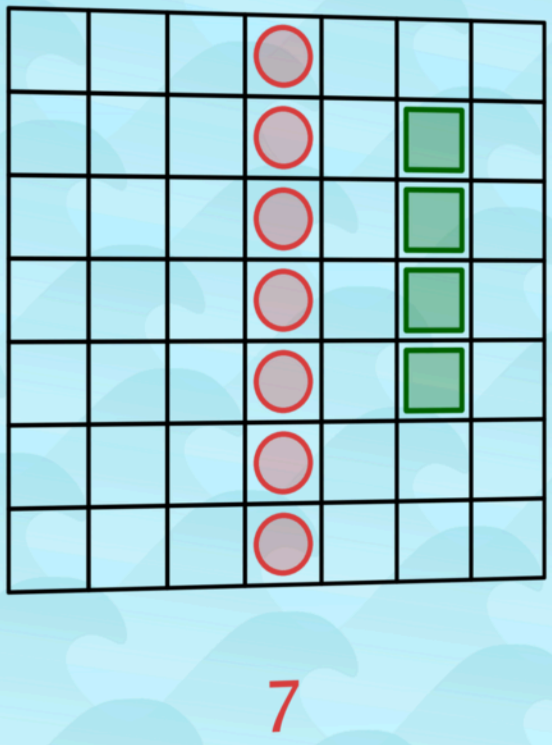

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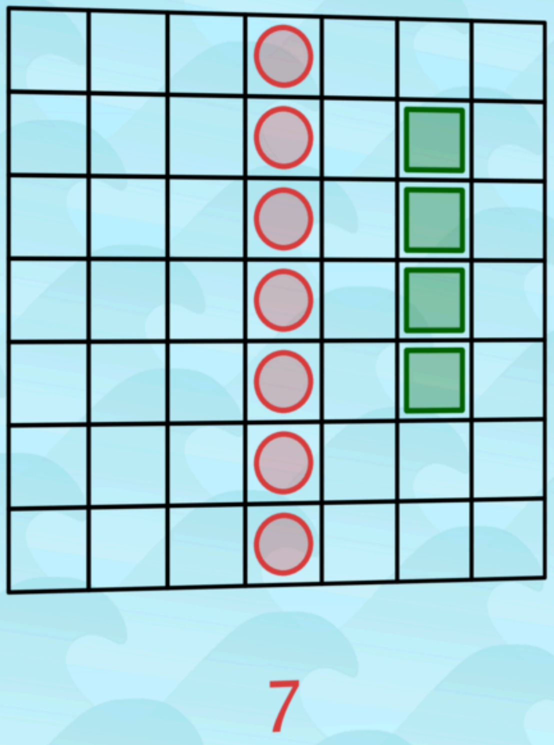

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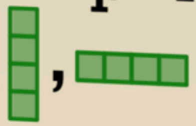
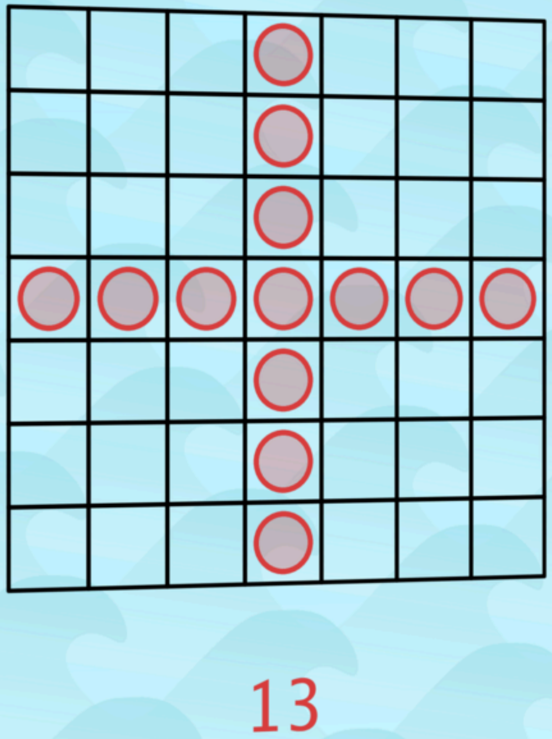
If so, we say the ship has *unknown shape*: S={}



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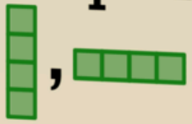
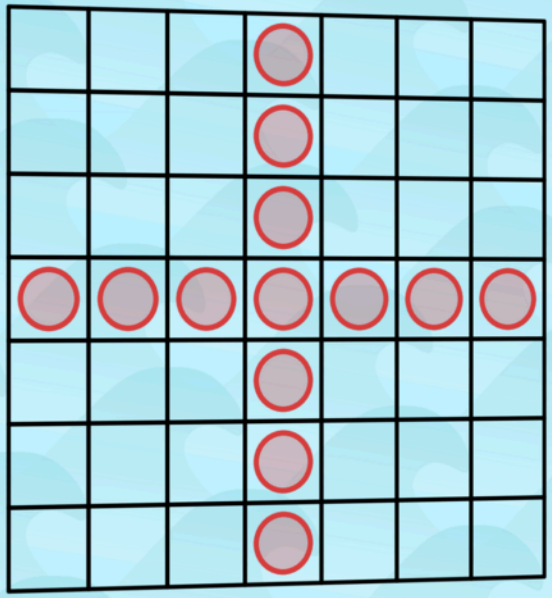
If so, we say the ship has *unknown shape*: S={  }



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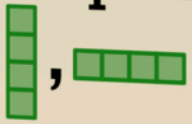
If so, we say the ship has *unknown shape*: S={  }

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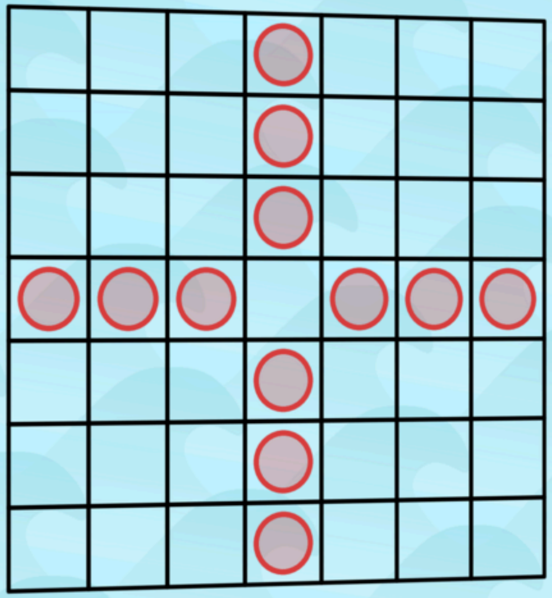

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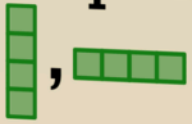
If so, we say the ship has *unknown shape*: S={  }

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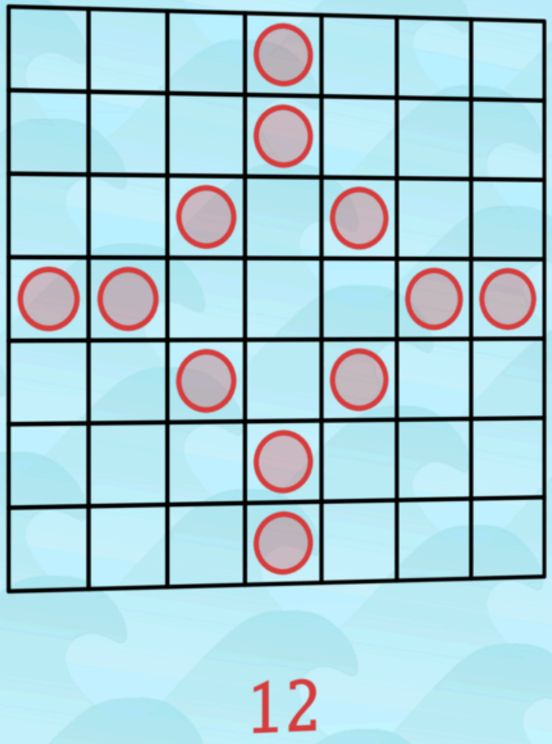

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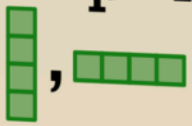
If so, we say the ship has *unknown shape*: S={  }

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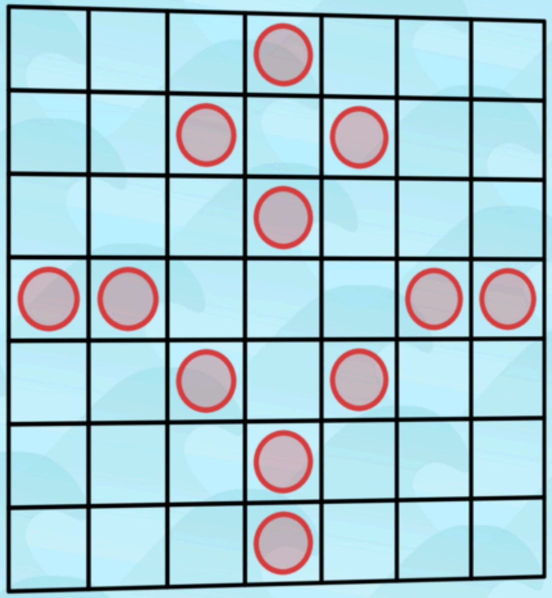

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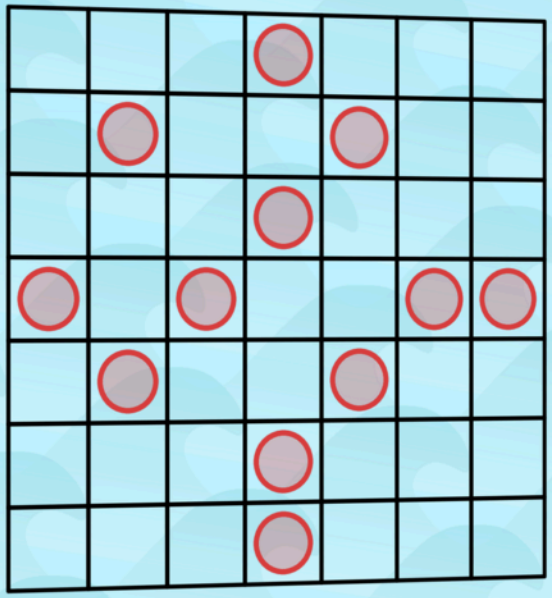

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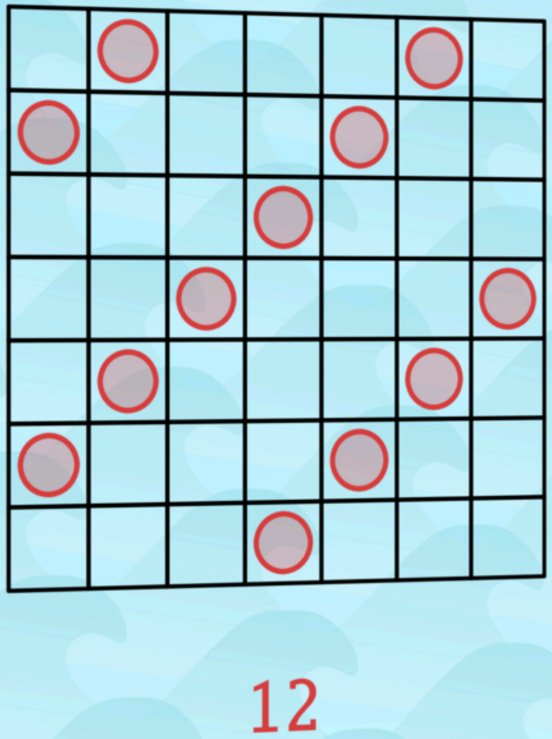

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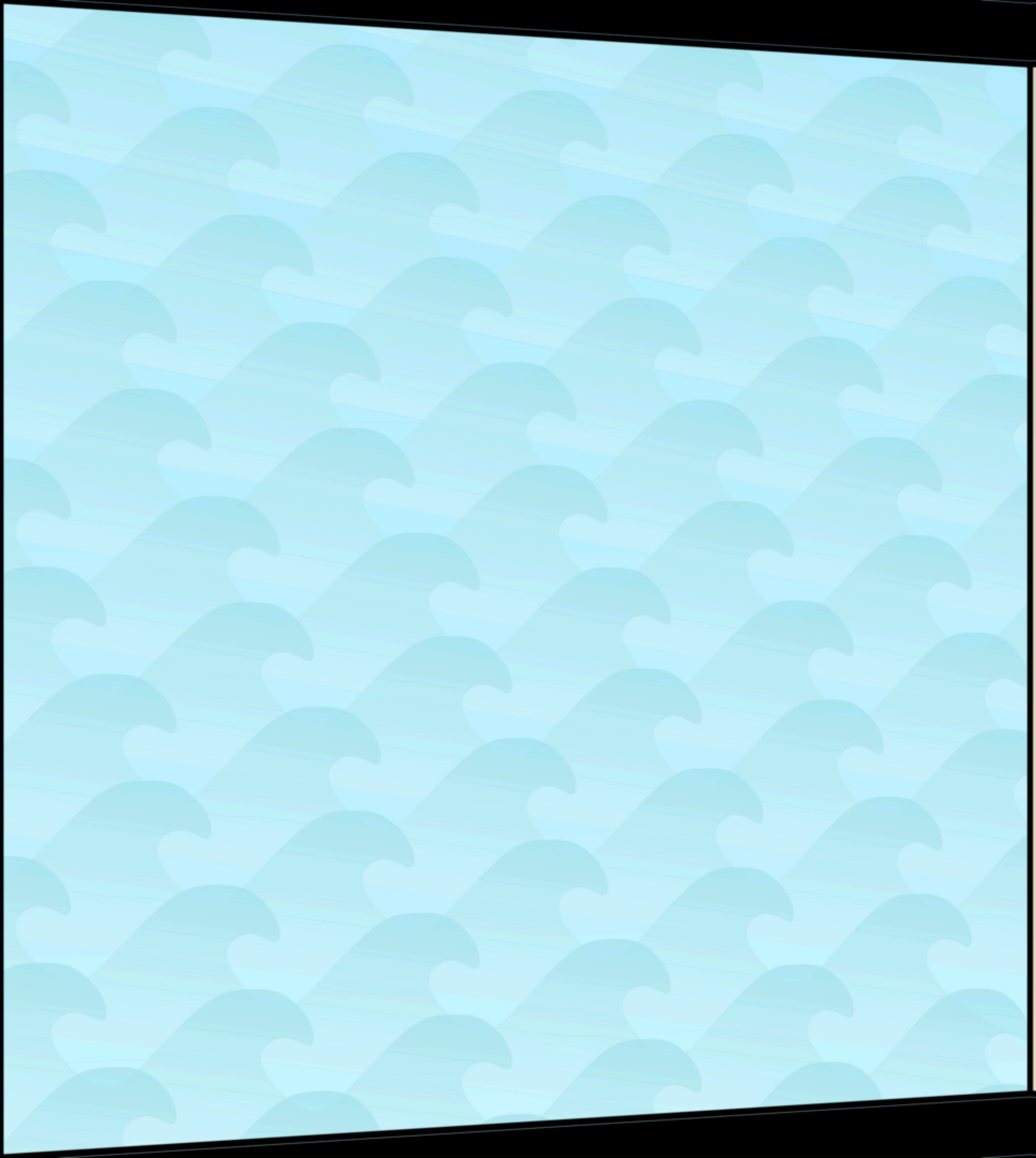
CHAPTER 3

DENSITY



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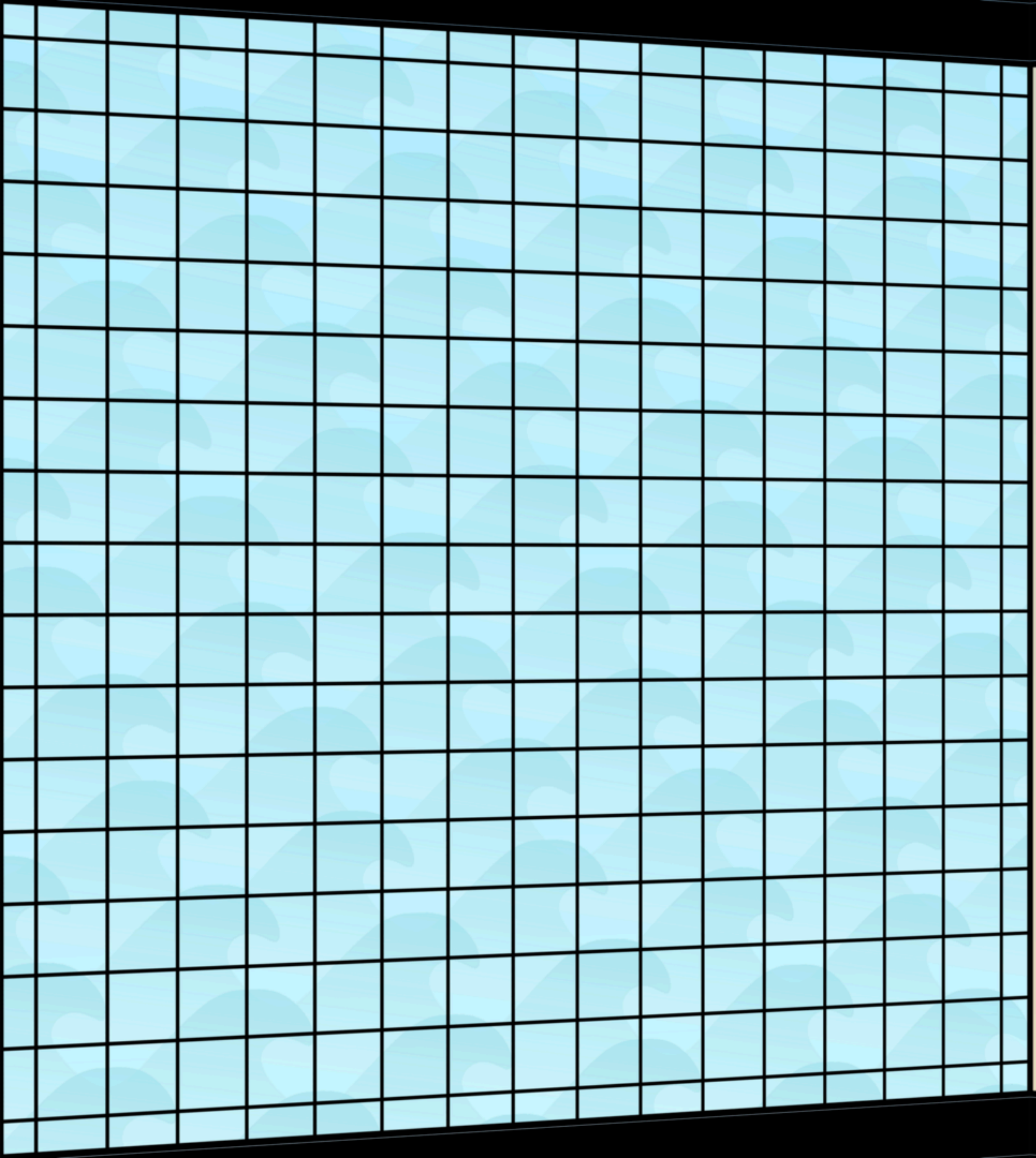

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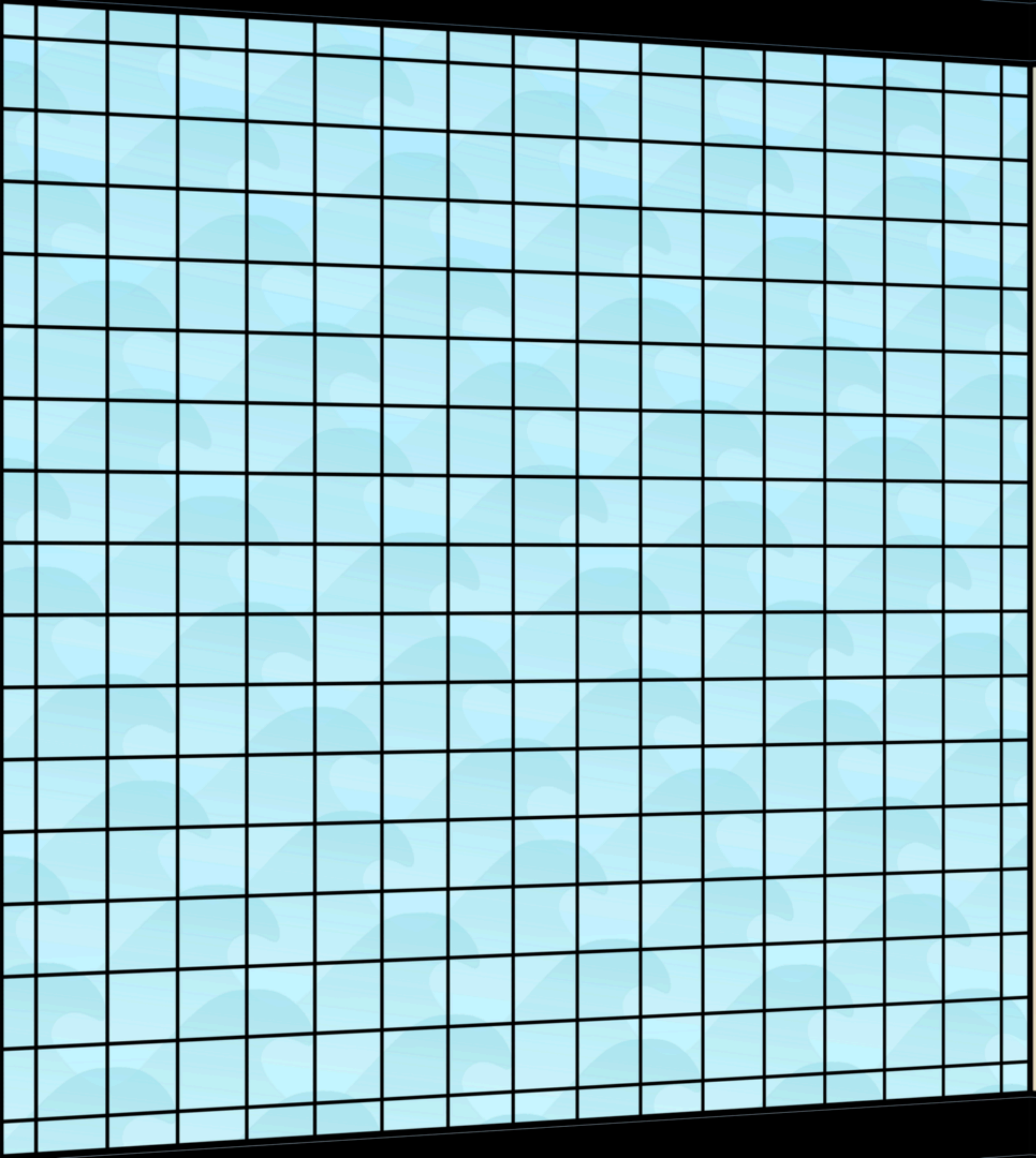

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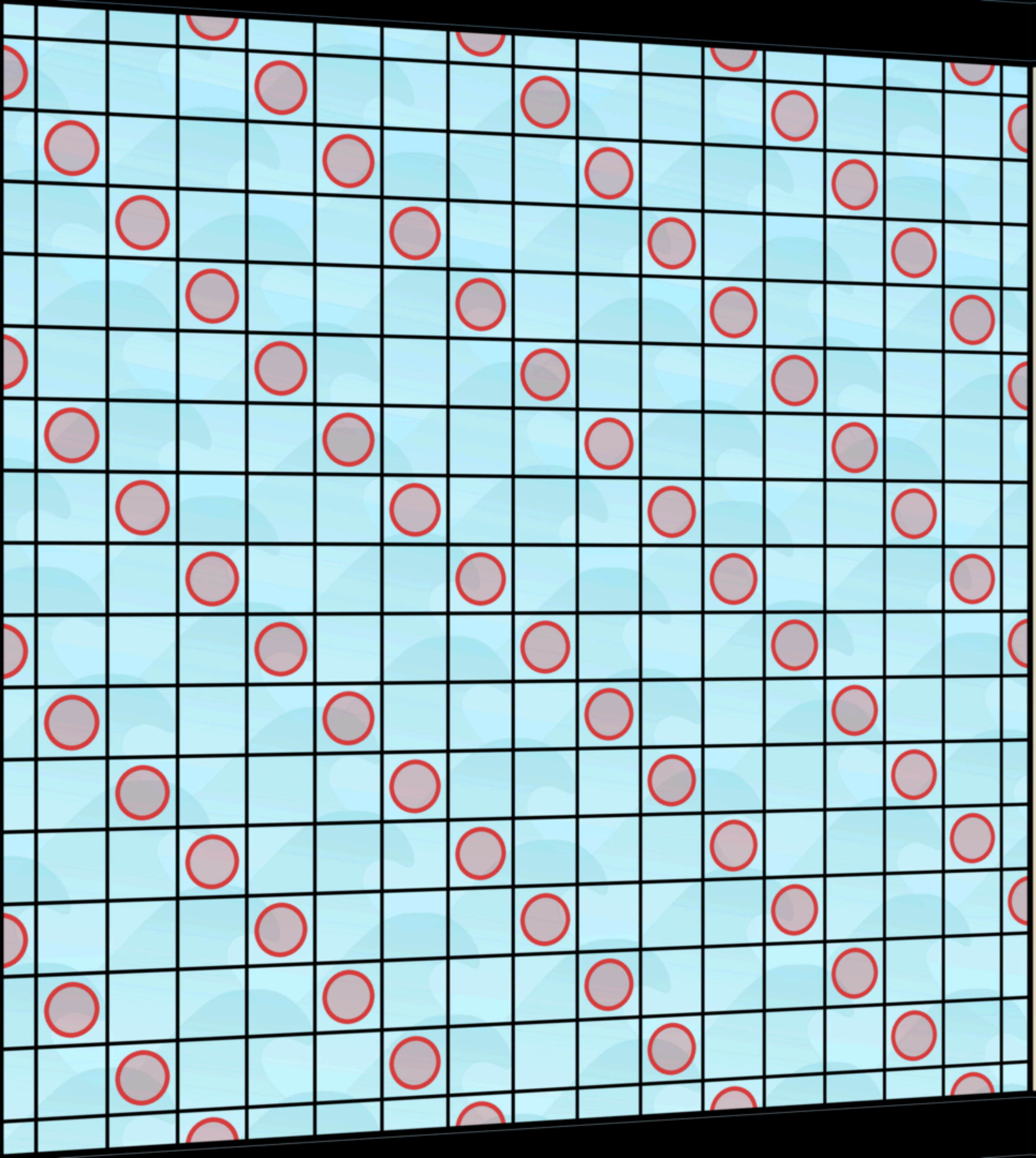

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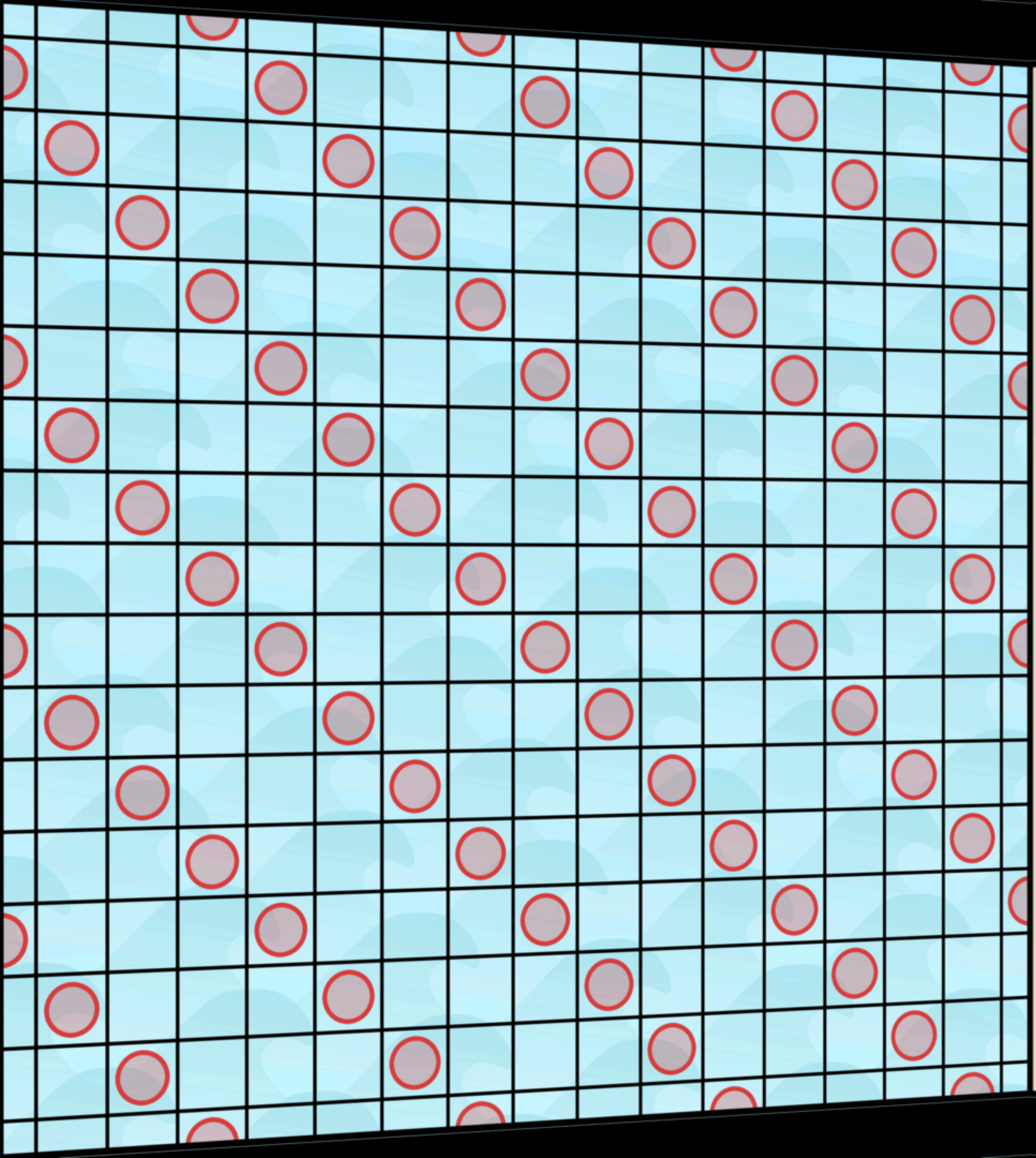

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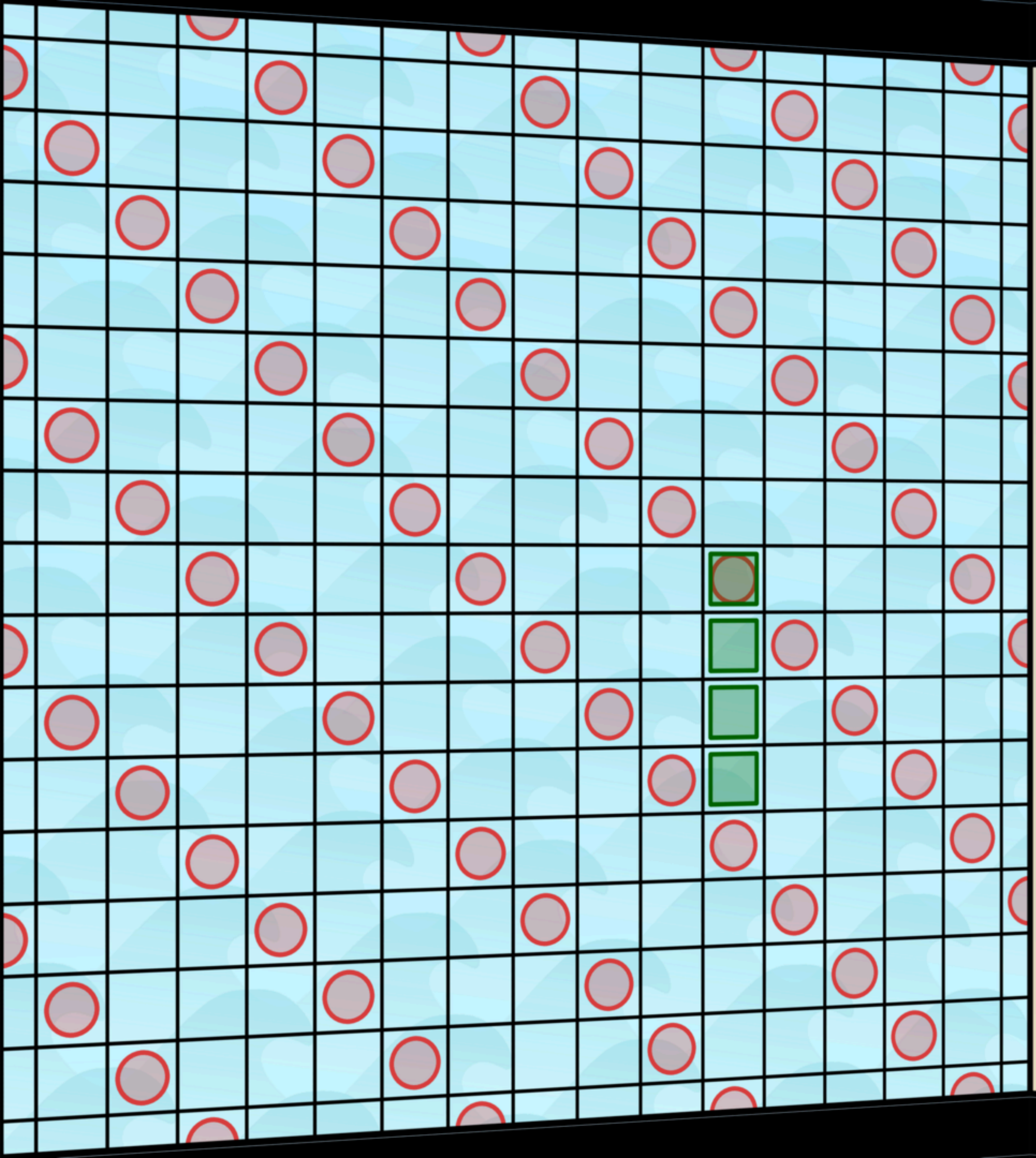

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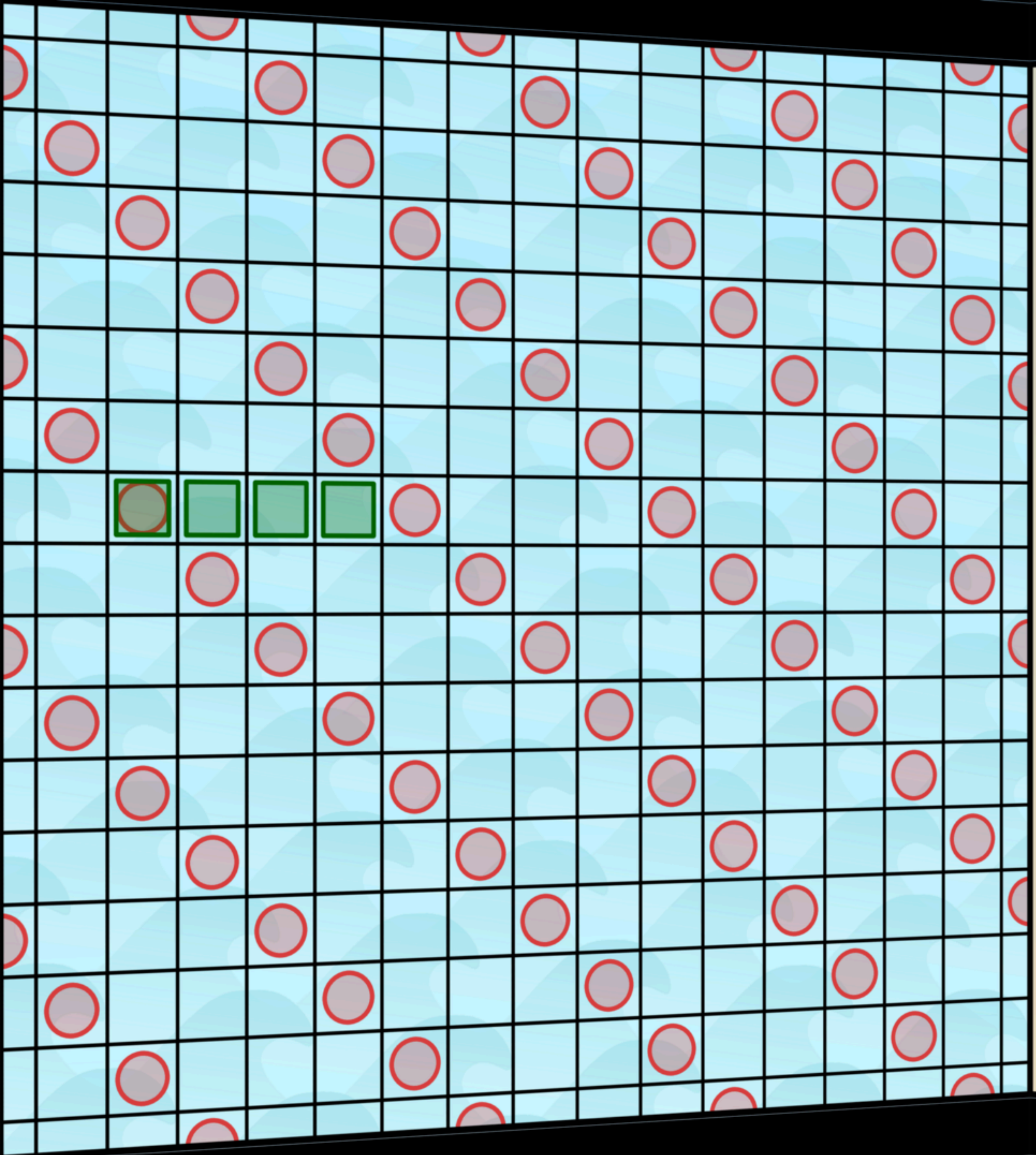

