

# COLORED SPANNING GRAPHS

*investigated by*

Ferran Hurtado



Matias Korman



Marc van Kreveld



Maarten Löffler



Vera Sacristán



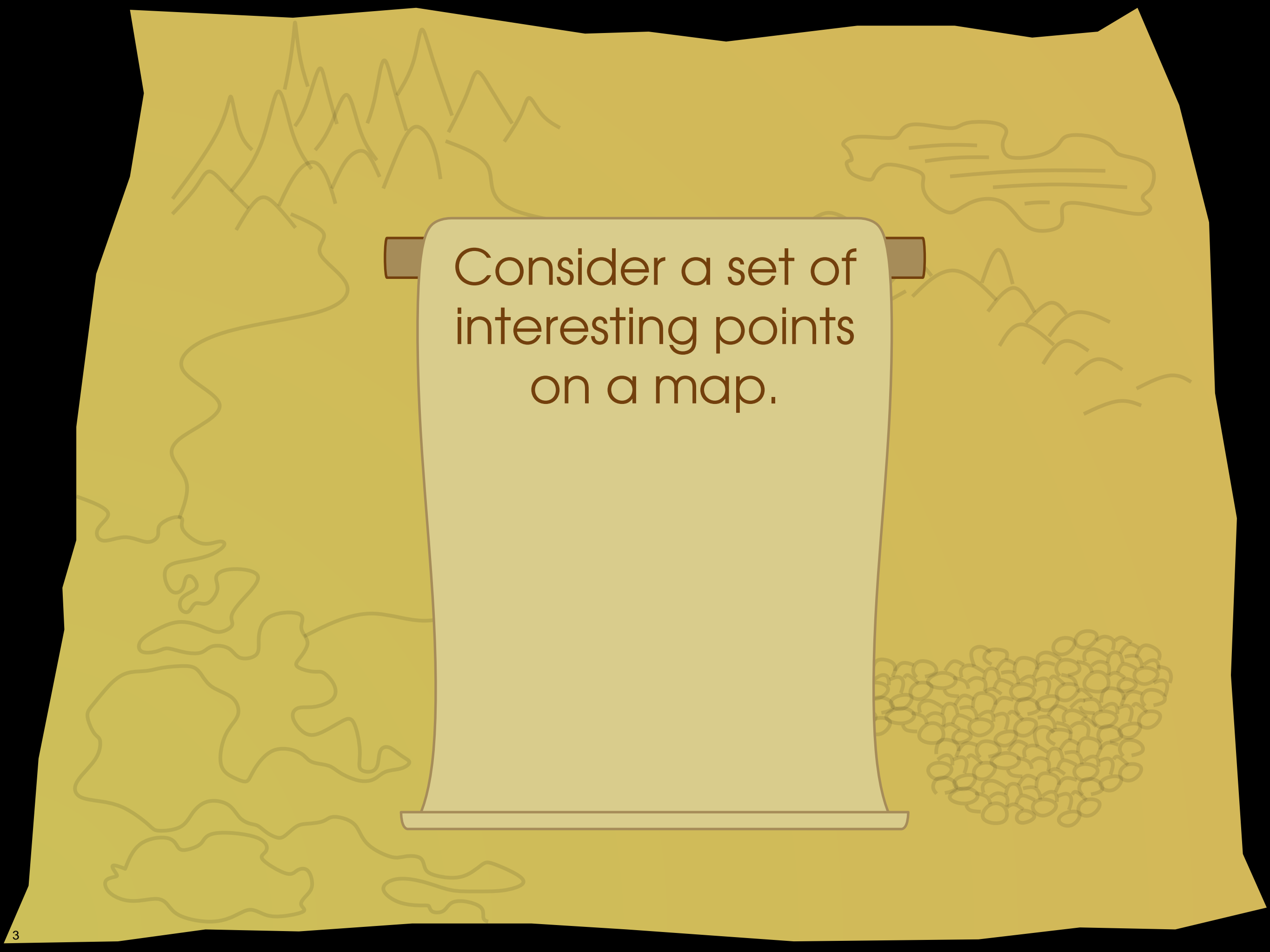
Rodrigo Silveira



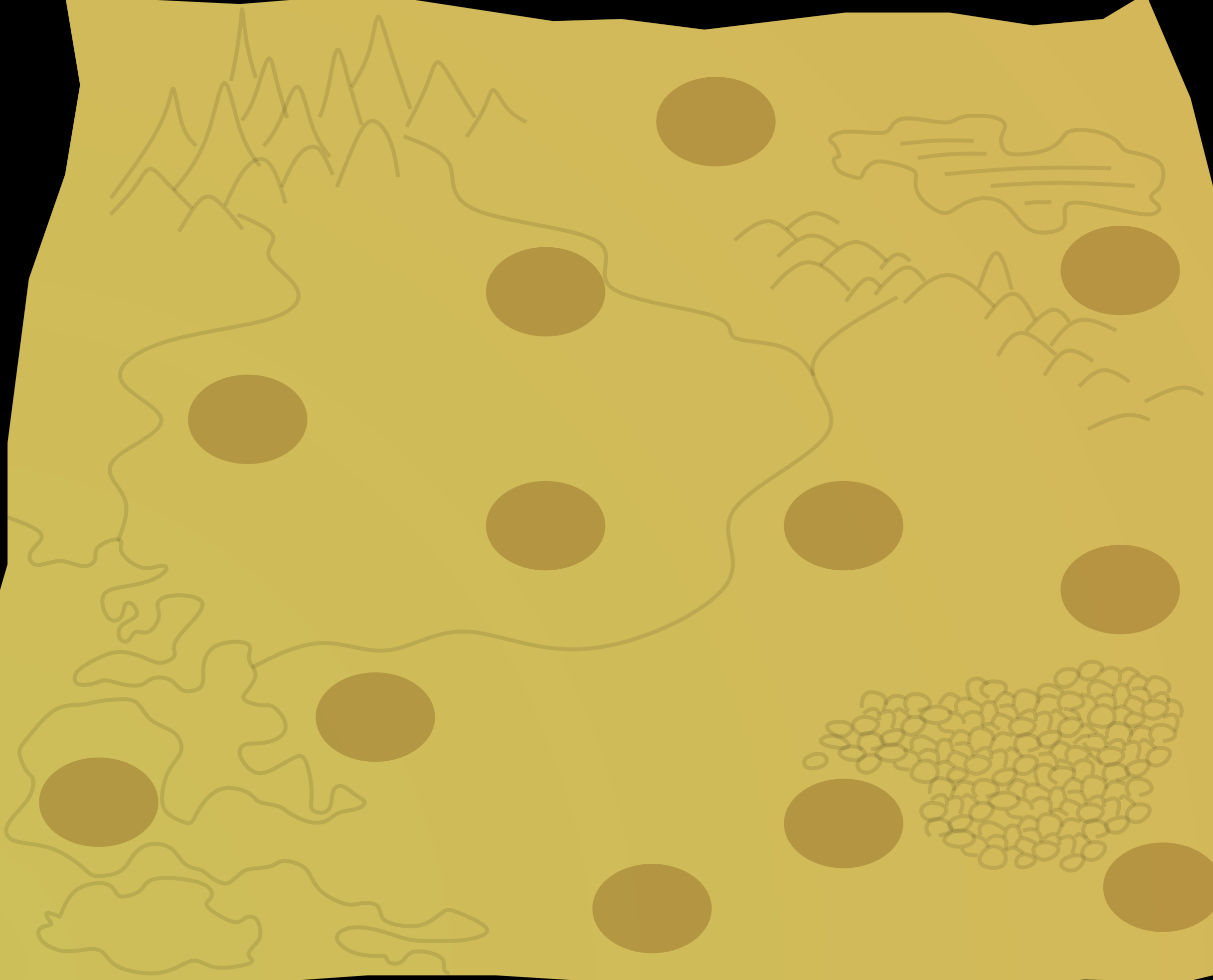
Bettina Speckmann

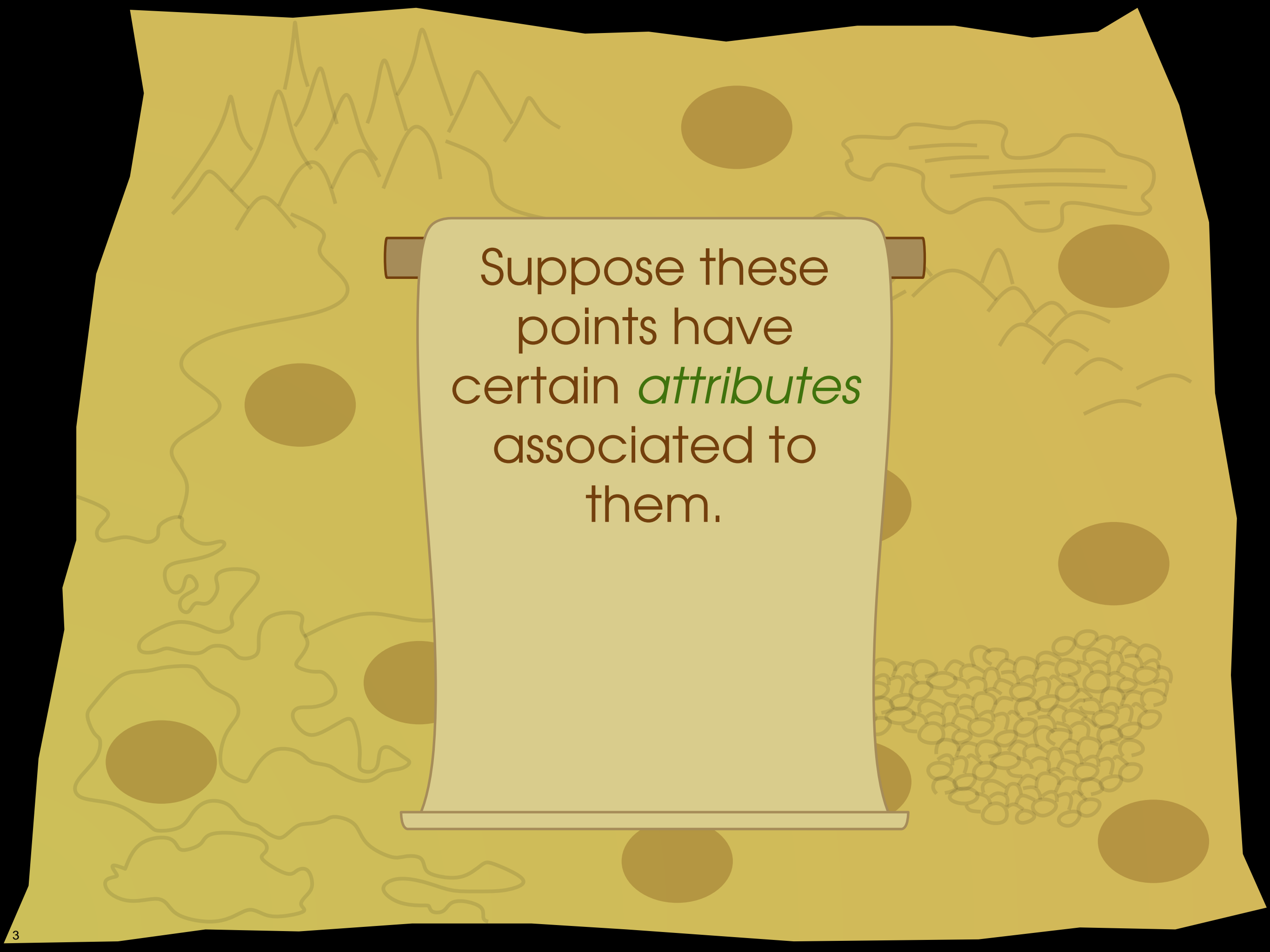


# INTRODUCTION



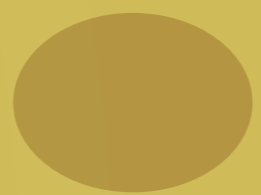
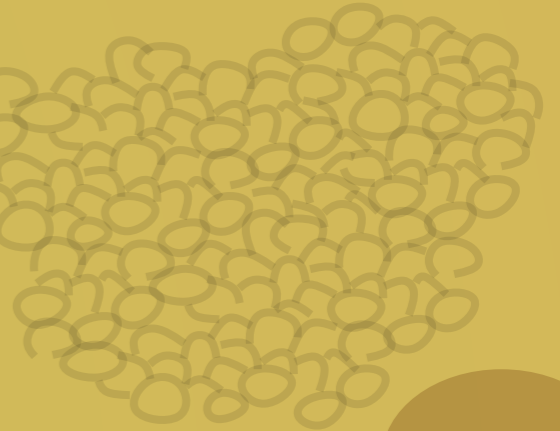
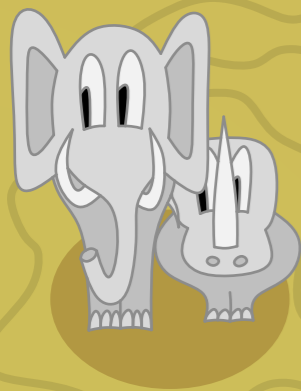
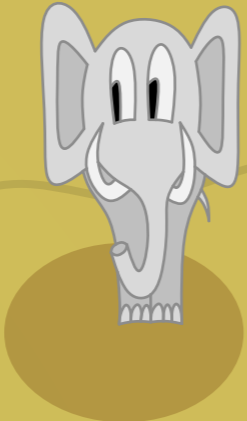
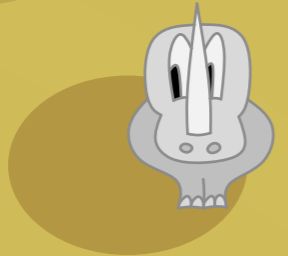
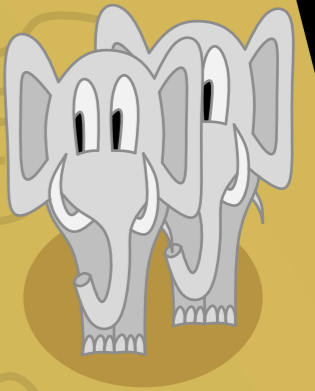
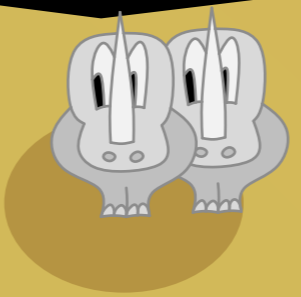
Consider a set of  
interesting points  
on a map.





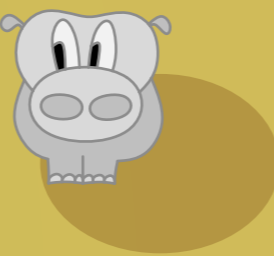
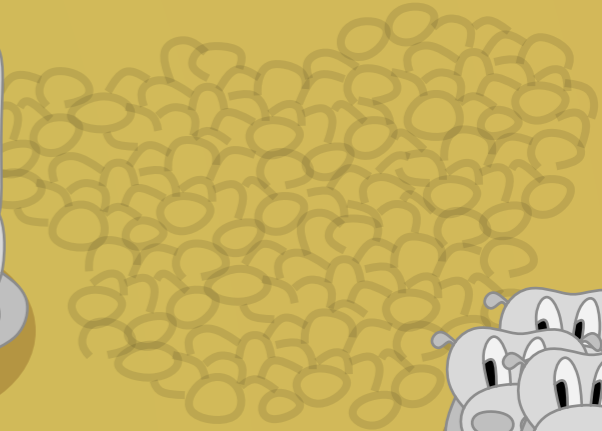
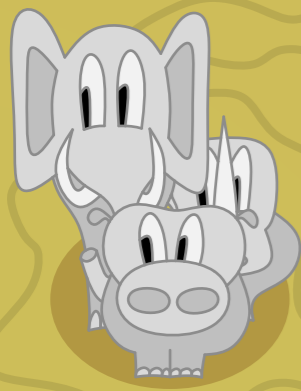
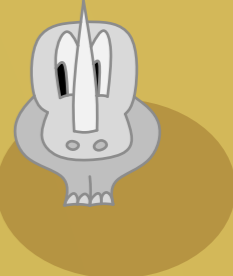
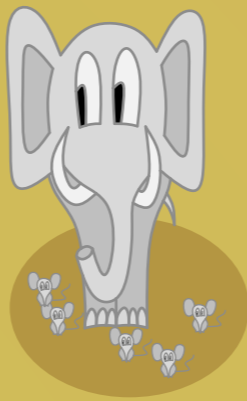
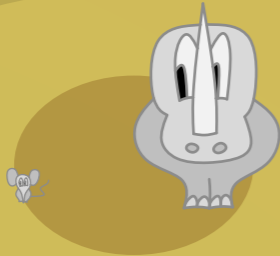
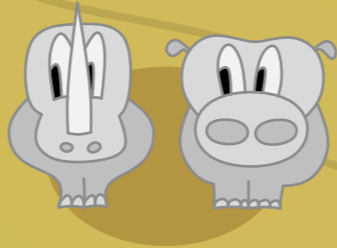
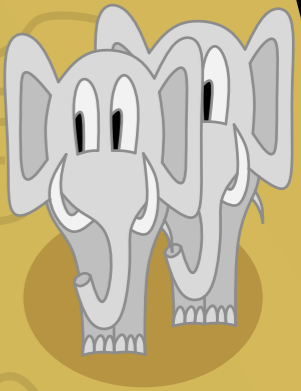
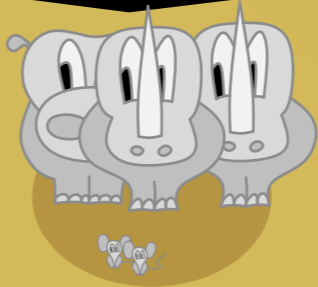
Suppose these  
points have  
certain *attributes*  
associated to  
them.

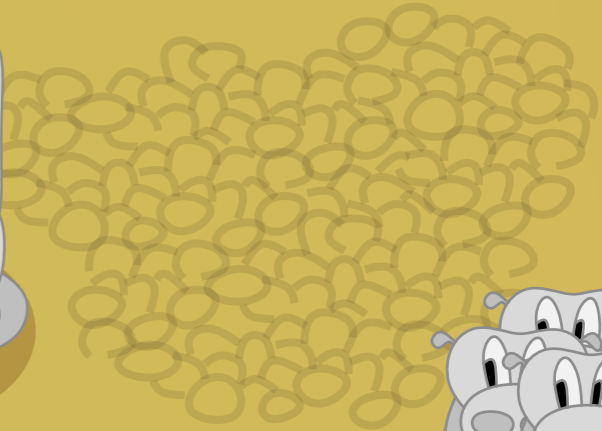
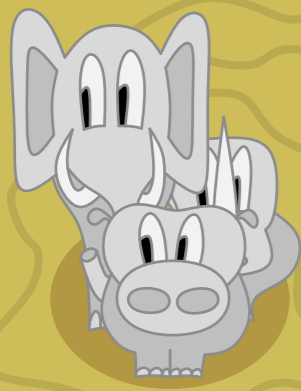
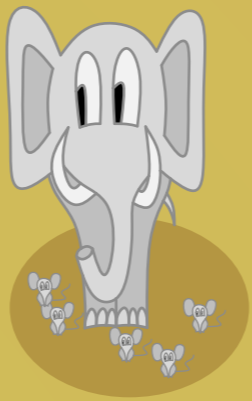
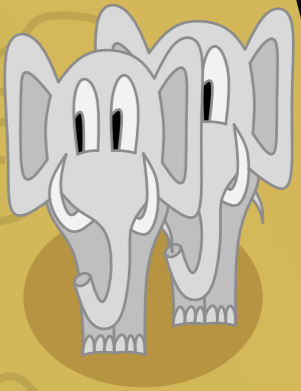
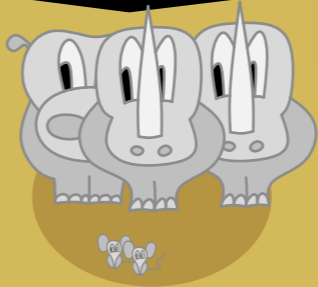


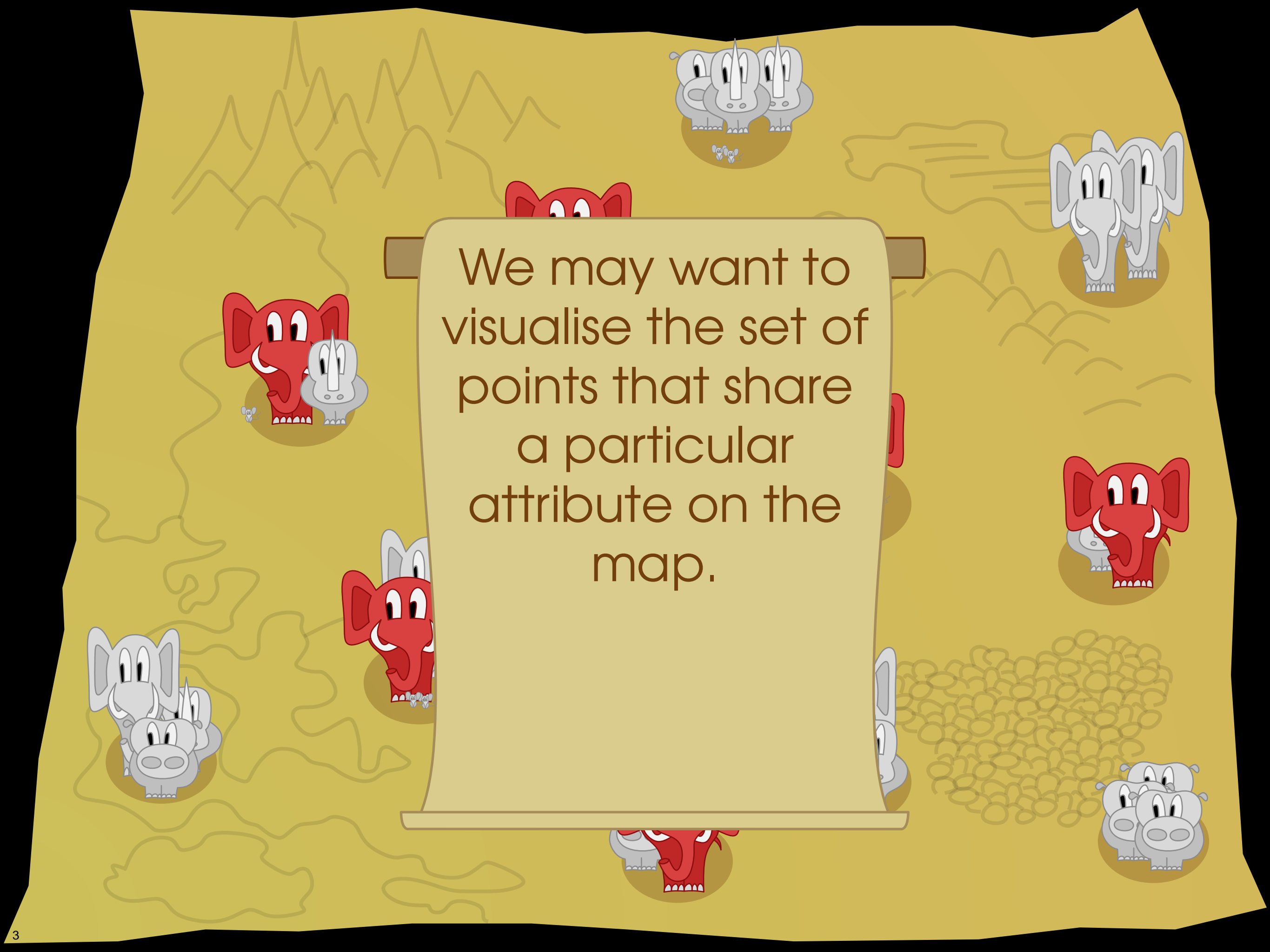




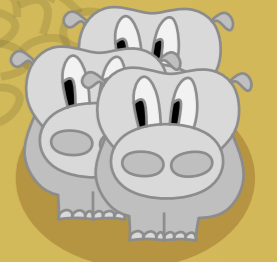
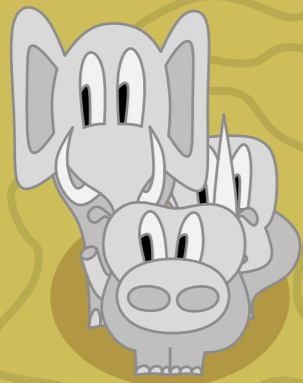
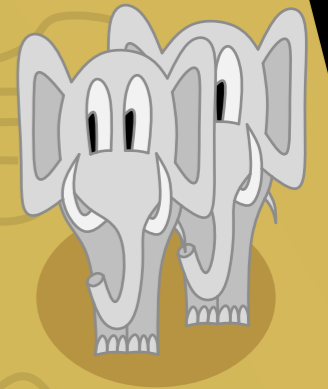
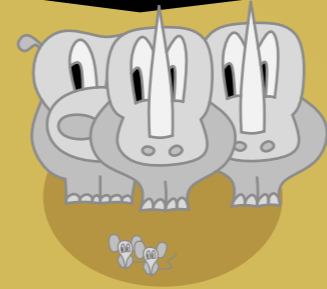


