COMPACTION GAMES

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Anika Rounds *Amazon*

Giovanni Viglietta *JAIST*

Trash Cubes designed by Mary Bogdan, 2008. Brooklyn Art Project.

GRID COMPACTION

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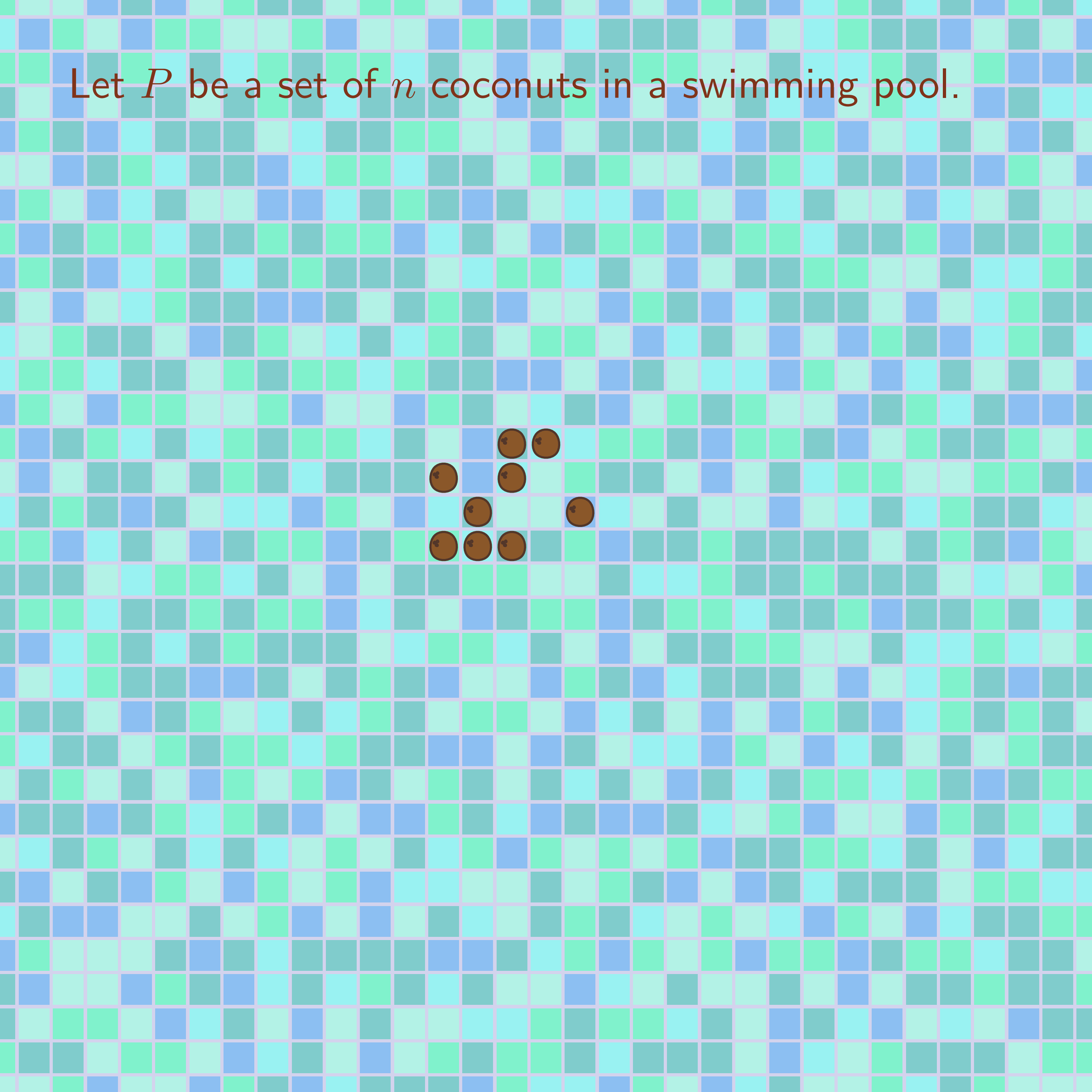

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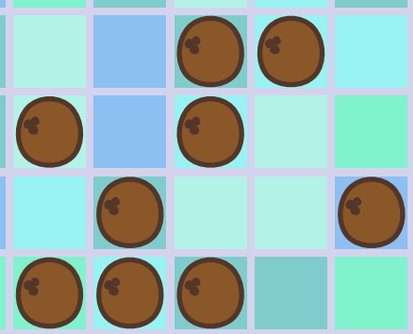

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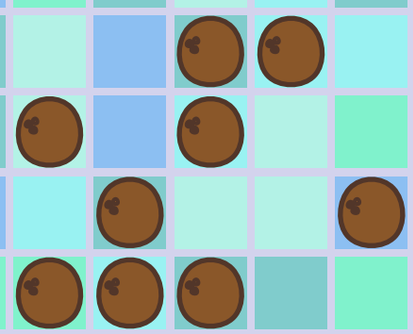

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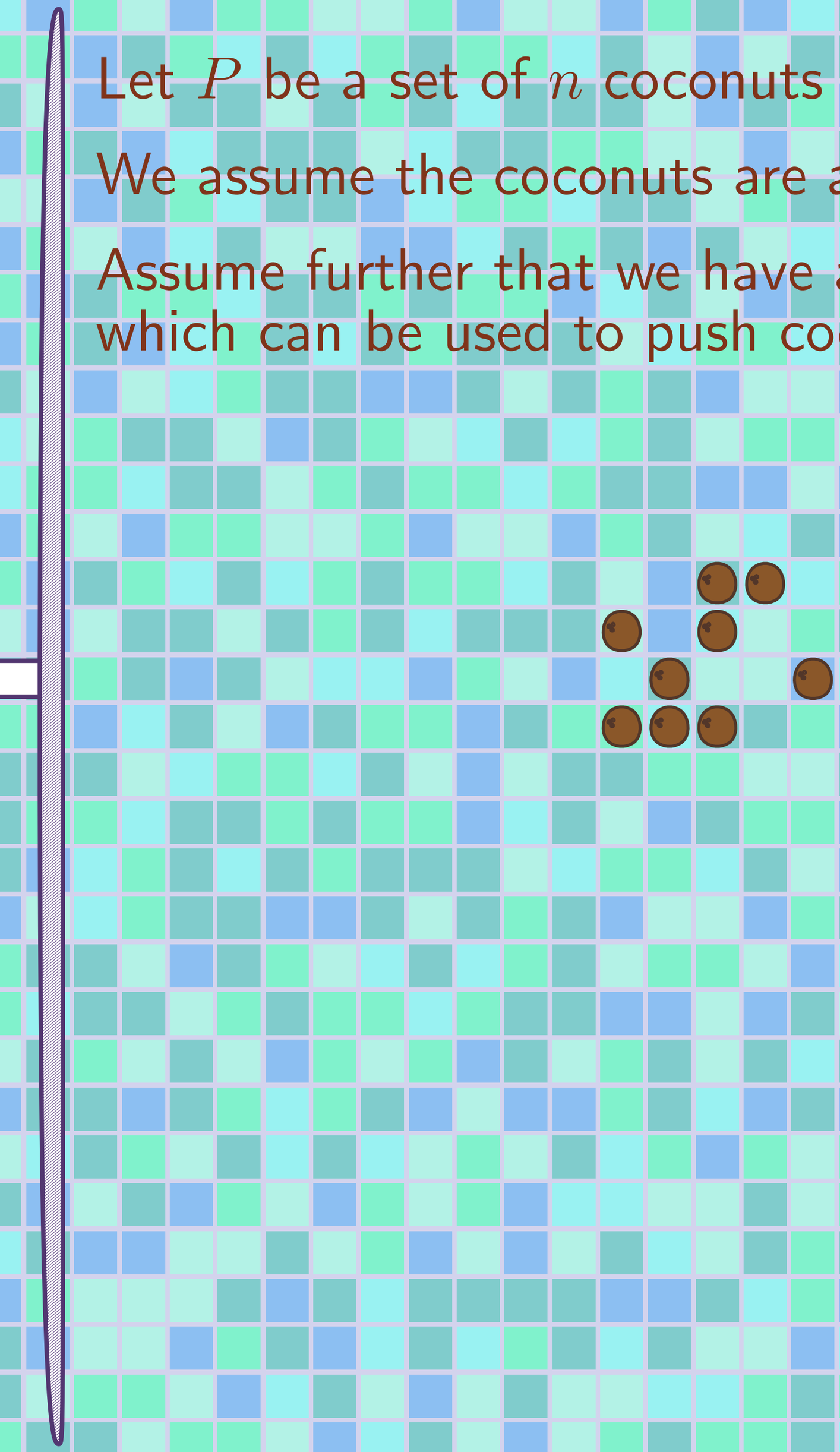

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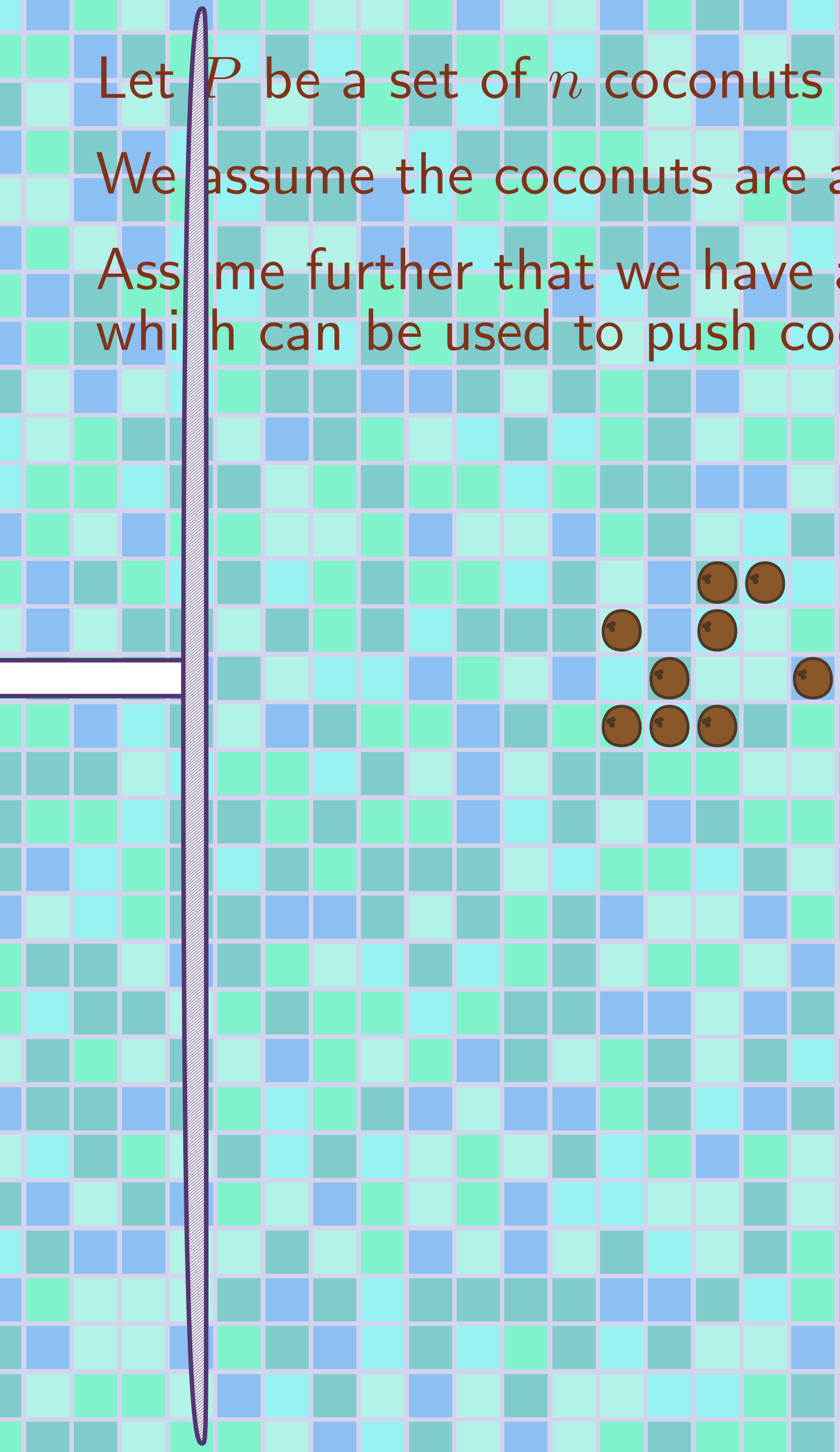

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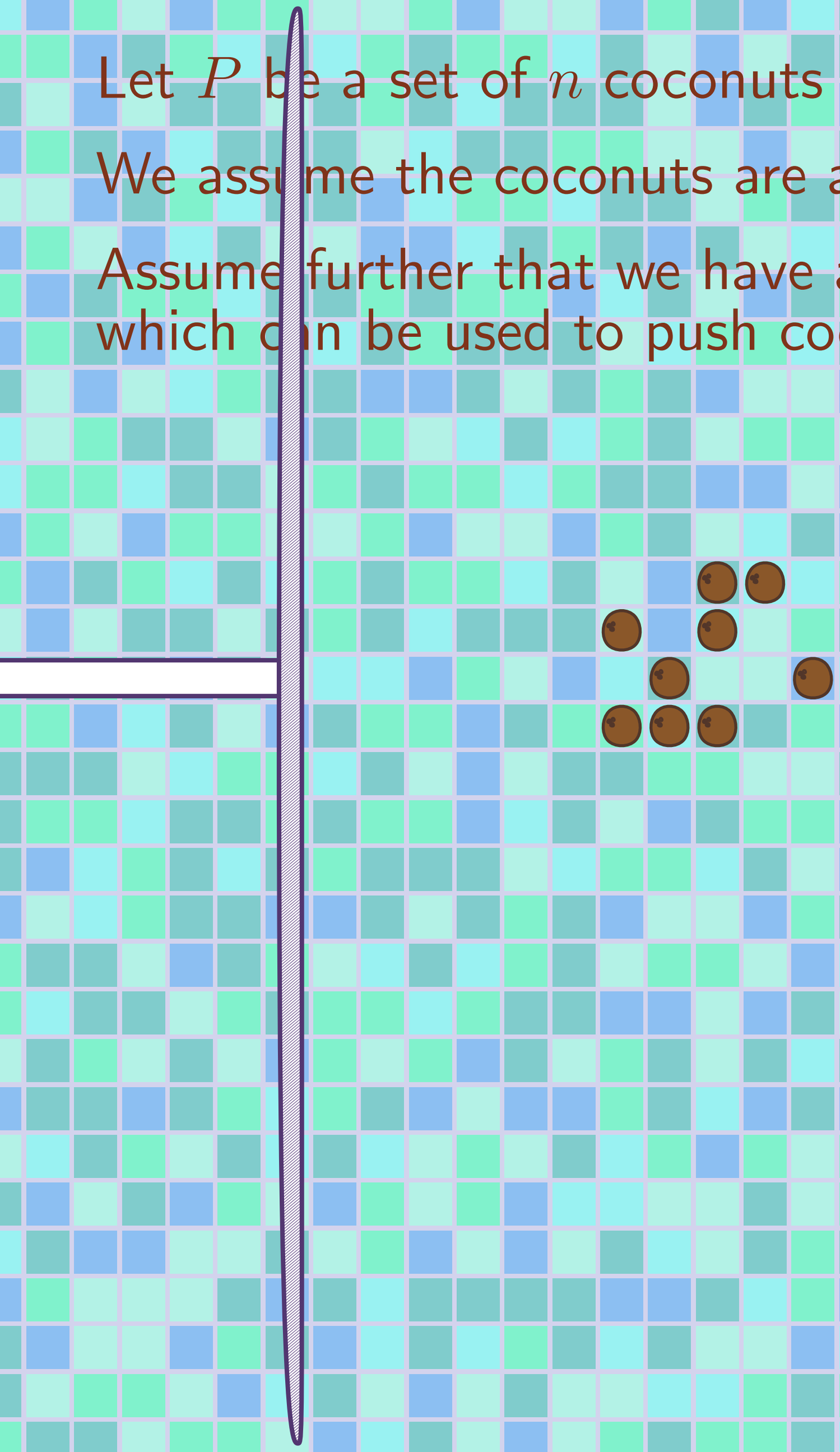

