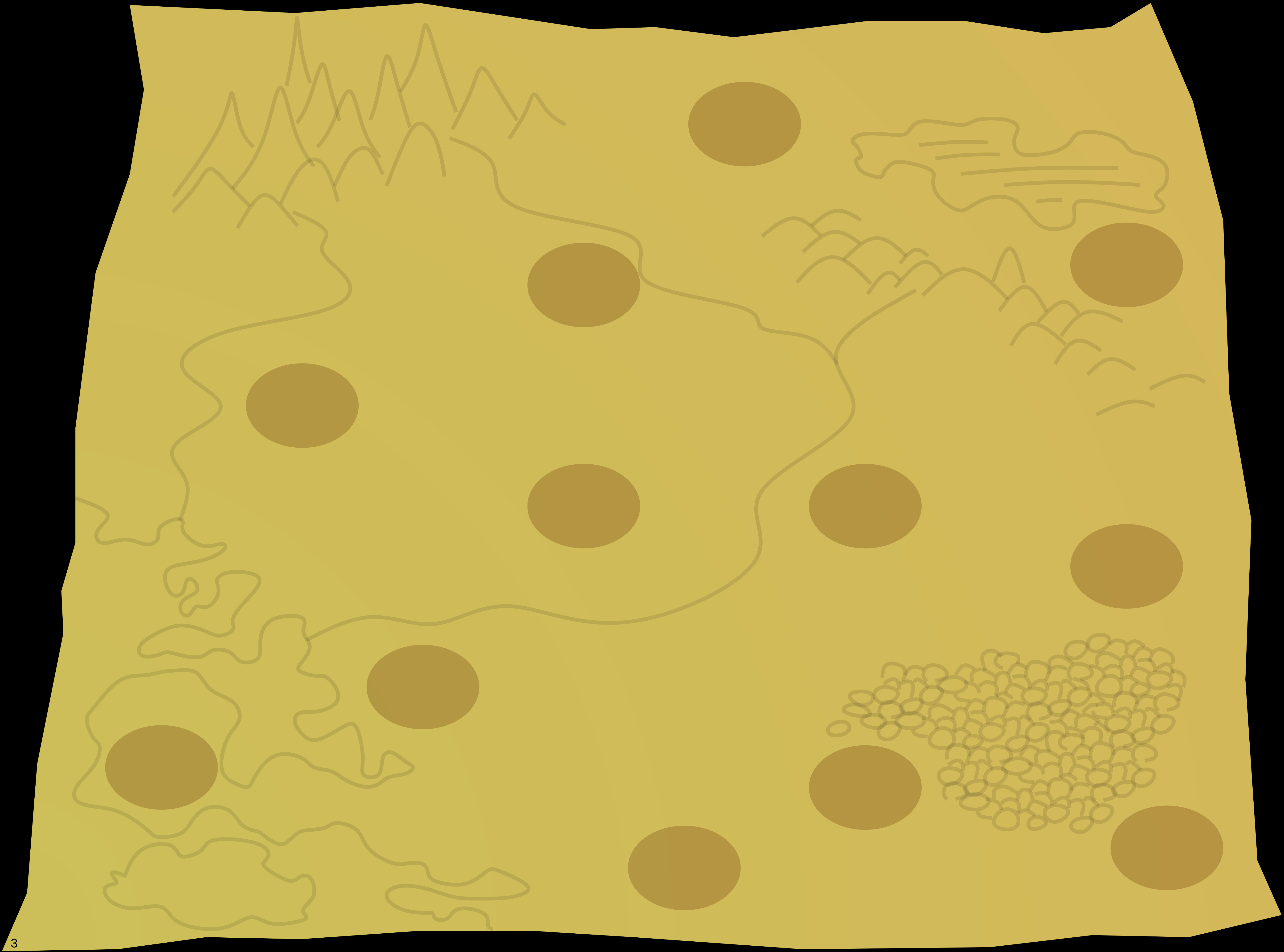
COLORED SPANNING GRAPHS

investigated by

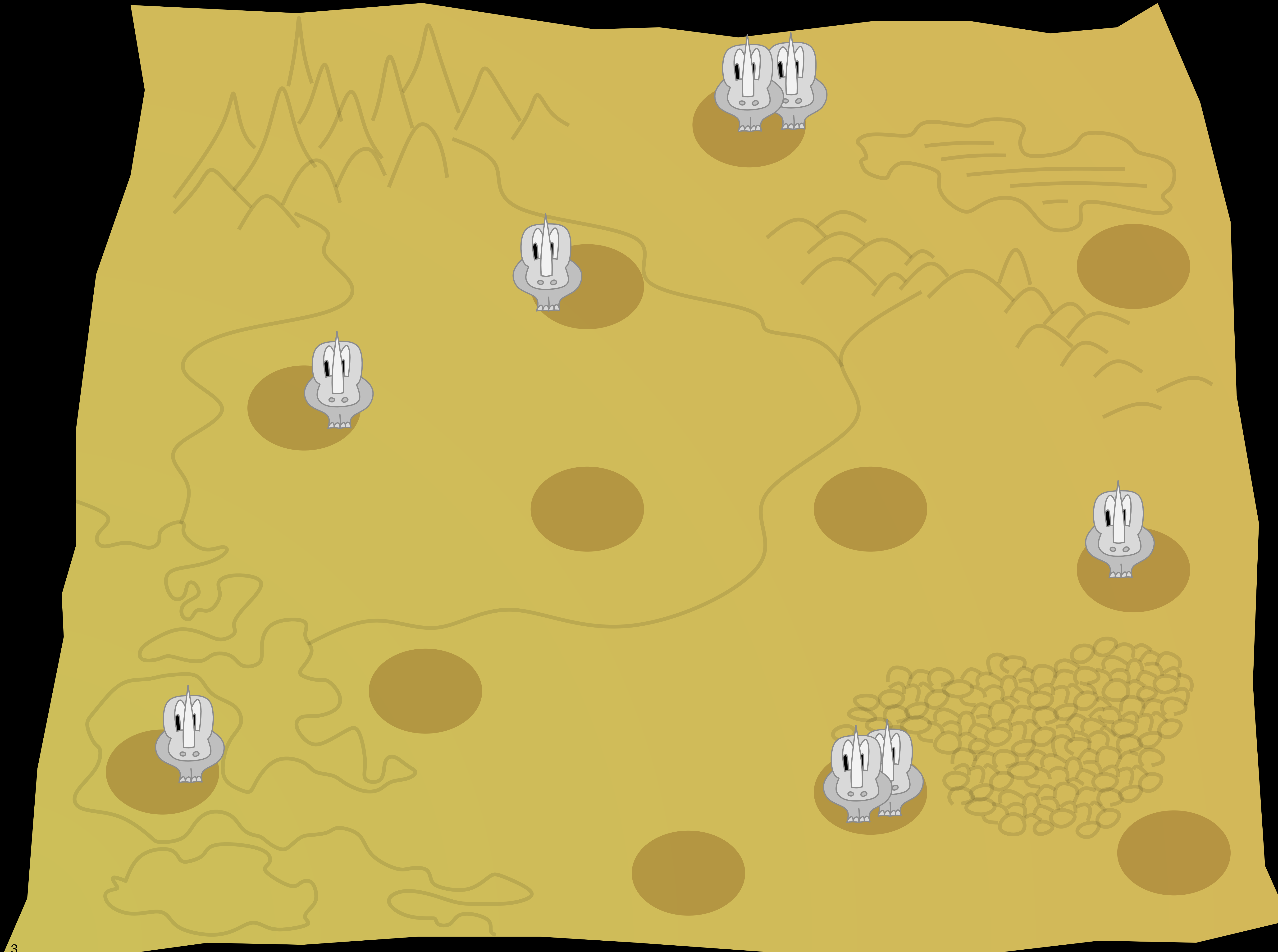
Ferran Hurtado
◇
Matias Korman
◇
Marc van Kreveld
◇
Maarten Löffler
◇
Vera Sacristán
◇
Rodrigo Silveira
◇
Bettina Speckmann

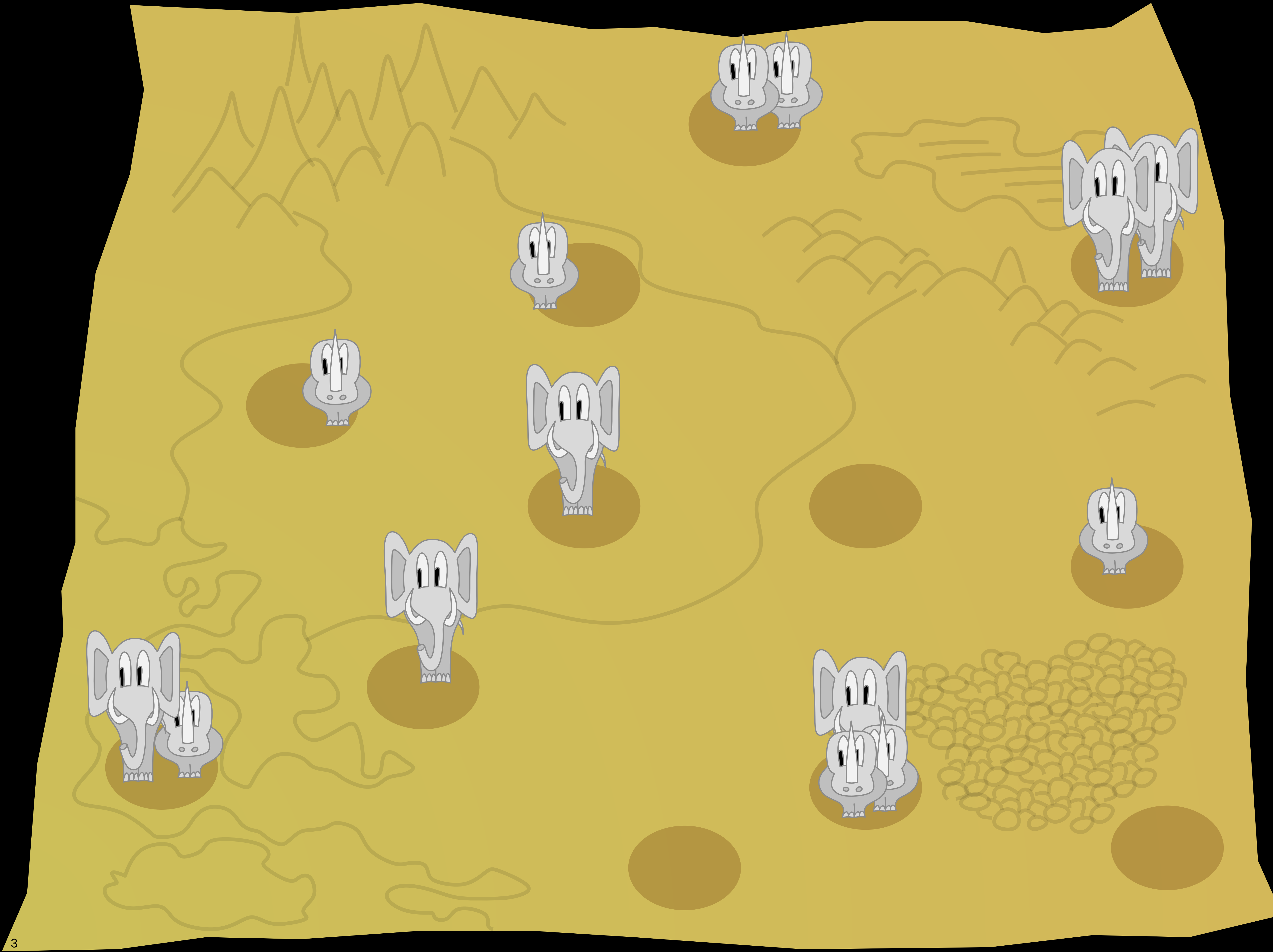
INTRODUCTION

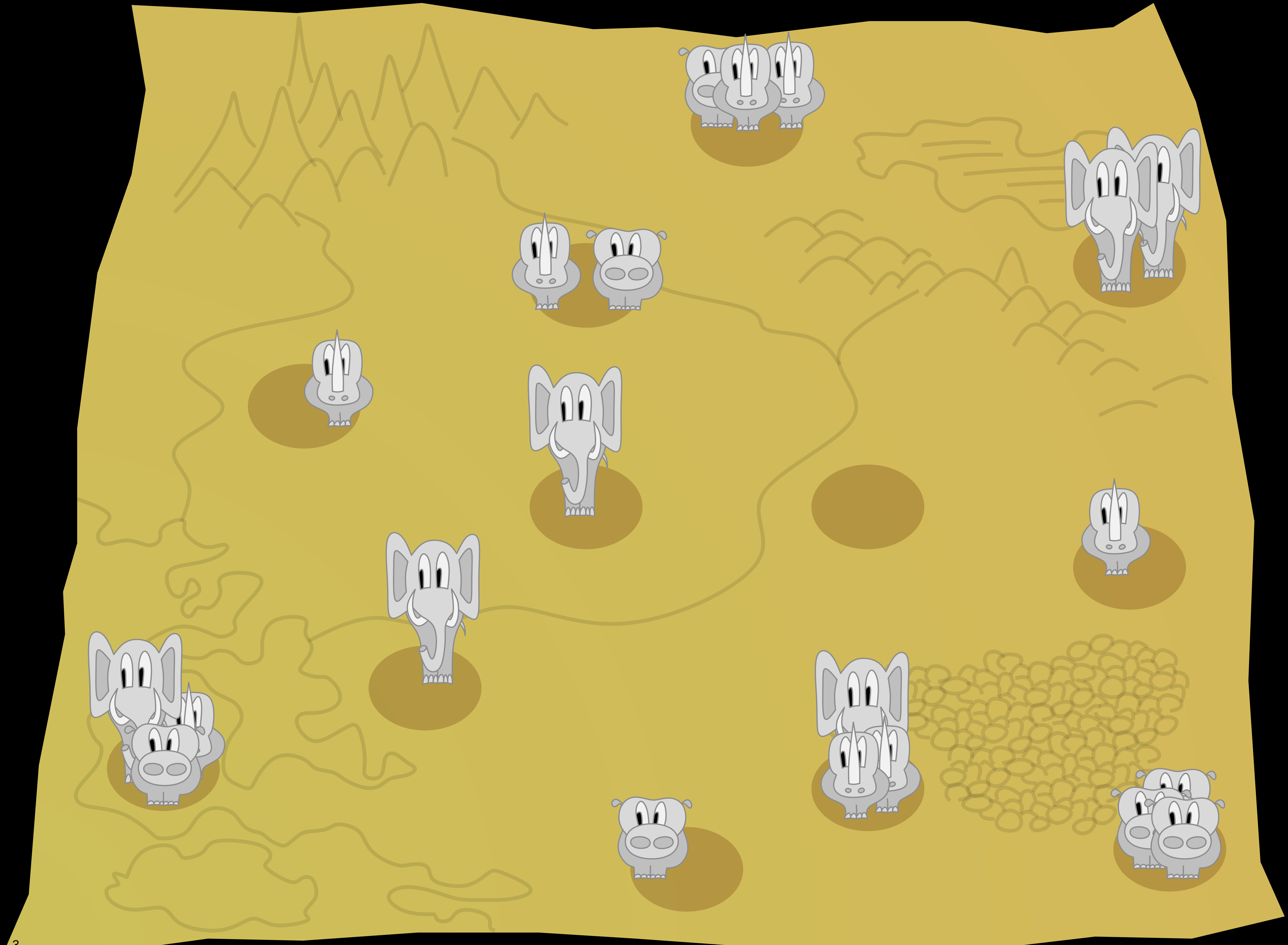
Consider a set of interesting points on a map.

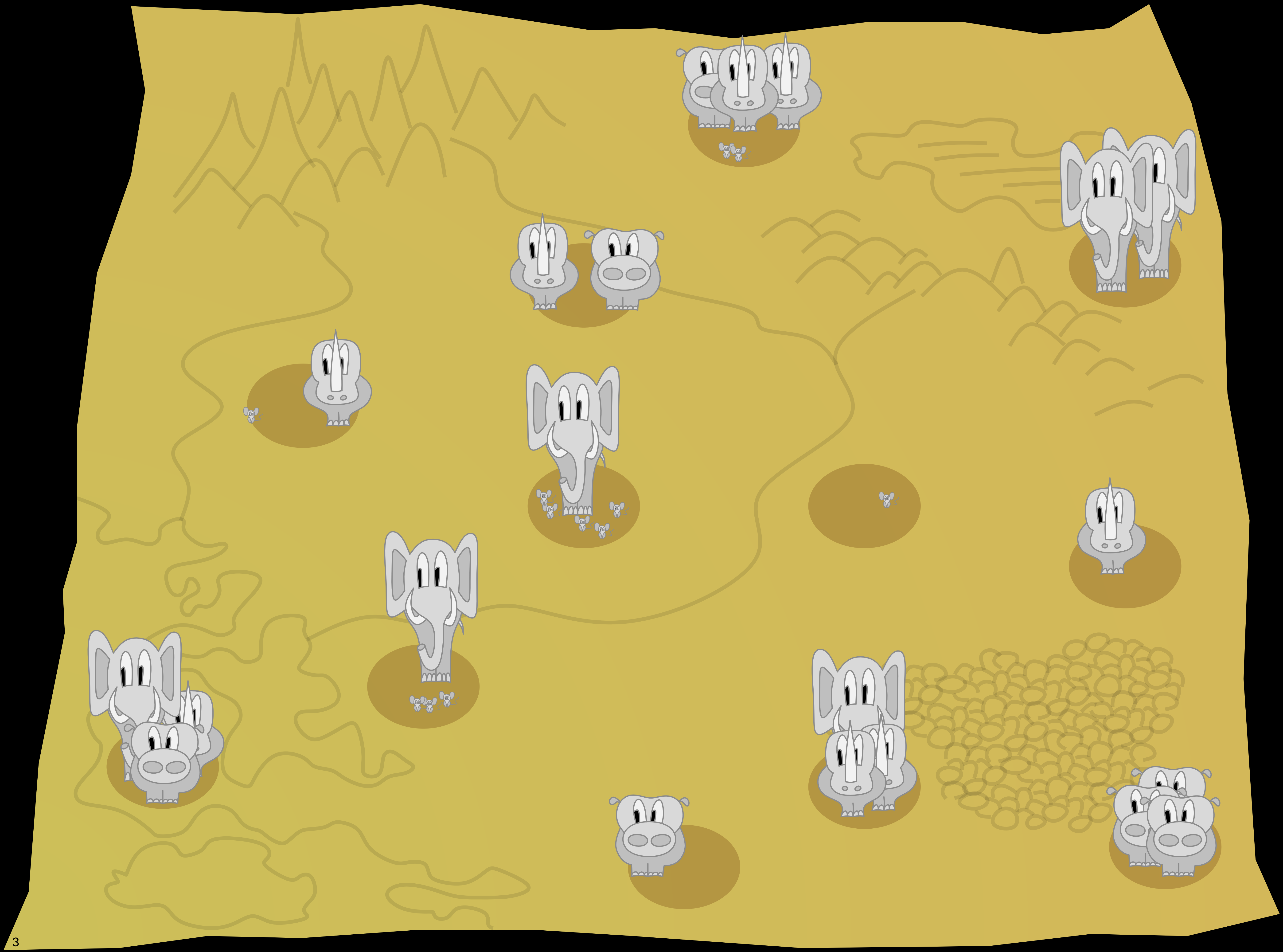


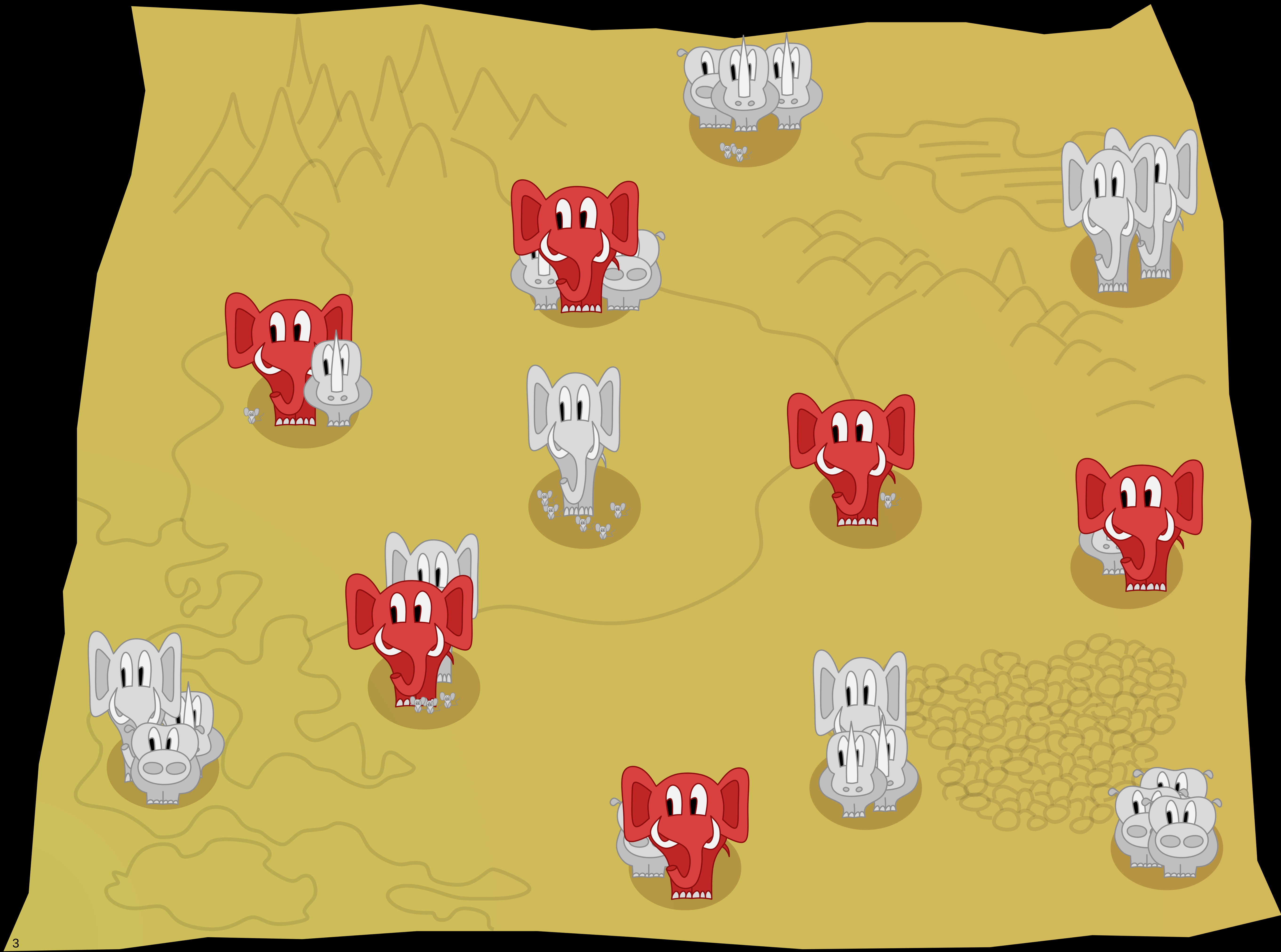
Suppose these points have certain *attributes* associated to them.