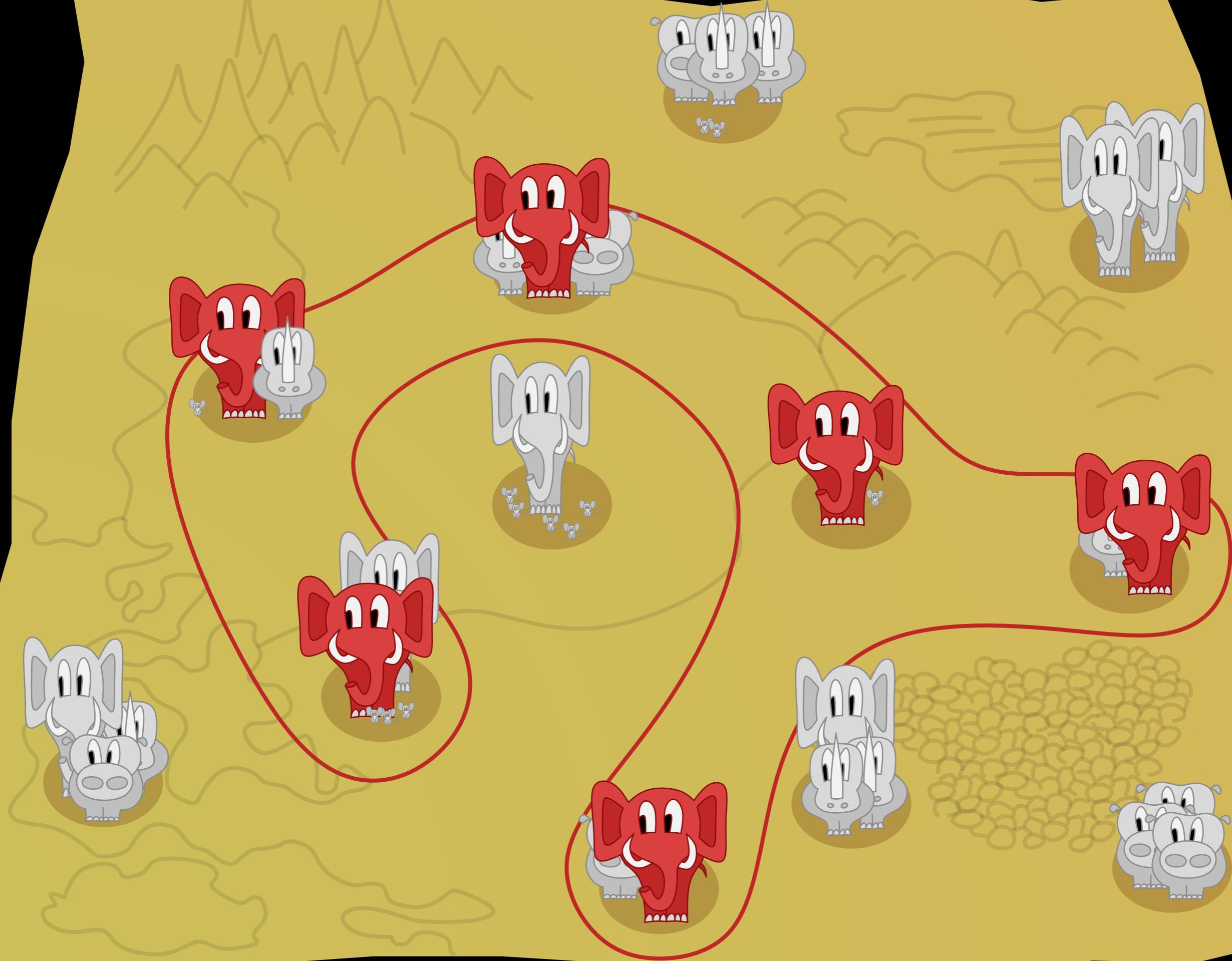




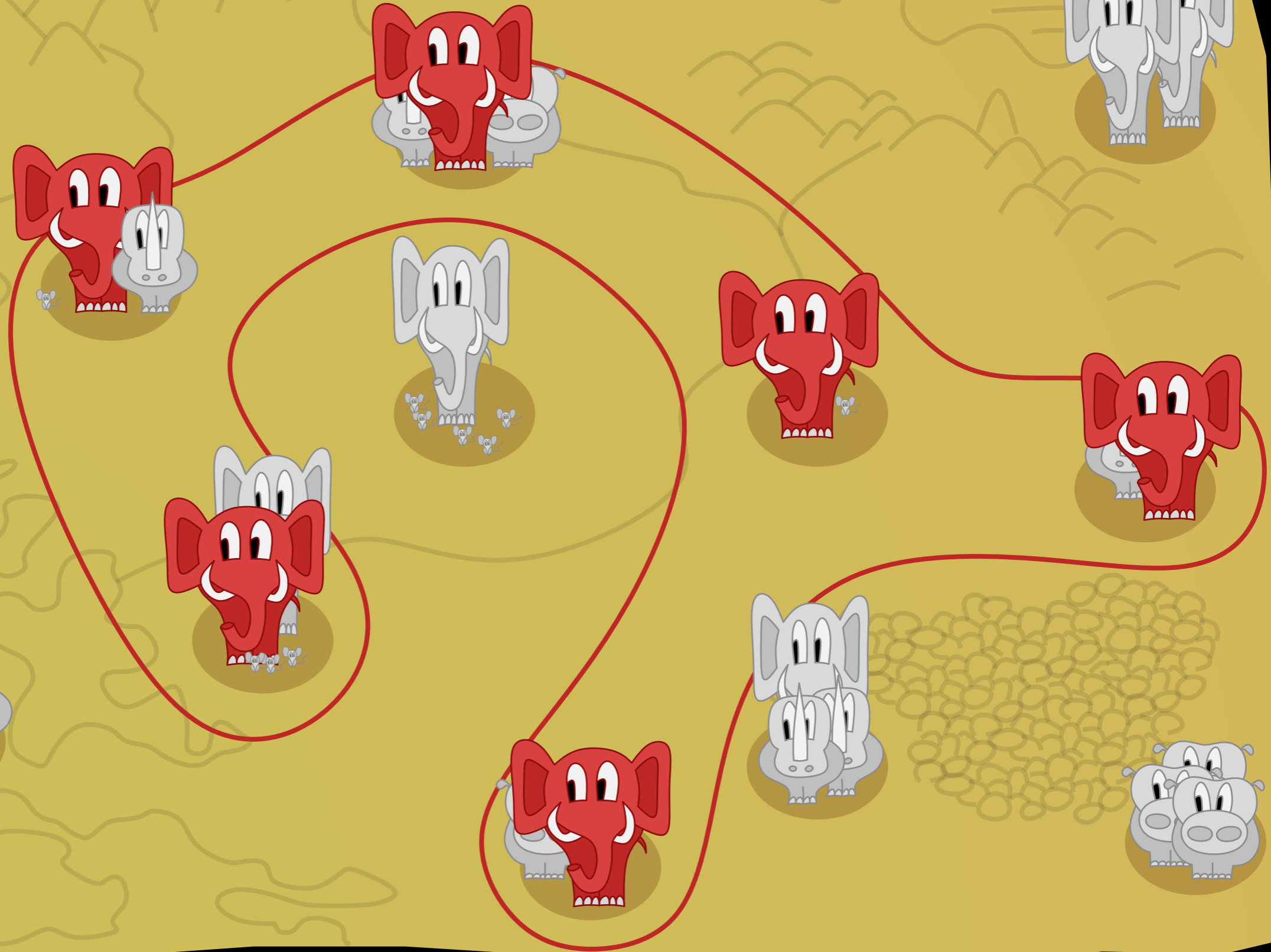
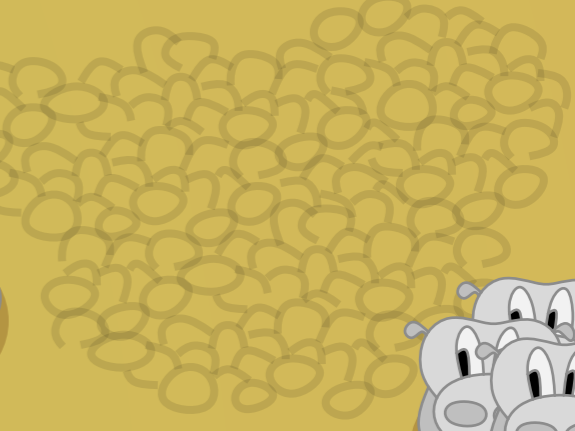
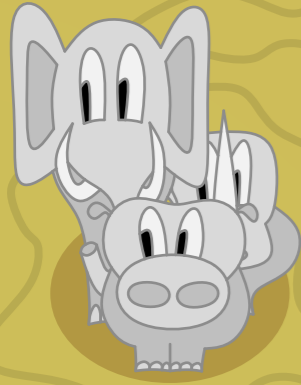
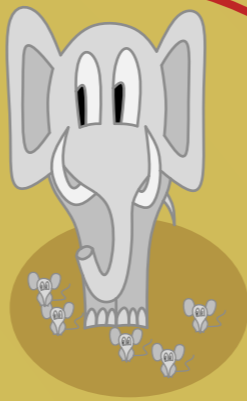
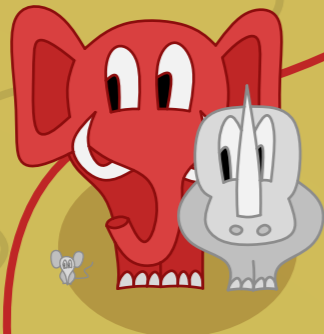
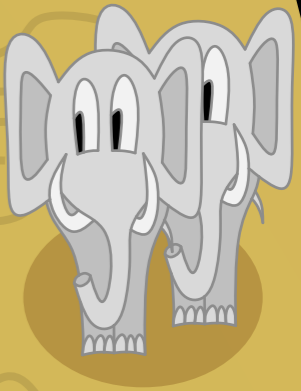
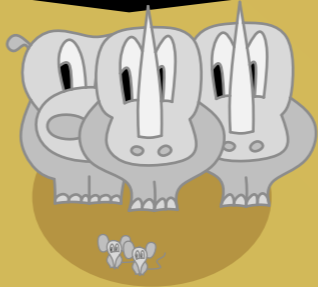


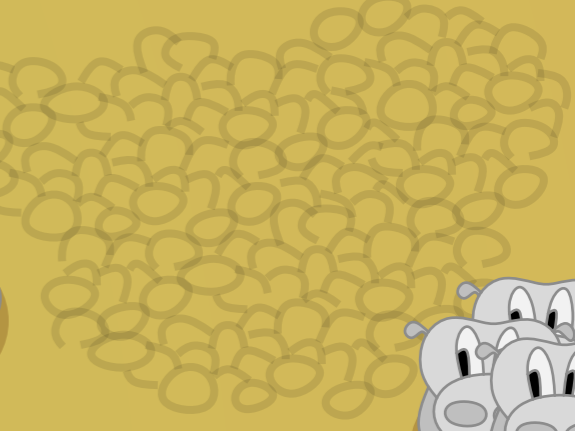
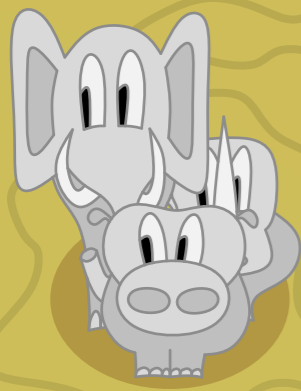
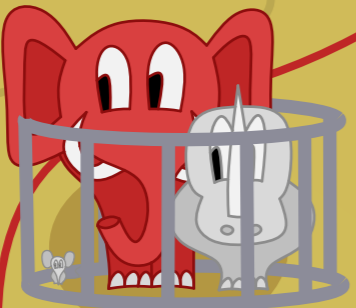
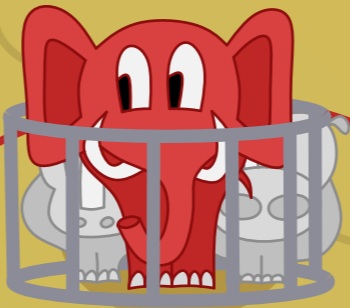
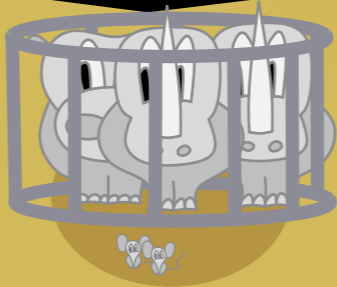
We may want to visualise the set of points that share a particular attribute on the map.

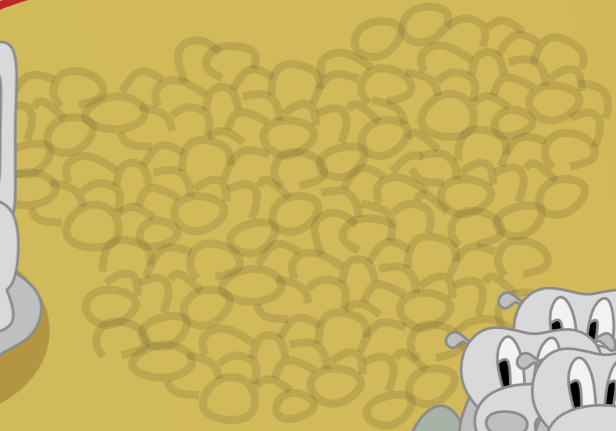
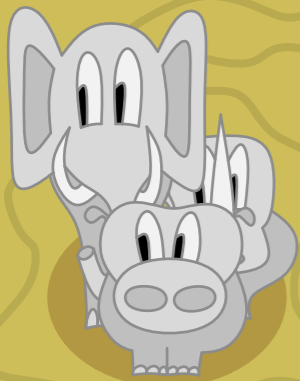
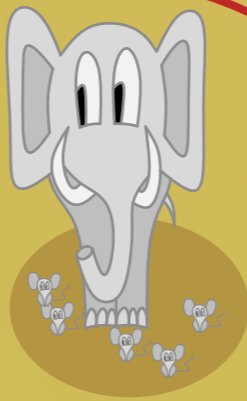
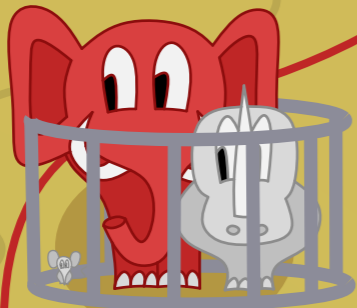
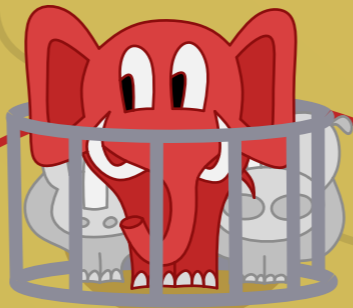
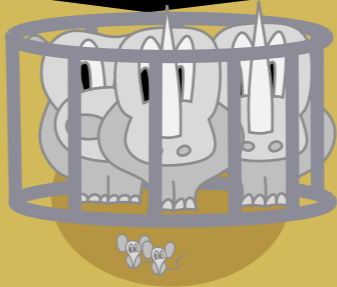




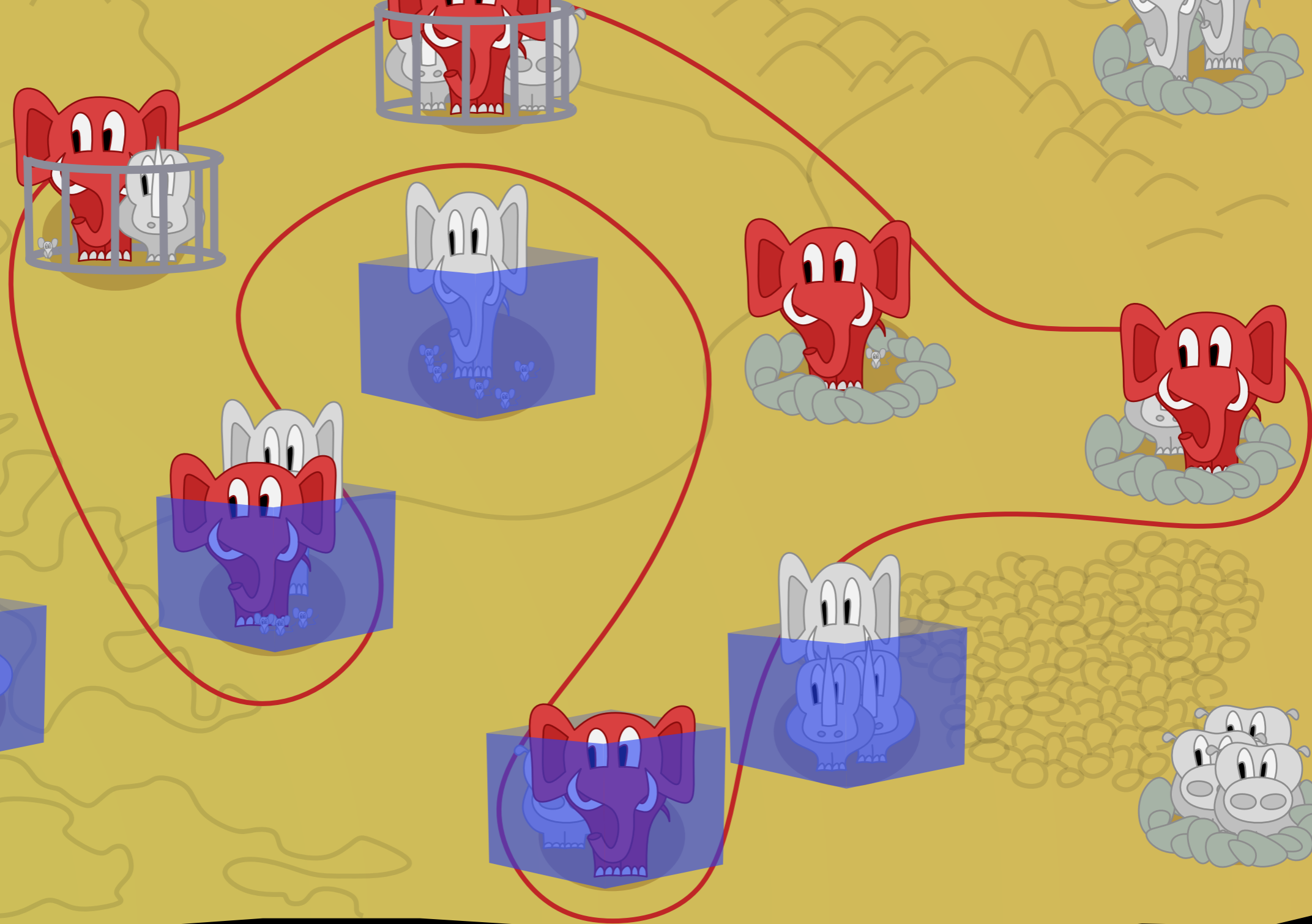
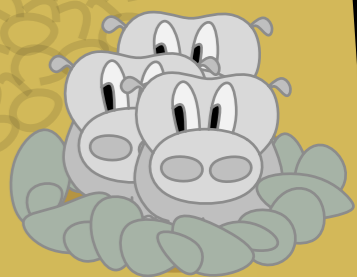
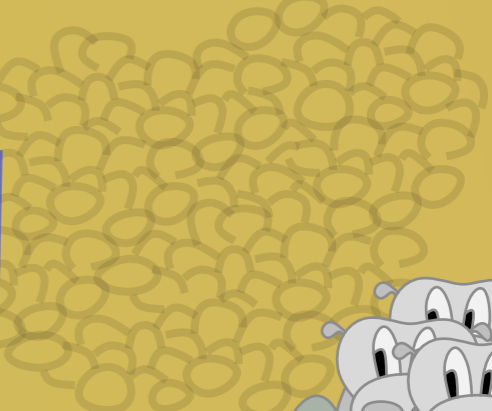
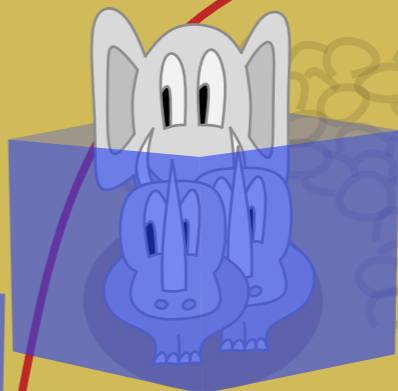
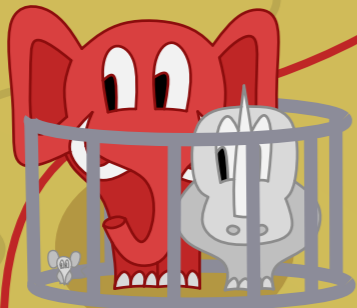
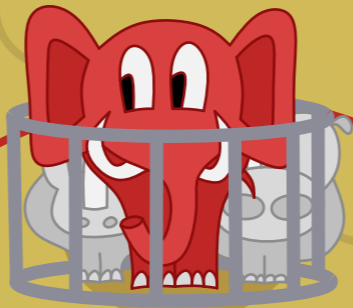
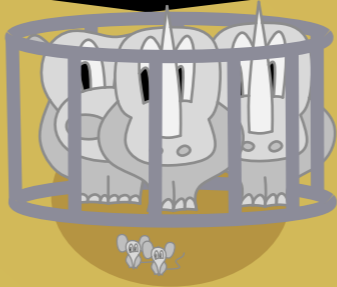




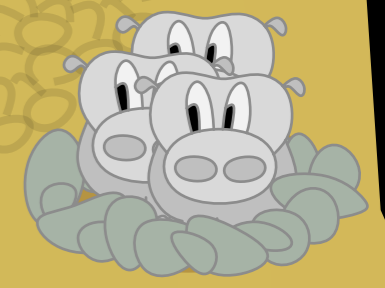
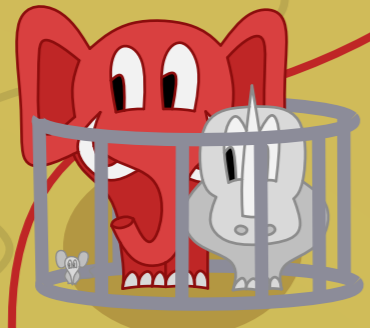
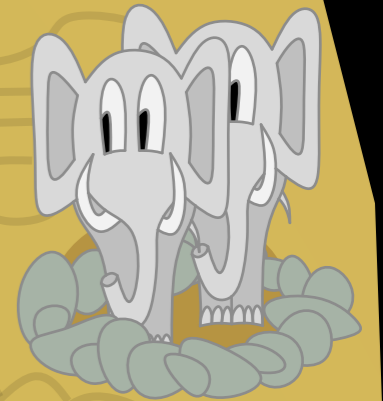
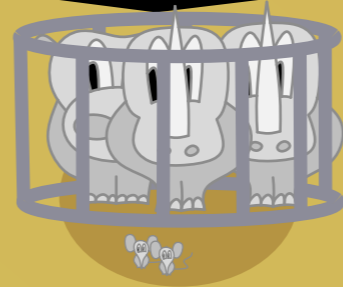


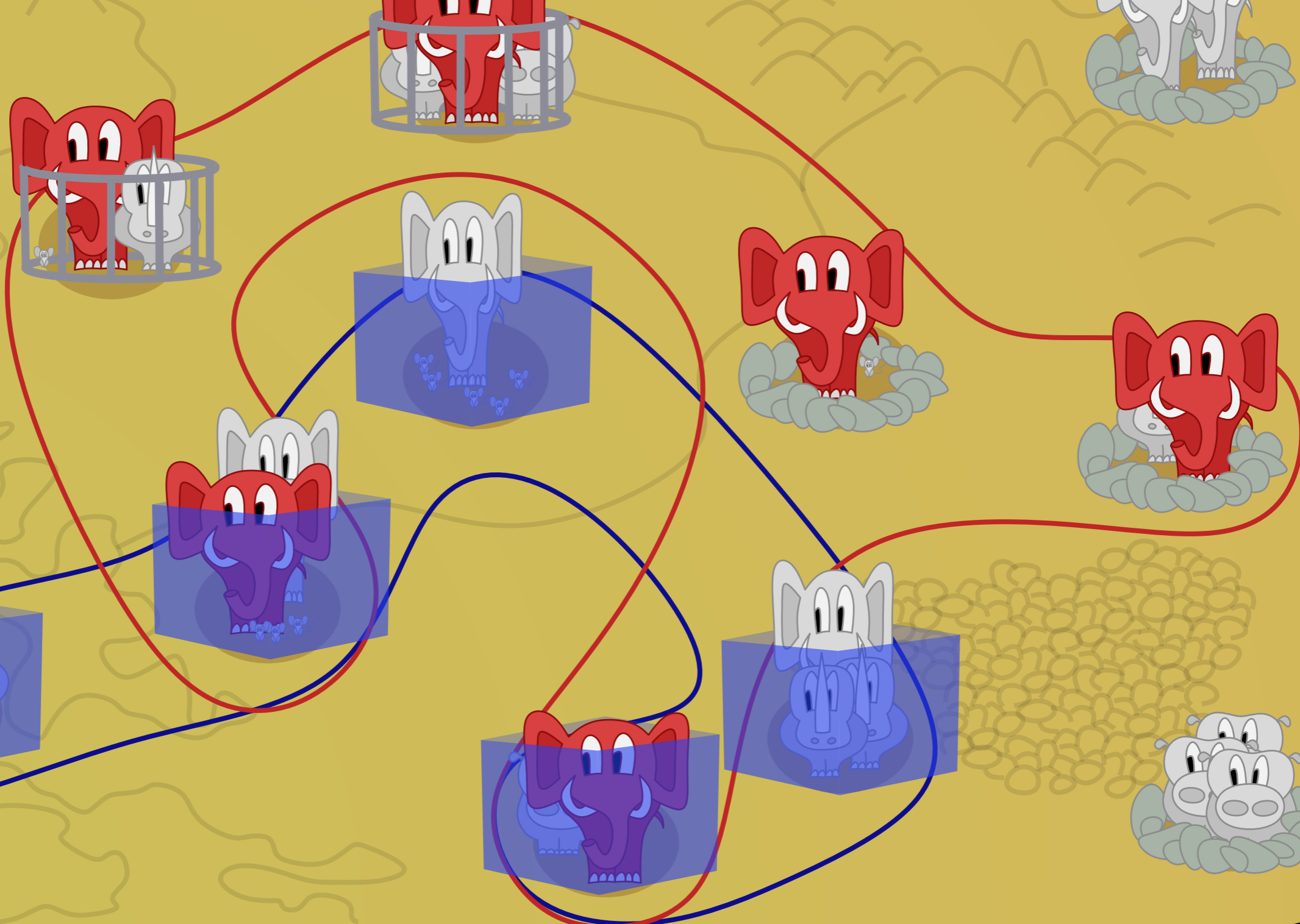
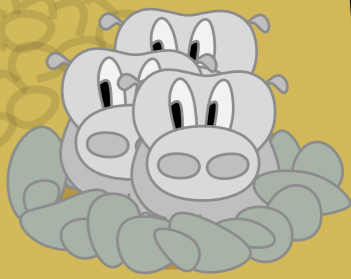
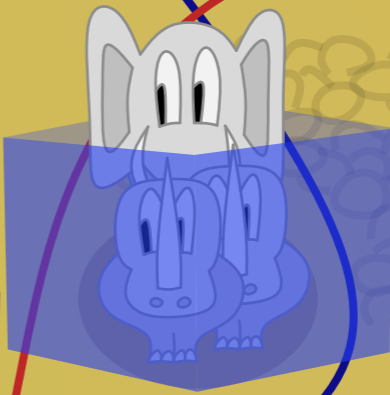
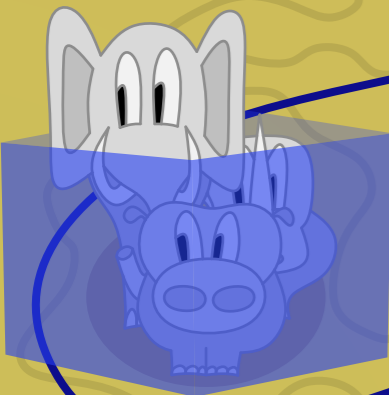
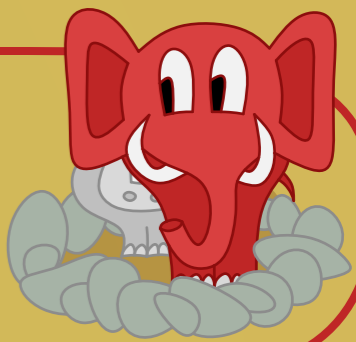
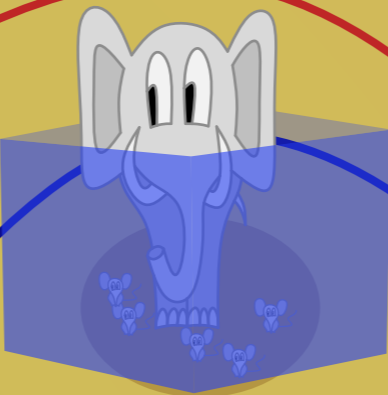
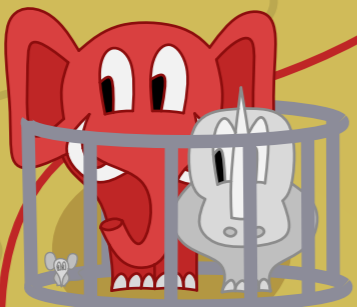
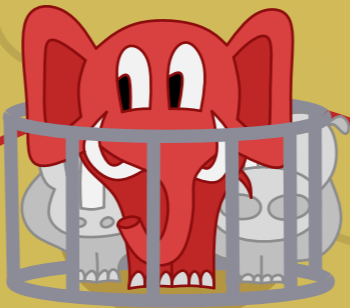
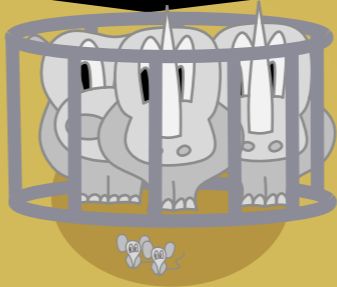