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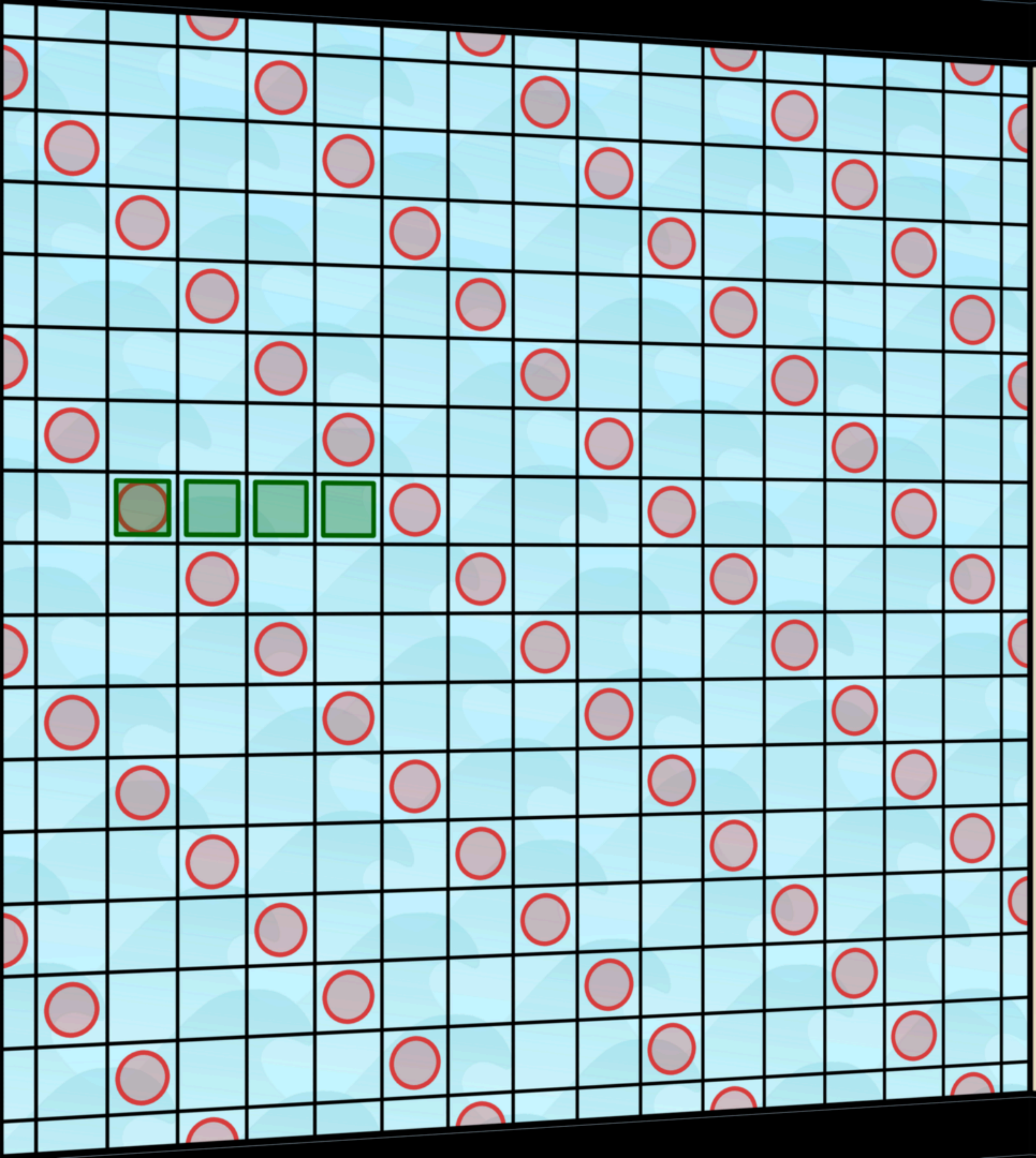

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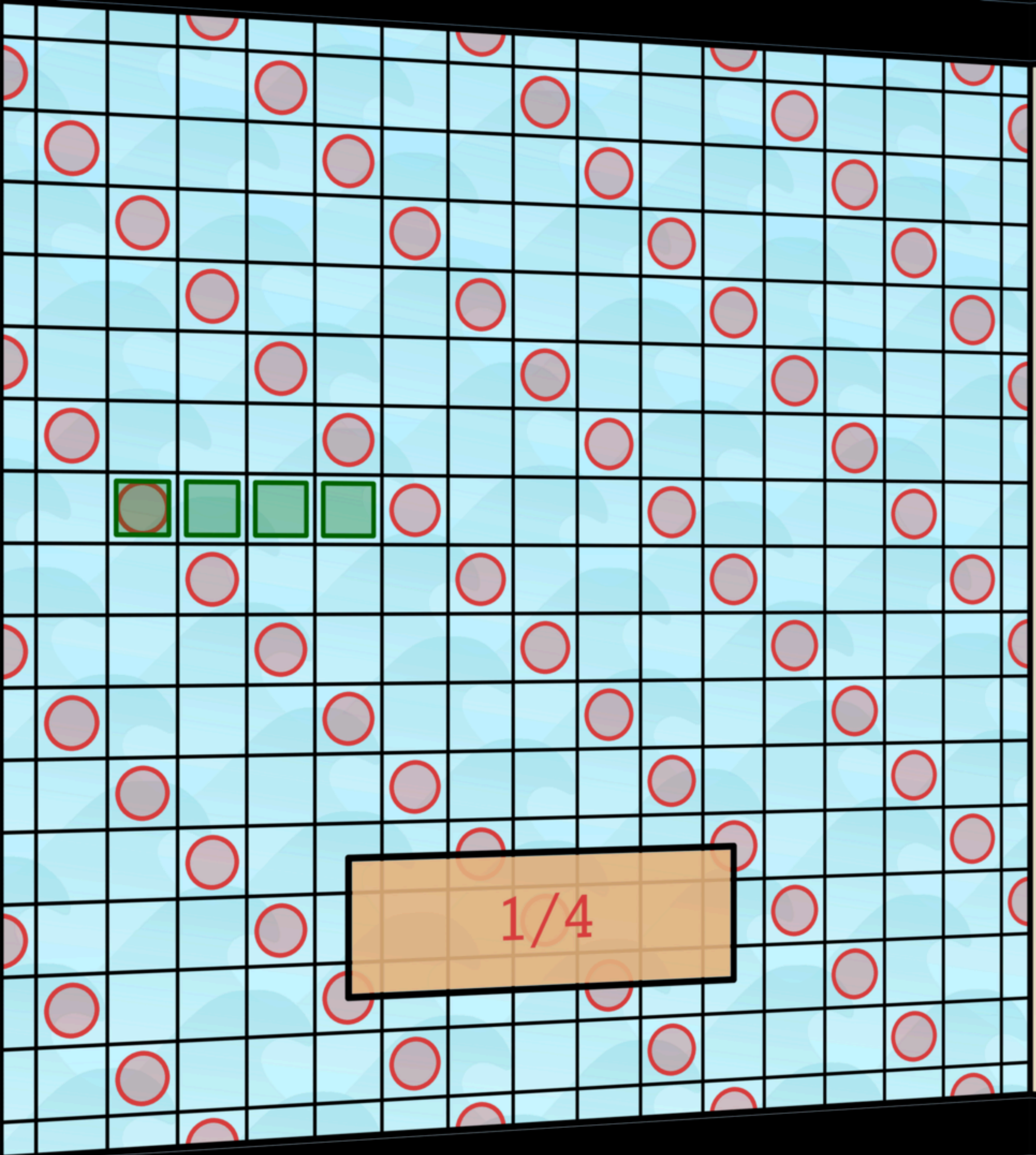

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What is the best pattern to find S={,}?

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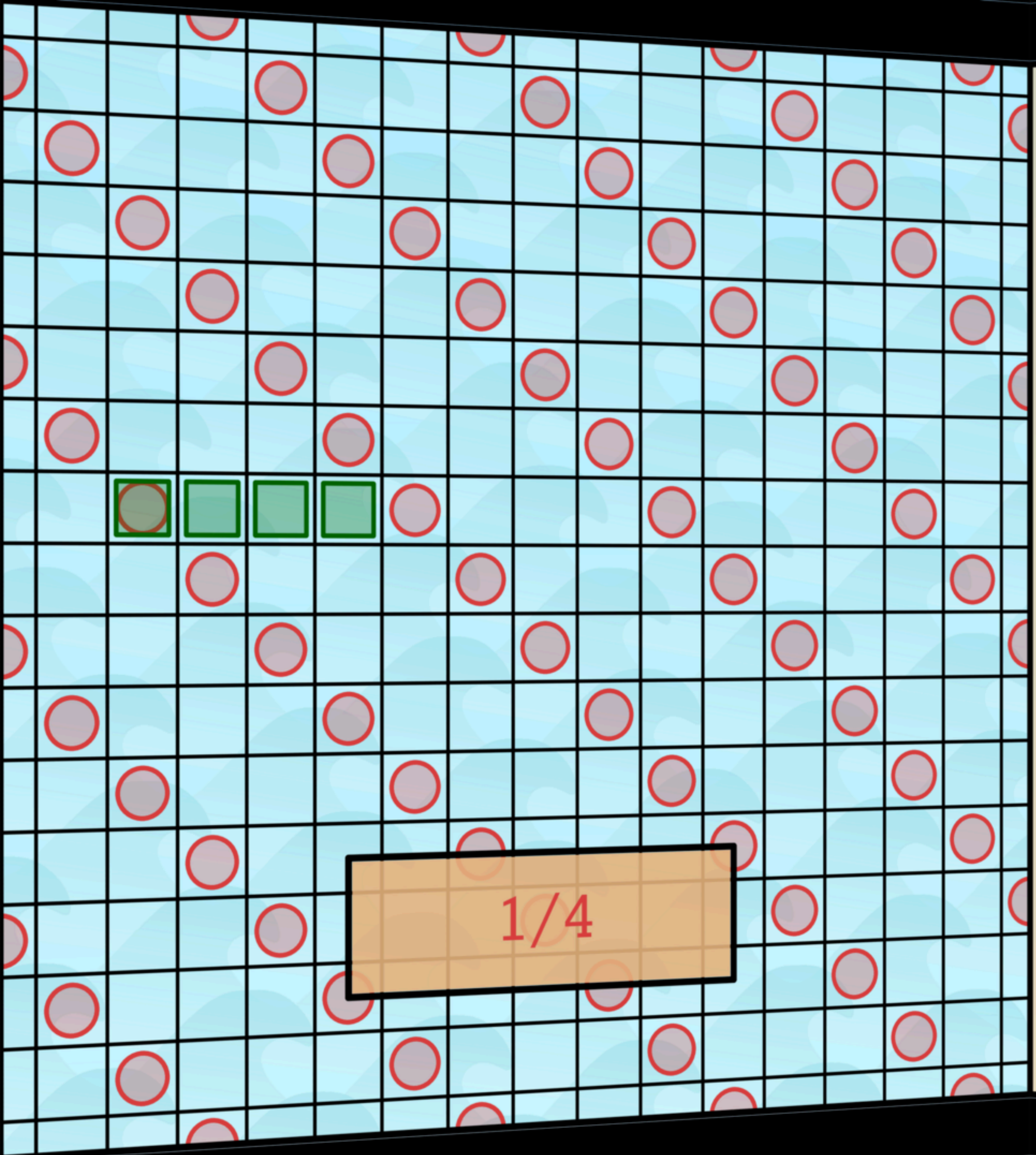

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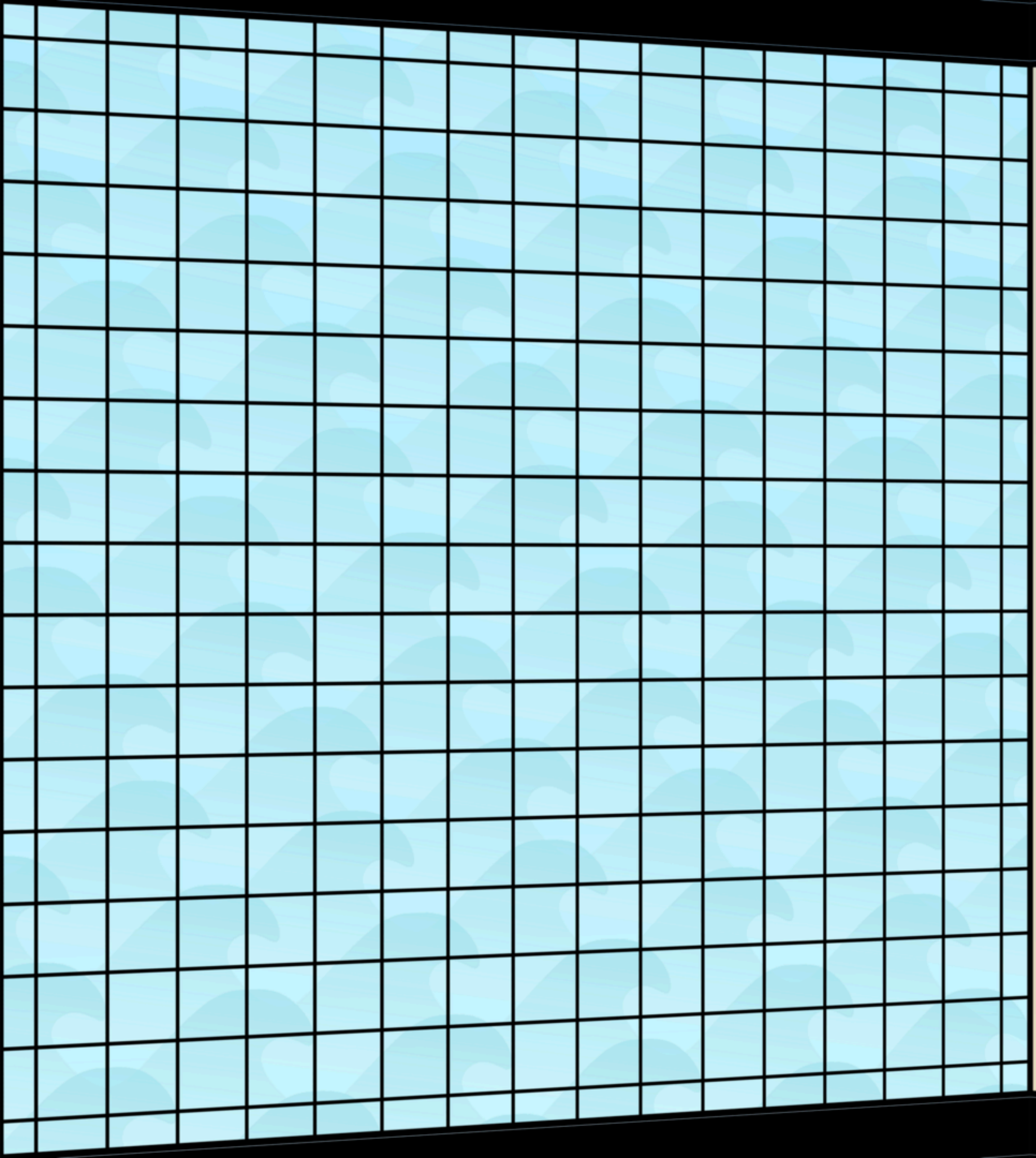
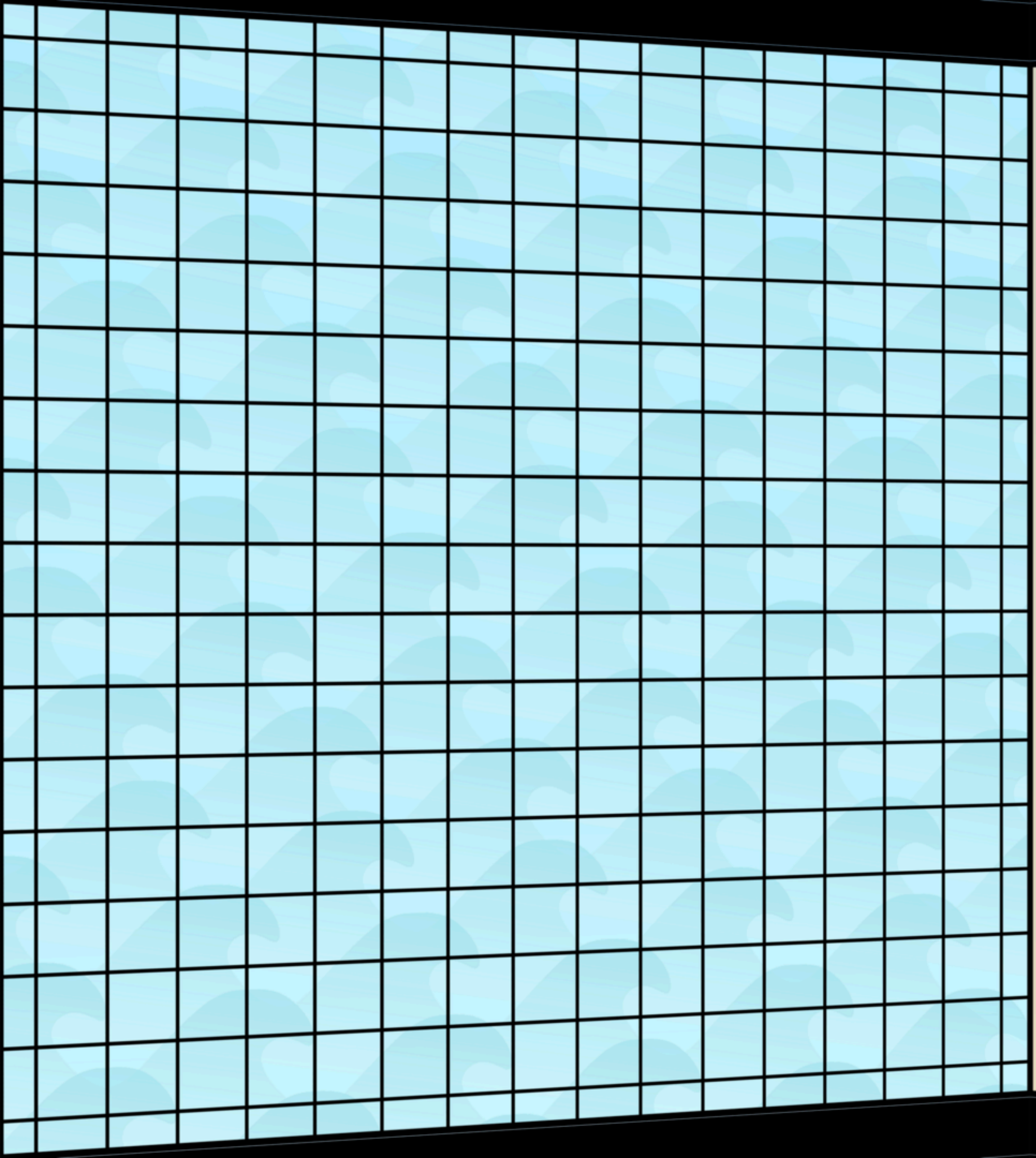
Note that the *same* pattern works for S={}!

Does rotation matter?



Does rotation matter?

For traditional ships ($1 \times k$ rectangles) not.



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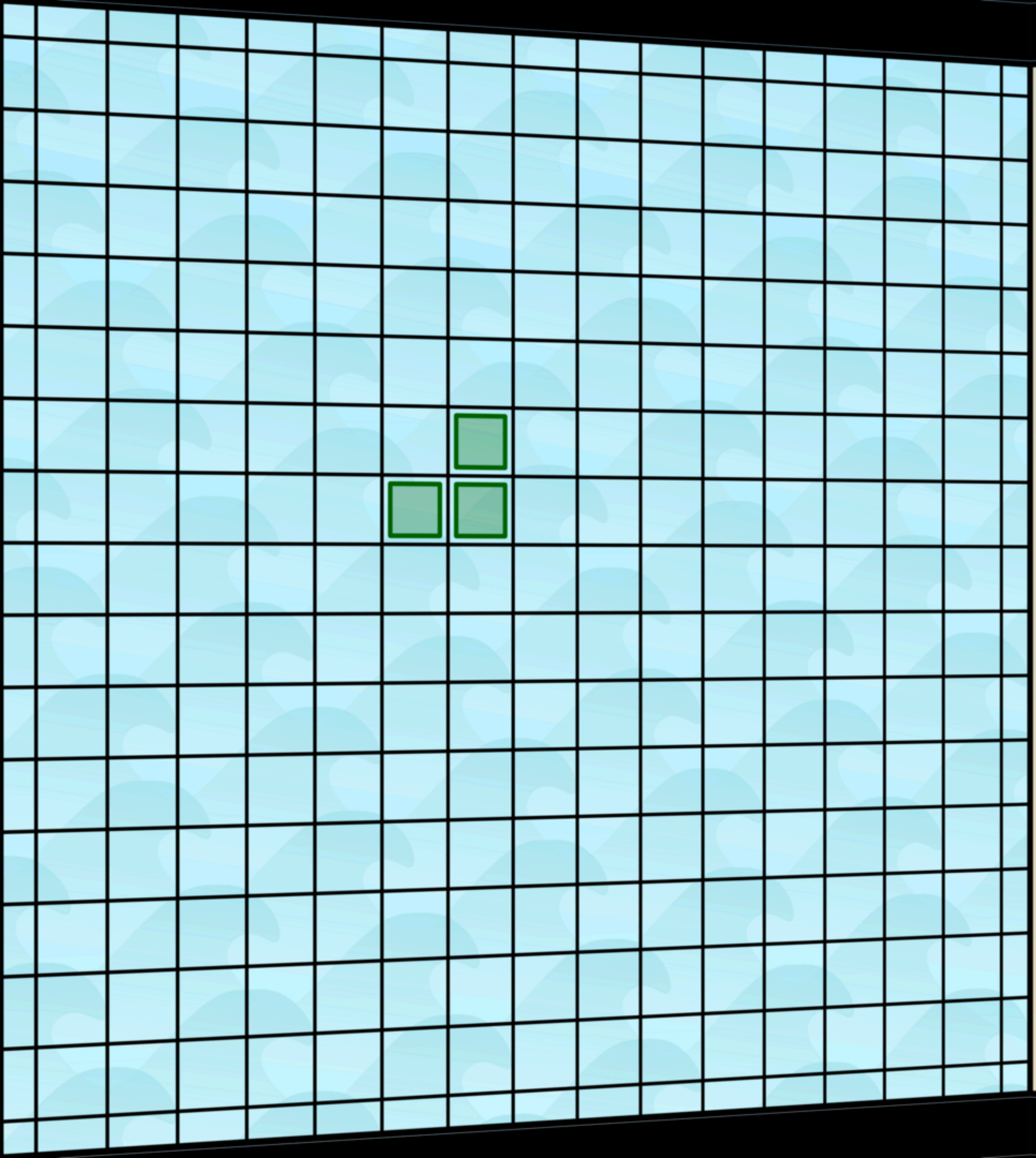

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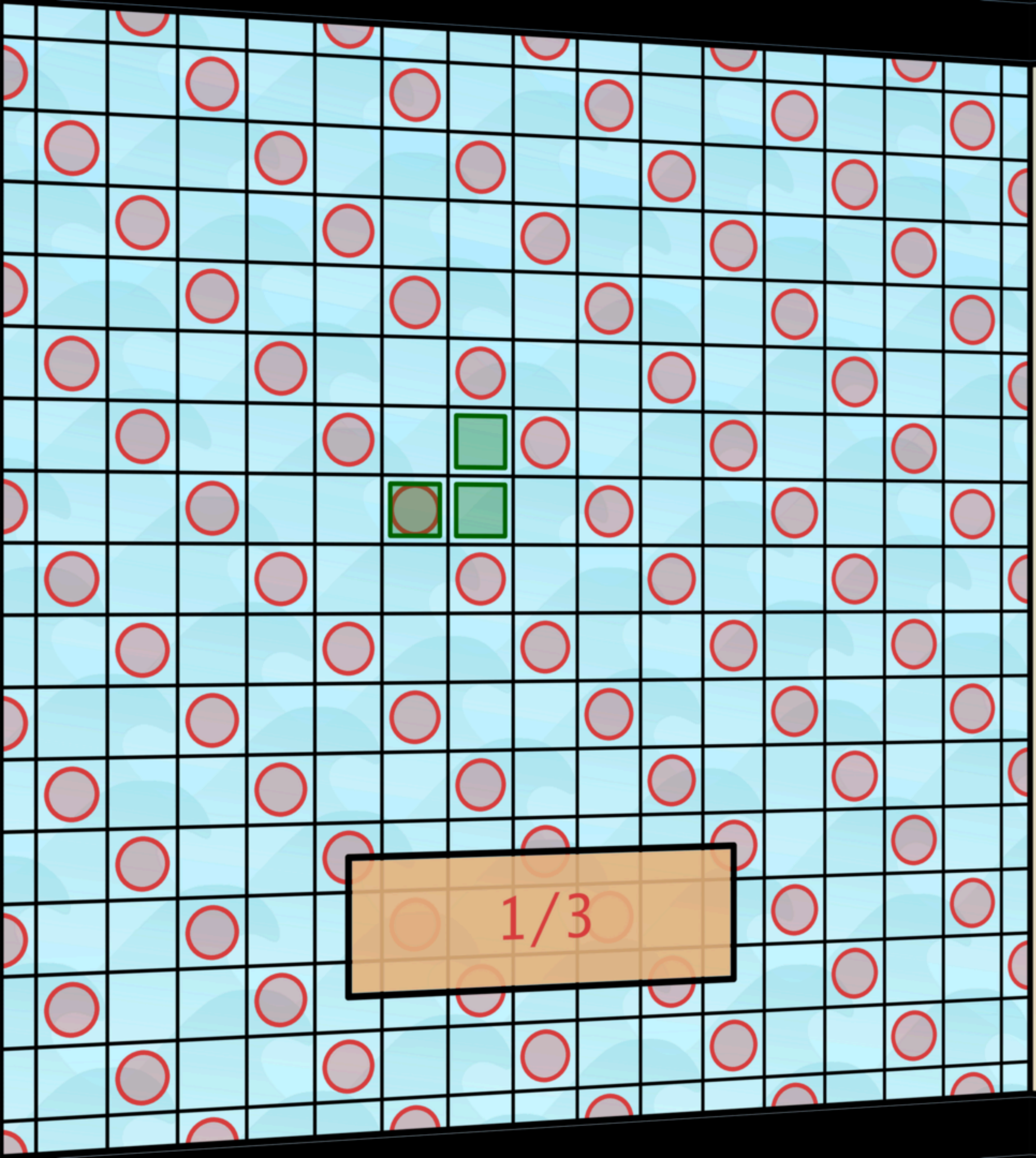

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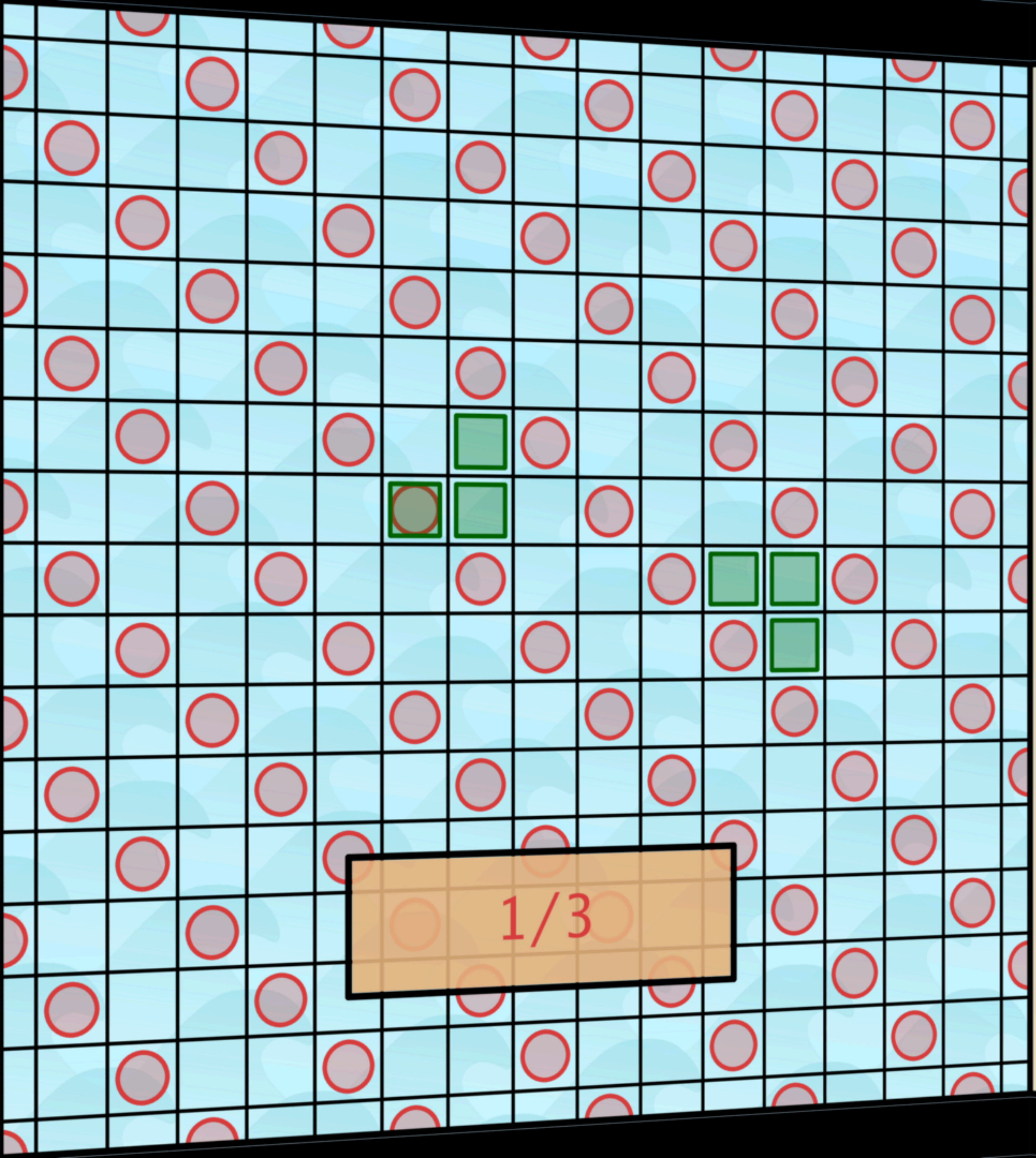

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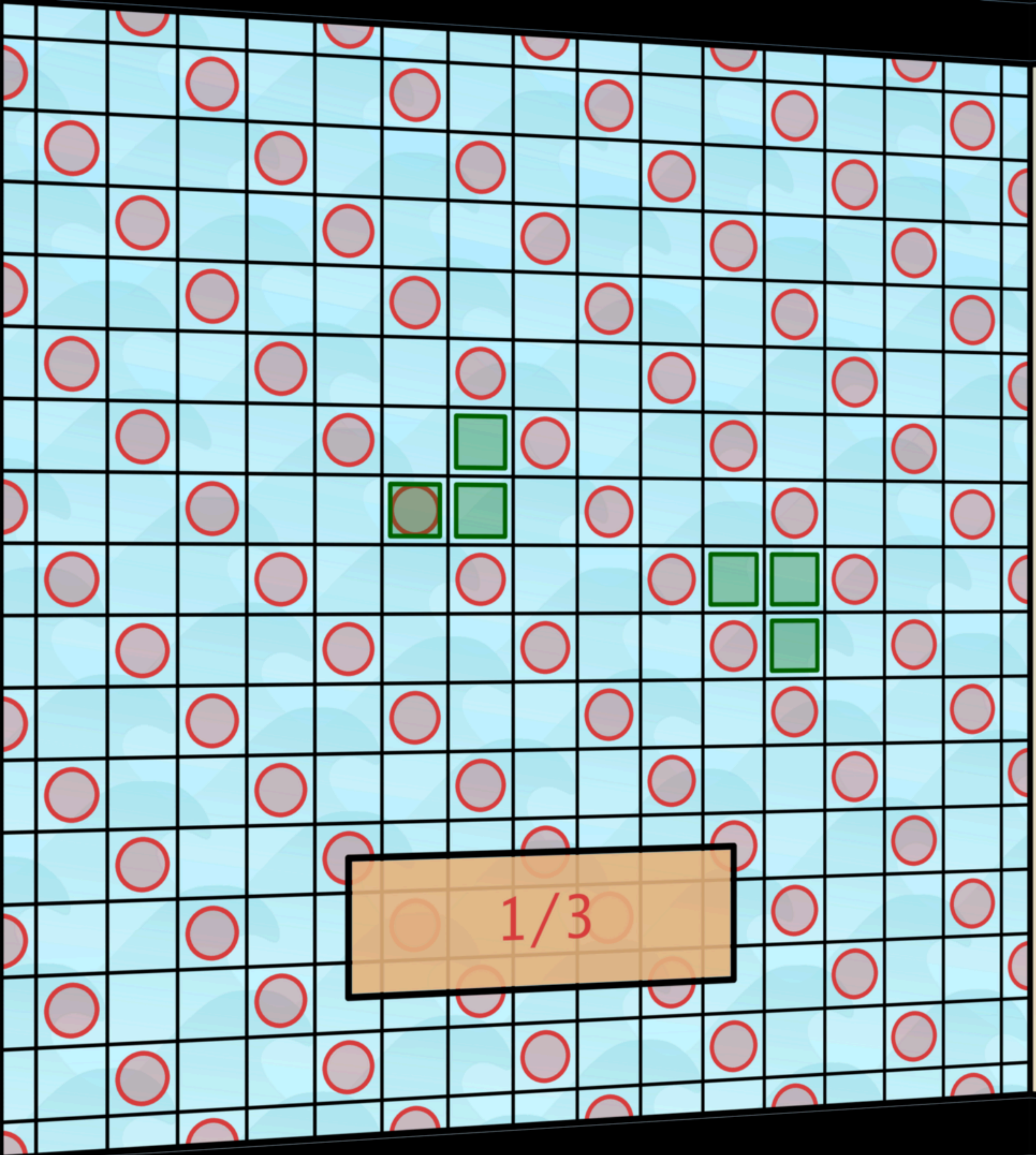

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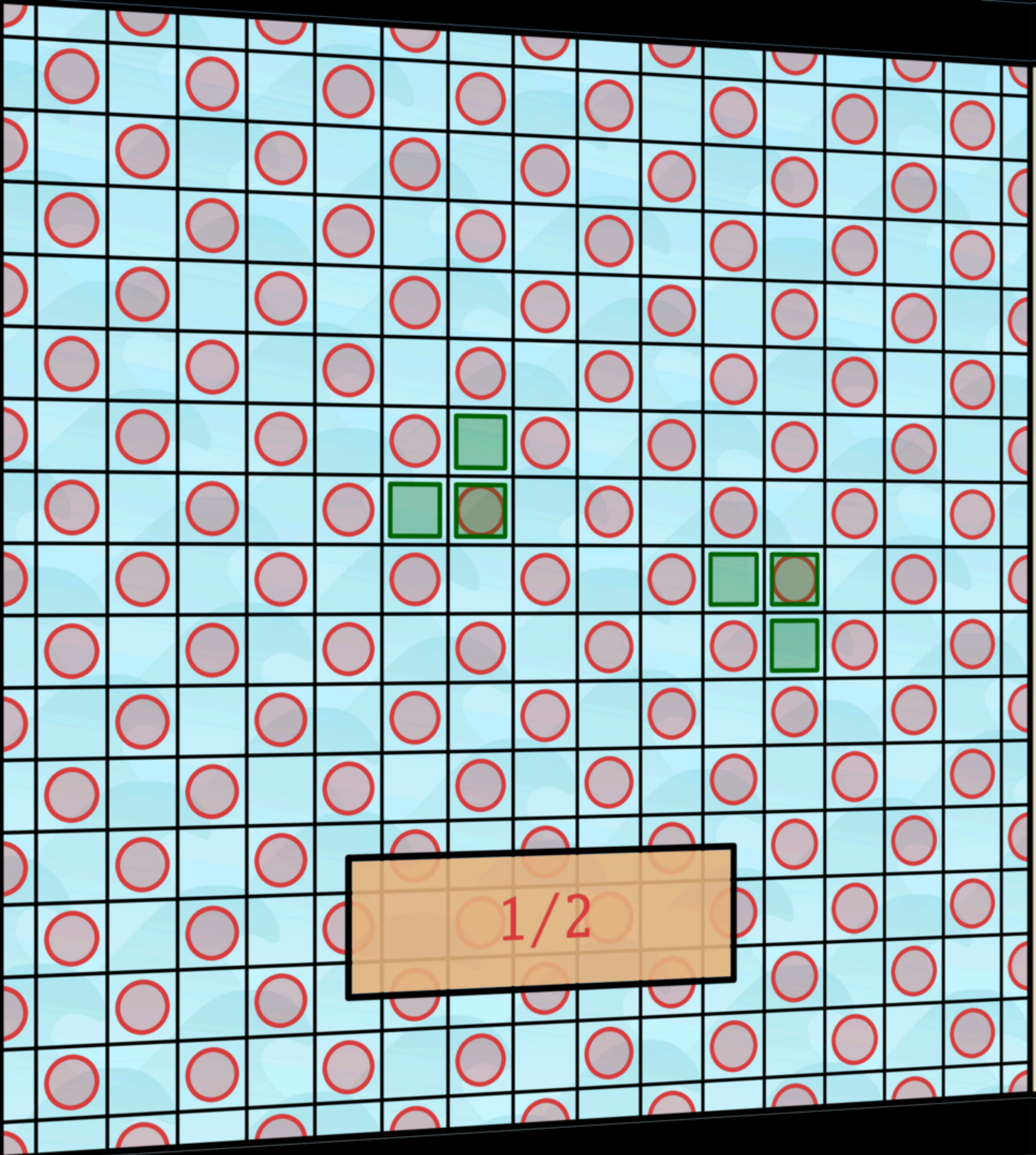