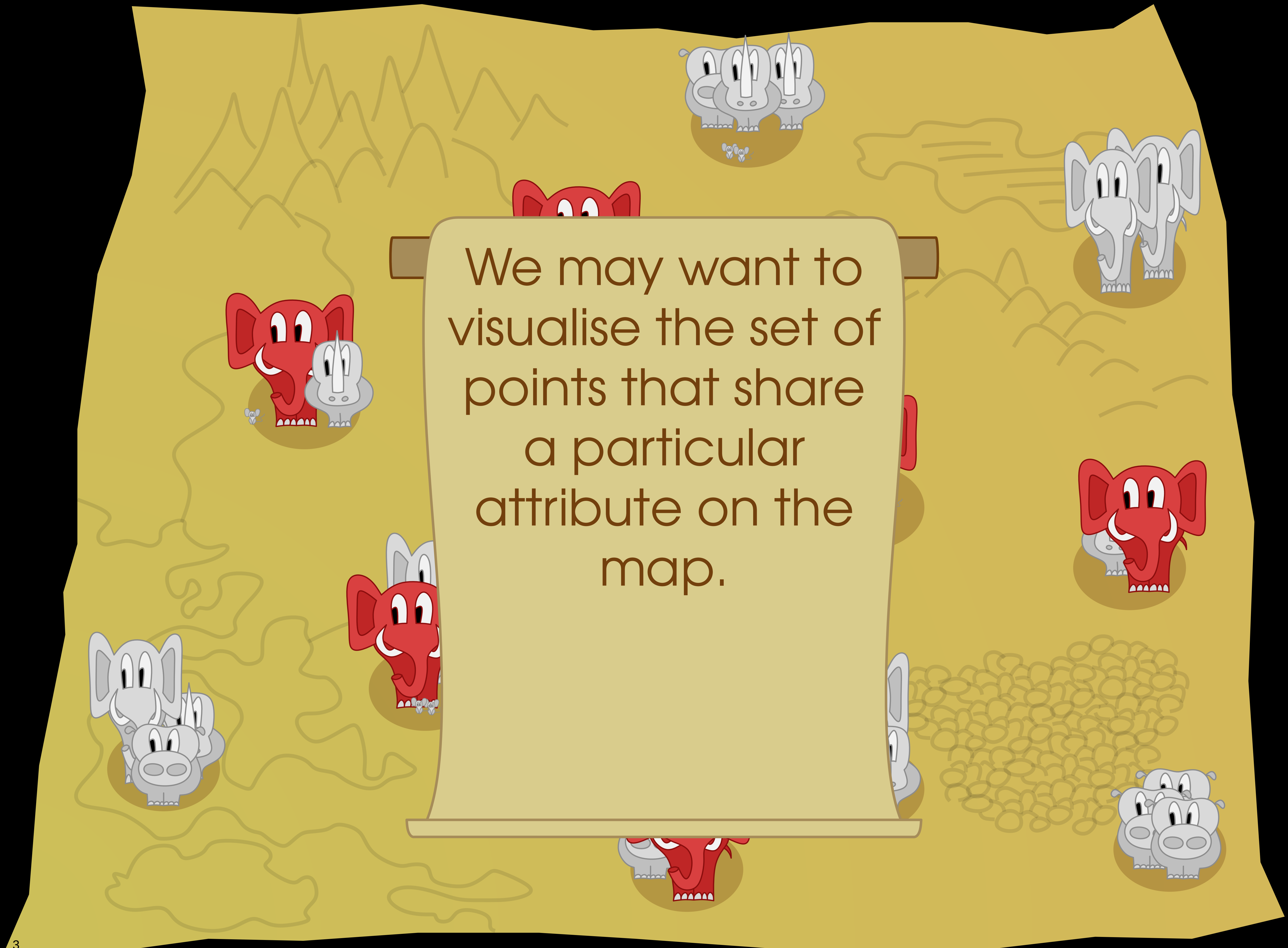


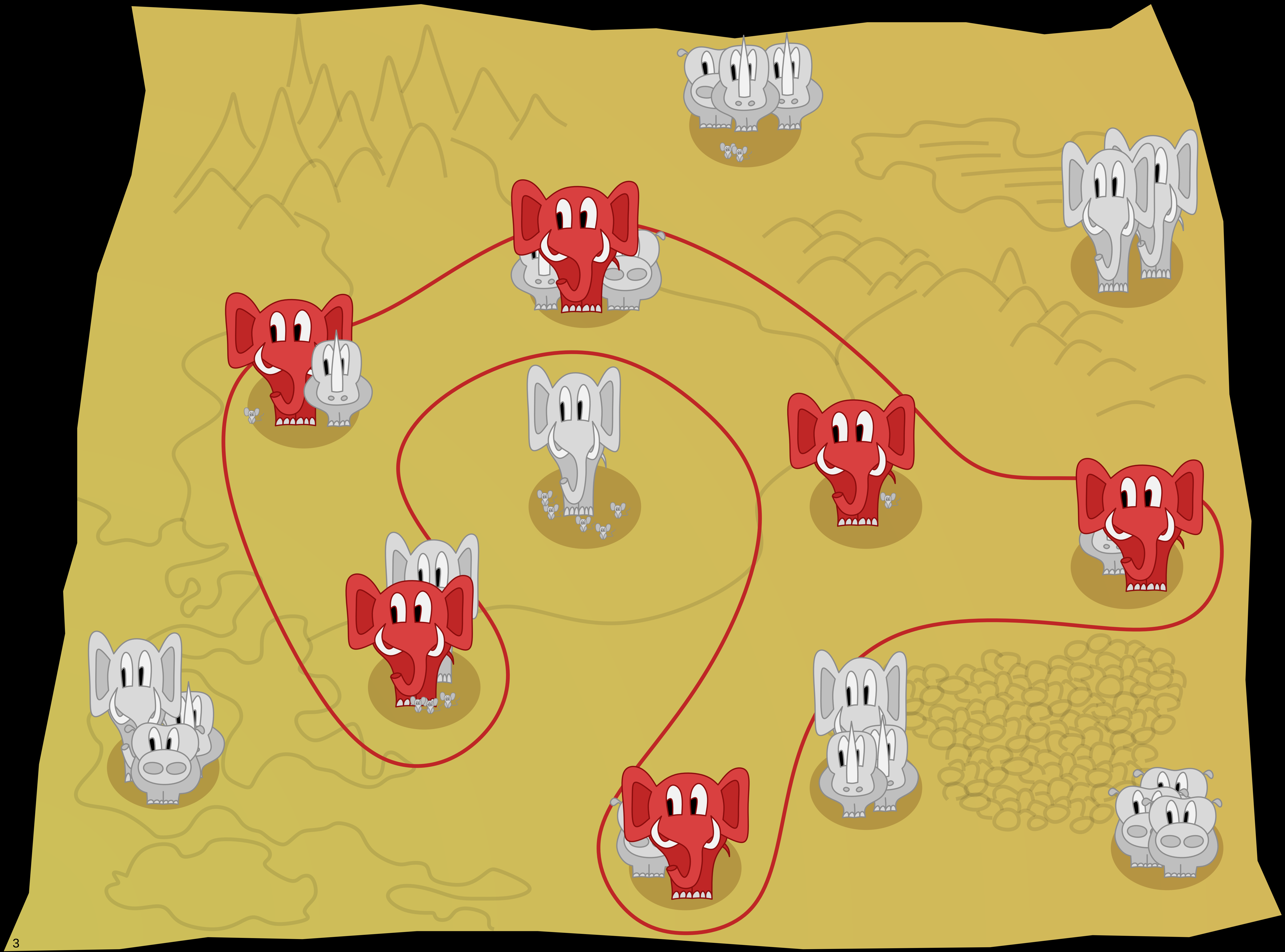


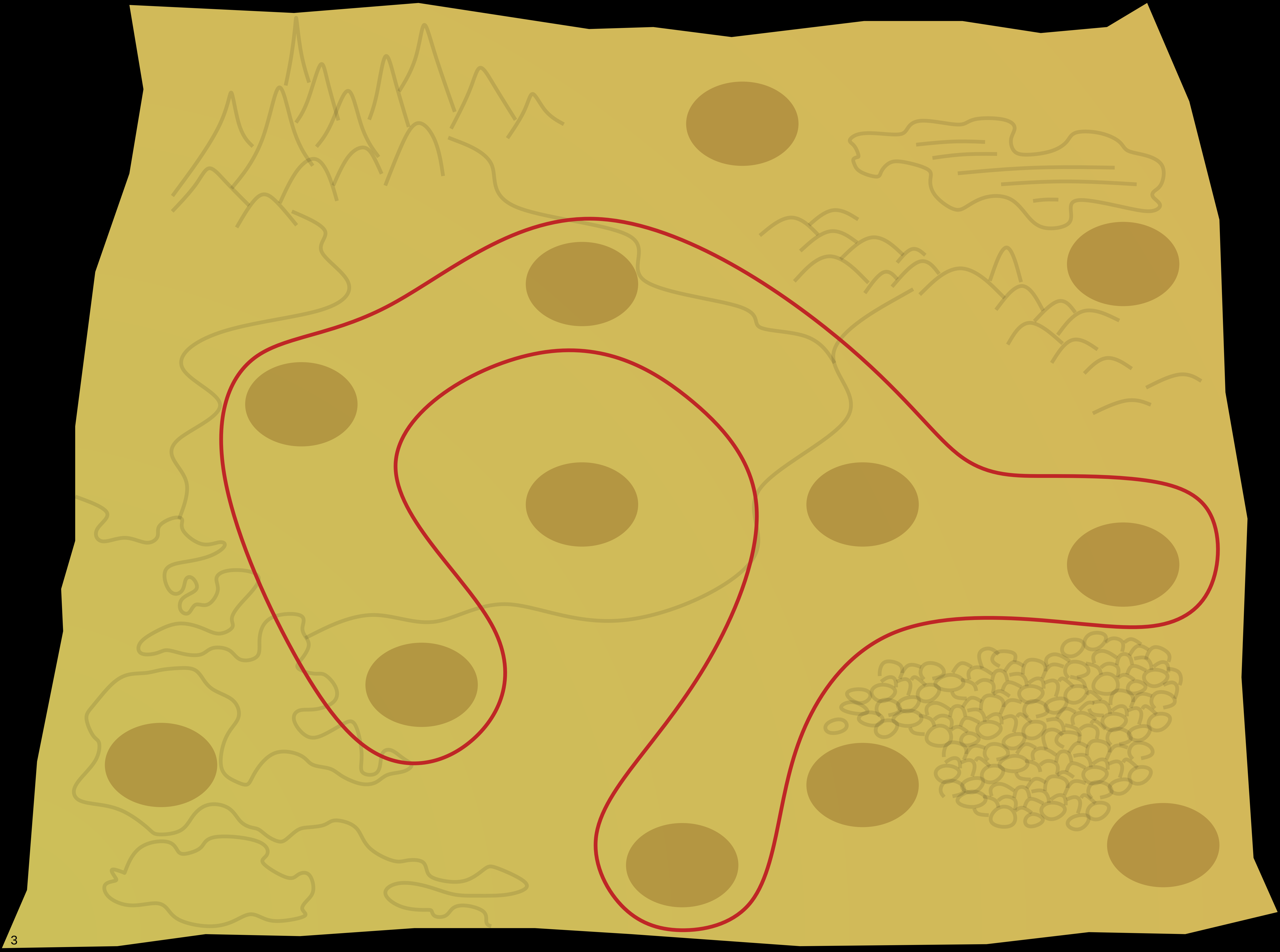


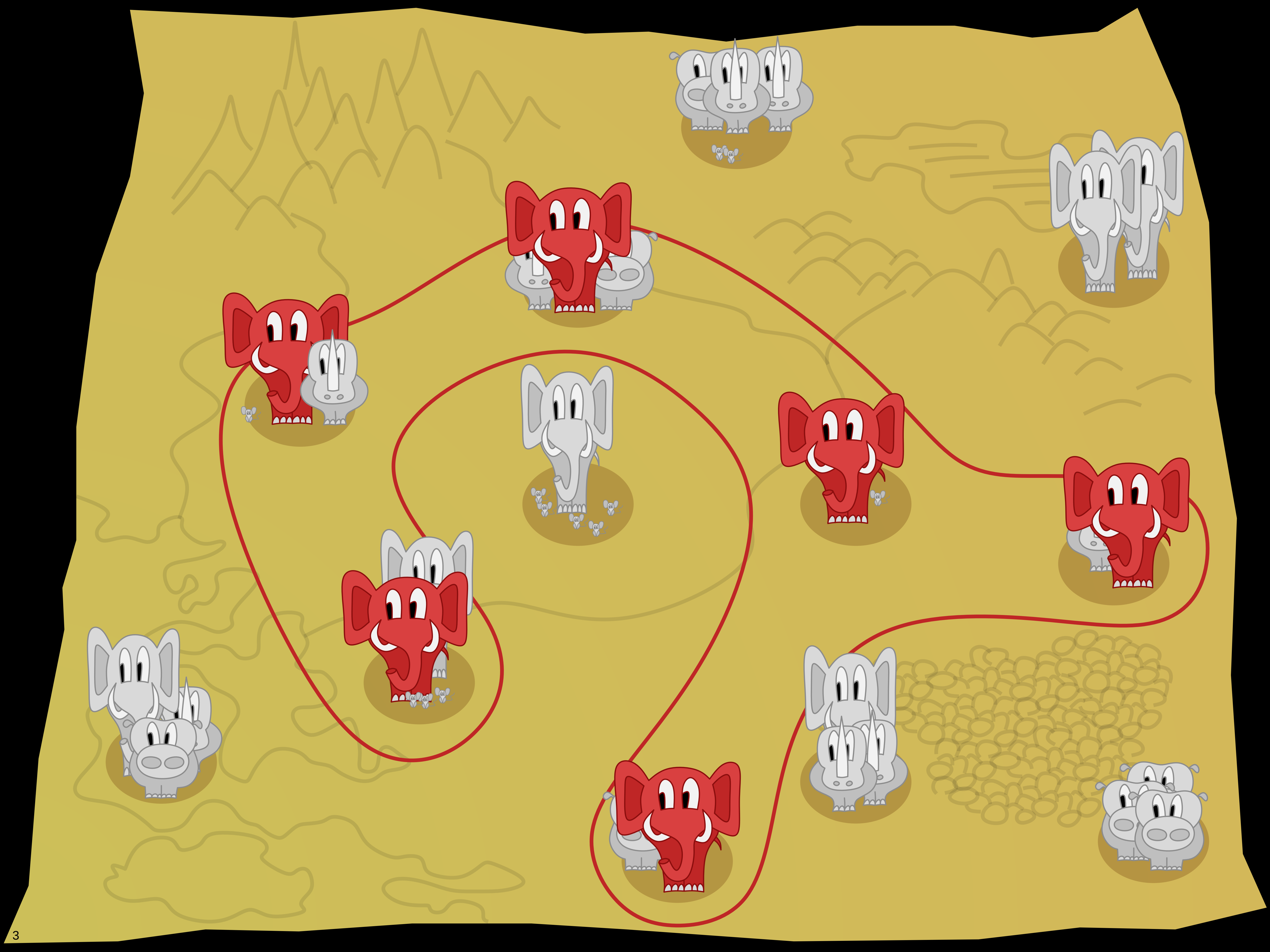


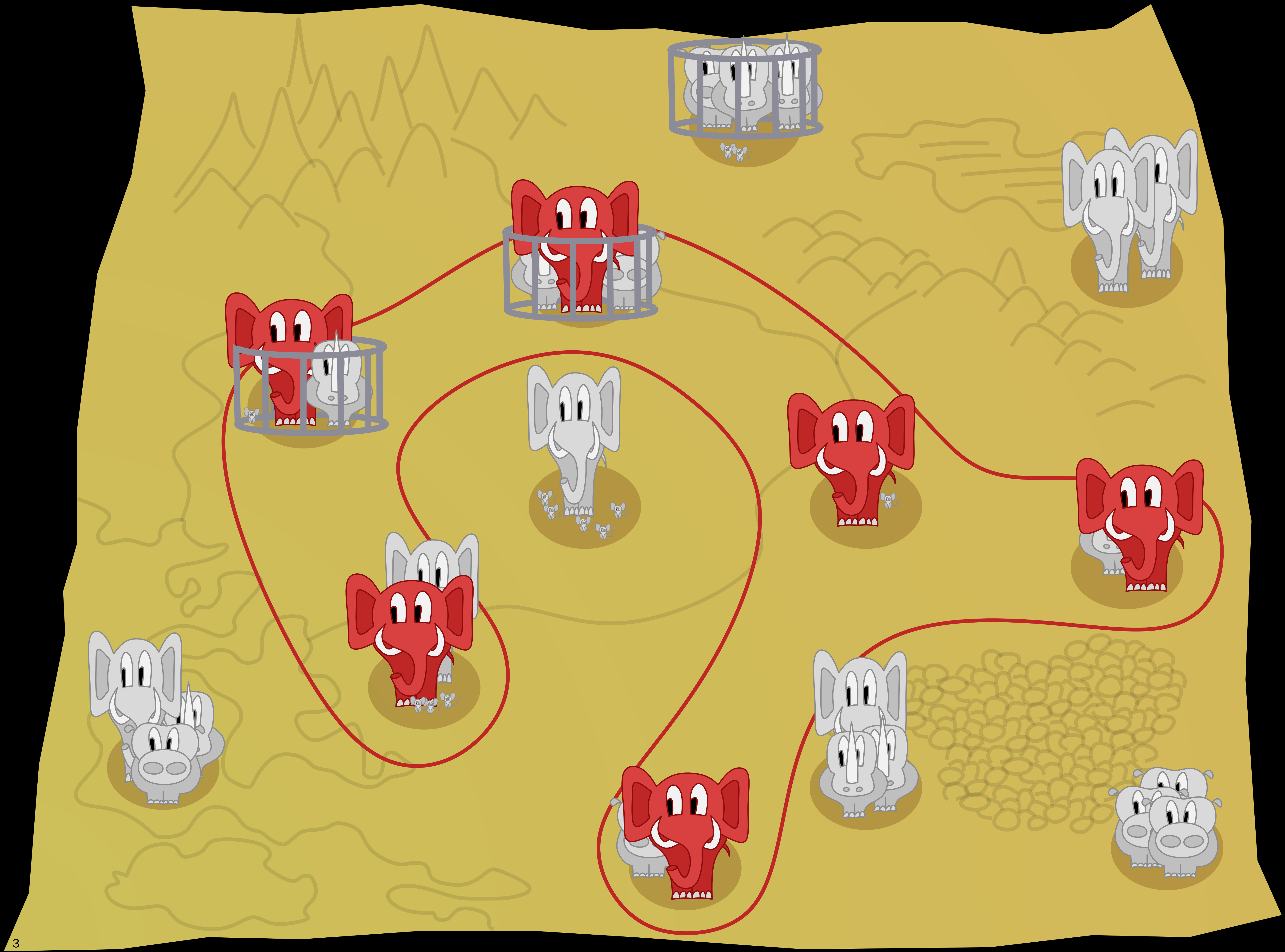


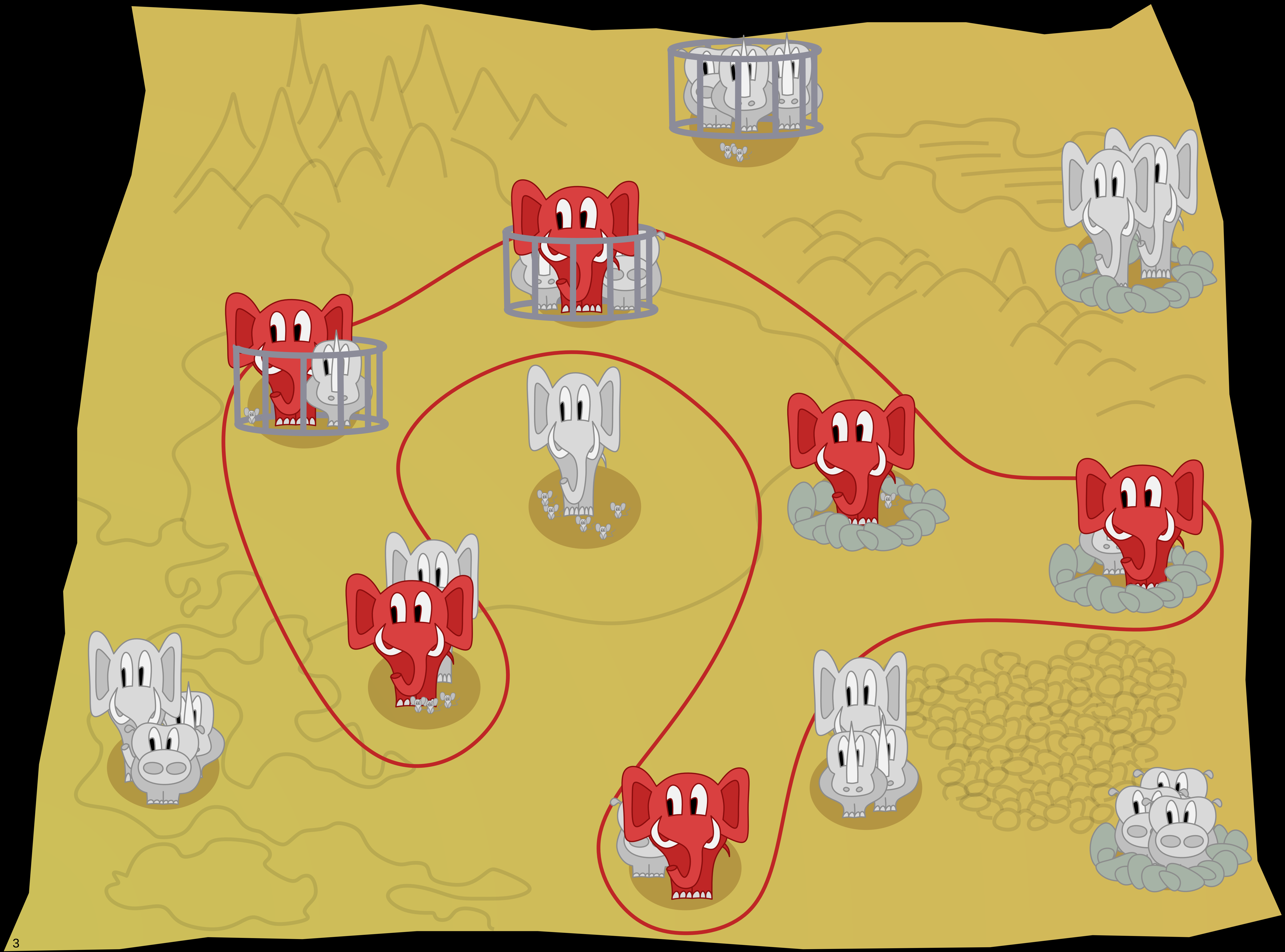

We may want to visualise the set of points that share a particular attribute on the map.

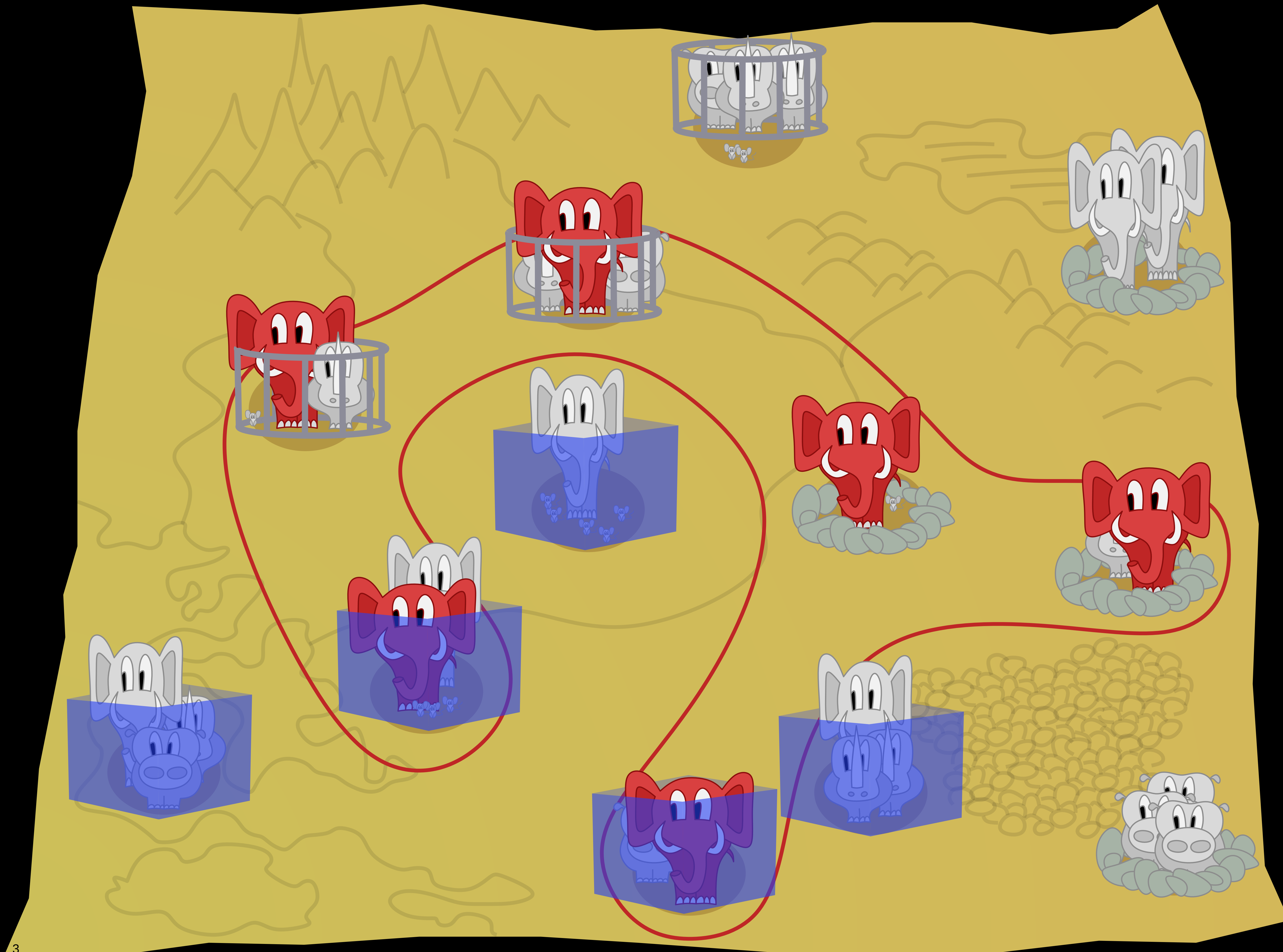


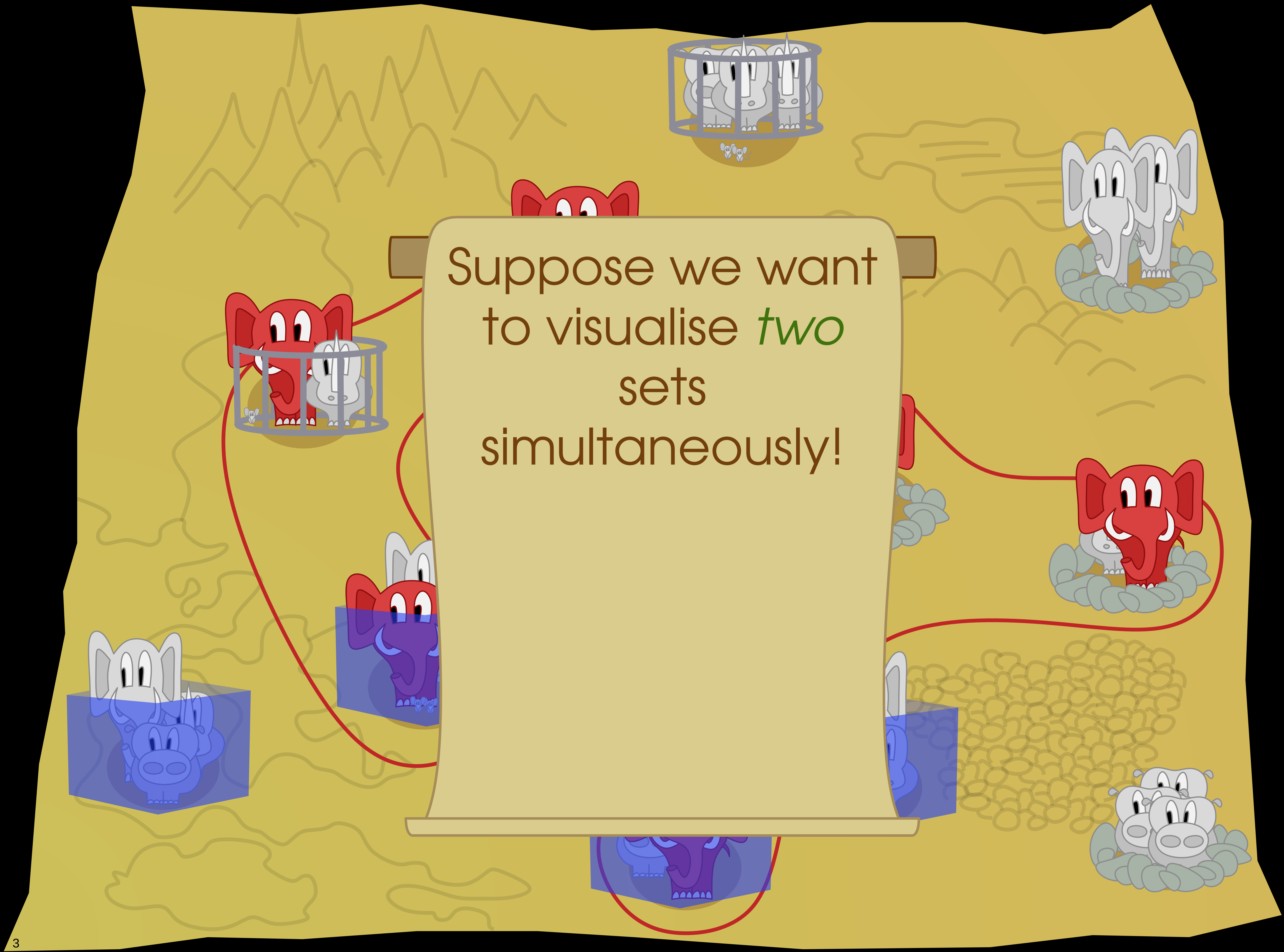


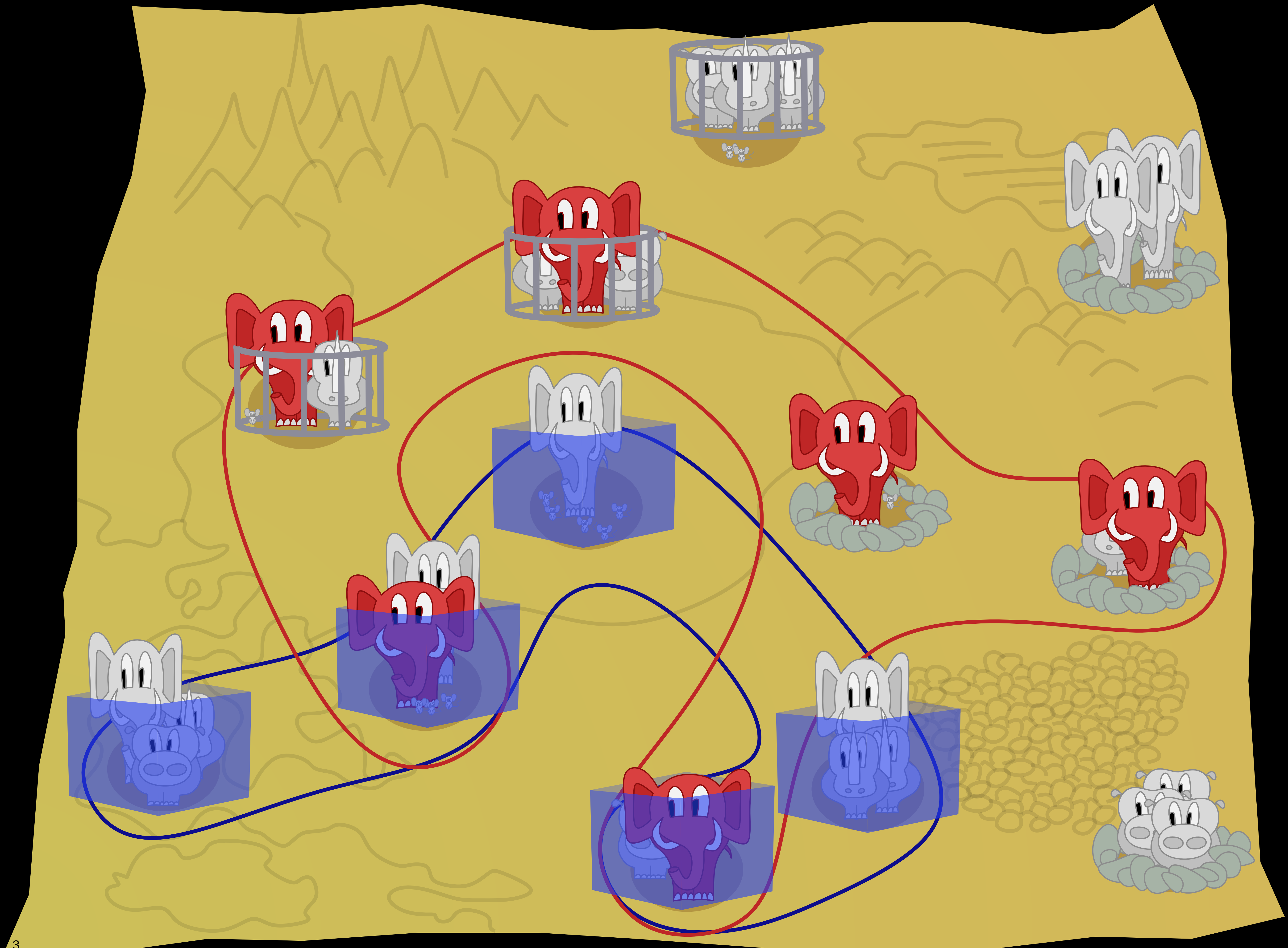




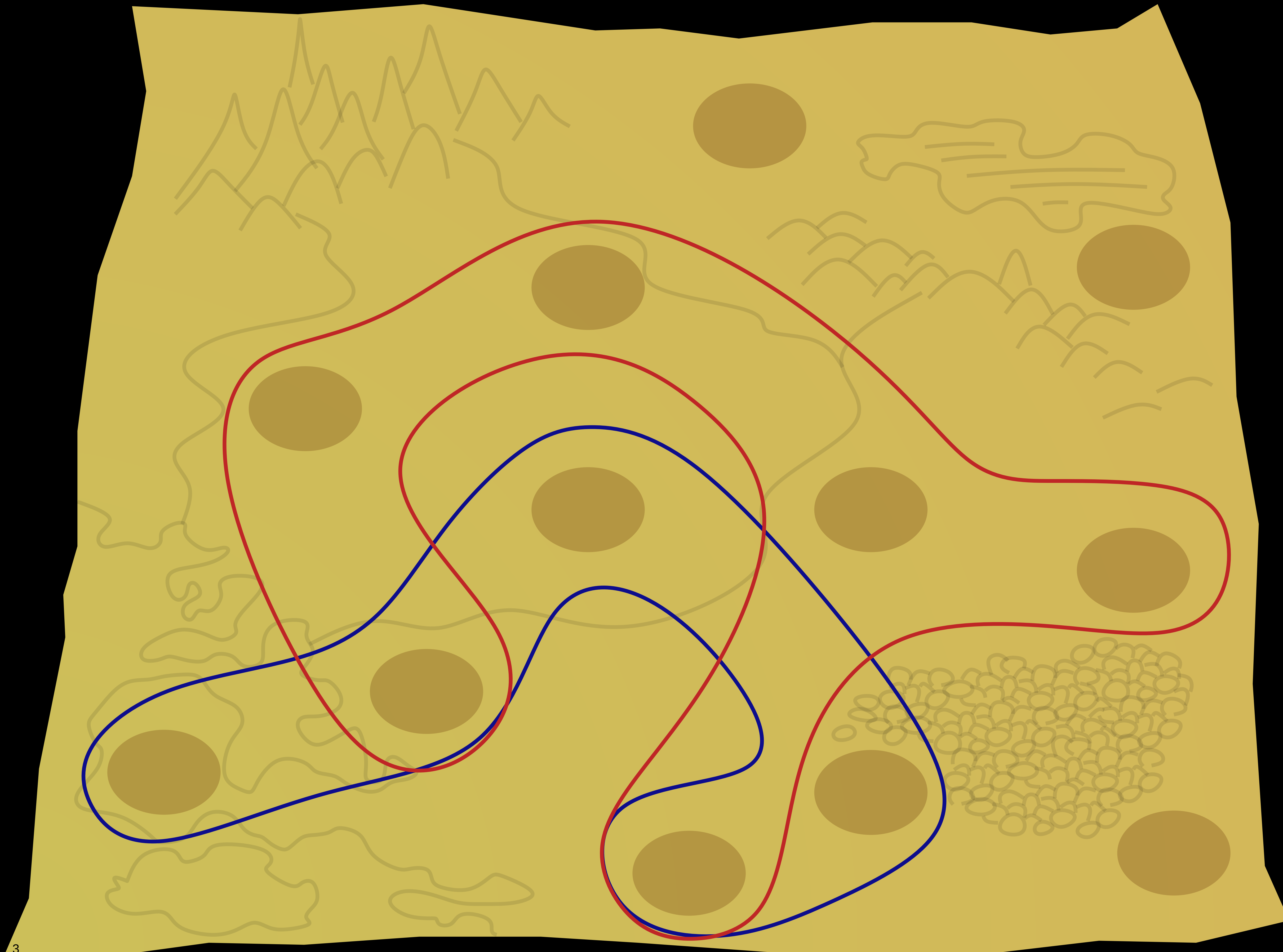




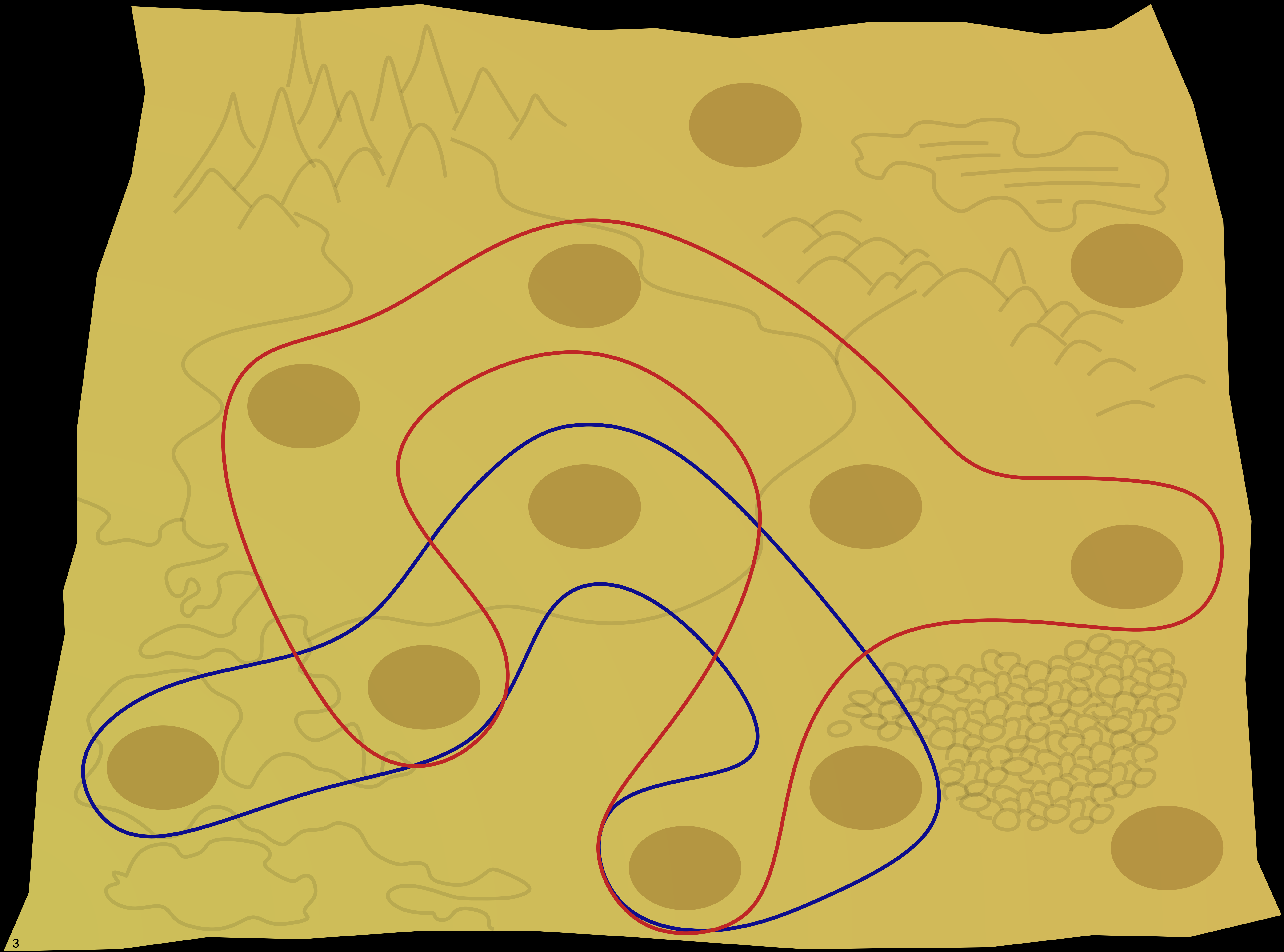

Suppose we want
to visualise *two*
sets
simultaneously!