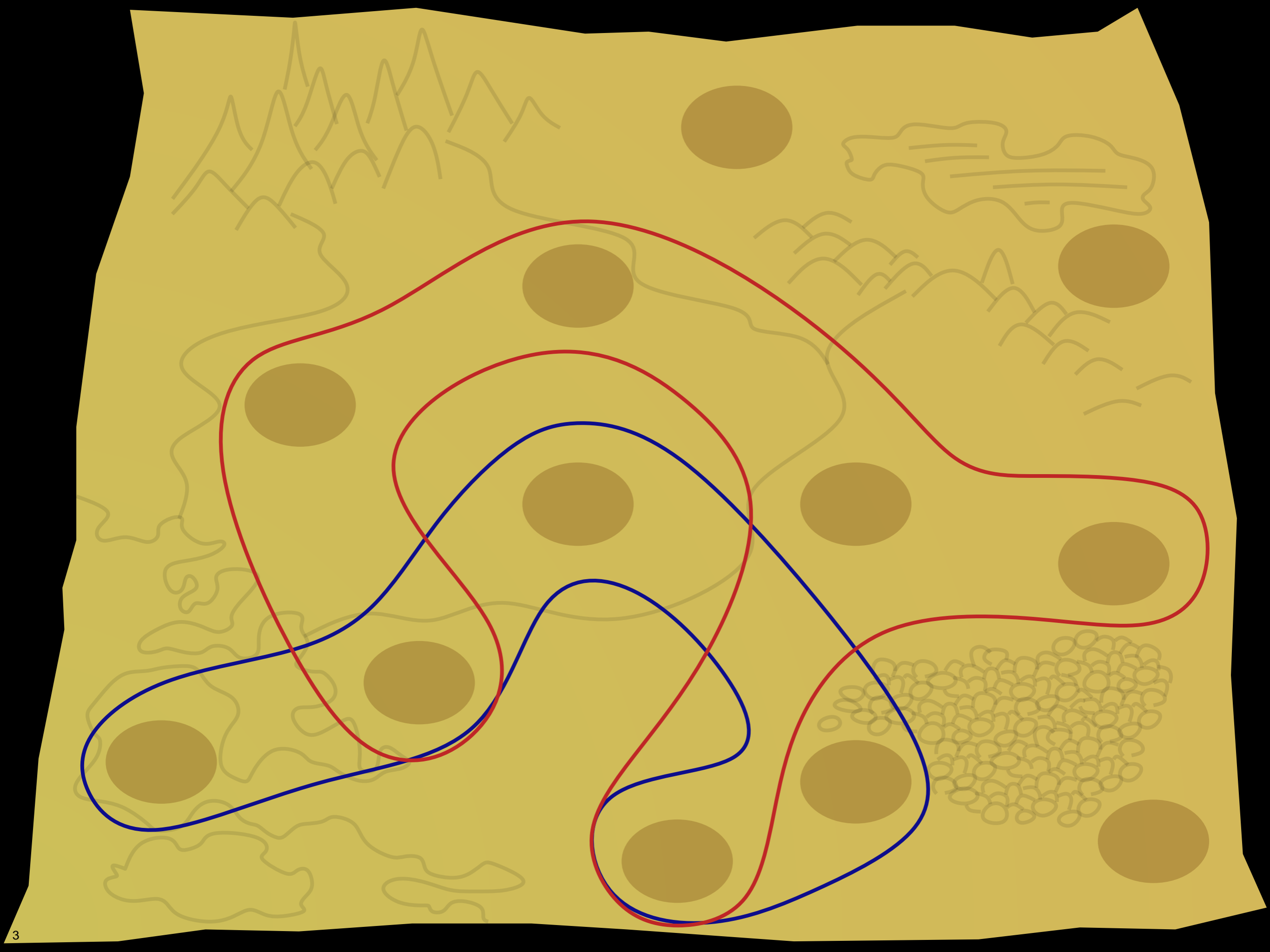





Suppose we want  
to visualise *two*  
sets  
simultaneously!

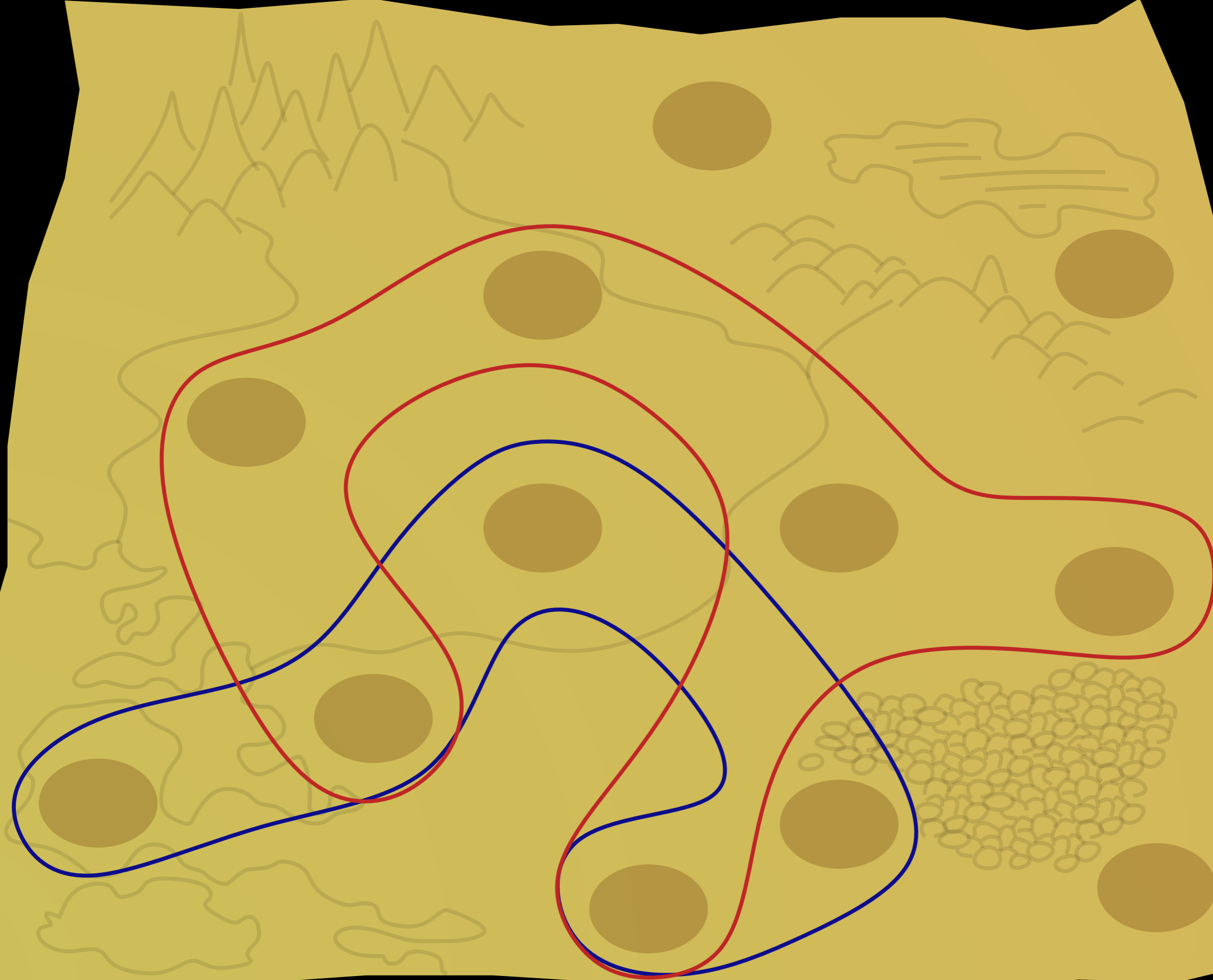


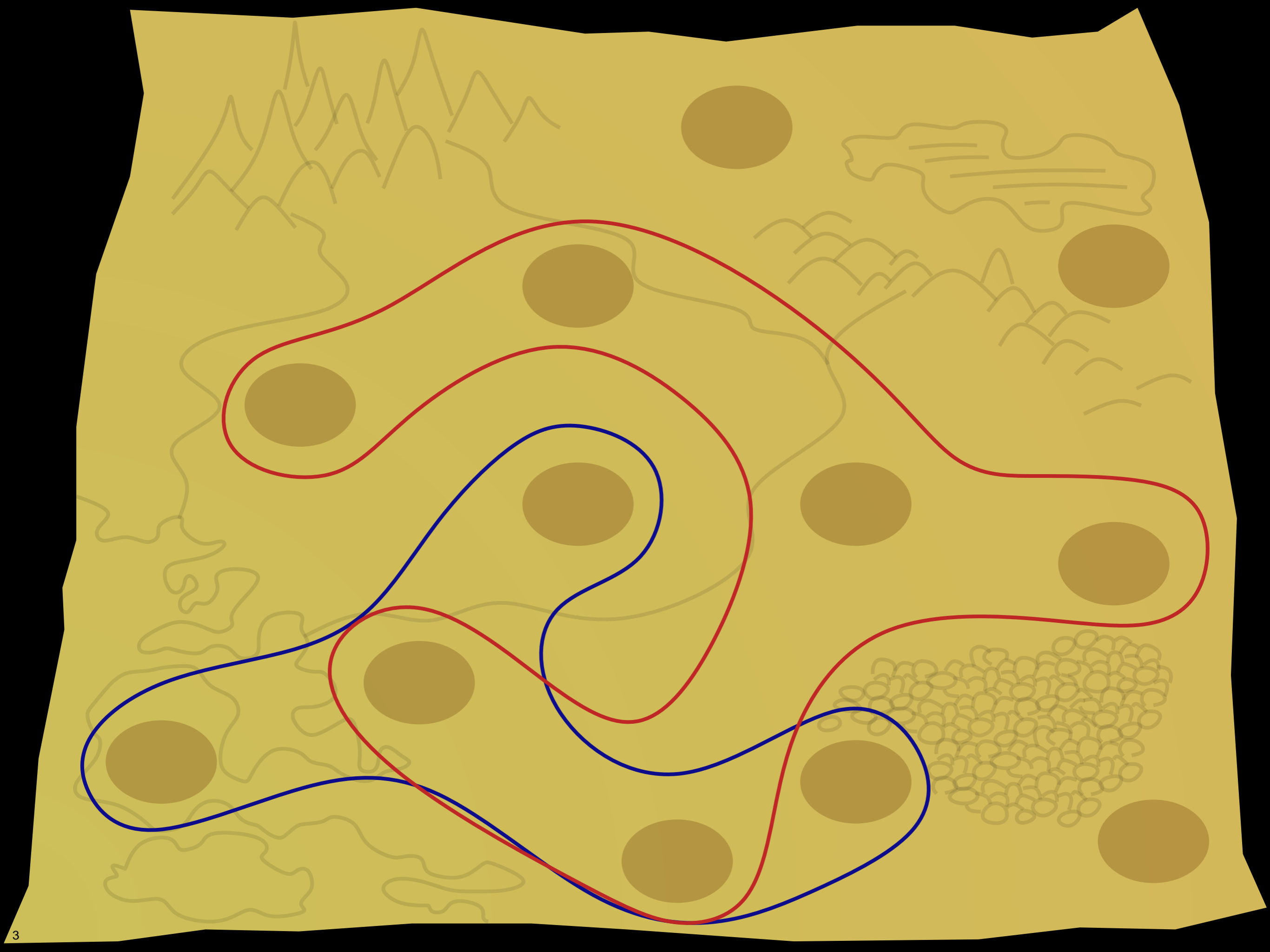


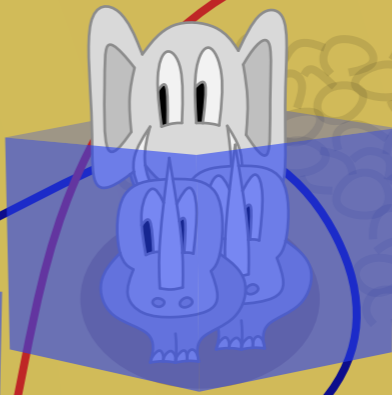
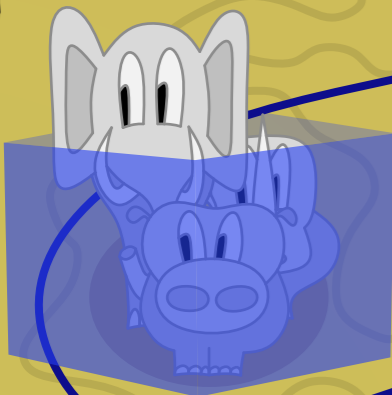
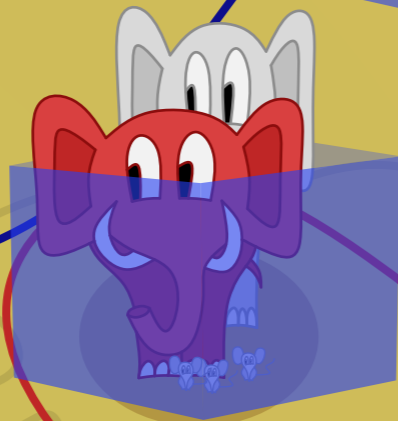
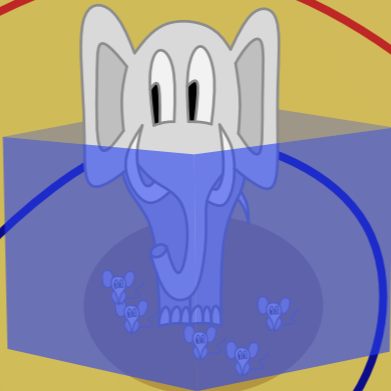
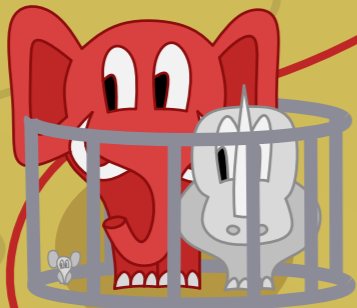
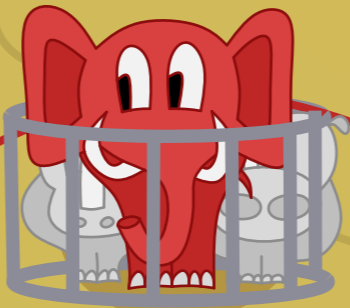
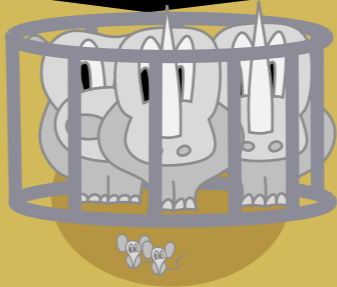




Two visualisations  
that are fine by  
themselves do not  
necessarily look  
good together.

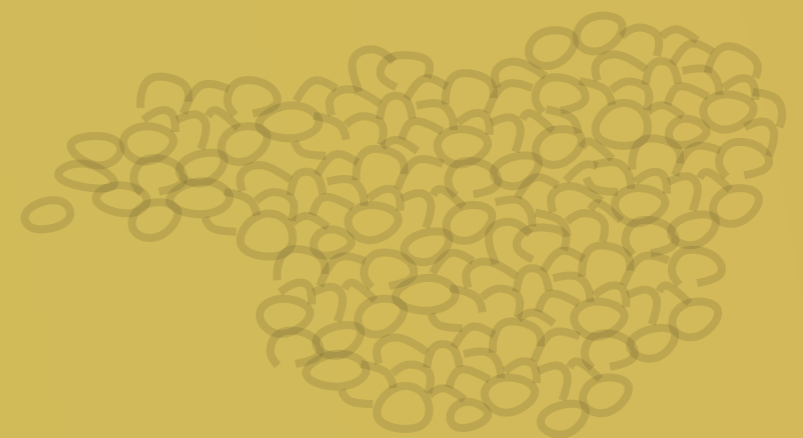


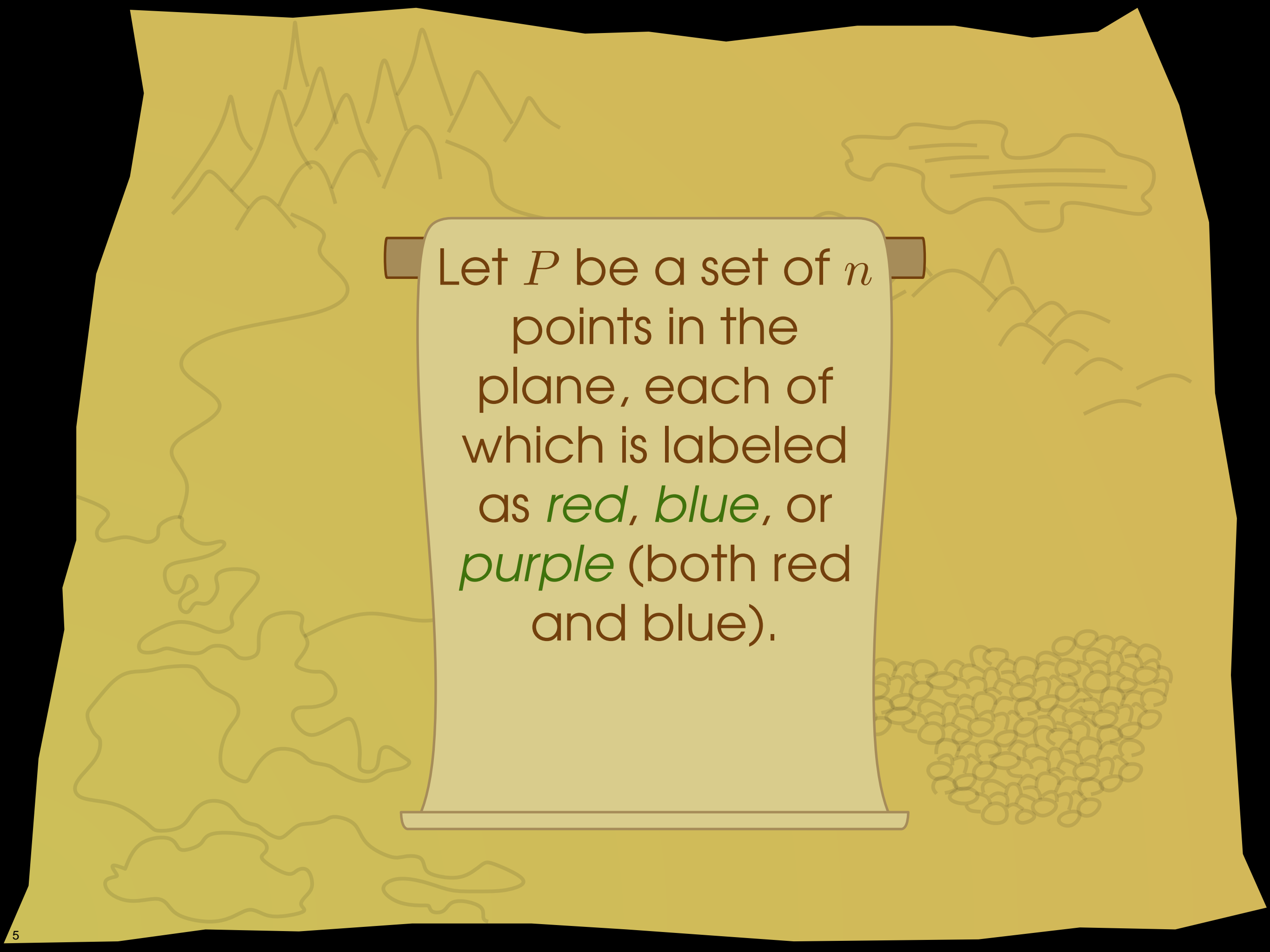




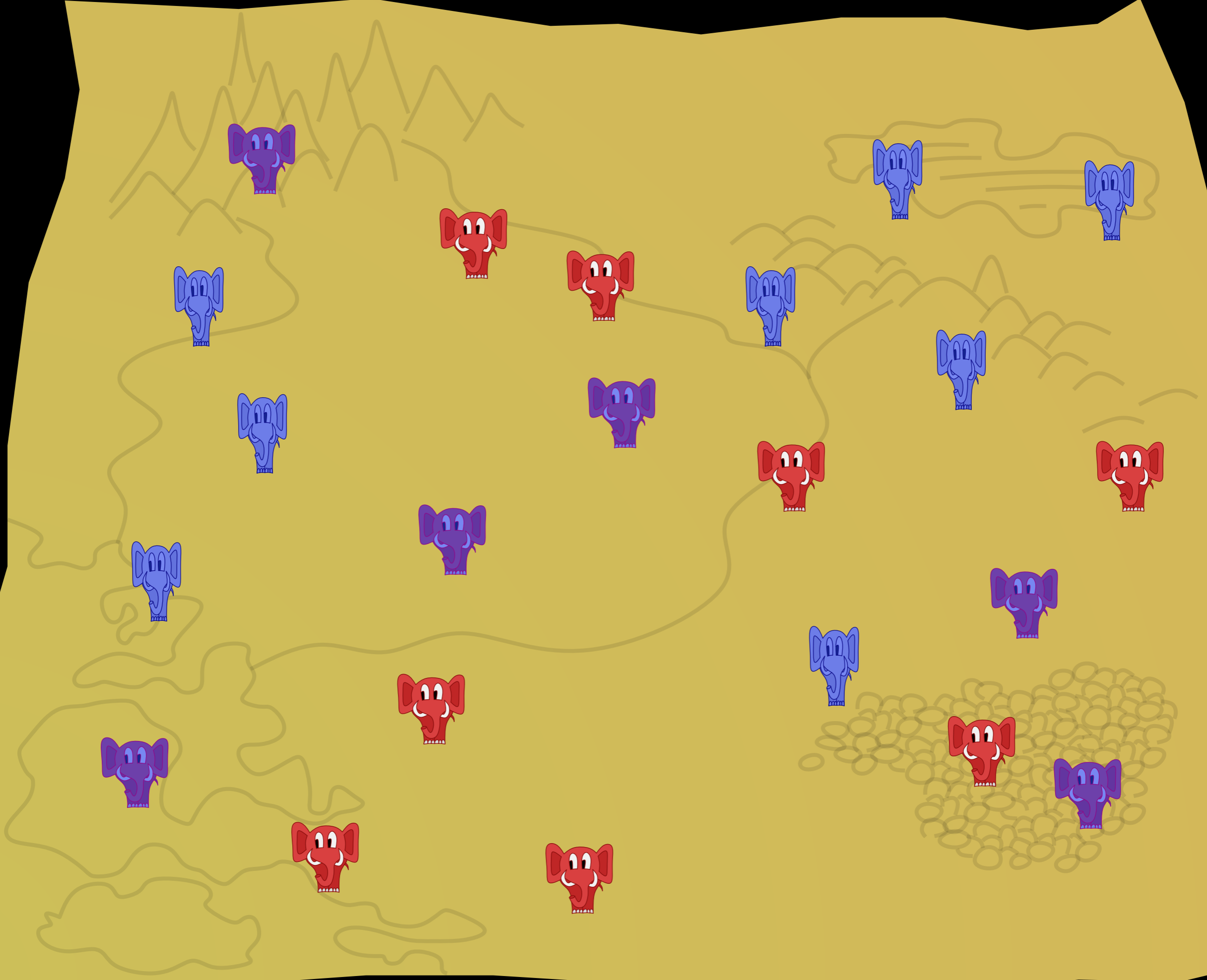


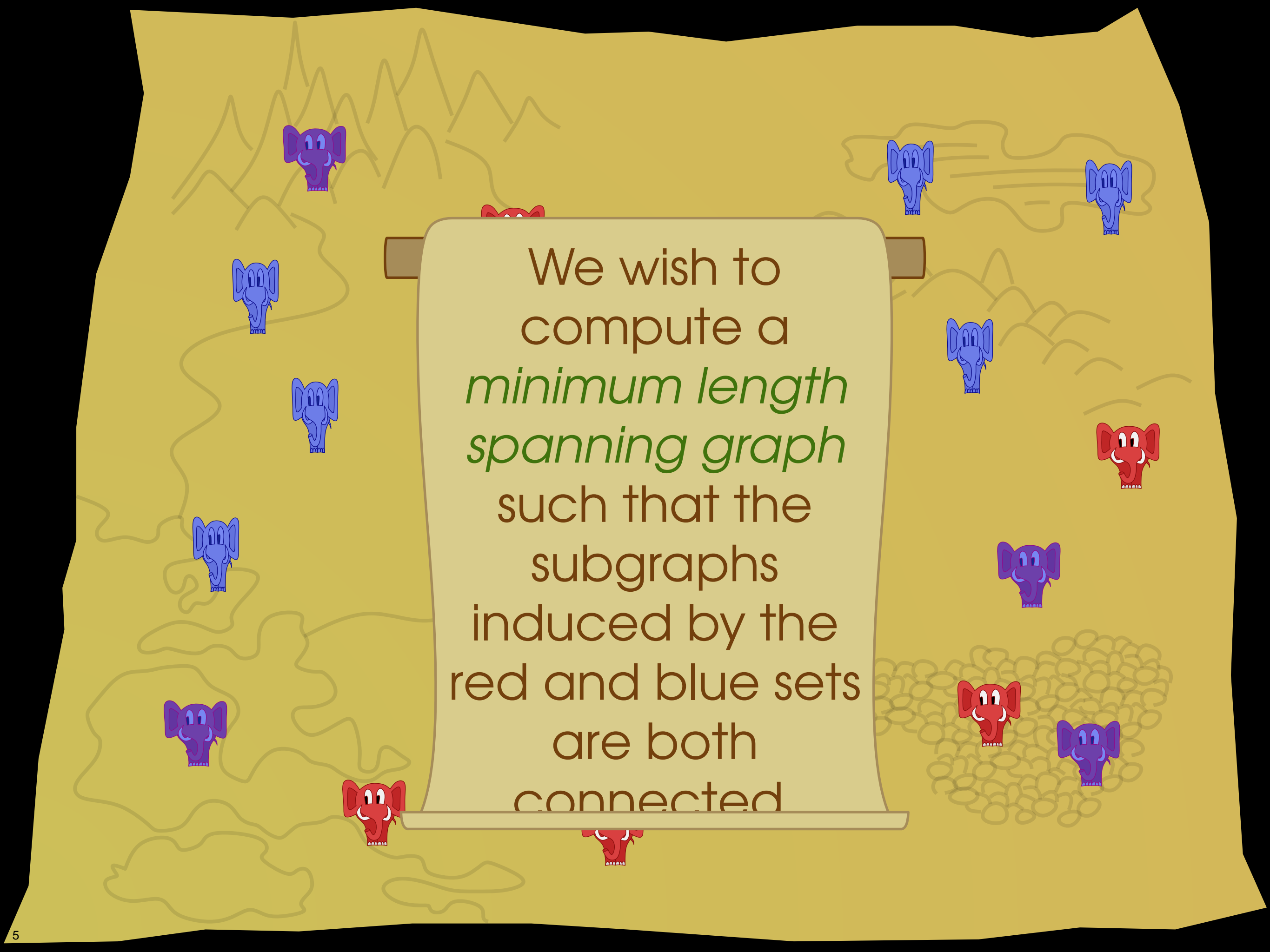
# PROBLEM STATEMENT & RESULTS





Let  $P$  be a set of  $n$  points in the plane, each of which is labeled as *red*, *blue*, or *purple* (both red and blue).