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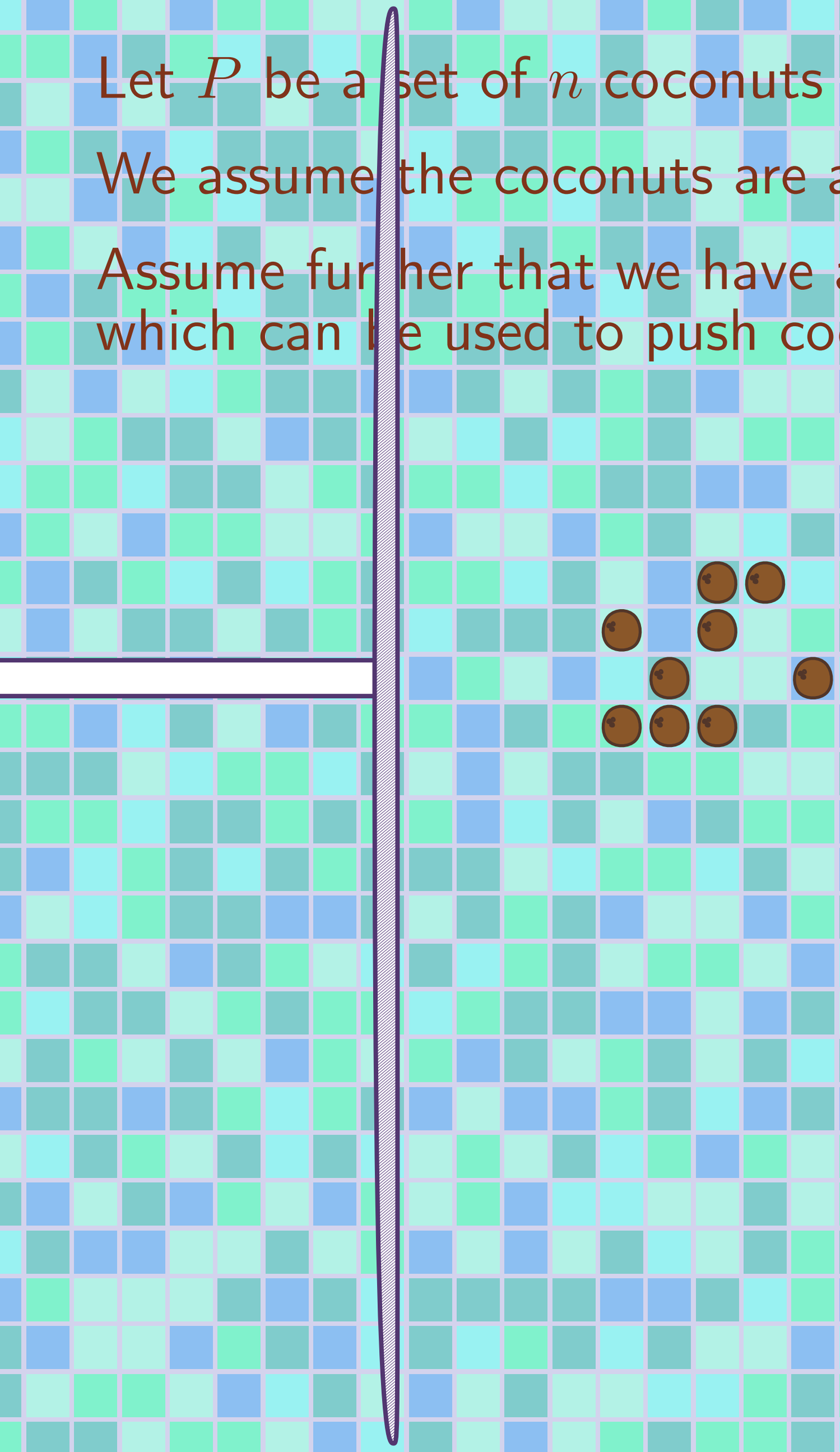

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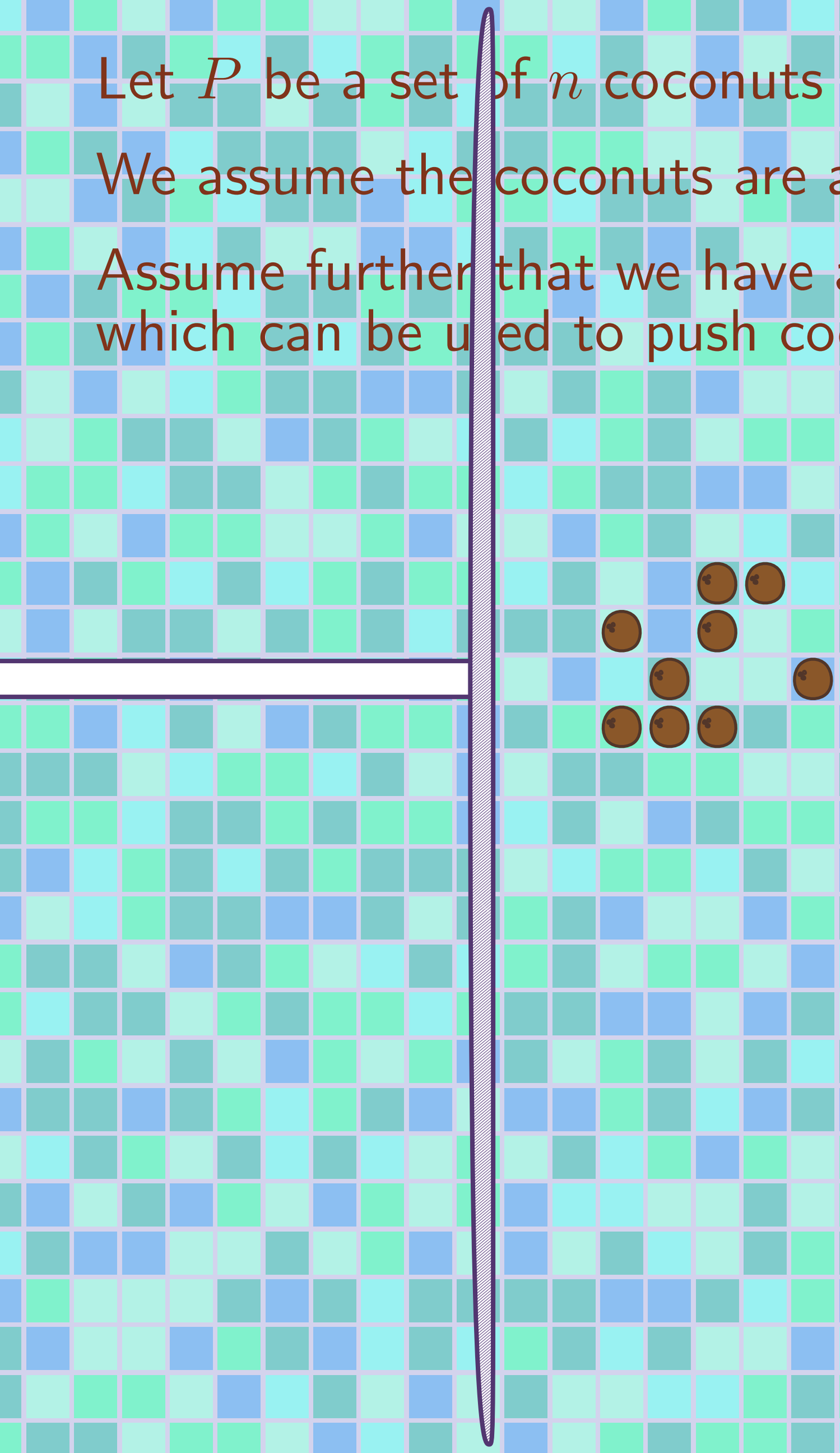

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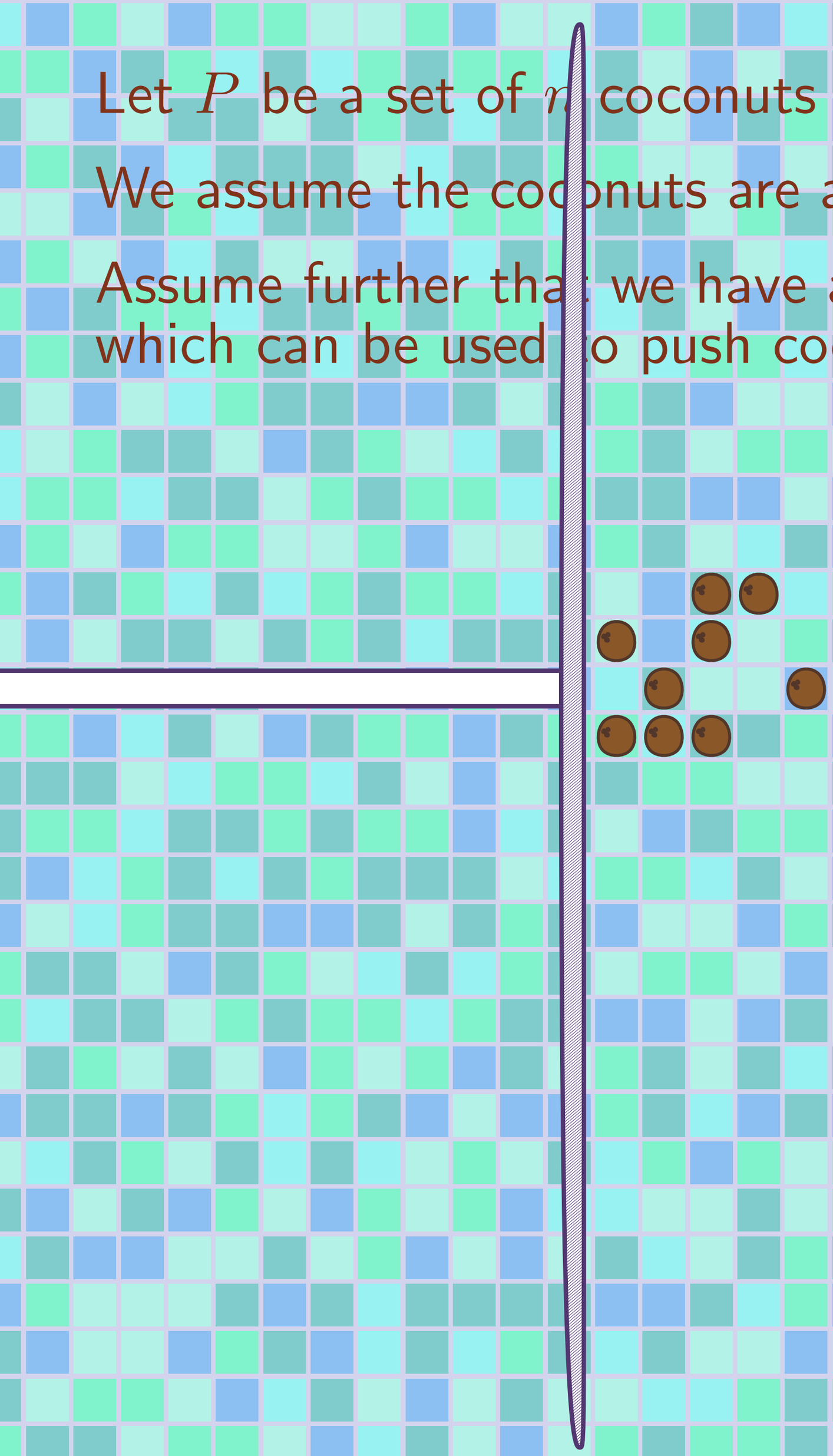

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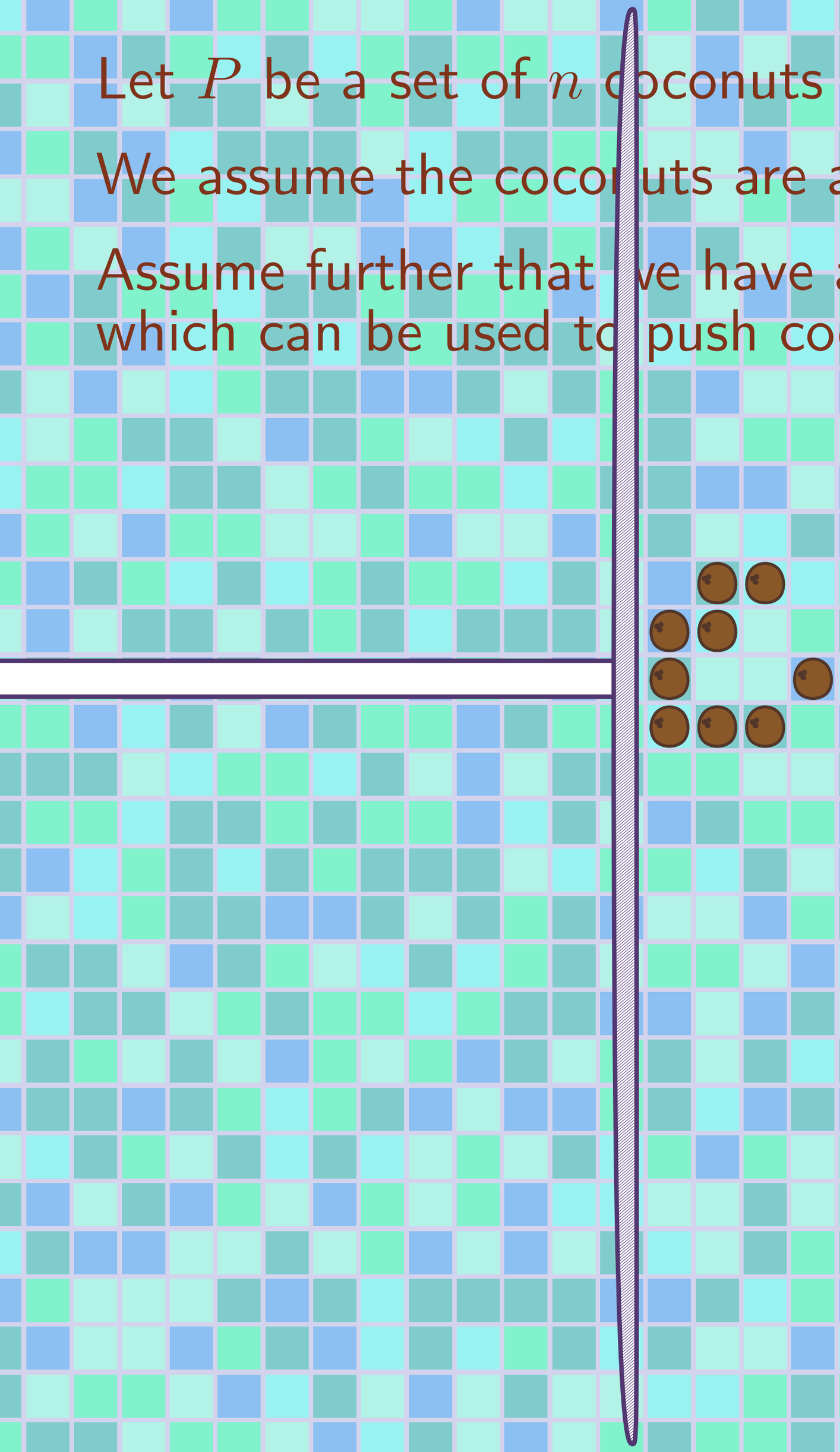

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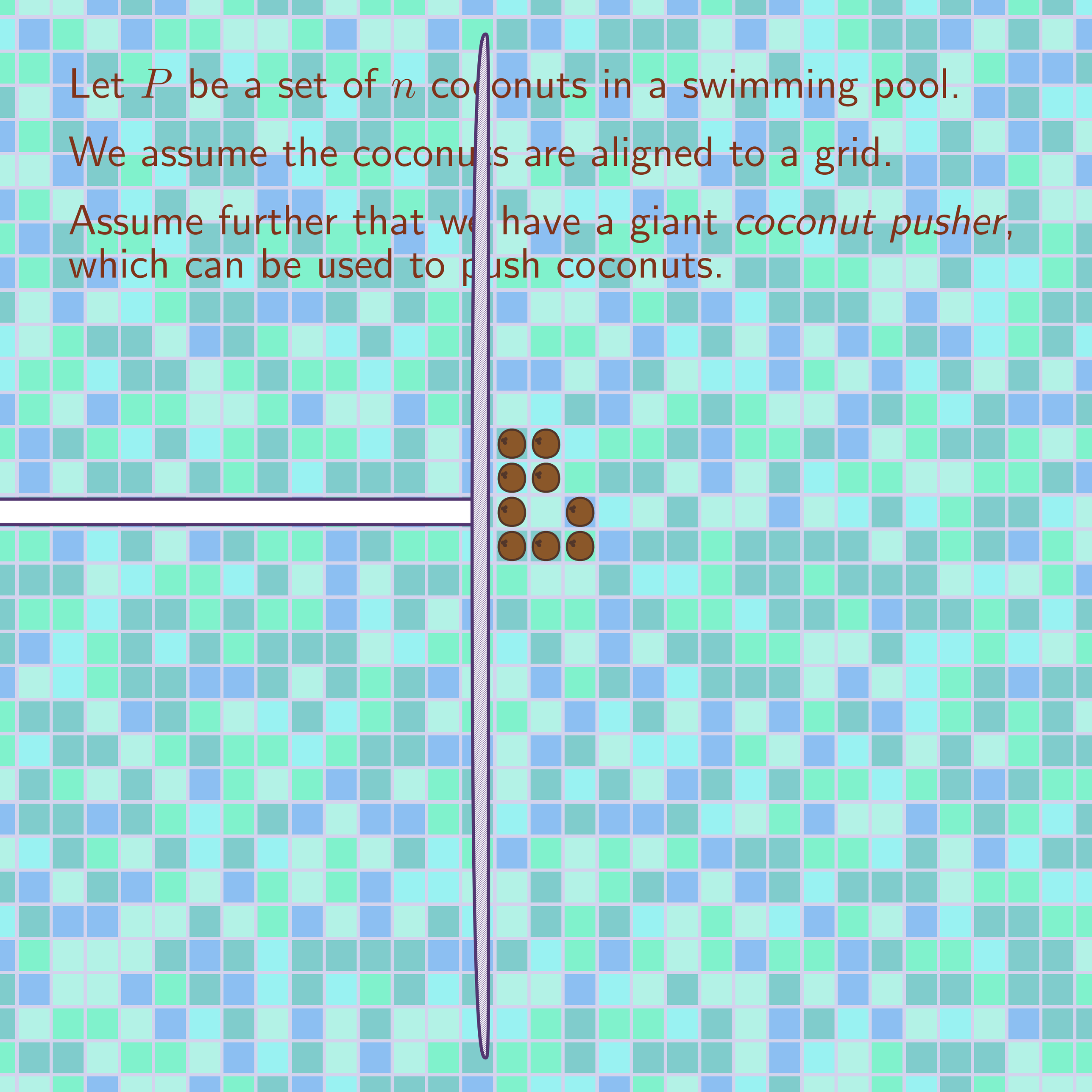