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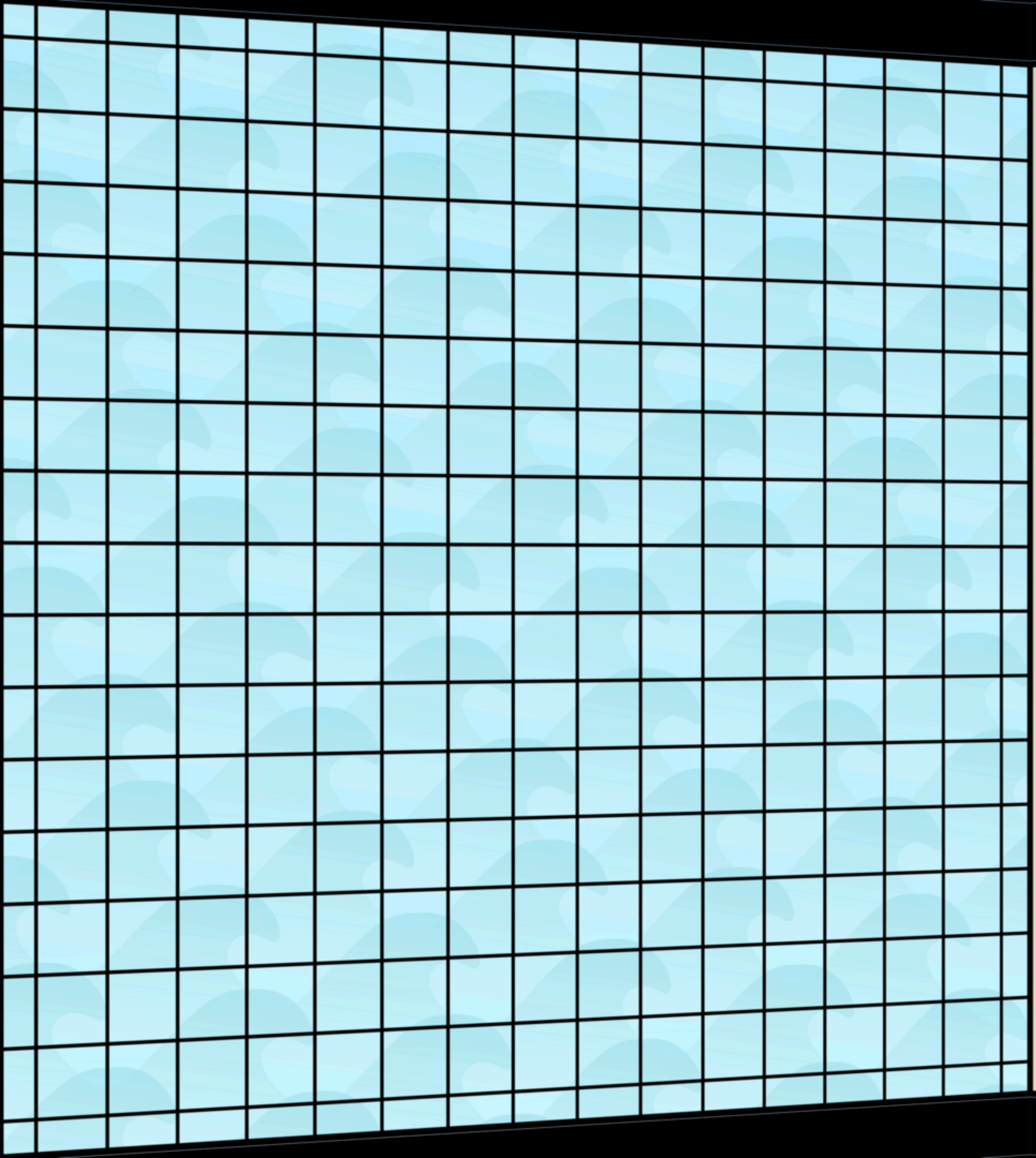

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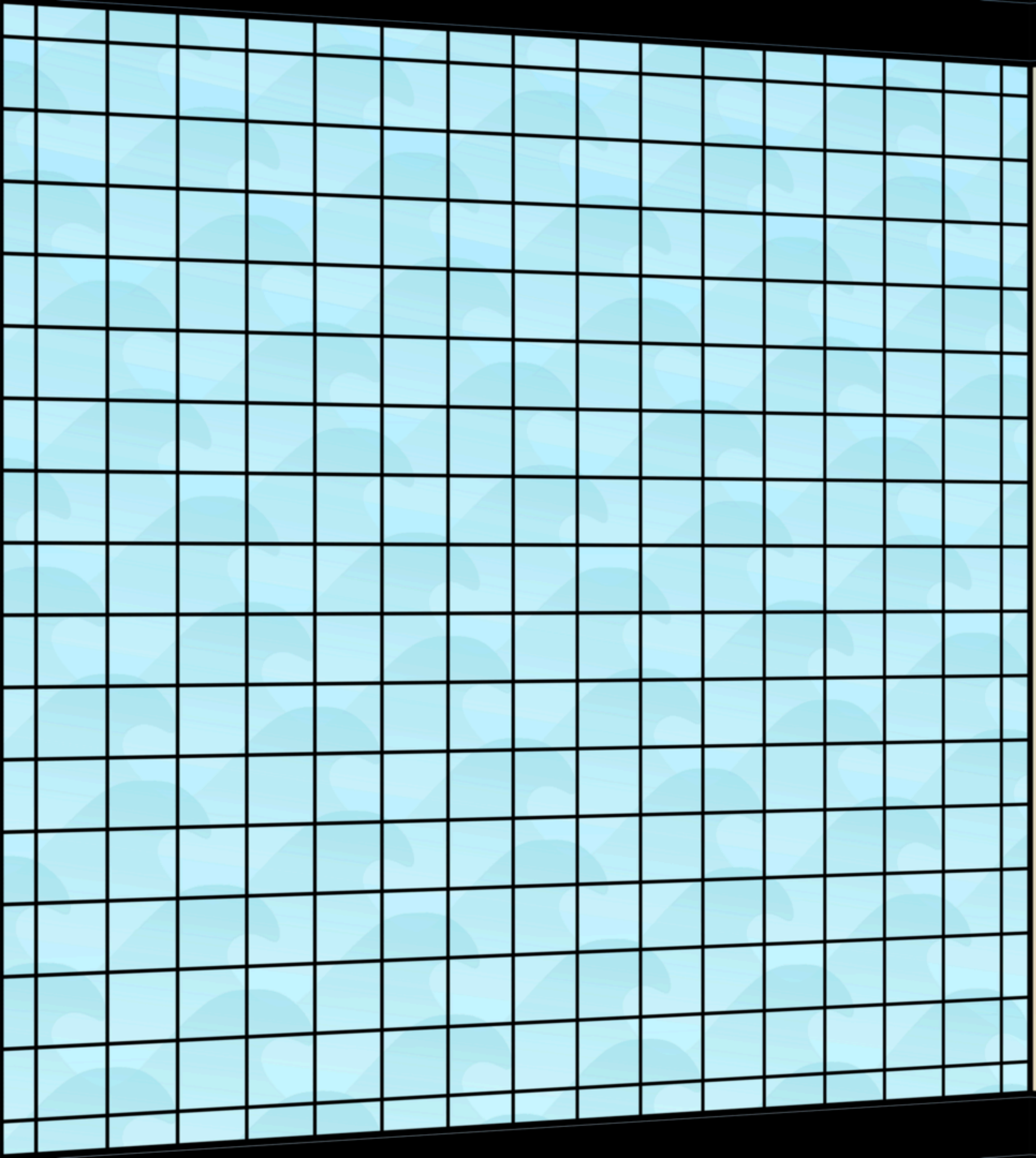

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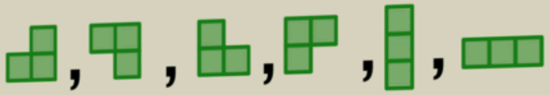
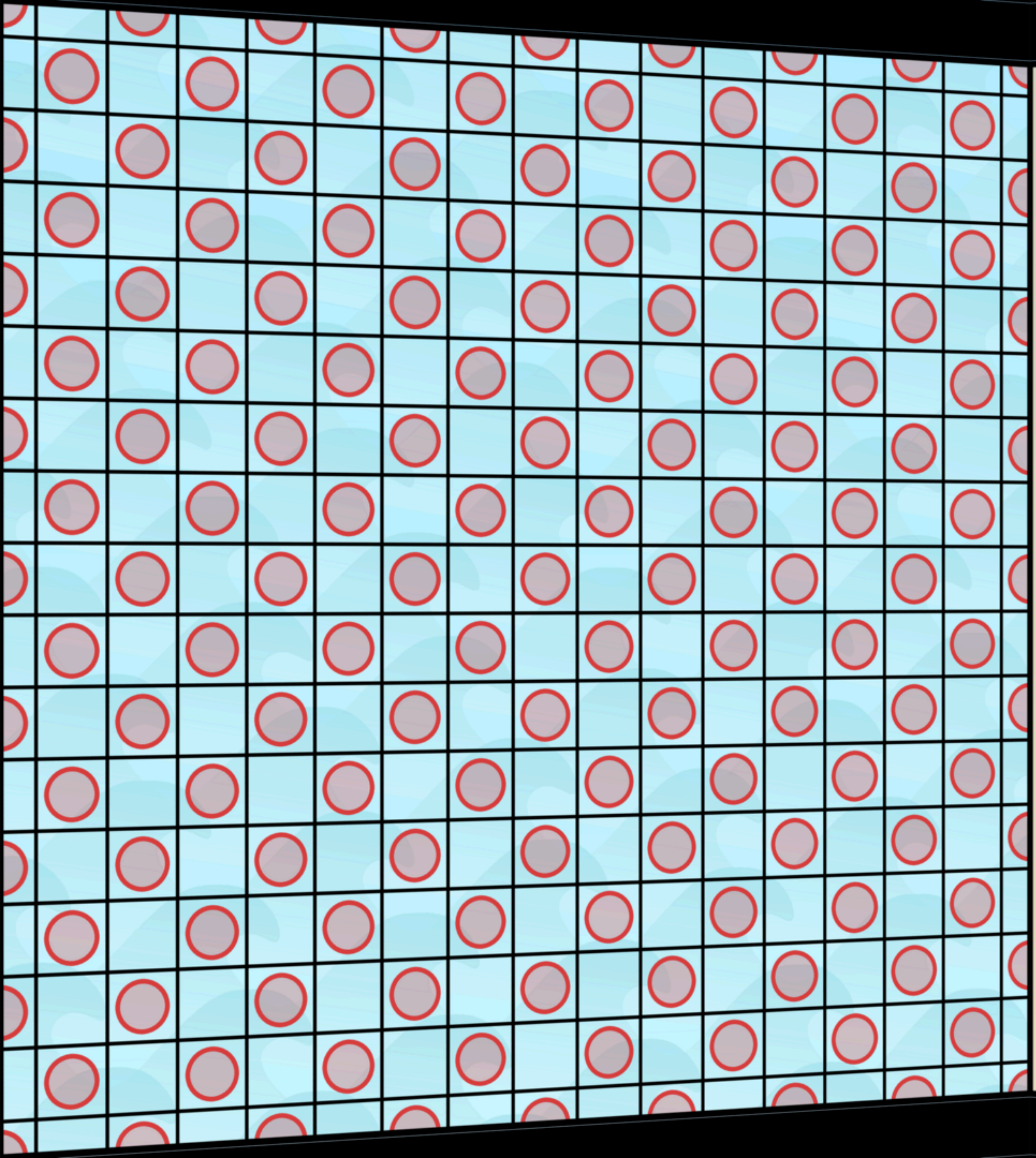
S={  }.



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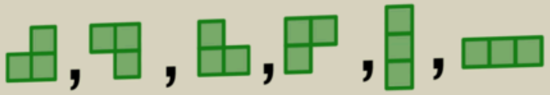
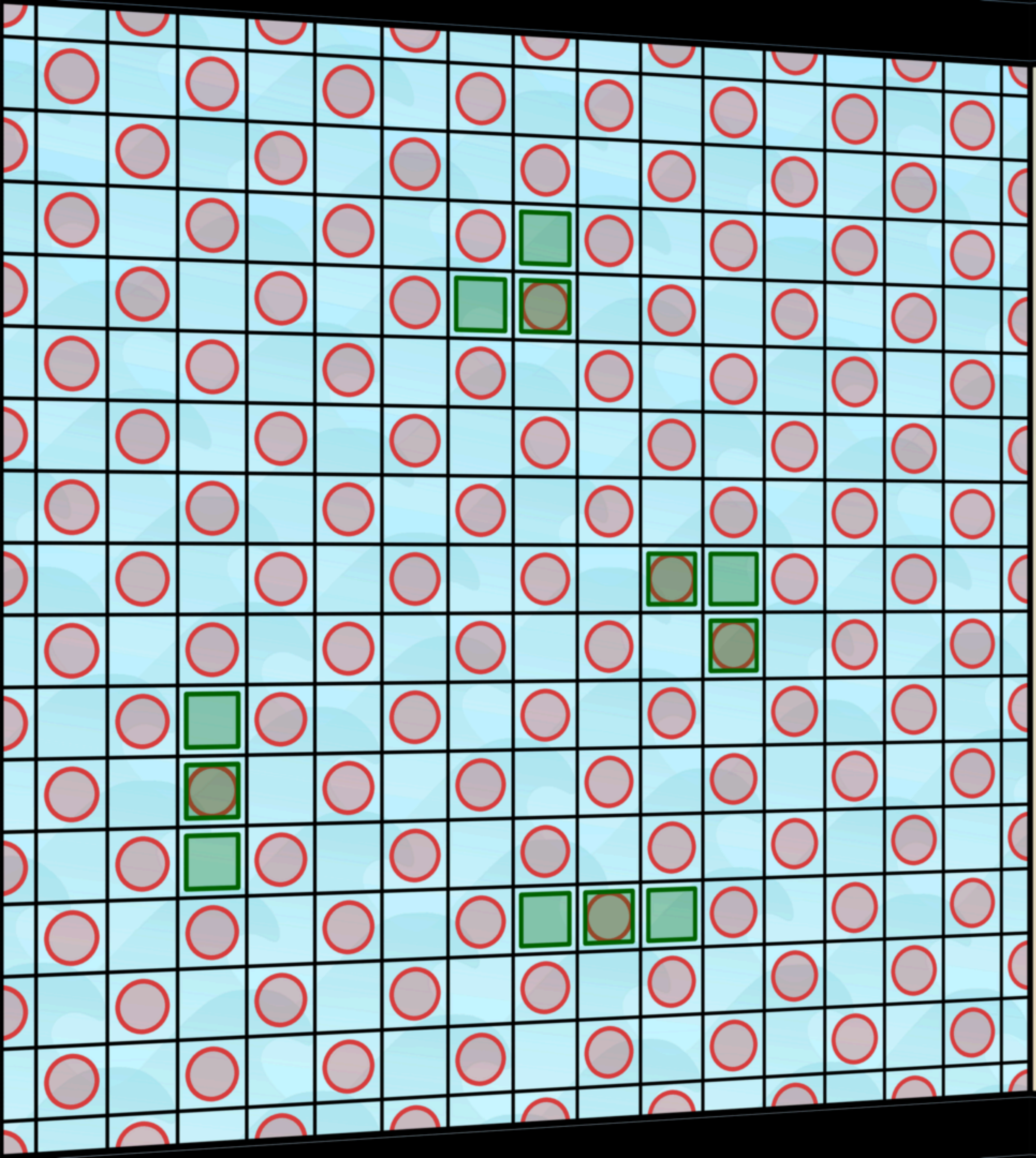
S={  }.



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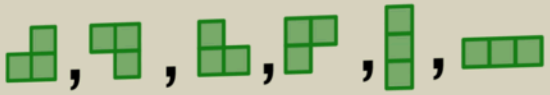
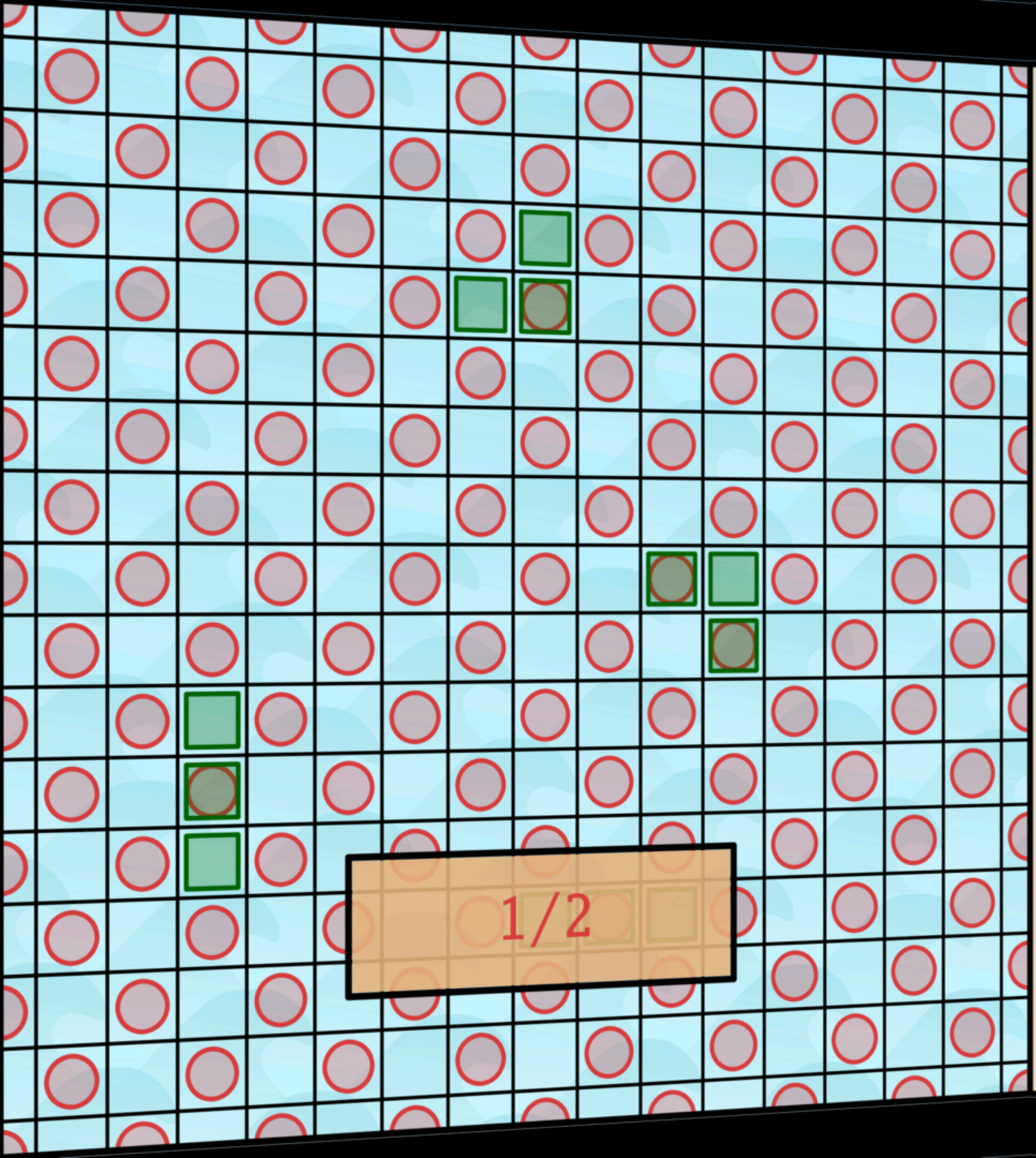
S={  }.



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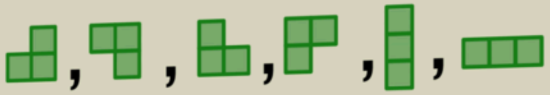
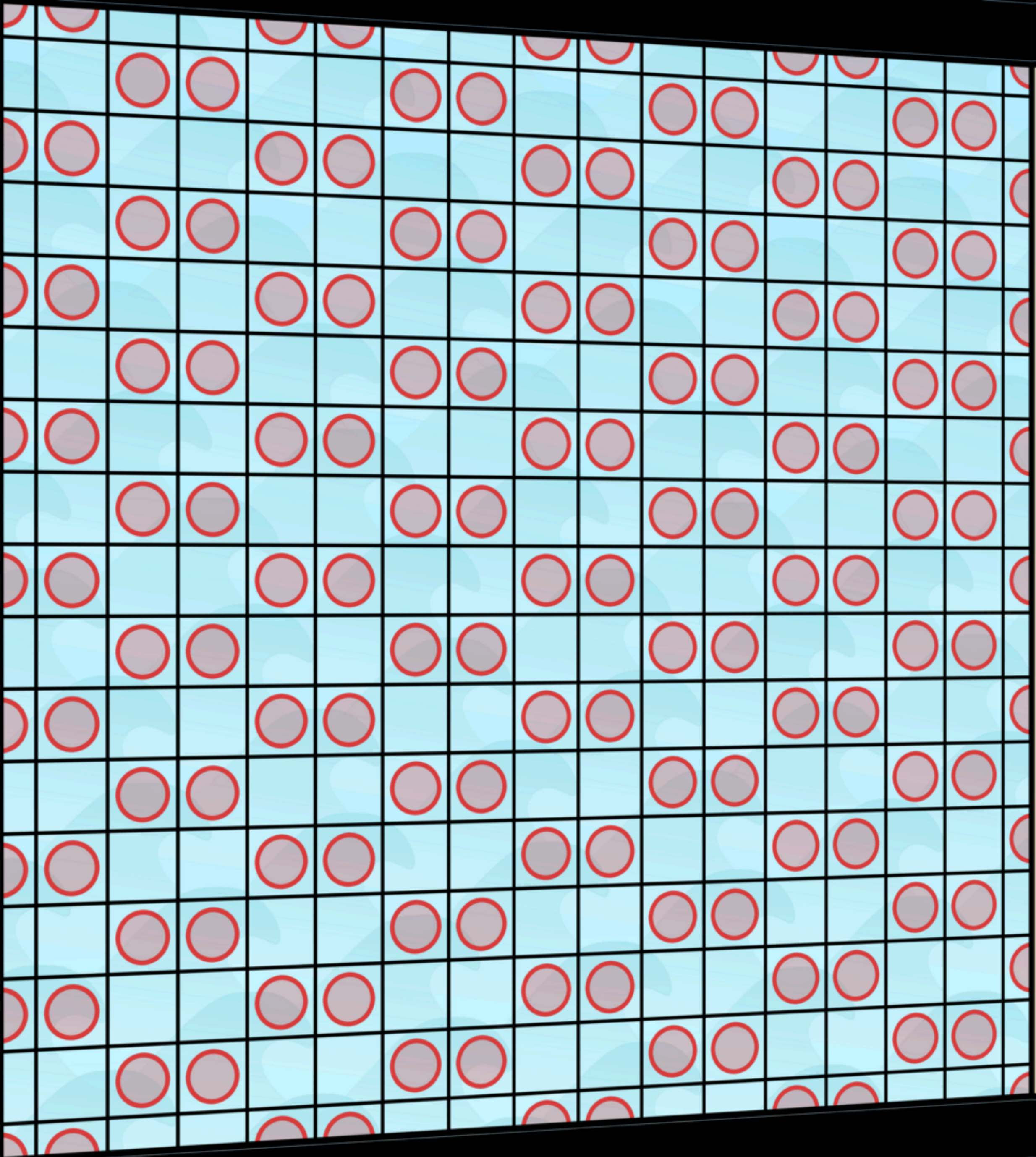
S={  }.



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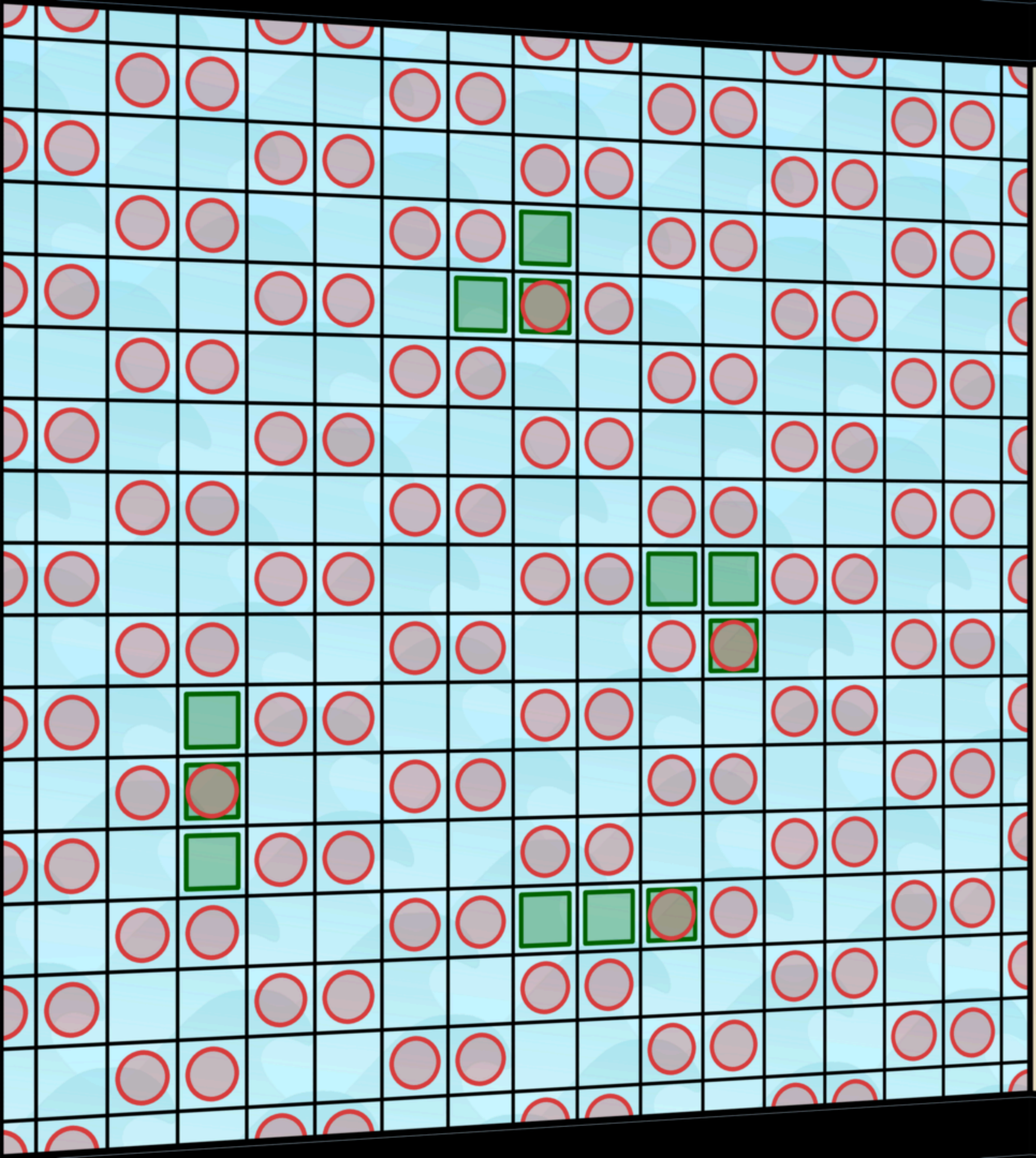

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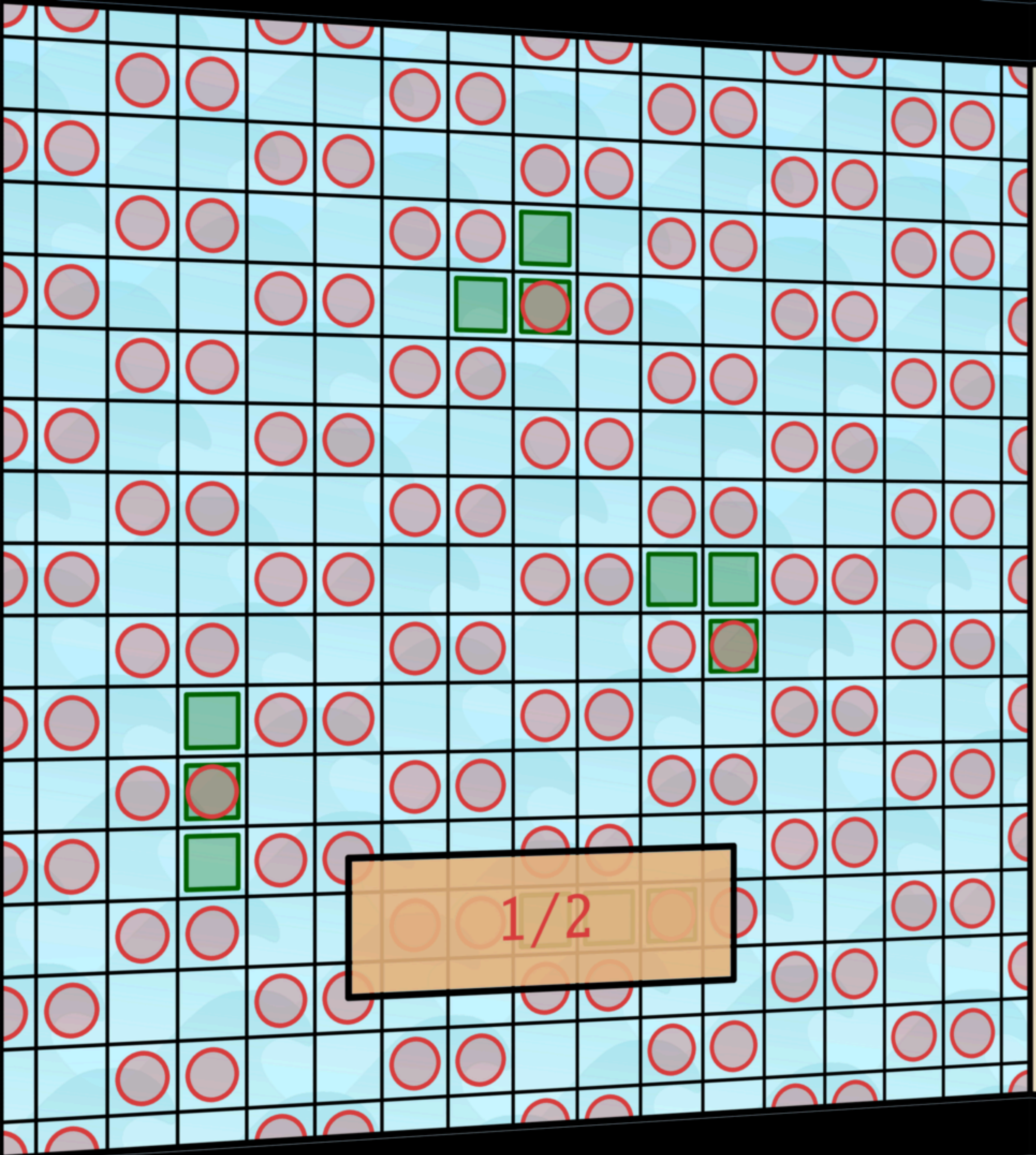

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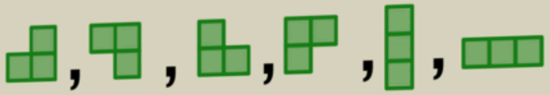
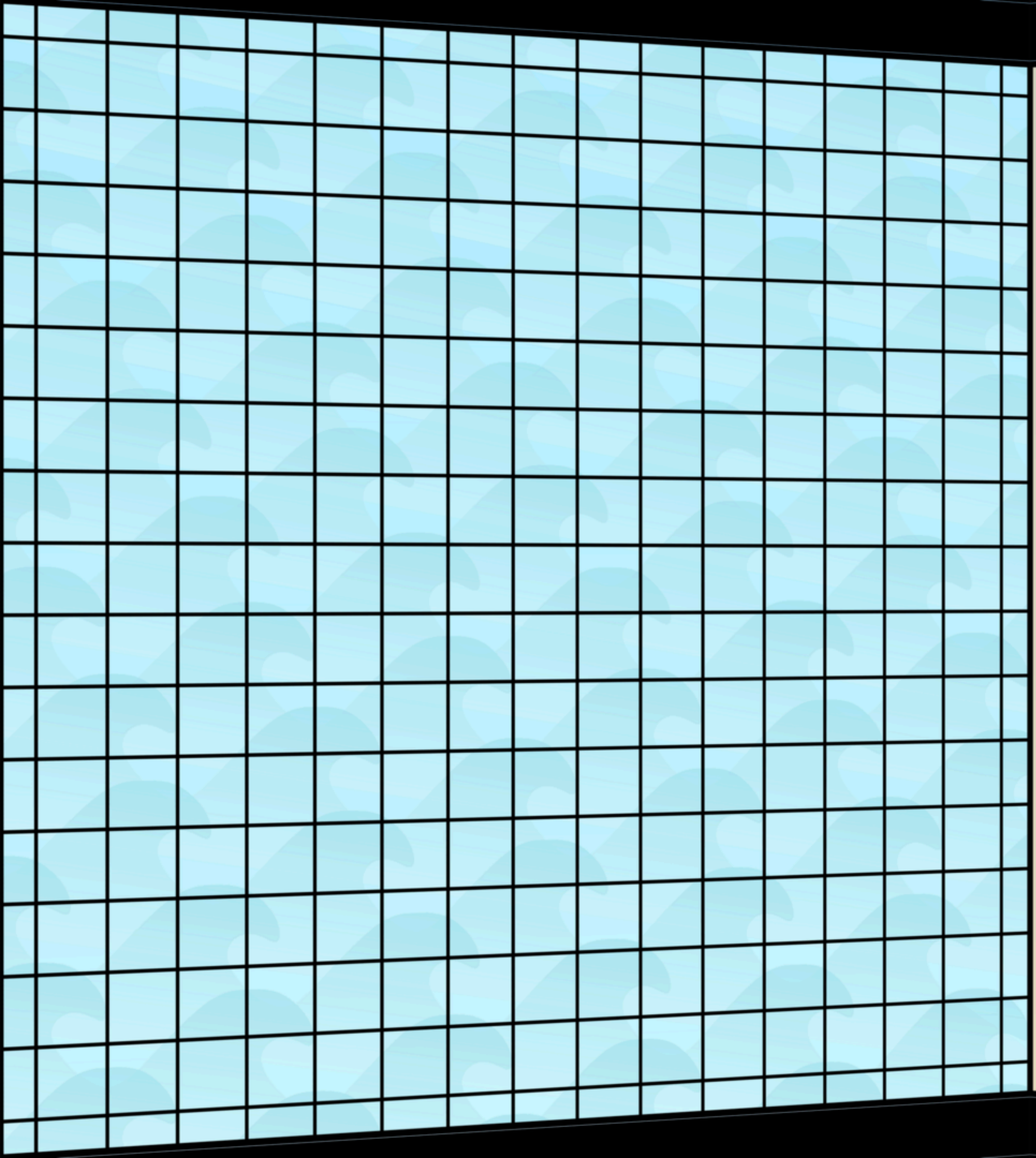
S={}.



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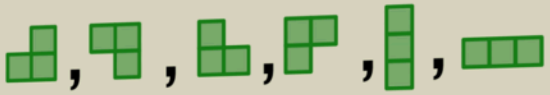
S={}.