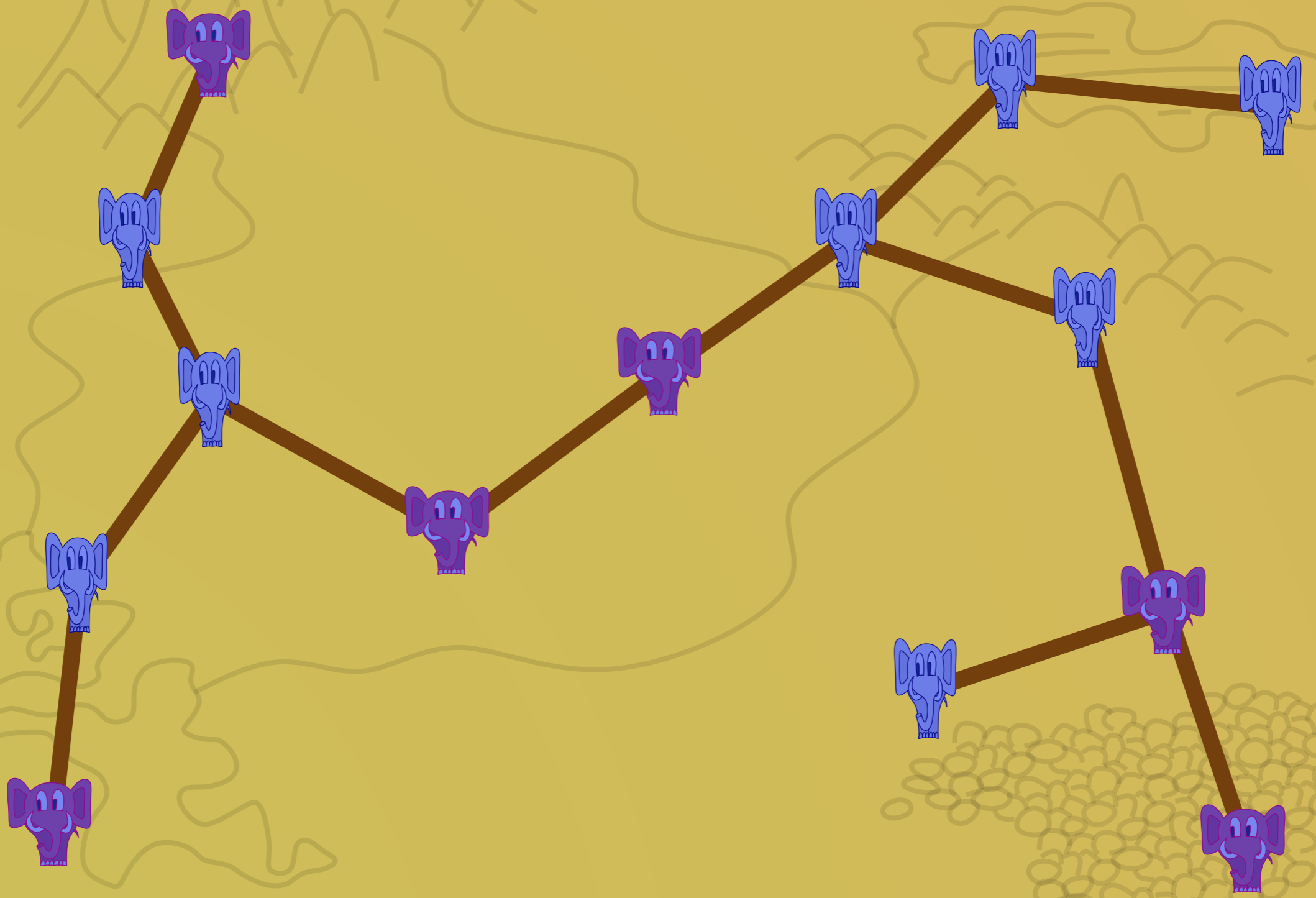


We wish to  
compute a  
*minimum length  
spanning graph*  
such that the  
subgraphs  
induced by the  
red and blue sets  
are both  
connected

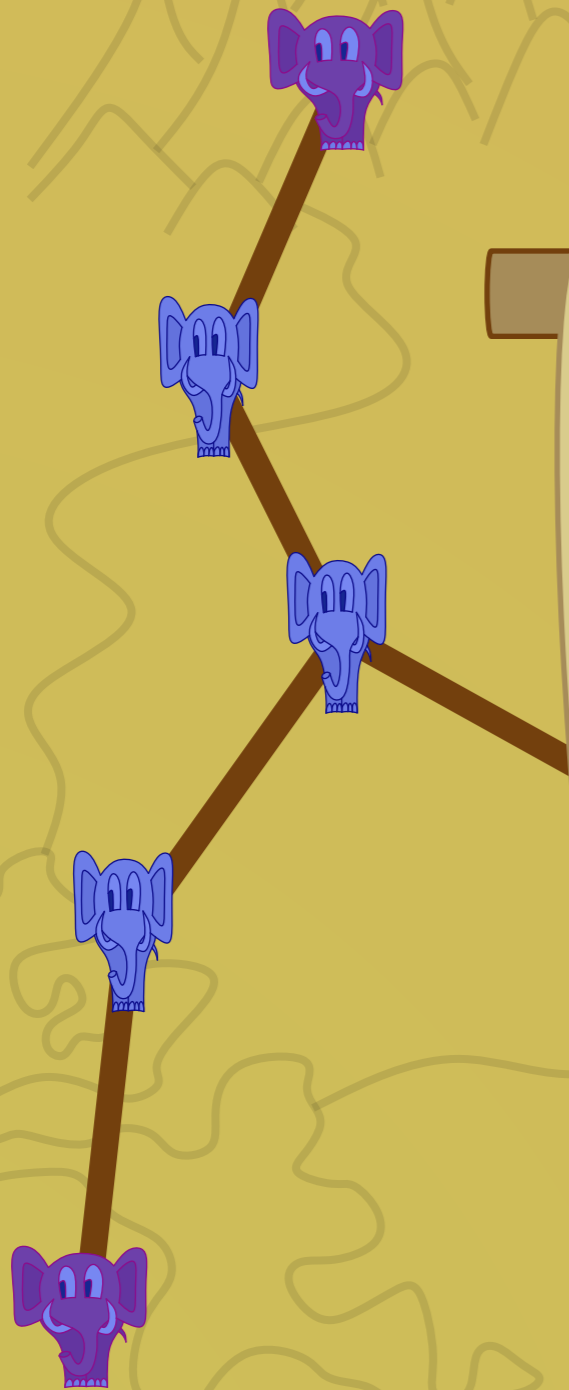




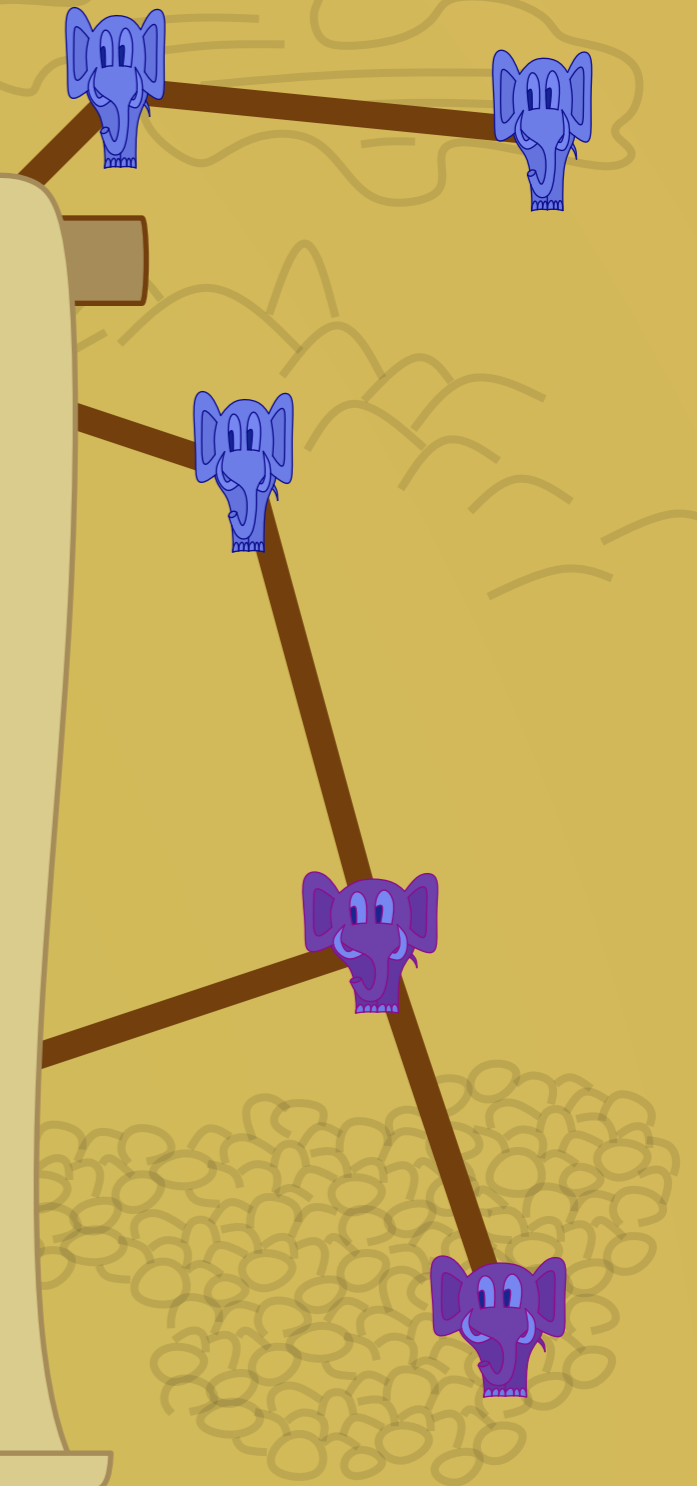


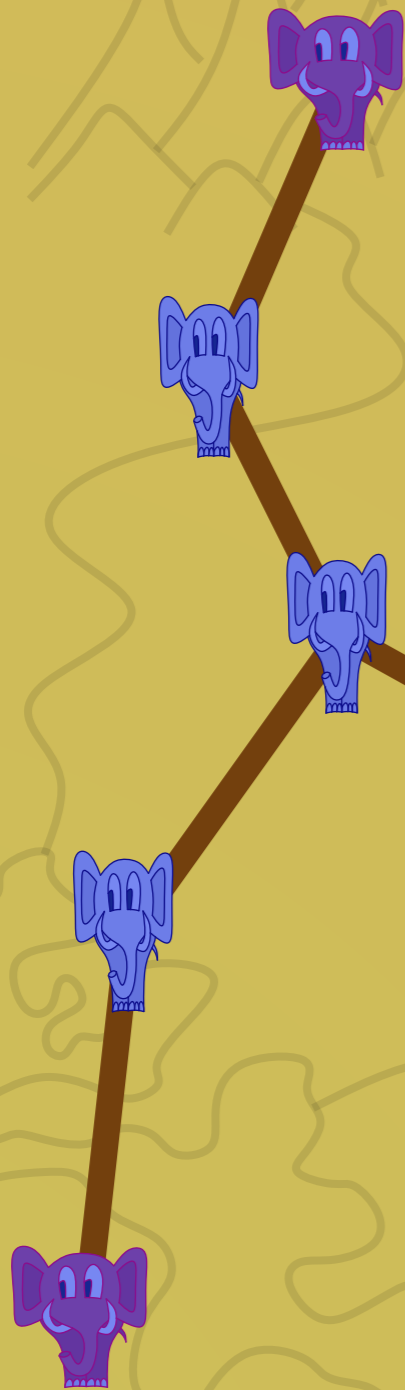






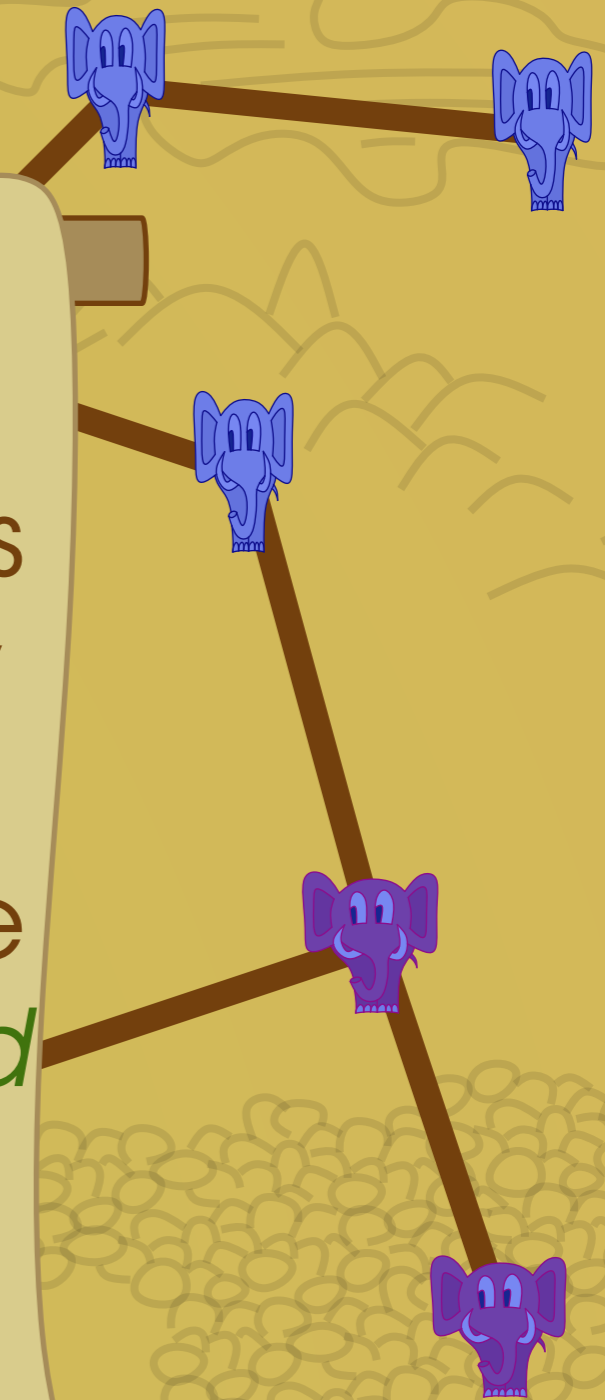
**OBSERVATION**  
In the optimal solution, these induced graphs are both trees.

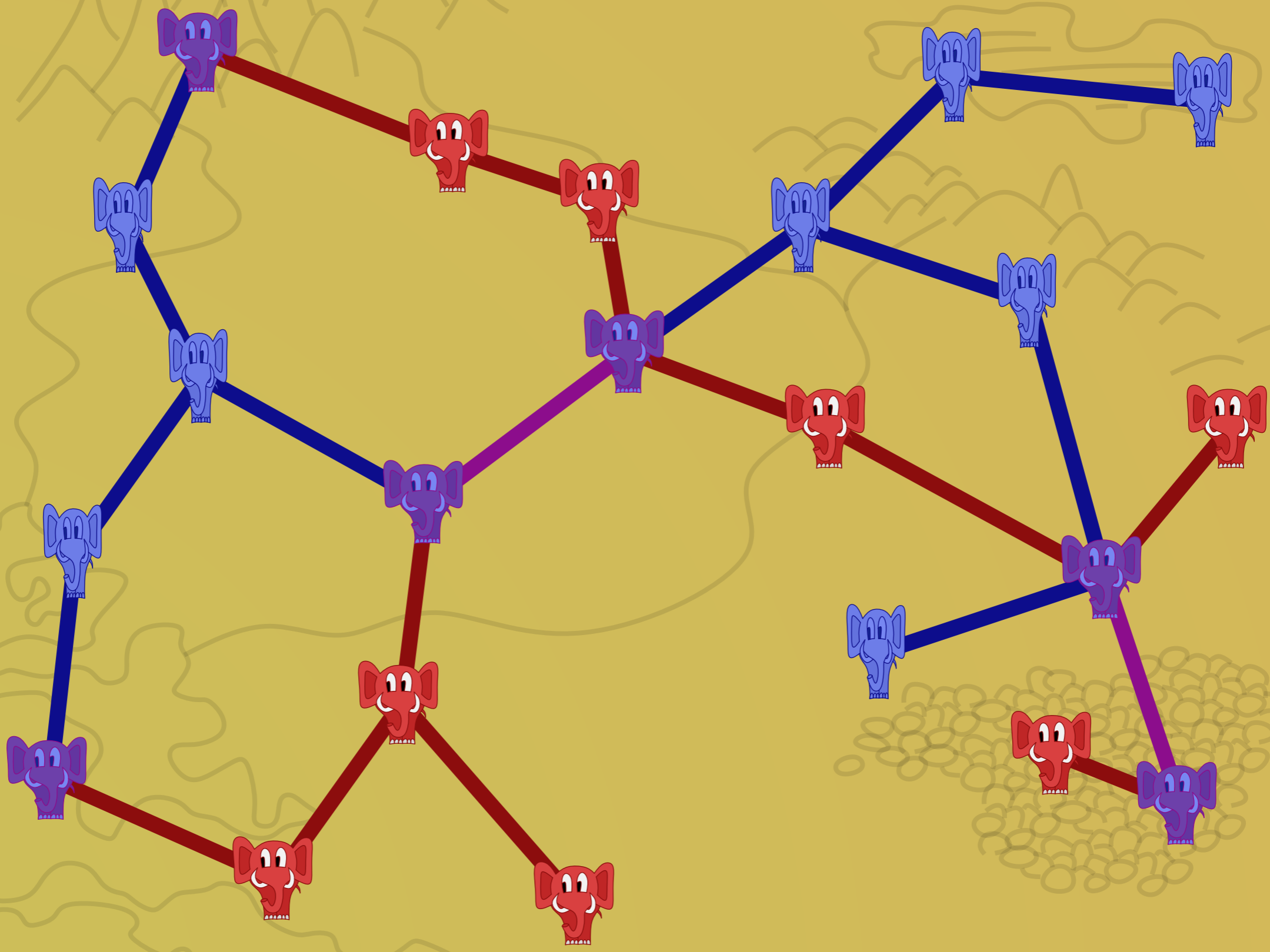




## OBSERVATION

Henceforth, we will call the edges that appear only in the blue tree *blue edges*, those in the red tree *red edges*, and the shared edges *purple edges*





## **RESULTS**

Finding the minimum weight graph that connects both the red points and the blue points is NP-hard.

## RESULTS

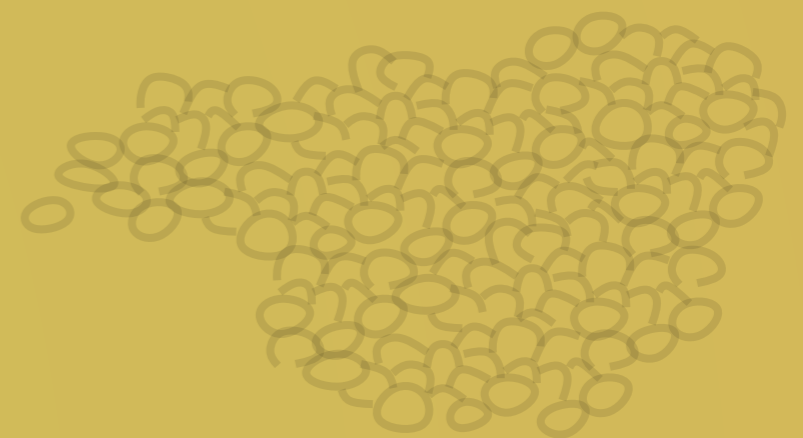
We can compute  
a  $\frac{1}{2}\rho + 1$ -  
approximation in  
 $O(n \log n)$  time,  
where  $\rho$  is the  
*Steiner ratio*.

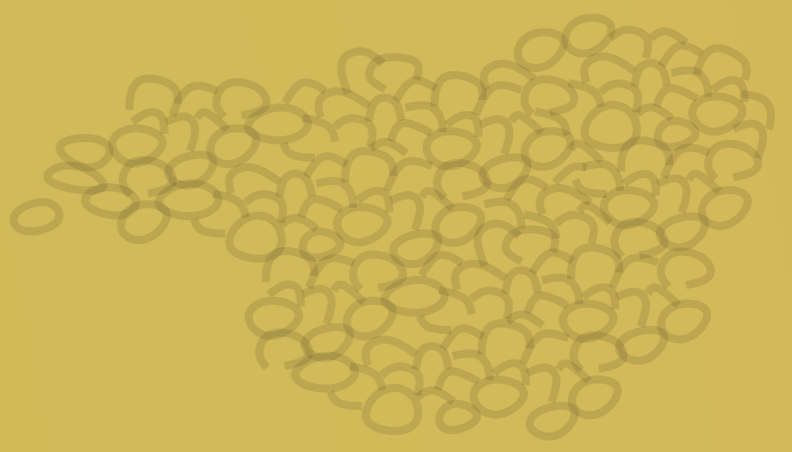
## RESULTS

If all points lie on a line, we can compute the optimal solution in  $O(n)$  time.

If they lie on a circle, in  $O(nk^3)$  time.

# PROPERTIES OF RBP GRAPHS

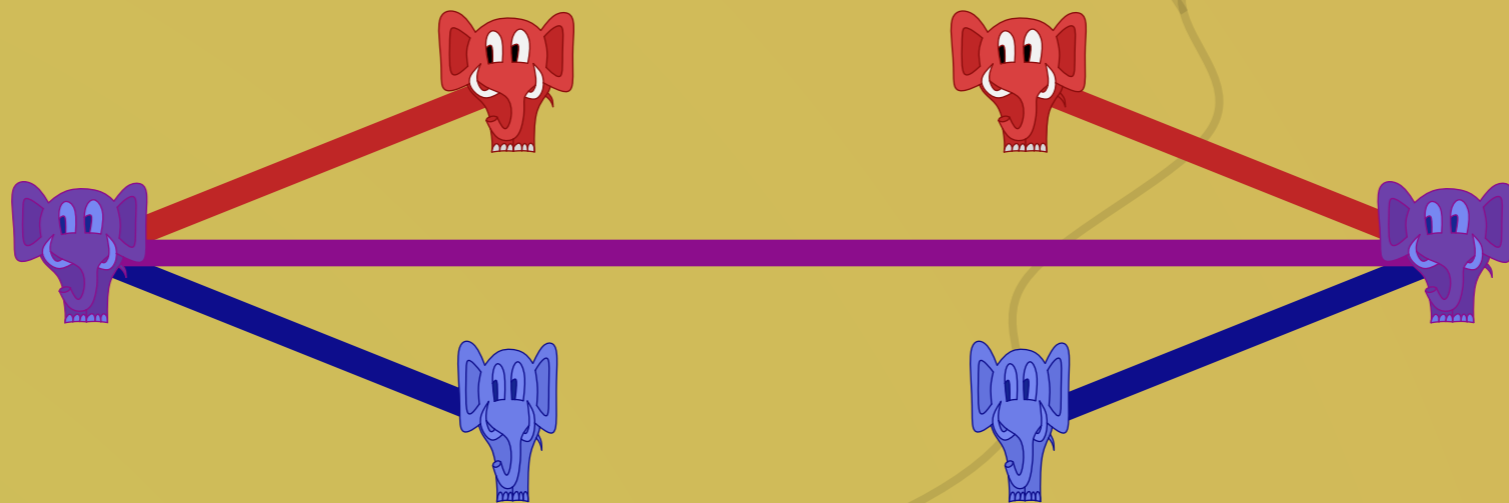


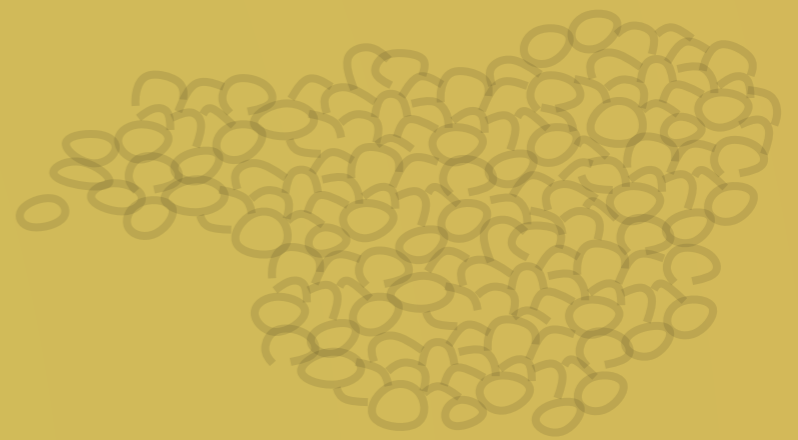
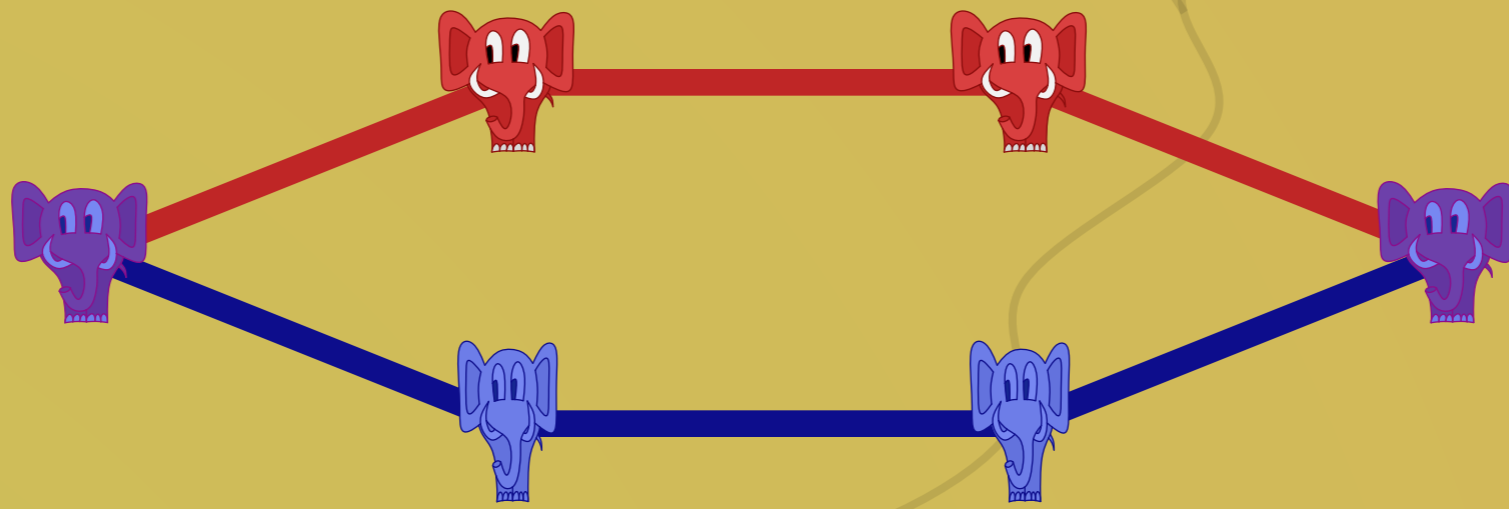