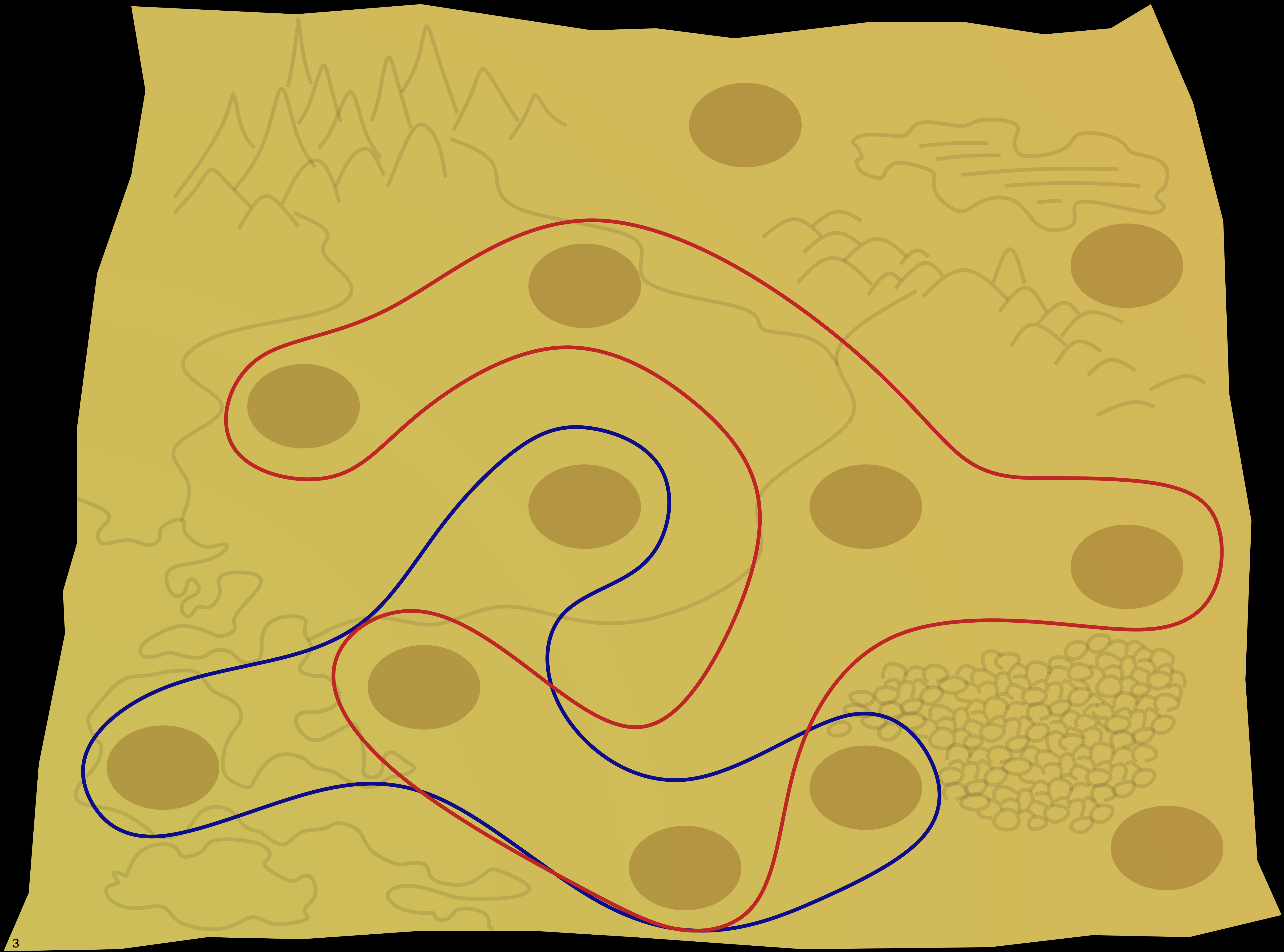


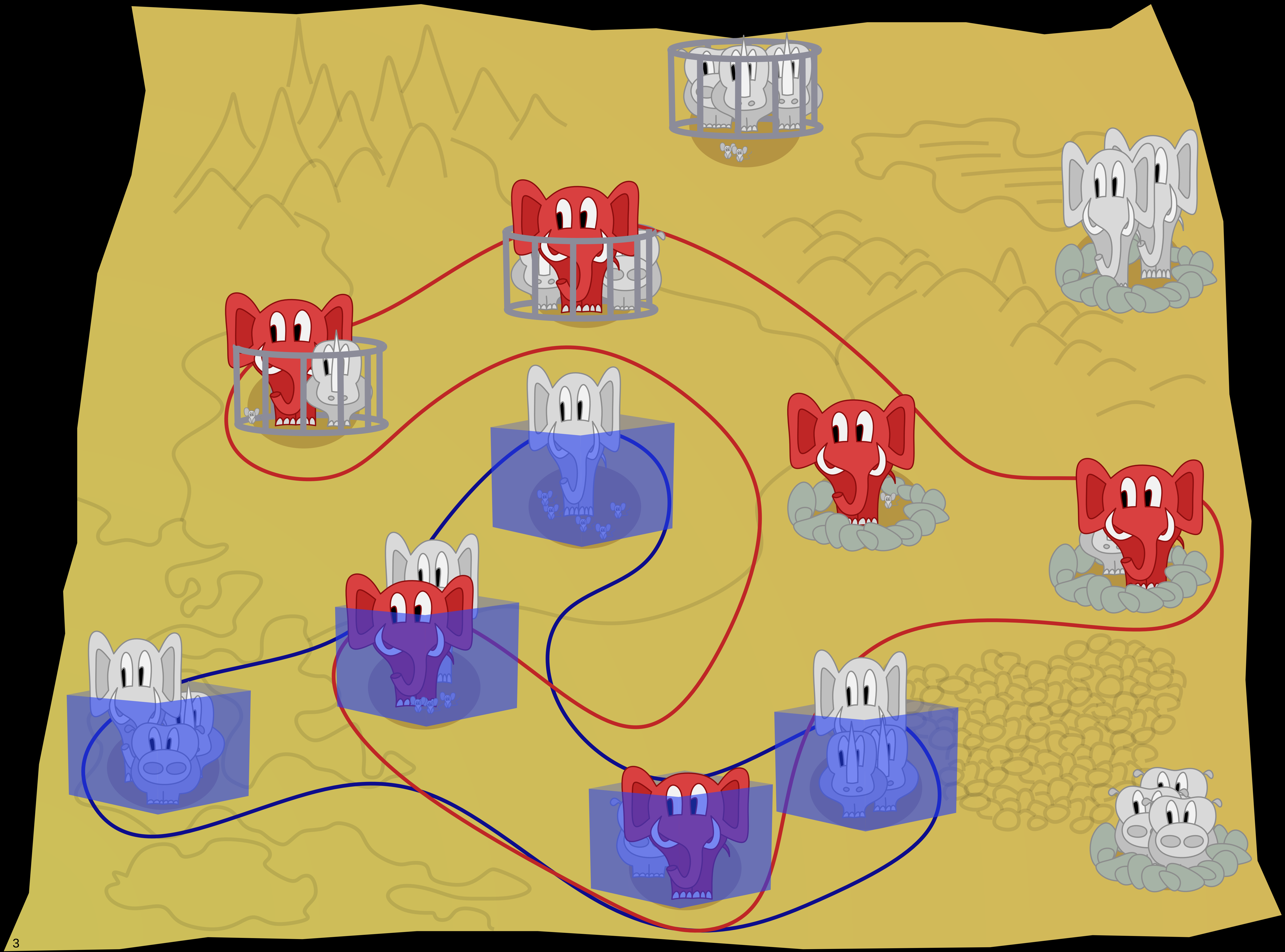
3



Two visualisations that are fine by themselves do not necessarily look good together.

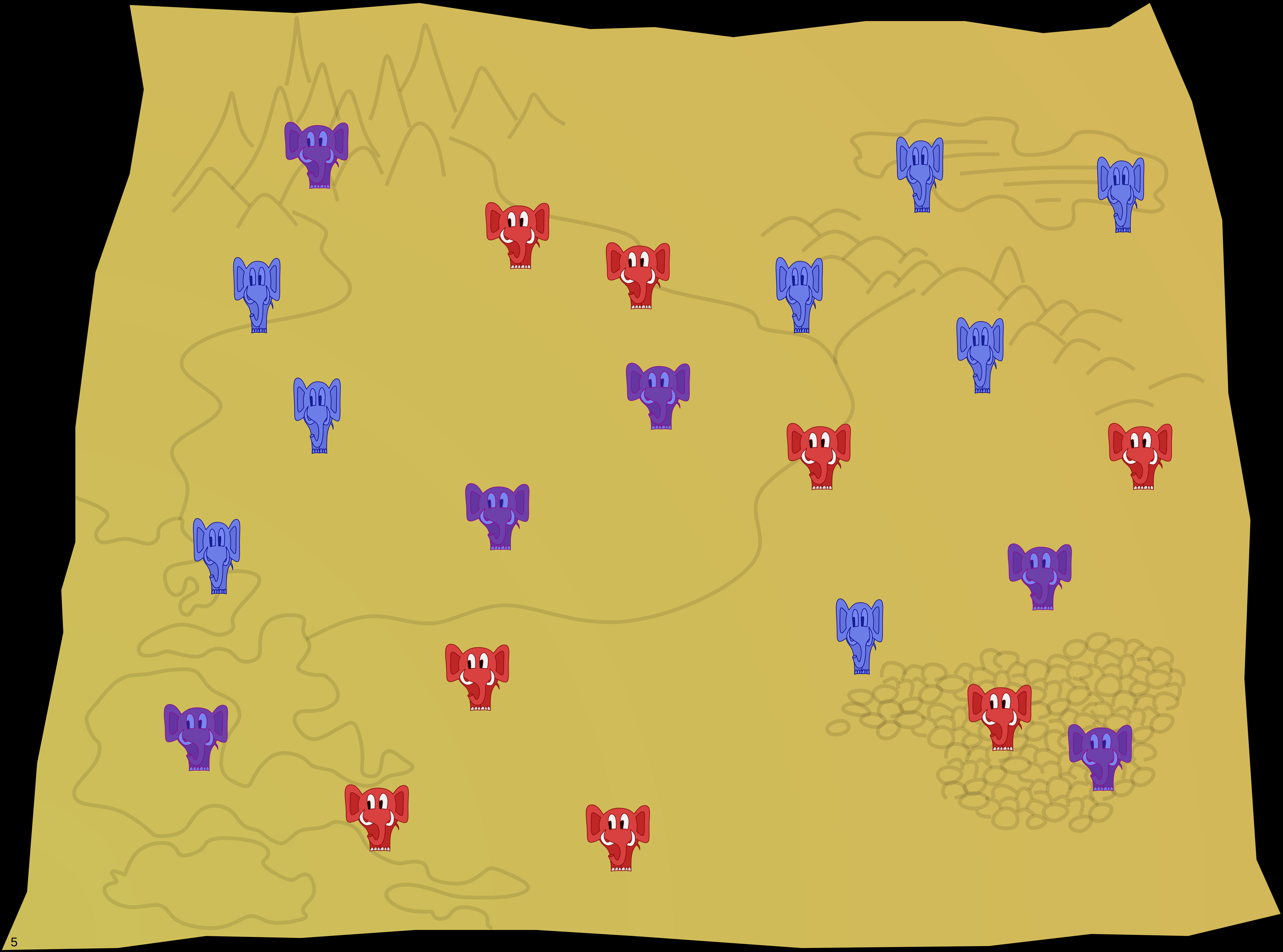


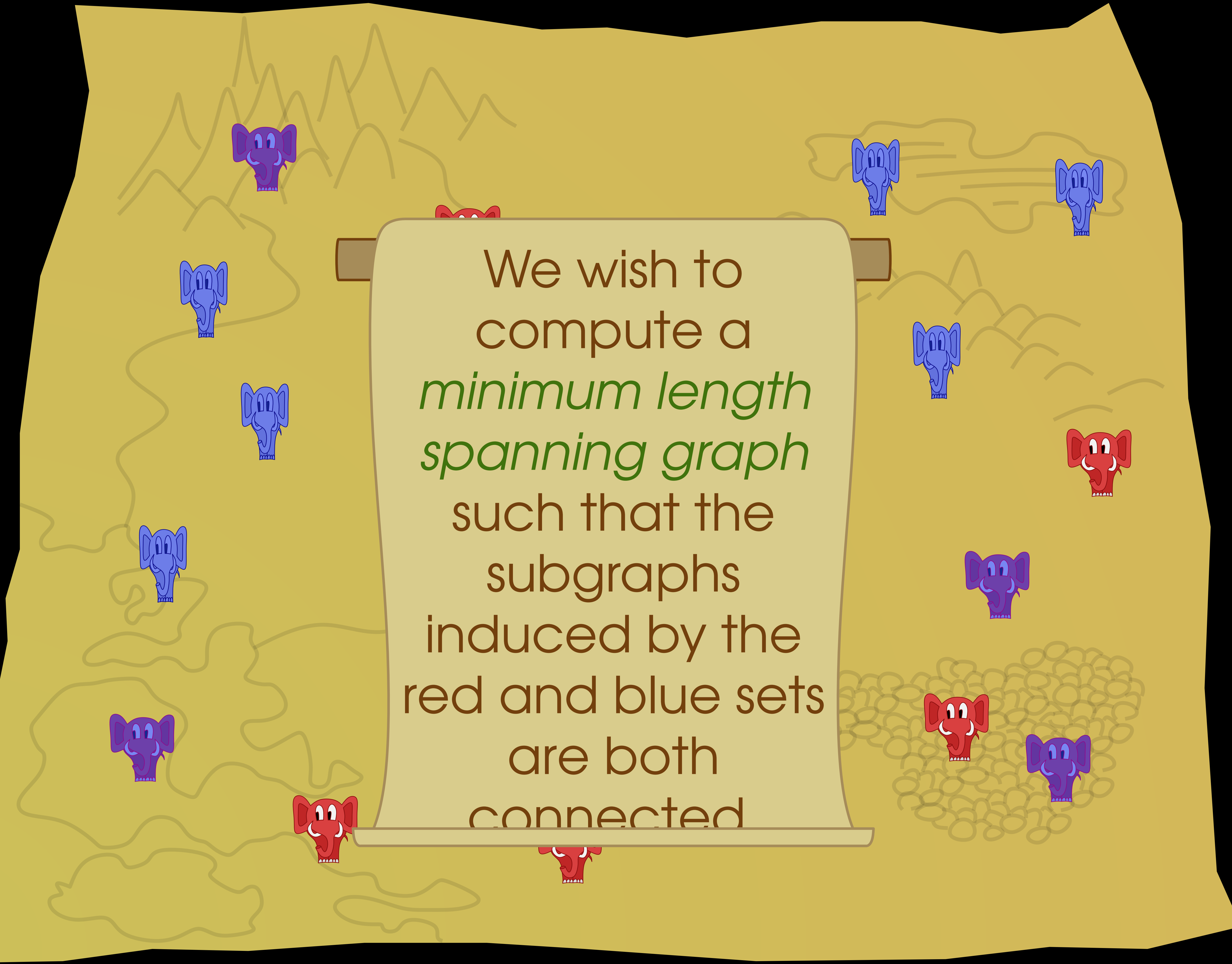


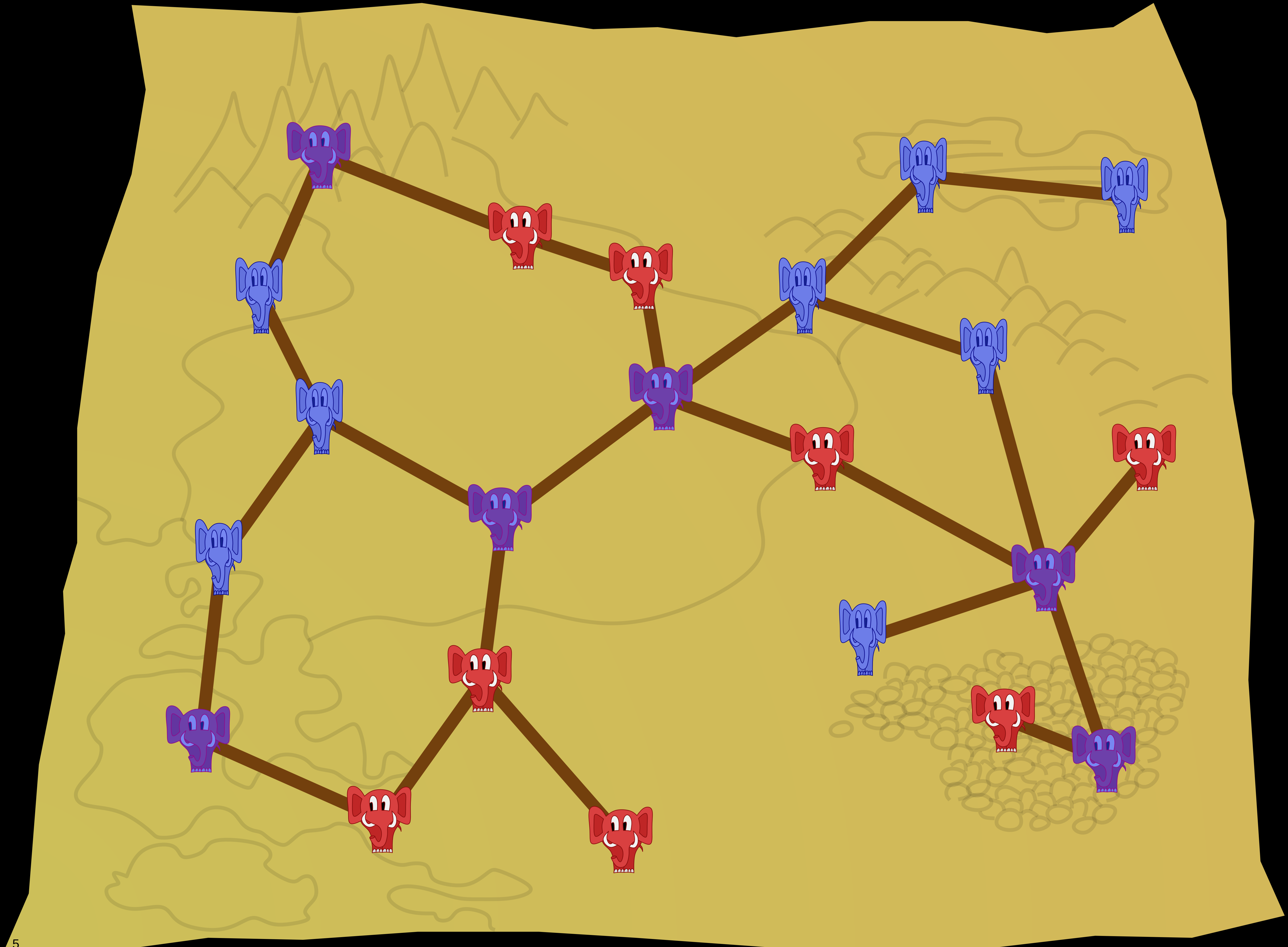


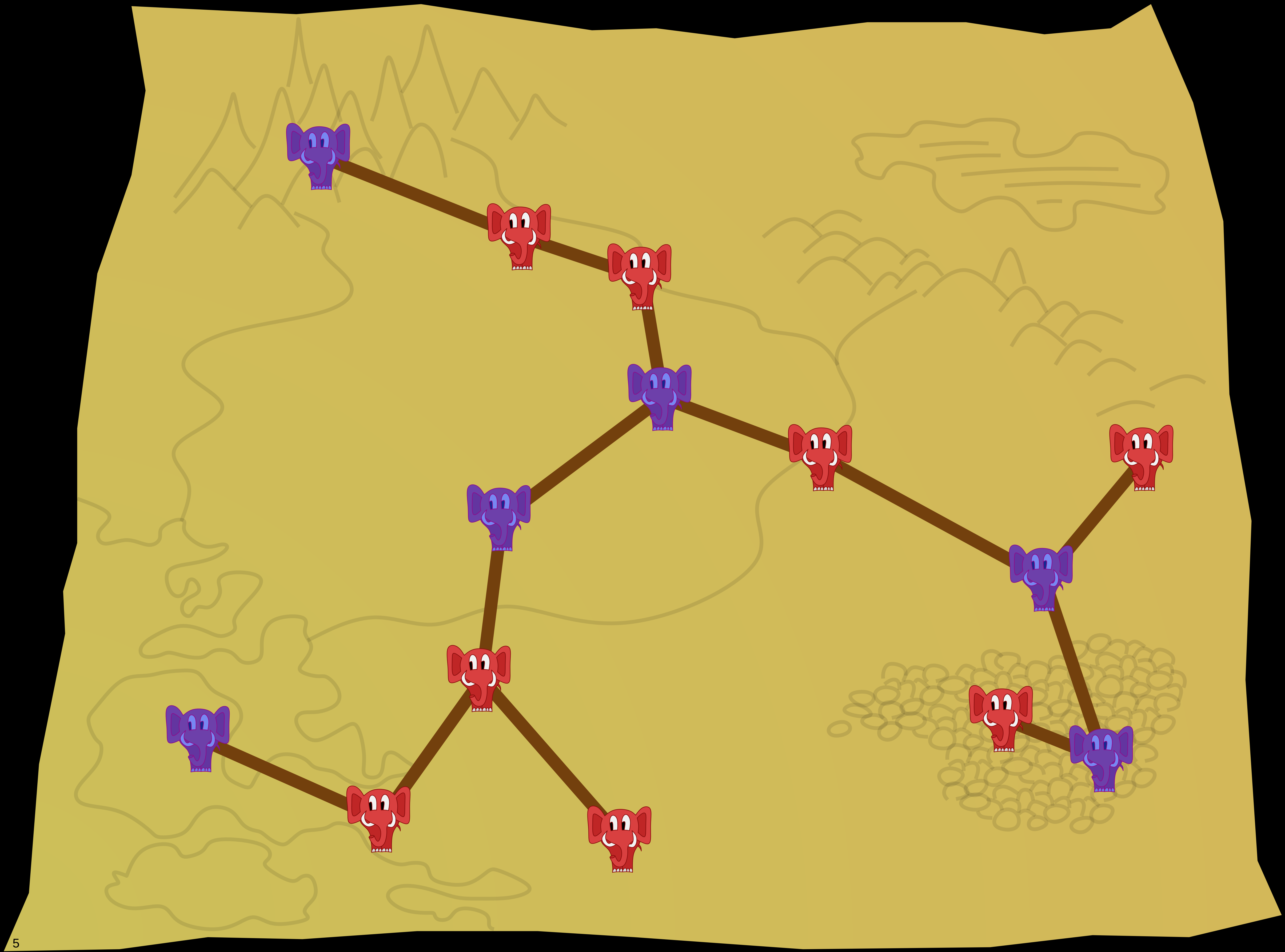
PROBLEM STATEMENT & RESULTS

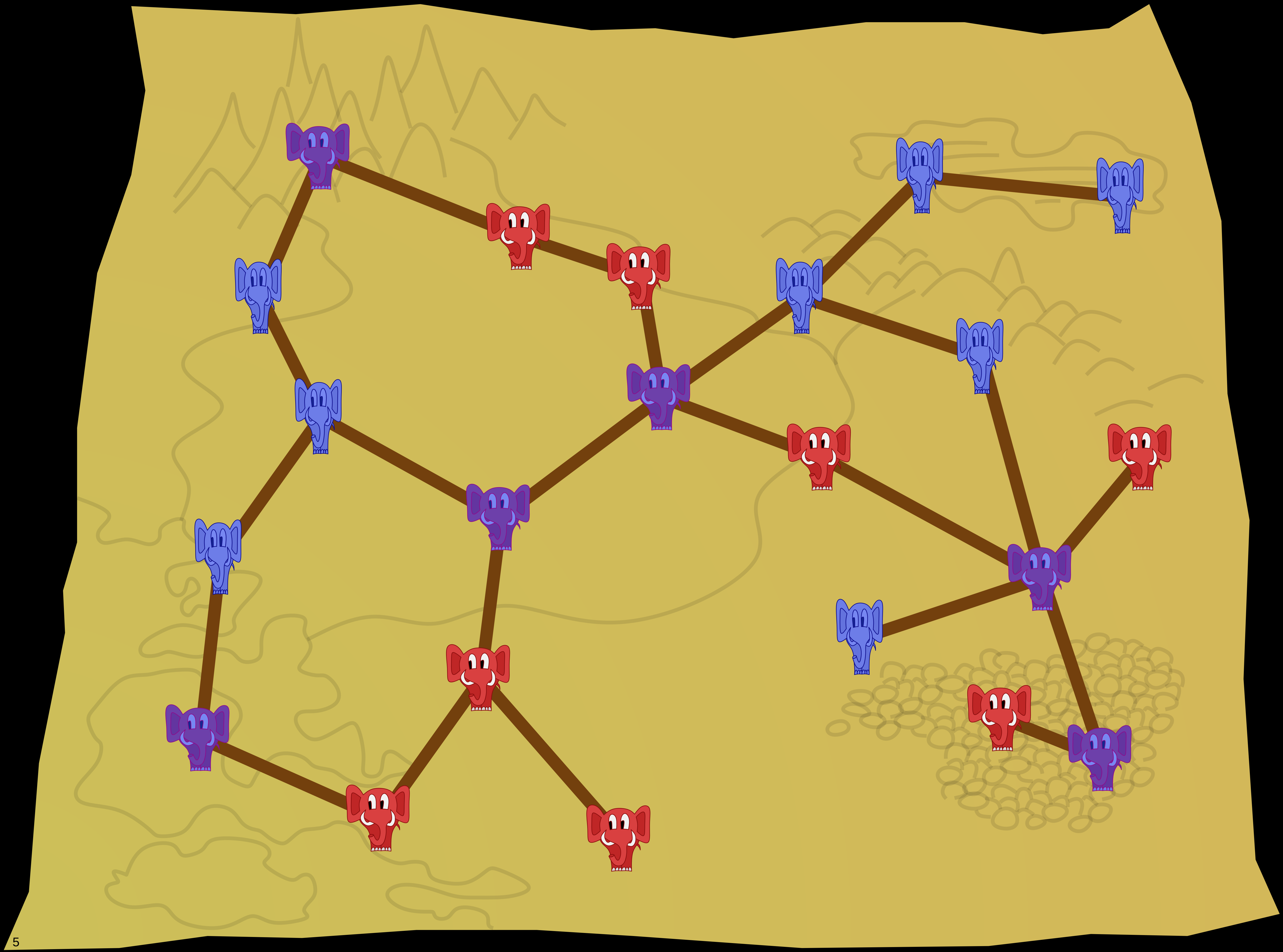
Let P be a set of n points in the plane, each of which is labeled as *red*, *blue*, or *purple* (both red and blue).

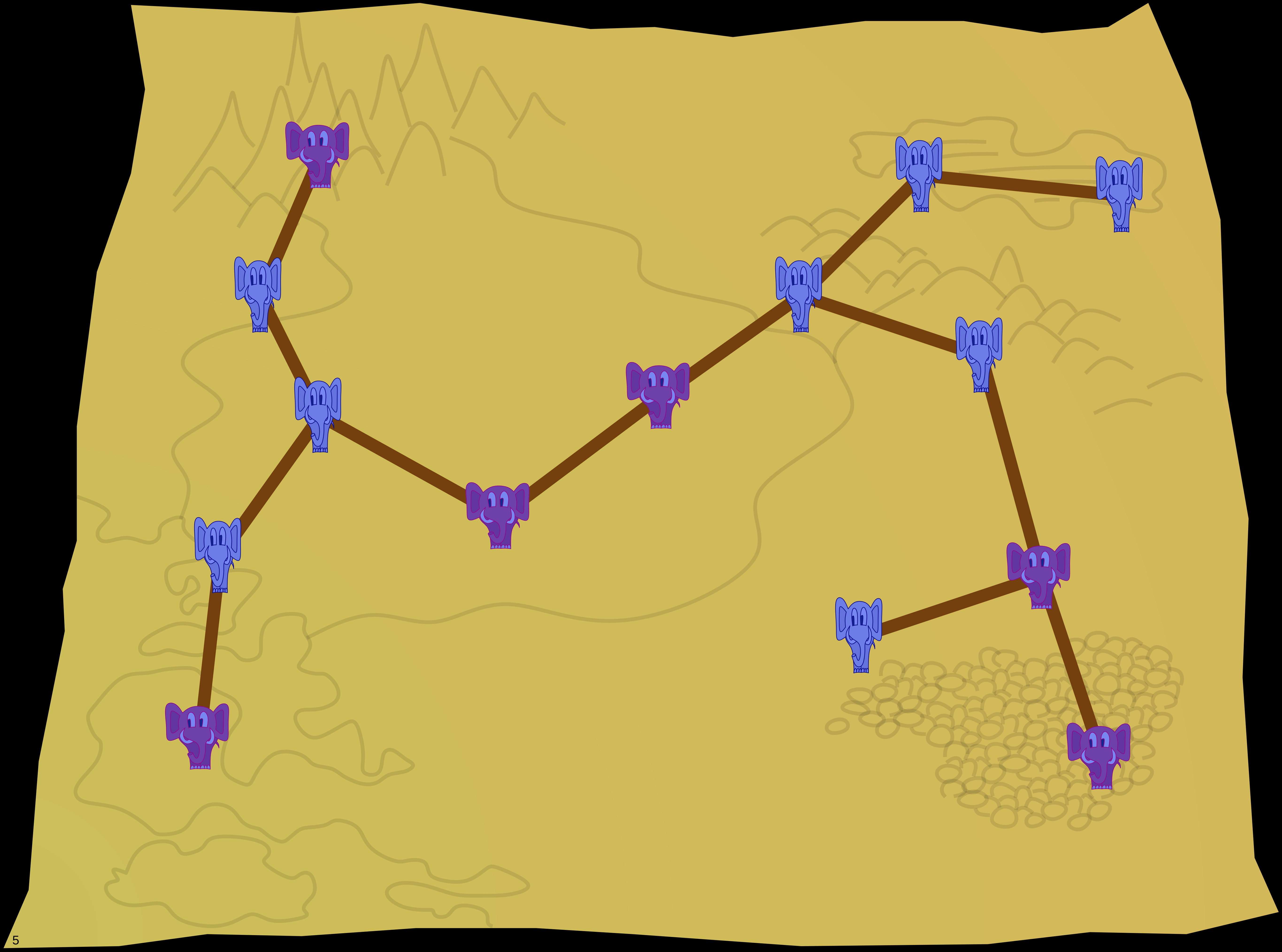


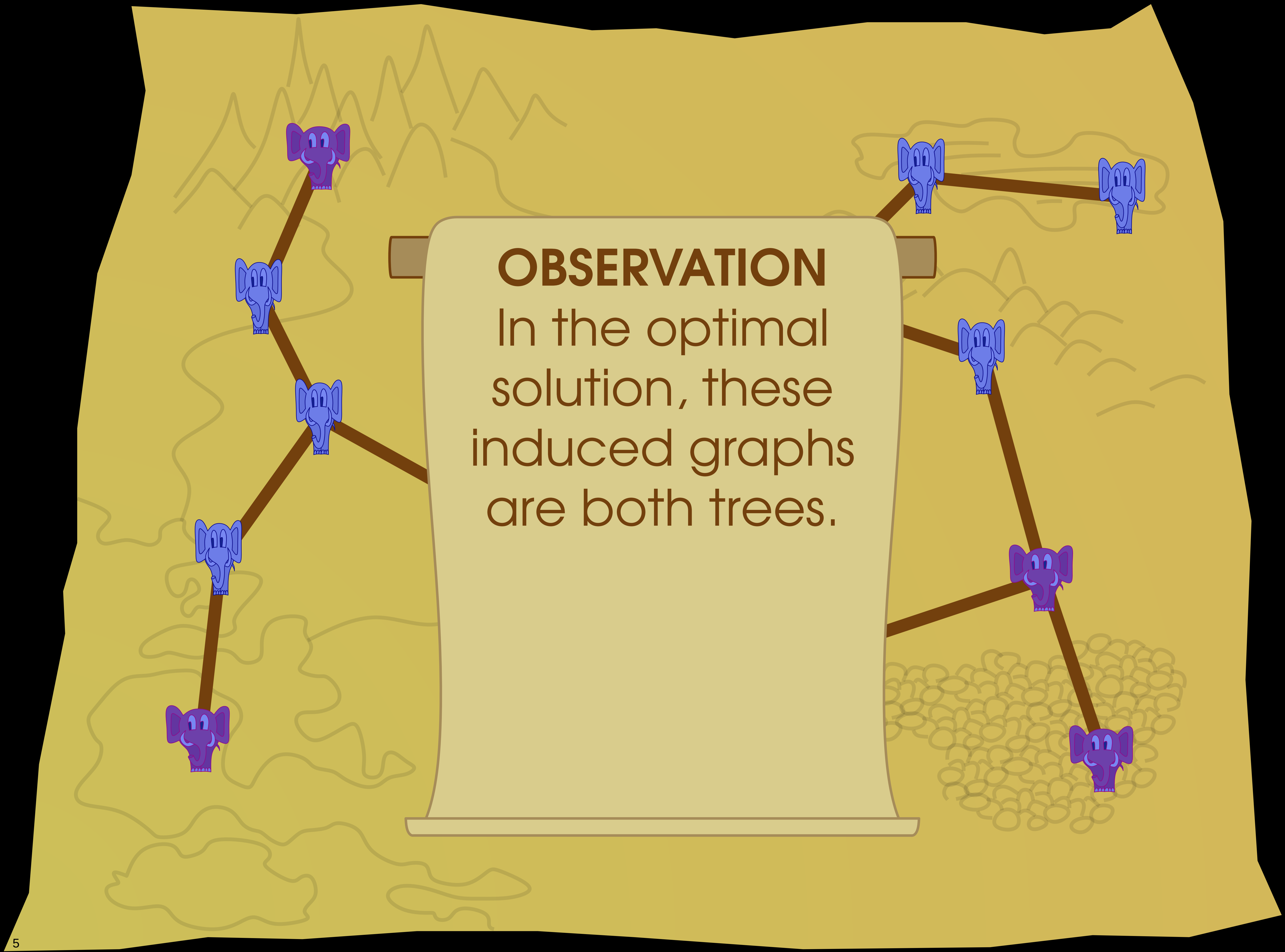

We wish to compute a *minimum length spanning graph* such that the subgraphs induced by the red and blue sets are both connected