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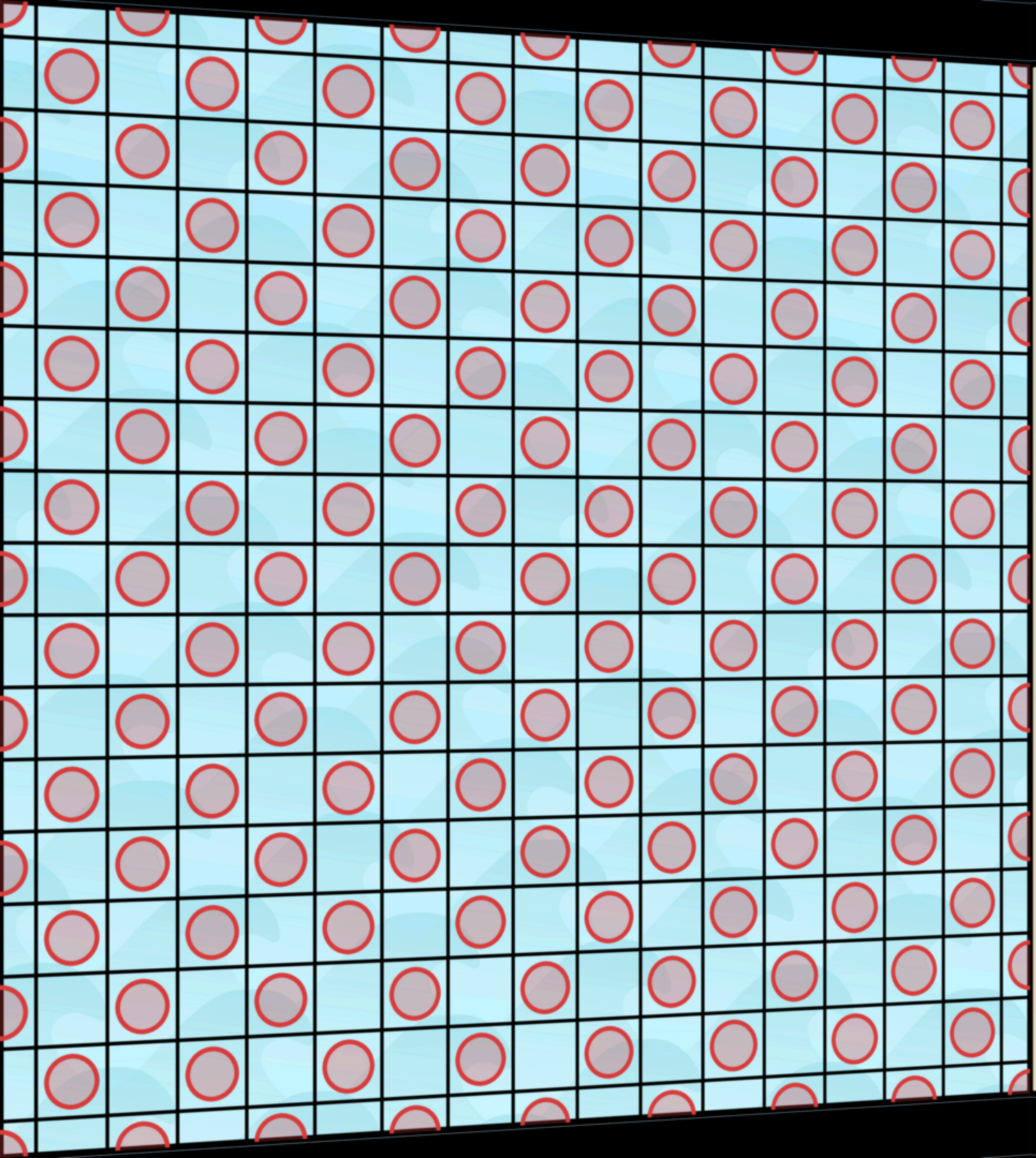

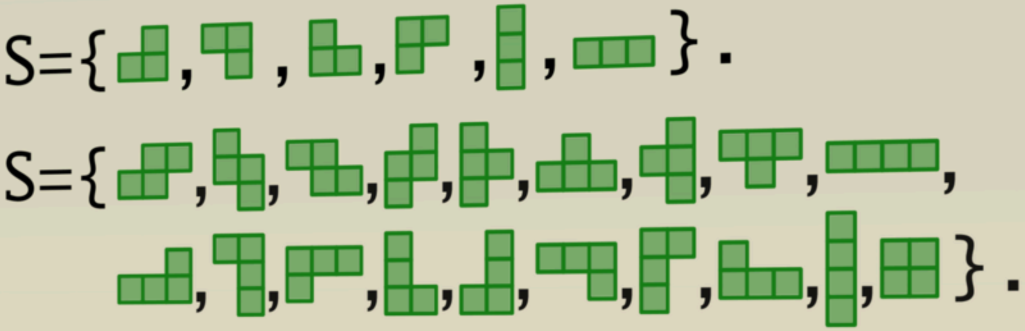

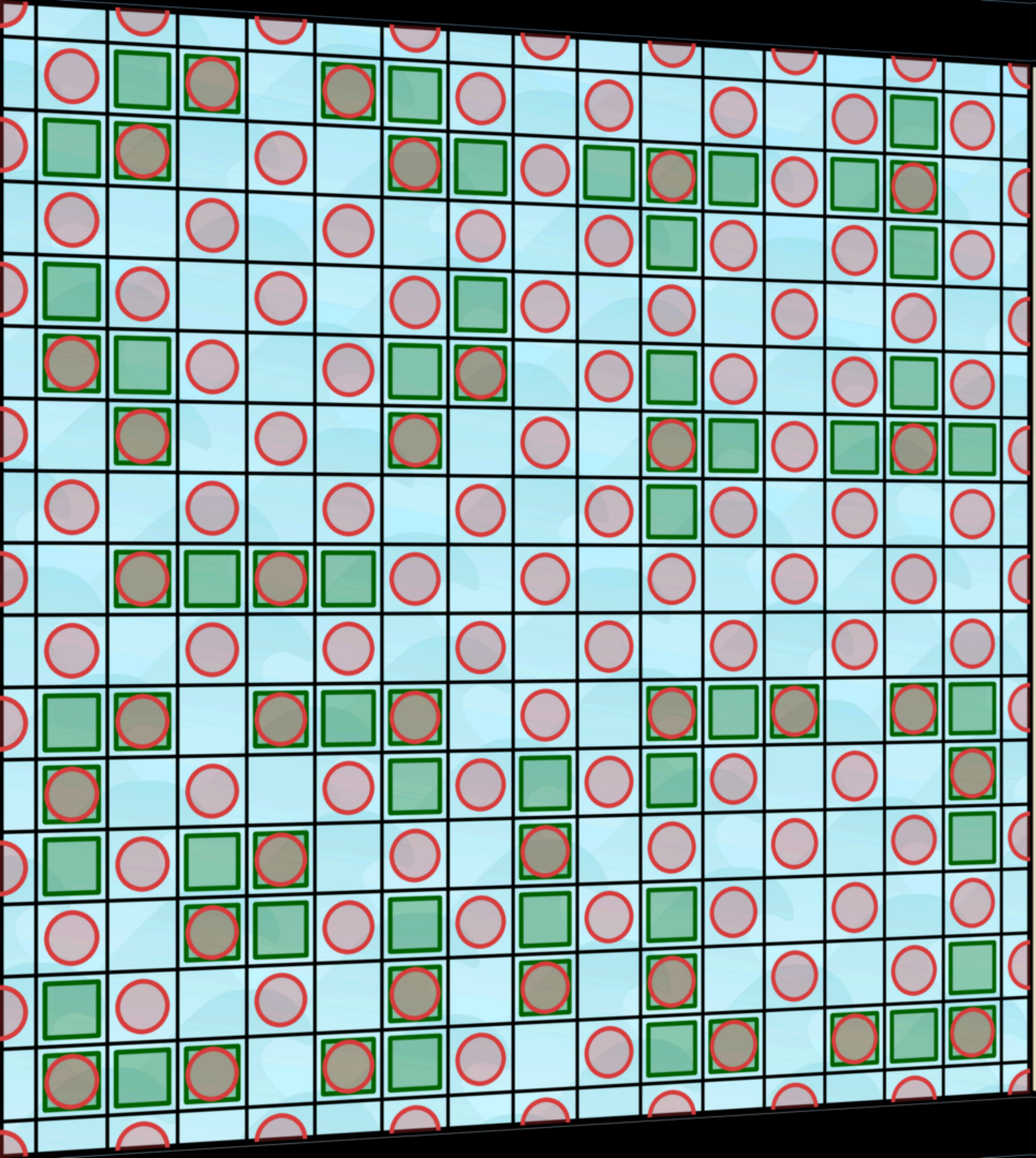

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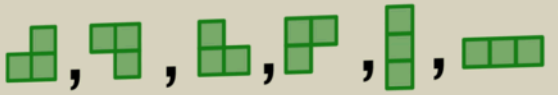
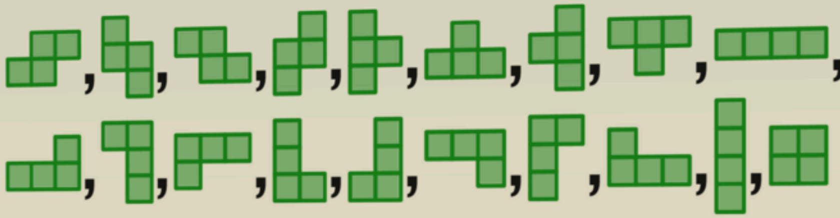
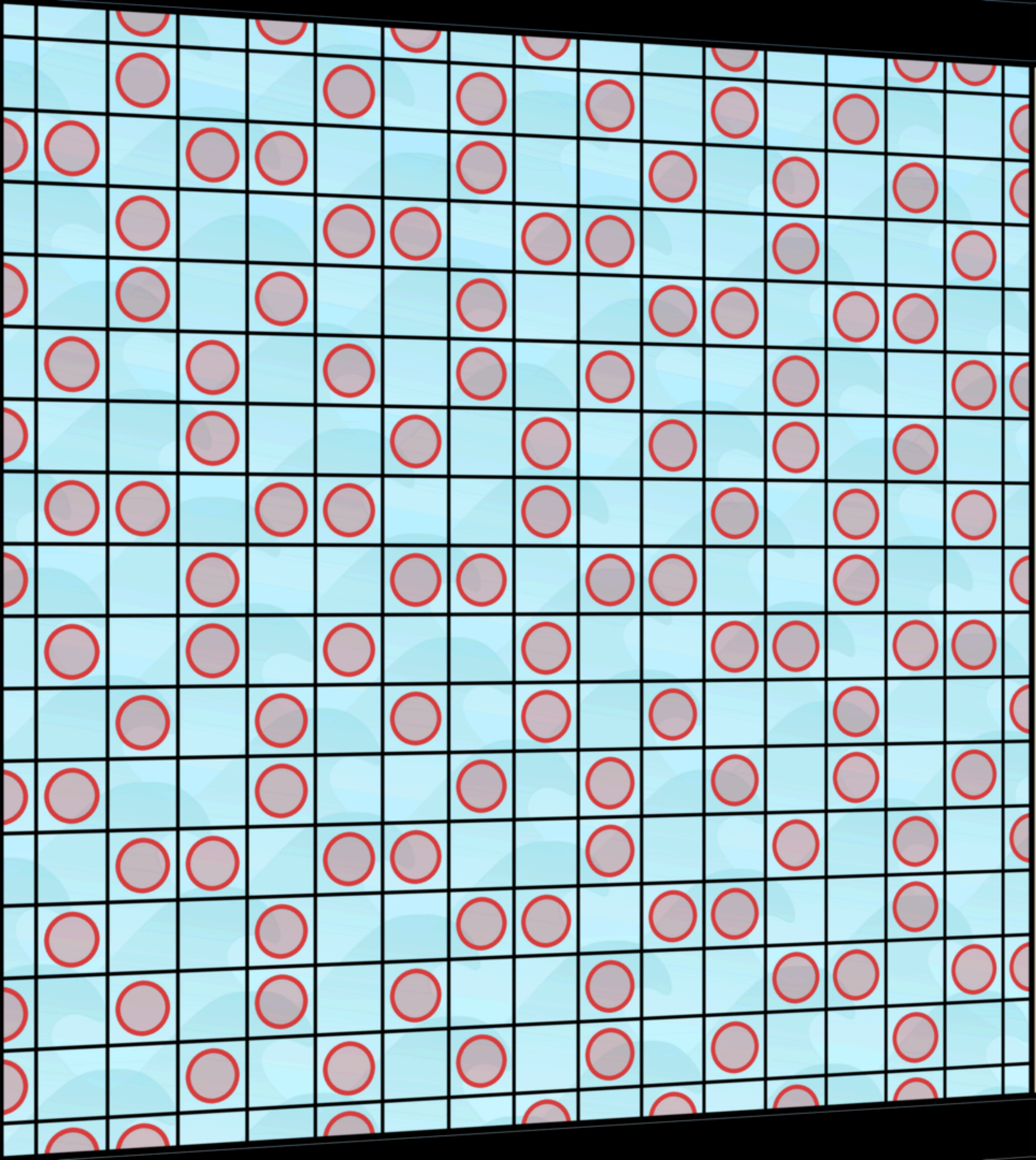
S={}.

S={}.



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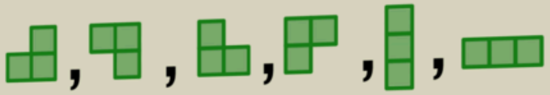
S={  }.

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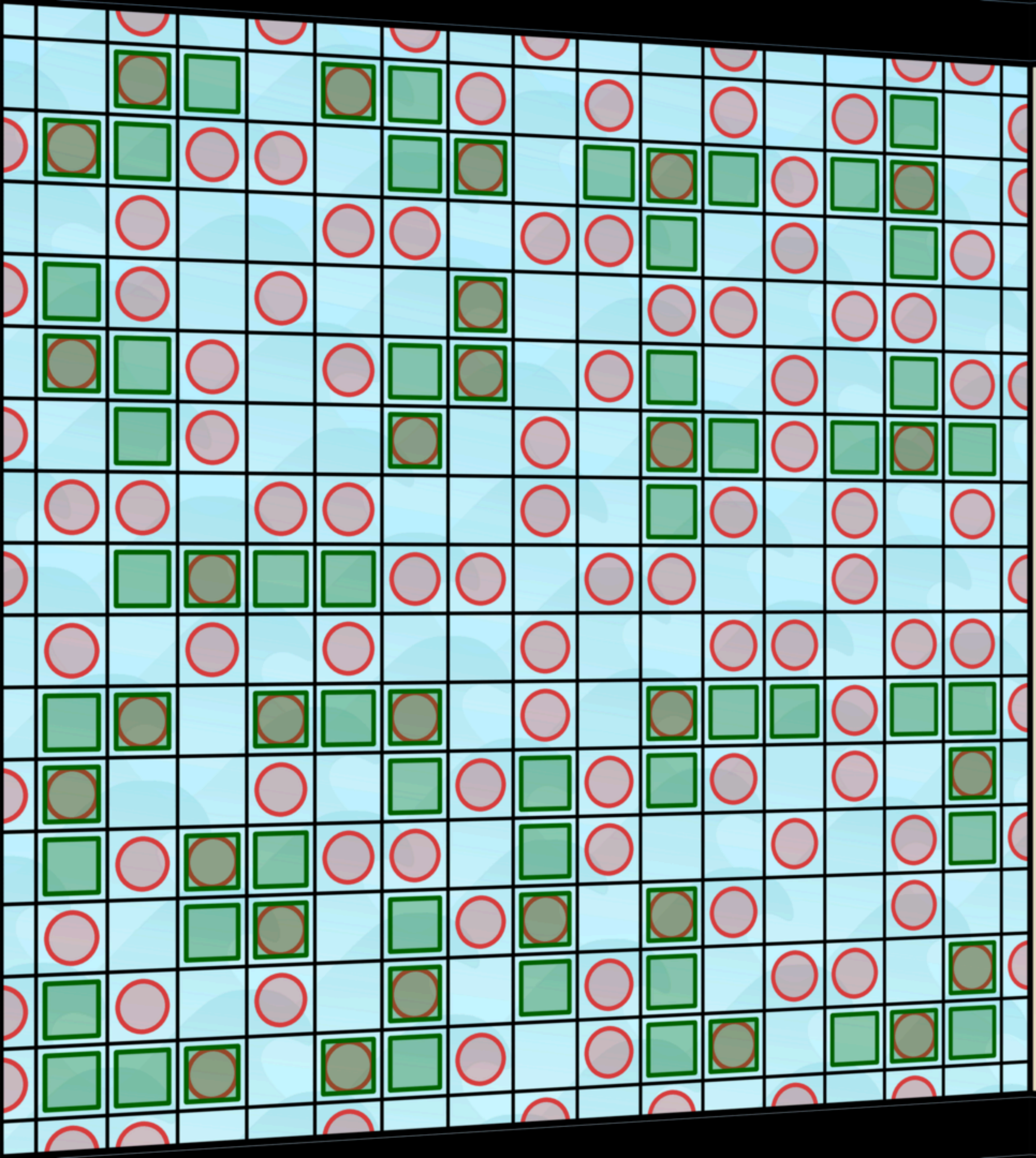

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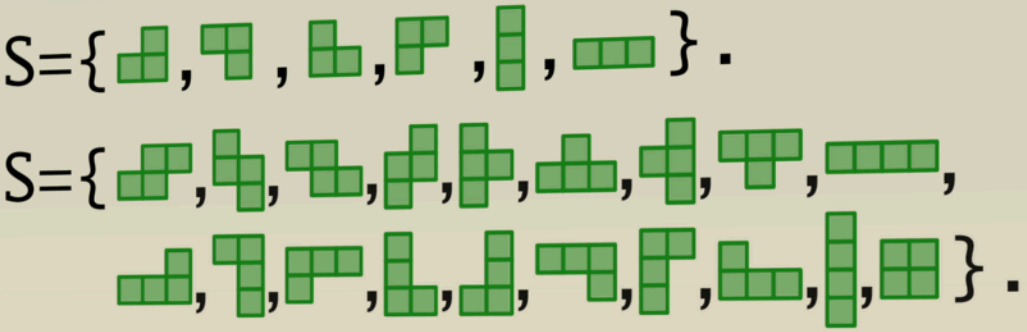

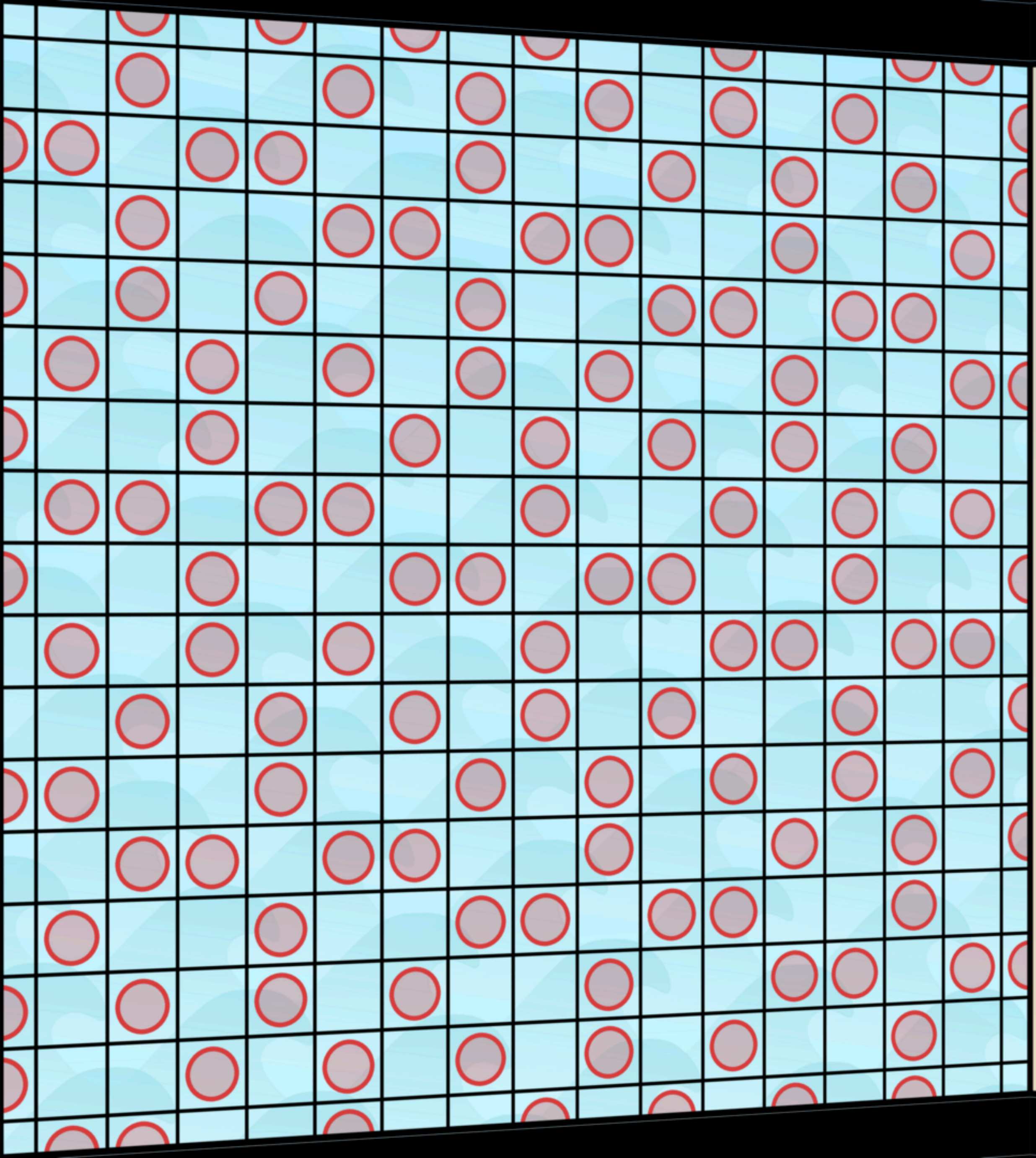

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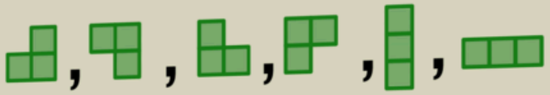
S={  }.

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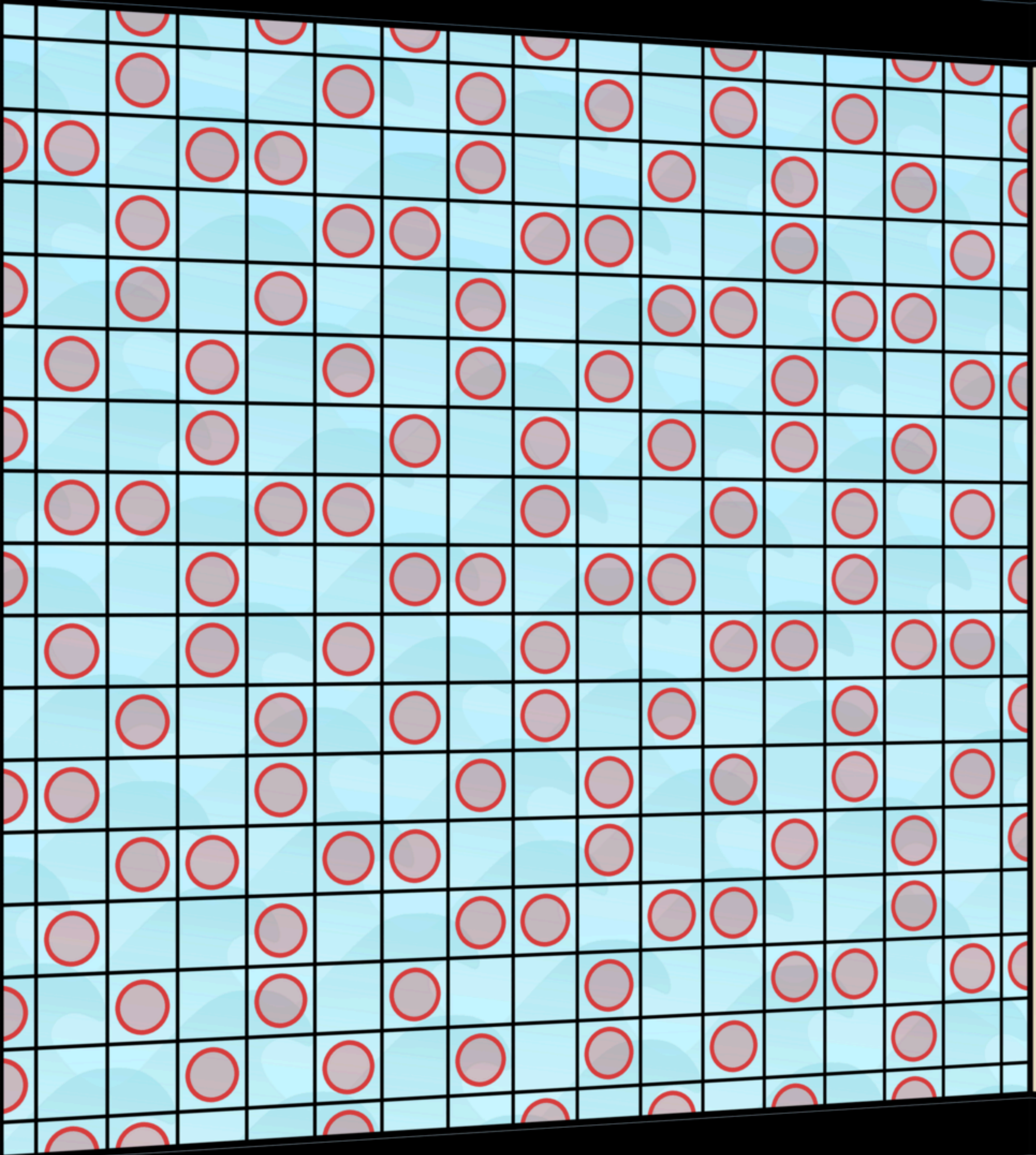

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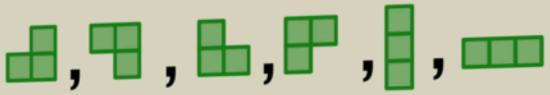
S={}.

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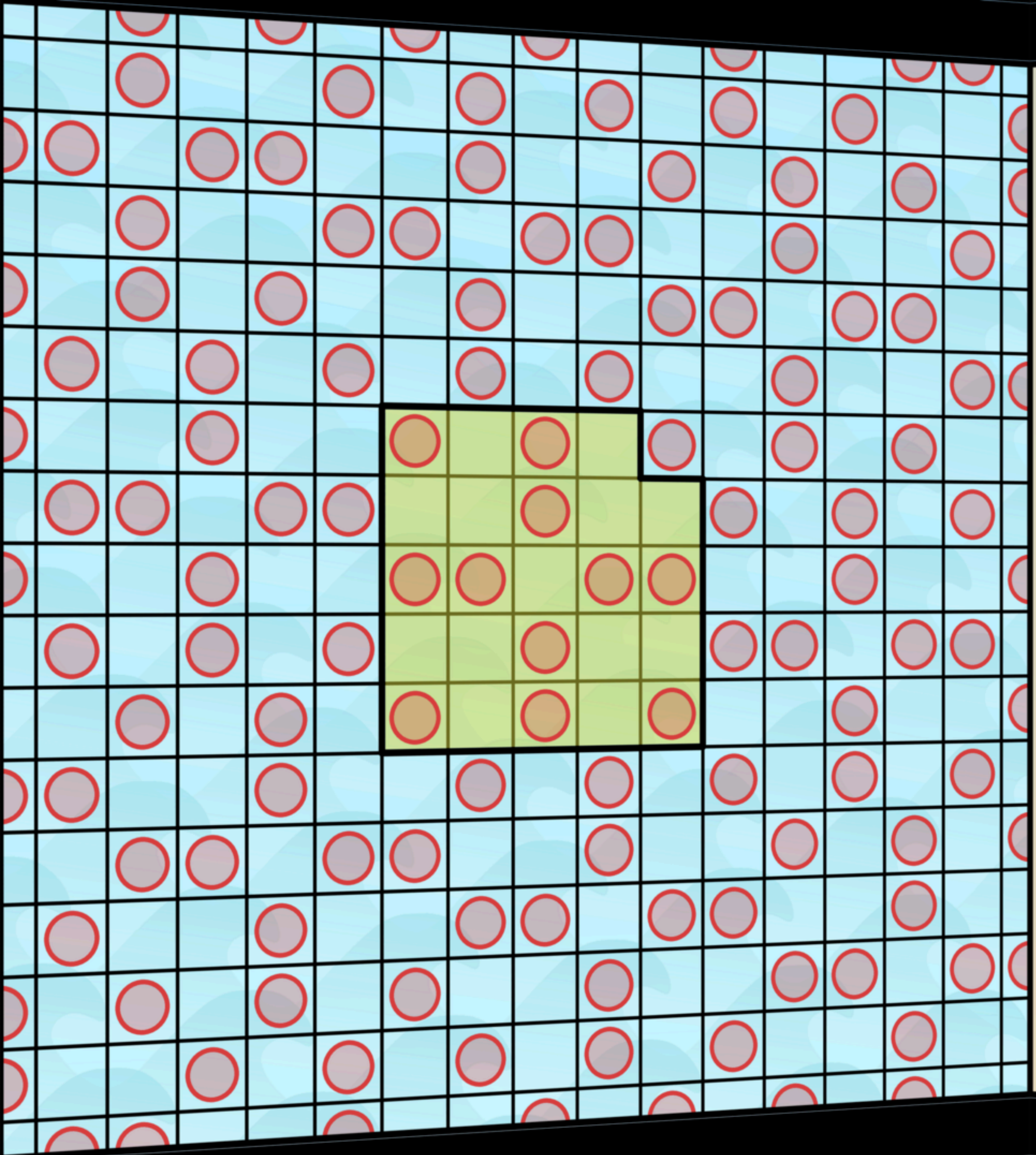

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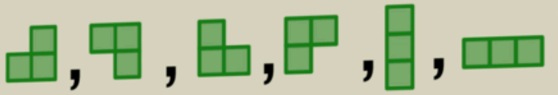
S={}.

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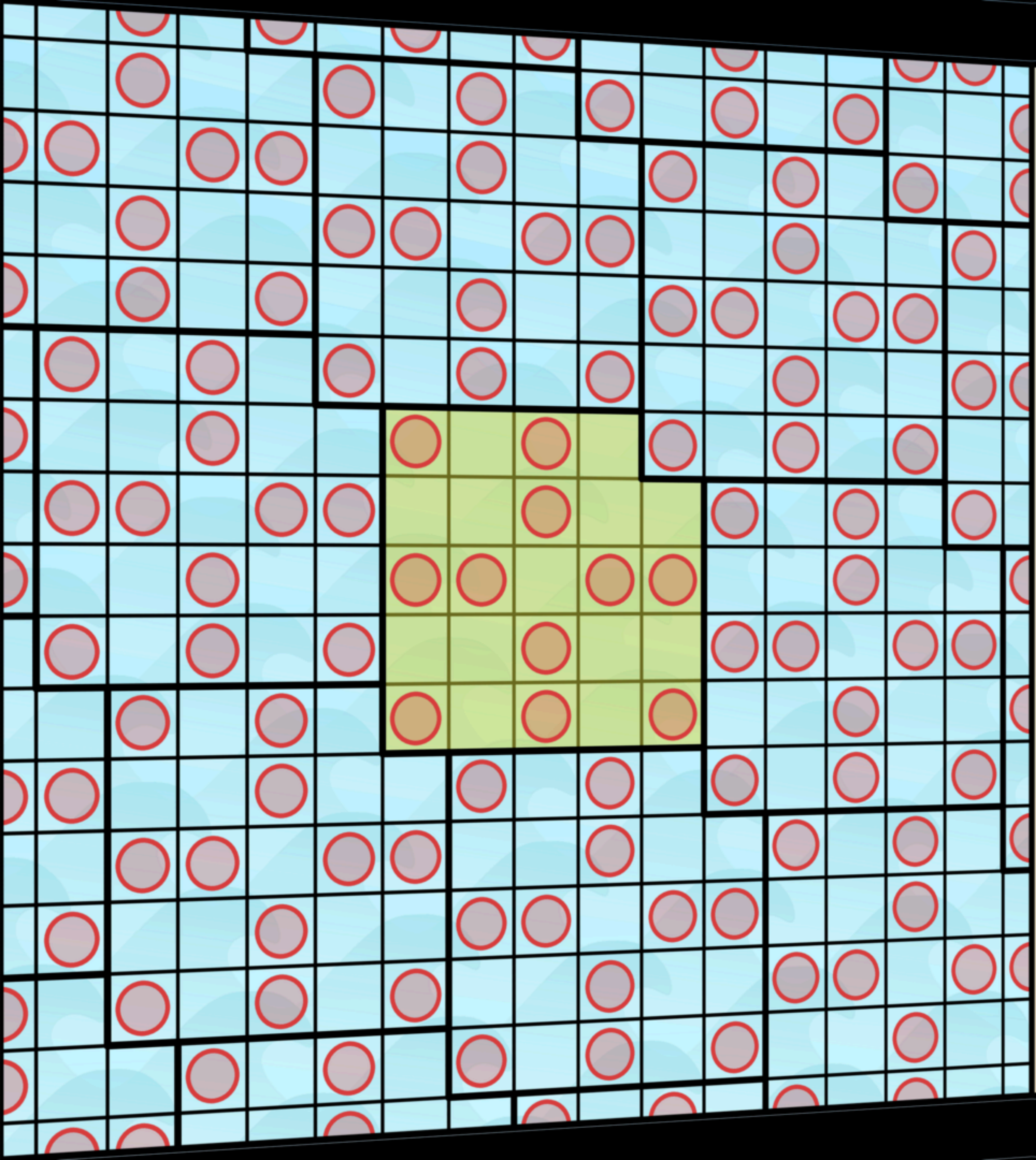

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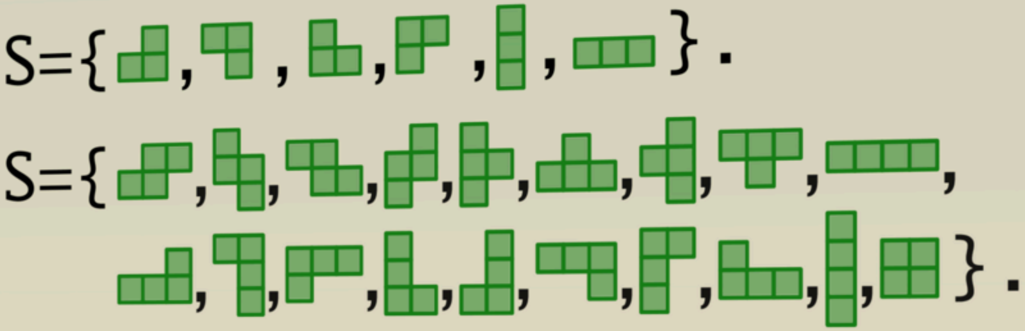

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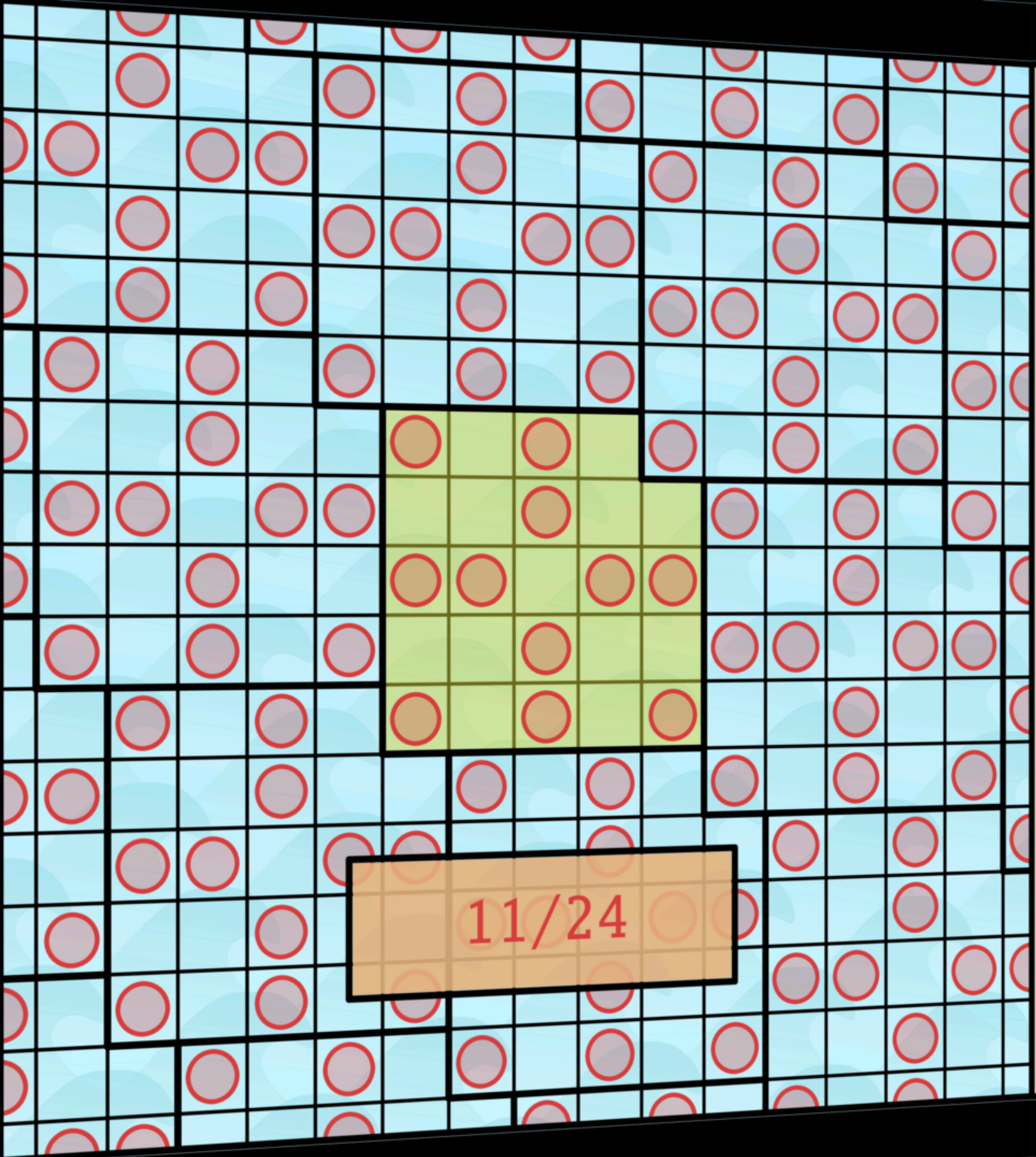

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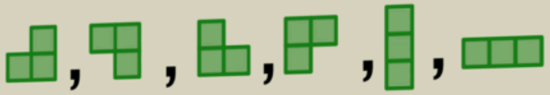
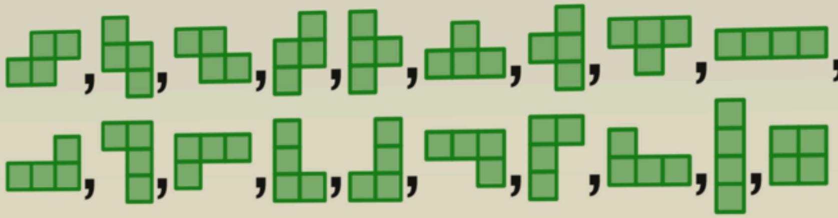
S={}.