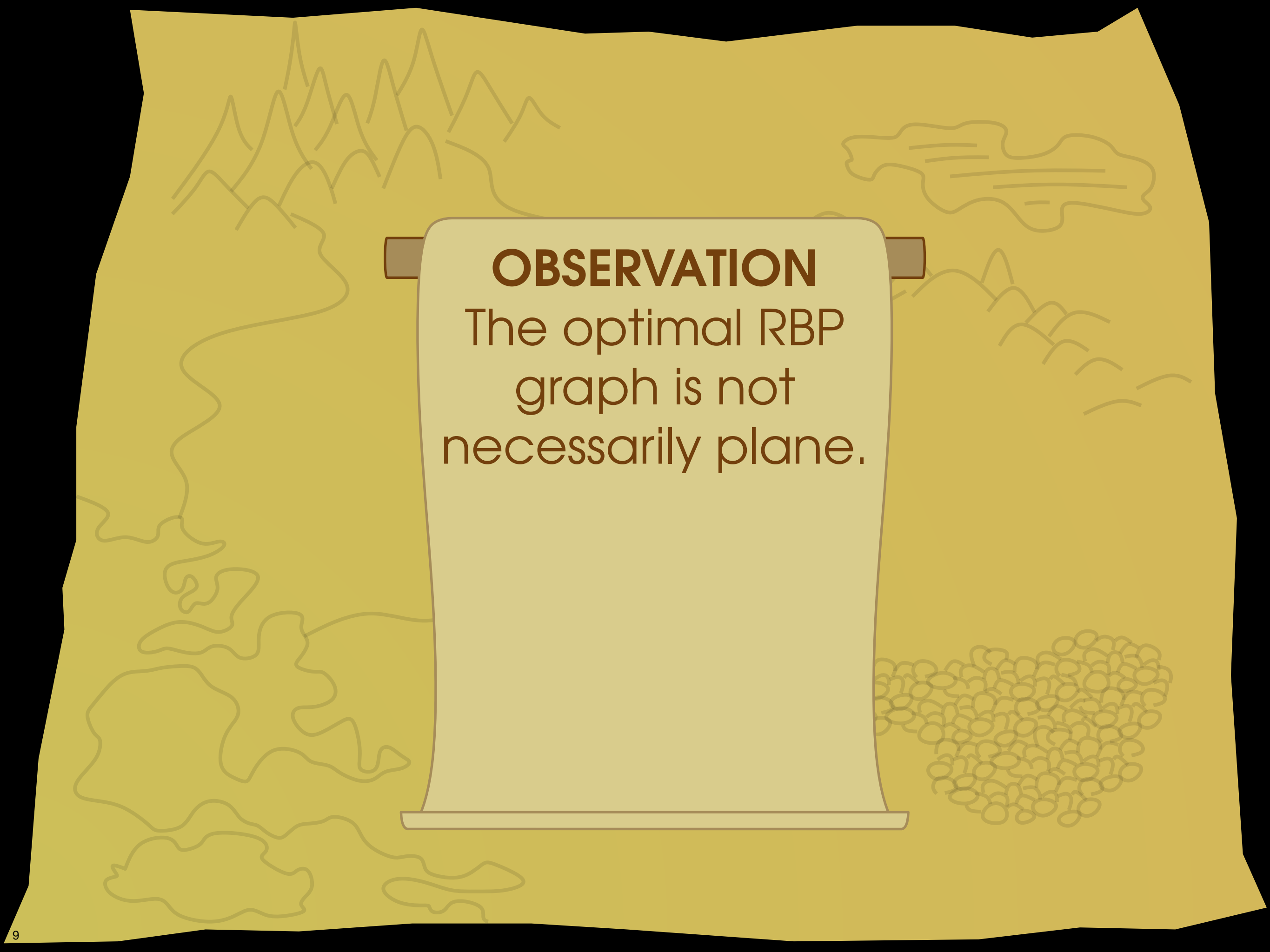




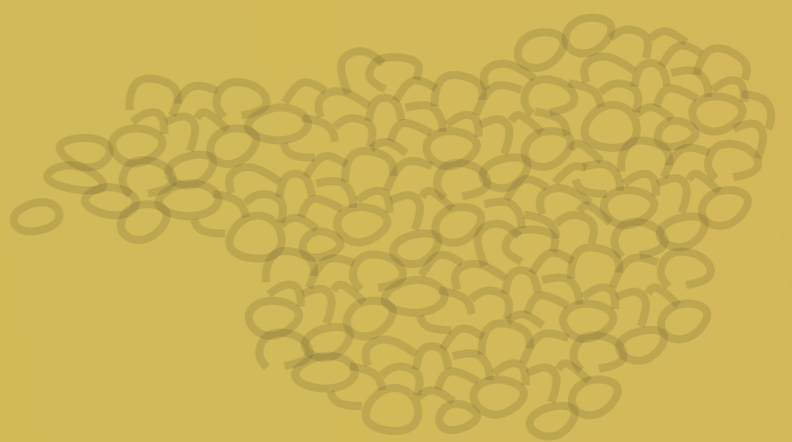


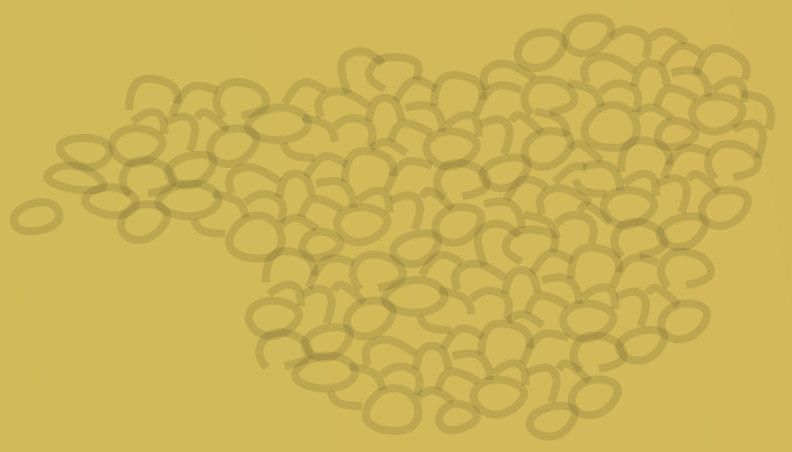
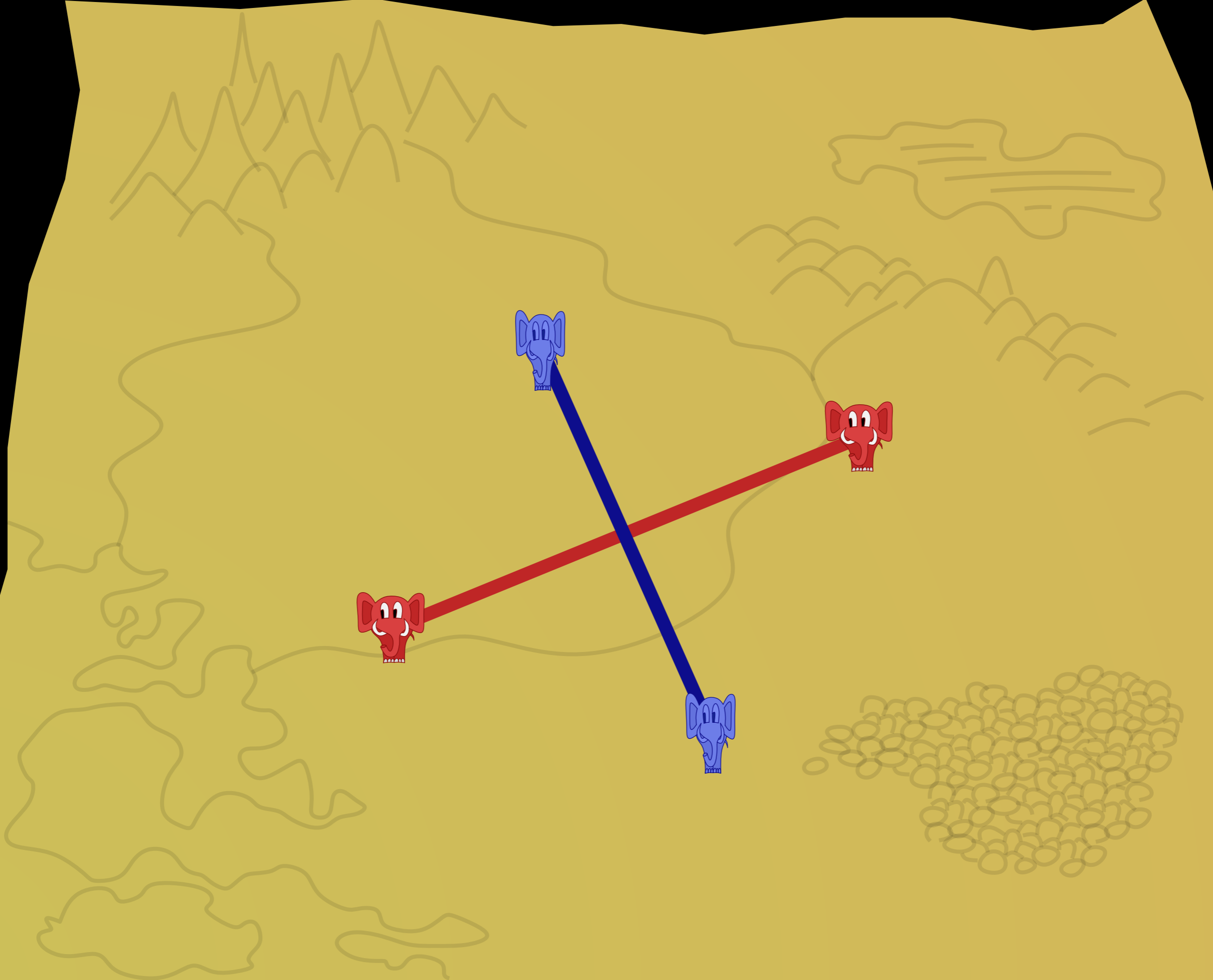







**OBSERVATION**  
The optimal RBP  
graph is not  
necessarily plane.



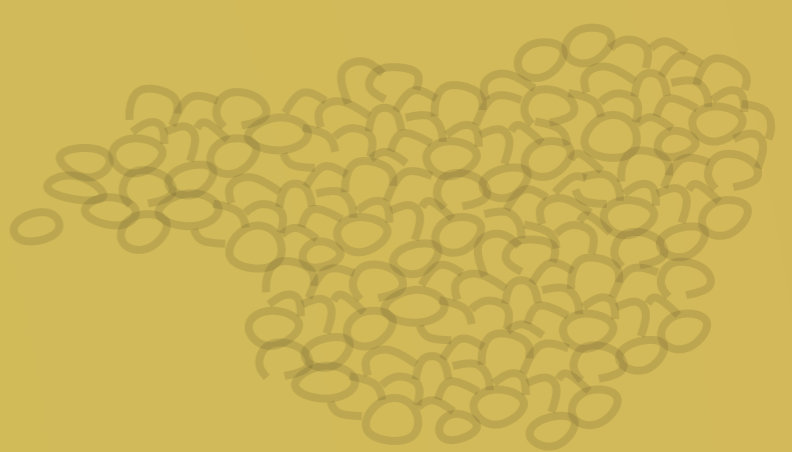
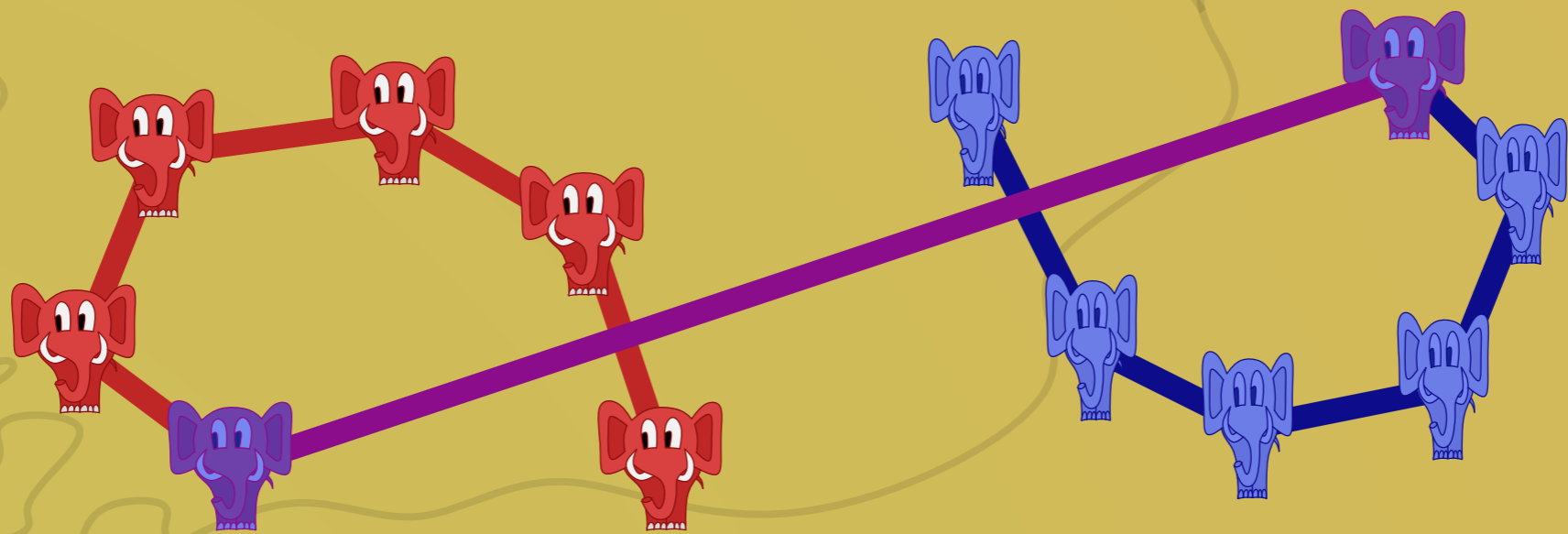




**OBSERVATION**  
Even *purple*  
edges can be  
crossed!

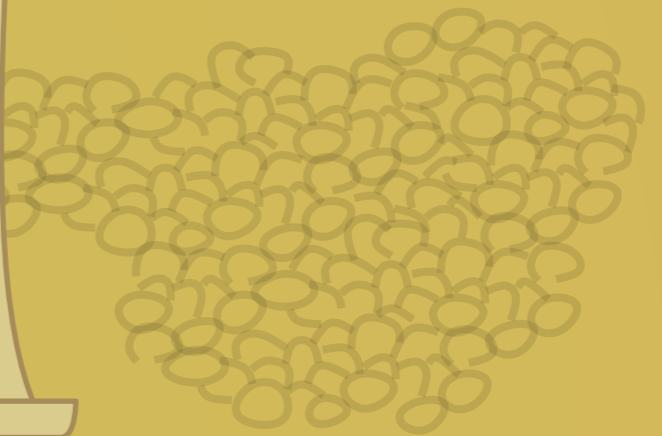
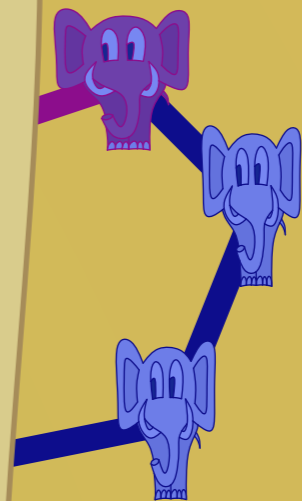
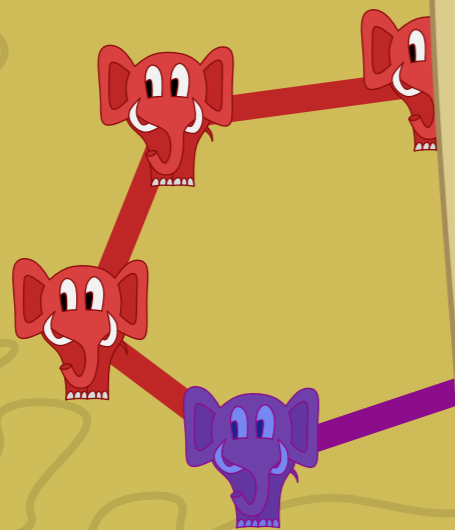






## QUESTION

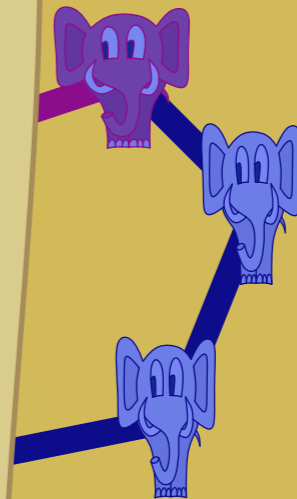
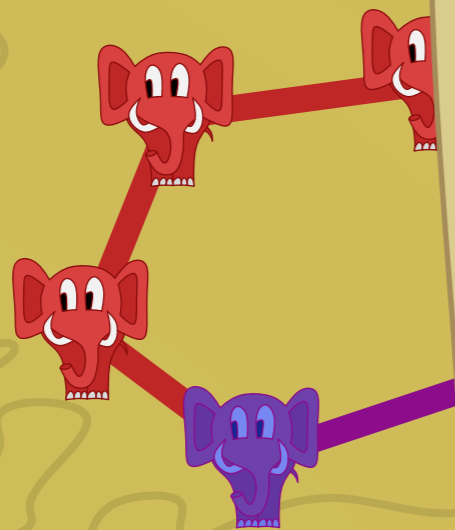
But can purple edges cross other purple edges?



## QUESTION

But can purple edges cross other purple edges?

Why, surely not!

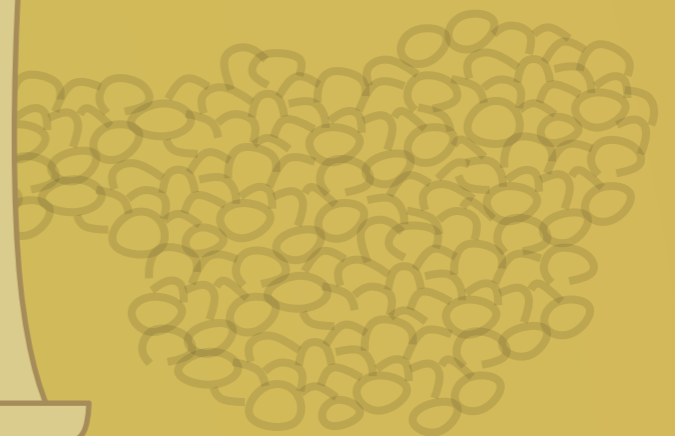
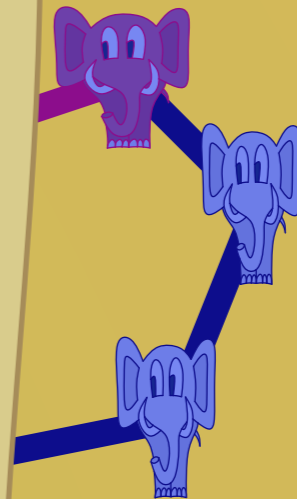
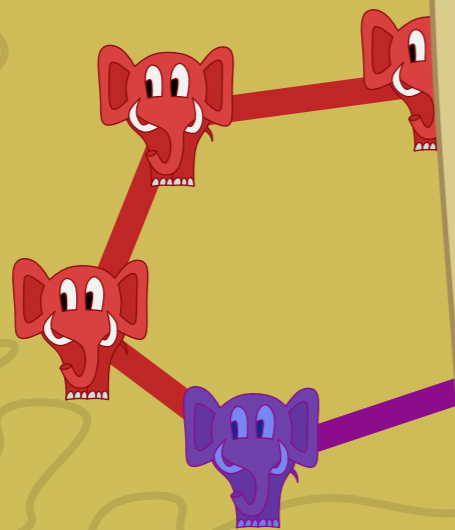


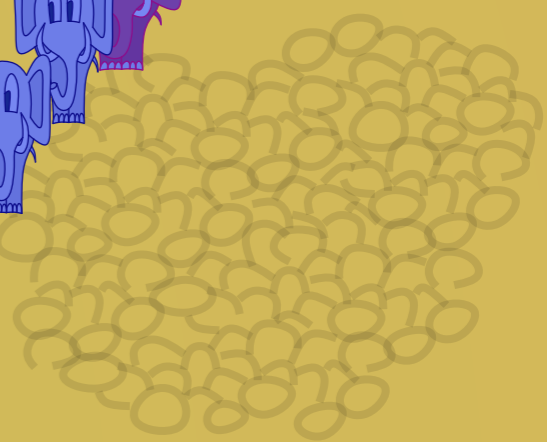
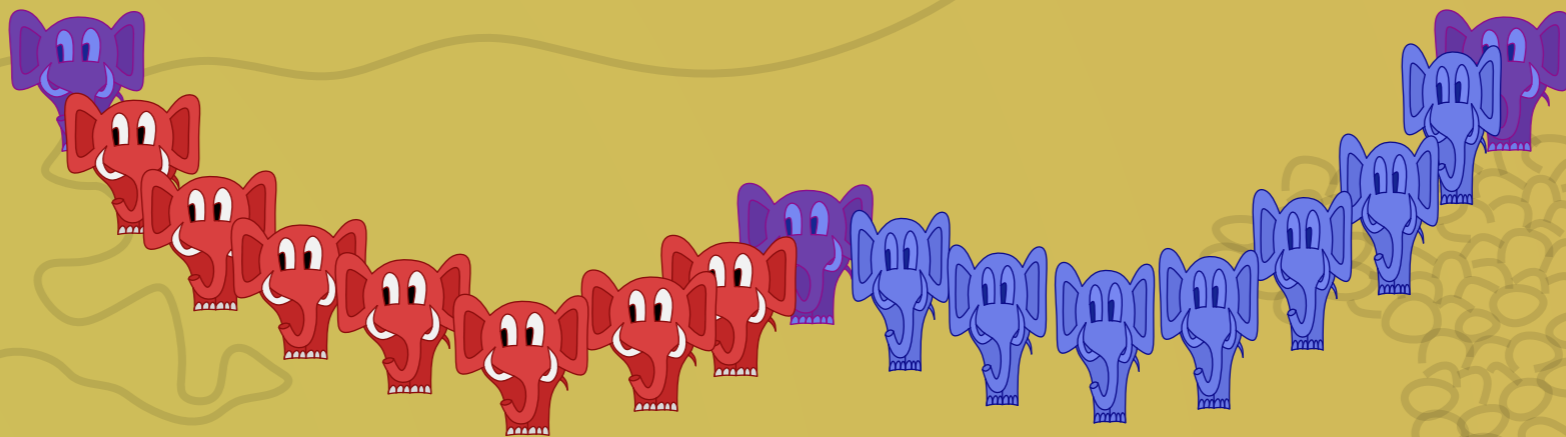
## QUESTION

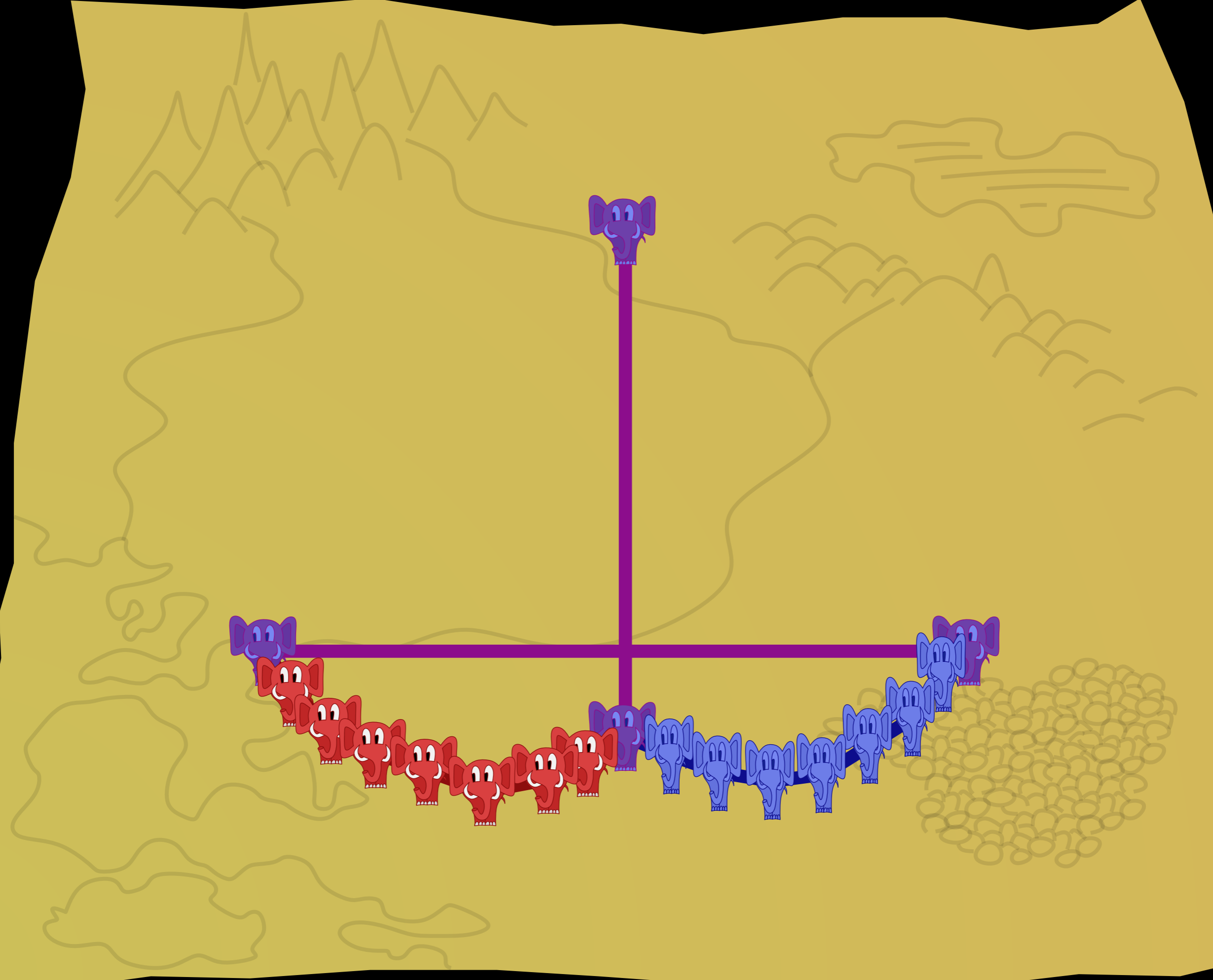
But can purple edges cross other purple edges?