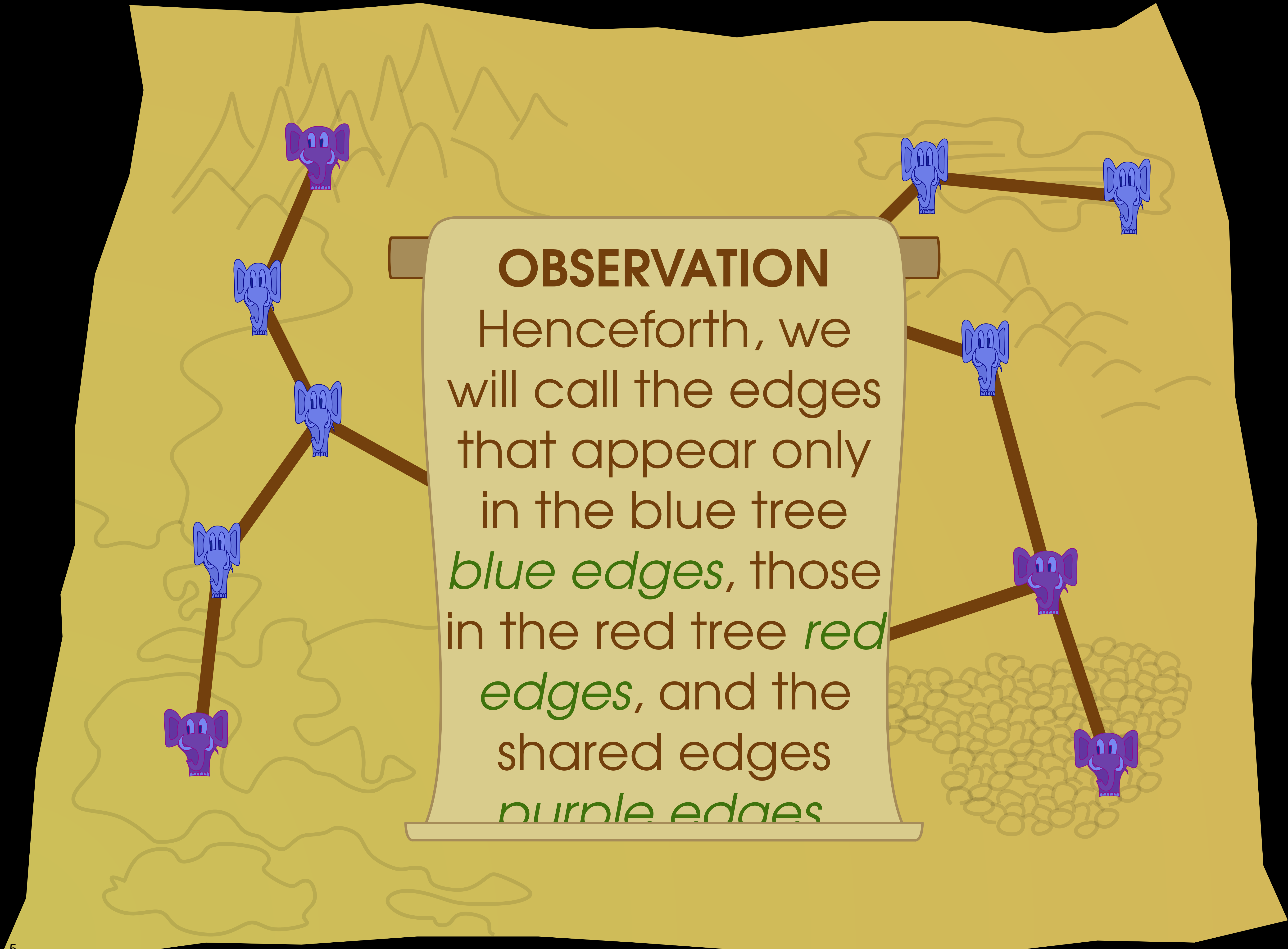


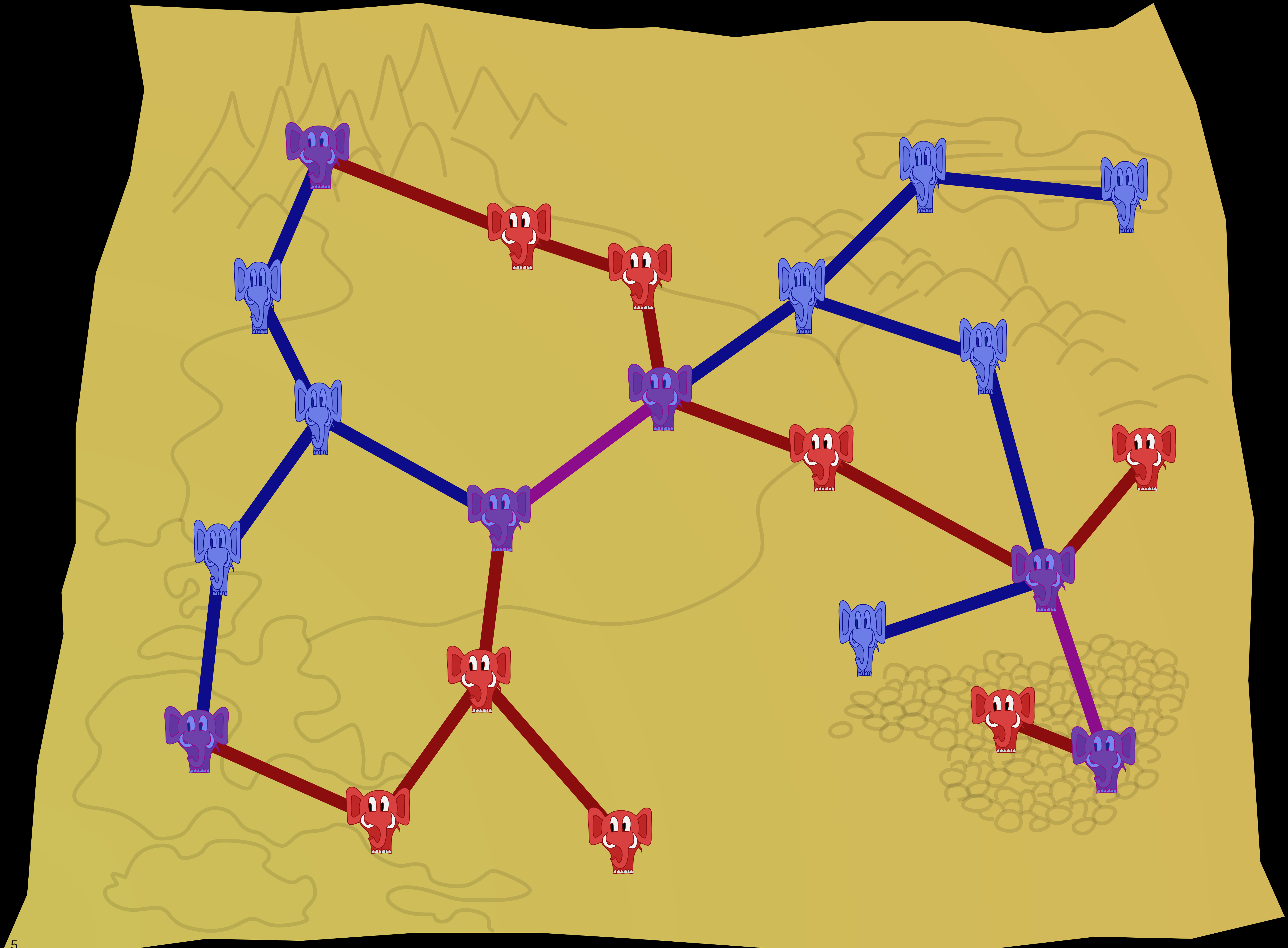







OBSERVATION
In the optimal solution, these induced graphs are both trees.


OBSERVATION
Henceforth, we will call the edges that appear only in the blue tree *blue edges*, those in the red tree *red edges*, and the shared edges *purple edges*



RESULTS

Finding the minimum weight graph that connects both the red points and the blue points is NP-hard.

RESULTS

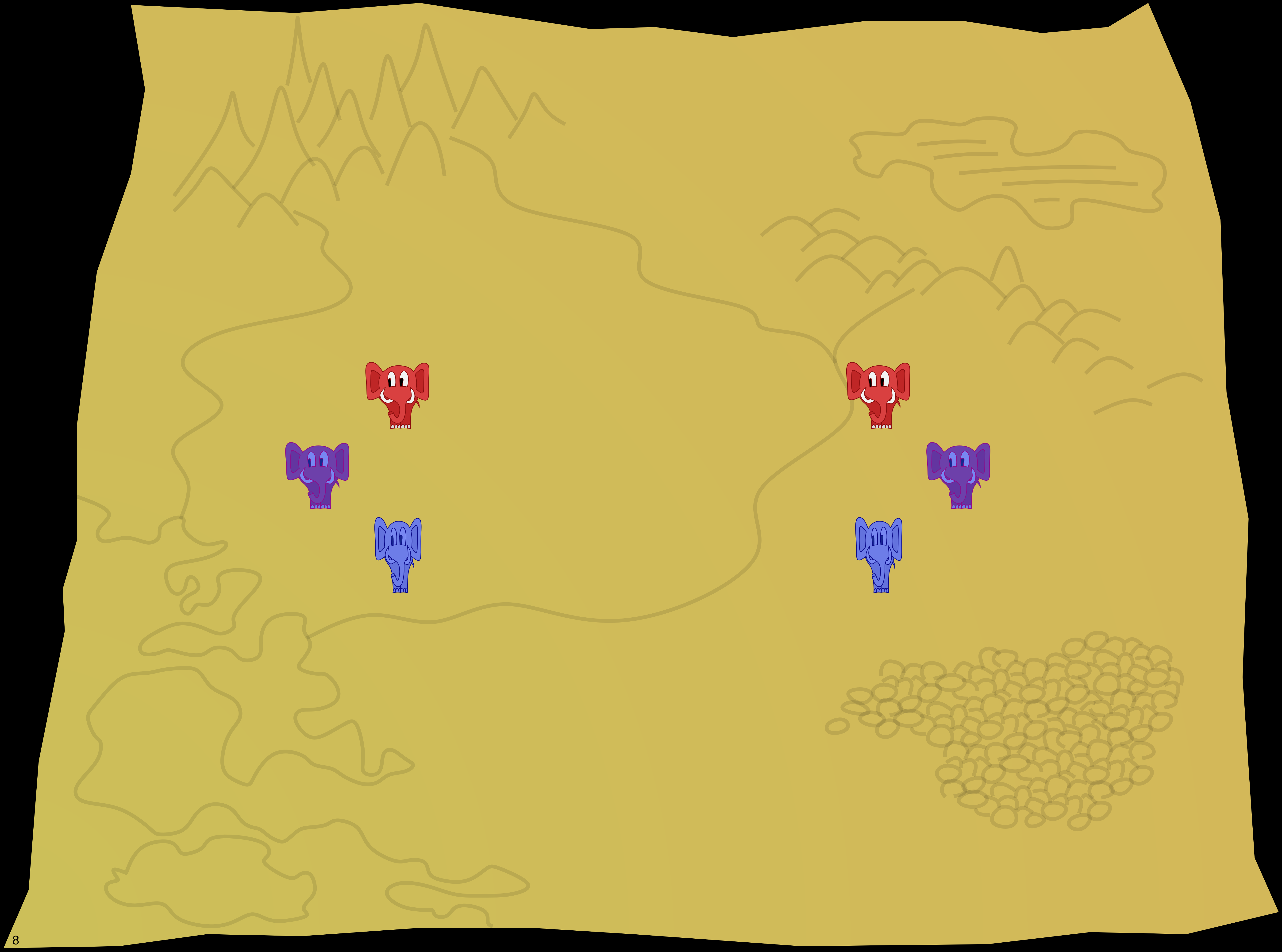
We can compute a $\frac{1}{2}\rho + 1$-approximation in $O(n \log n)$ time, where ρ is the *Steiner ratio*.

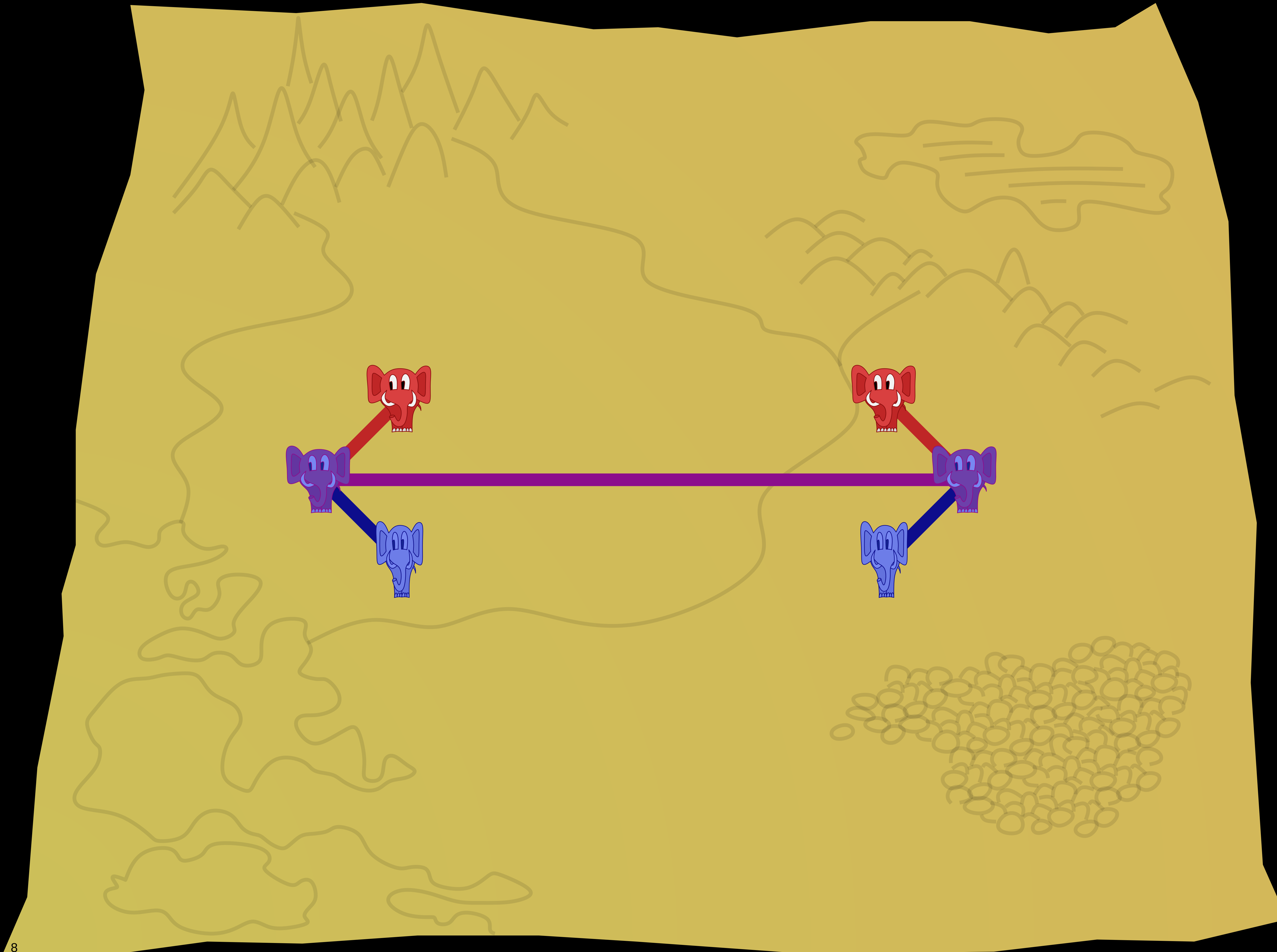
RESULTS

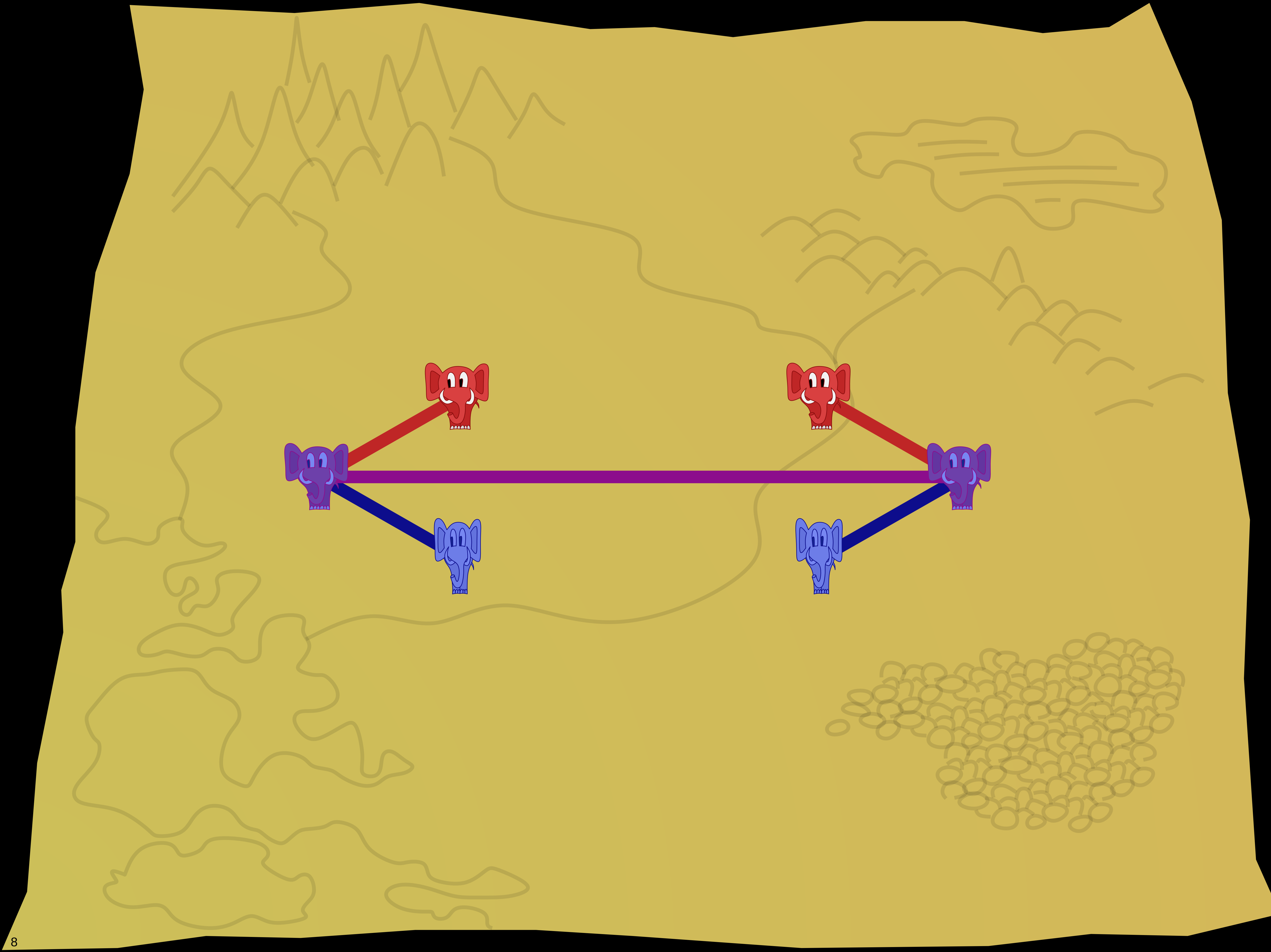
If all points lie on a line, we can compute the optimal solution in $O(n)$ time.

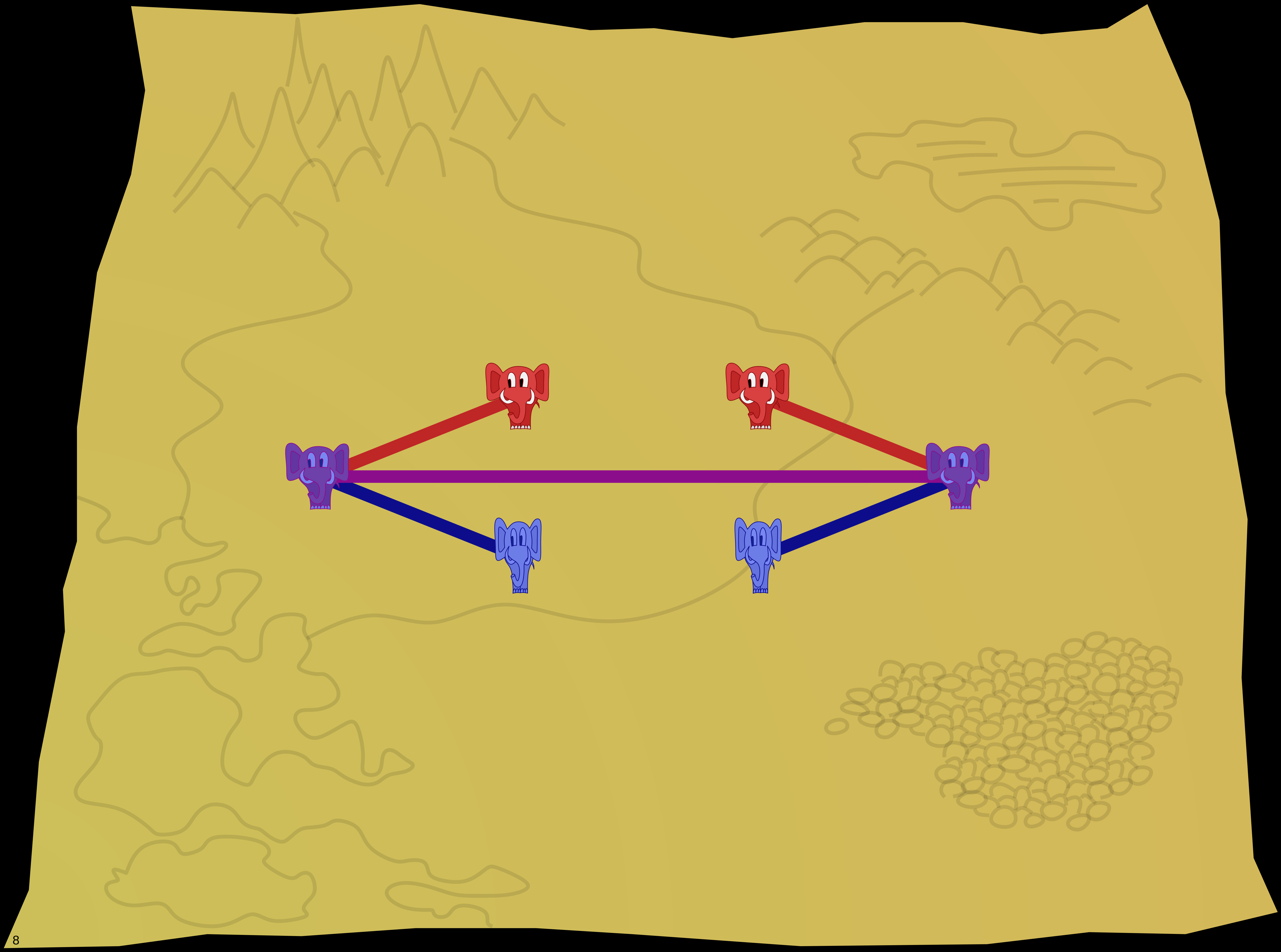
If they lie on a circle, in $O(nk^3)$ time.