S={}.

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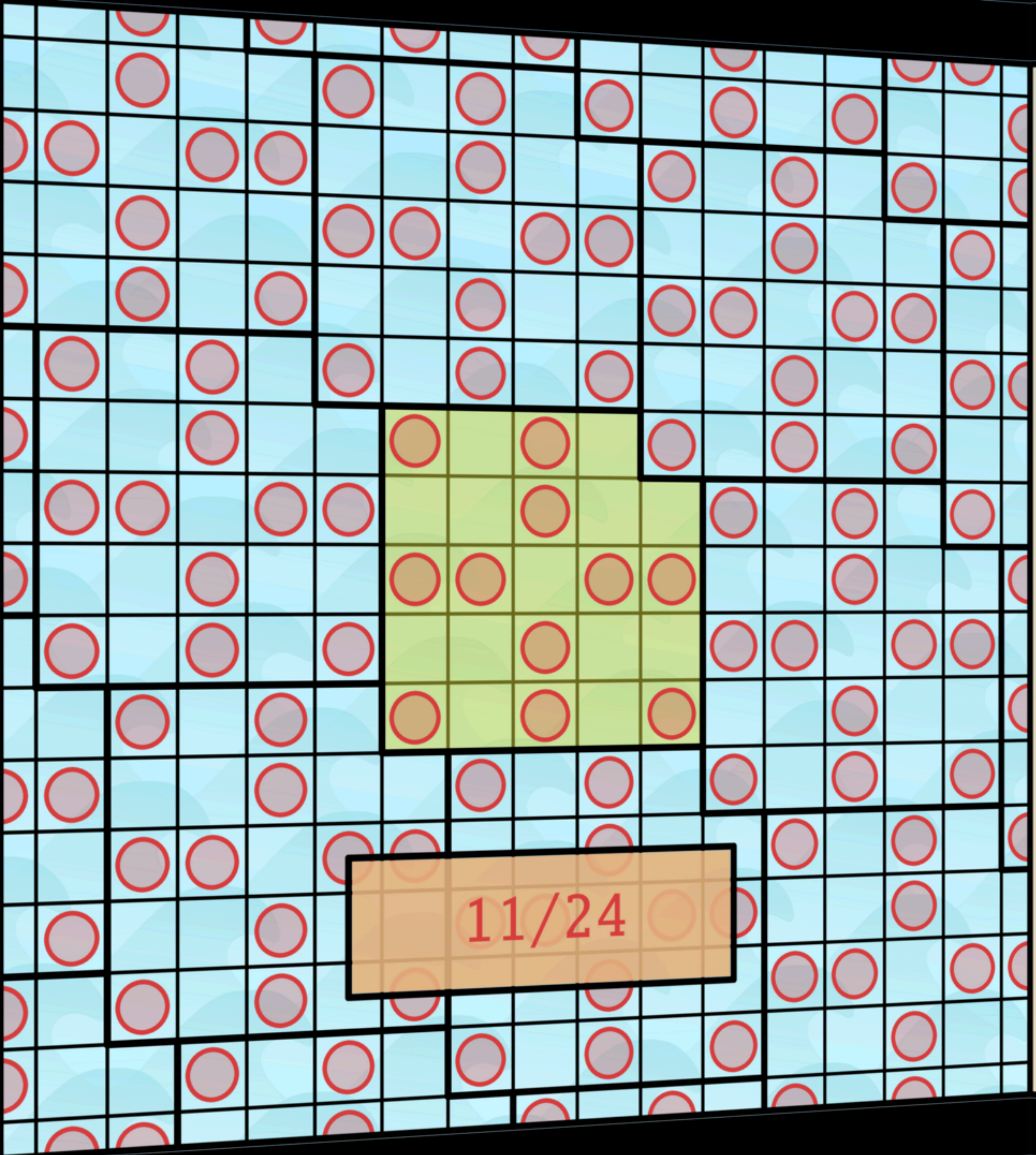

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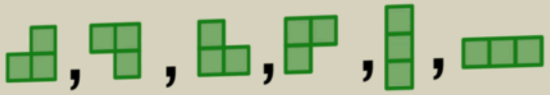
S={}.

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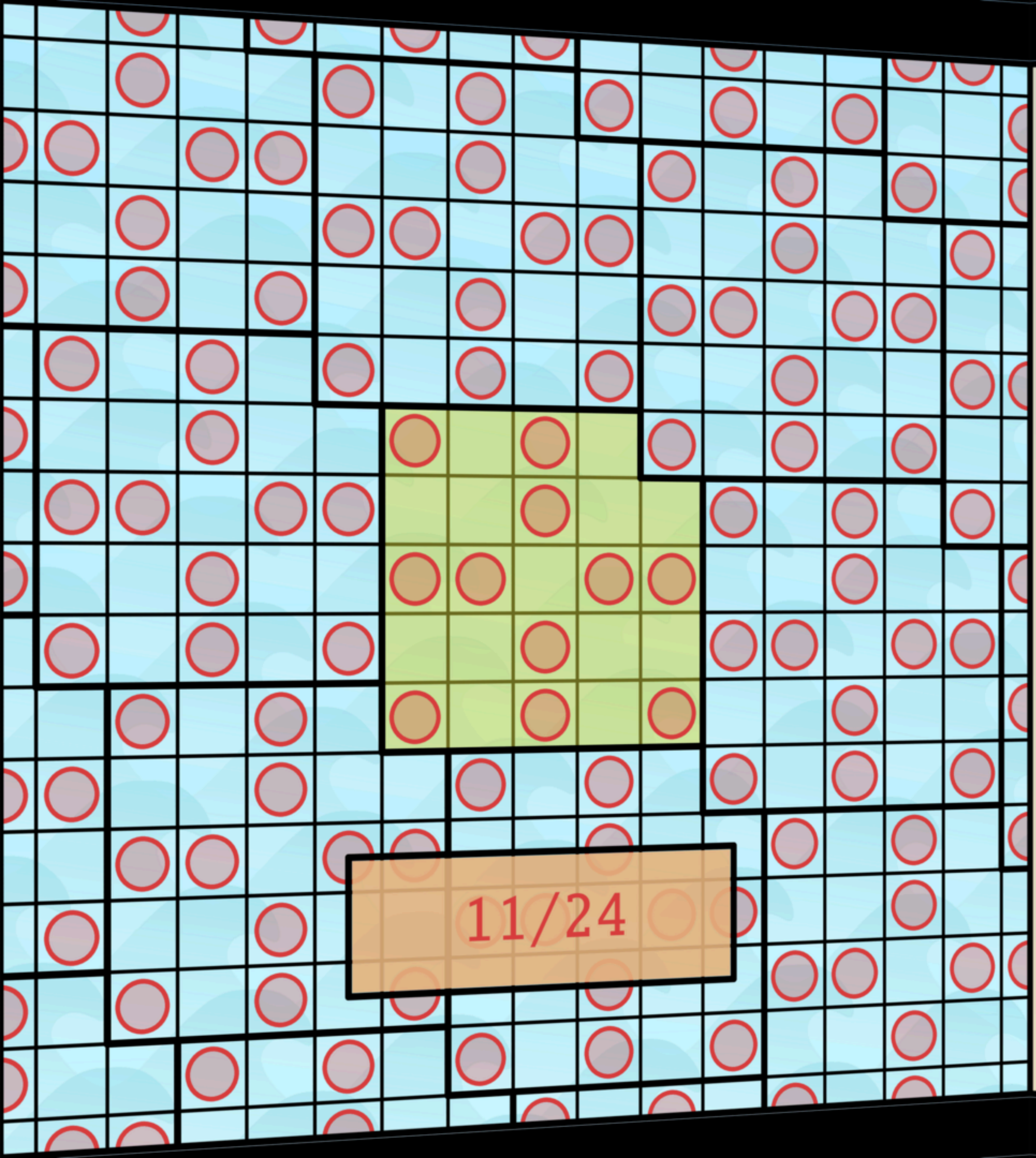
Is this optimal?



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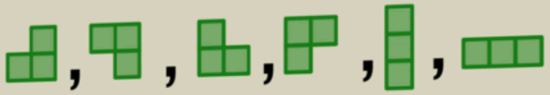
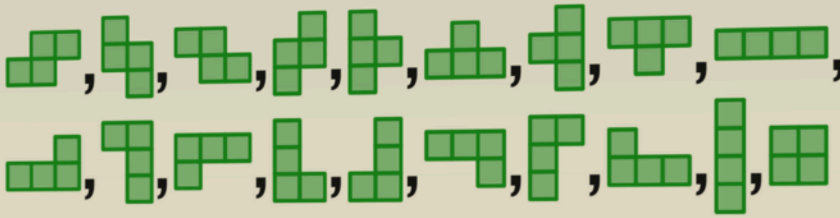
S={}.

S={}.

What is the density of this pattern?

Is this optimal?

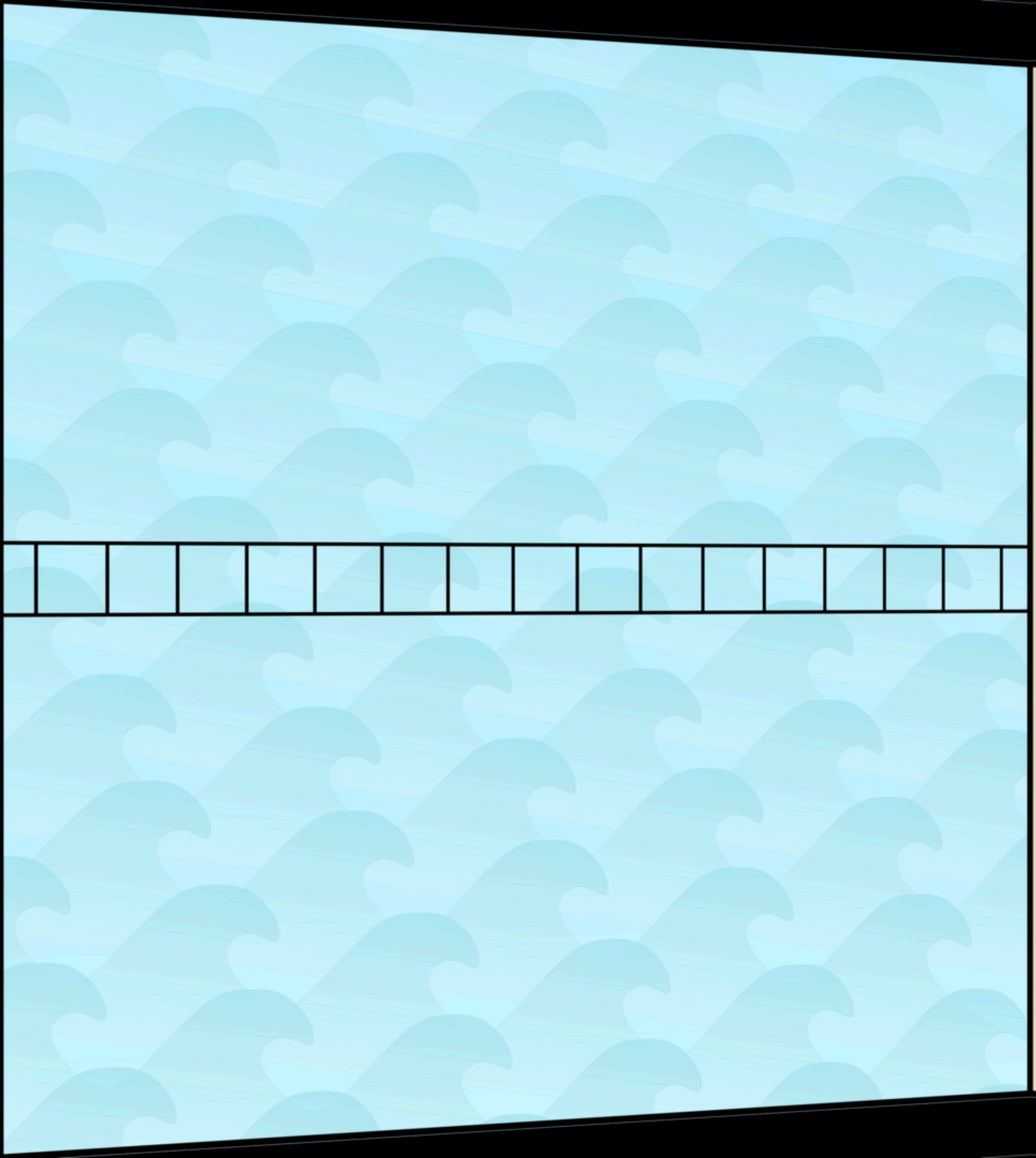
How does this continue?
How do we *find* such patterns?

4

CHAPTER

1-DIMENSIONAL SHIPS

Consider an infinite
1-dimensional grid.



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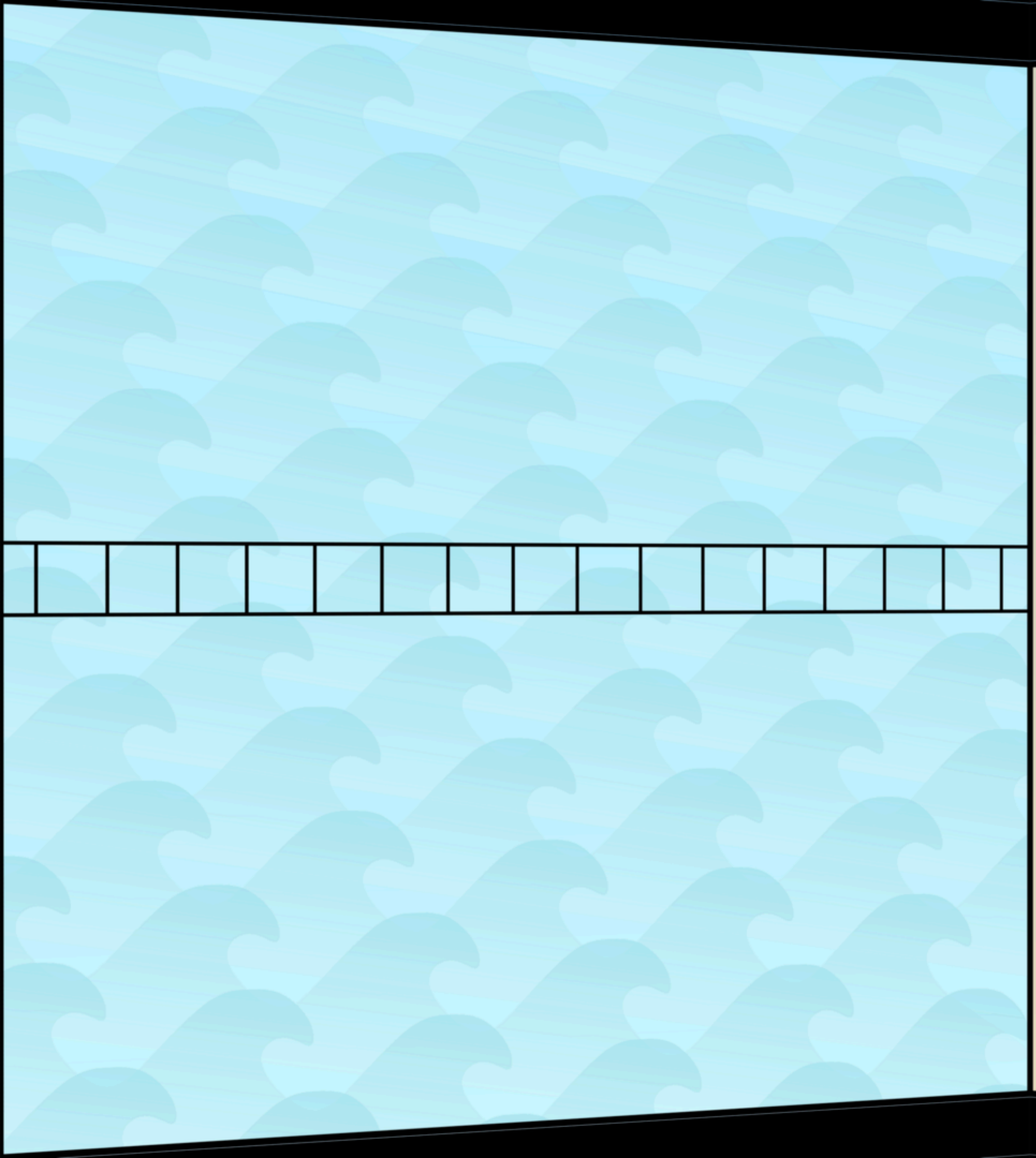
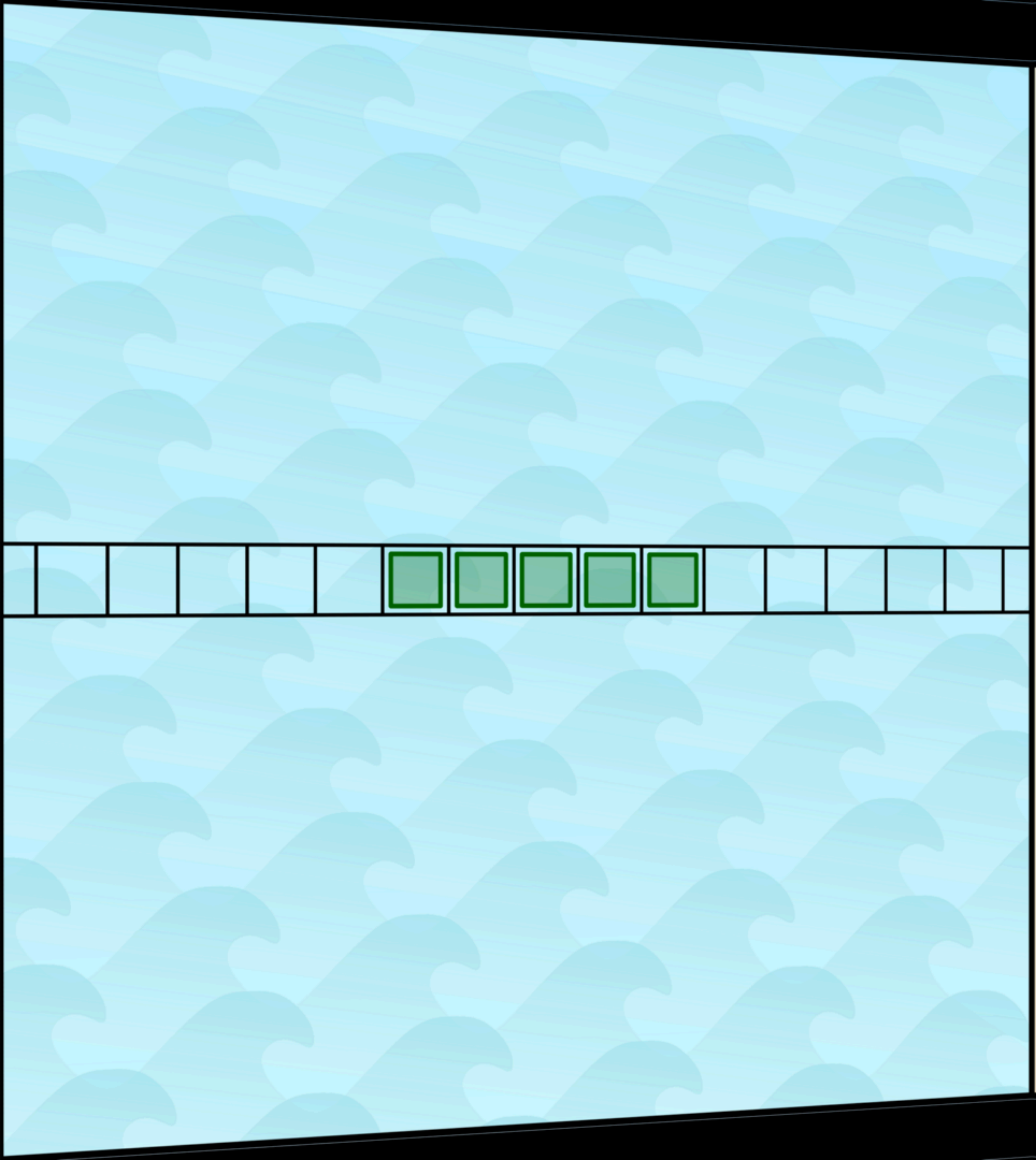


Consider an infinite 1-dimensional grid.

Ships of length k can be hit with density 1/k...



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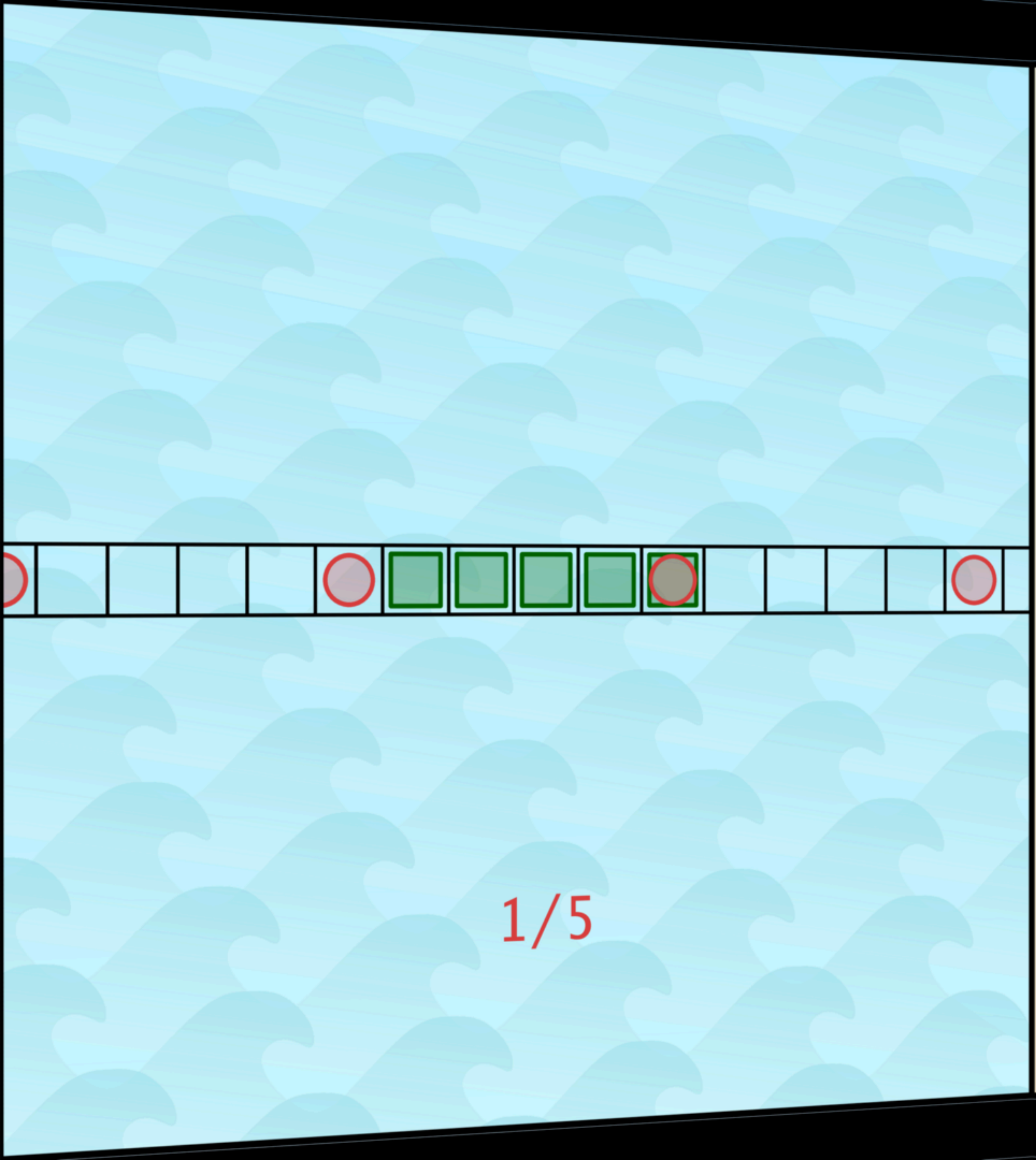

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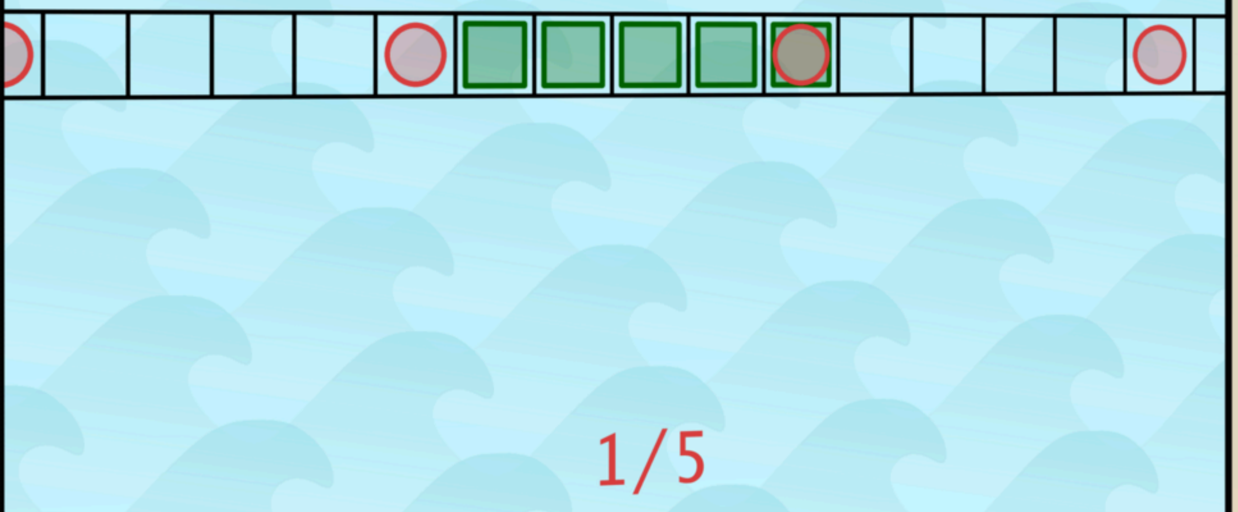

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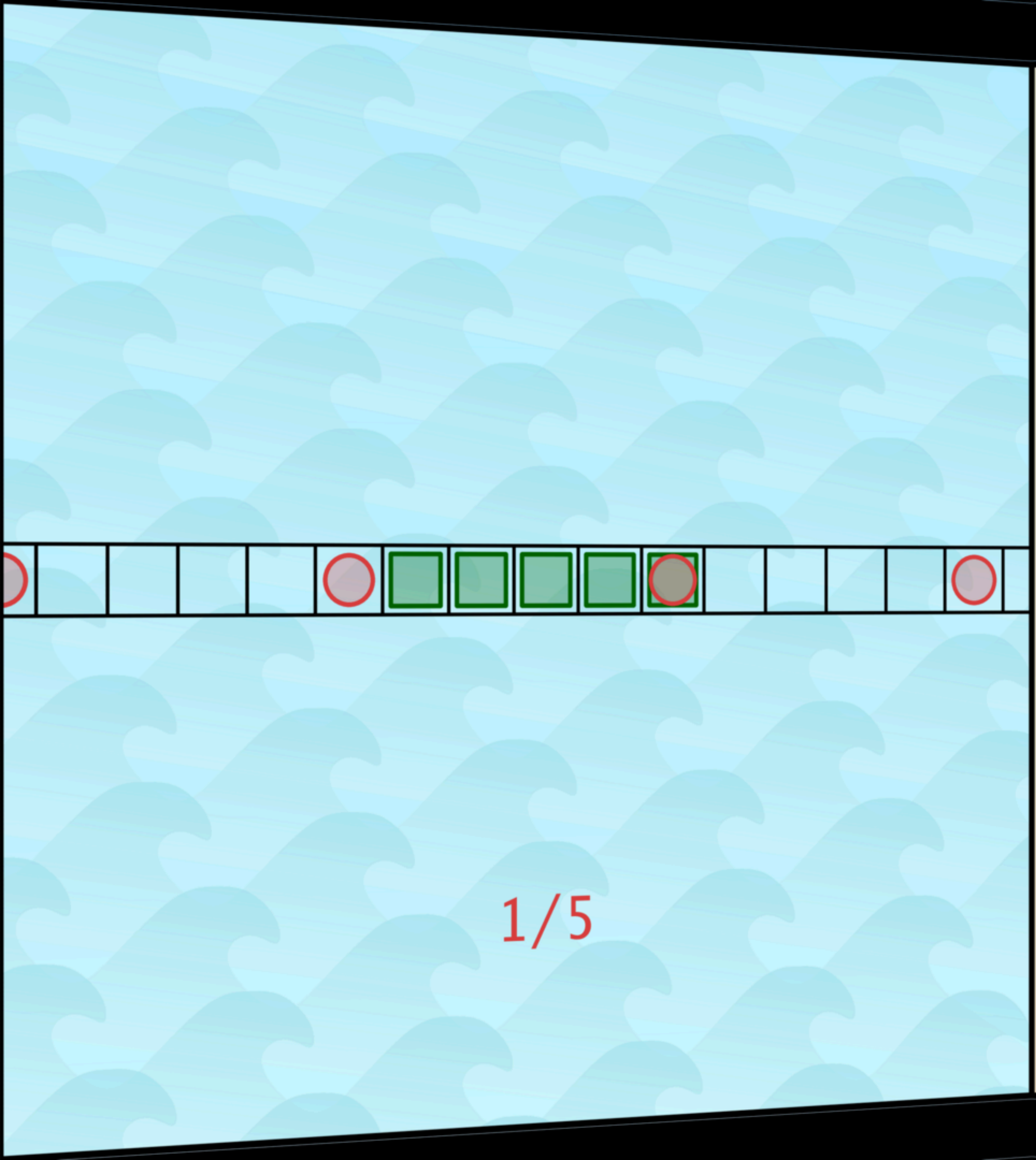
Ships of length k can be hit with density 1/k...

...if they are *connected*.



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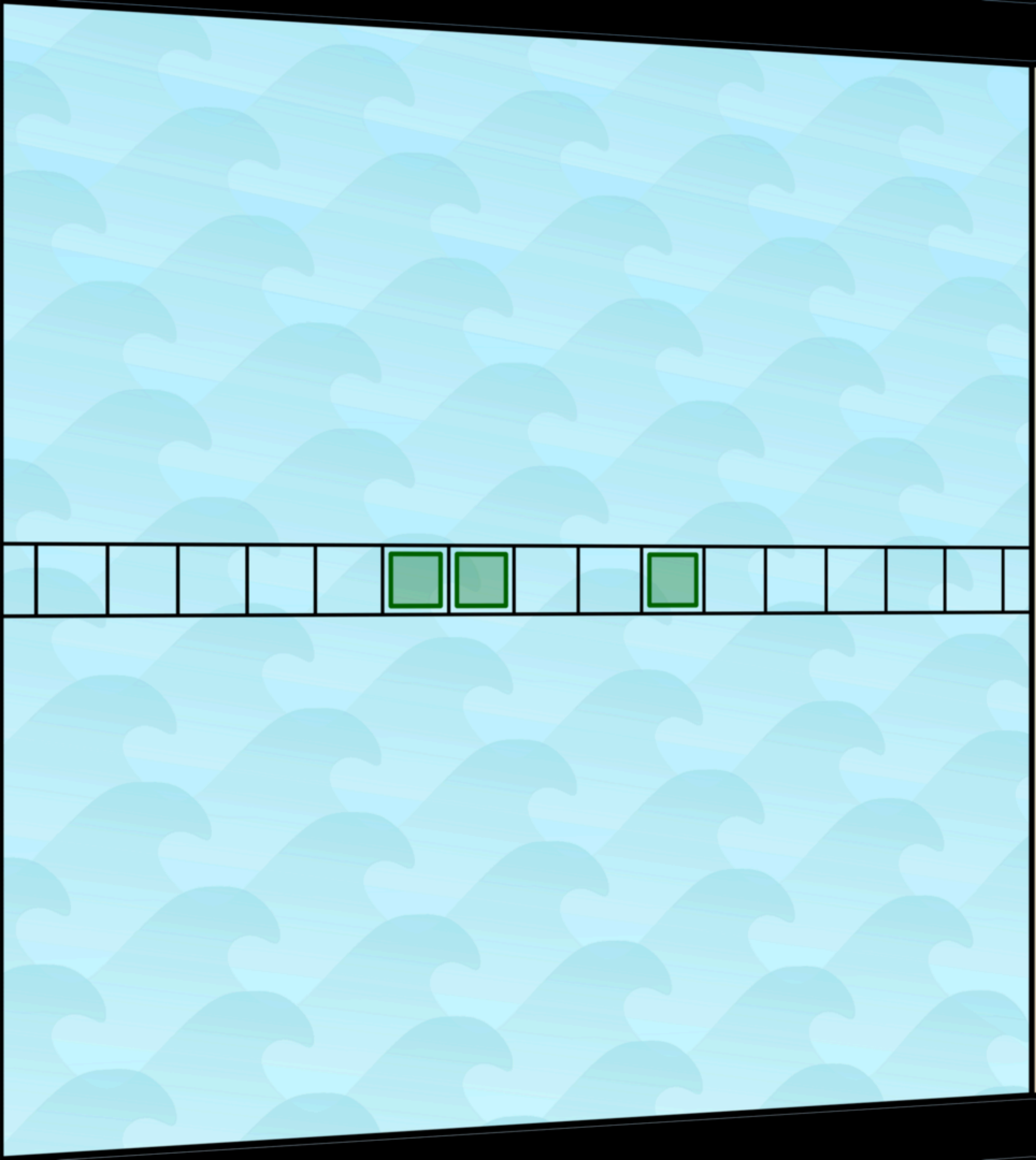
Disconnected ships behave in less predictable ways.