Why, surely not!

...right?



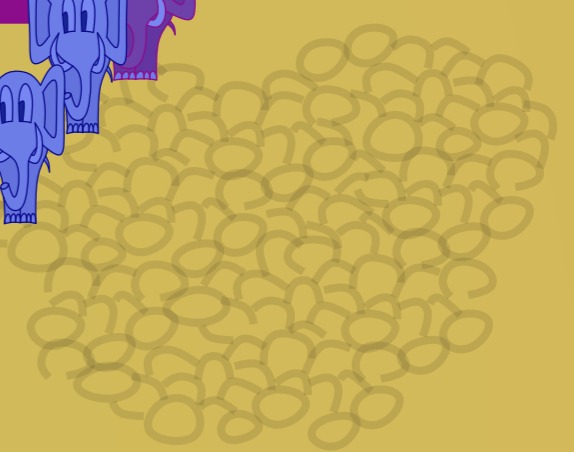
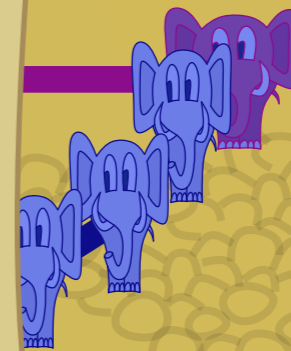






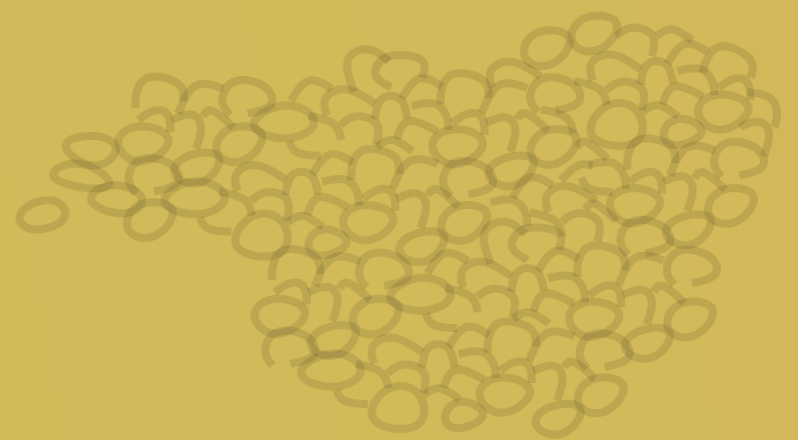
## OBSERVATION

Life is even worse.  
A single purple  
edge can in fact  
intersect arbitrarily  
many other  
purple edges!



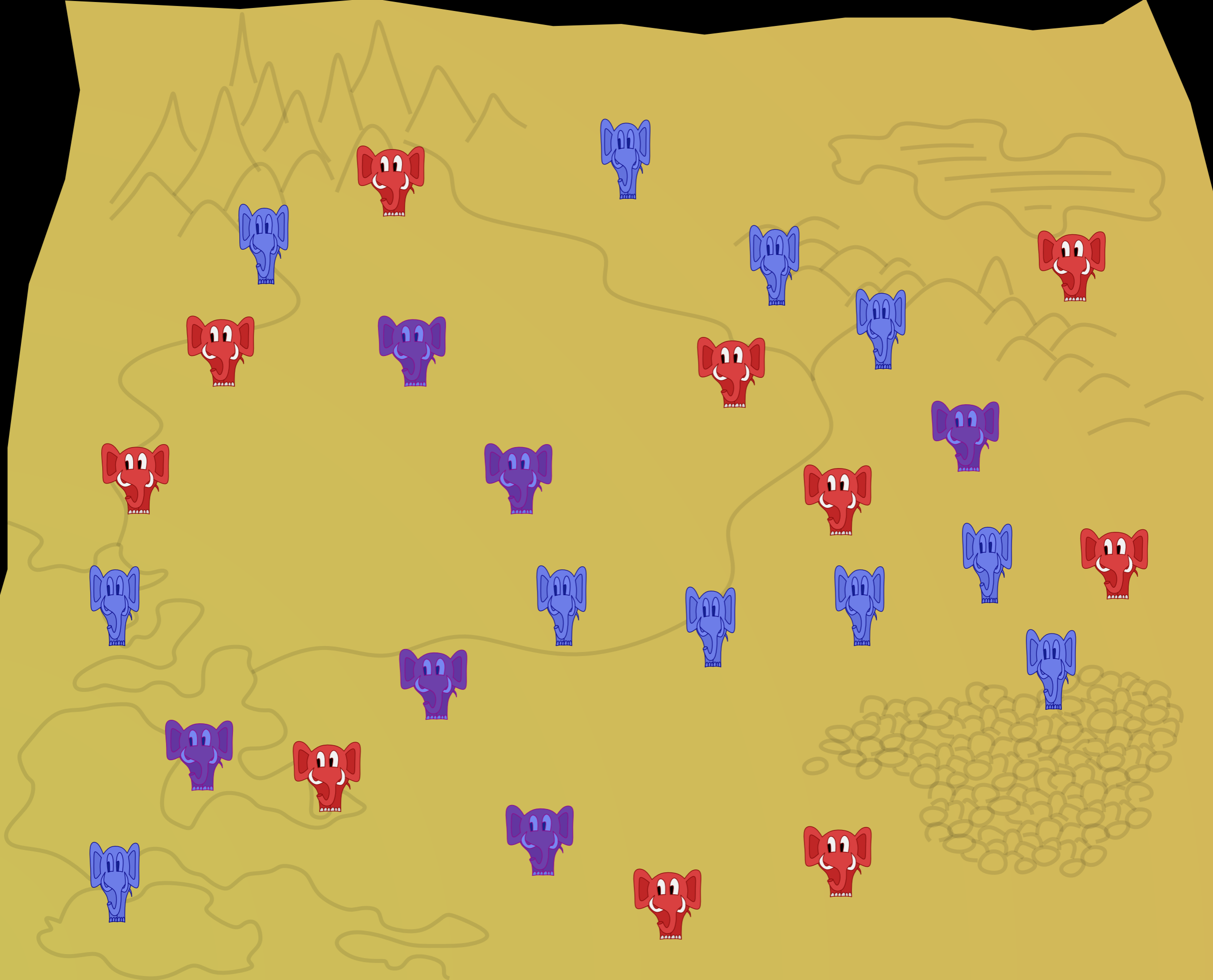


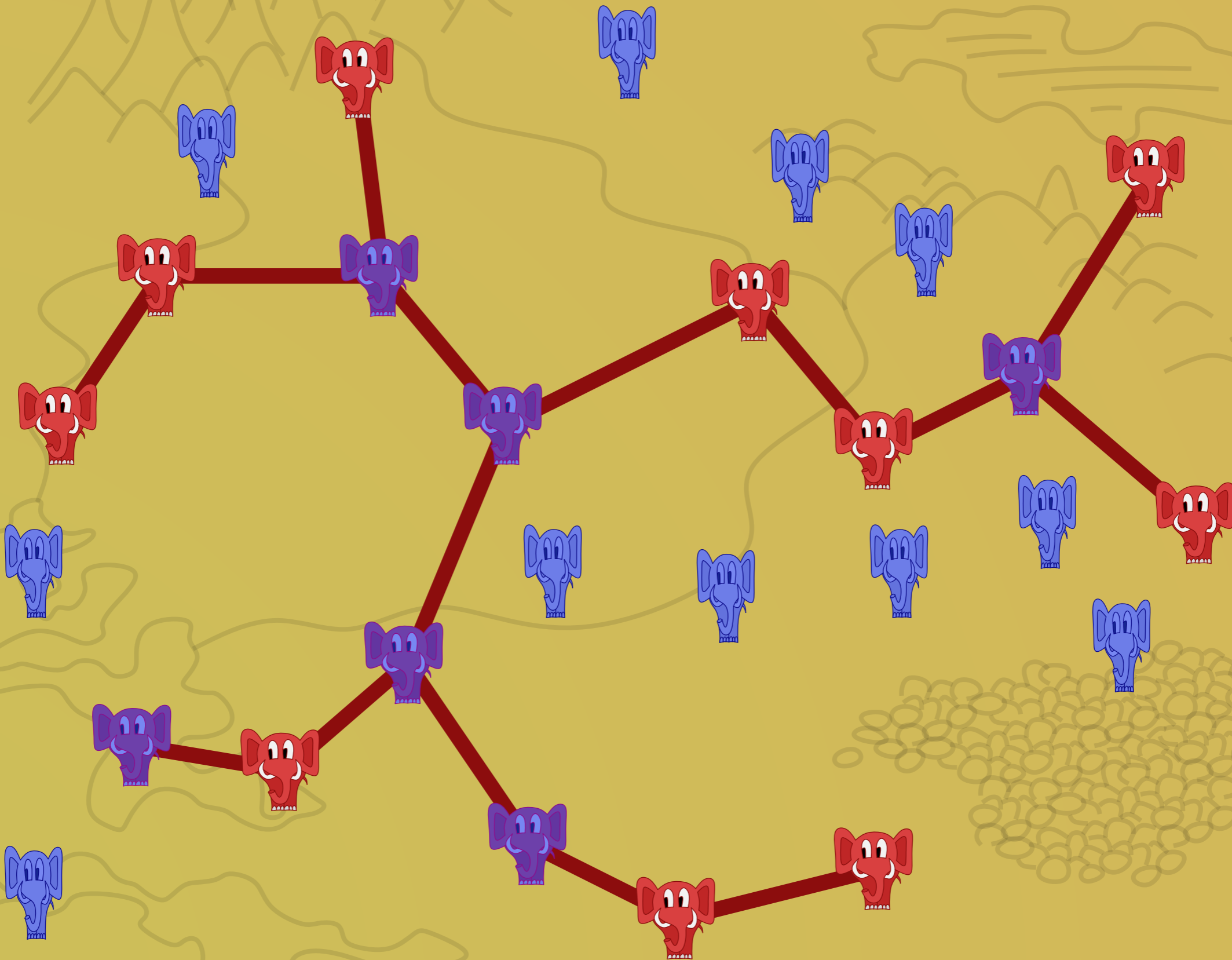
# APPROXIMATION ALGORITHMS

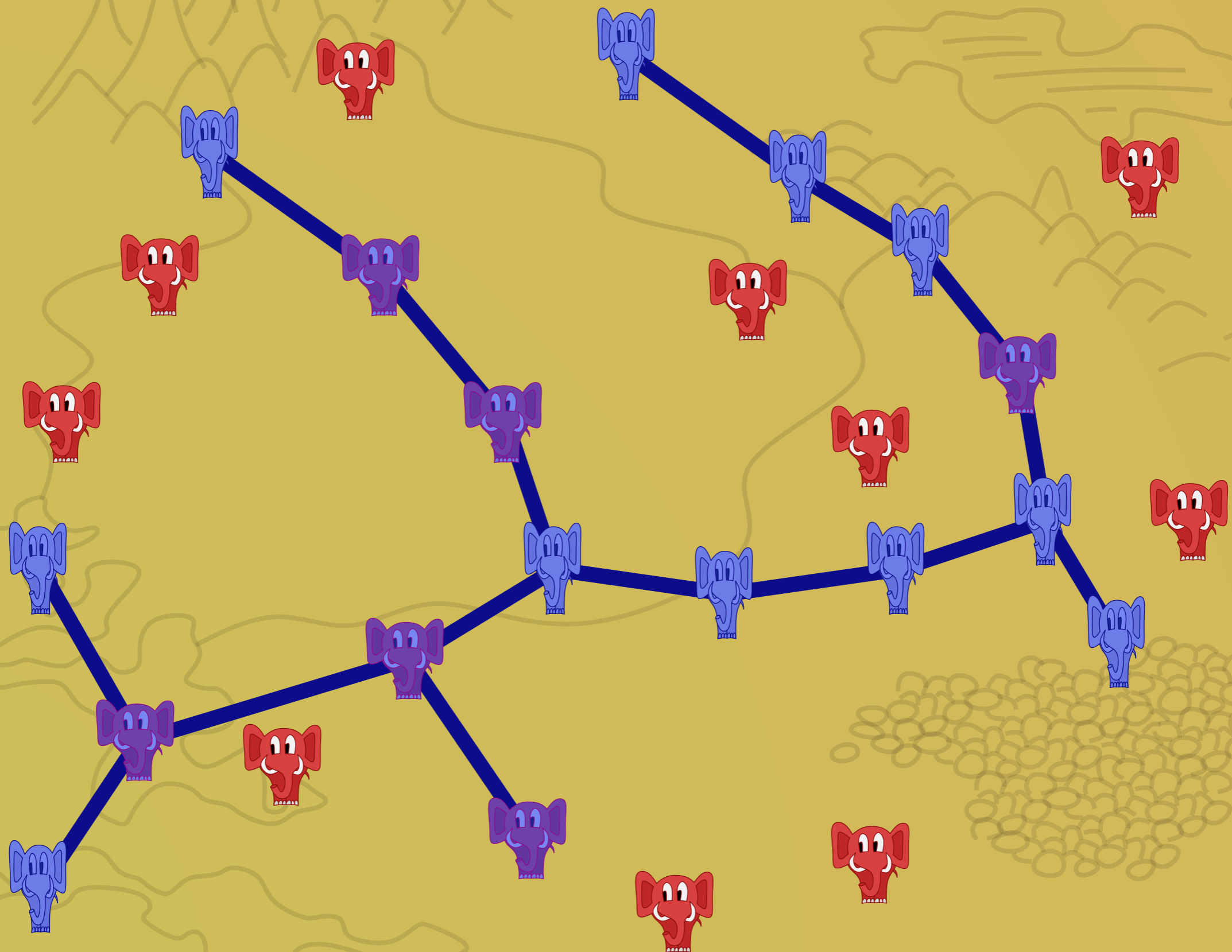


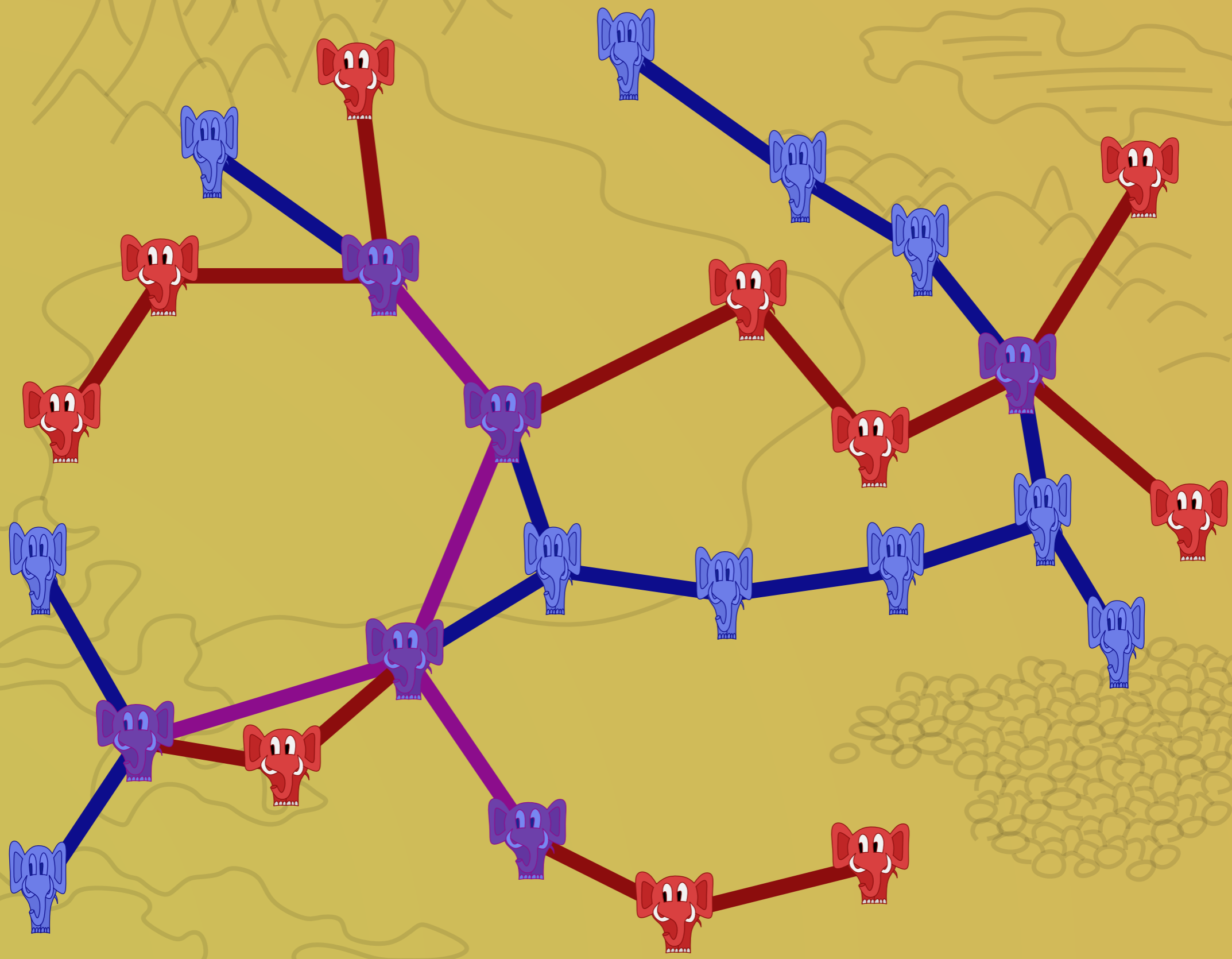
## **ALGORITHM 1**

Compute the MST  
of the red points.  
Compute the MST  
of the blue points.  
Take union of  
edges.

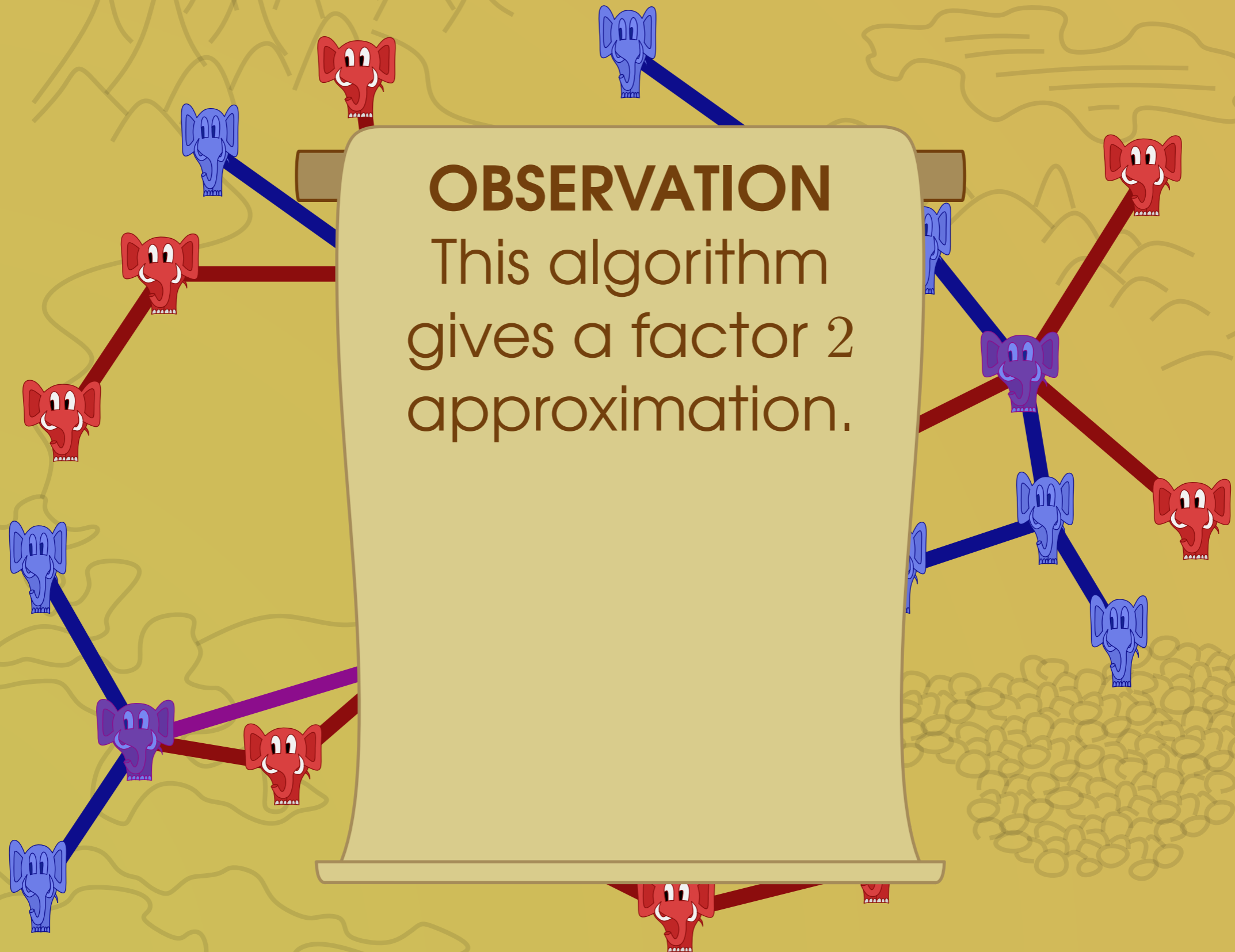


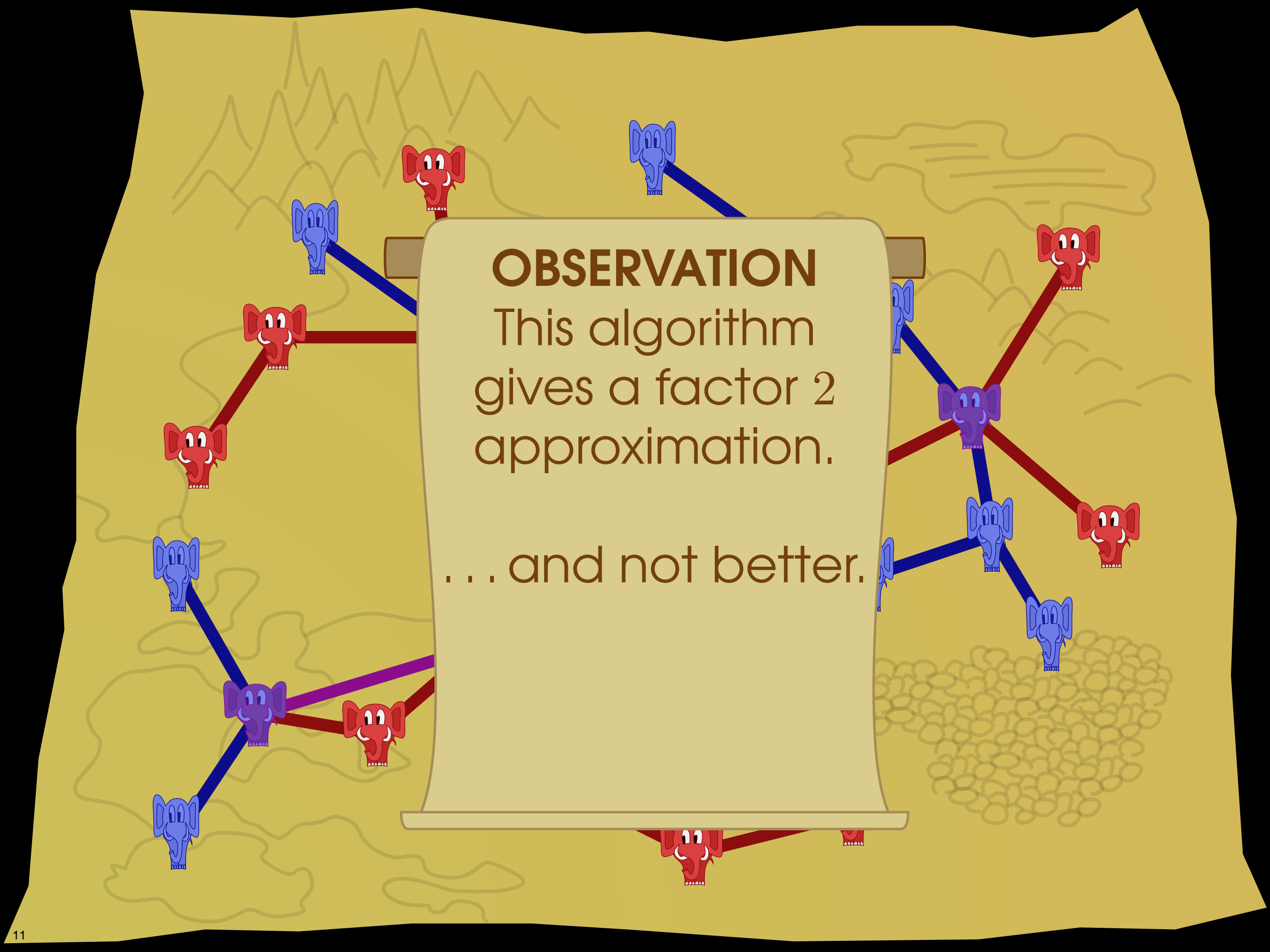






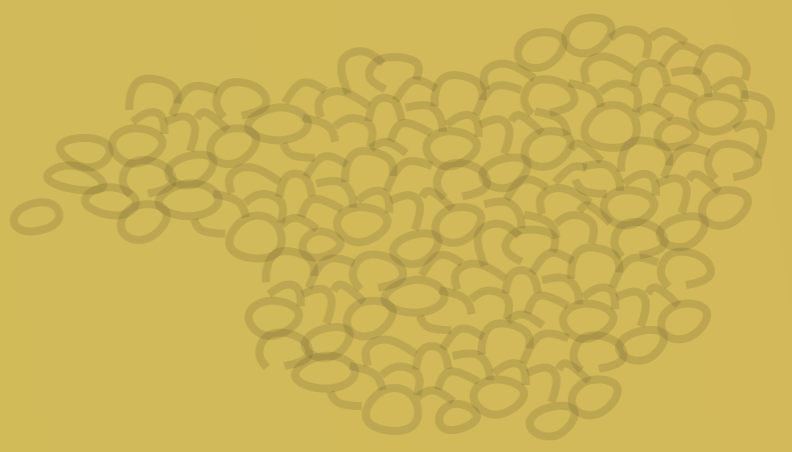
**OBSERVATION**  
This algorithm gives a factor 2 approximation.