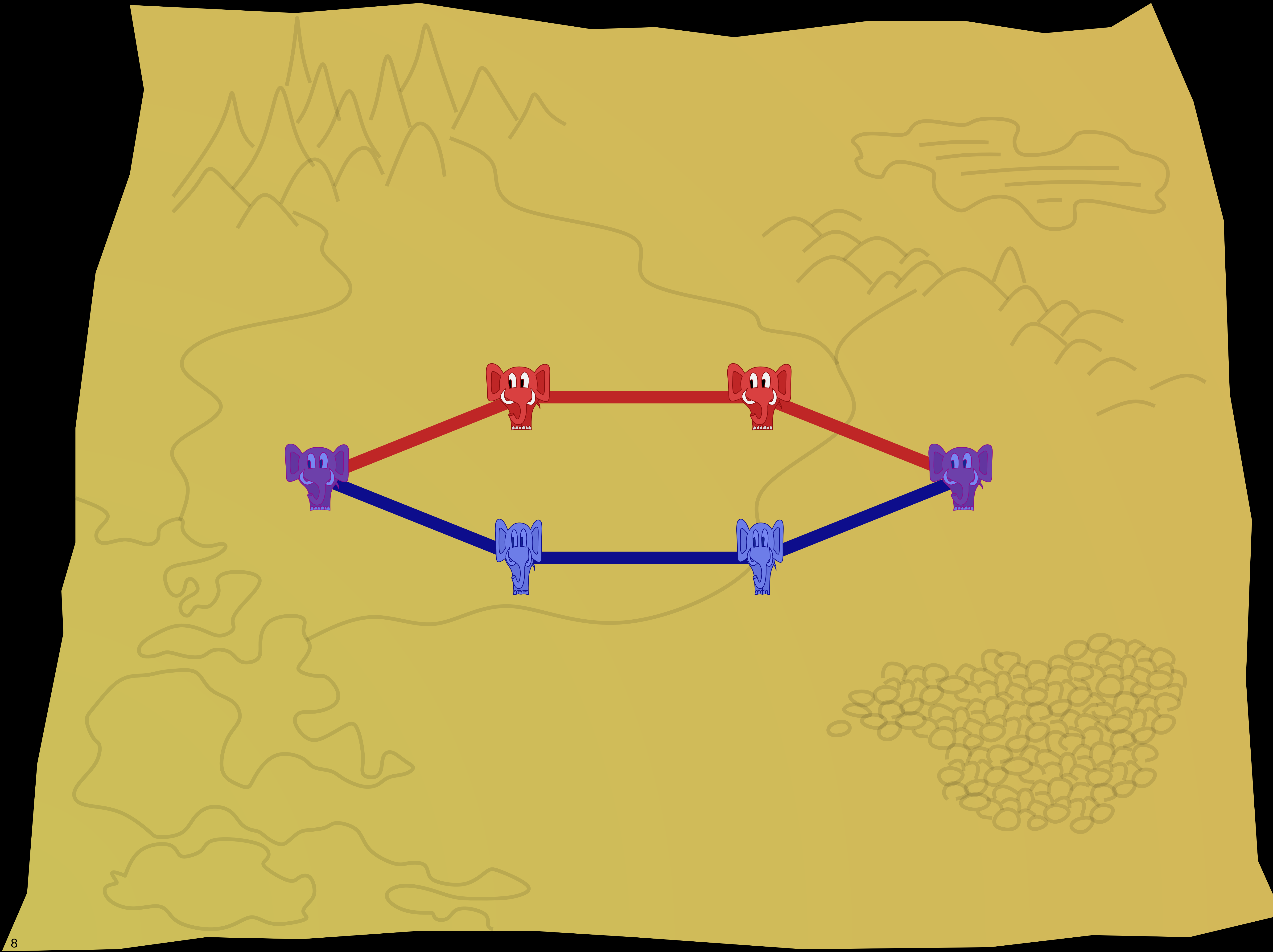
PROPERTIES OF RBP GRAPHS





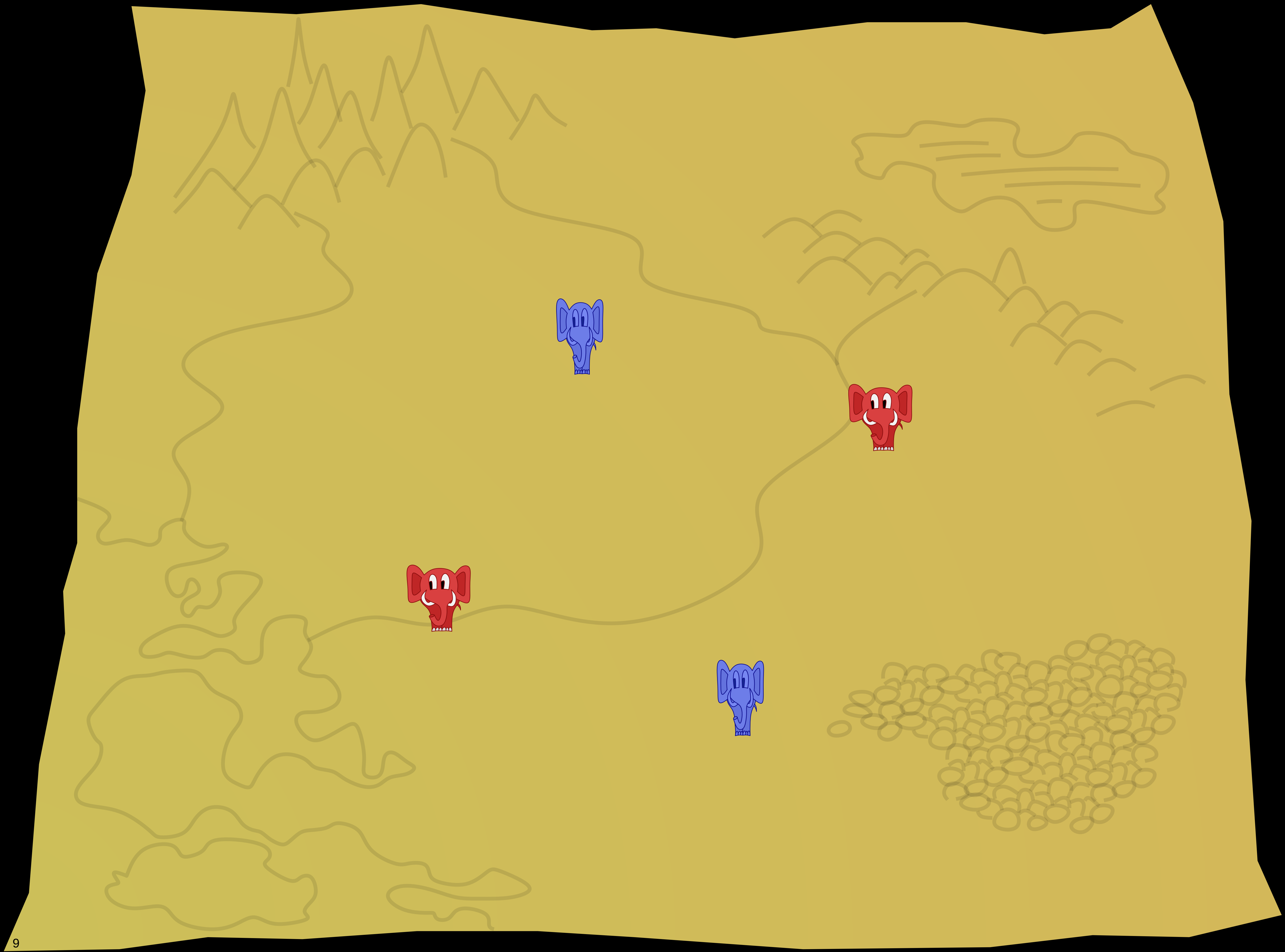


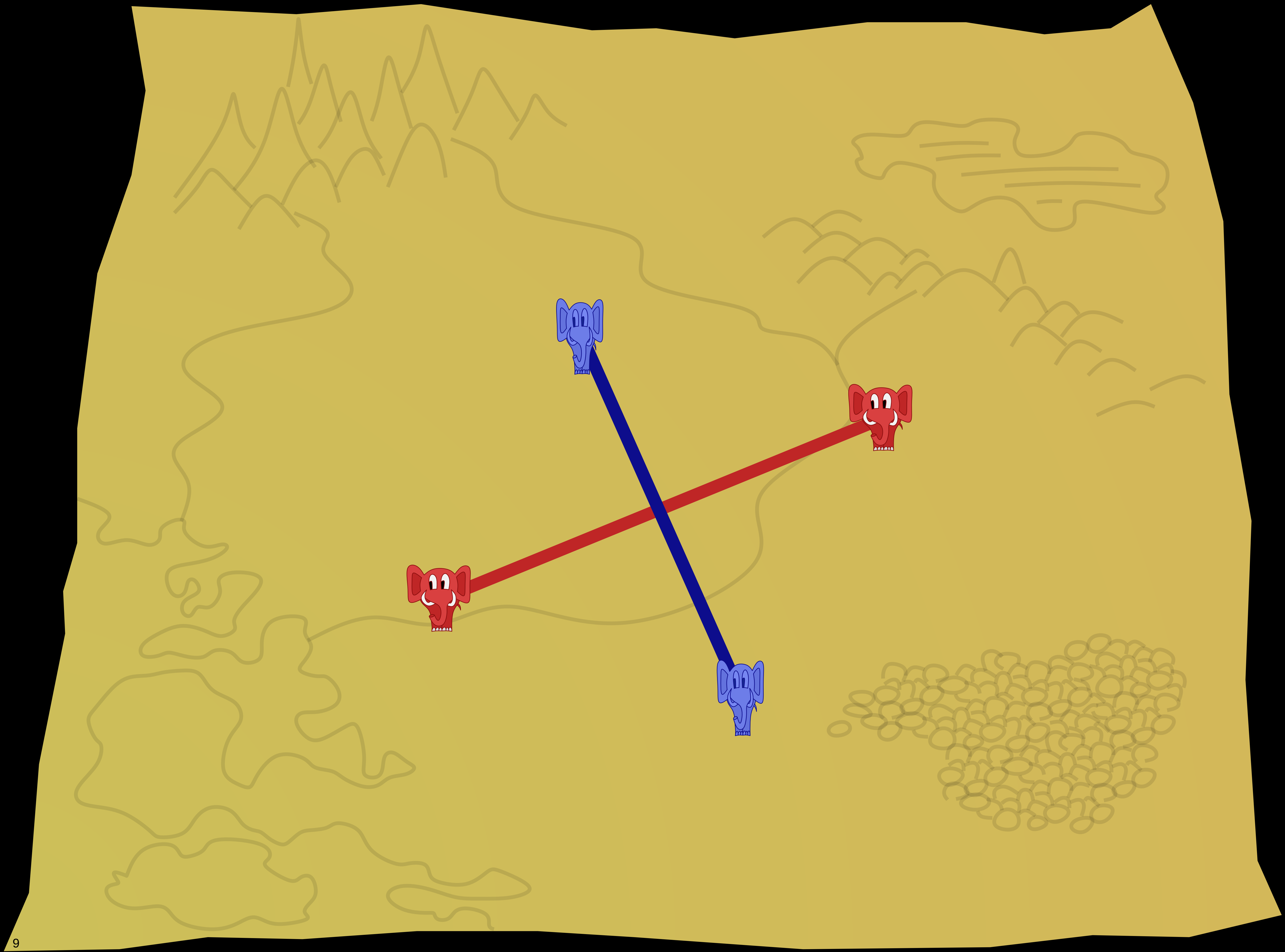




OBSERVATION

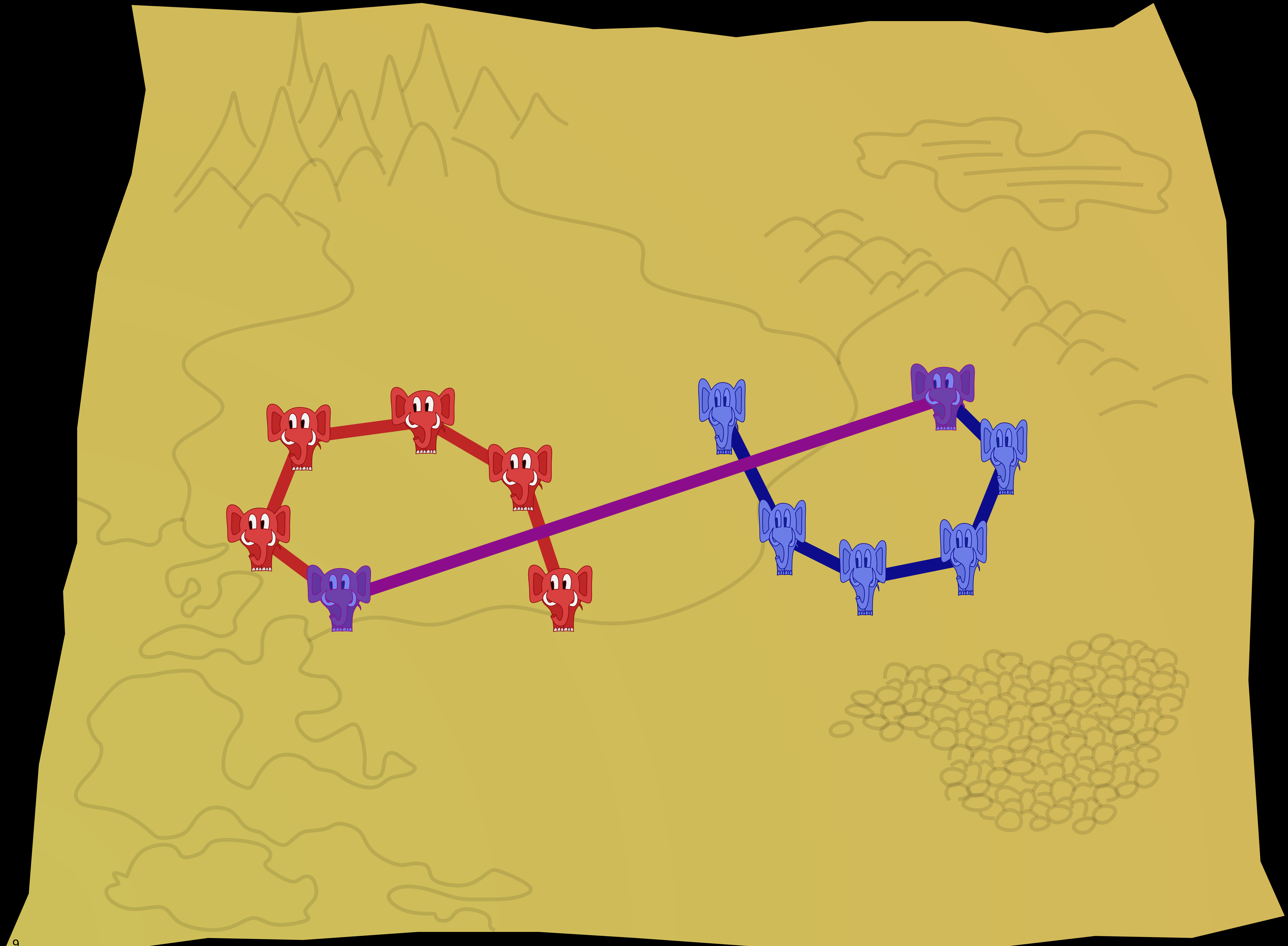
The optimal RBP graph is not necessarily plane.





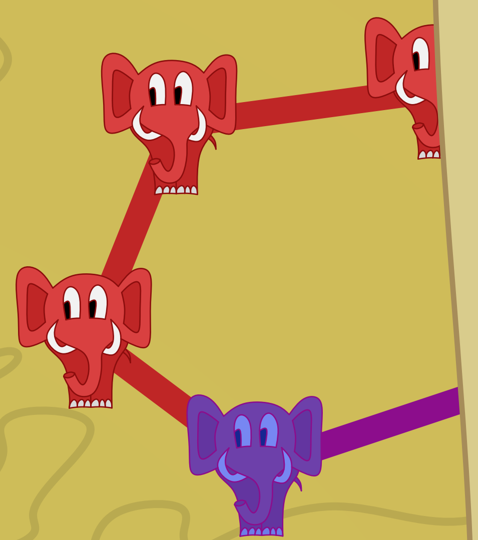
OBSERVATION

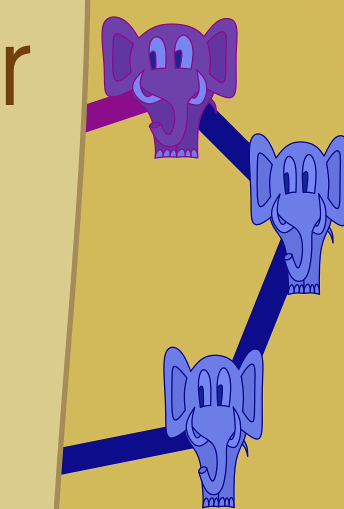
Even *purple* edges can be crossed!

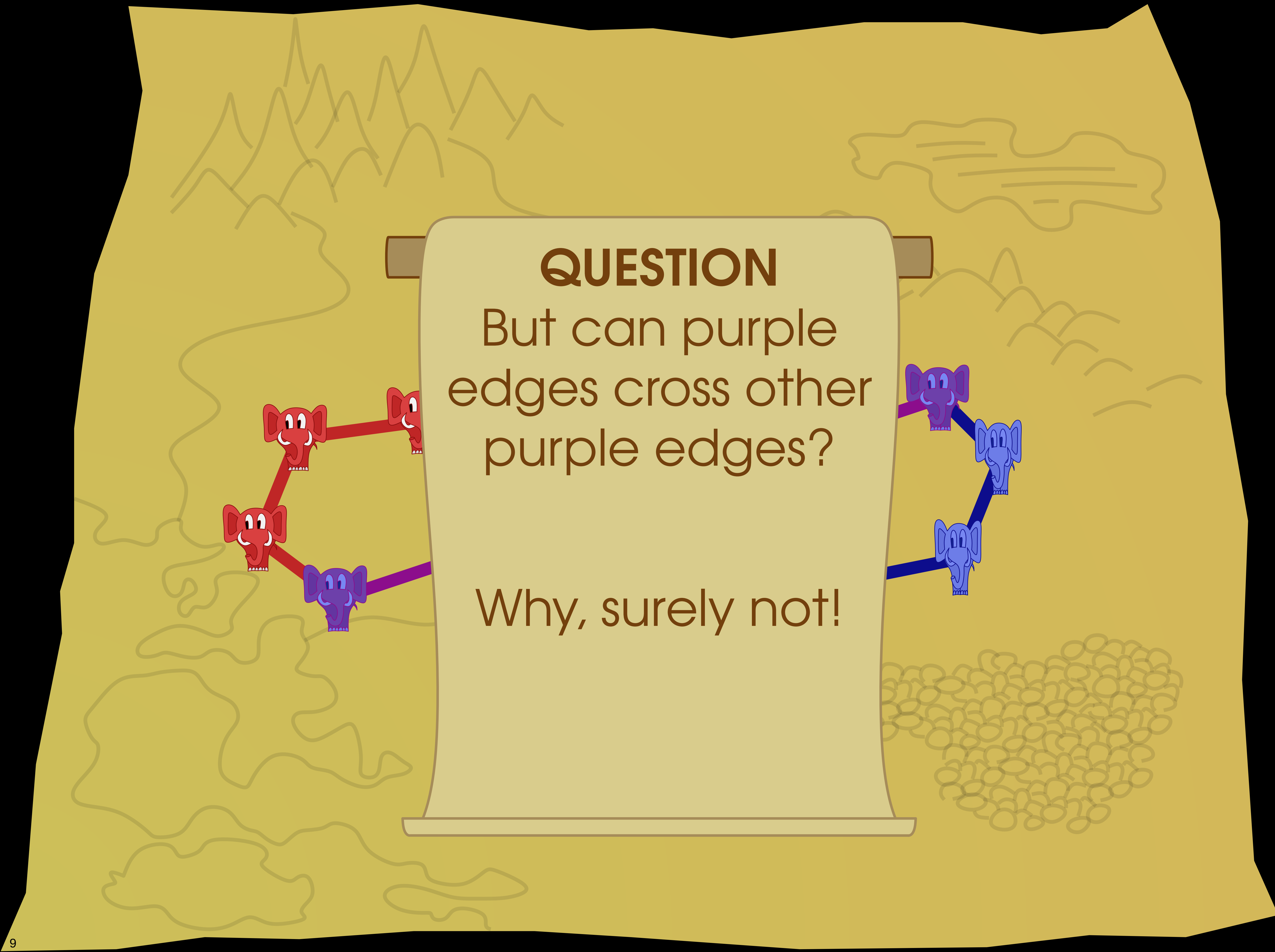


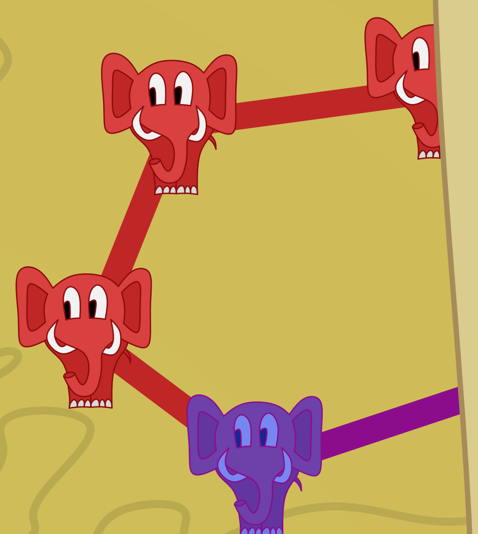
QUESTION

But can purple edges cross other purple edges?






QUESTION
But can purple edges cross other purple edges?
Why, surely not!

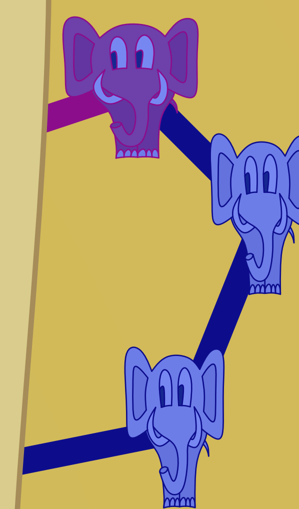


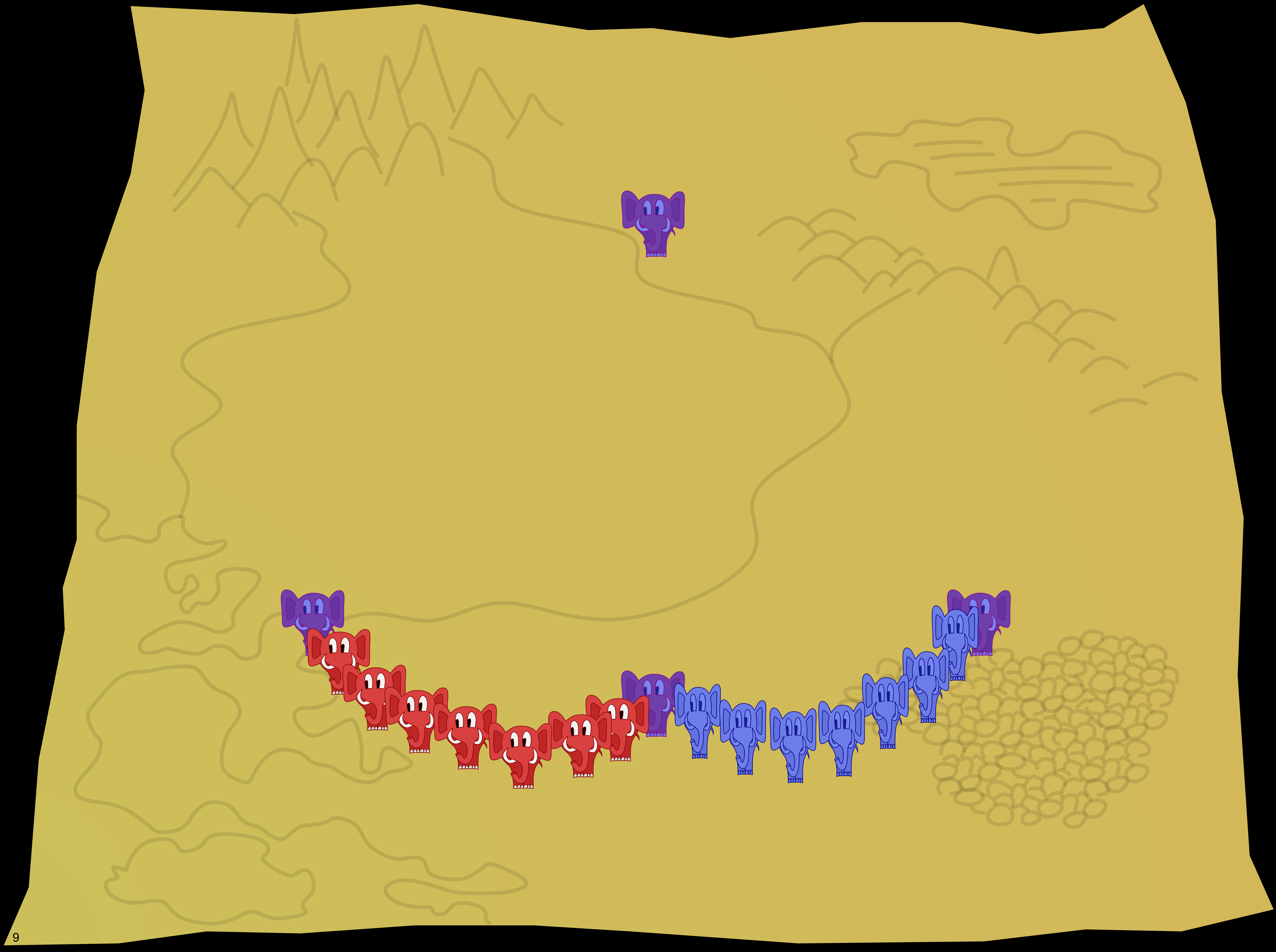
QUESTION

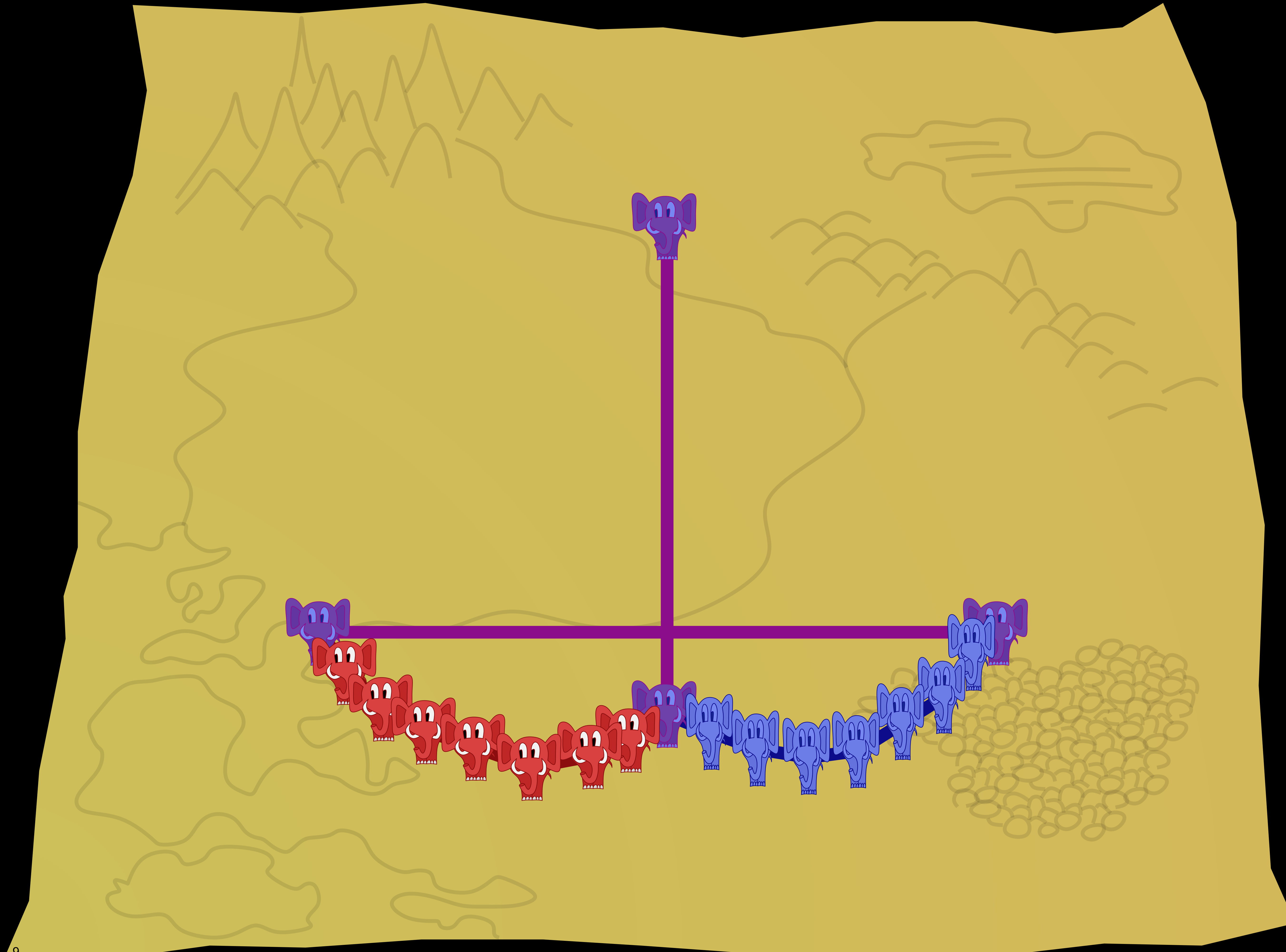
But can purple edges cross other purple edges?

Why, surely not!

. . . right?







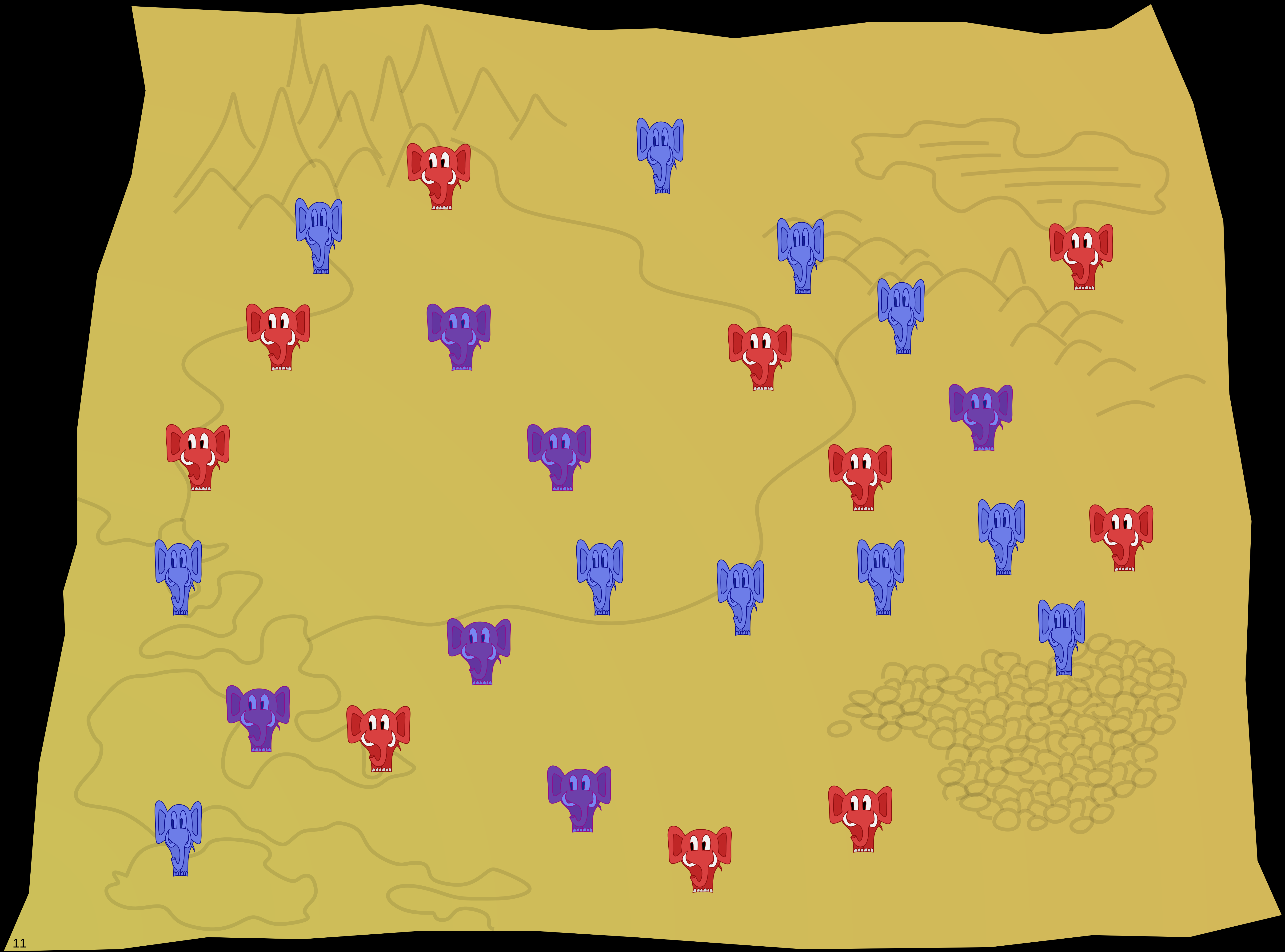
OBSERVATION

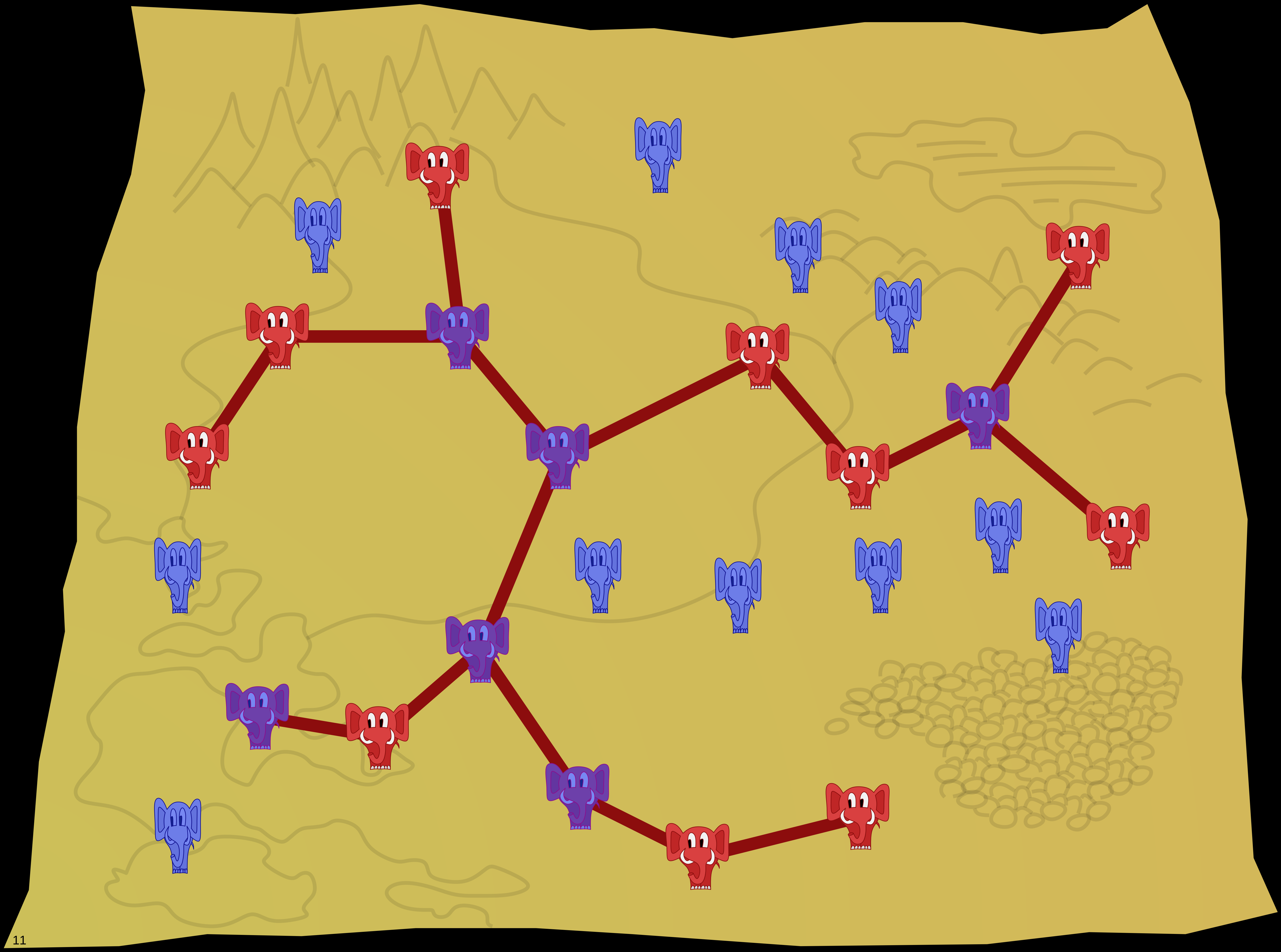
Life is even worse. A single purple edge can in fact intersect arbitrarily many other purple edges!