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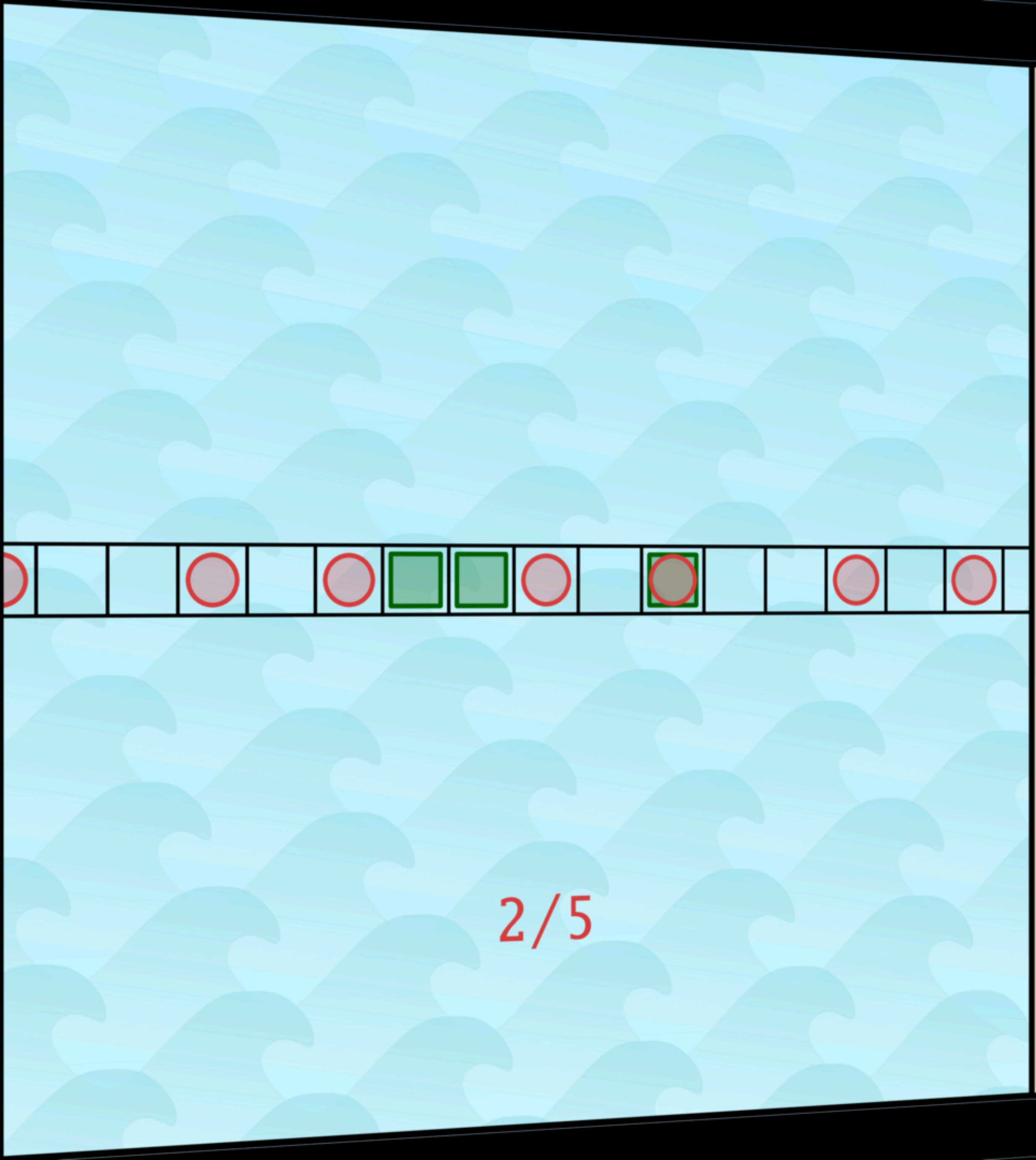

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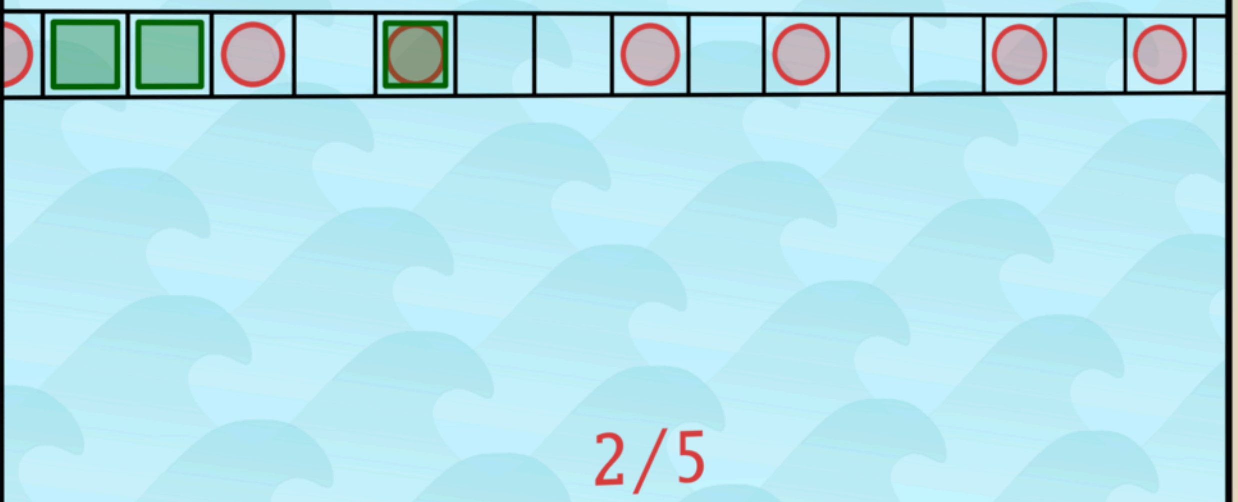

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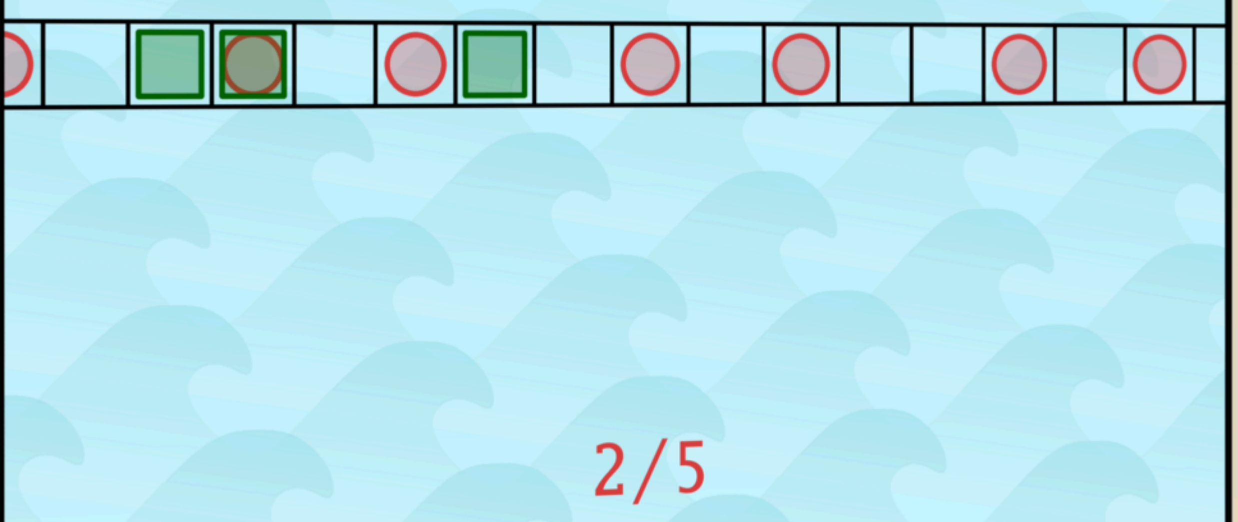

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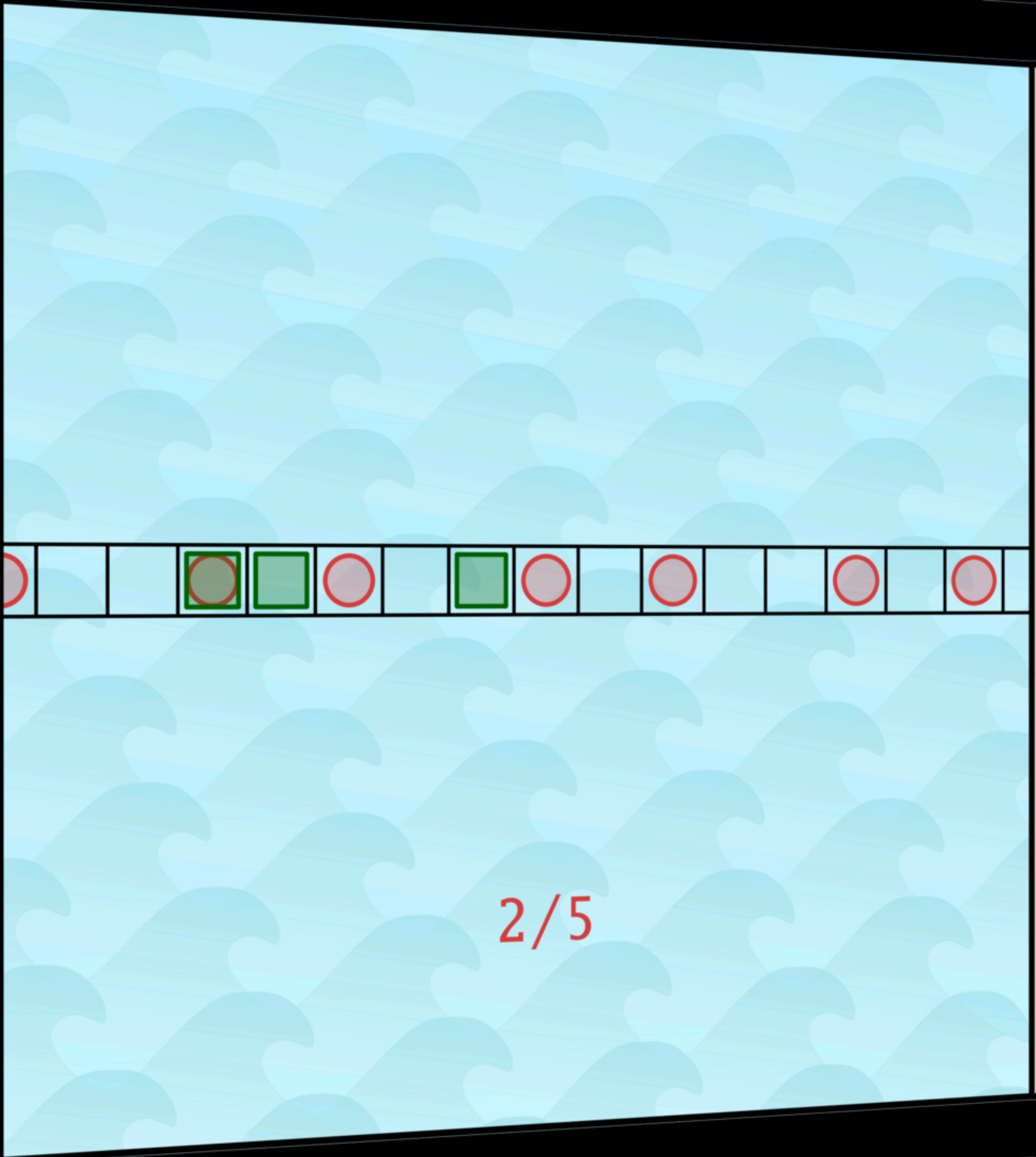

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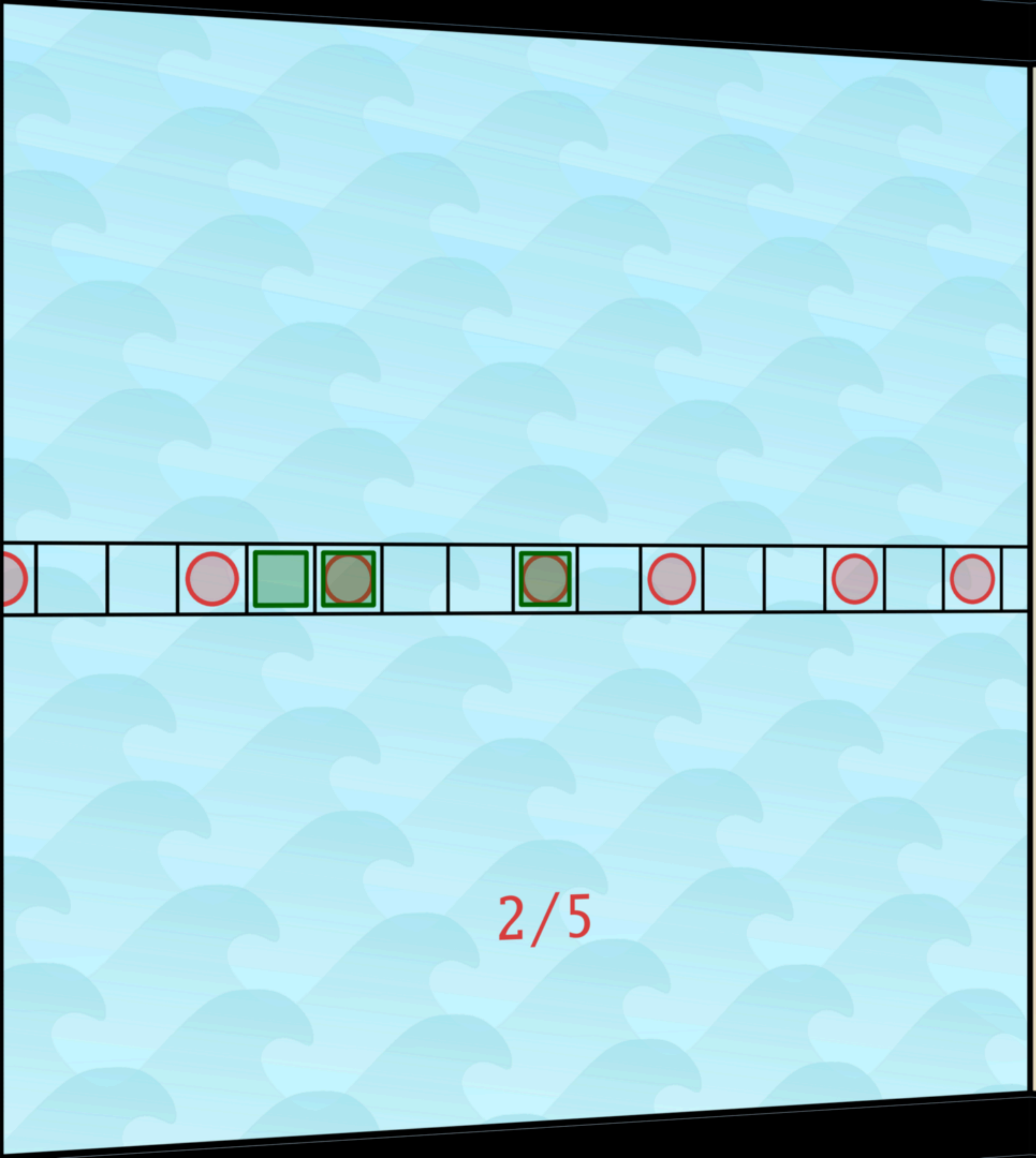

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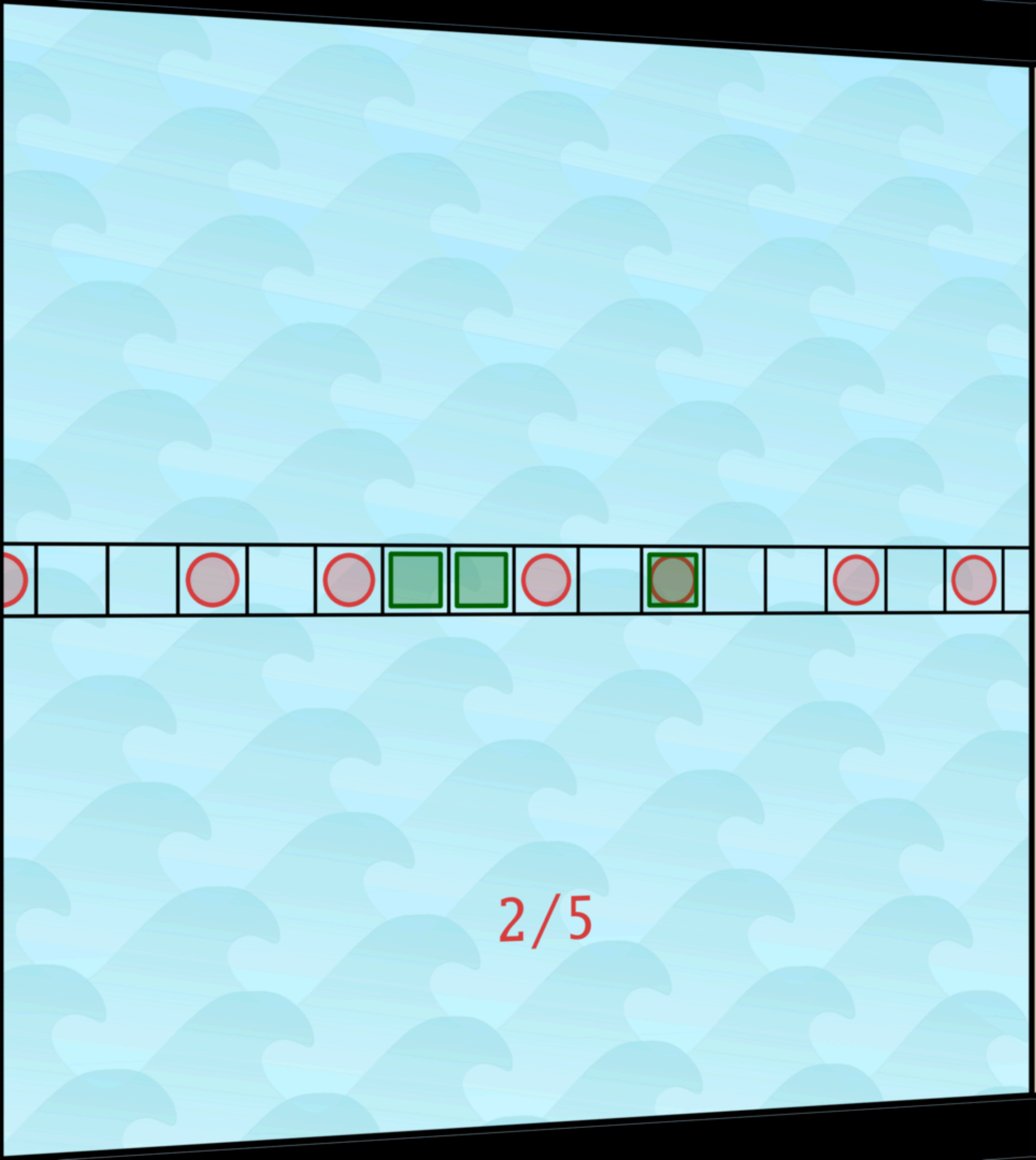

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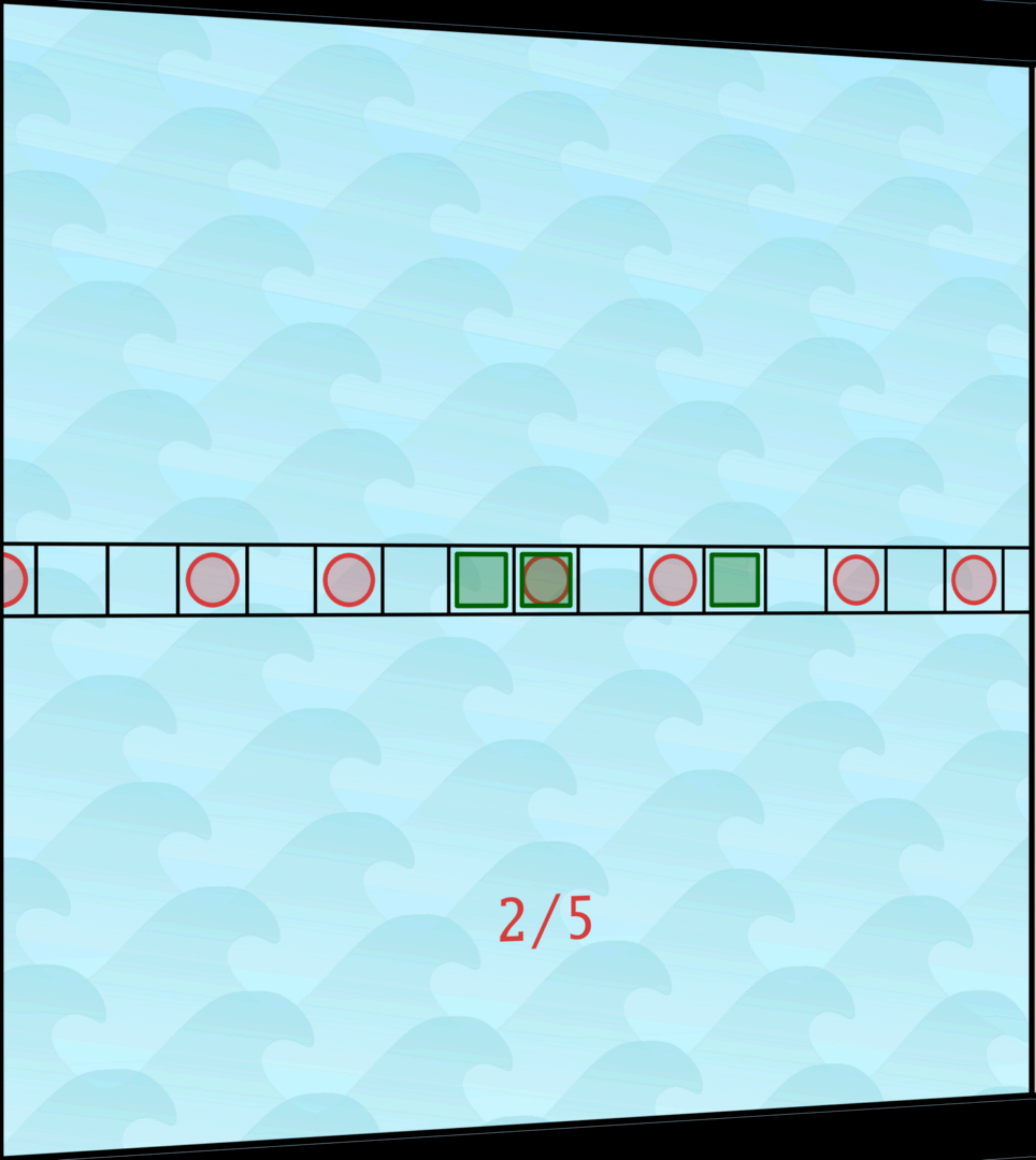

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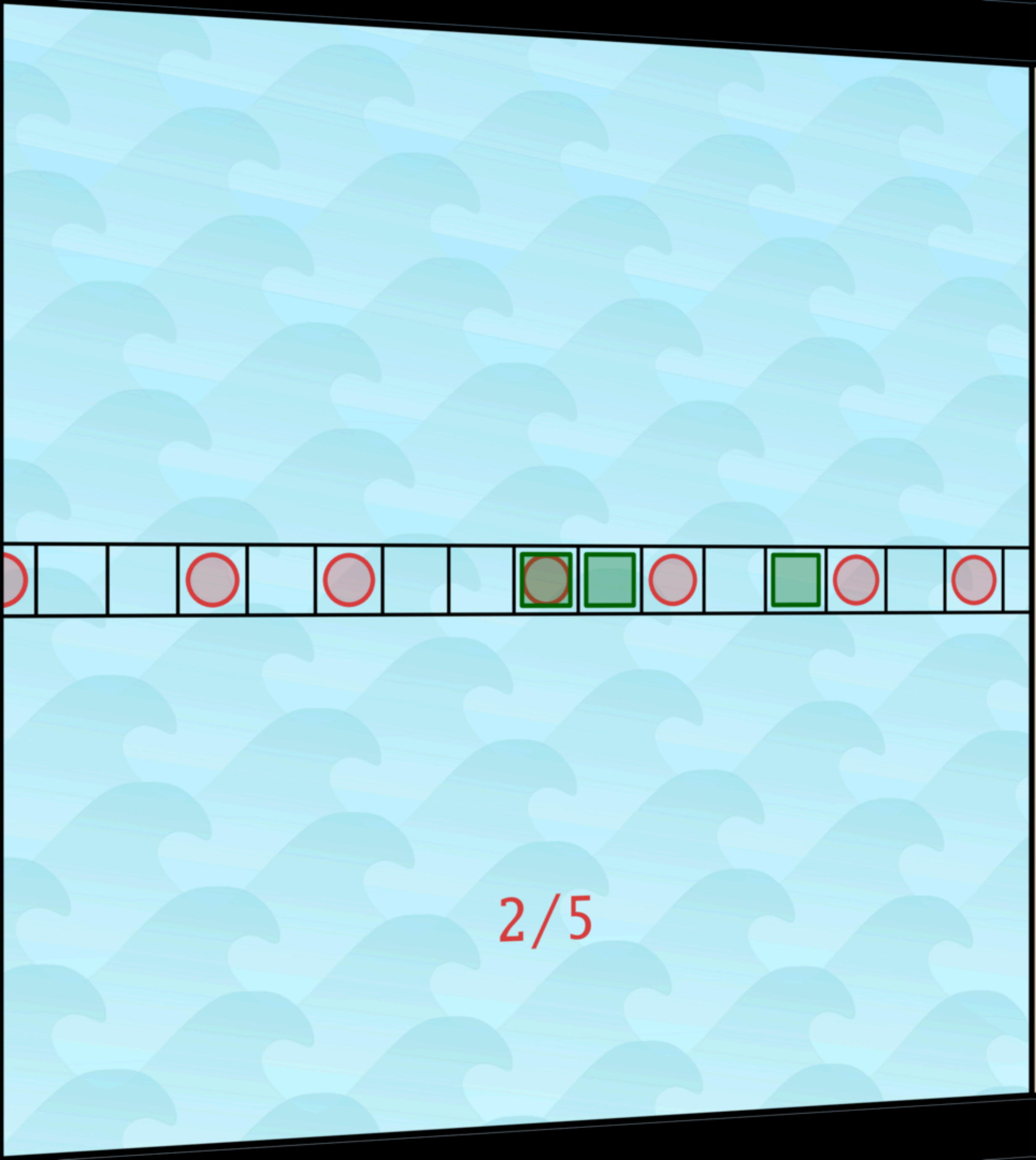

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Ships of length k can be hit with density 1/k...

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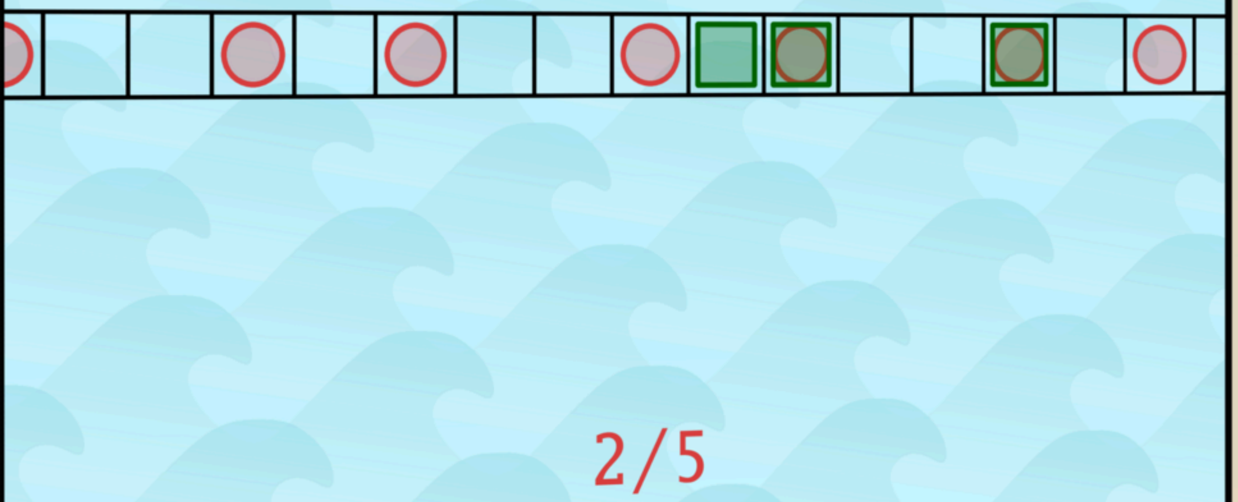

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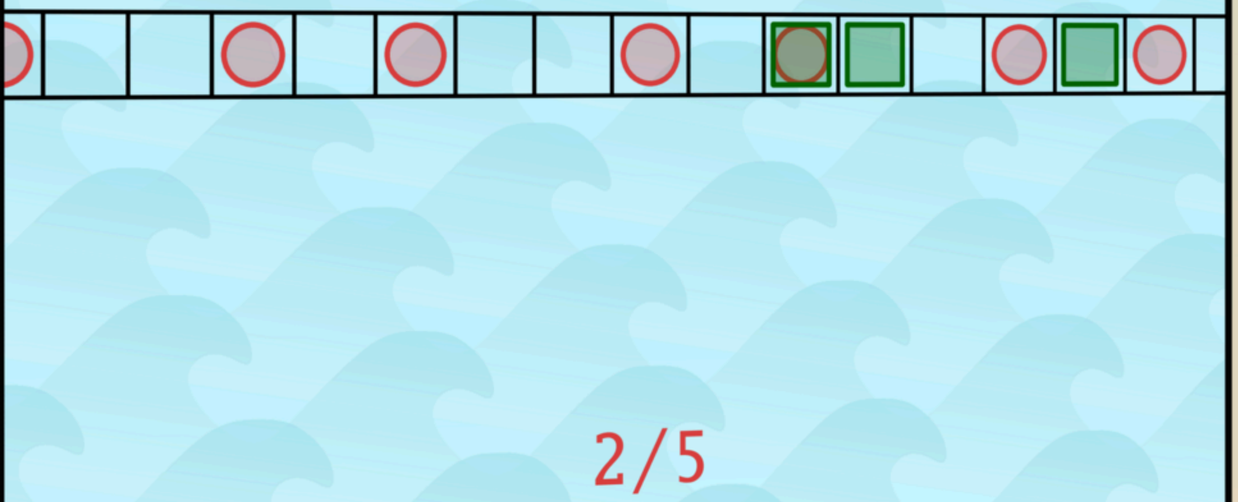

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2/5

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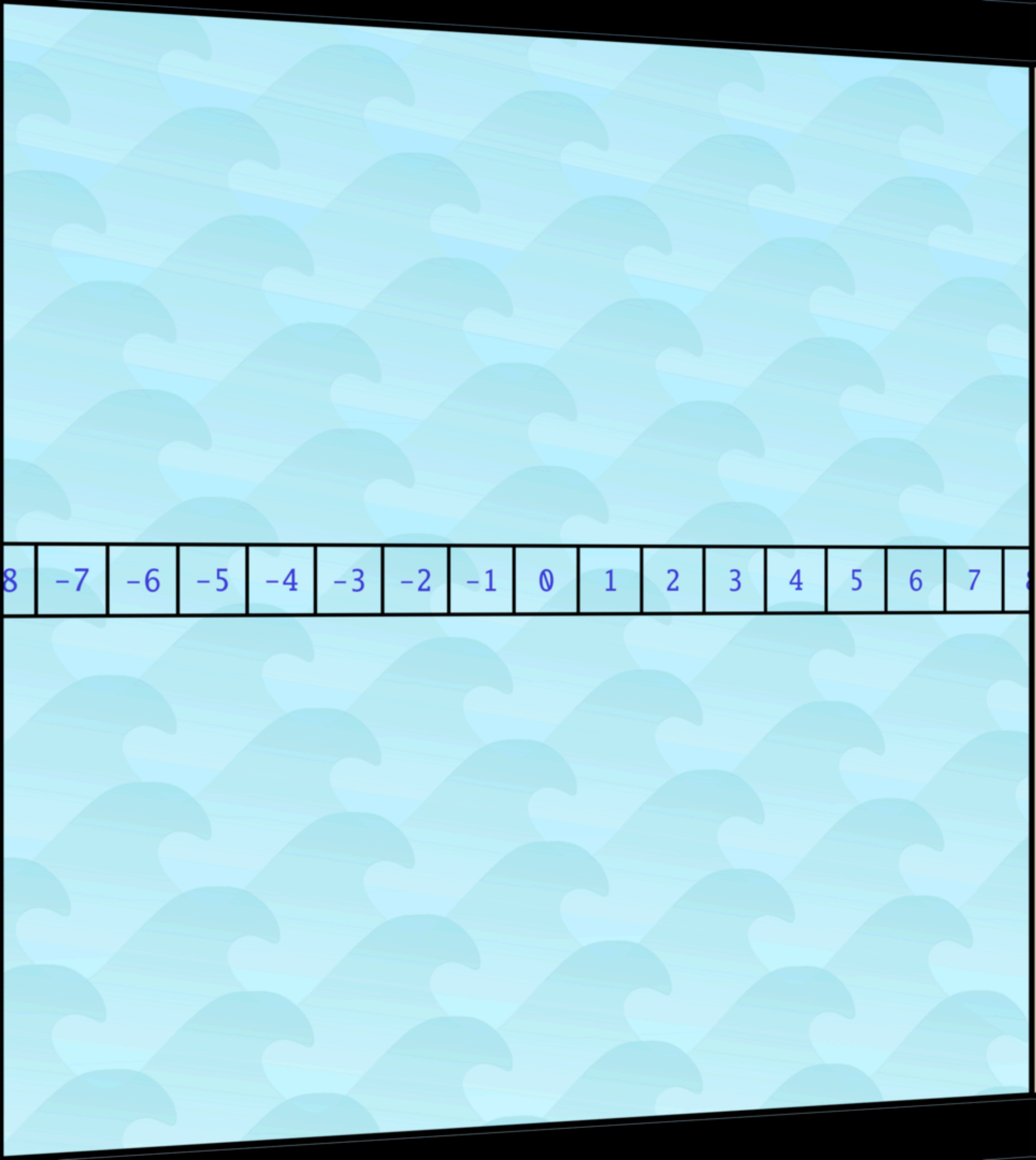
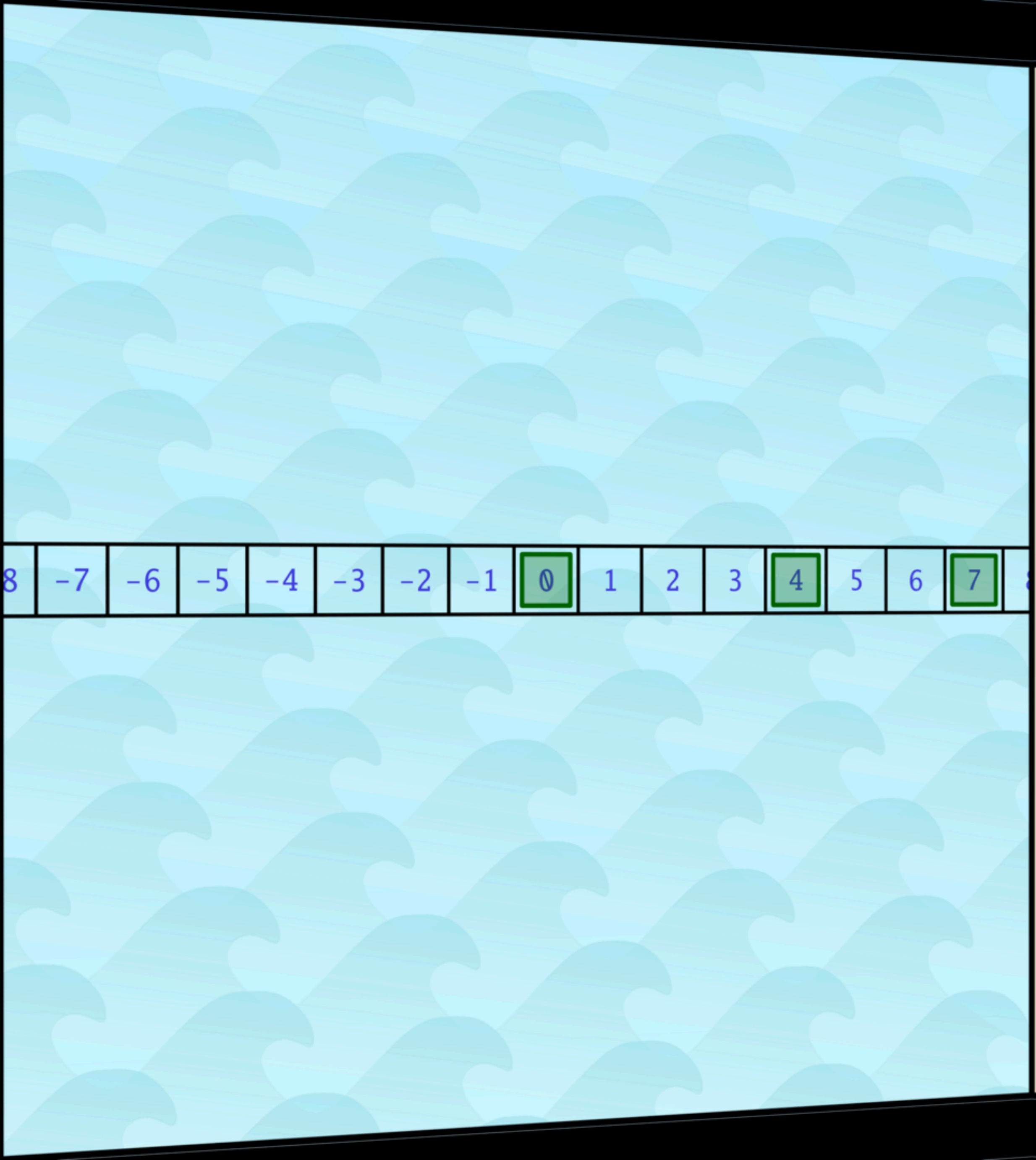


Ships $[0, a, b]$ with a and b coprime.