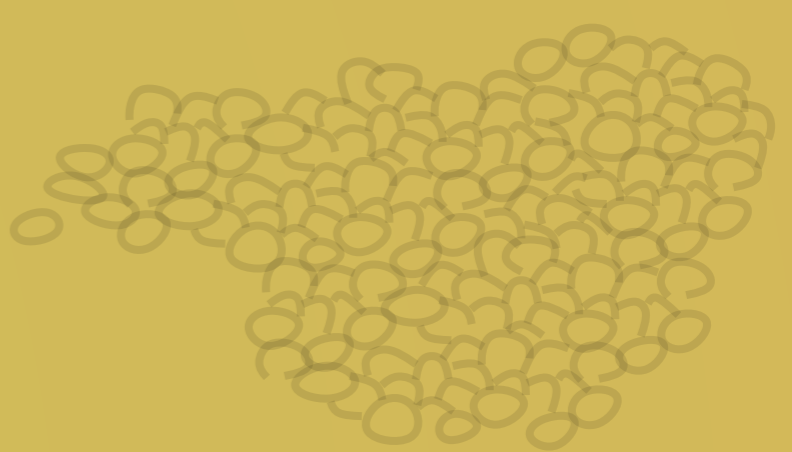


**OBSERVATION**  
This algorithm  
gives a factor 2  
approximation.  
... and not better.



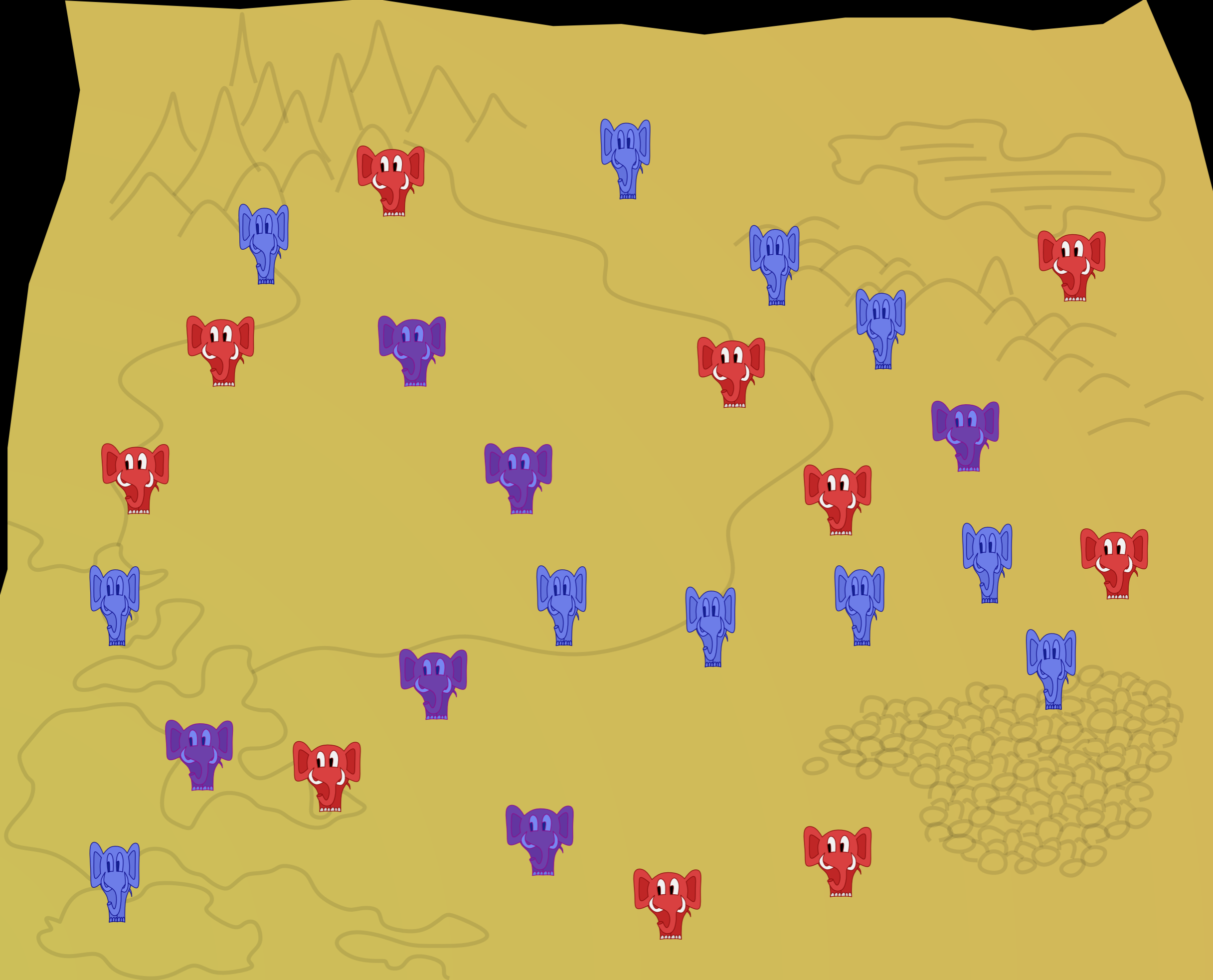


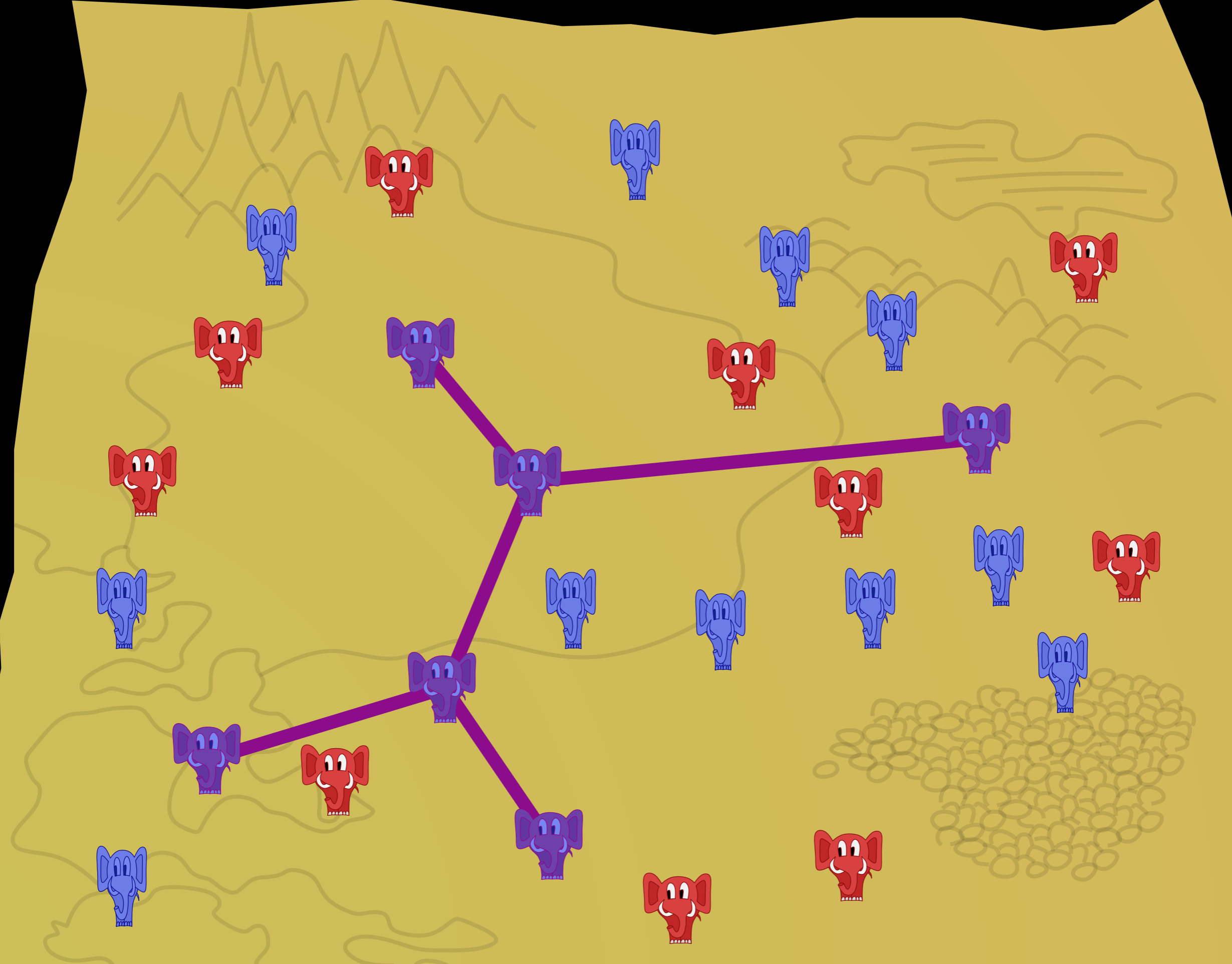




## ALGORITHM 2

Compute the MST of the *purple* points. Optimally connect the red and blue points to the purple skeleton.





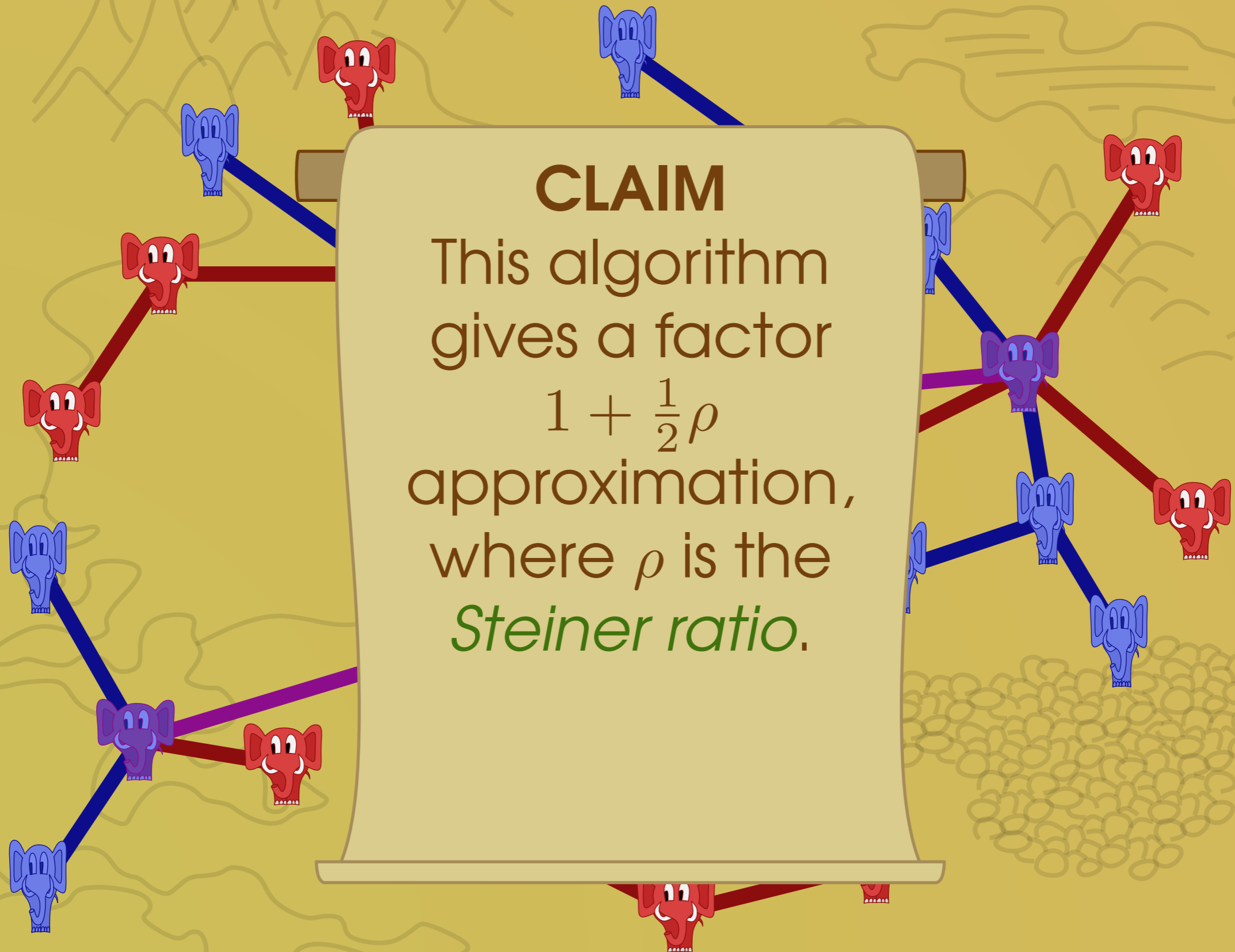




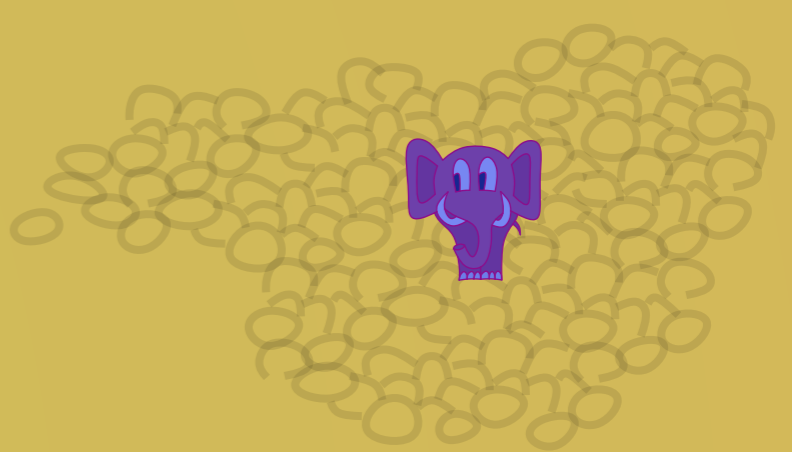


# CLAIM

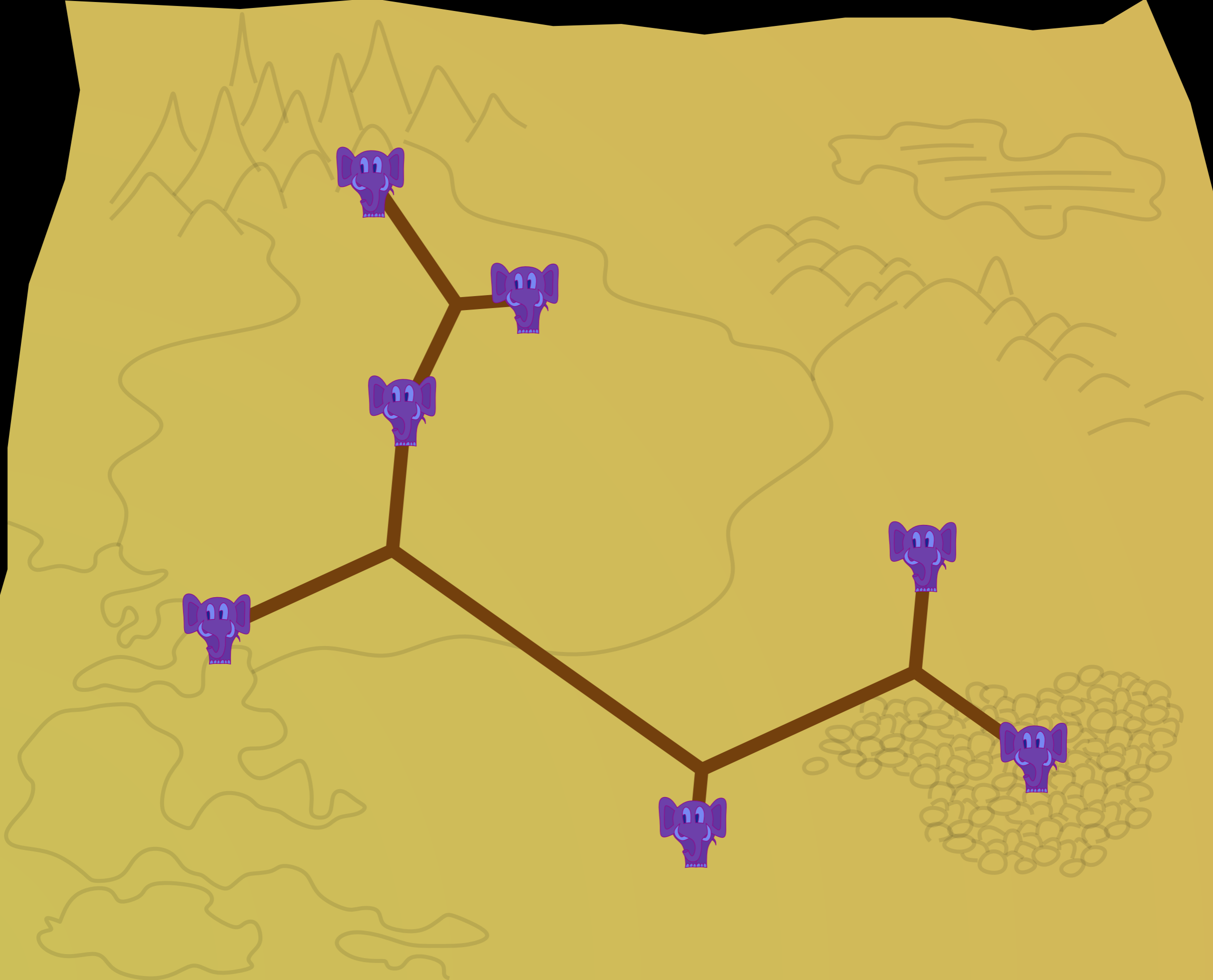
This algorithm gives a factor  $1 + \frac{1}{2}\rho$  approximation, where  $\rho$  is the *Steiner ratio*.

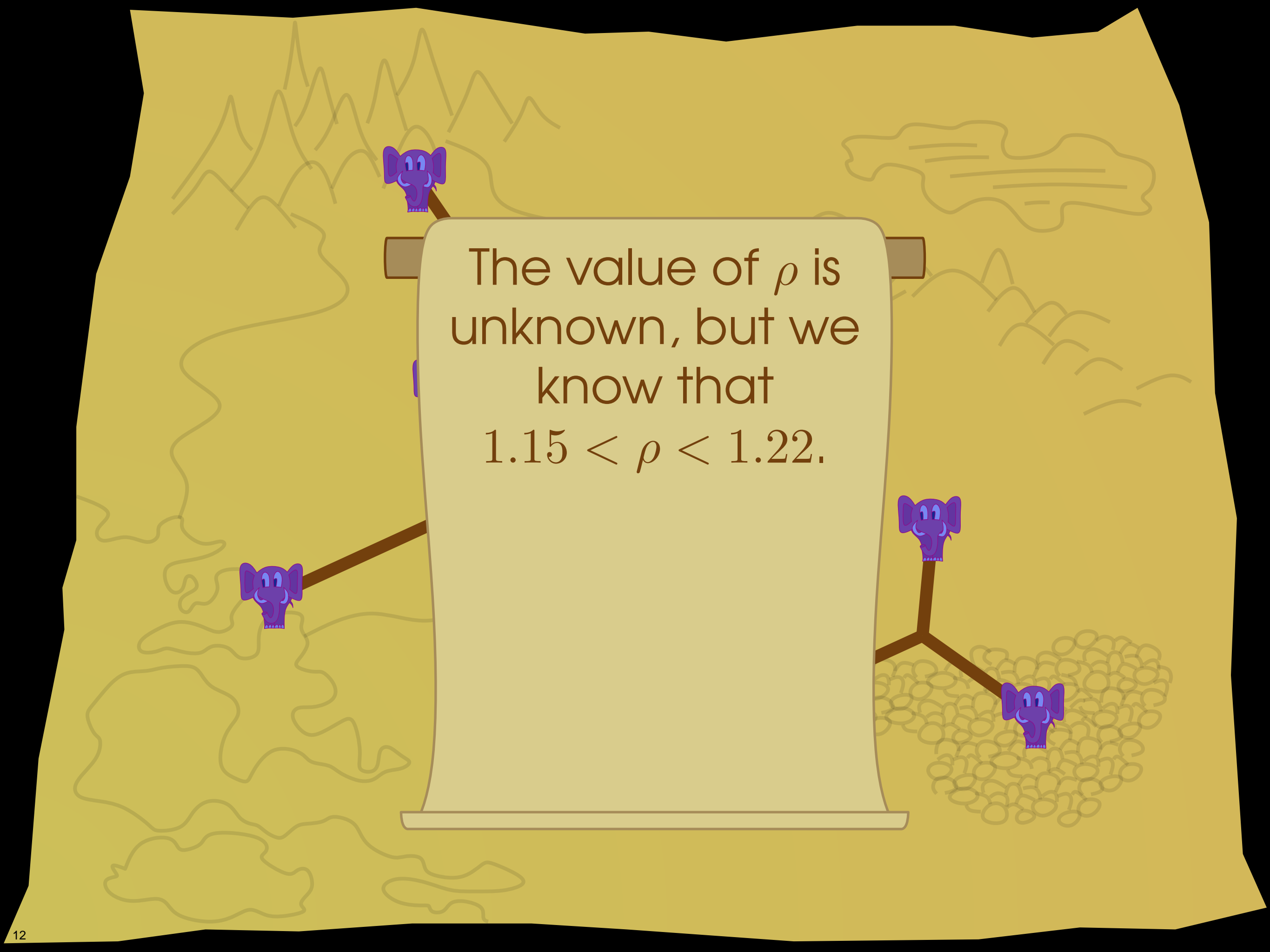


The *Steiner ratio*  $\rho$  is the maximum ratio between the weight of the MST and the minimum weight Steiner tree of a set of points.