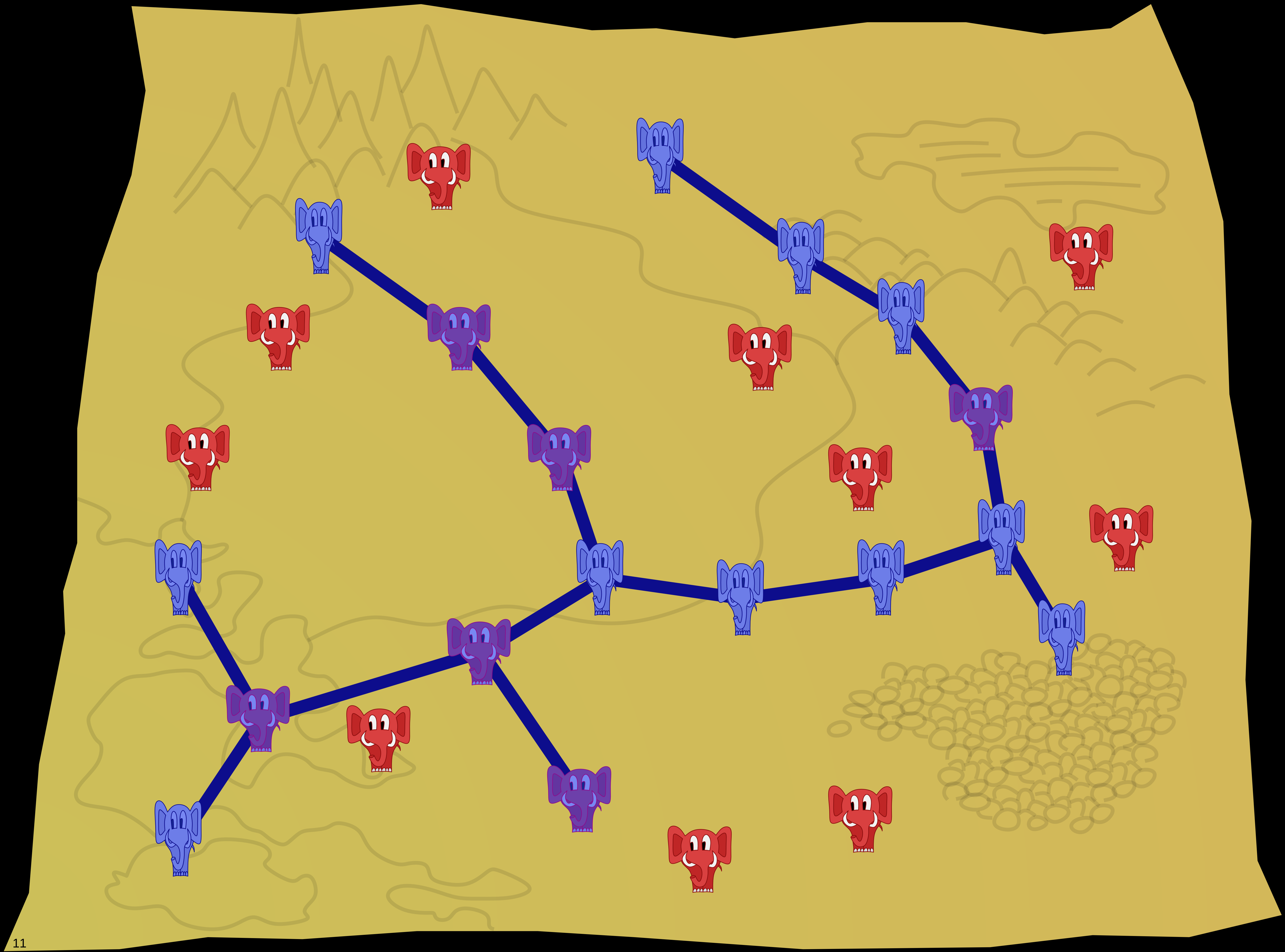
APPROXIMATION ALGORITHMS

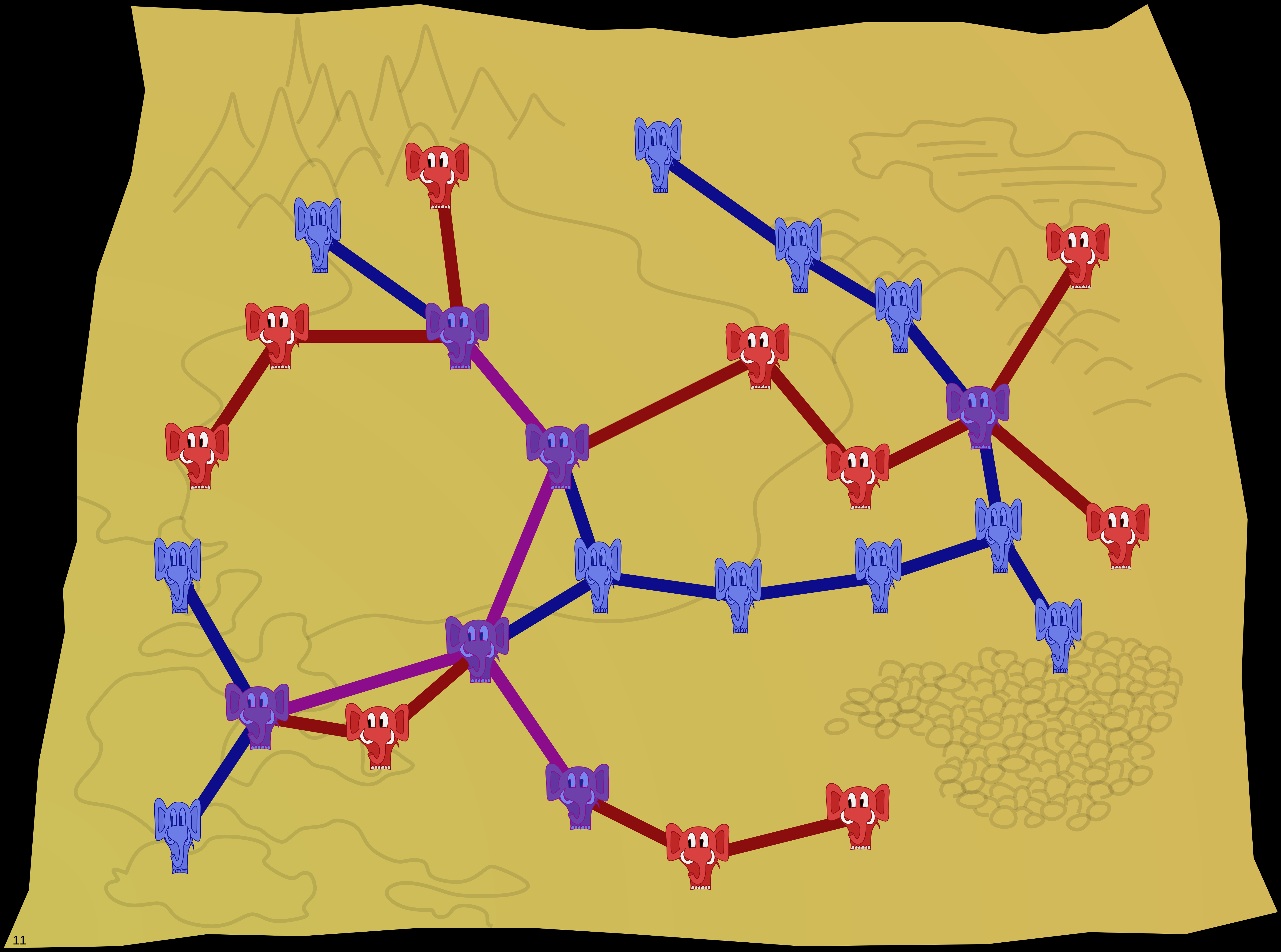
ALGORITHM 1

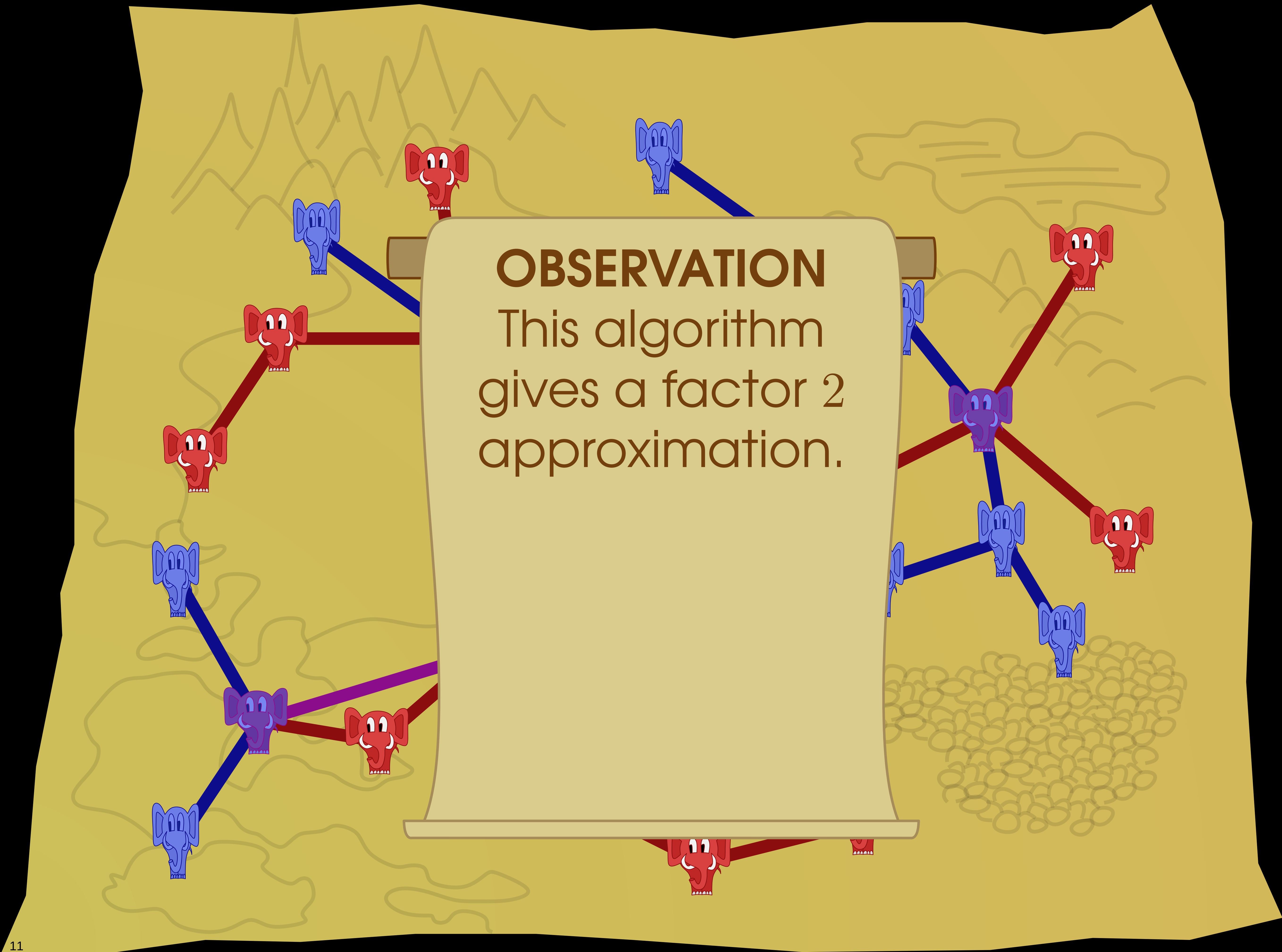
Compute the MST of the red points.
Compute the MST of the blue points.
Take union of edges.







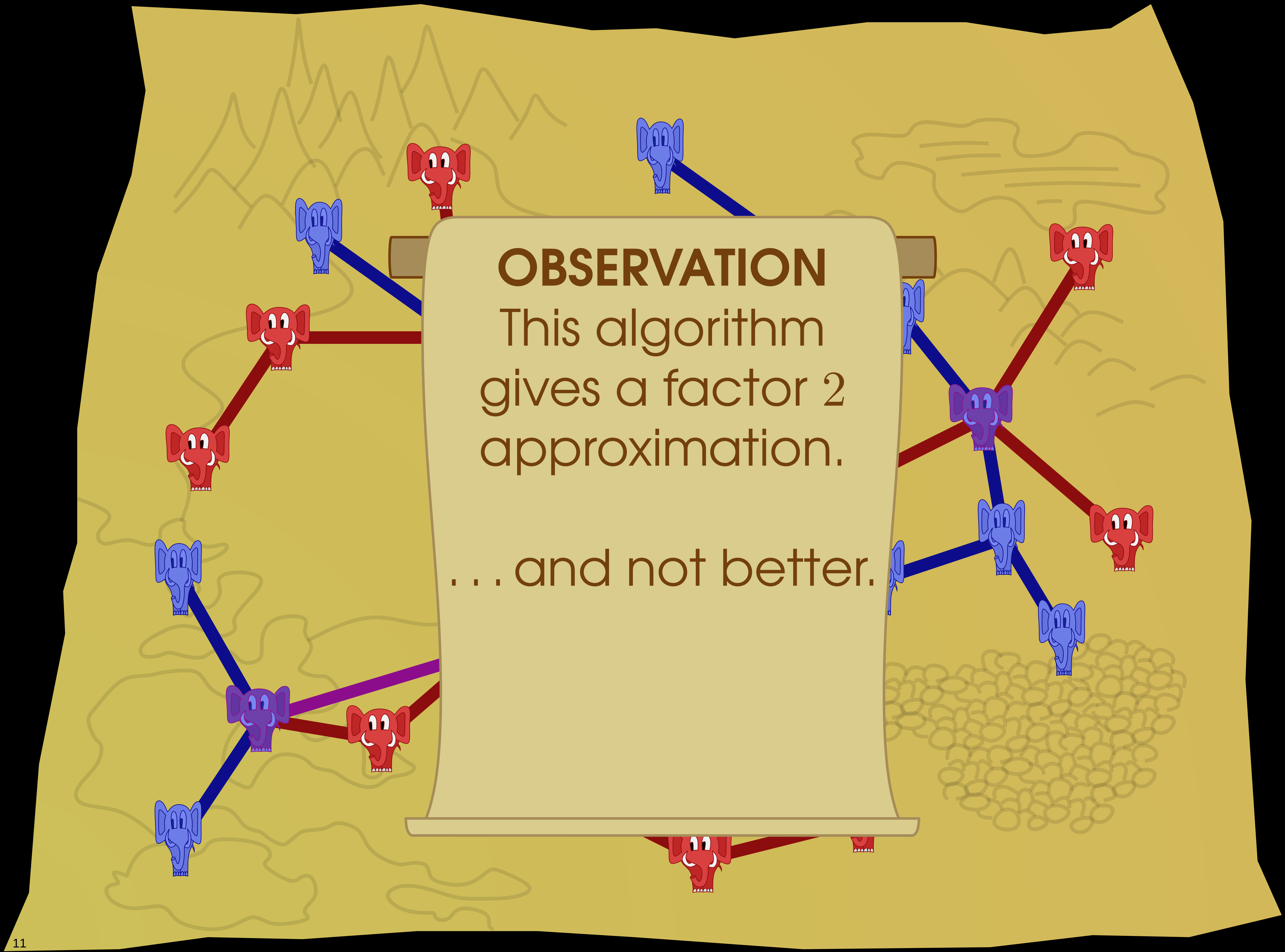


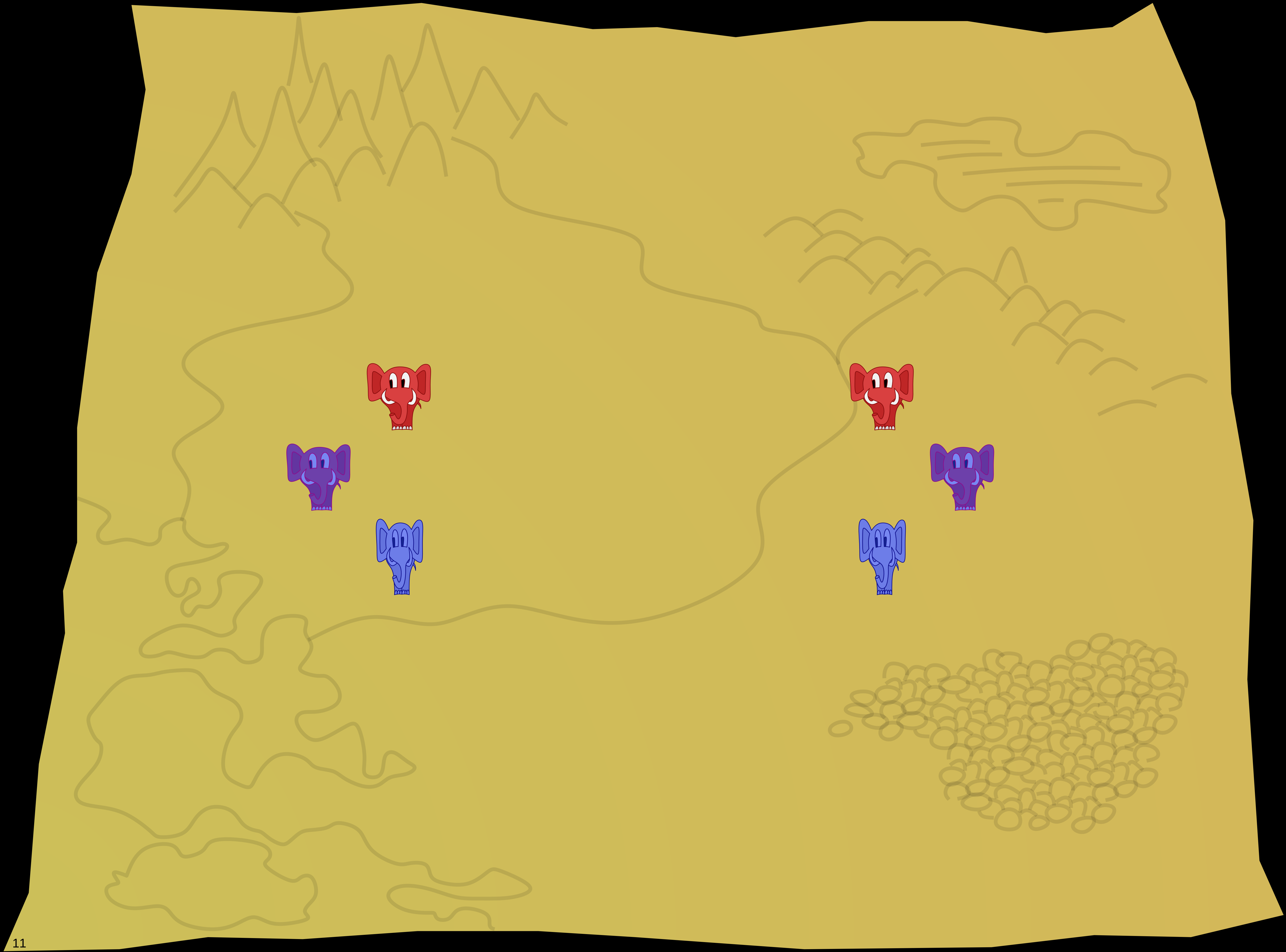

OBSERVATION
This algorithm gives a factor 2 approximation.

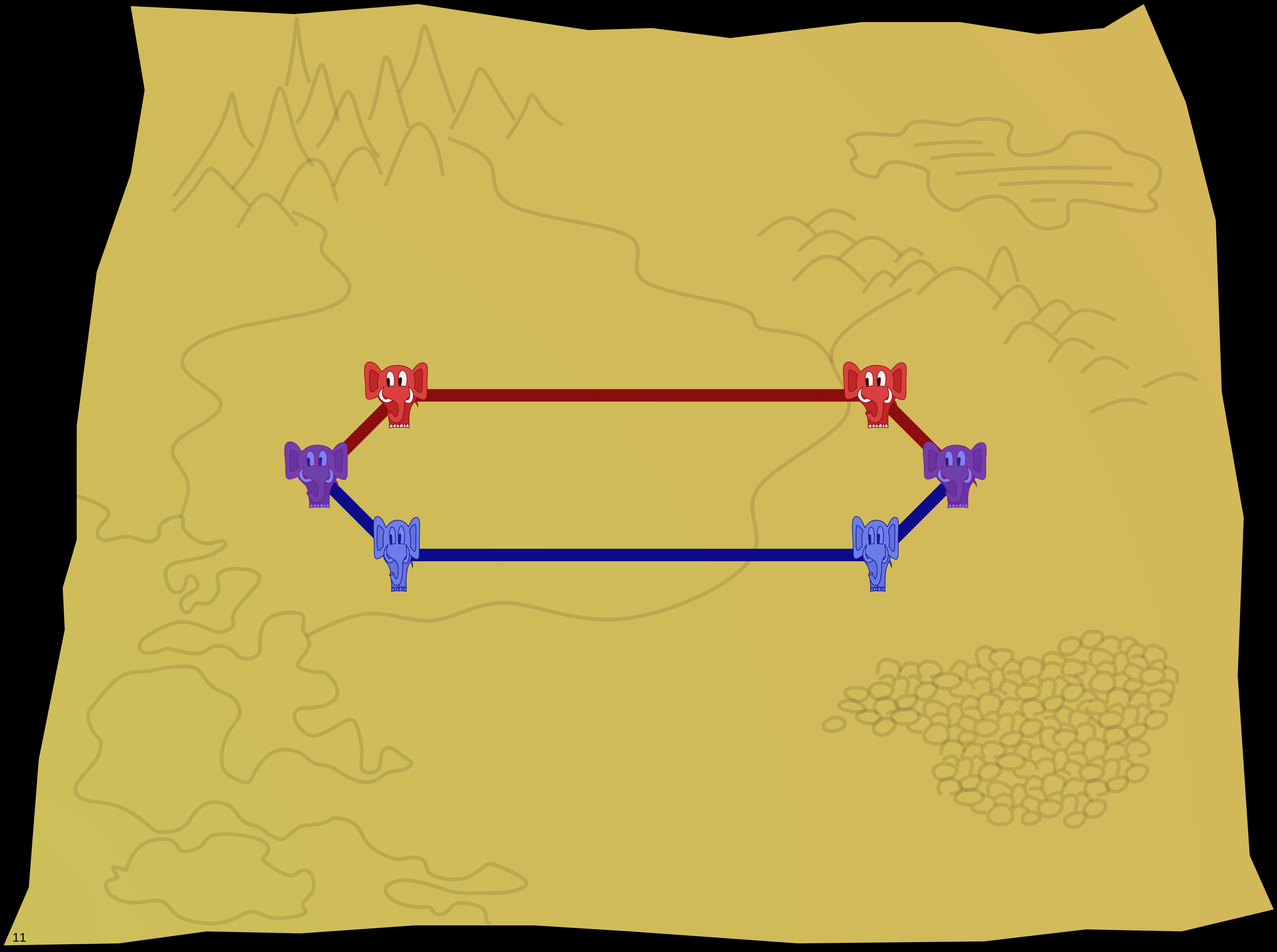
OBSERVATION

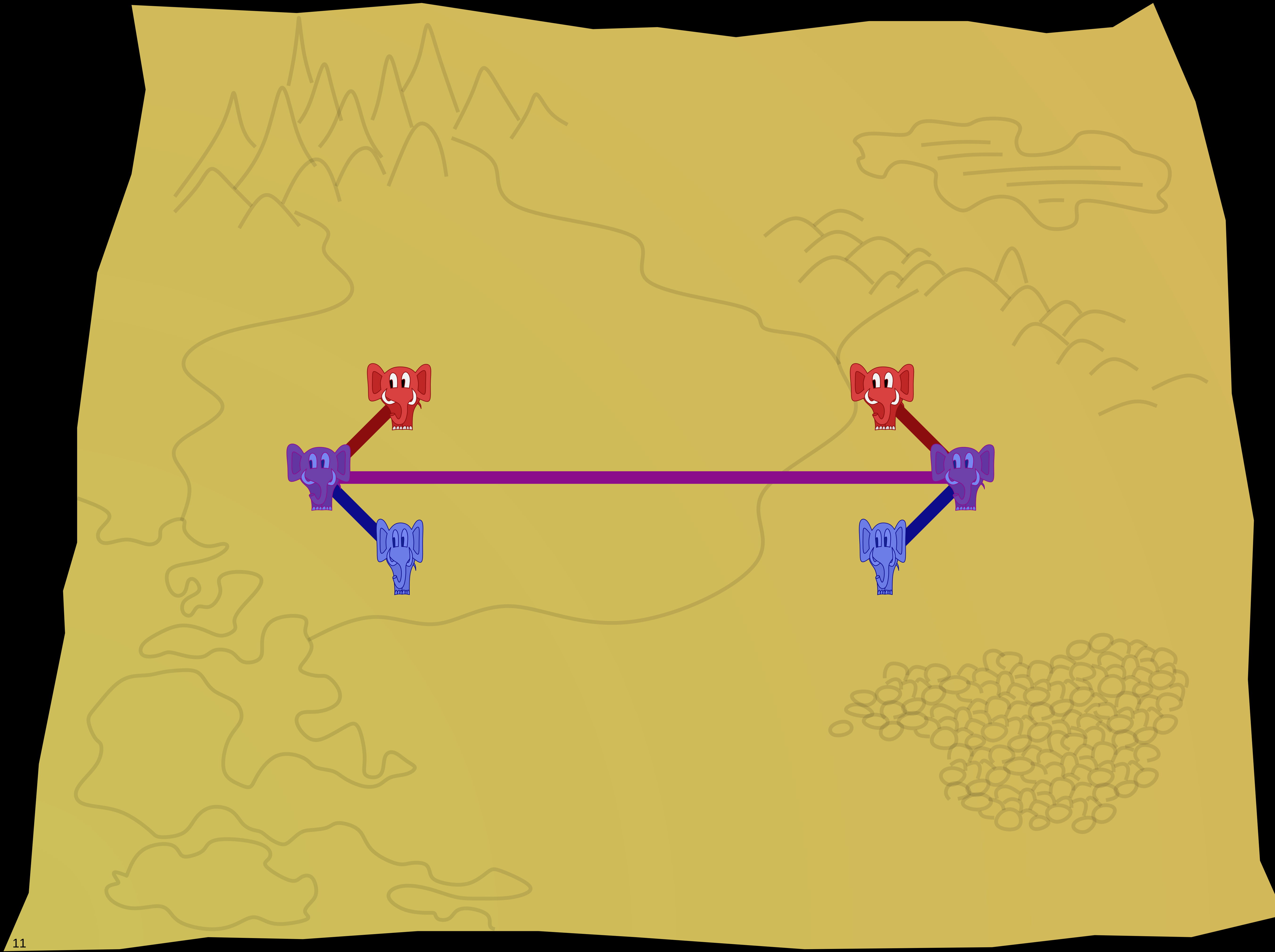
This algorithm gives a factor 2 approximation.

. . . and not better.