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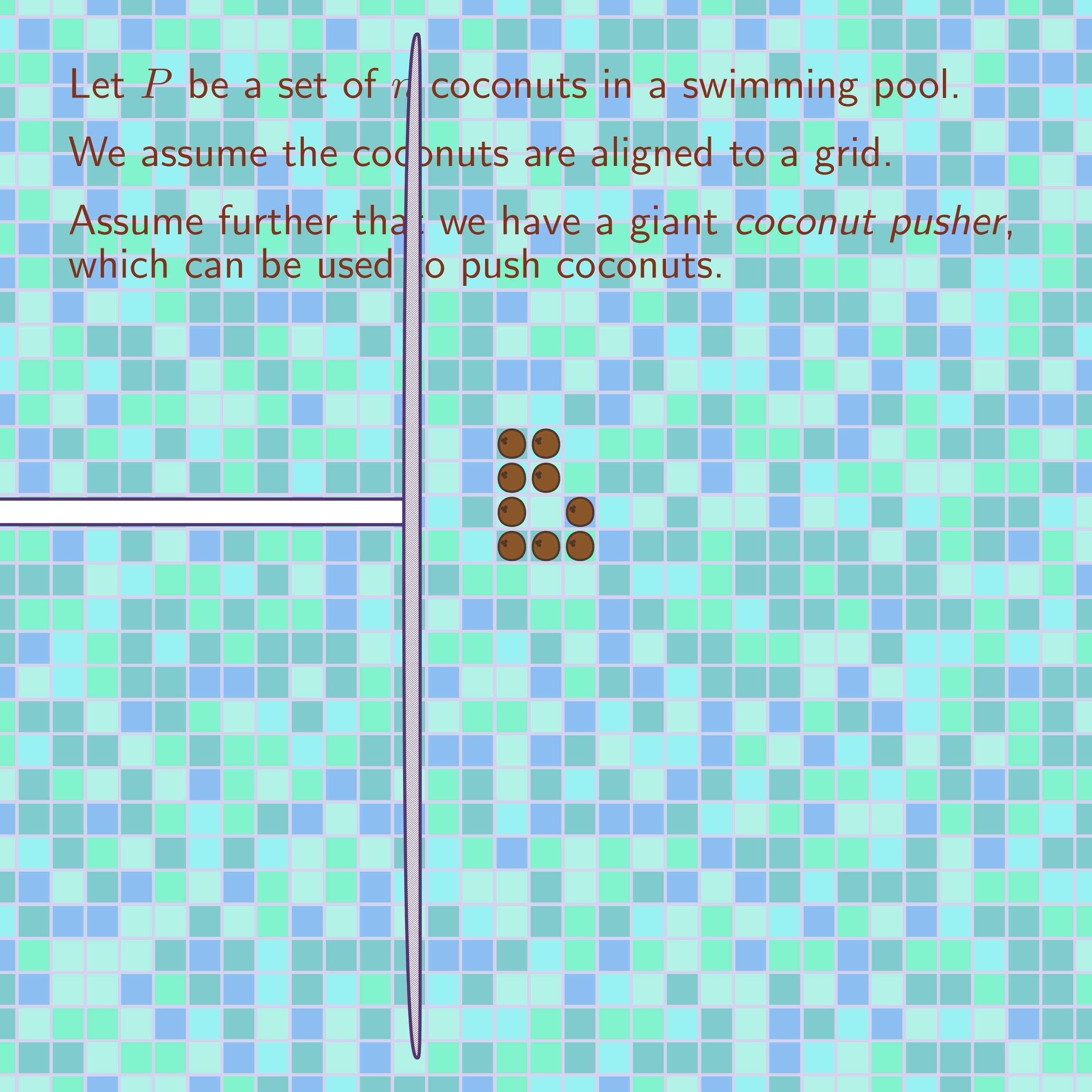

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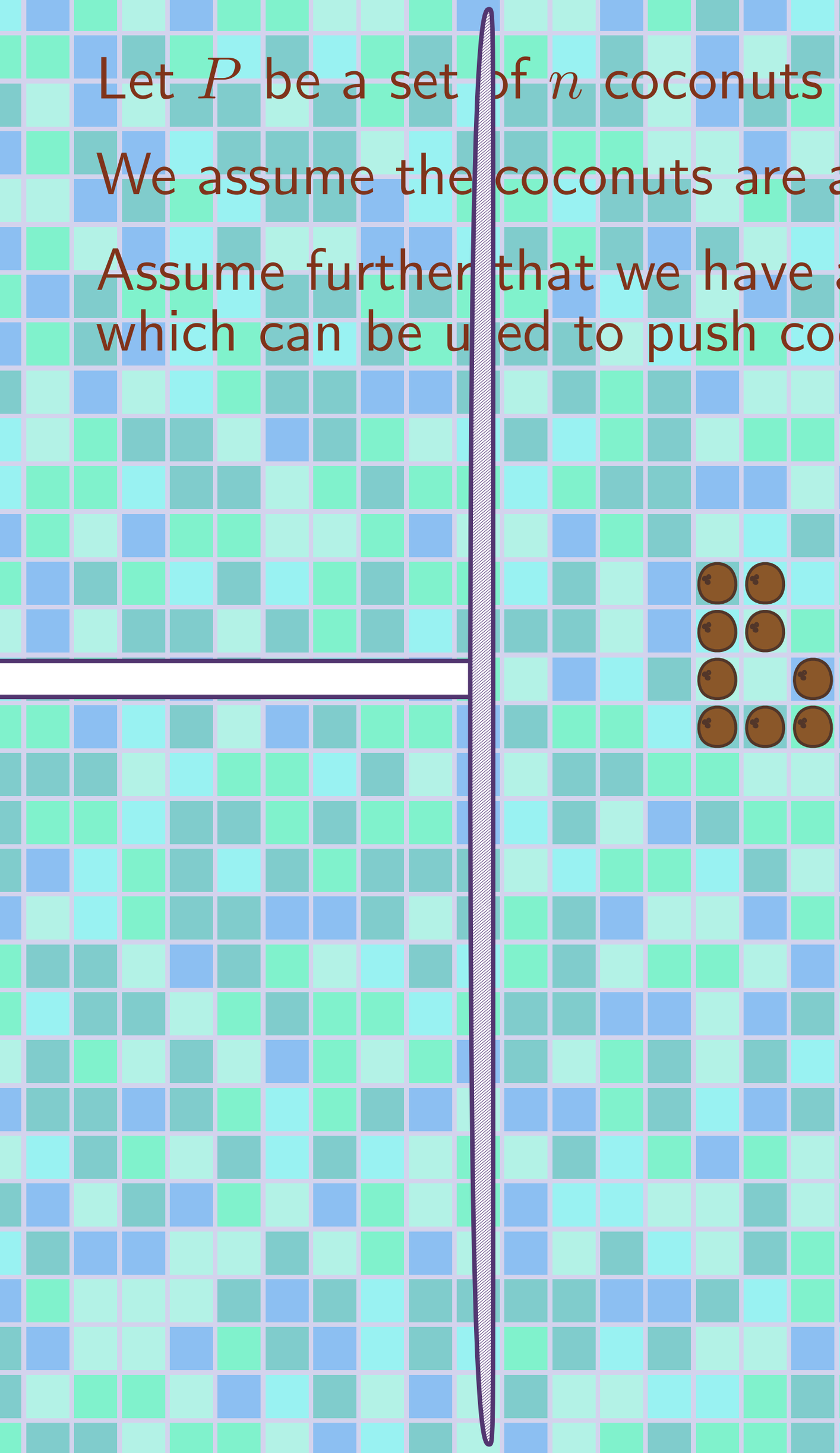

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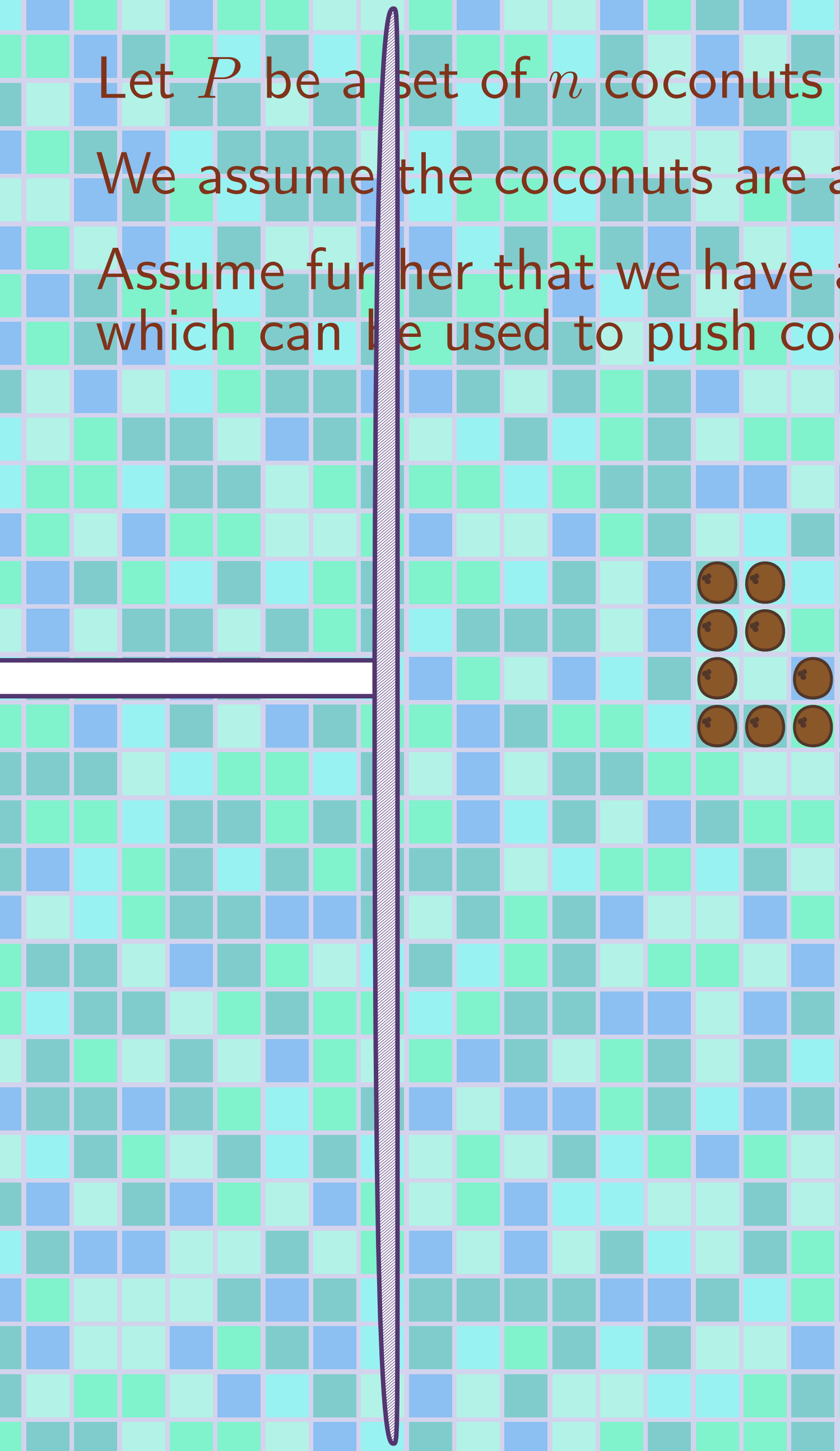

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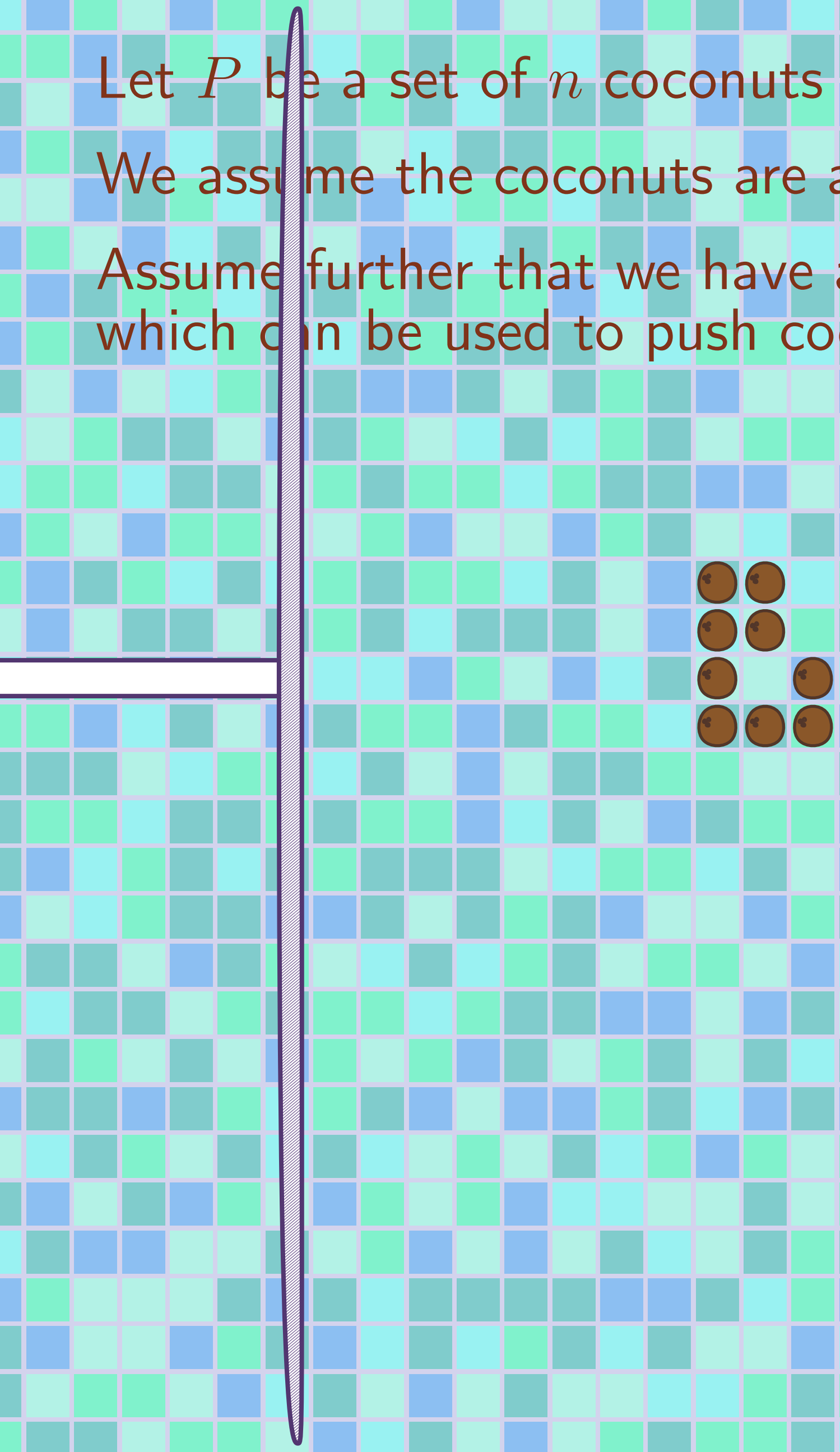

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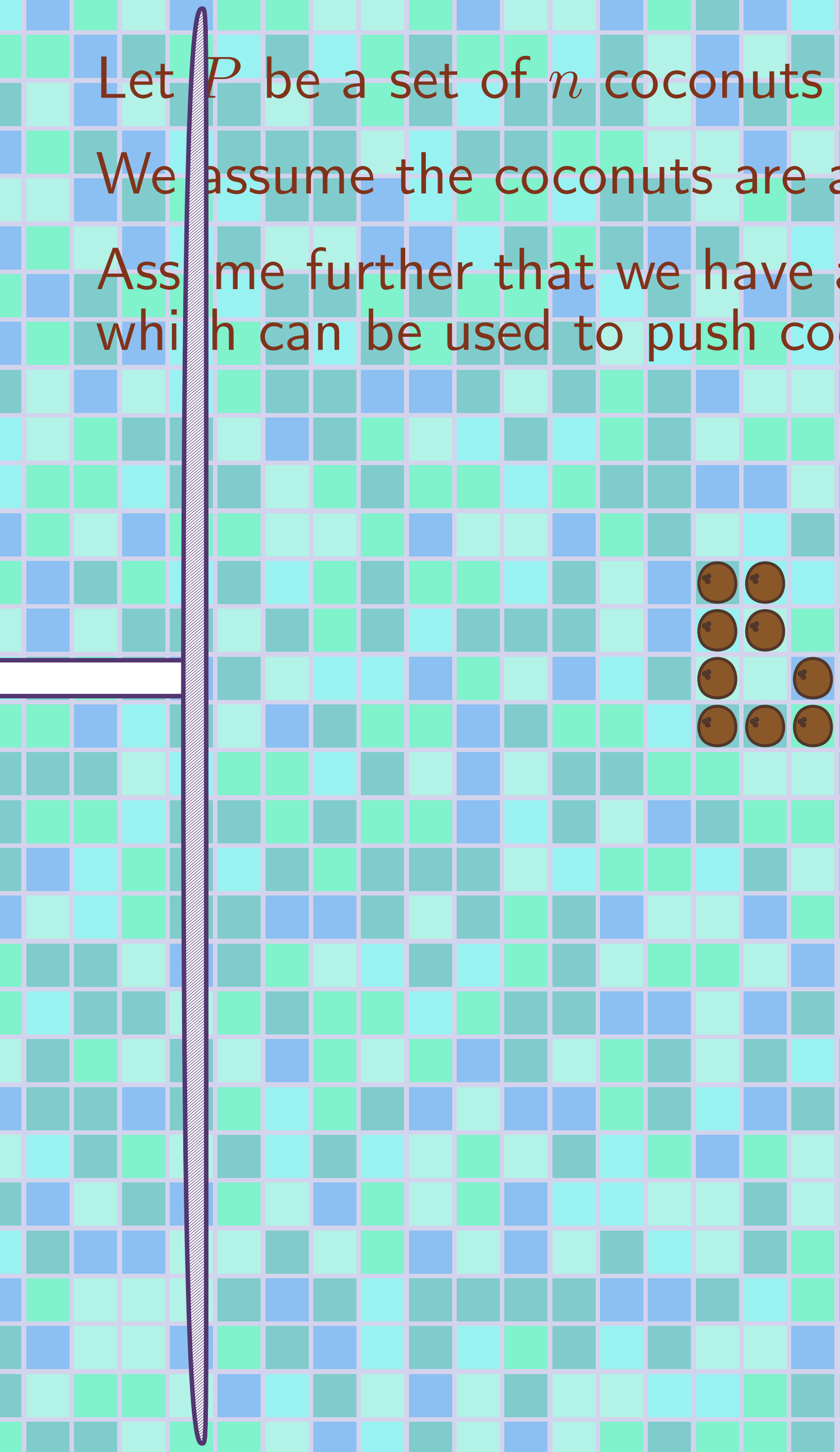