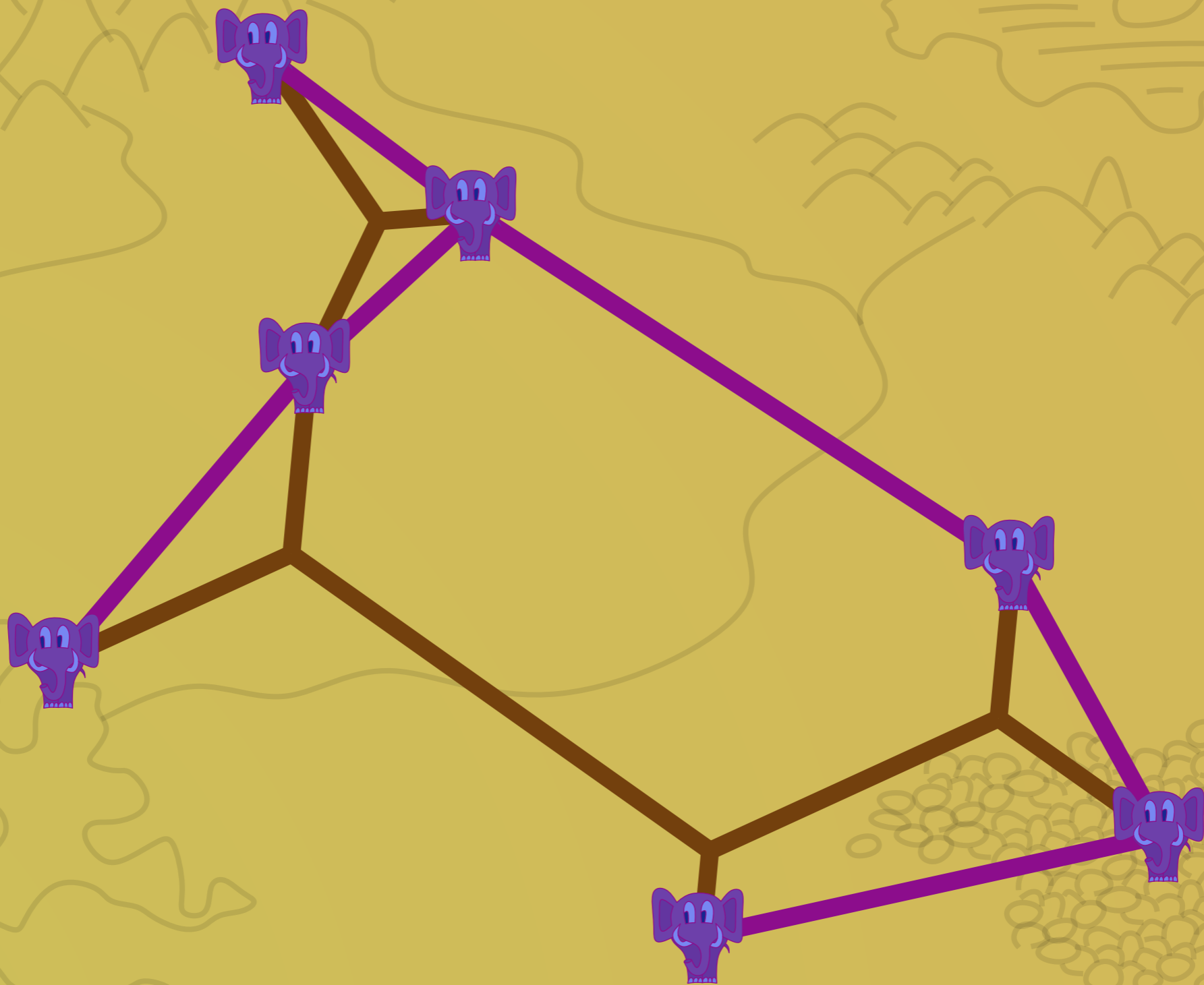


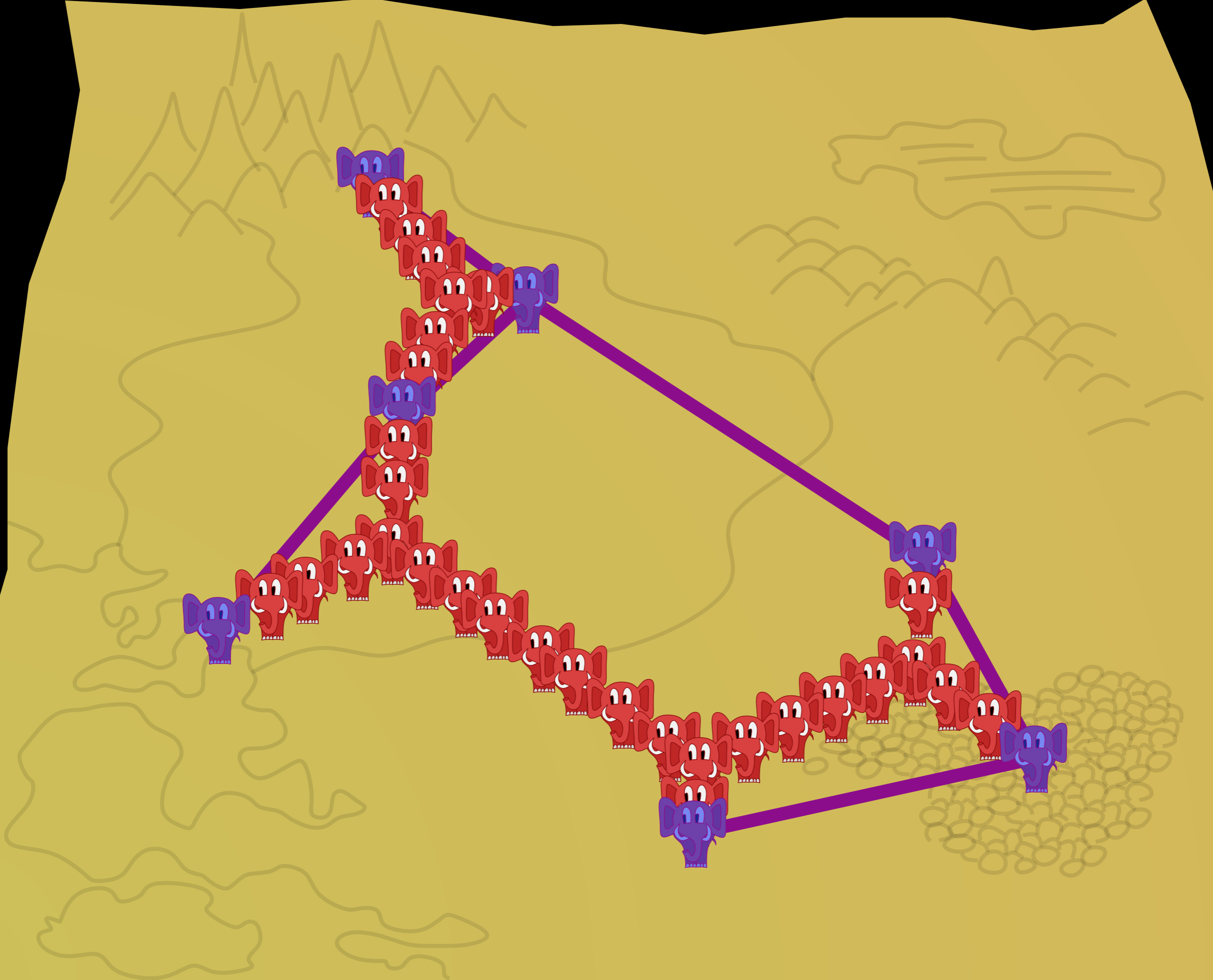






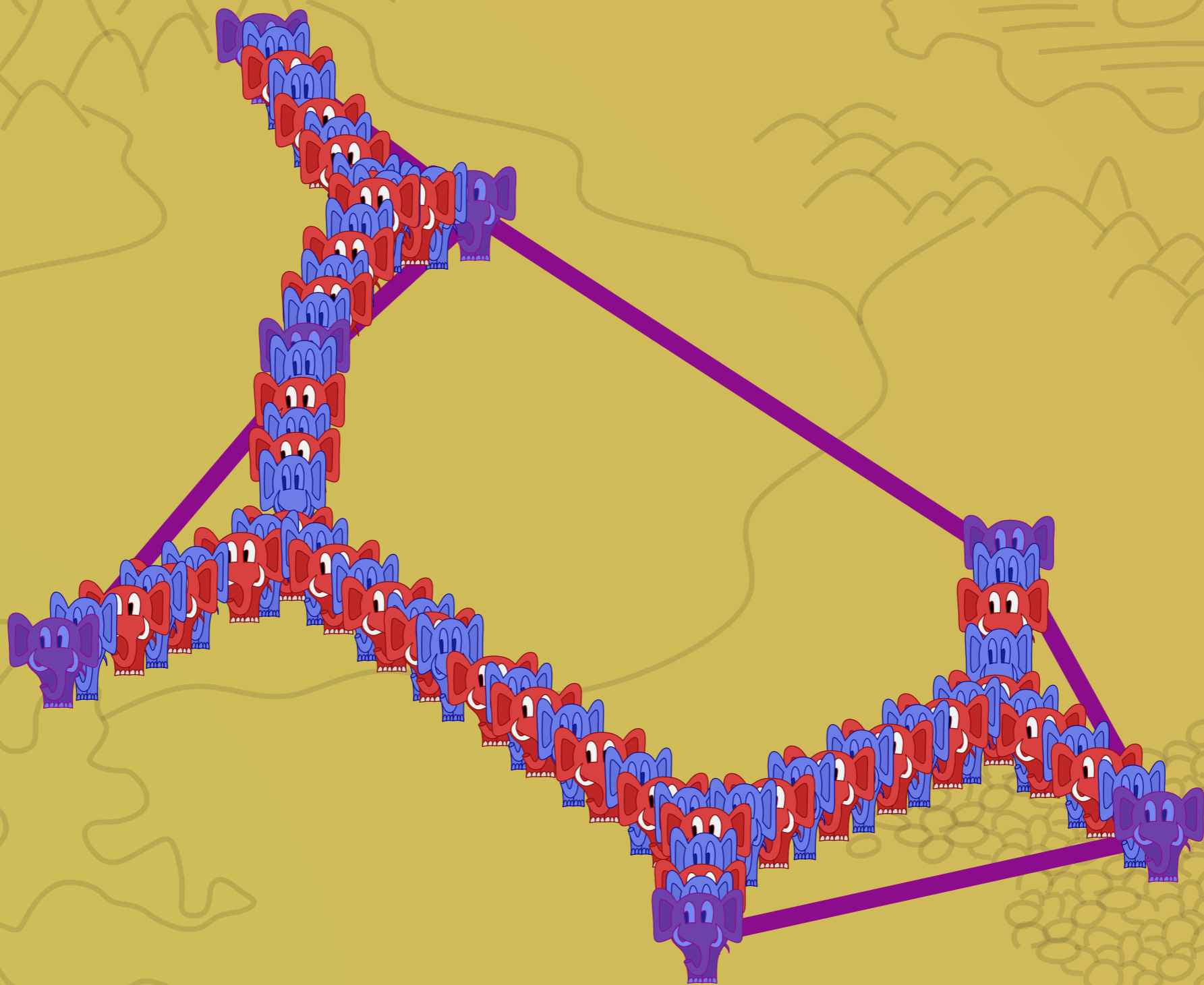
The value of  $\rho$  is unknown, but we know that  $1.15 < \rho < 1.22$ .

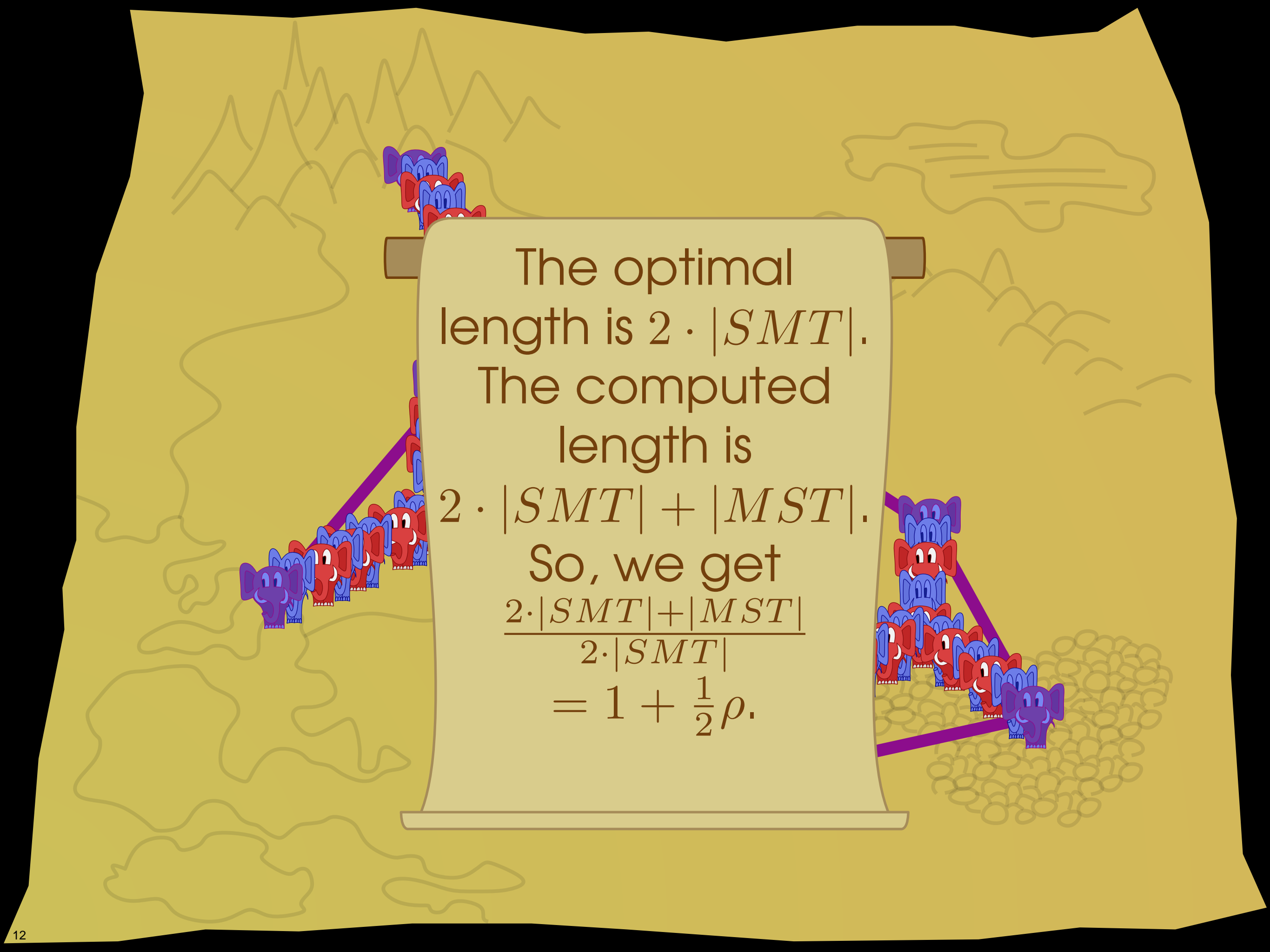










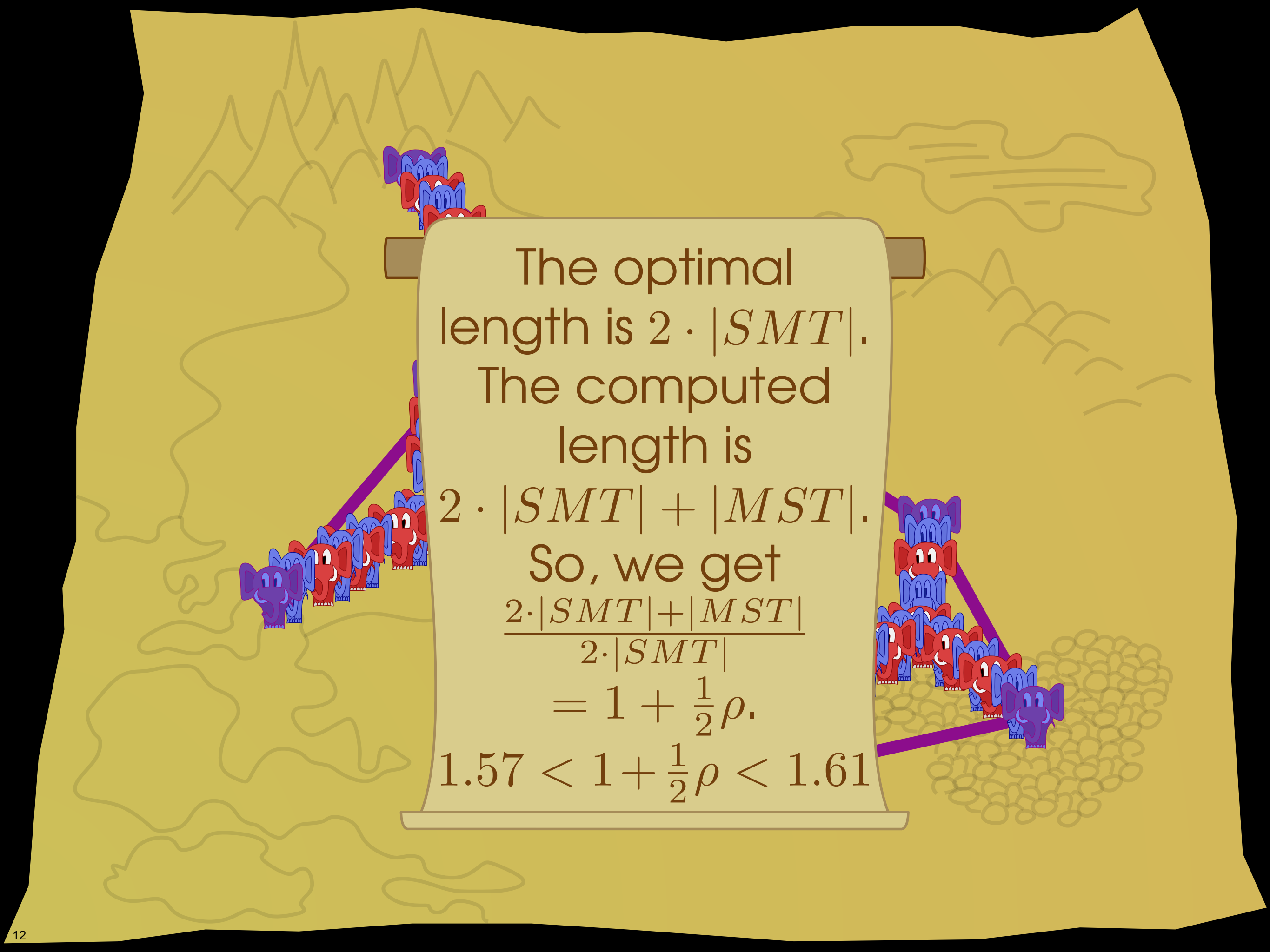


The optimal  
length is  $2 \cdot |SMT|$ .  
The computed  
length is

$$2 \cdot |SMT| + |MST|.$$

So, we get

$$\frac{2 \cdot |SMT| + |MST|}{2 \cdot |SMT|} \\ = 1 + \frac{1}{2} \rho.$$



The optimal  
length is  $2 \cdot |SMT|$ .  
The computed  
length is

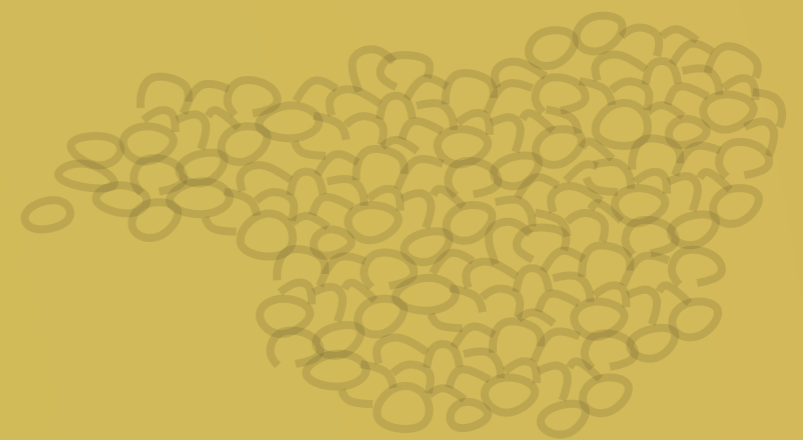
$$2 \cdot |SMT| + |MST|.$$


So, we get

$$\frac{2 \cdot |SMT| + |MST|}{2 \cdot |SMT|}$$
$$= 1 + \frac{1}{2} \rho.$$

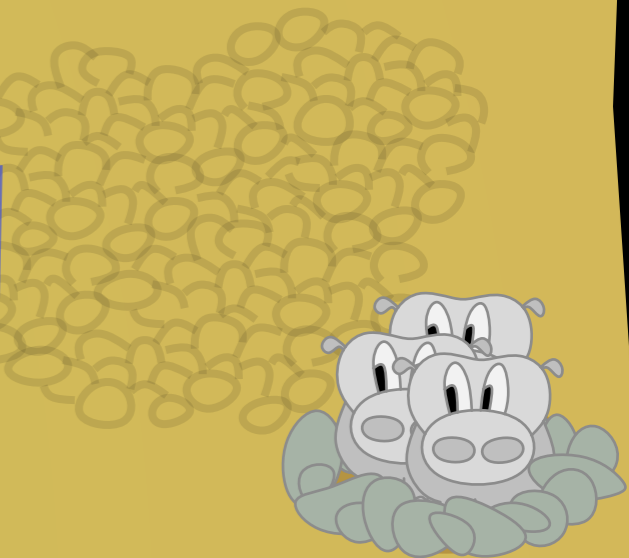
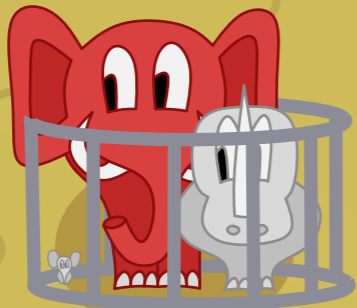
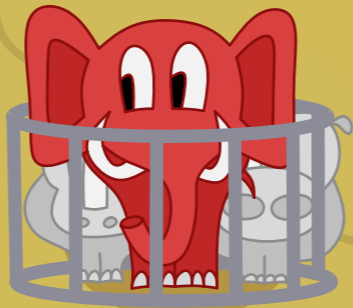
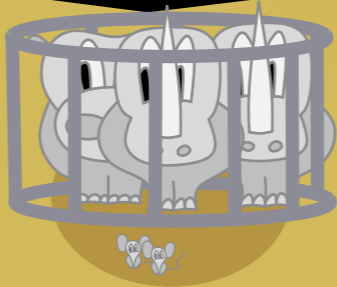
$$1.57 < 1 + \frac{1}{2} \rho < 1.61$$

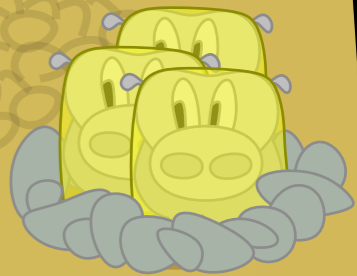
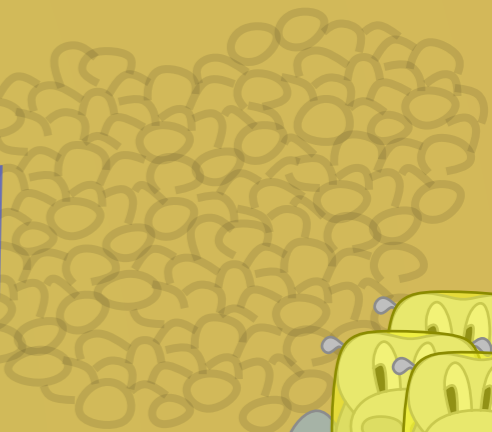
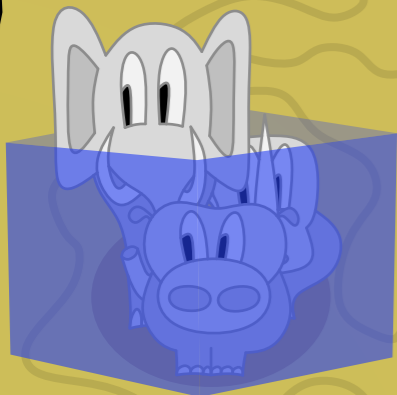
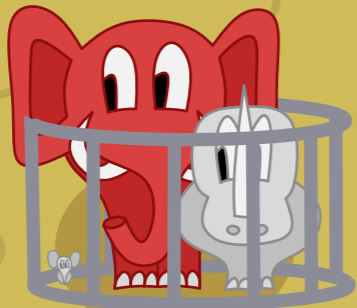
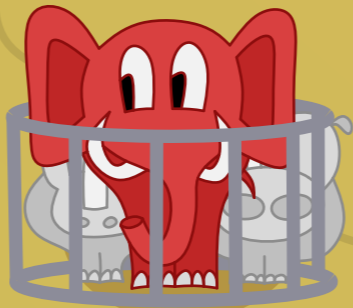
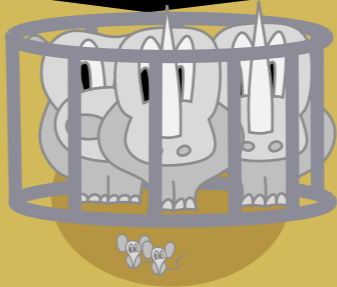
# EXTENSIONS



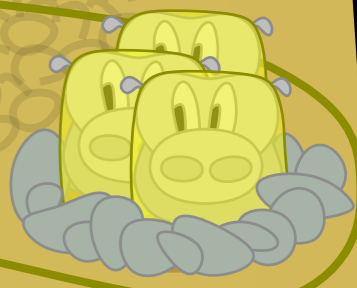
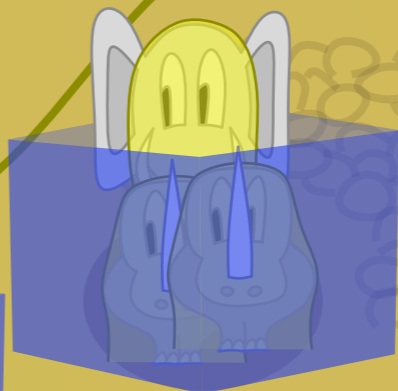
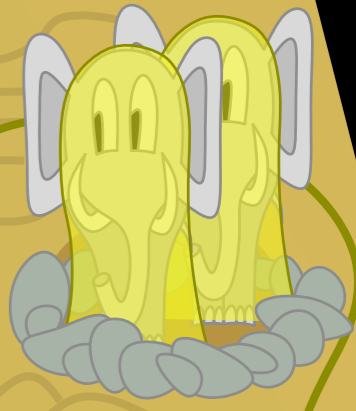
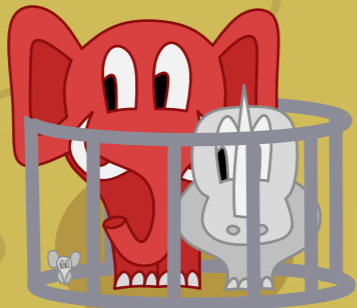
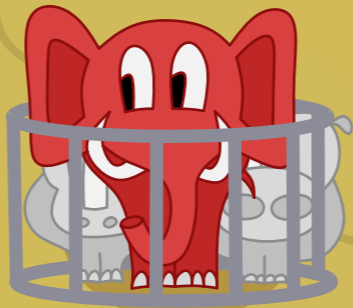
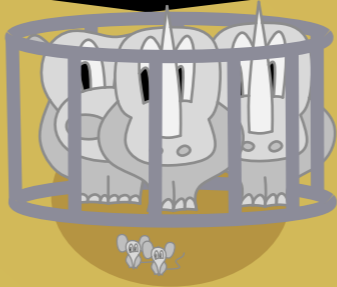


We may want to  
visualise more  
than two  
attributes at the  
same time.










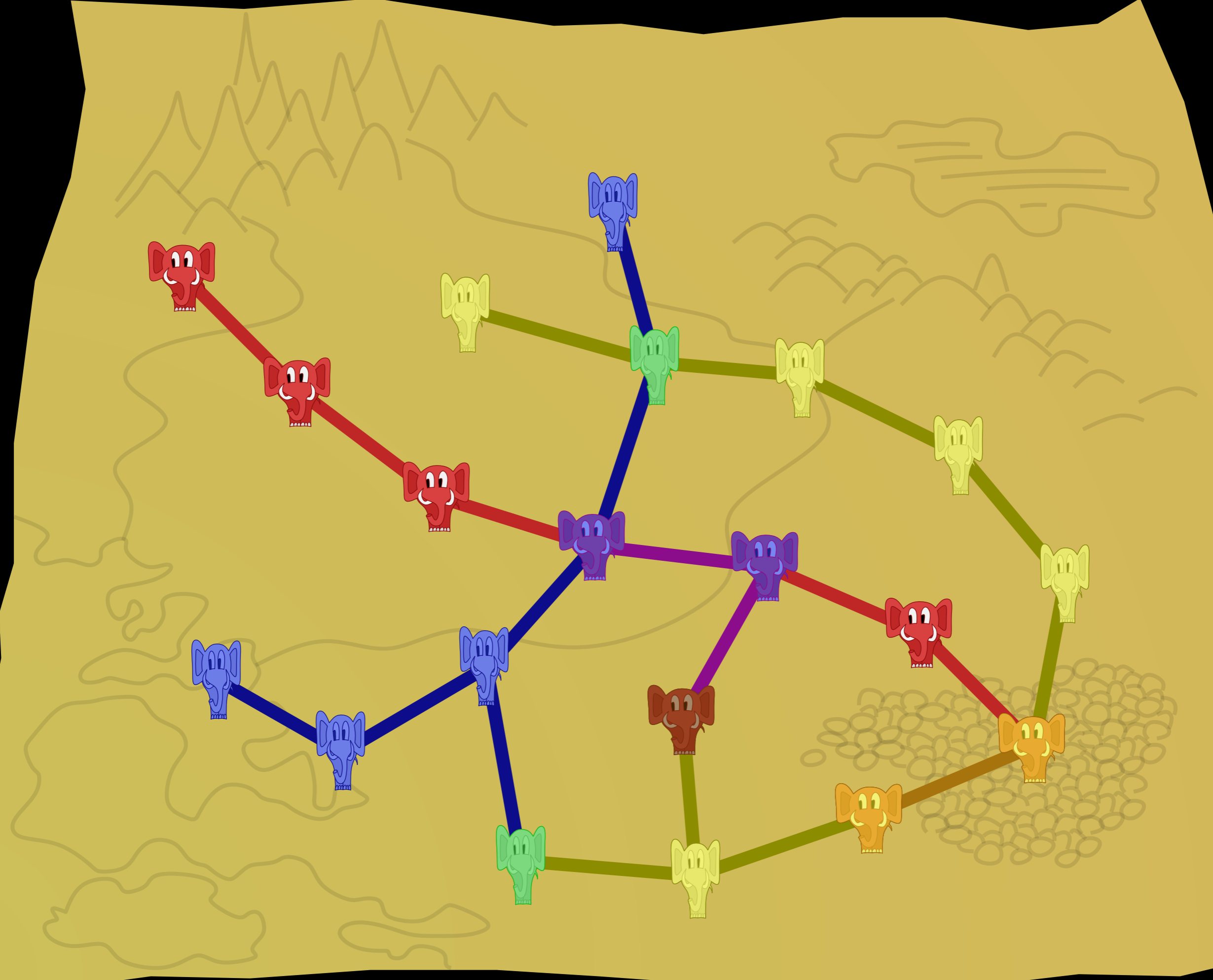



Now we have a set of points with three colours, say *red, blue* and *yellow*, and corresponding secondary and tertiary colours.



The image features a central scroll with text, set against a yellow background with faint outlines of mountains and a path. The scroll is held by two brown brackets on its left and right sides. The text on the scroll is in a brown, serif font. Surrounding the scroll are several cartoon elephant heads in various colors: red, blue, yellow, and orange. One blue elephant head is positioned at the top center of the scroll. A red elephant head is on the left side, and another red one is below it. A blue elephant head is on the left side, and another blue one is below it. A yellow elephant head is on the right side, and another yellow one is below it. An orange elephant head is on the right side, and another orange one is below it. A green elephant head is at the bottom center, and a yellow one is at the bottom right. The background includes a path leading from the bottom left towards the top left, and a mountain range in the upper left corner. A large, irregular shape resembling a cloud or a patch of ground is on the right side. The overall style is simple and colorful, typical of a children's educational book or presentation.

As before, we  
want to find the  
minimum weight  
graph that  
connects each  
primary colour.





Some results carry over (we can get a  $\lceil \frac{1}{2}k \rceil + \lfloor \frac{1}{2}k \rfloor \frac{1}{2}\rho$  approximation), but mostly the problem is open.

THANKS!