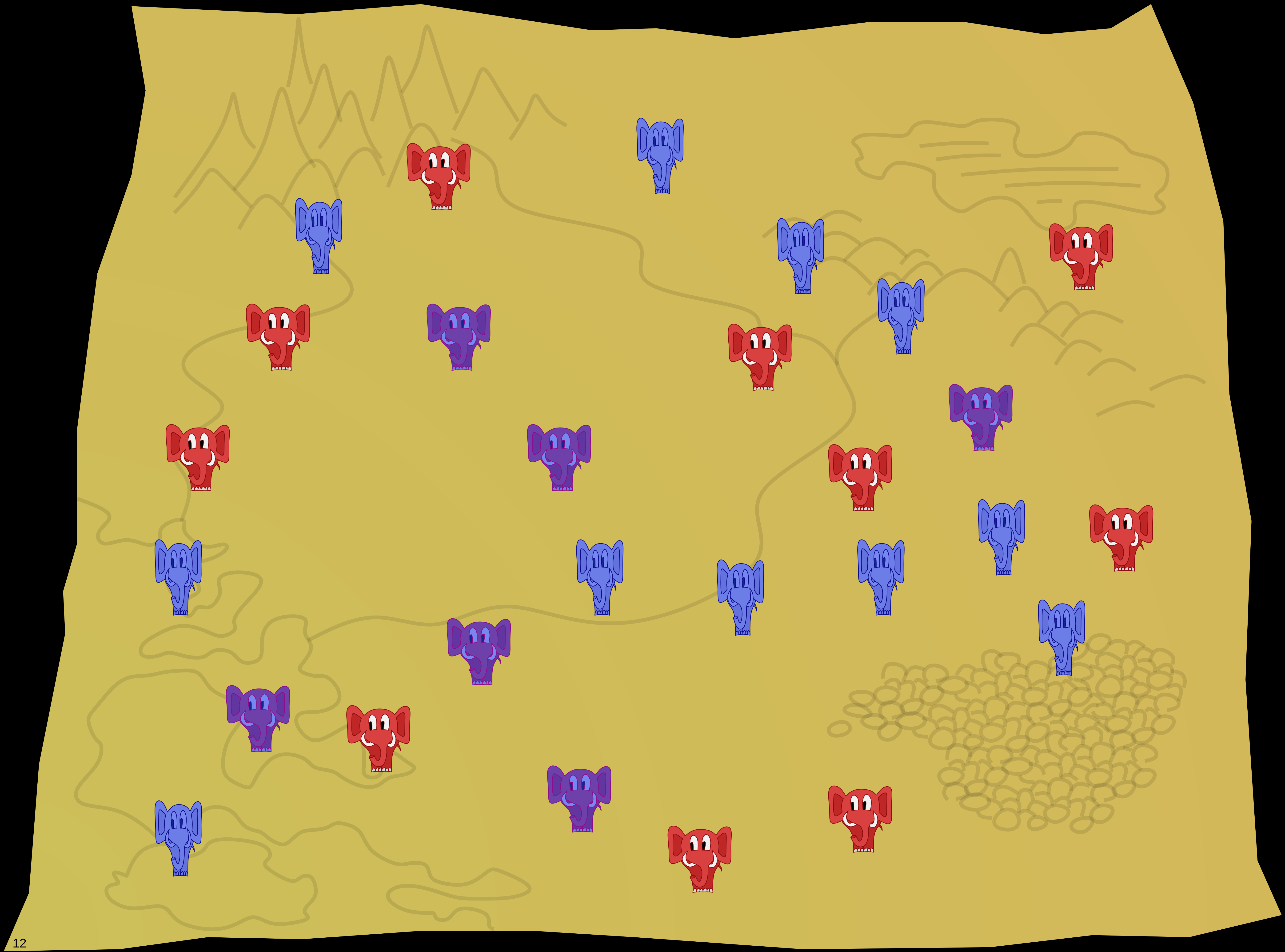


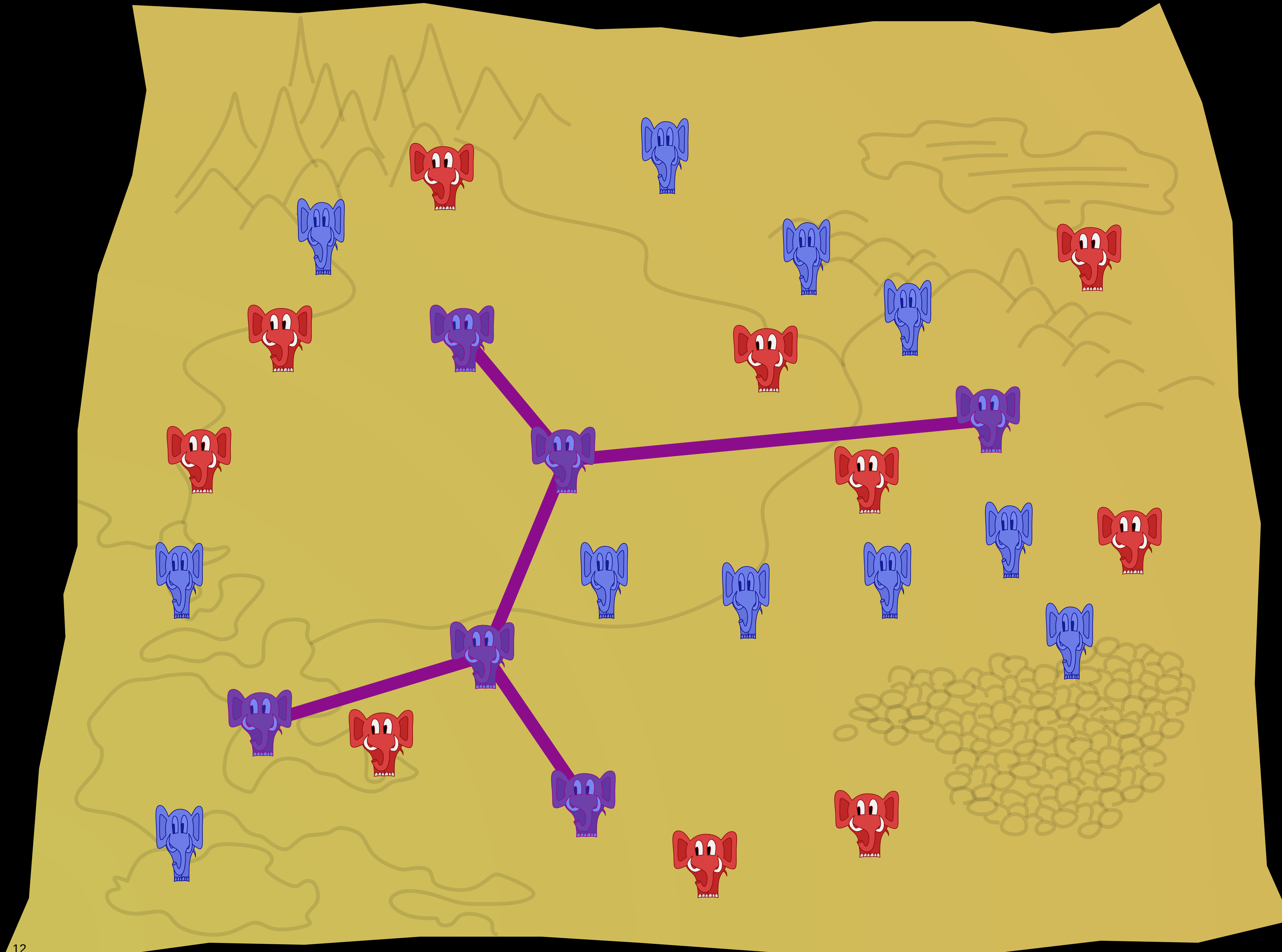


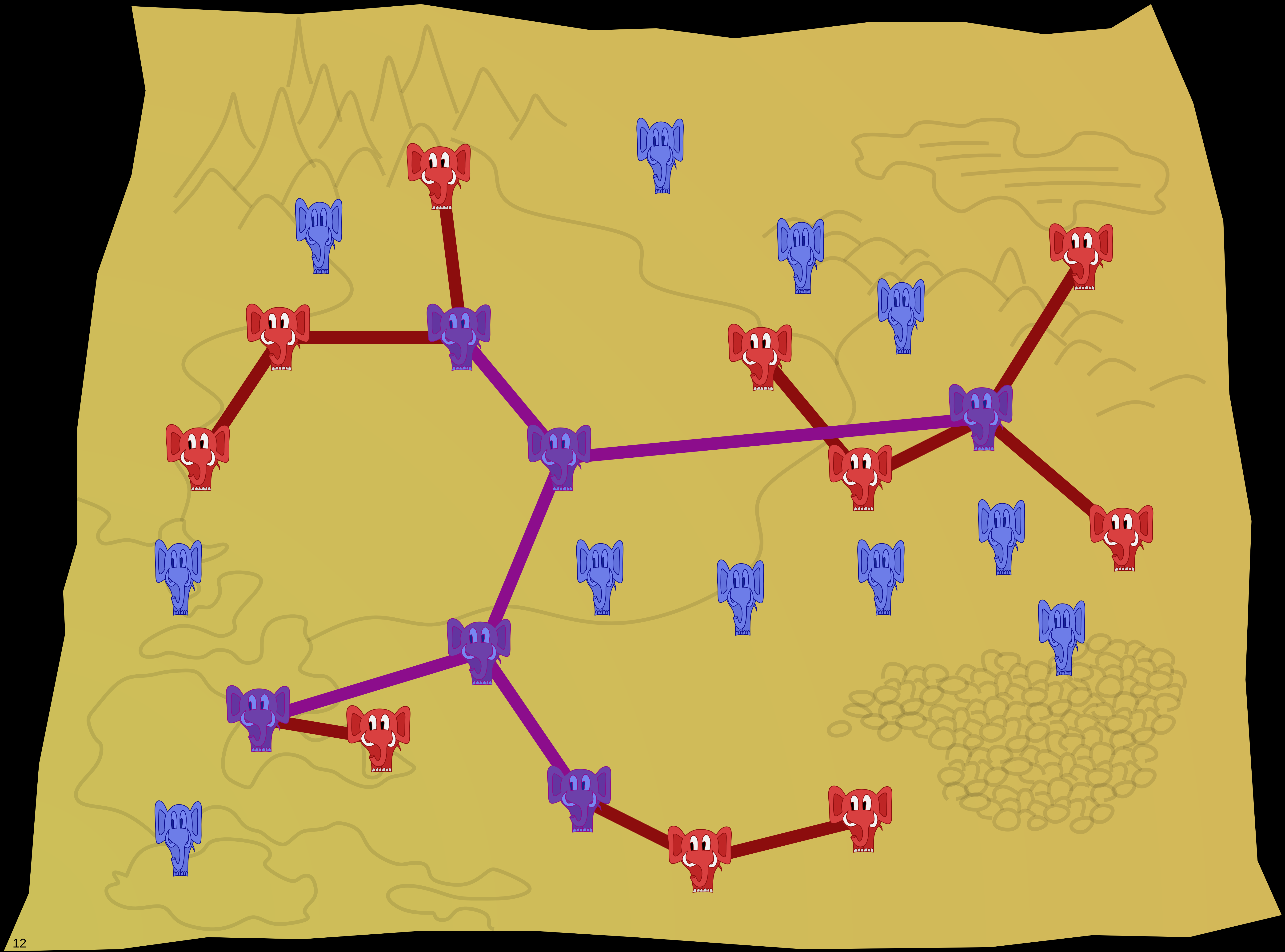


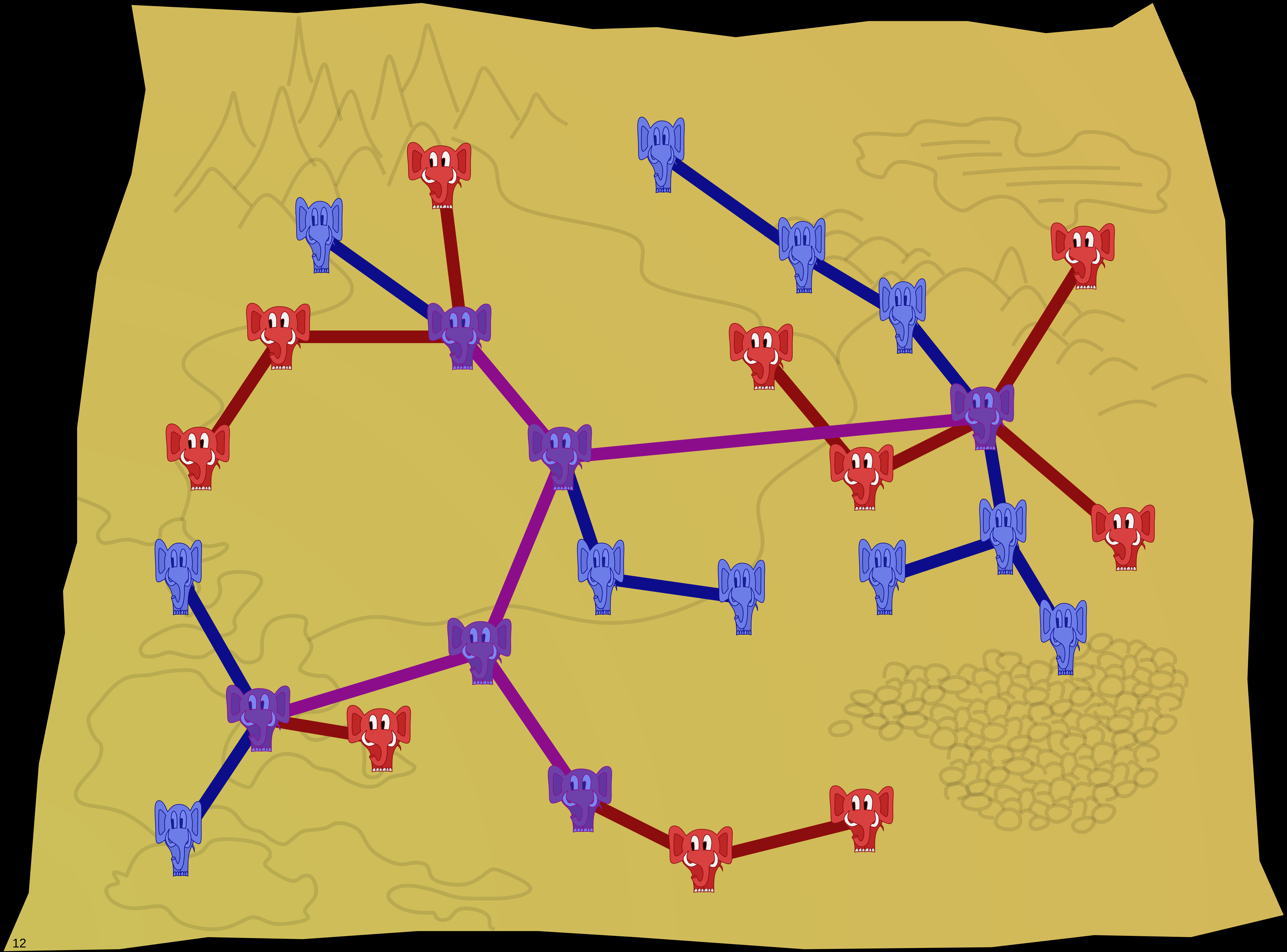
ALGORITHM 2

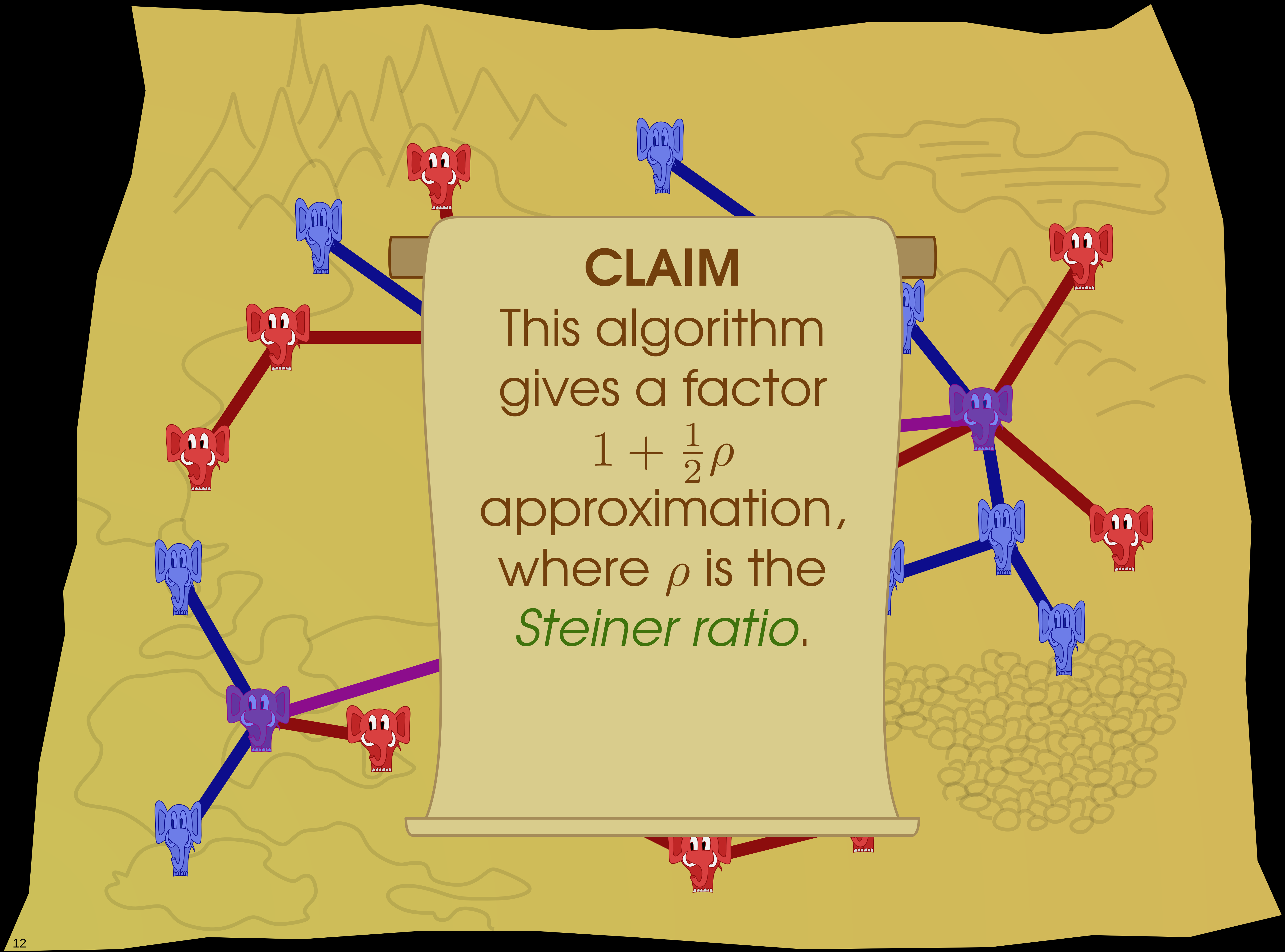
Compute the MST of the *purple* points. Optimally connect the red and blue points to the purple skeleton.



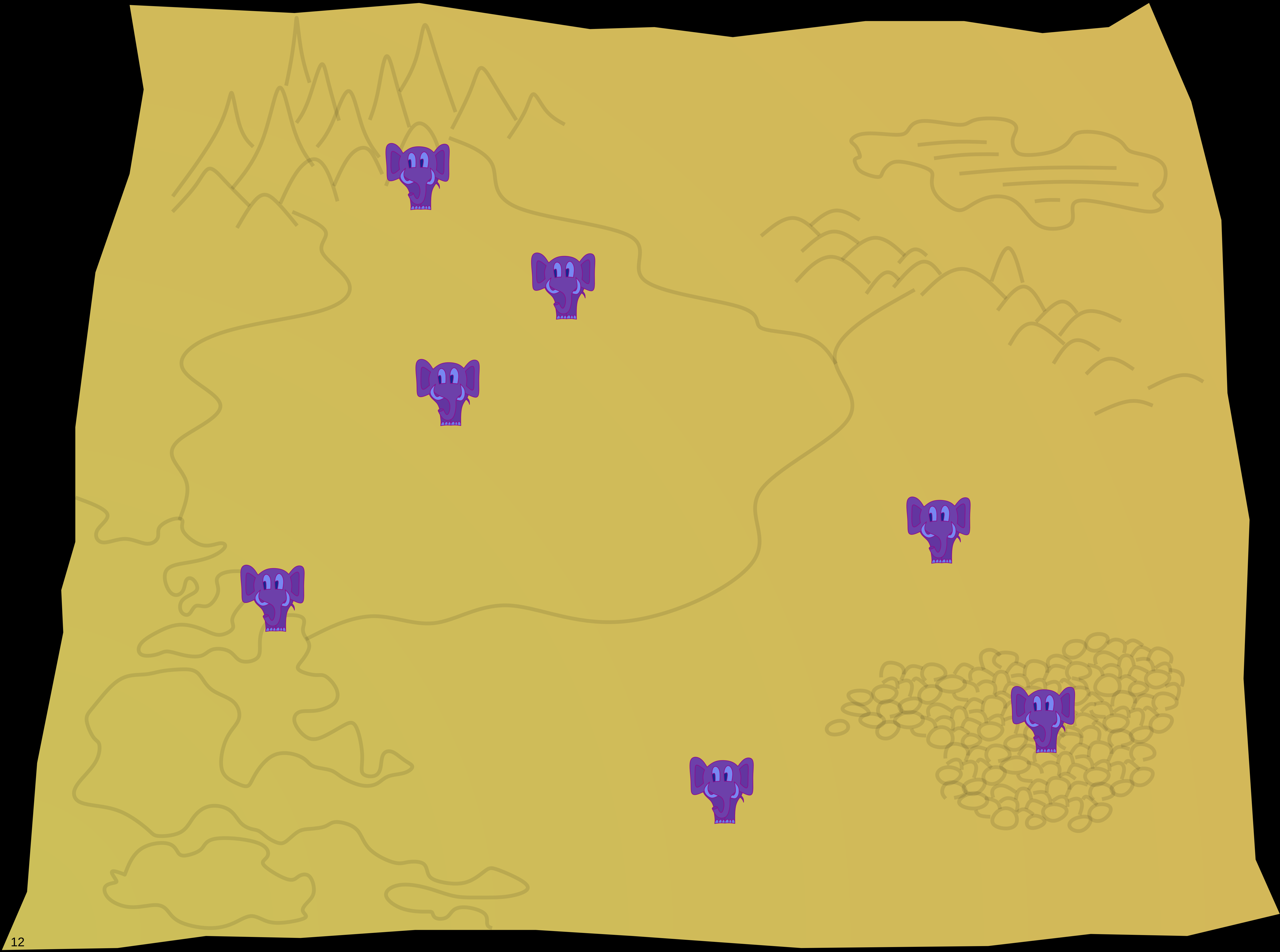


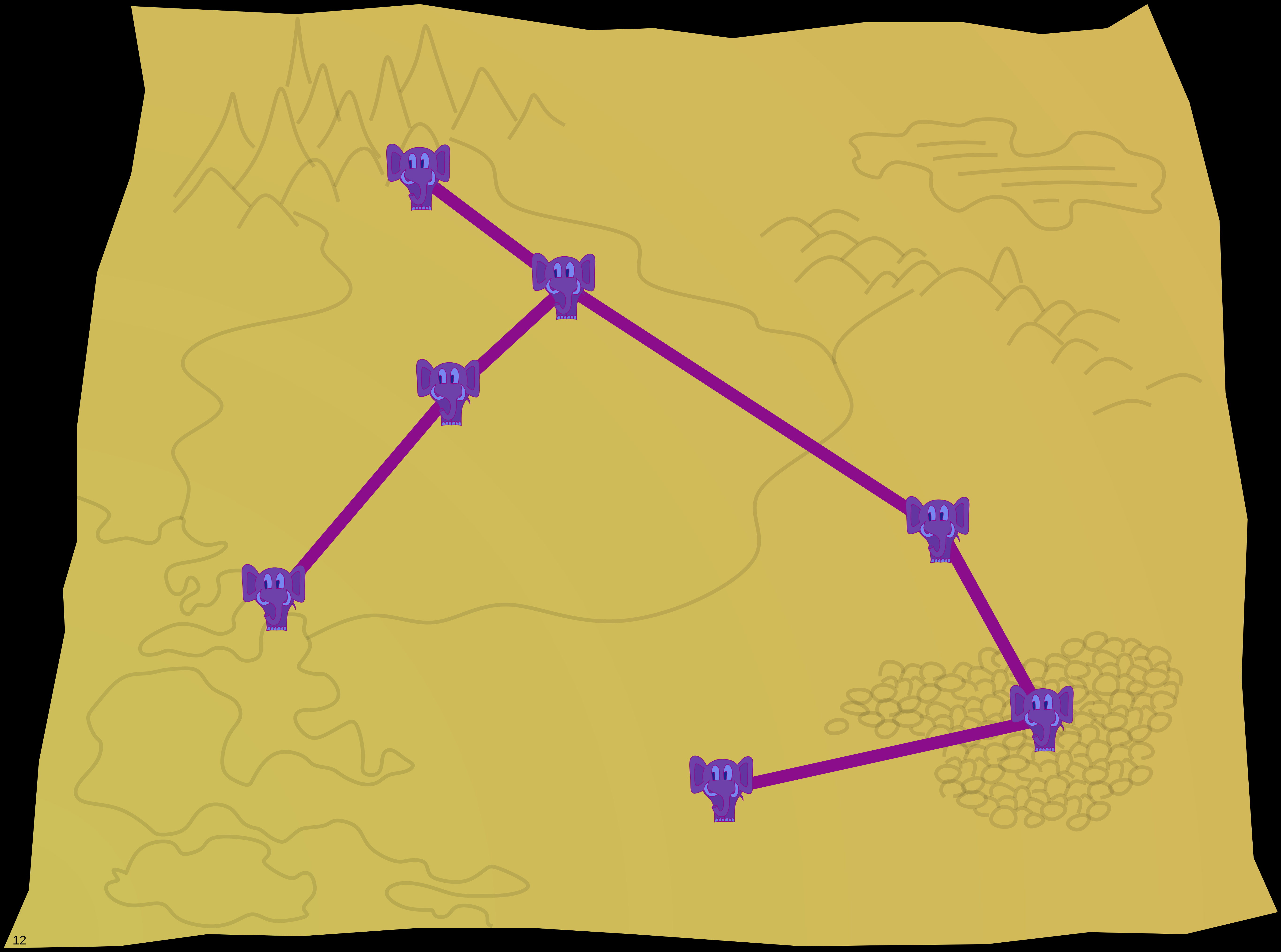


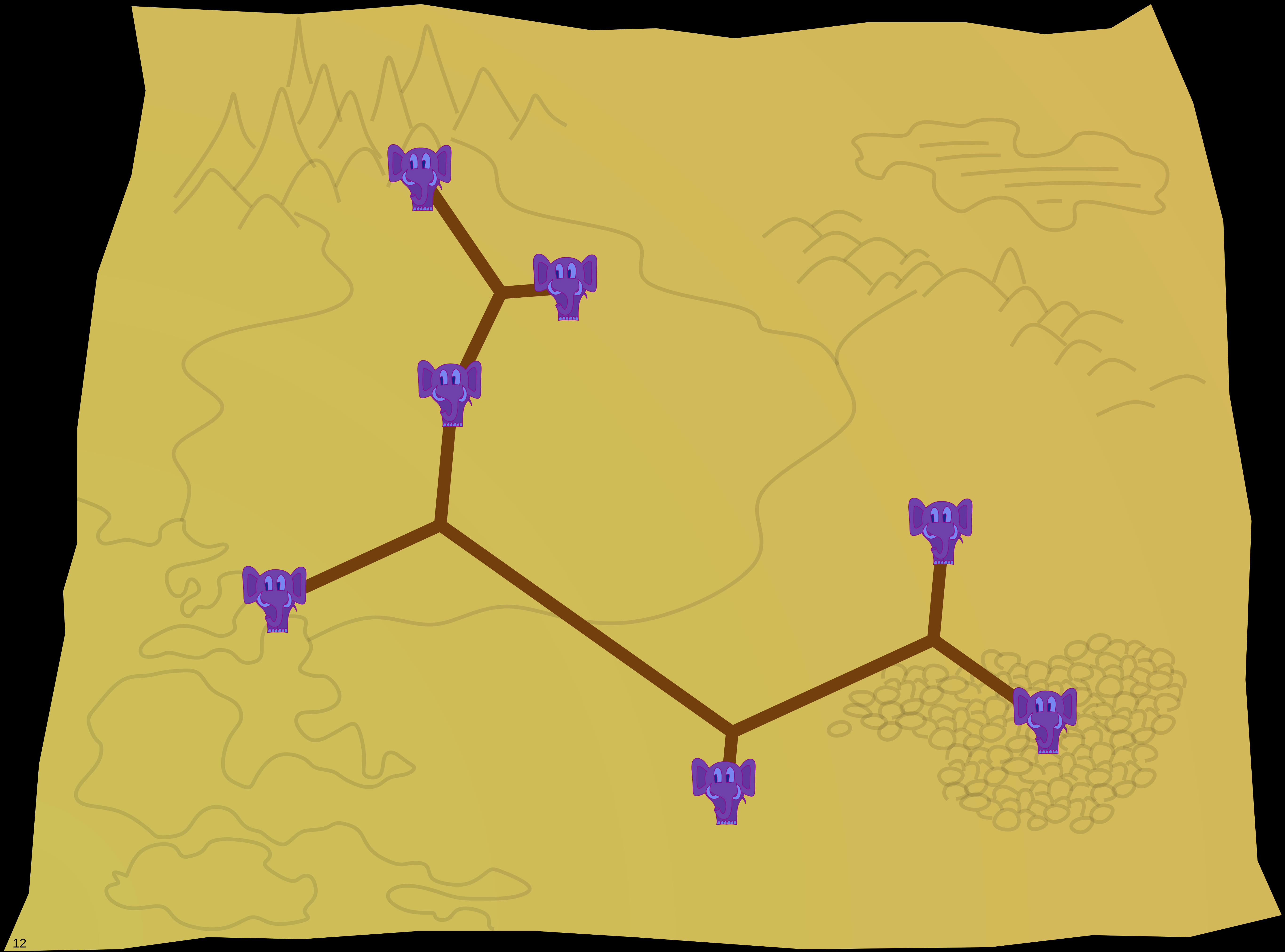


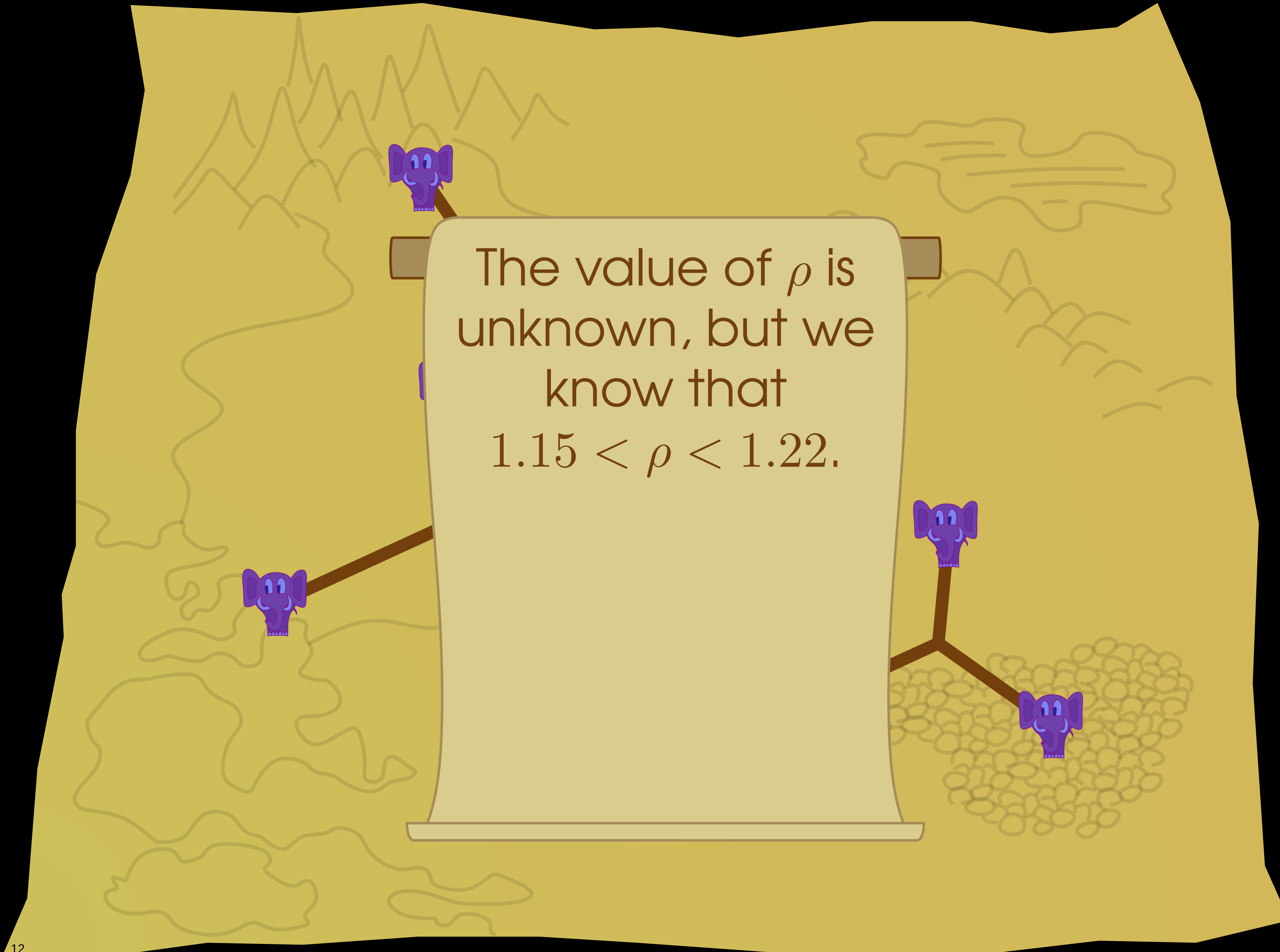

CLAIM
This algorithm gives a factor $1 + \frac{1}{2}\rho$ approximation, where ρ is the *Steiner ratio*.

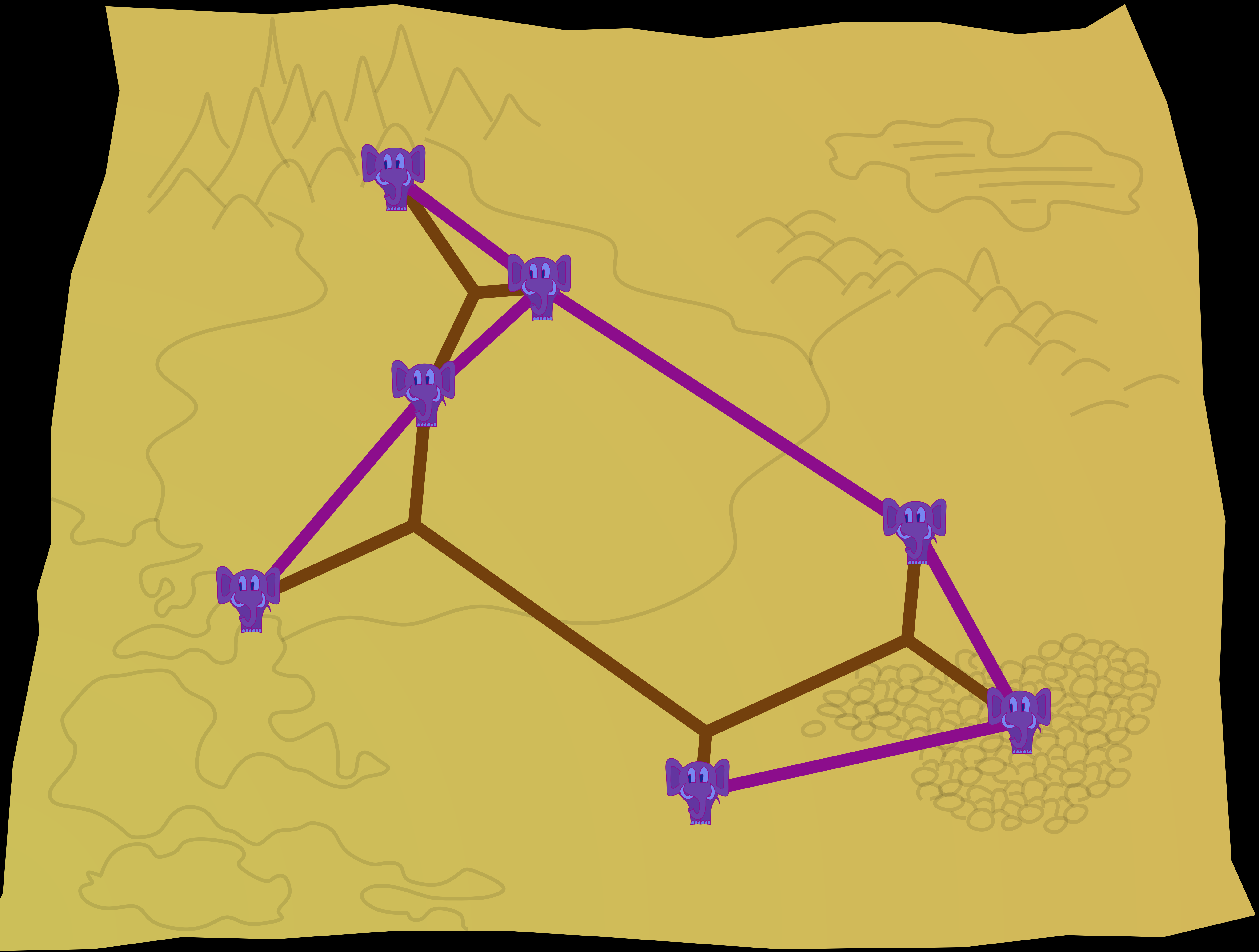
The *Steiner ratio* ρ is the maximum ratio between the weight of the MST and the minimum weight Steiner tree of a set of points.