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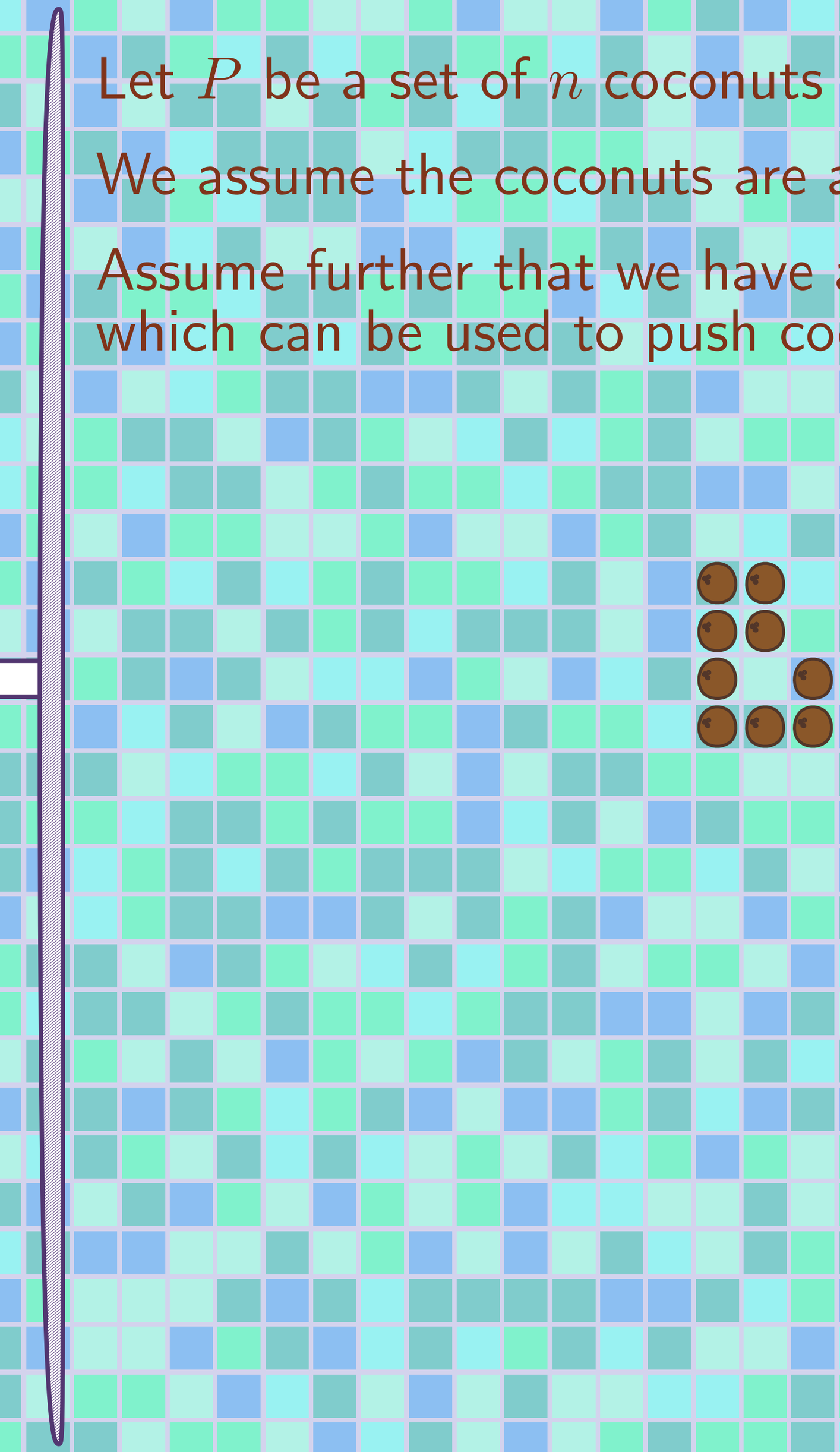

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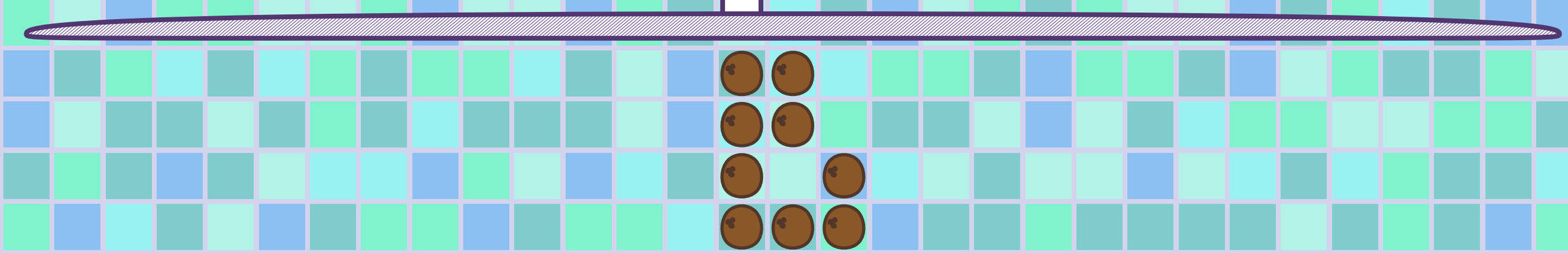

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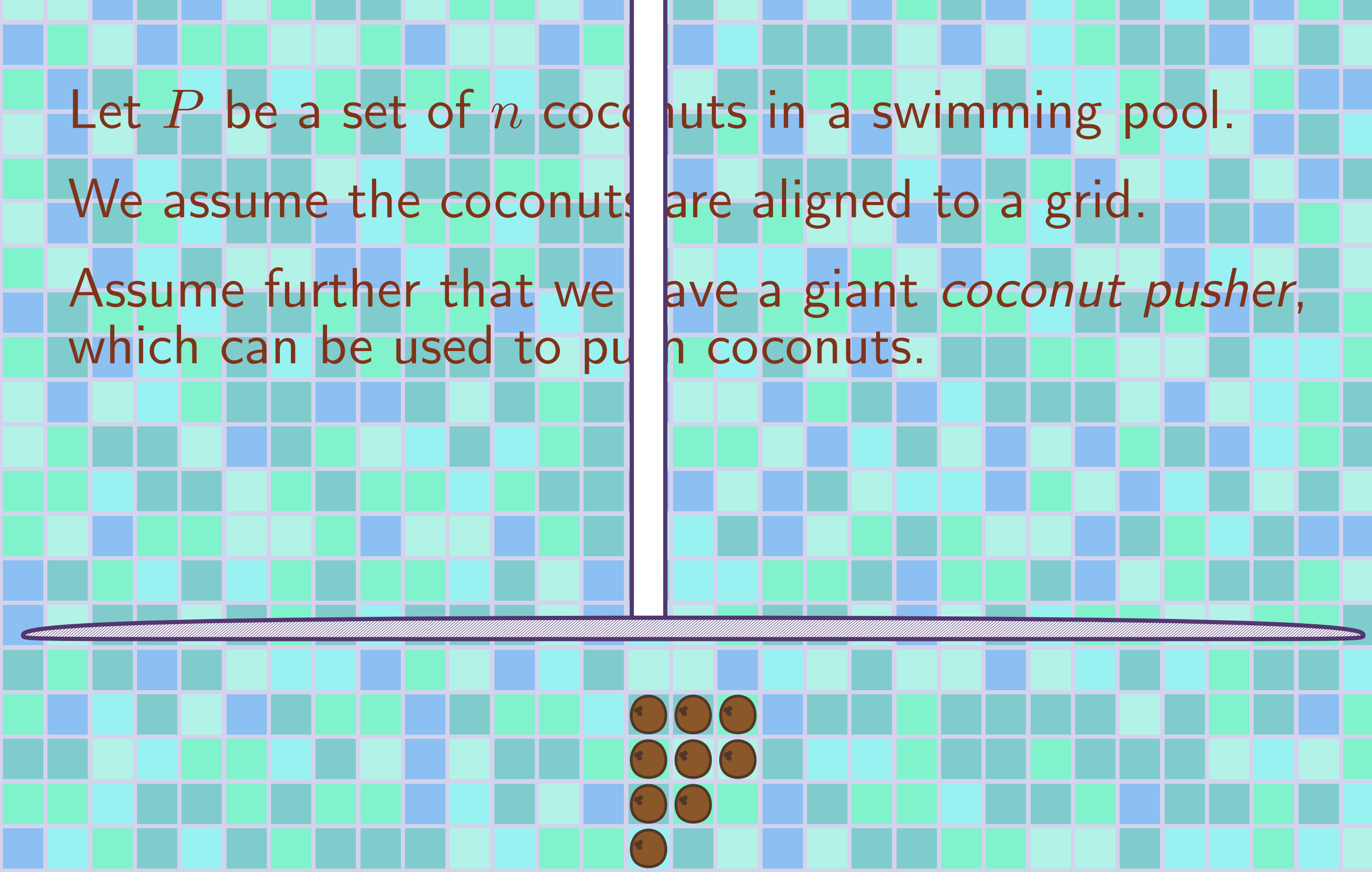

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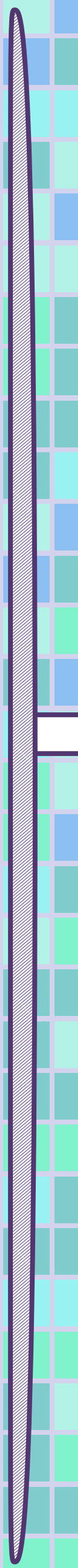

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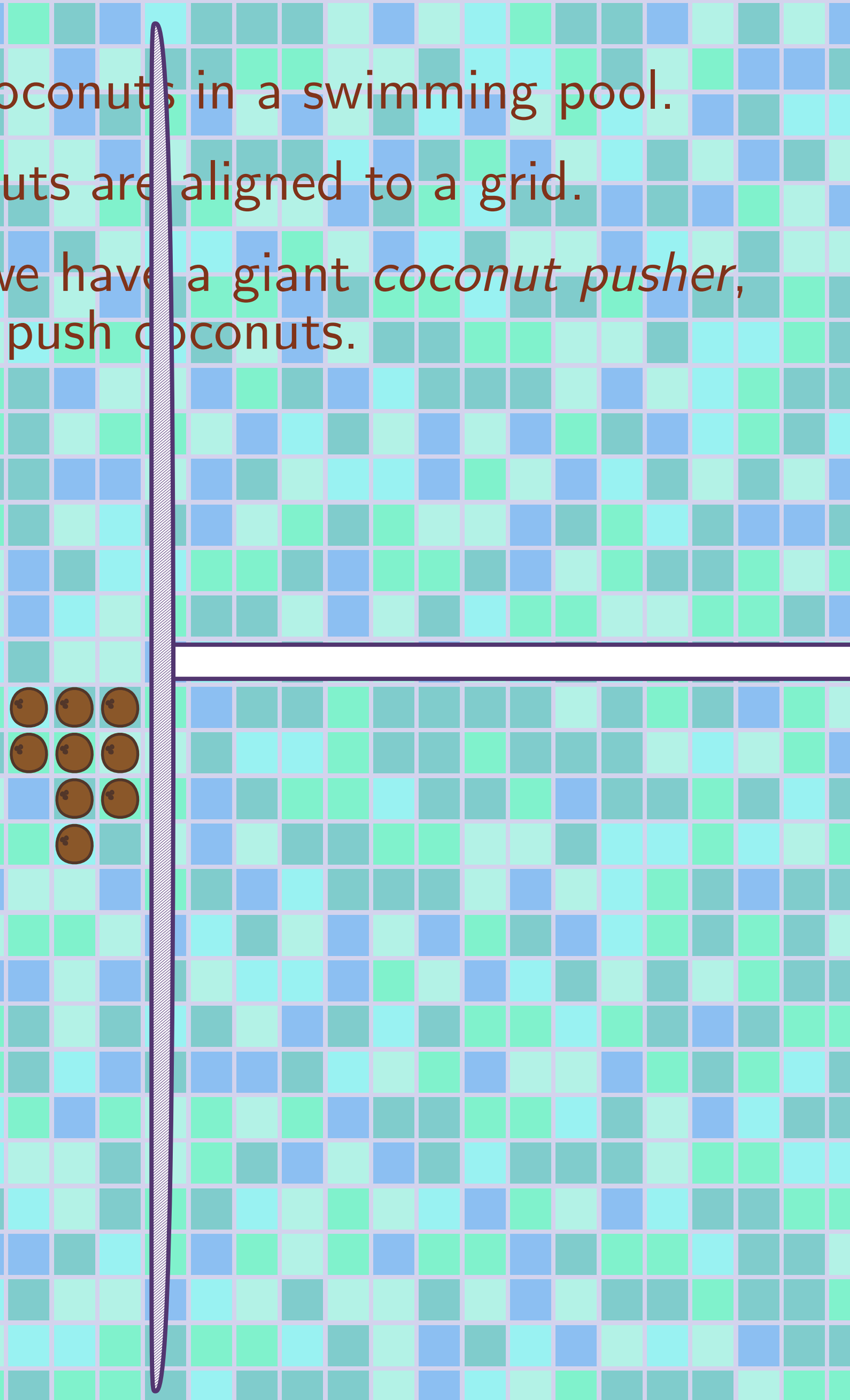

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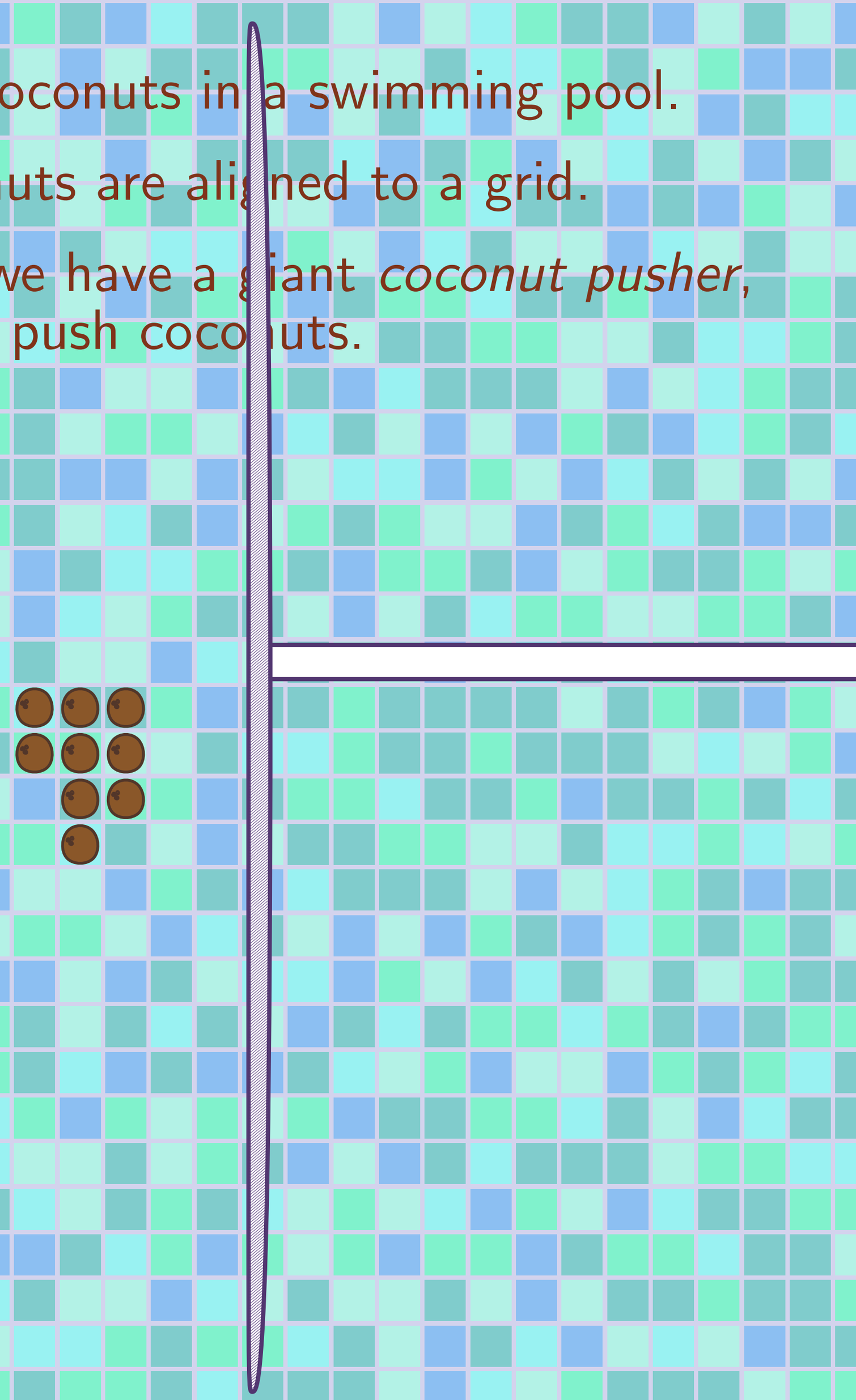