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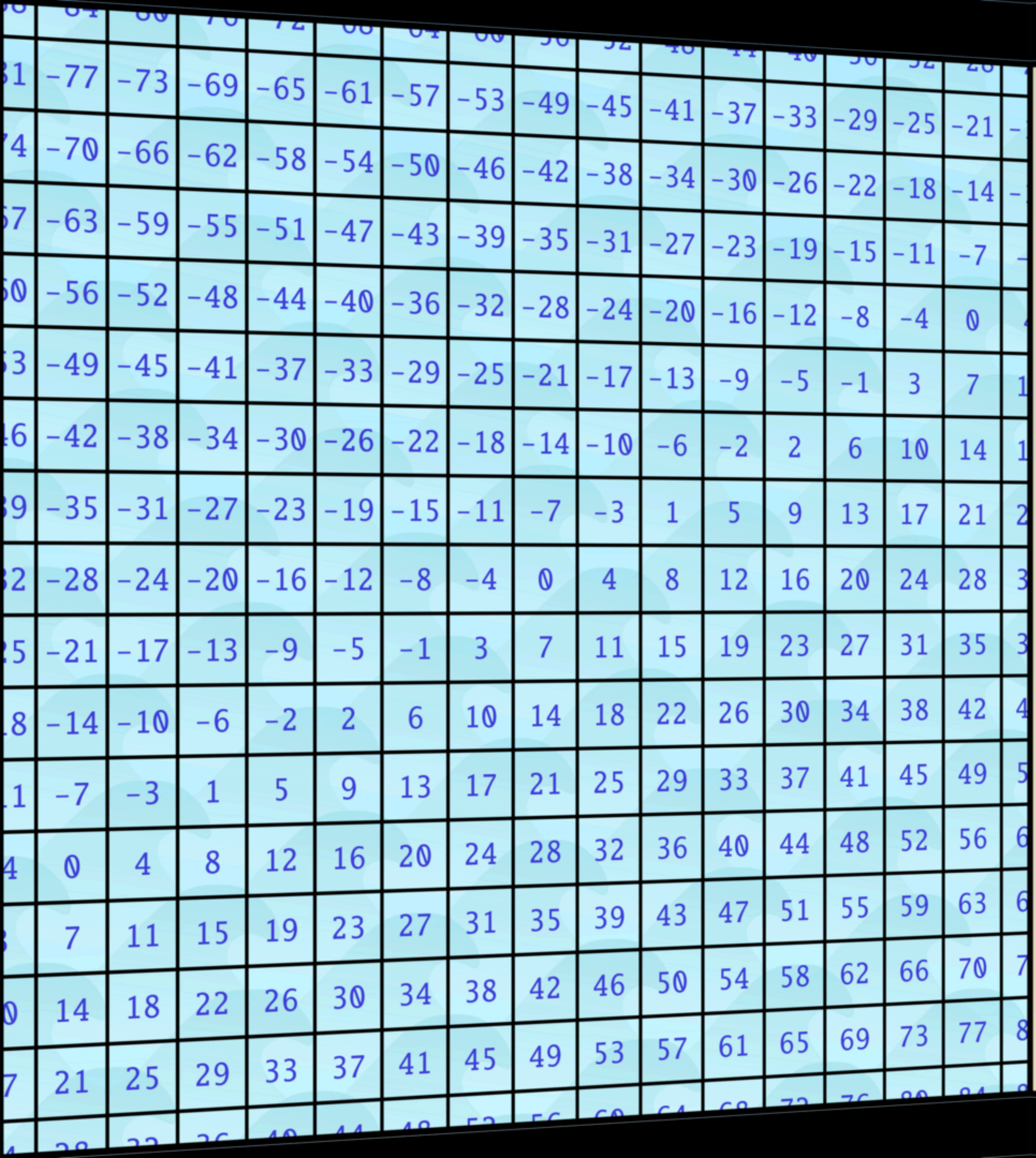

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Since a and b are coprime, every strip of width b contains all integers exactly once!

-68	-64	-60	-56	-52	-48	-44
-61	-57	-53	-49	-45	-41	-37
-54	-50	-46	-42	-38	-34	-30
-47	-43	-39	-35	-31	-27	-23
-40	-36	-32	-28	-24	-20	-16
-33	-29	-25	-21	-17	-13	-9
-26	-22	-18	-14	-10	-6	-2
-19	-15	-11	-7	-3	1	5
-12	-8	-4	0	4	8	12
-5	-1	3	7	11	15	19
2	6	10	14	18	22	26
9	13	17	21	25	29	33
16	20	24	28	32	36	40
23	27	31	35	39	43	47
30	34	38	42	46	50	54
37	41	45	49	53	57	61
44	48	52	56	60	64	68

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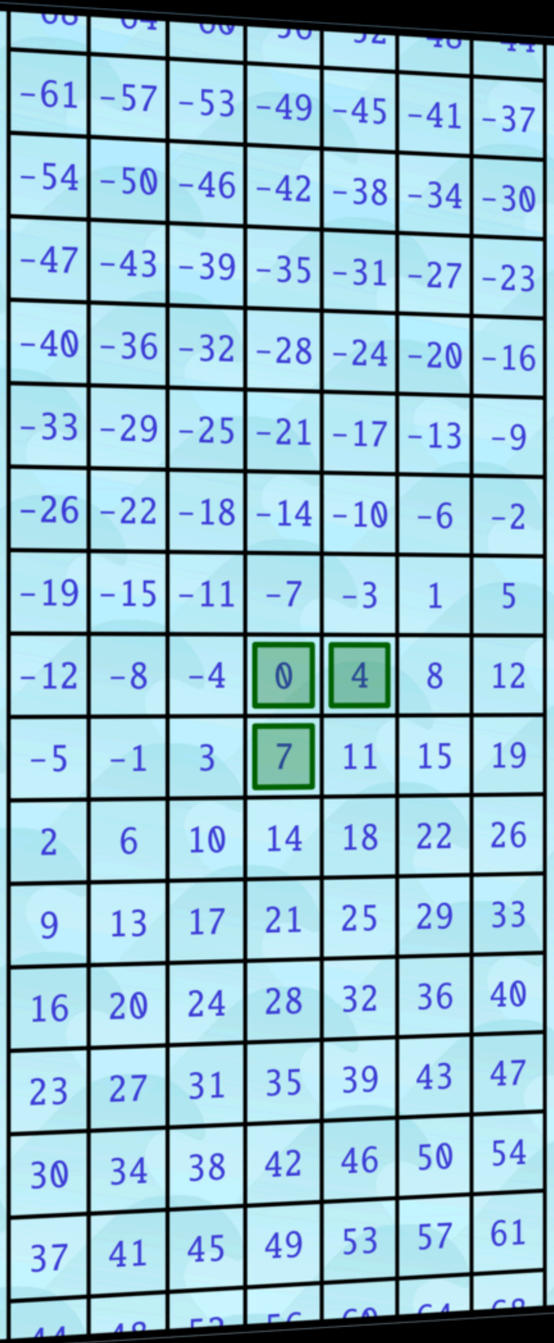

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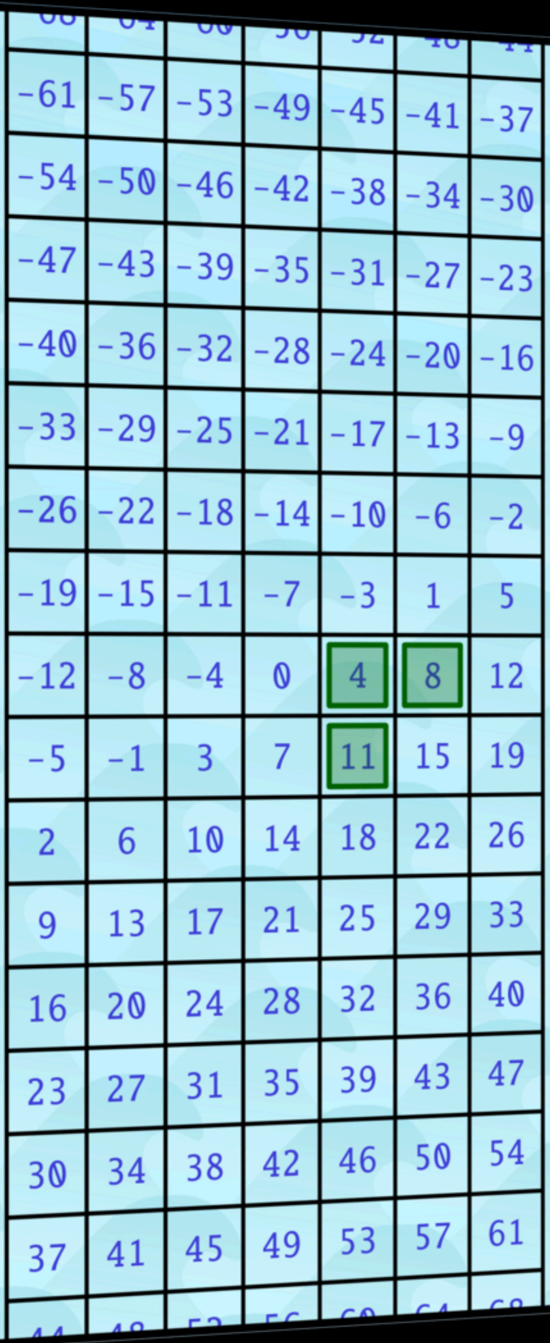

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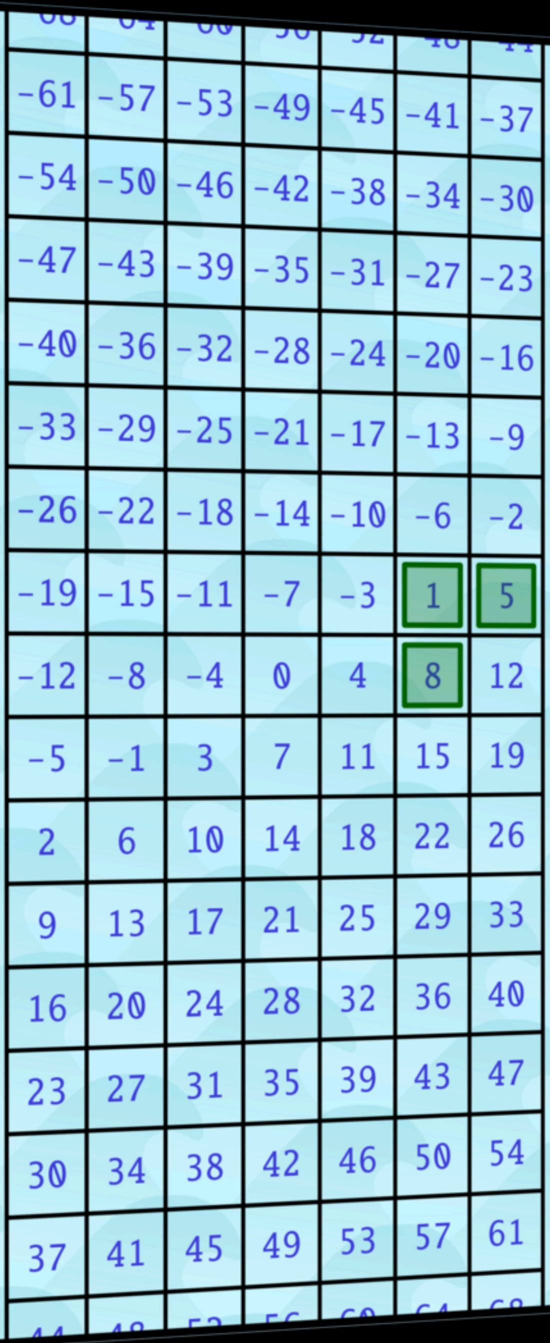

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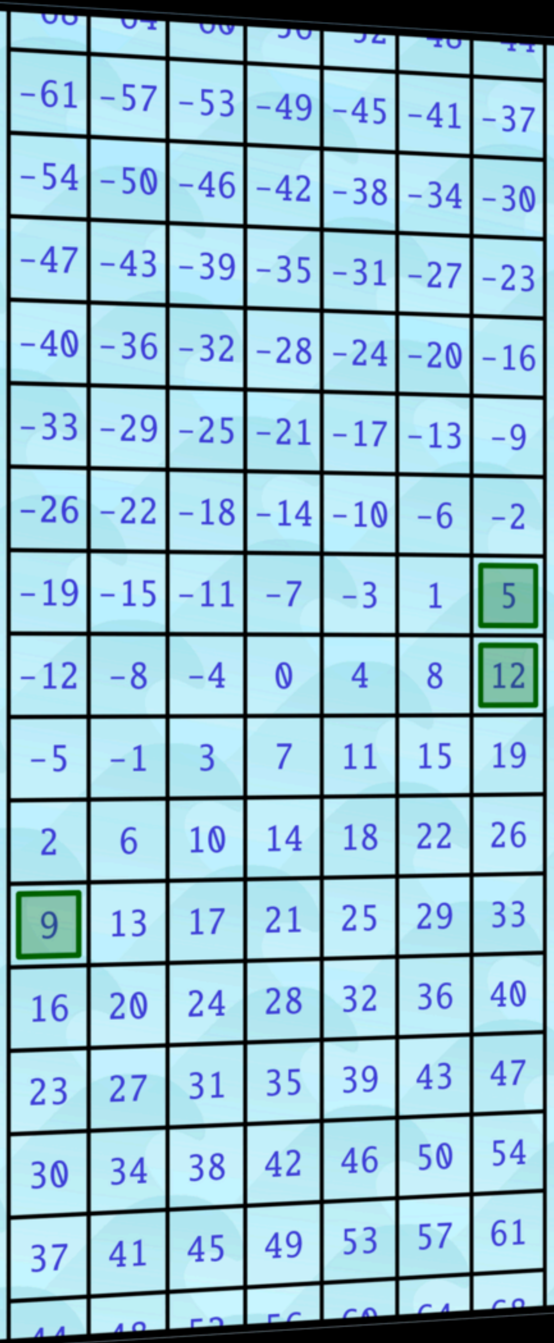

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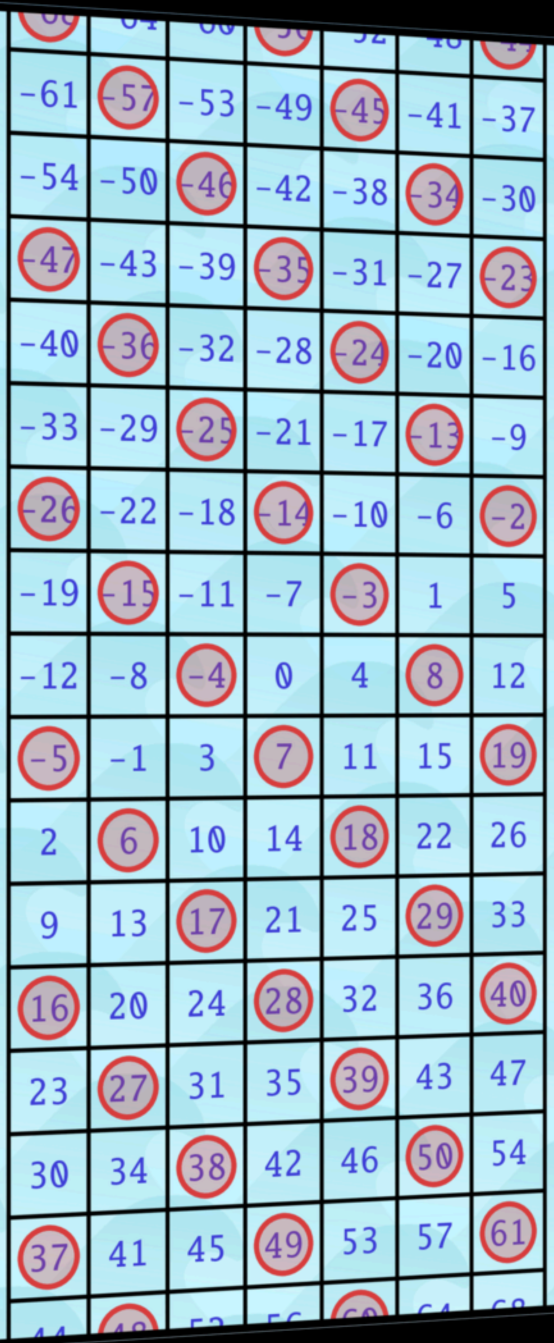

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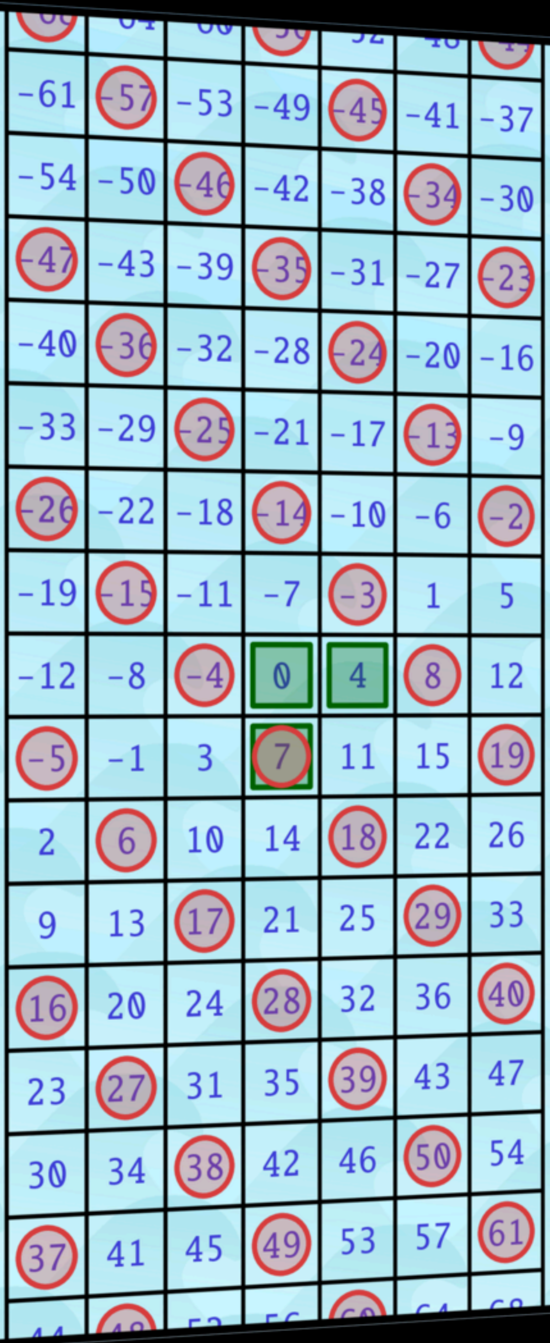

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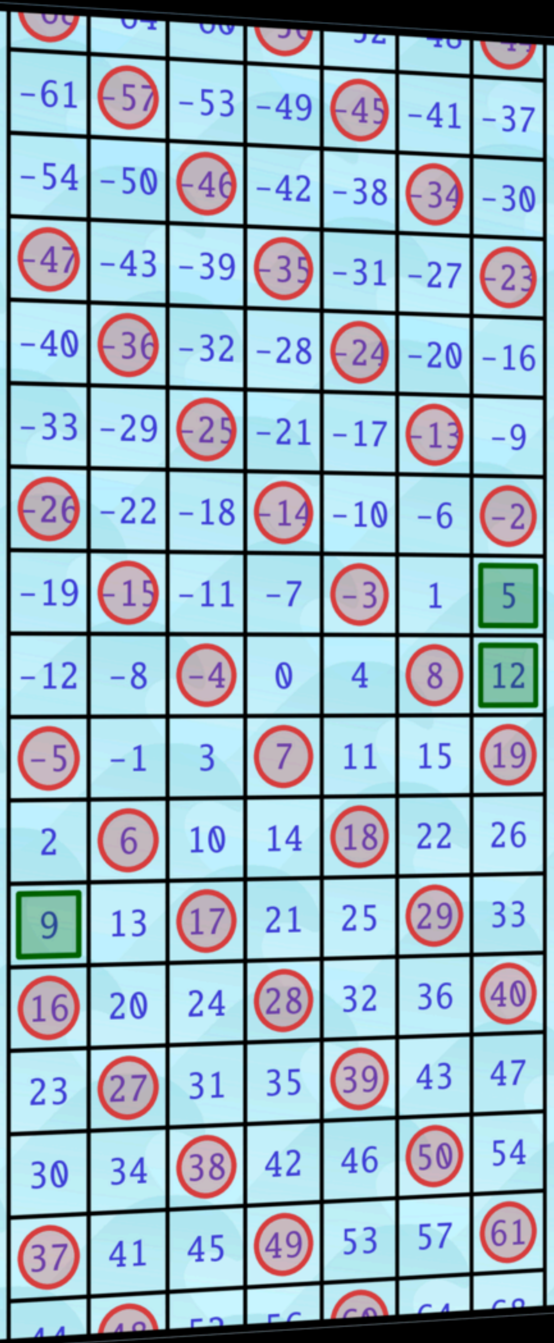

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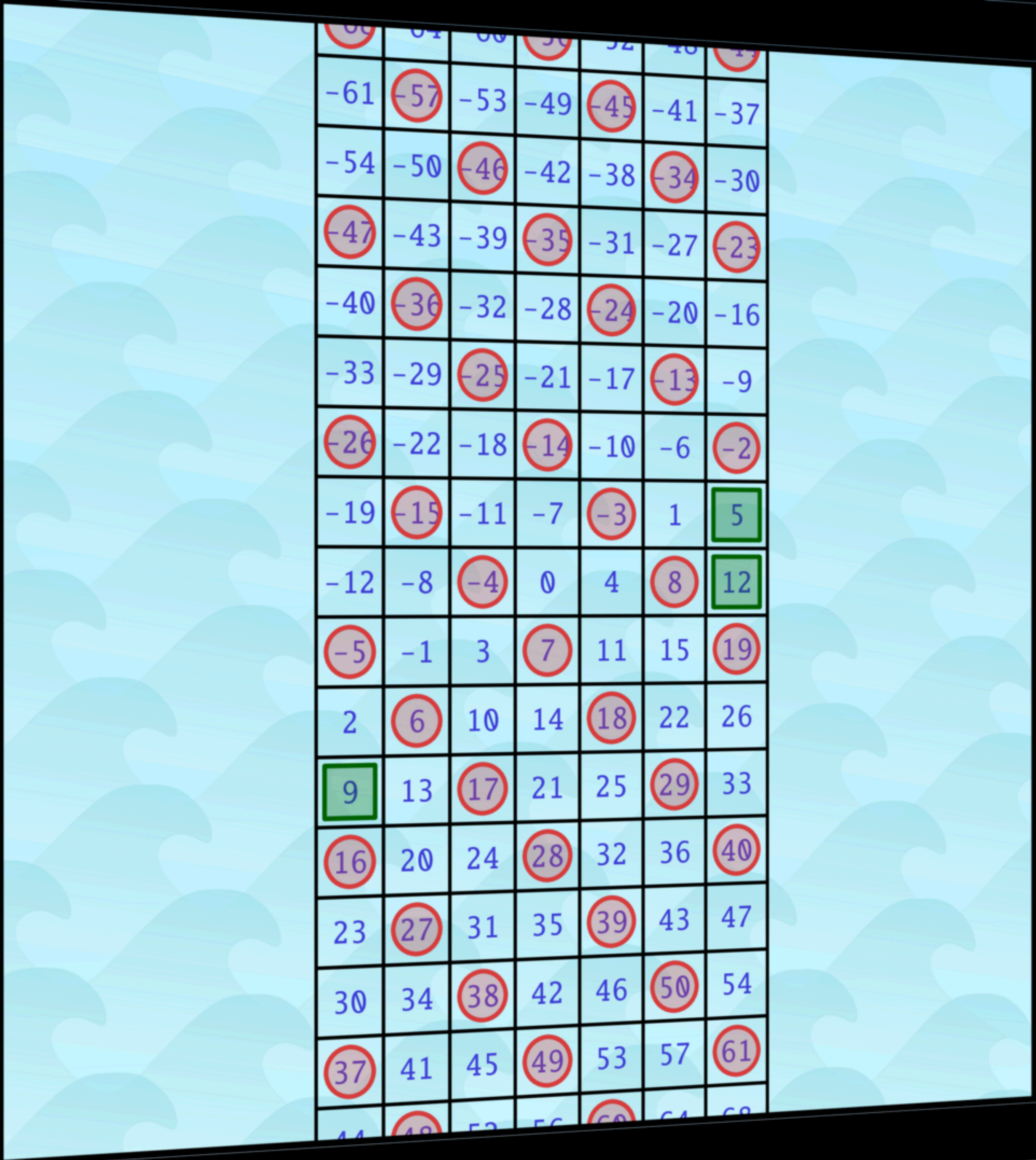

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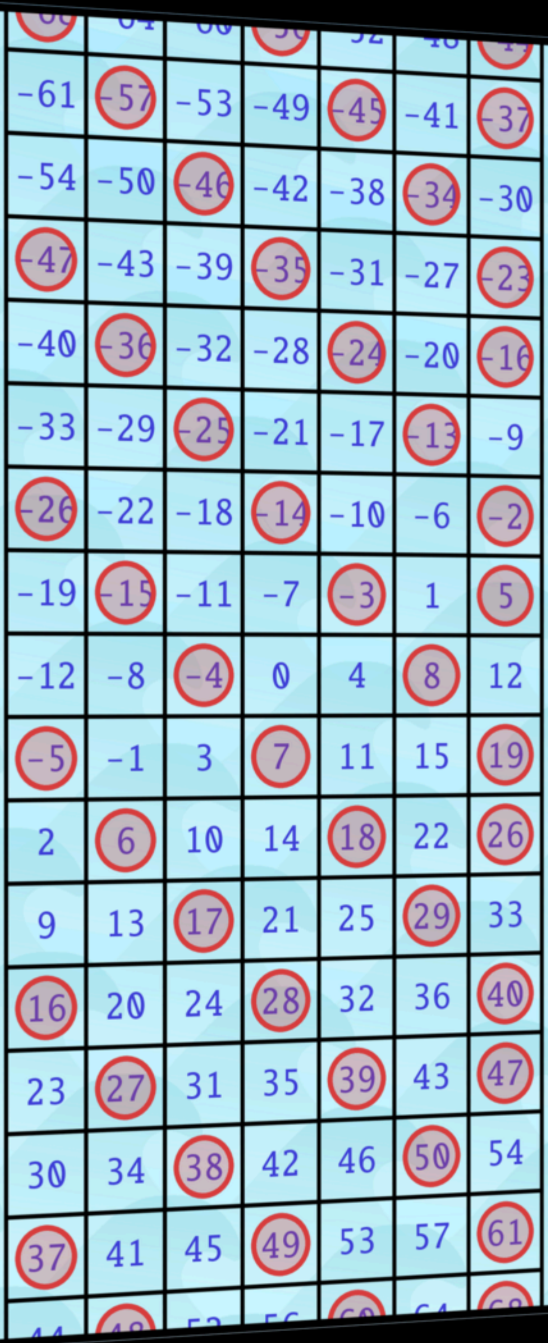

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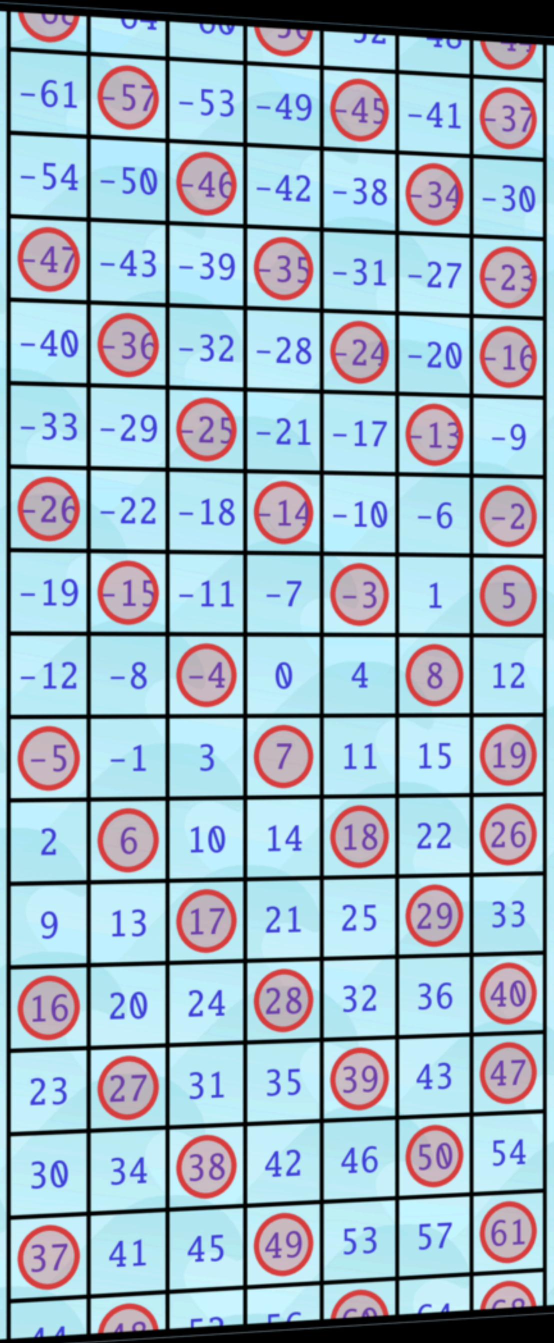

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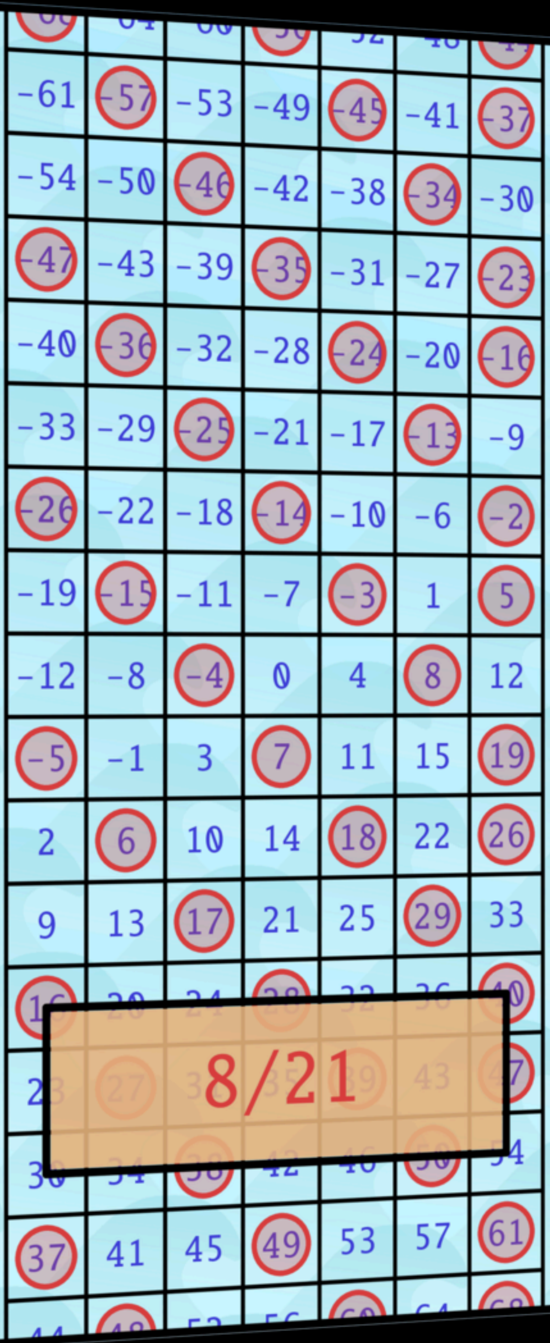
This pattern works. What is the density?



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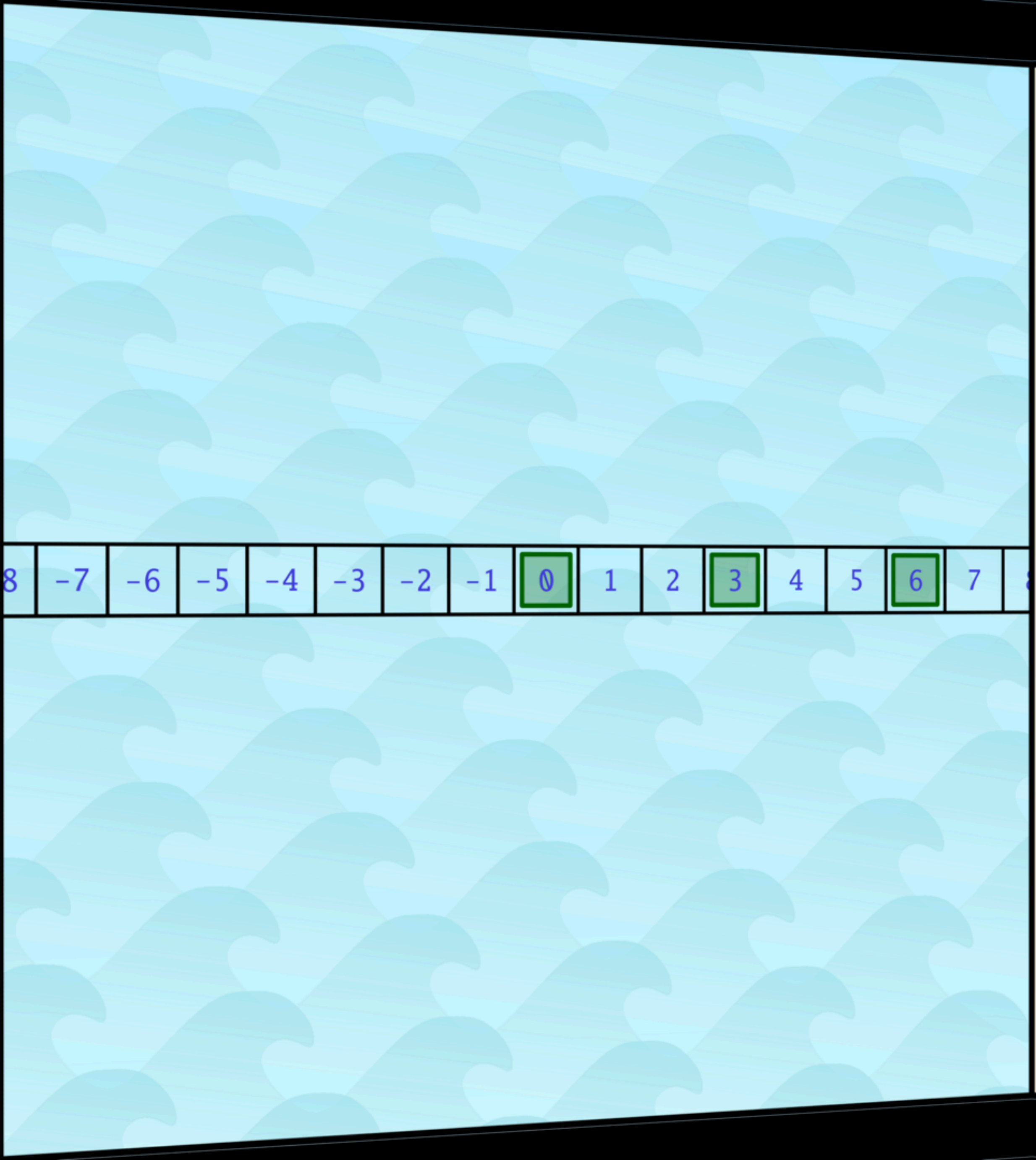
density = $\frac{b-1}{b} \times \frac{1}{3} + \frac{1}{b} \times \frac{2}{3}$

density $\to 1/3$ as $b \to \infty$

Small disconnected ships are harder to hit!