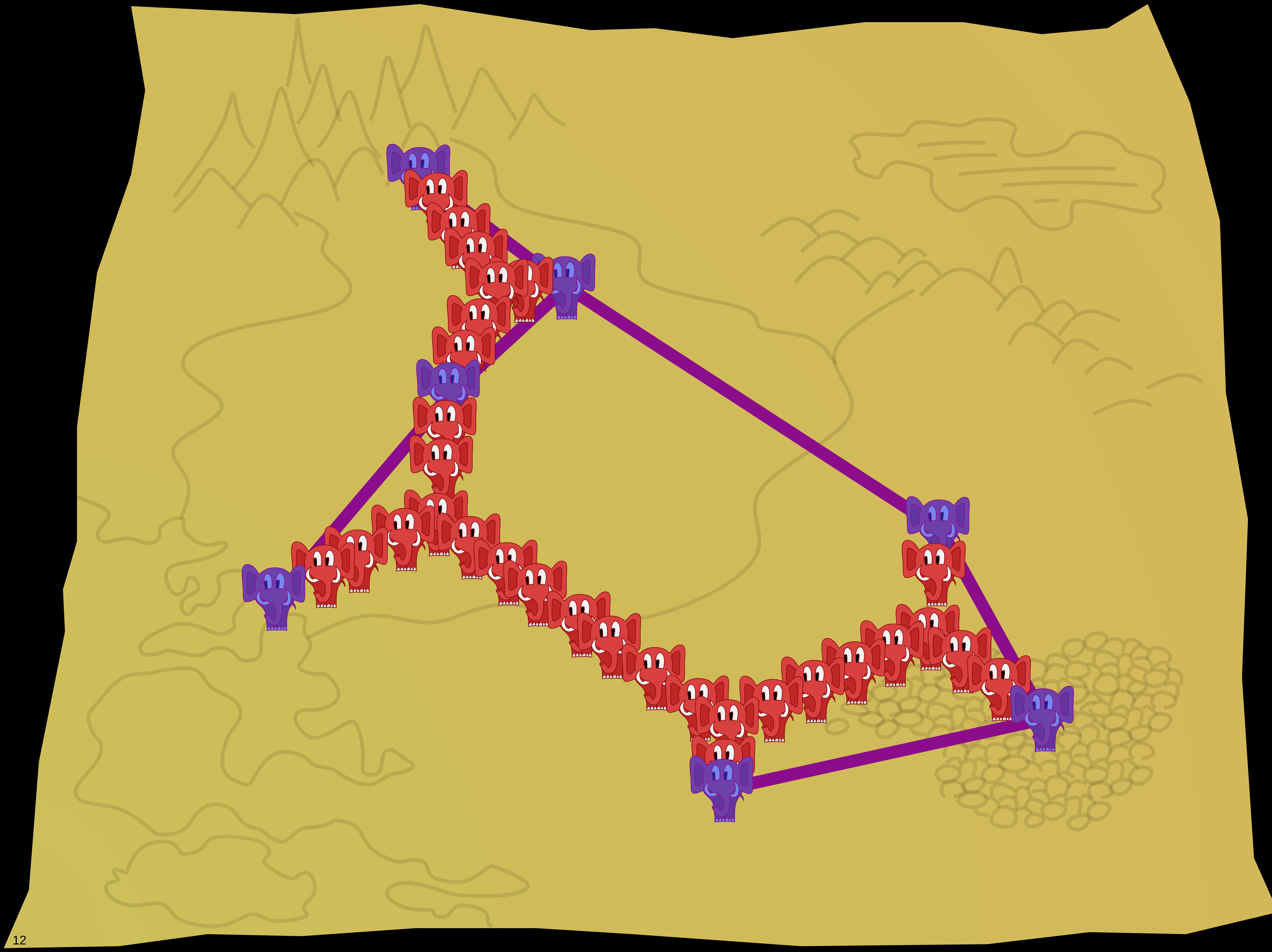


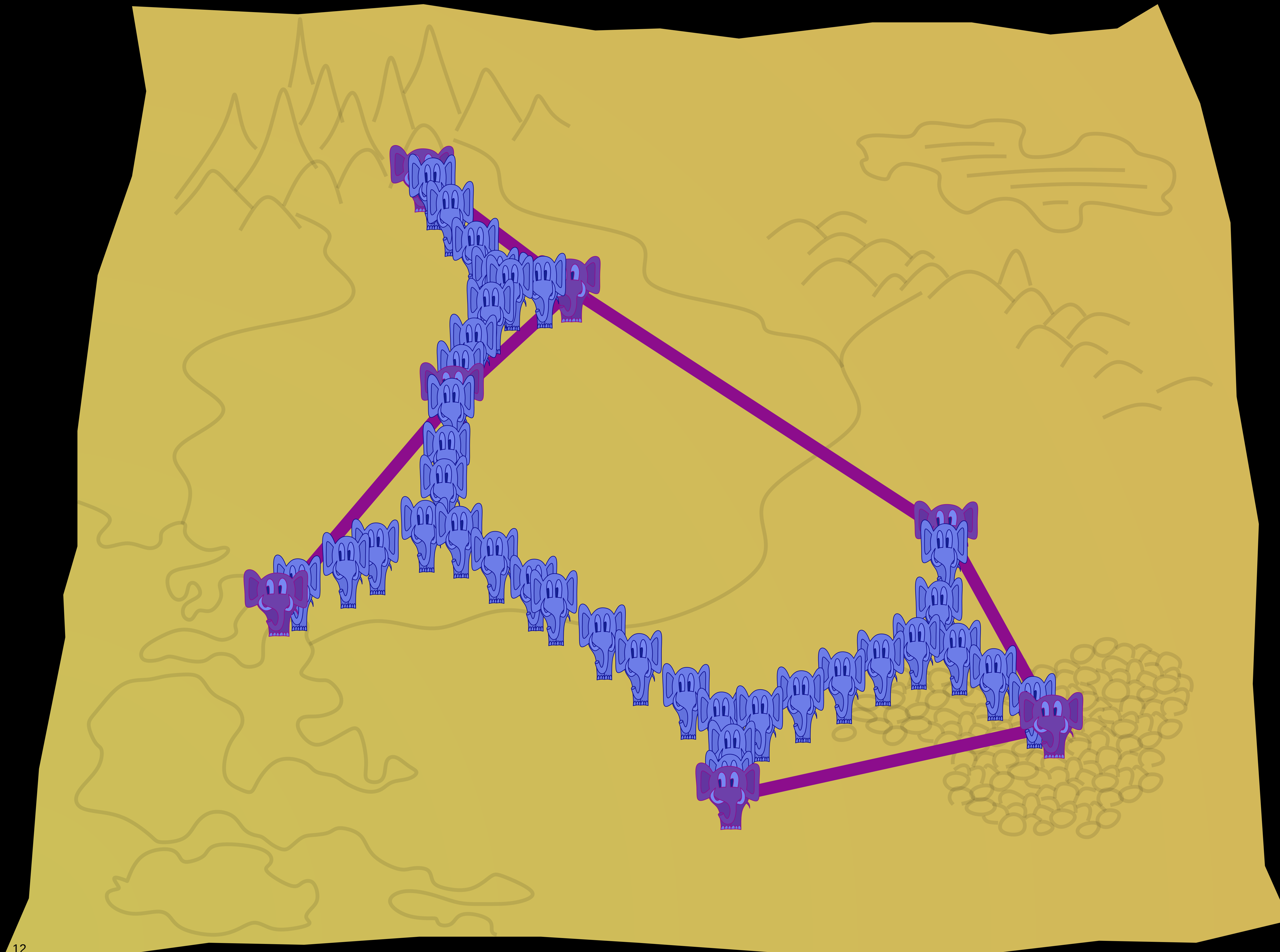


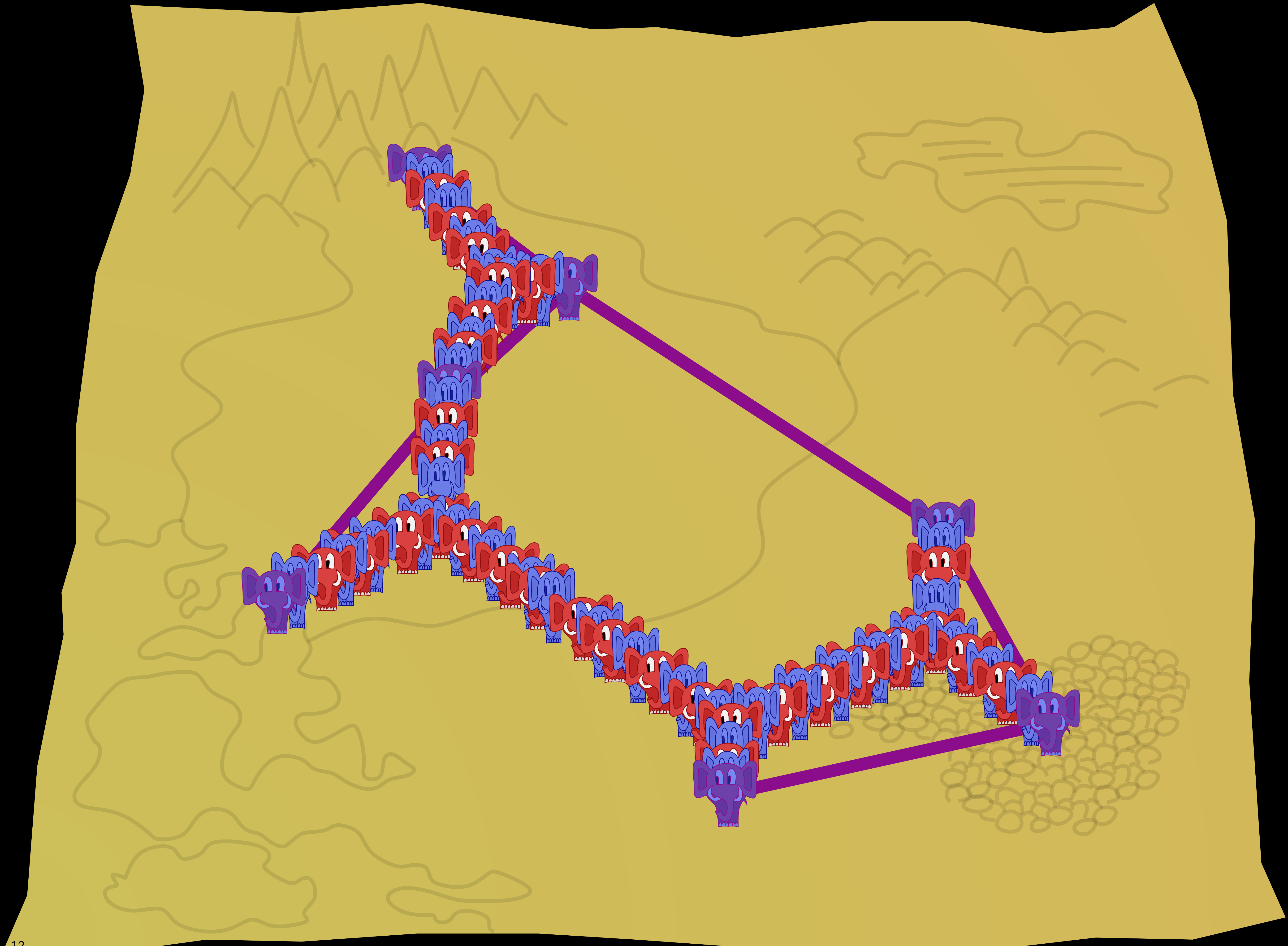



The value of ρ is unknown, but we know that
$1.15 < \rho < 1.22.$









The optimal length is $2 \cdot |SMT|$.
The computed length is $2 \cdot |SMT| + |MST|$.
So, we get

$$\frac{2 \cdot |SMT| + |MST|}{2 \cdot |SMT|} = 1 + \frac{1}{2}\rho.$$

The optimal length is $2 \cdot |SMT|$.

The computed length is $2 \cdot |SMT| + |MST|$.

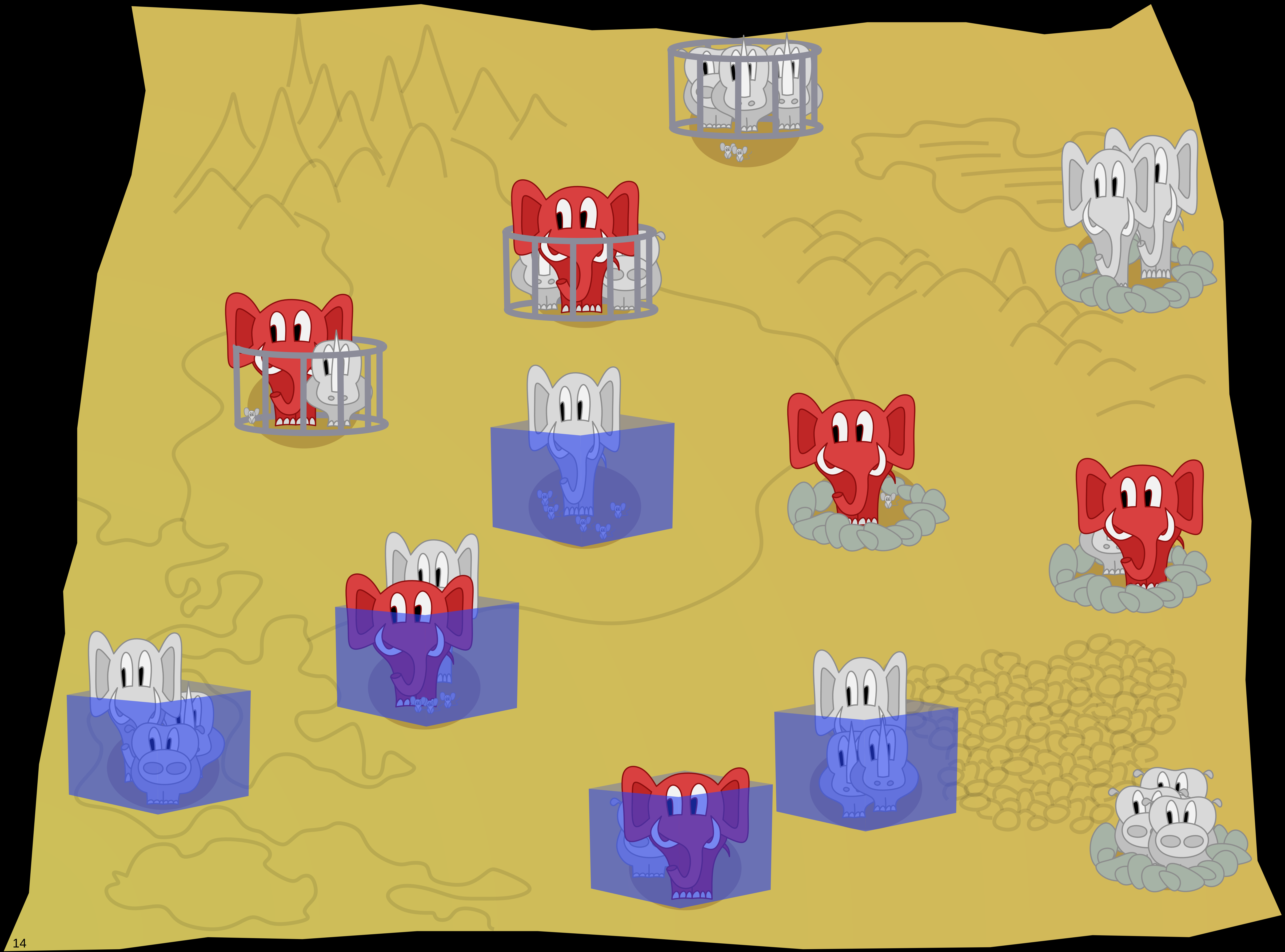
So, we get

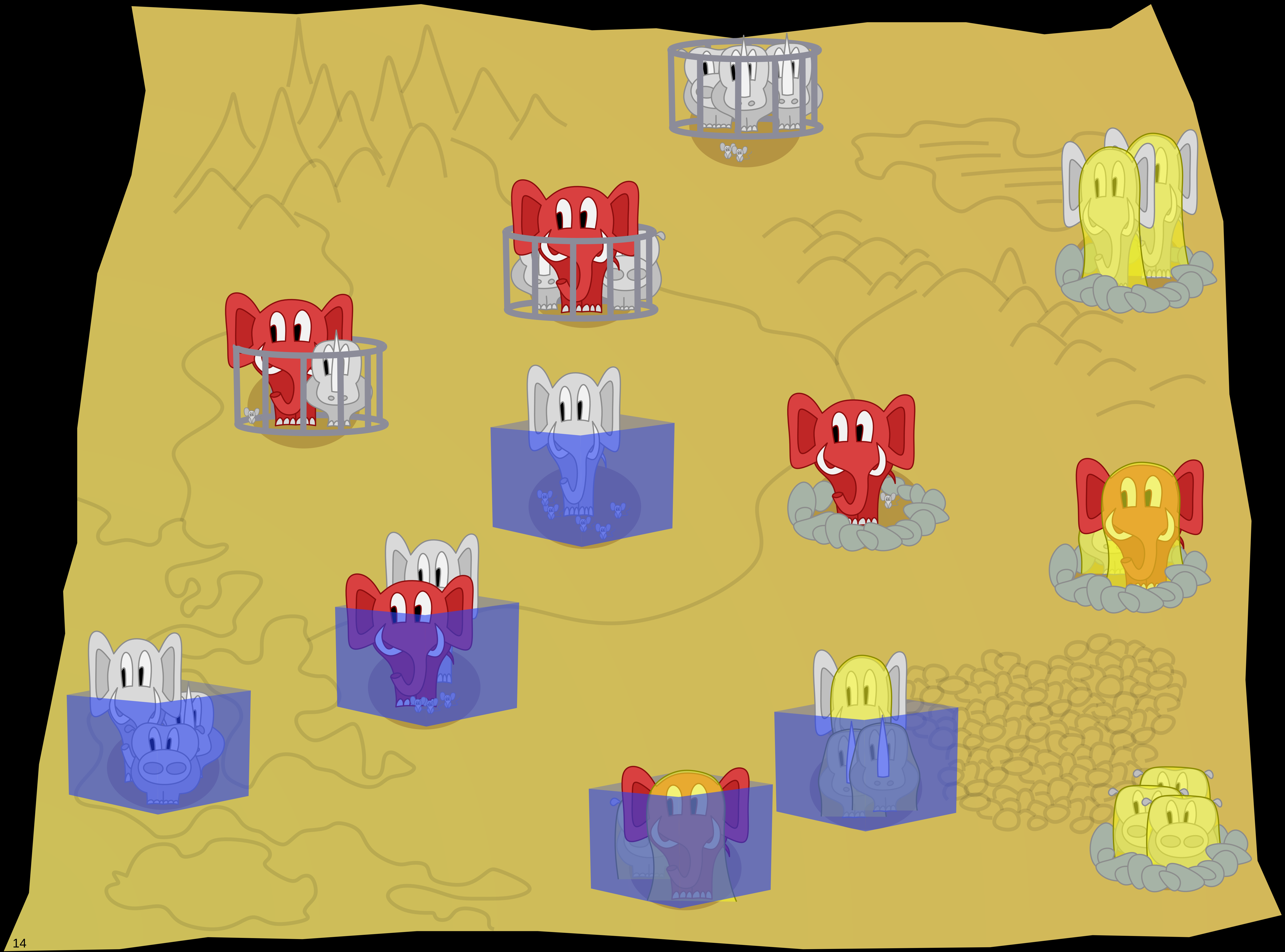
$$\frac{2 \cdot |SMT| + |MST|}{2 \cdot |SMT|} = 1 + \frac{1}{2}\rho.$$

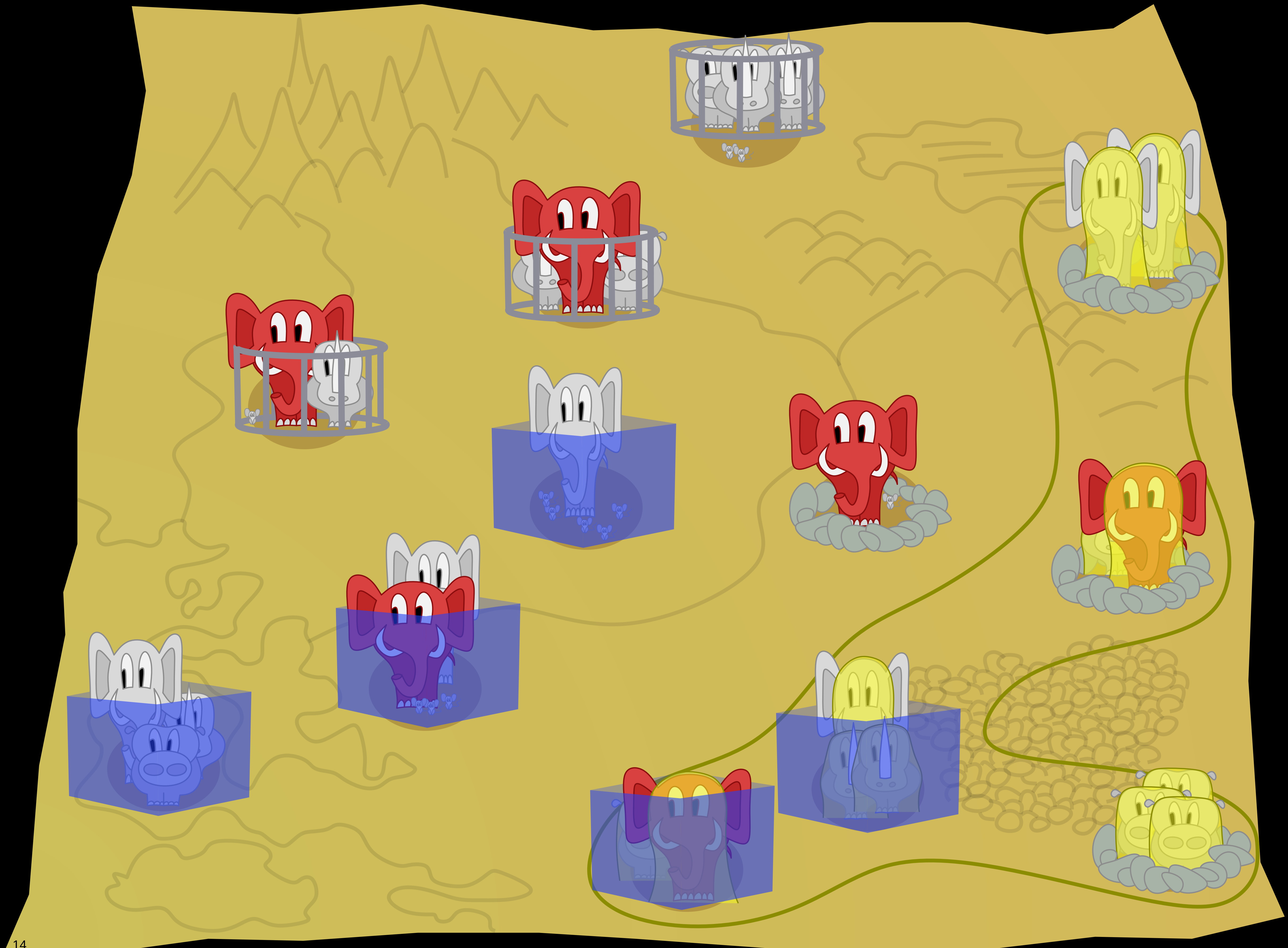
$$1.57 < 1 + \frac{1}{2}\rho < 1.61$$

EXTENSIONS

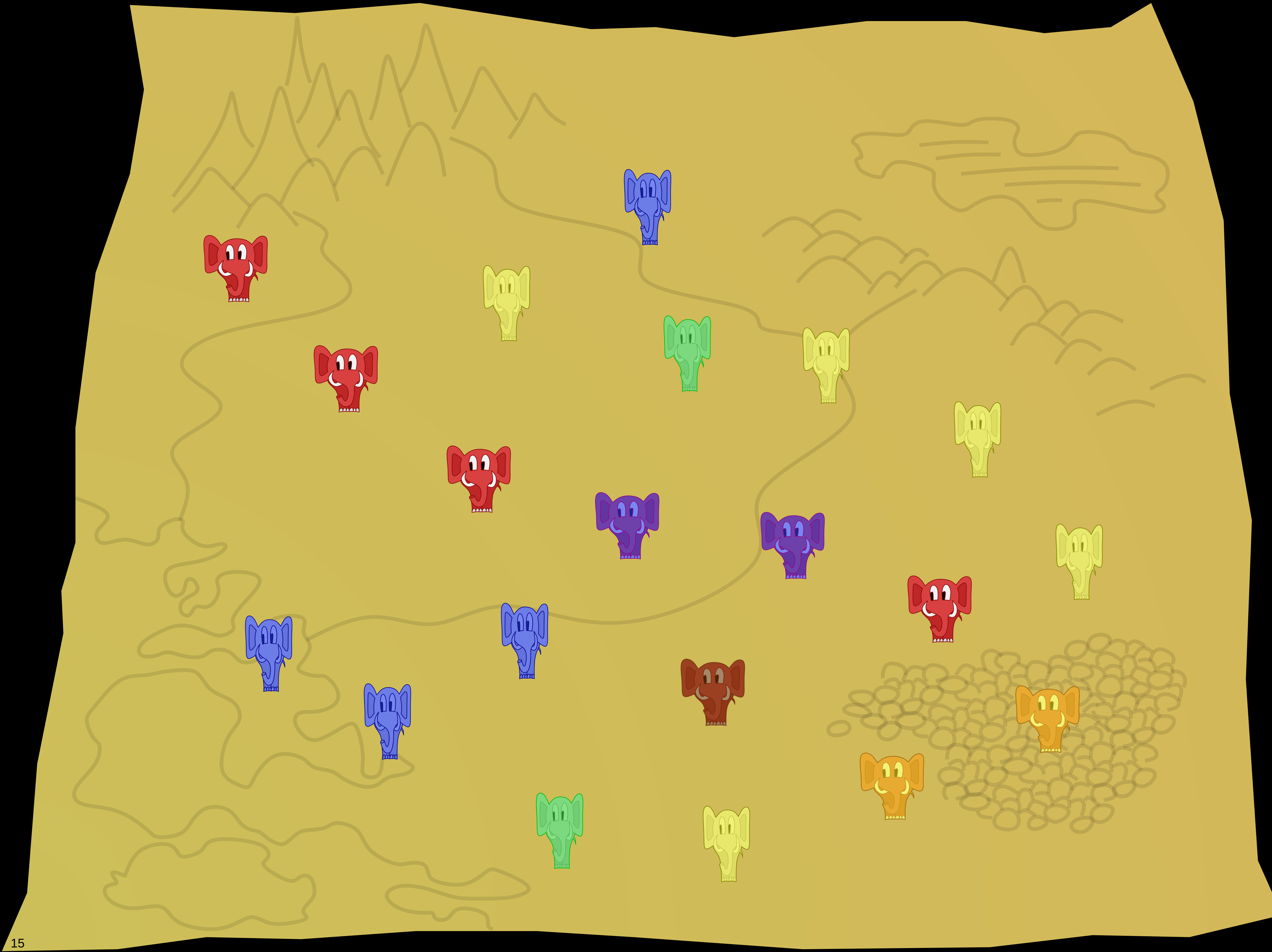
We may want to visualise more than two attributes at the same time.



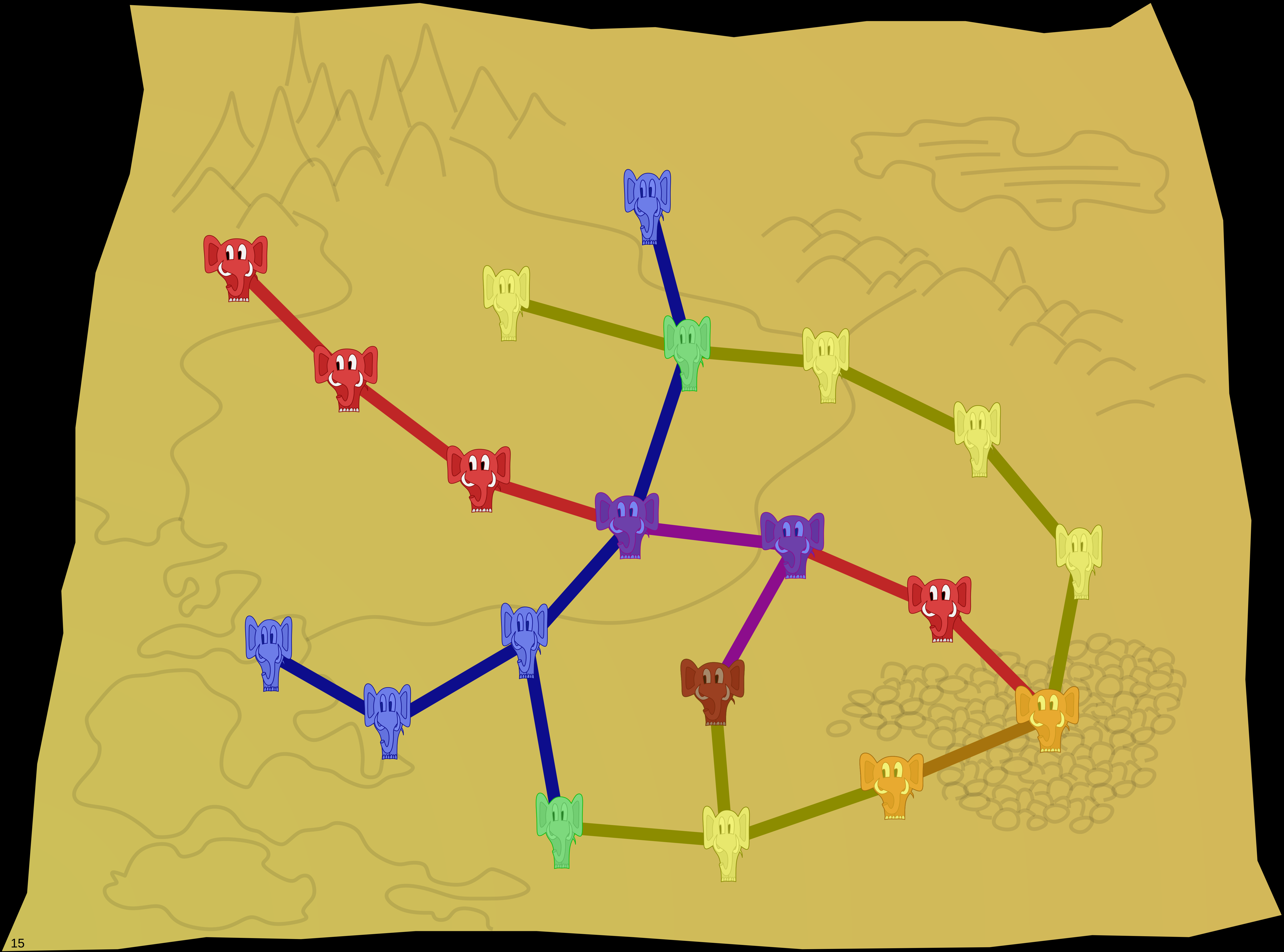


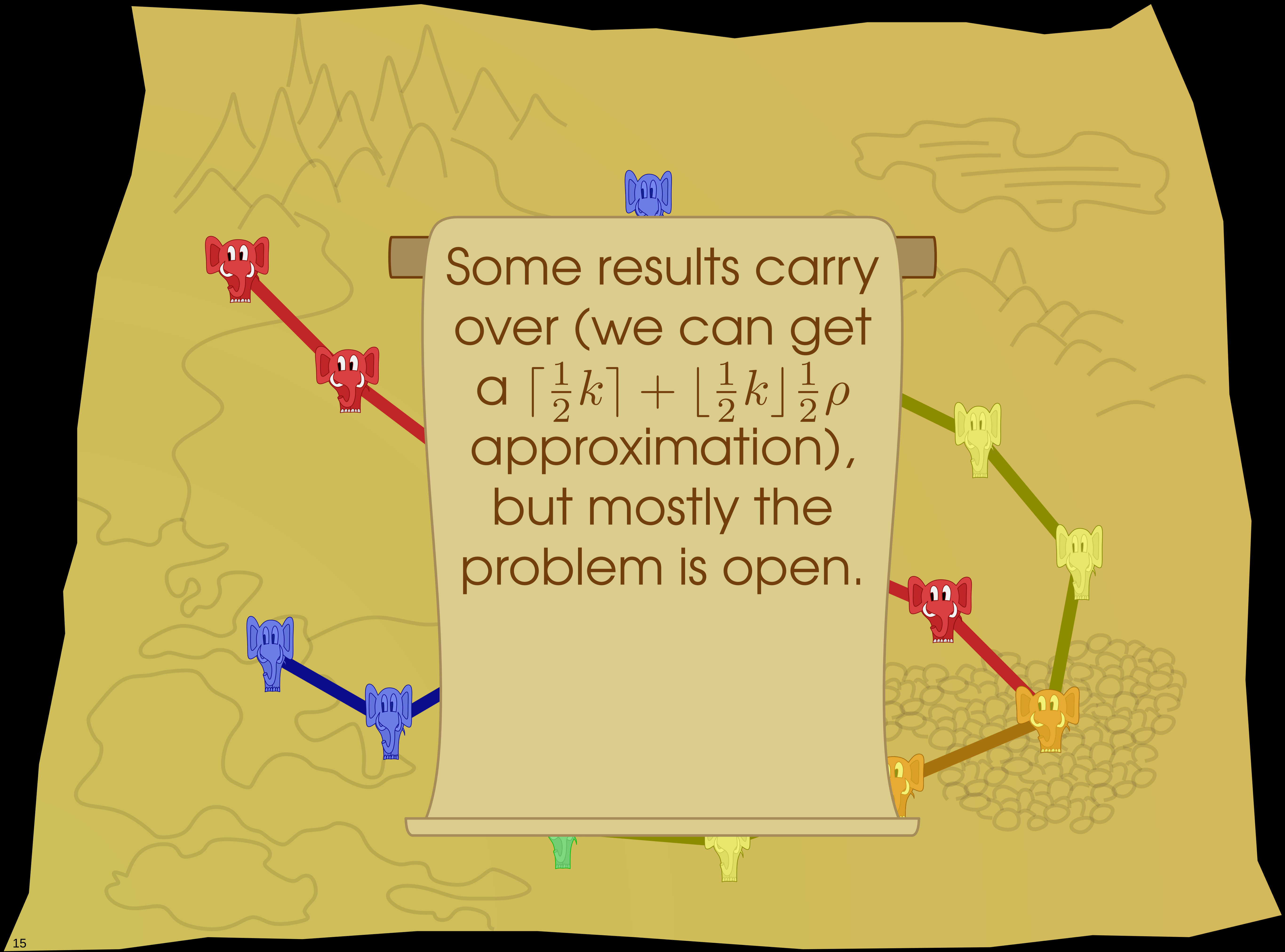


Now we have a set of points with three colours, say *red*, *blue* and *yellow*, and corresponding secondary and tertiary colours.




As before, we want to find the minimum weight graph that connects each primary colour.




Some results carry over (we can get a $\lceil \frac{1}{2}k \rceil + \lfloor \frac{1}{2}k \rfloor \frac{1}{2}\rho$ approximation), but mostly the problem is open.

THANKS!