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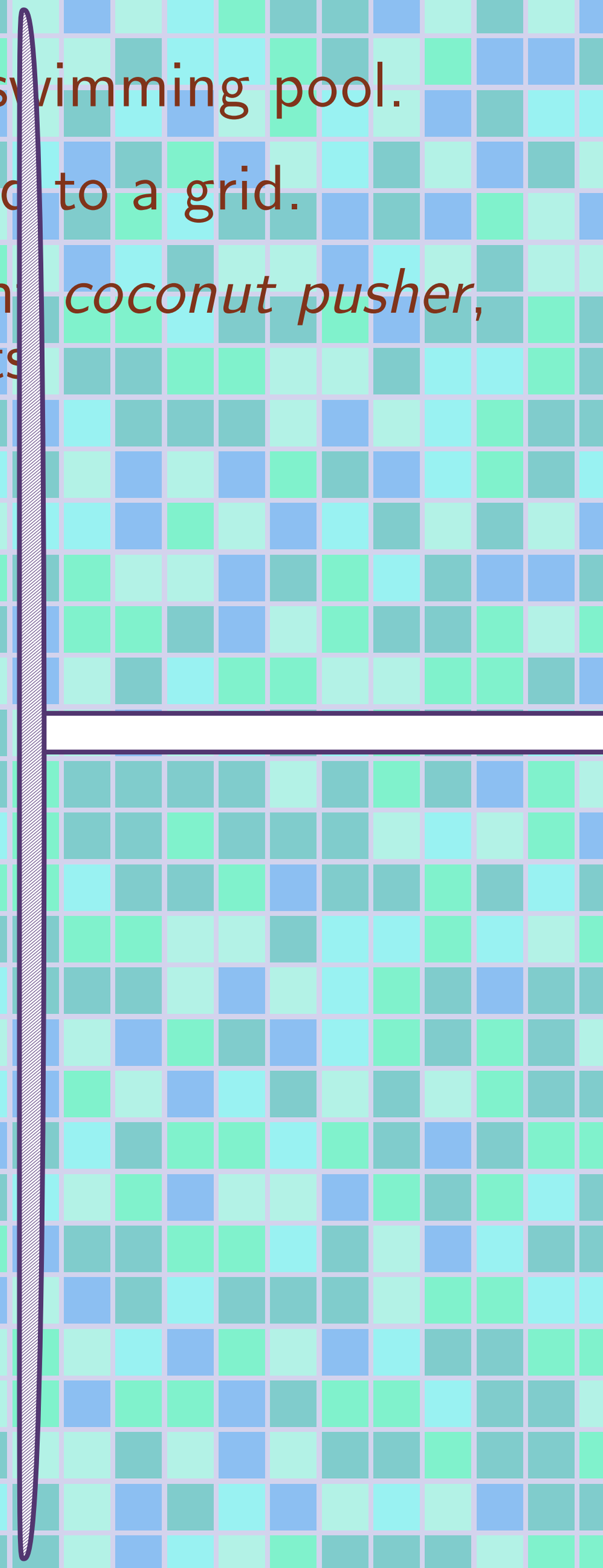

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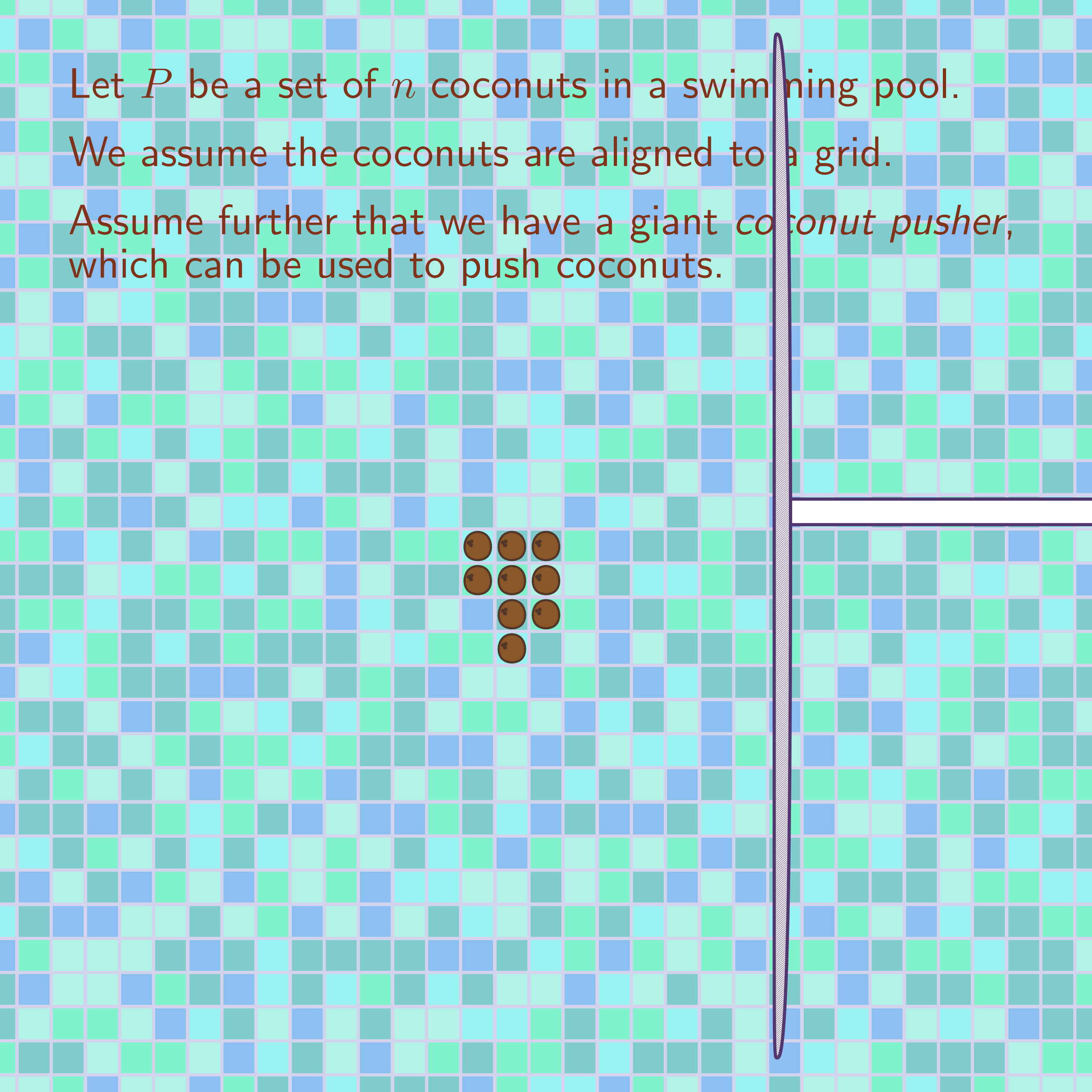

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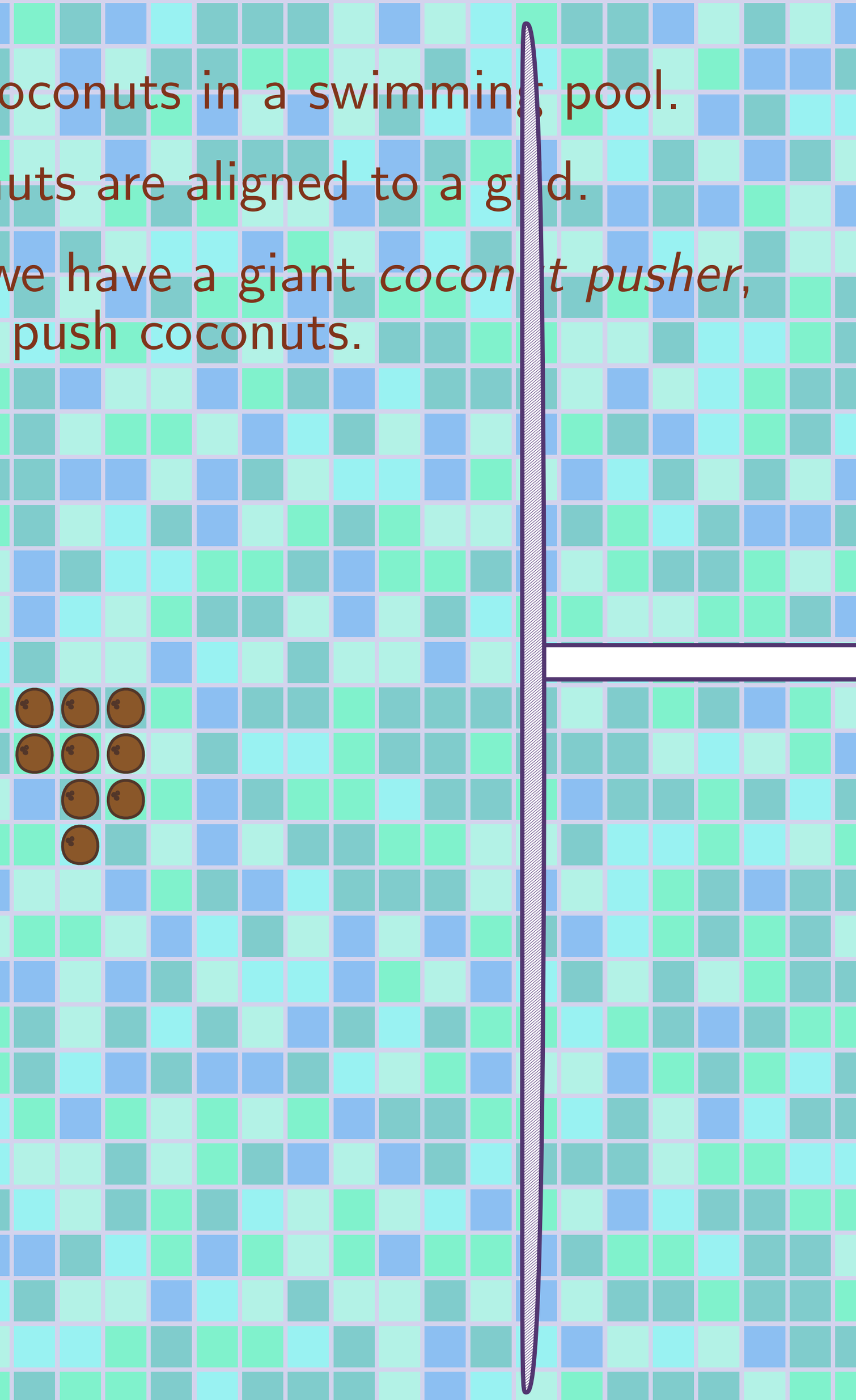

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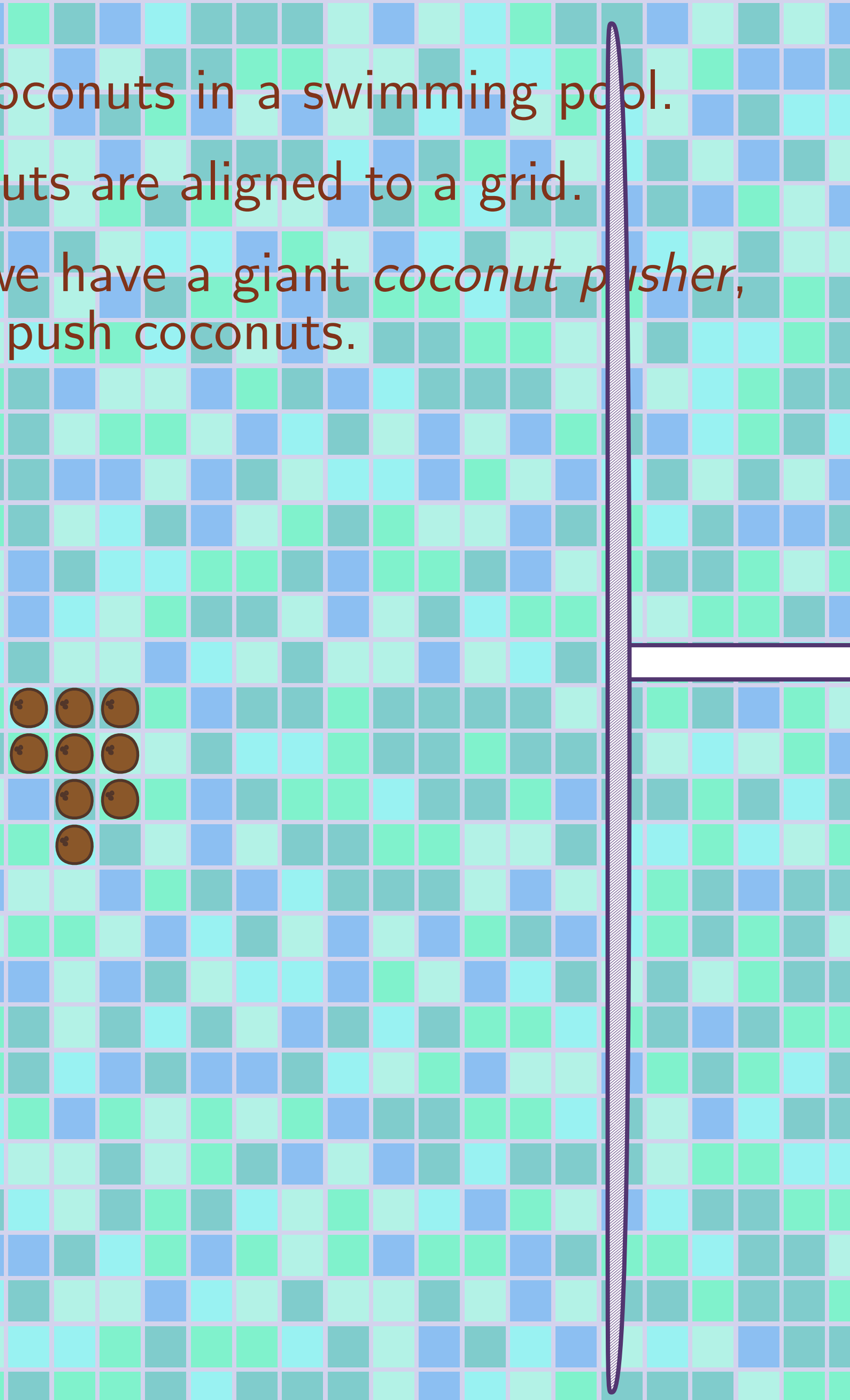

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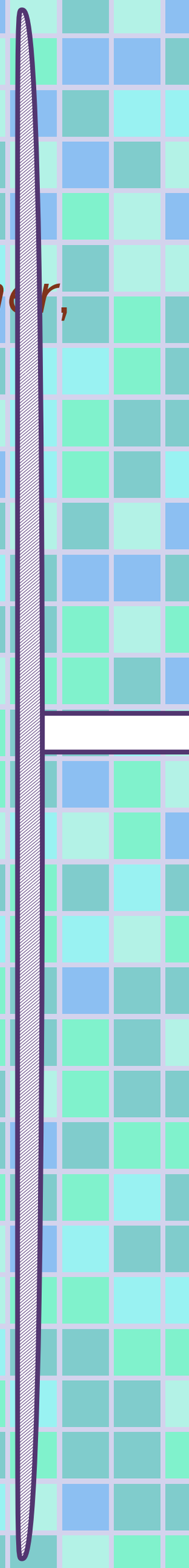

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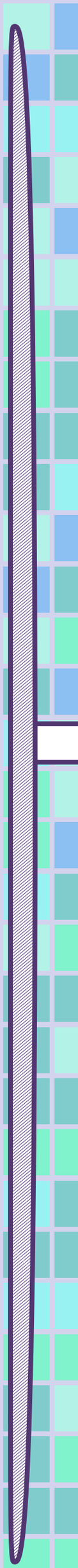

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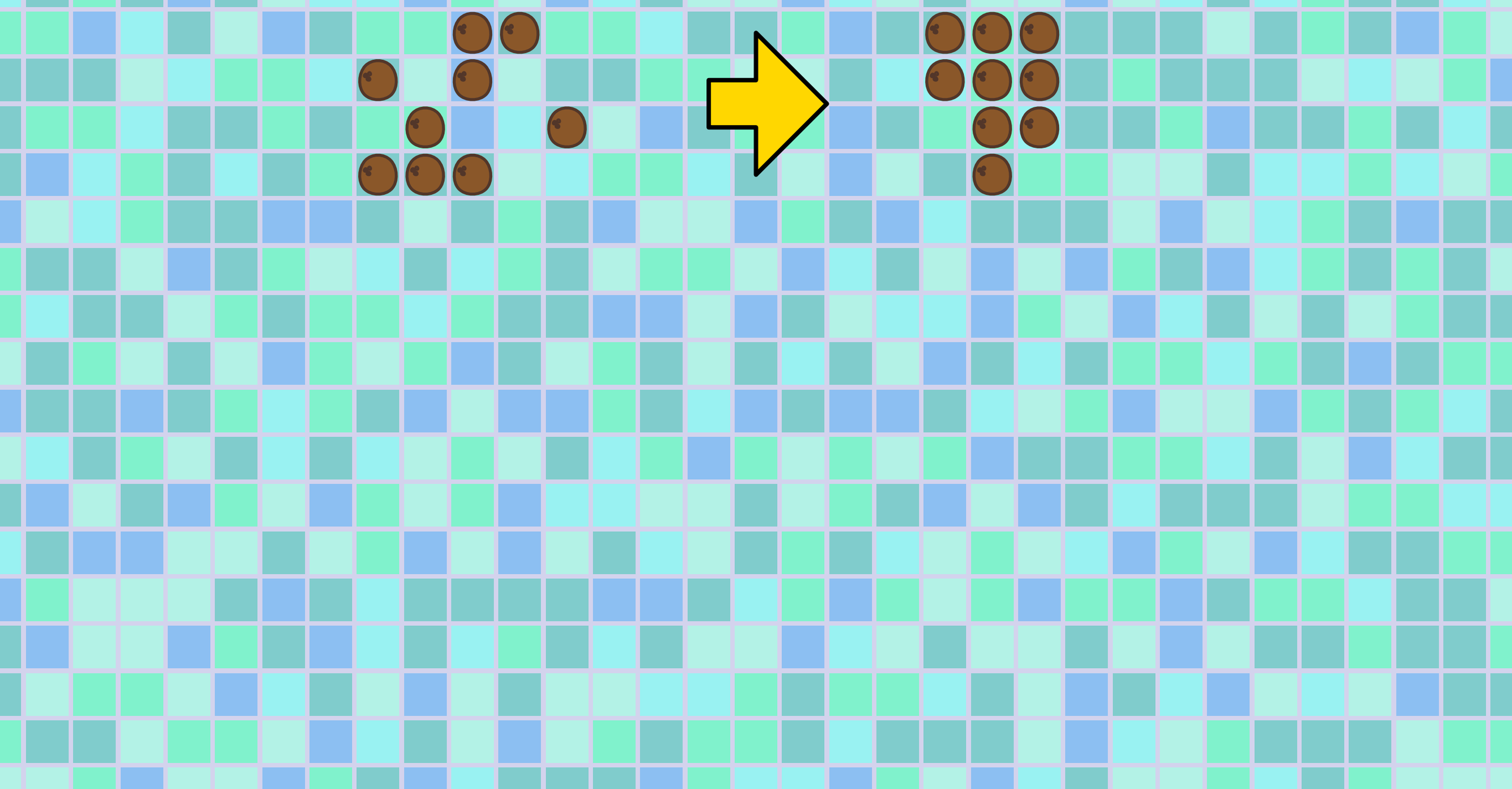

KNOWN RESULTS

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Can we always push our coconuts into tidy rectangles?