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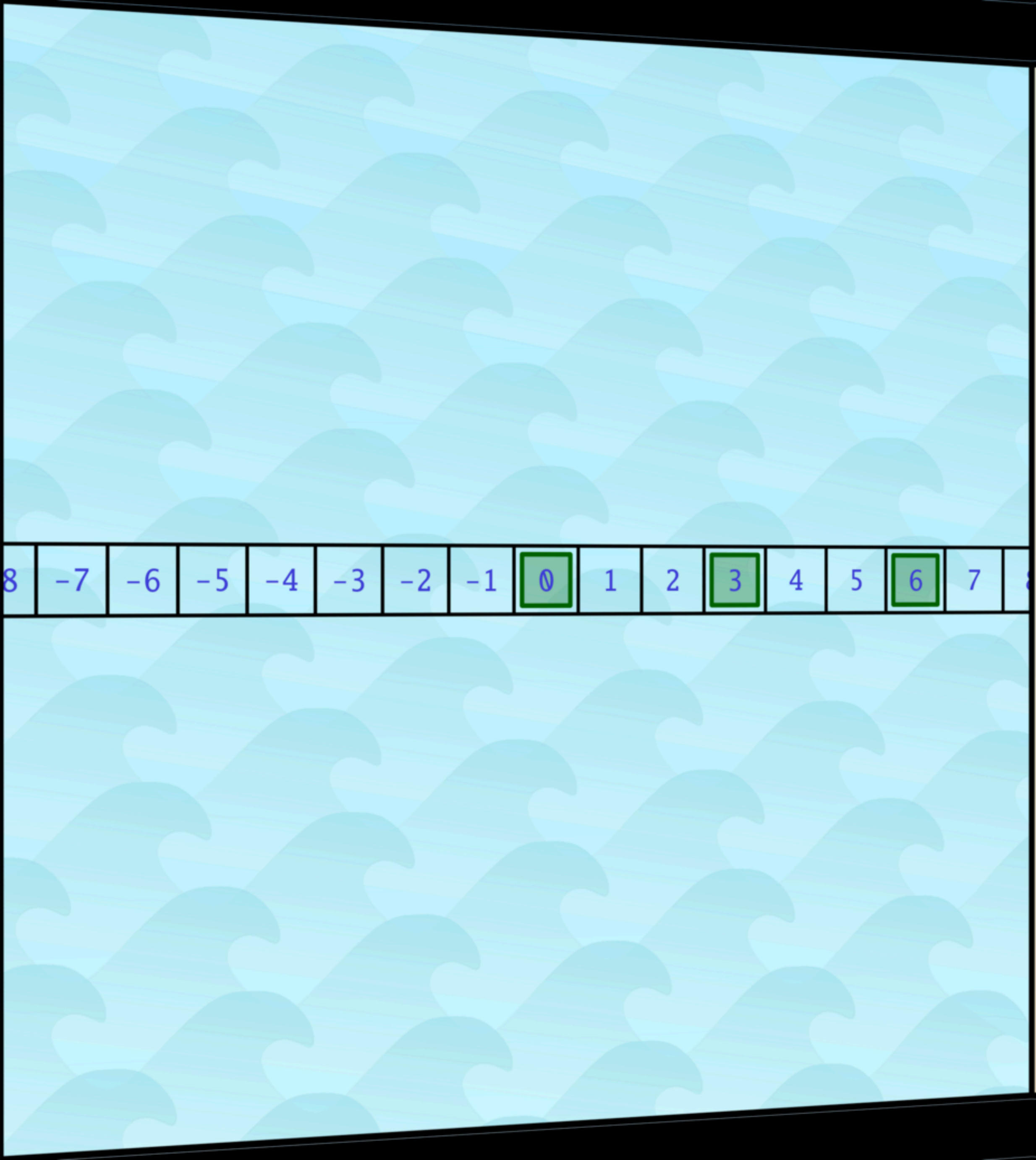

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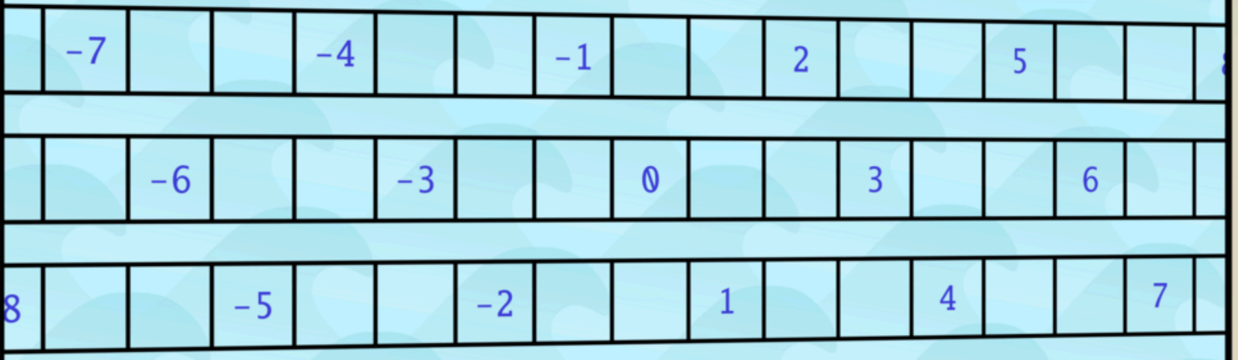

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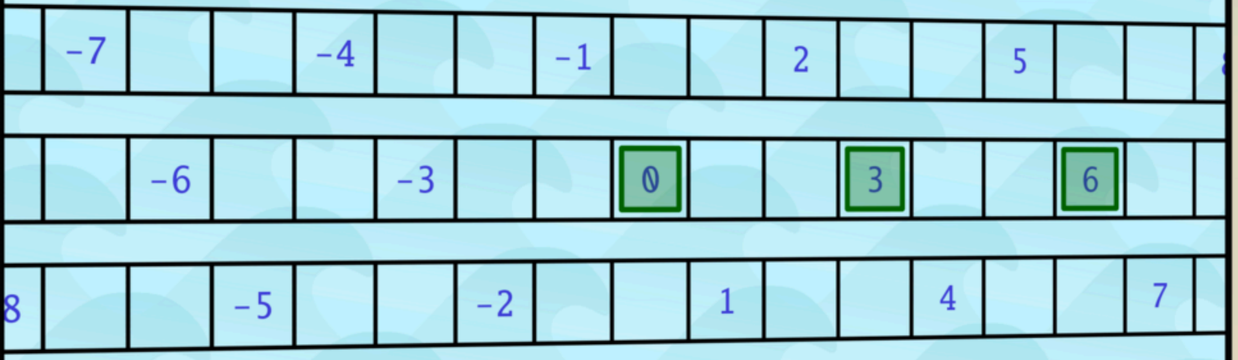

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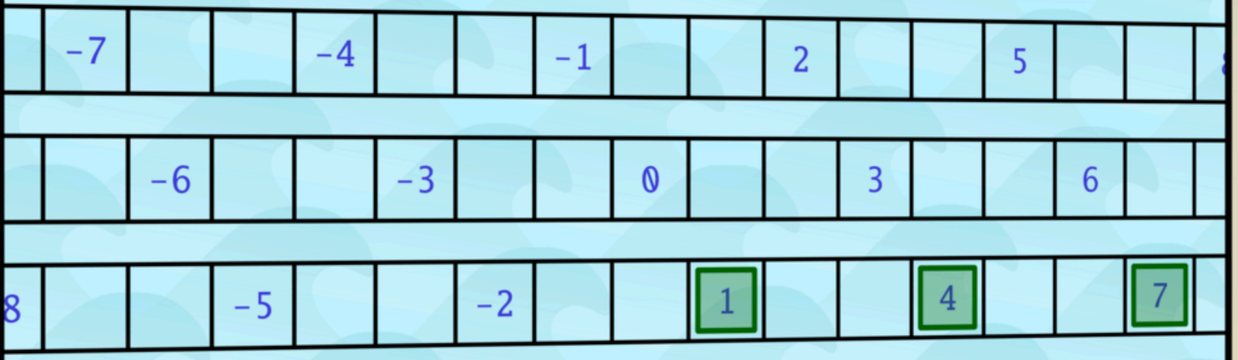

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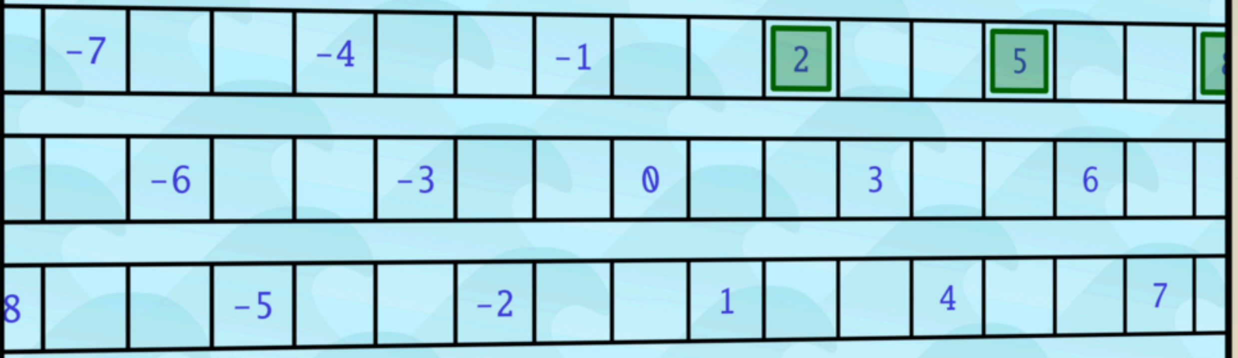

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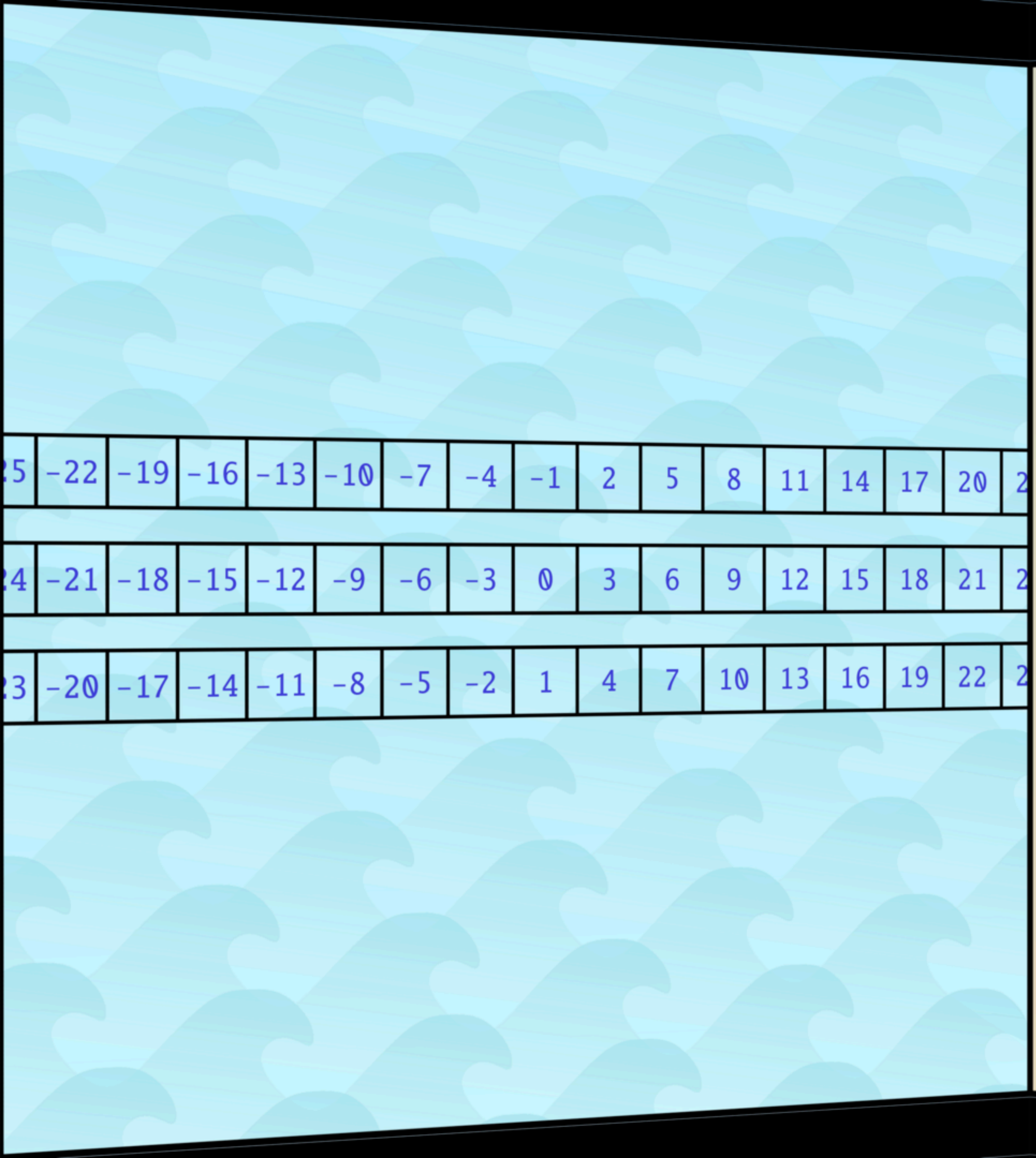

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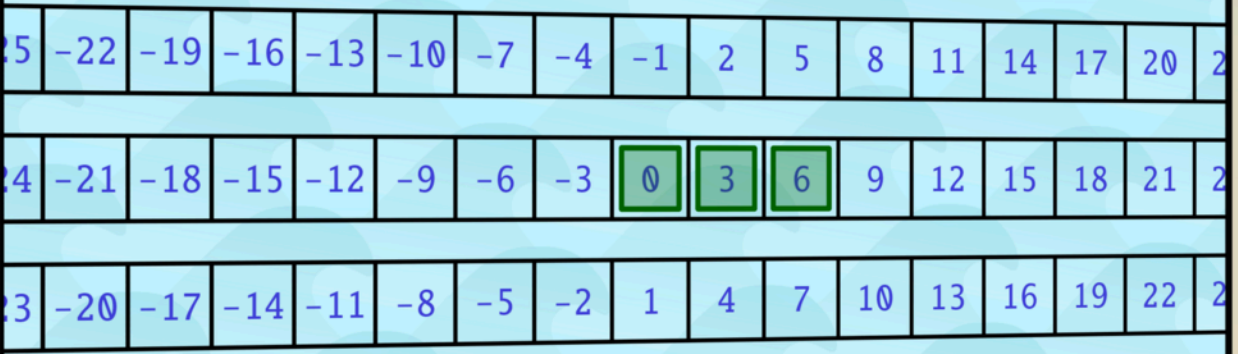

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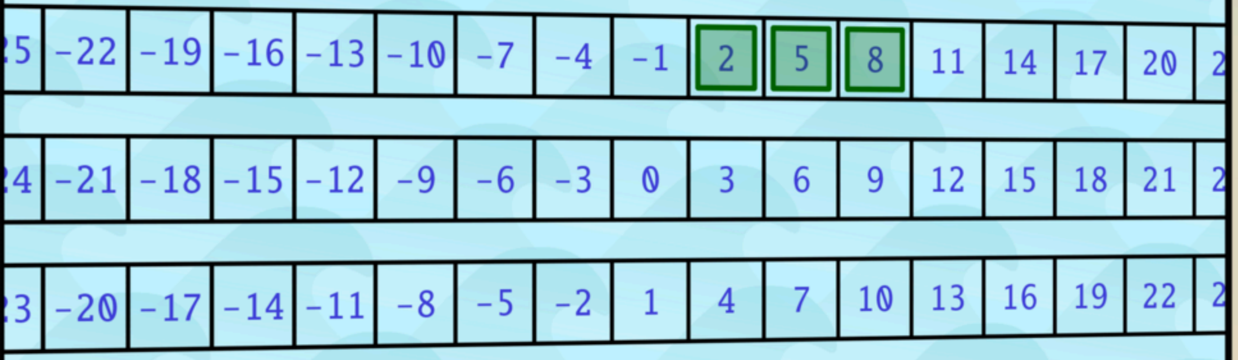

Ships $[0, a, b]$ with a and b *not* coprime

$a = k \cdot a' \;\; b = k \cdot b'$

Same density as $[0, a', b']$!

CHAPTER 5

CONTRIBUTION

Theorem 1 *Given a ship S with span s, the density of S can be computed in time polynomial in 2^s.*

Theorem 2 $S = \{[0, da], [0, db]\}$, a, b *coprime. Density is*

$$\begin{cases} 1/2 & \text{if } a \text{ and } b \text{ odd,} \\ \frac{a+b+1}{2(a+b)} & \text{otherwise.} \end{cases}$$

Theorem 3 *The toughest set of n shapes of size 2 has density $n/(n+1)$.*

Theorem 4 *Let S be a 3-shape and its reflection. Then the density is $\leq \frac{2}{5}$.*

Theorem 5 *The simplest set of n shapes of size k has density $\frac{1}{k}$.*

Theorem 6 *The toughest set of n shapes of size k has density d where*

$$1 - \frac{k-1}{\sqrt[k]{nk!}} \leq d \leq \min\left\{\frac{n}{n+1}, \frac{1+\log(kn)}{k}\right\}$$

Generalizations to 2D

CHAPTER 6

OPEN PROBLEMS

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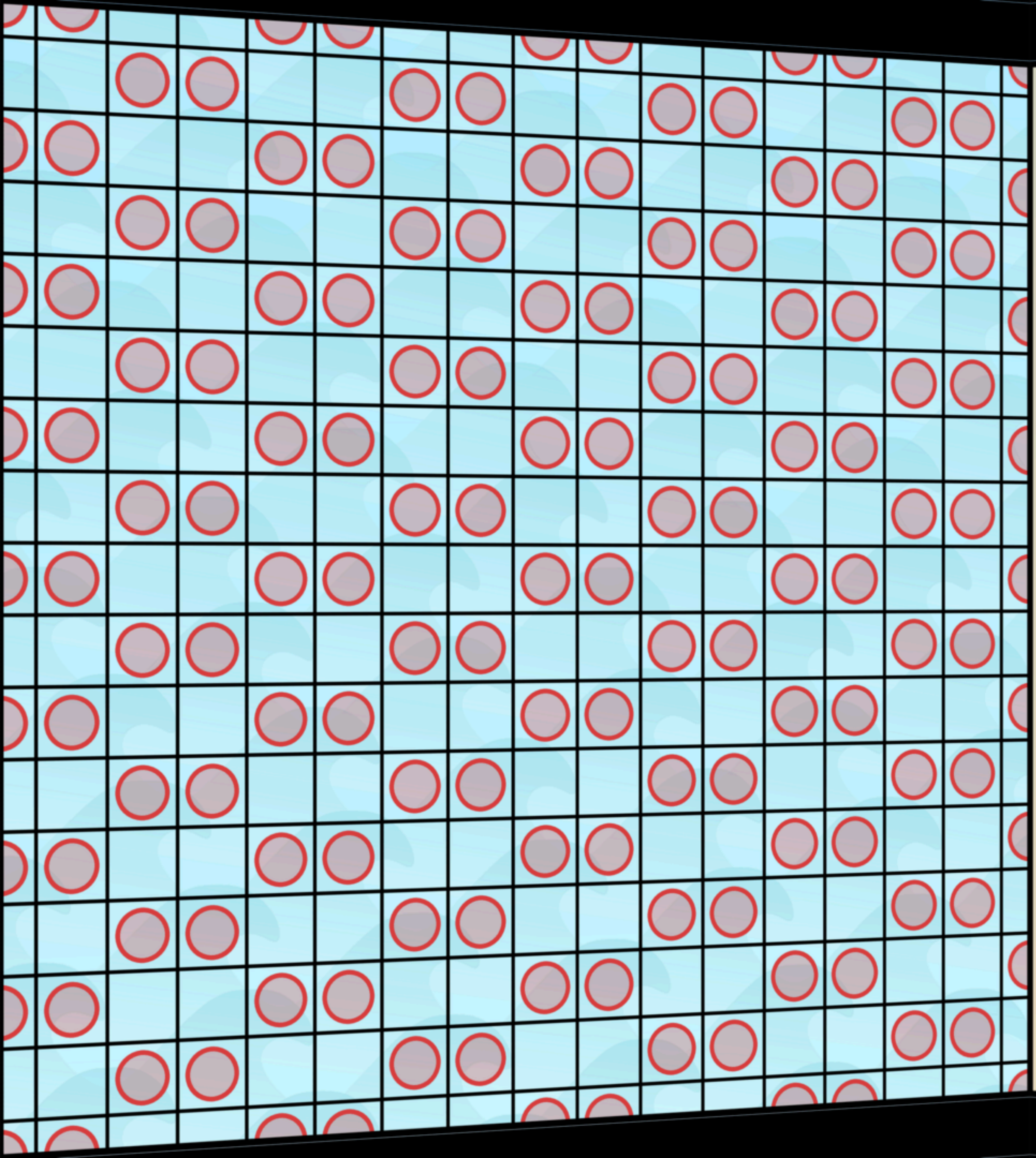

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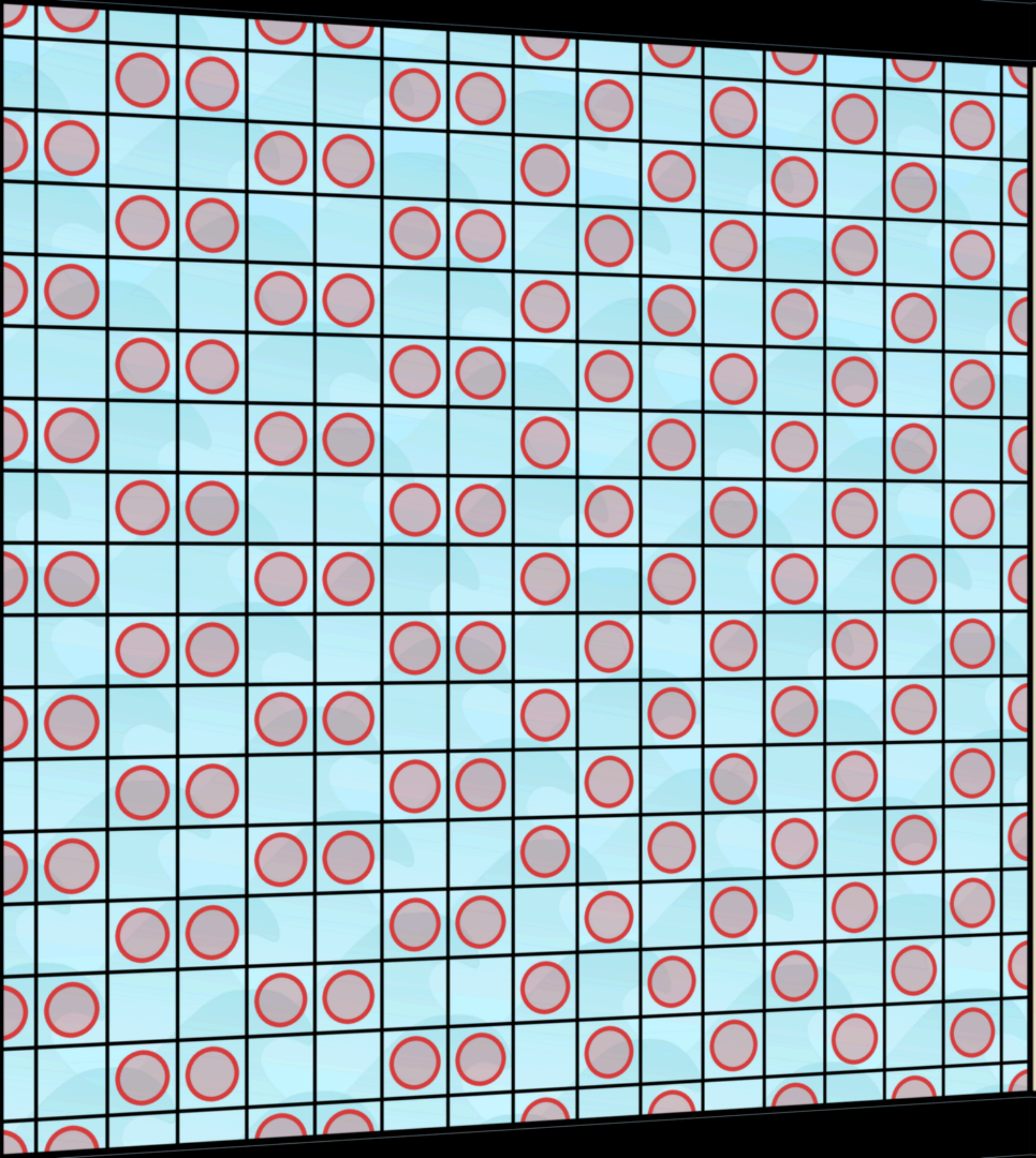

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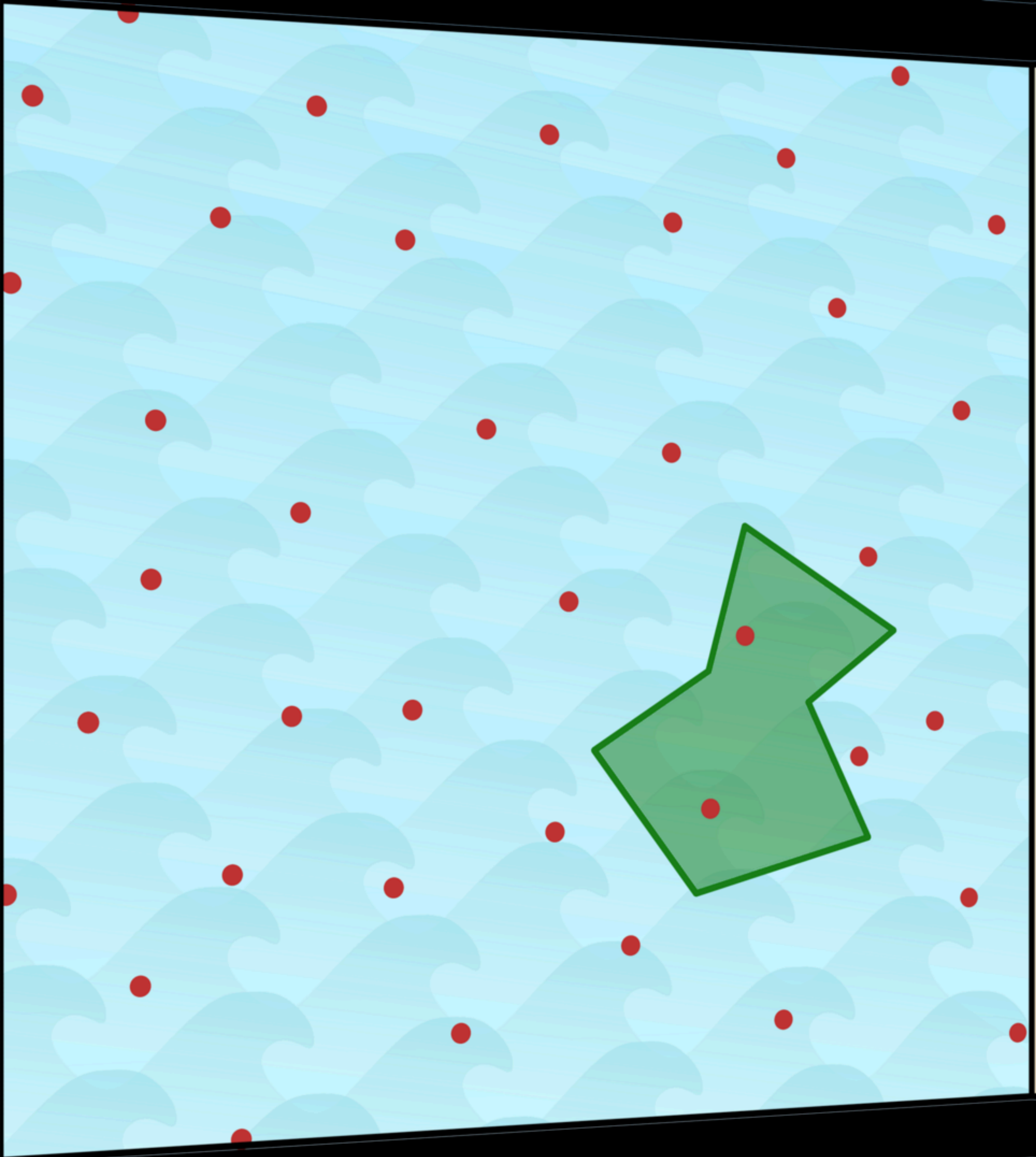

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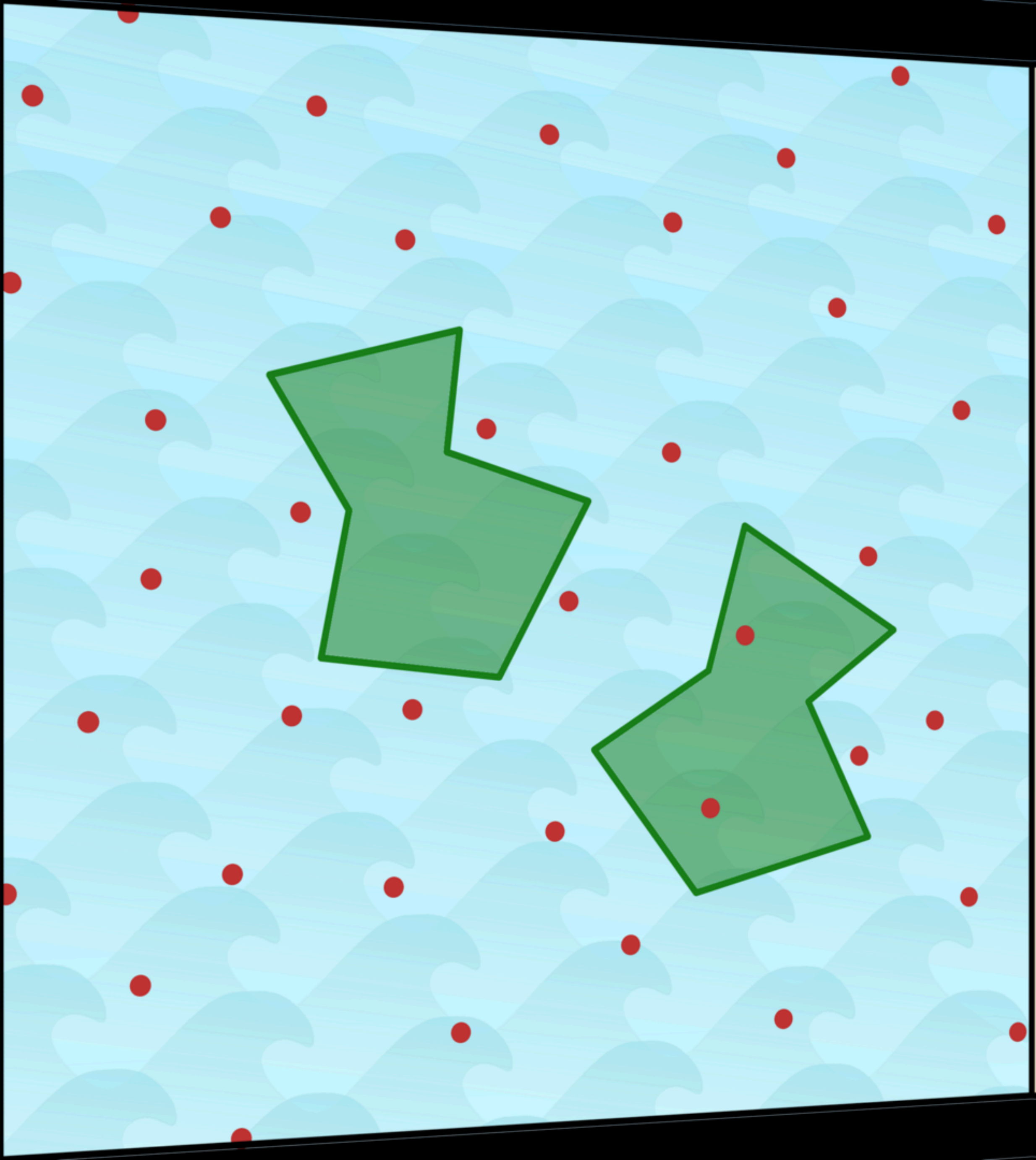

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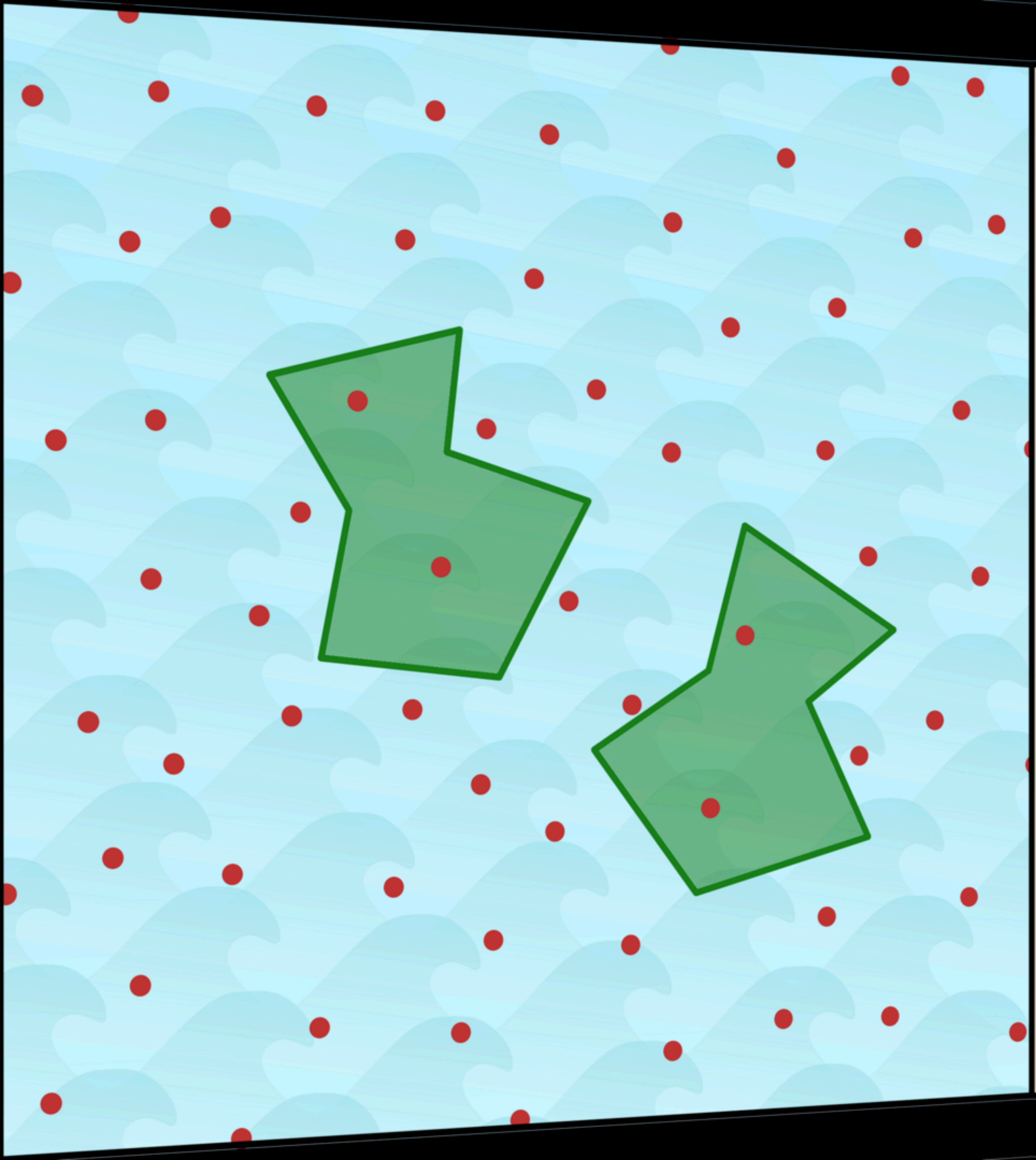

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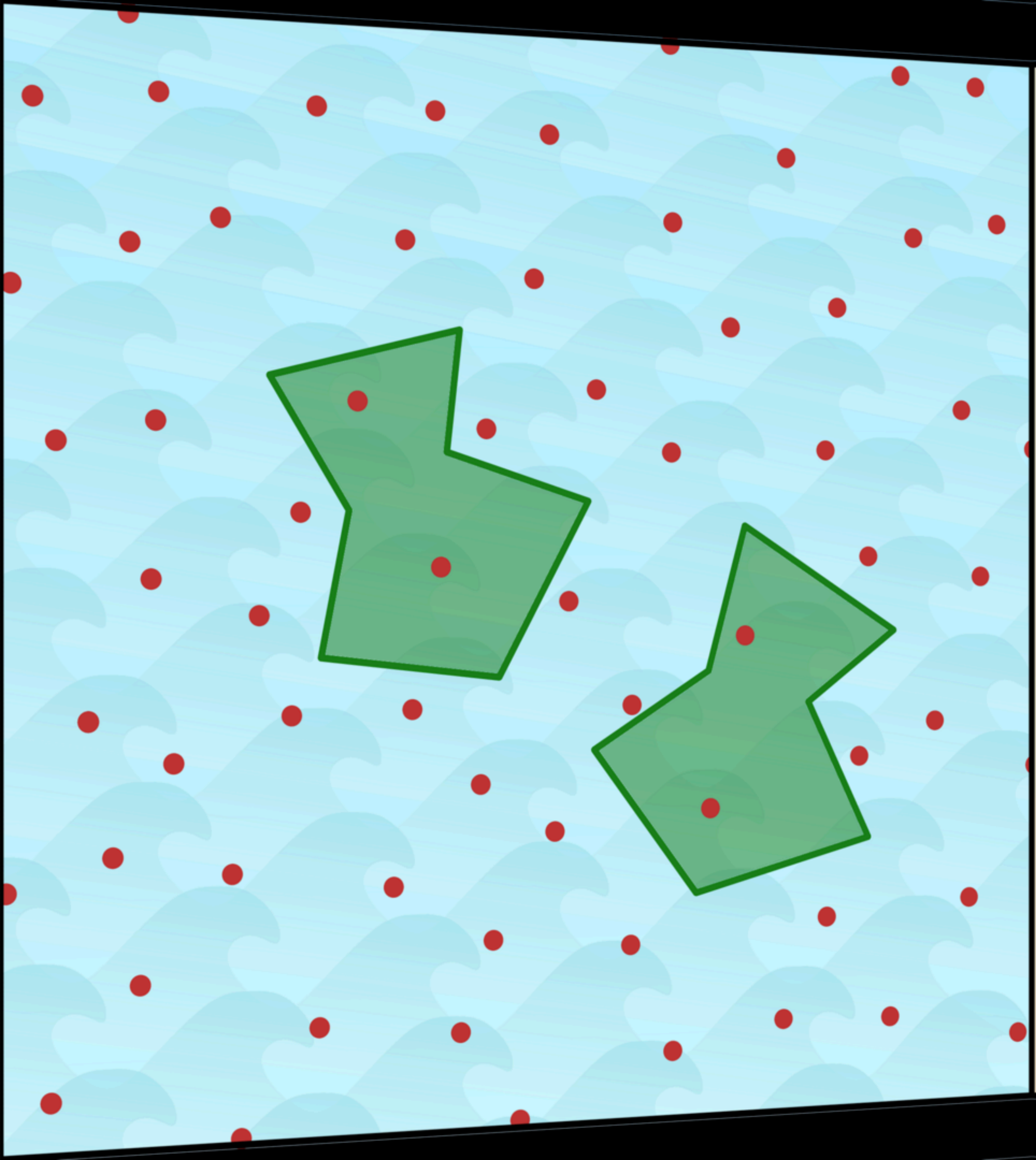

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Not much is known!

Geometry,
Uncertainty,
&
BATTLESHIPS!!!

Eva Hainzl
Maarten Löffler
Daniel Perz
Josef Tkadlec