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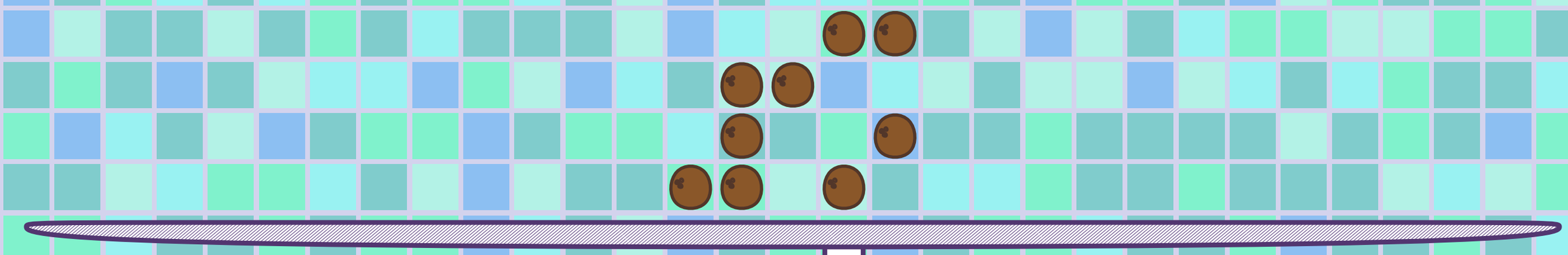

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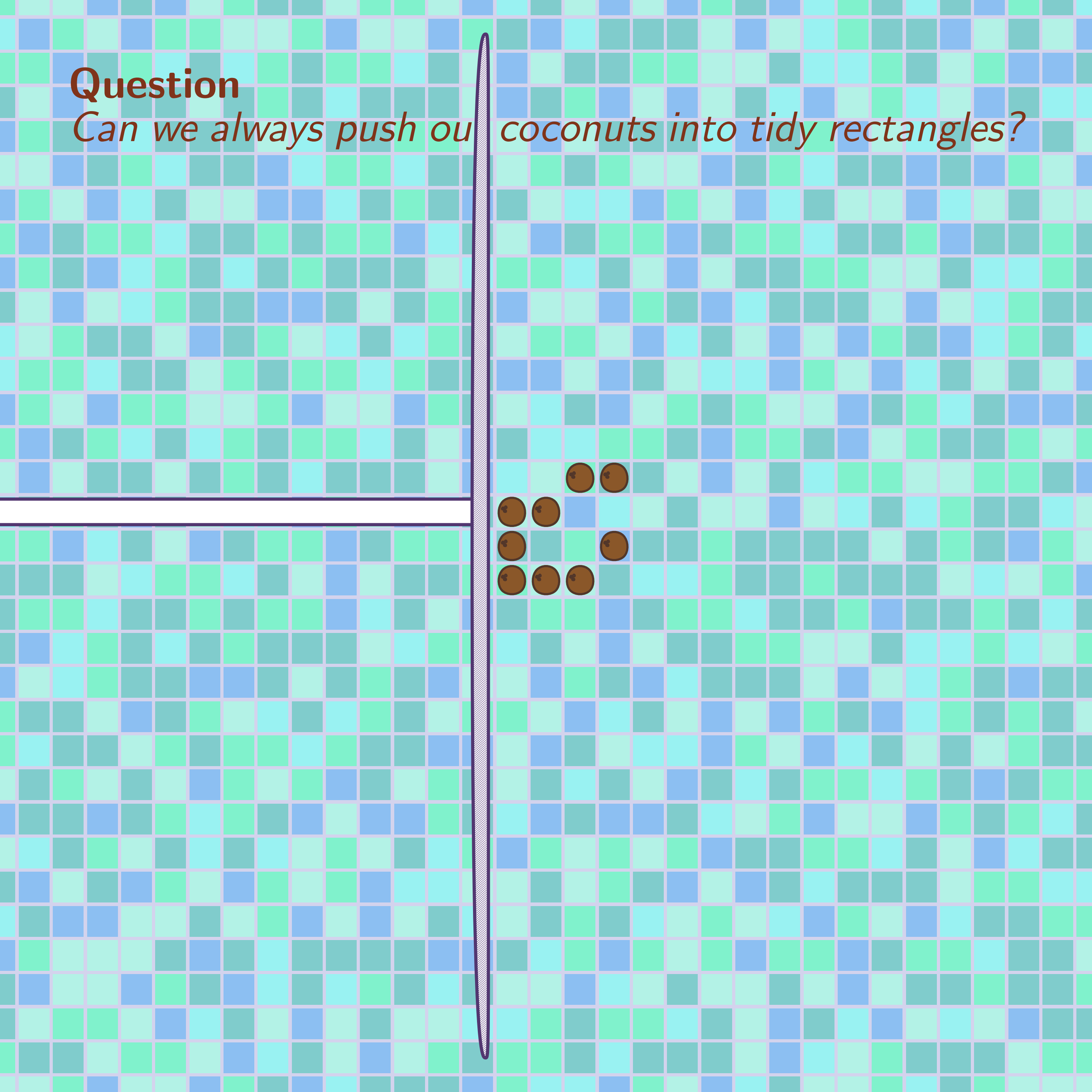

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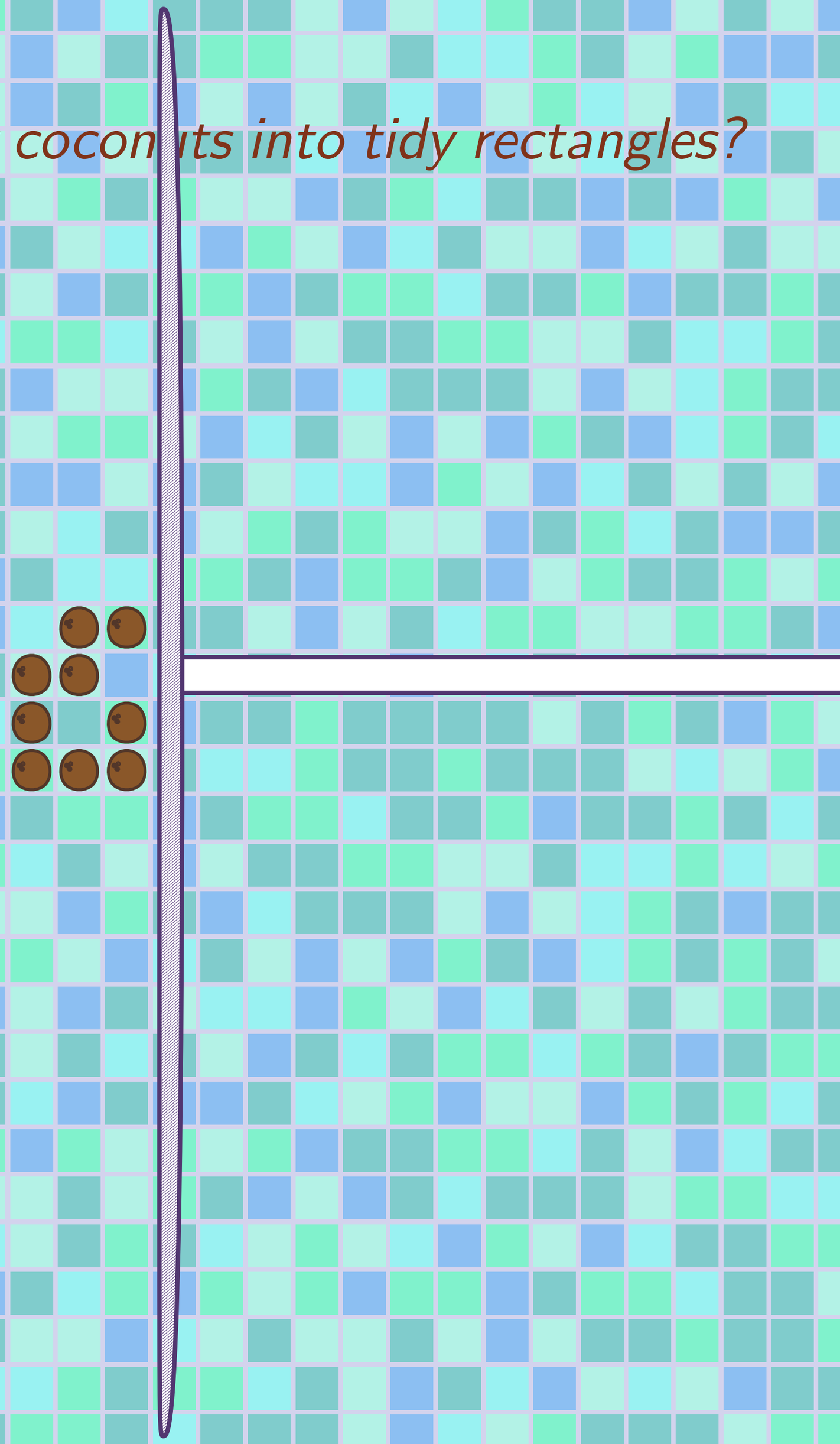

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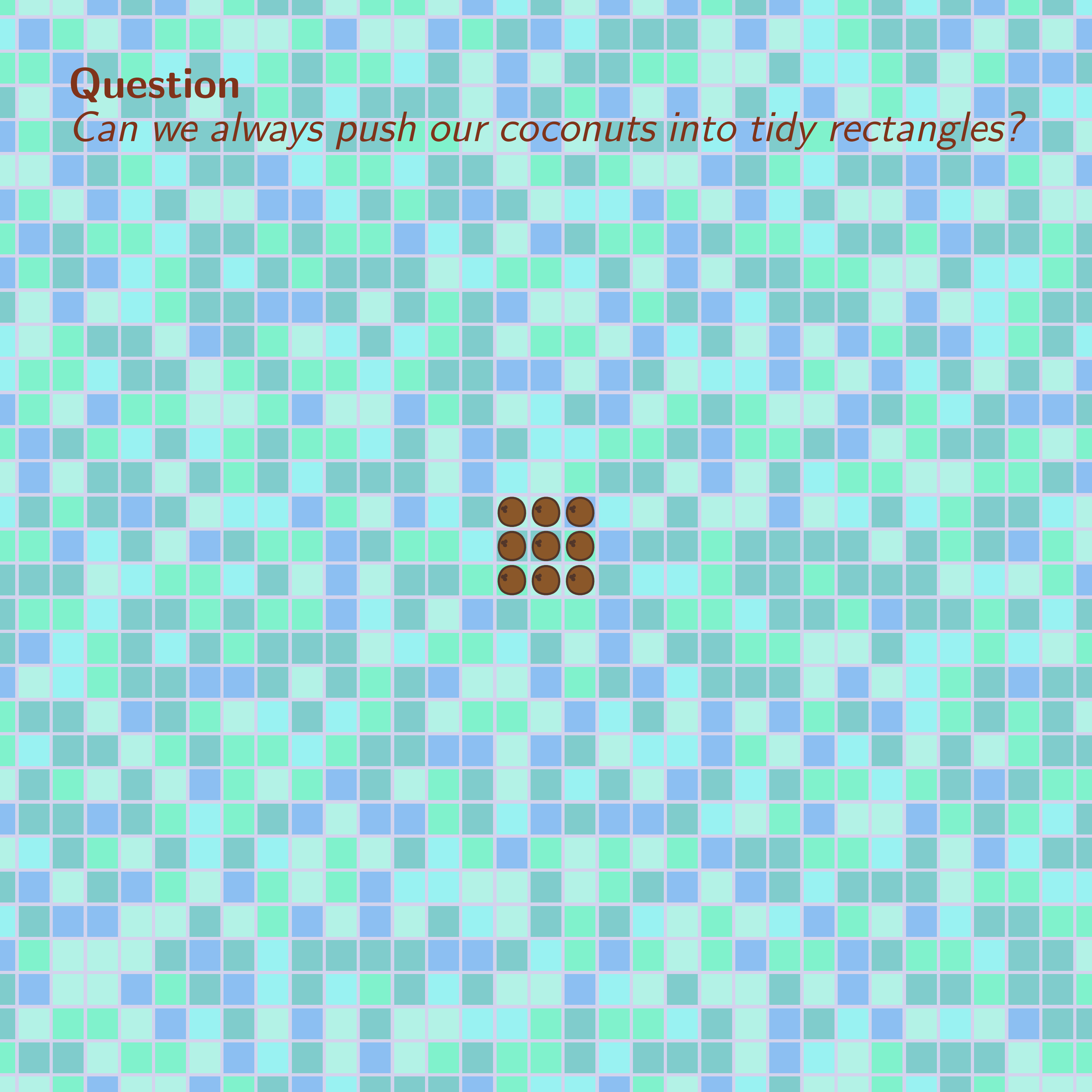

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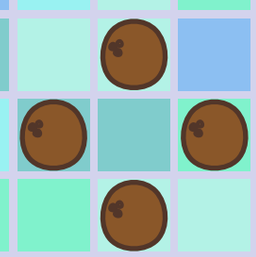

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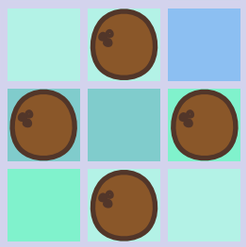

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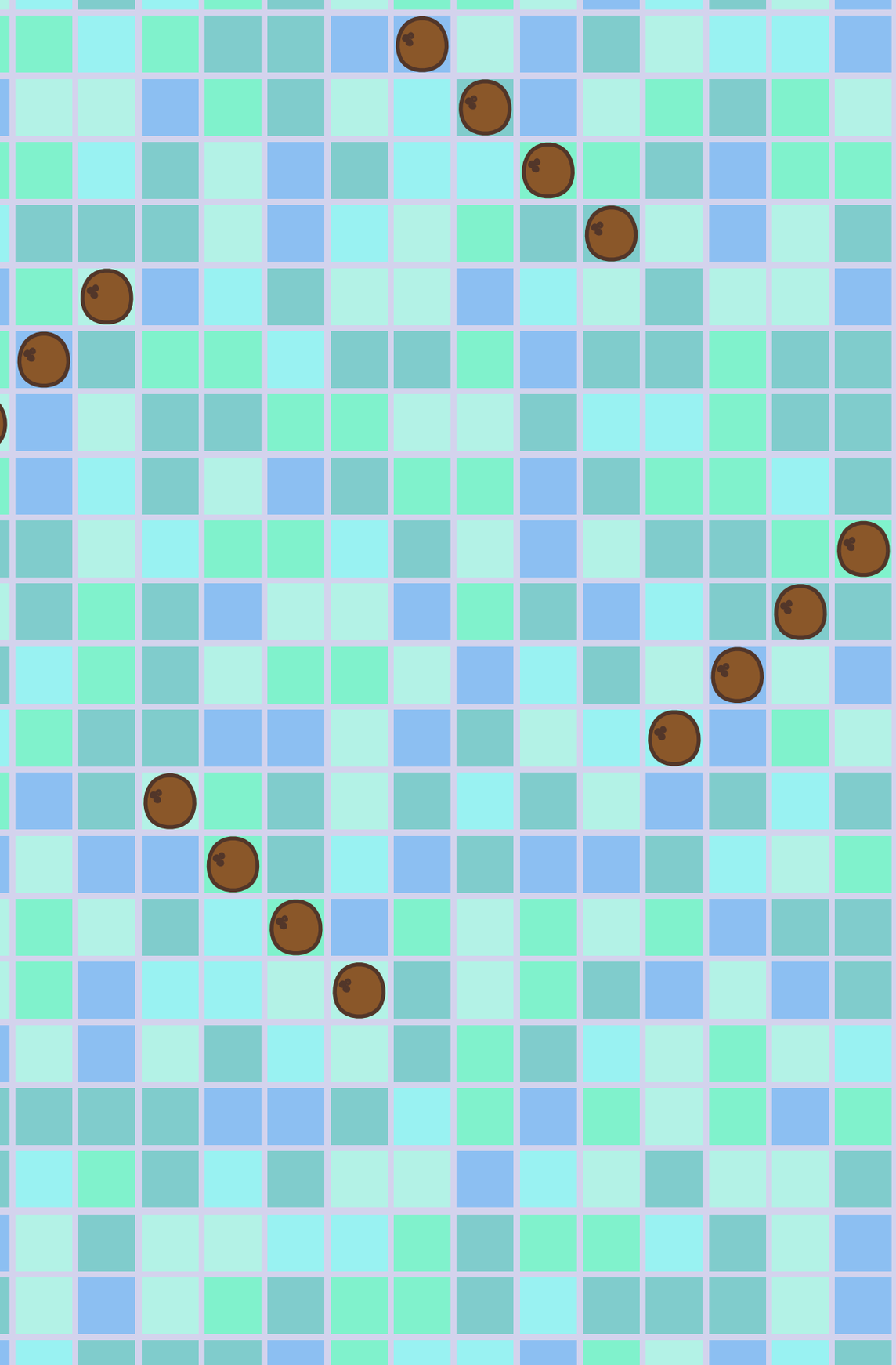

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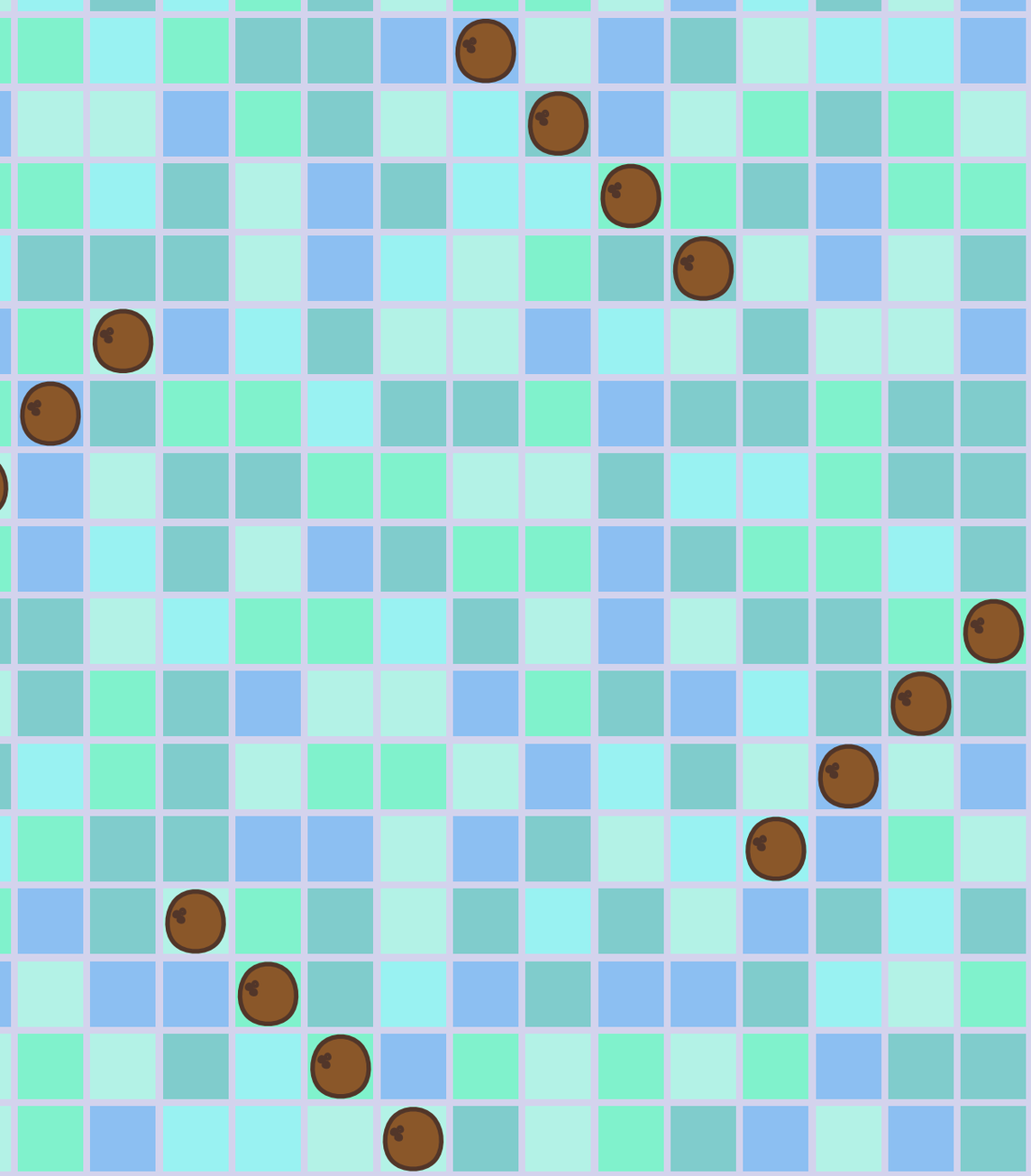

...and *some* aspect ratios are not even possible if we start with at most one coconut per row and column.

Results

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Deciding whether n coconuts can be pushed into an $a \times b$ rectangle is NP-complete.

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Everything inbetween is still open!

Results

Deciding whether n coconuts can be pushed into an $a \times b$ rectangle is NP-complete.

Deciding whether n coconuts which occupy at most k rows can be pushed into a $2 \times n/2$ rectangle is polynomial.

Everything inbetween is still open!

[Akitaya, Aloupis, Löffler, Rounds, 2016]

GAMES

In compaction *games*, players take turns pushing.

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We will consider 2-player games, *red* against *blue*.

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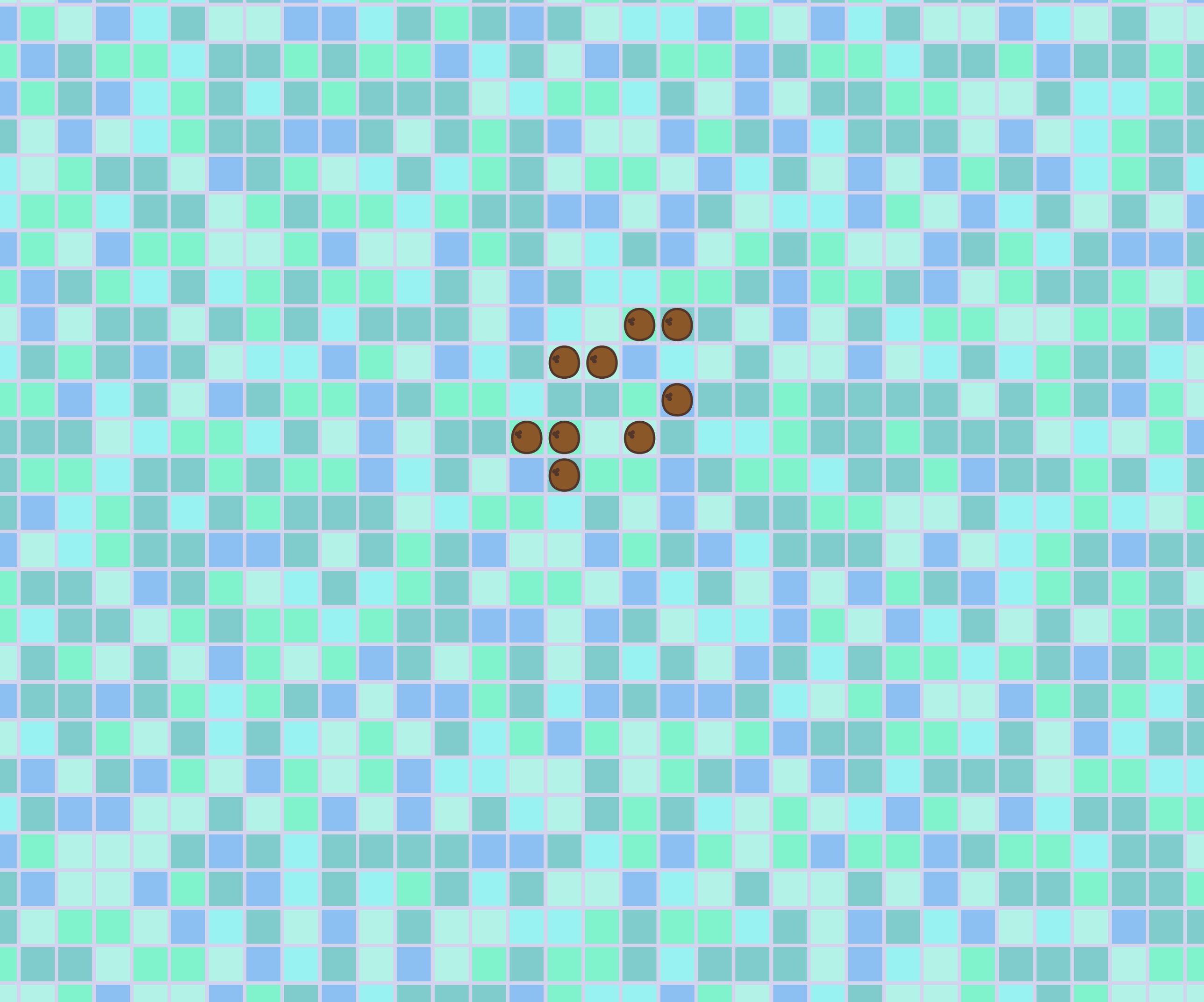

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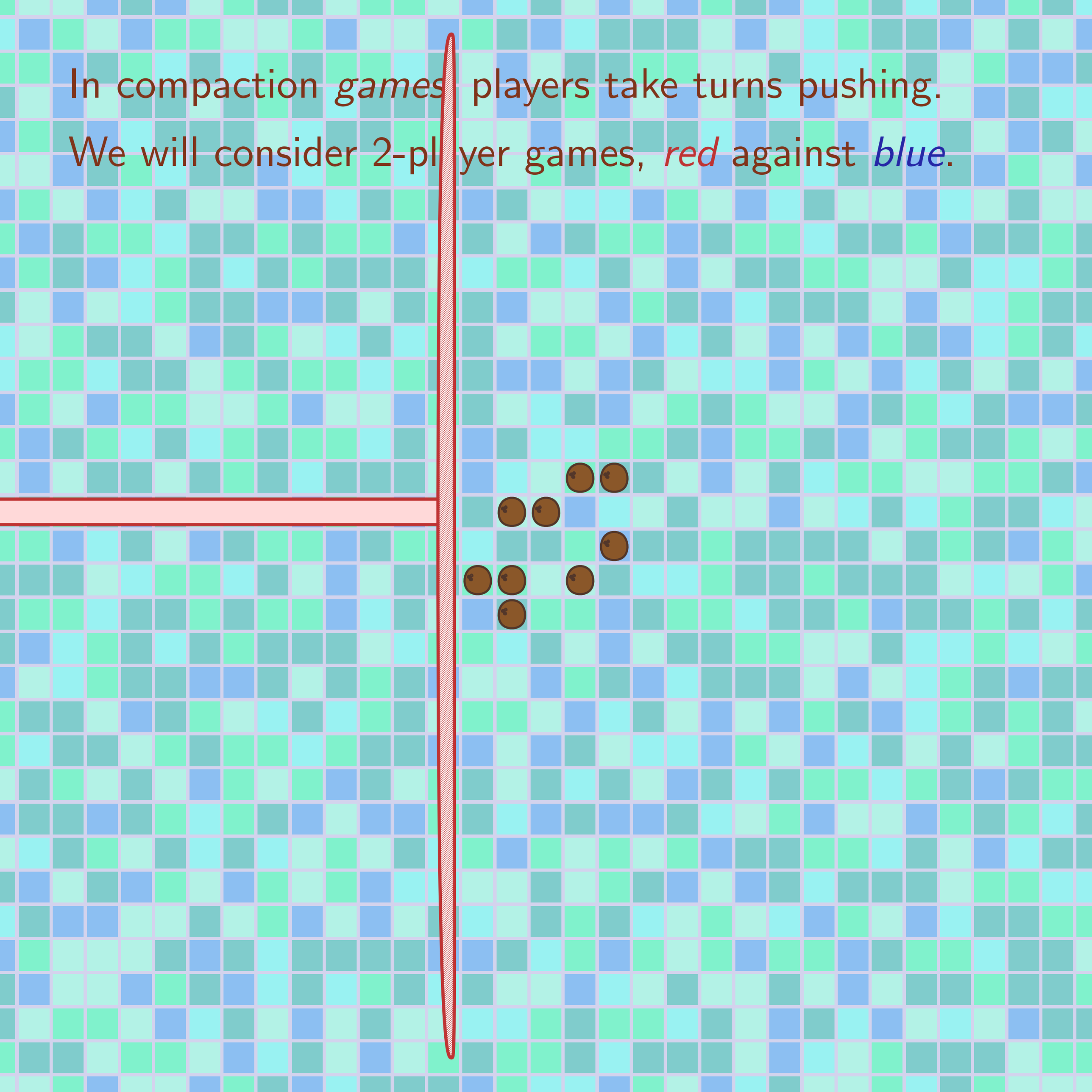

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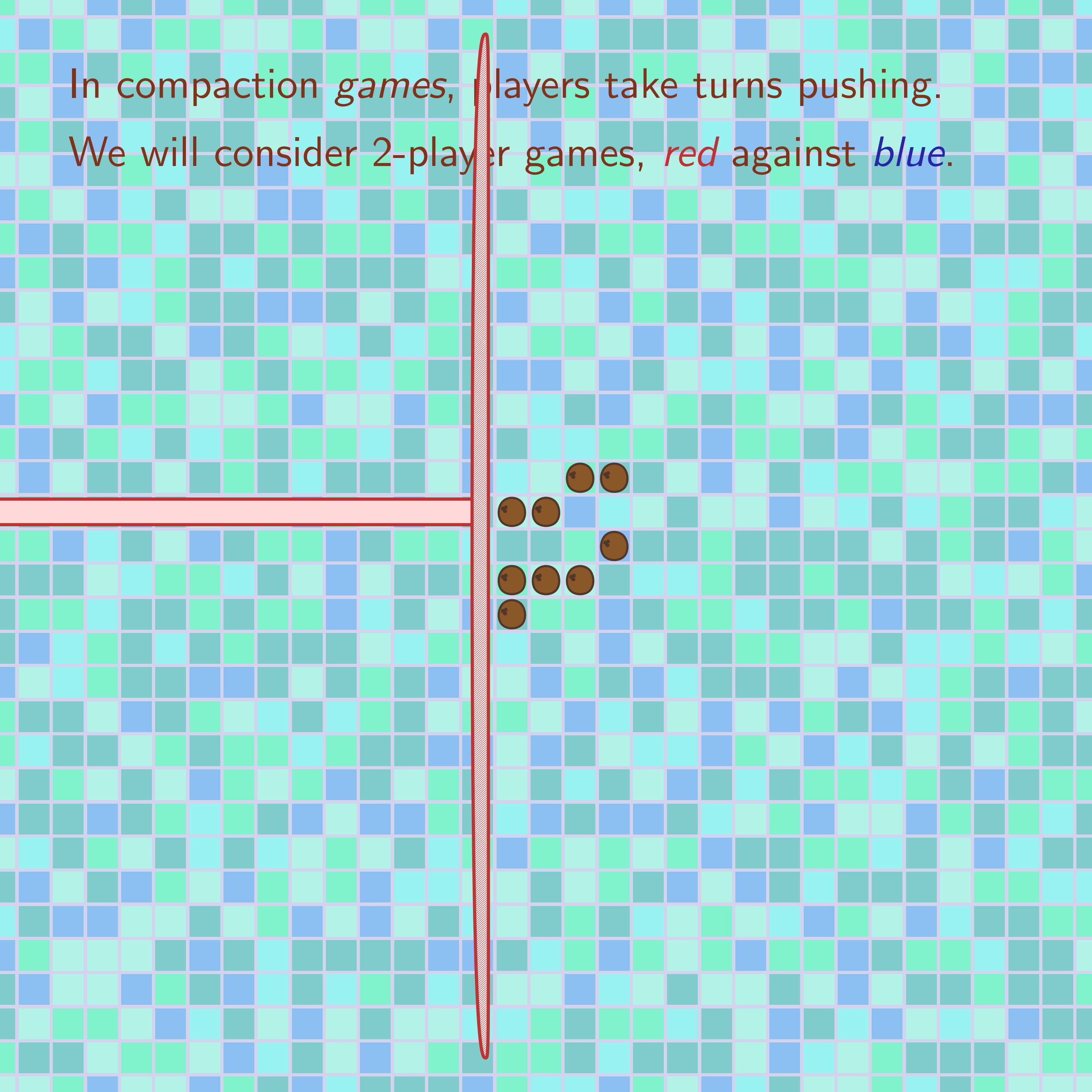

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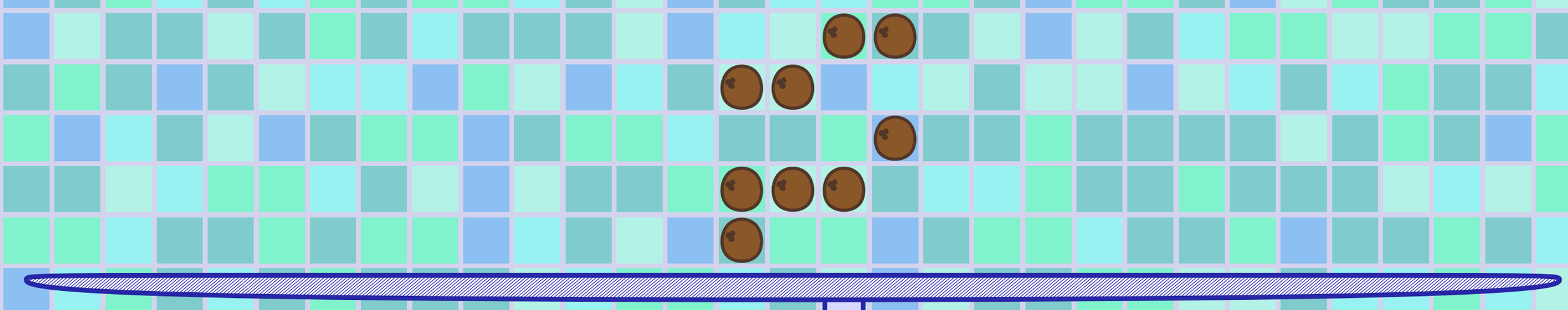

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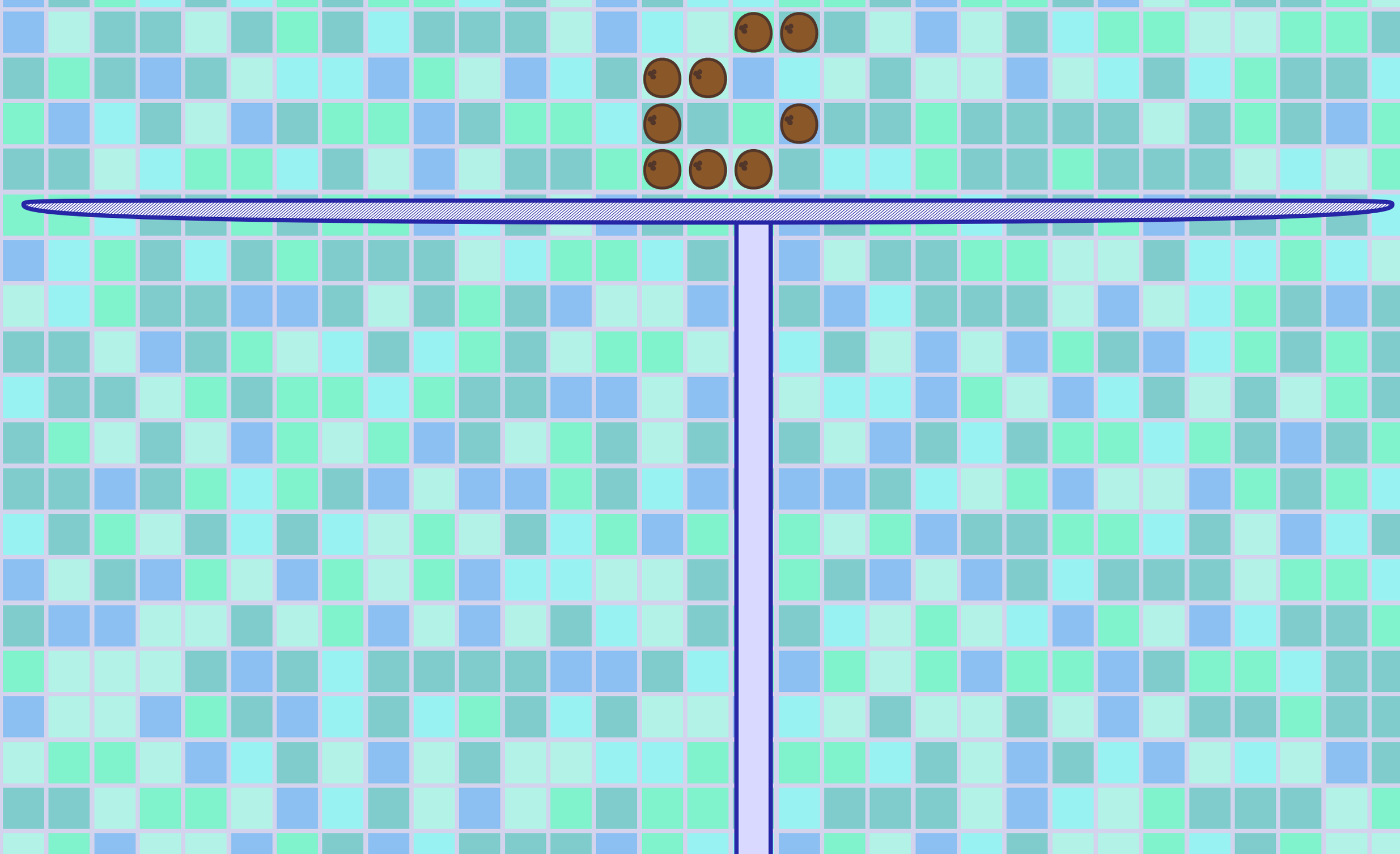

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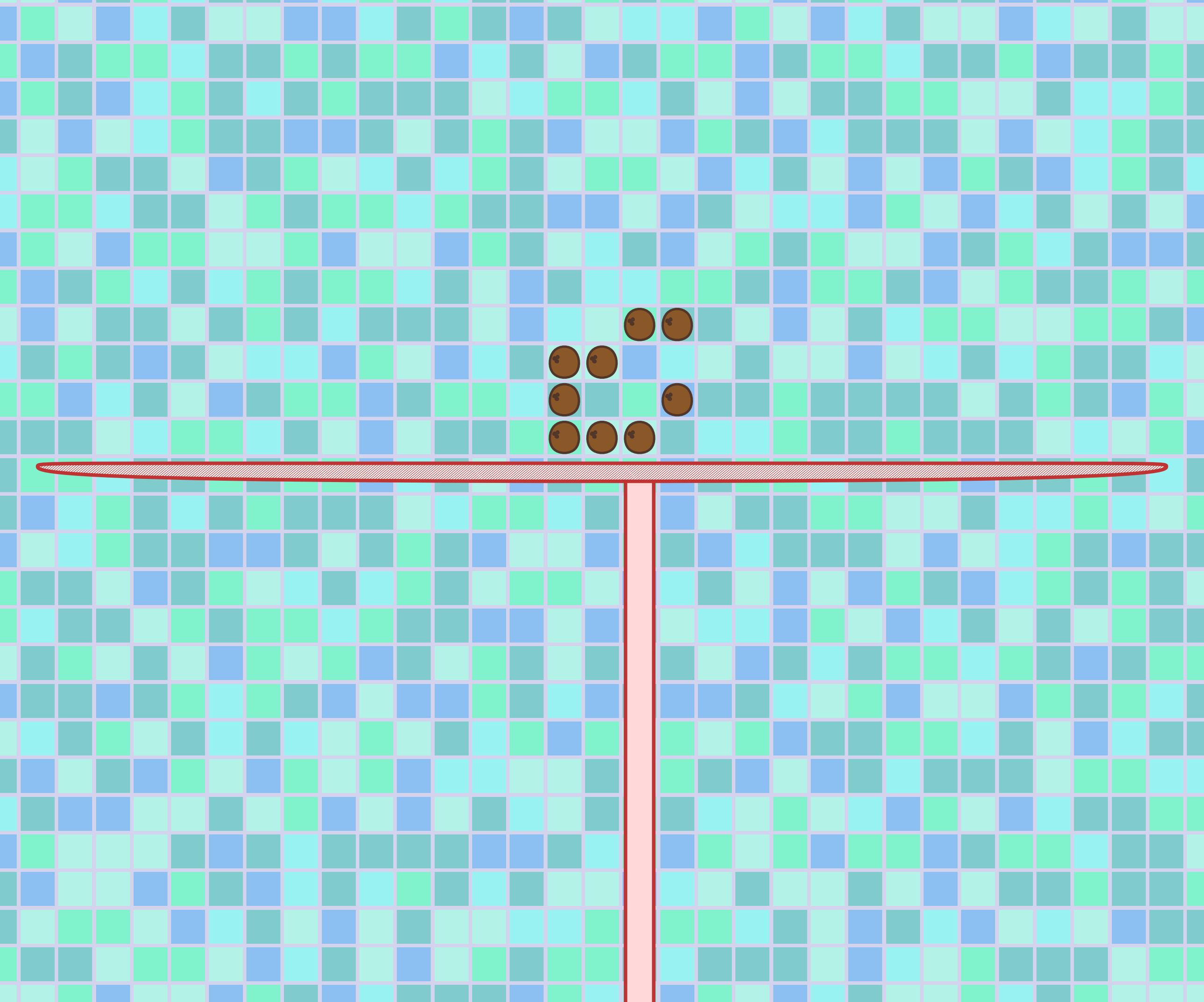

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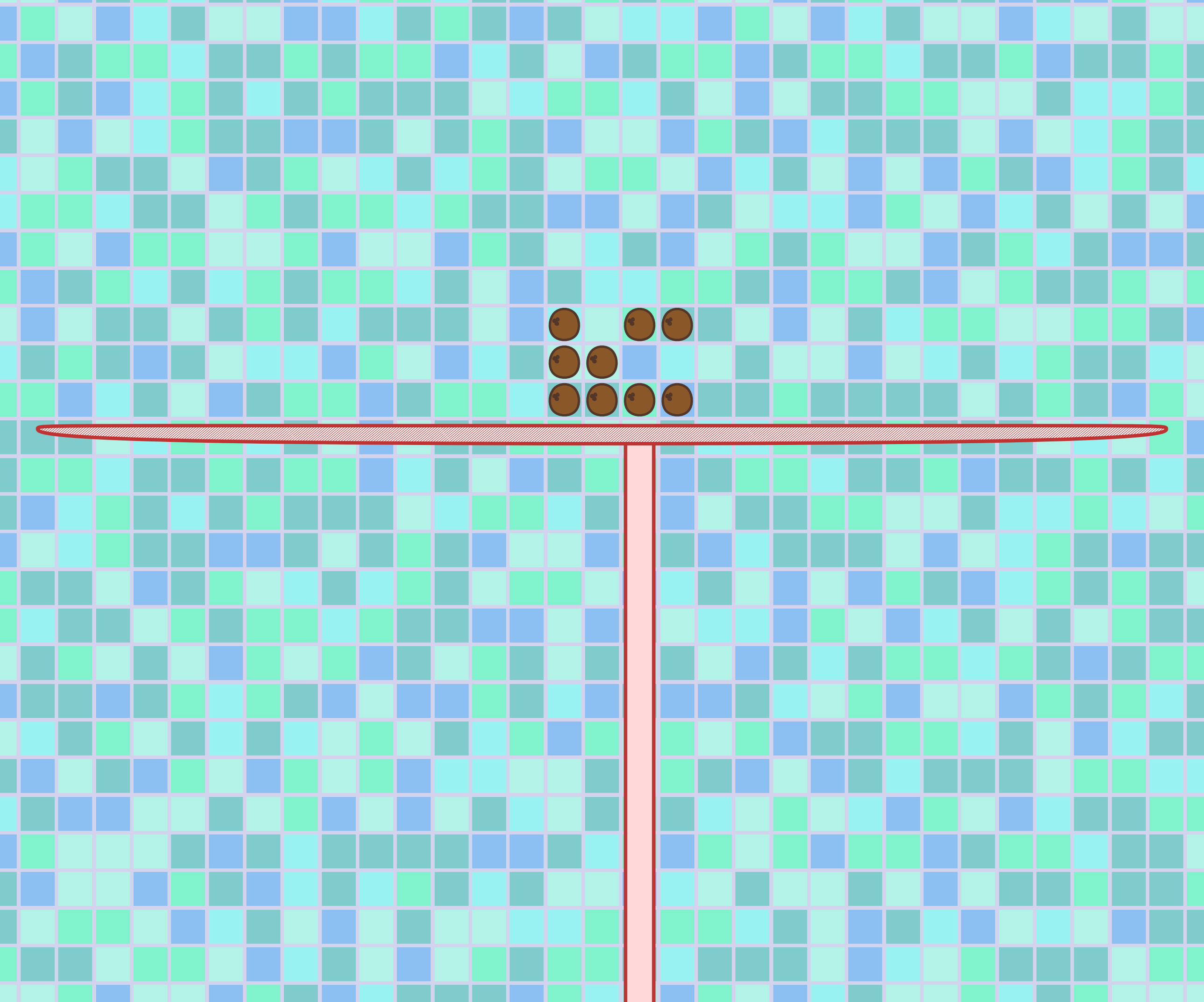

GAME 1

LAST MOVE WINS

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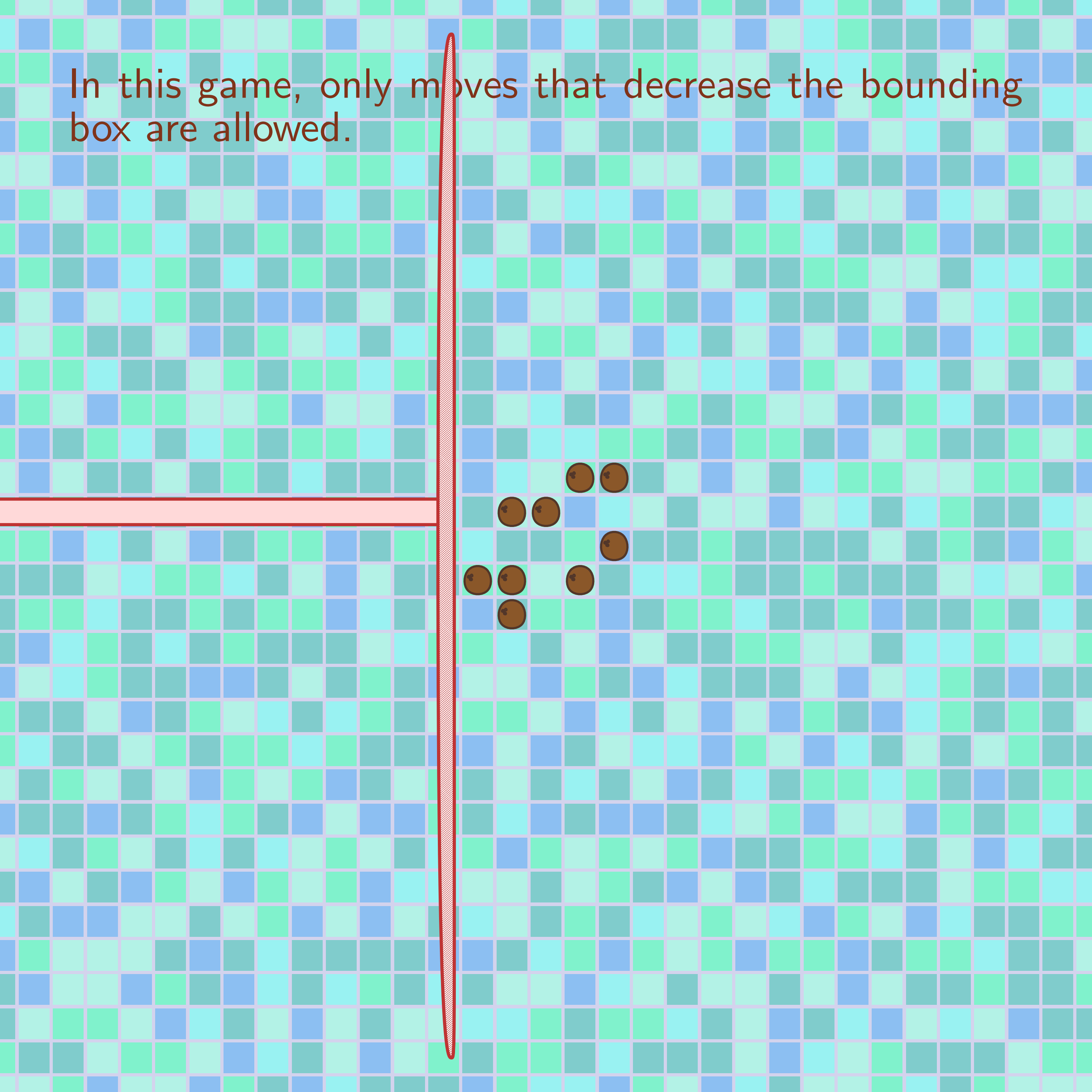

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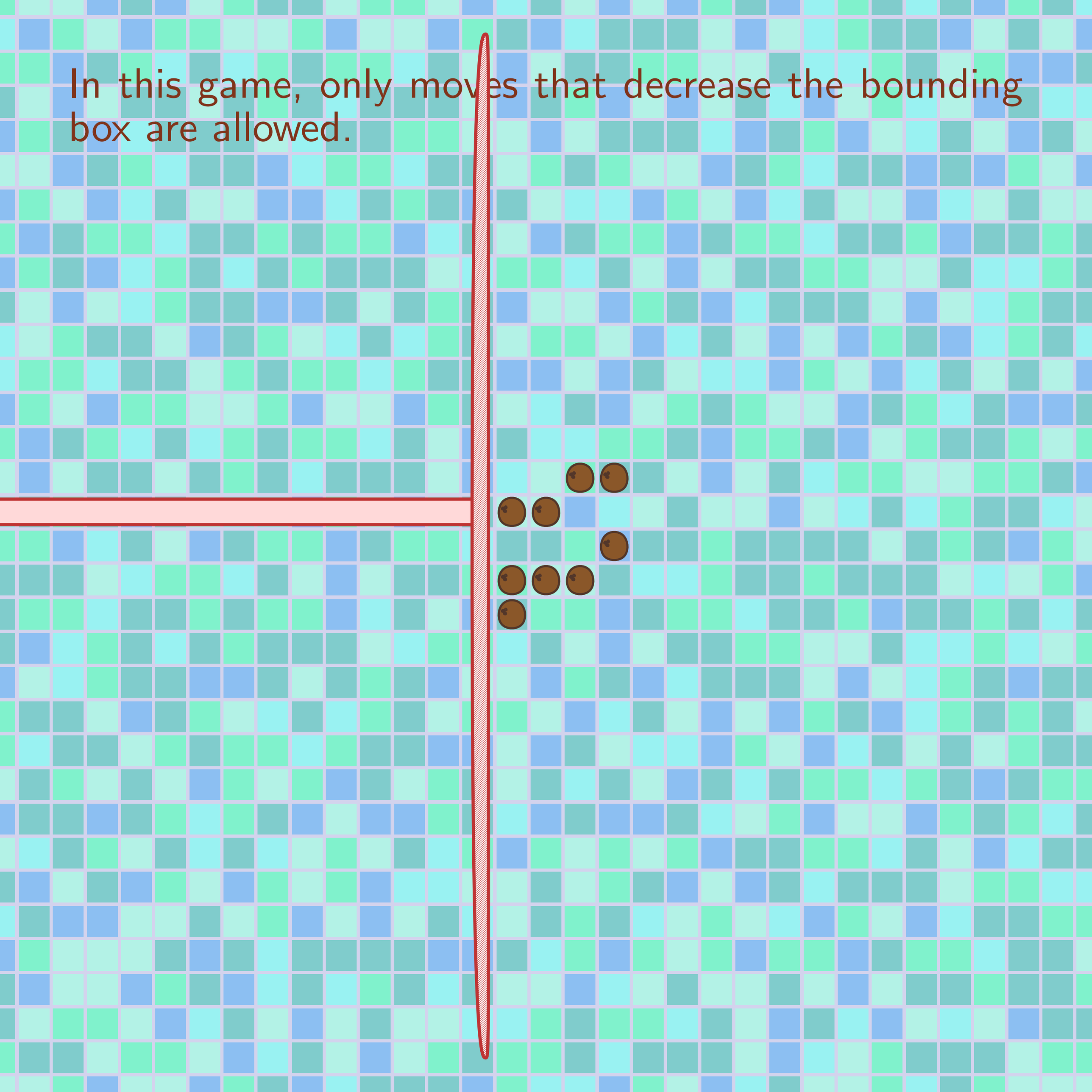

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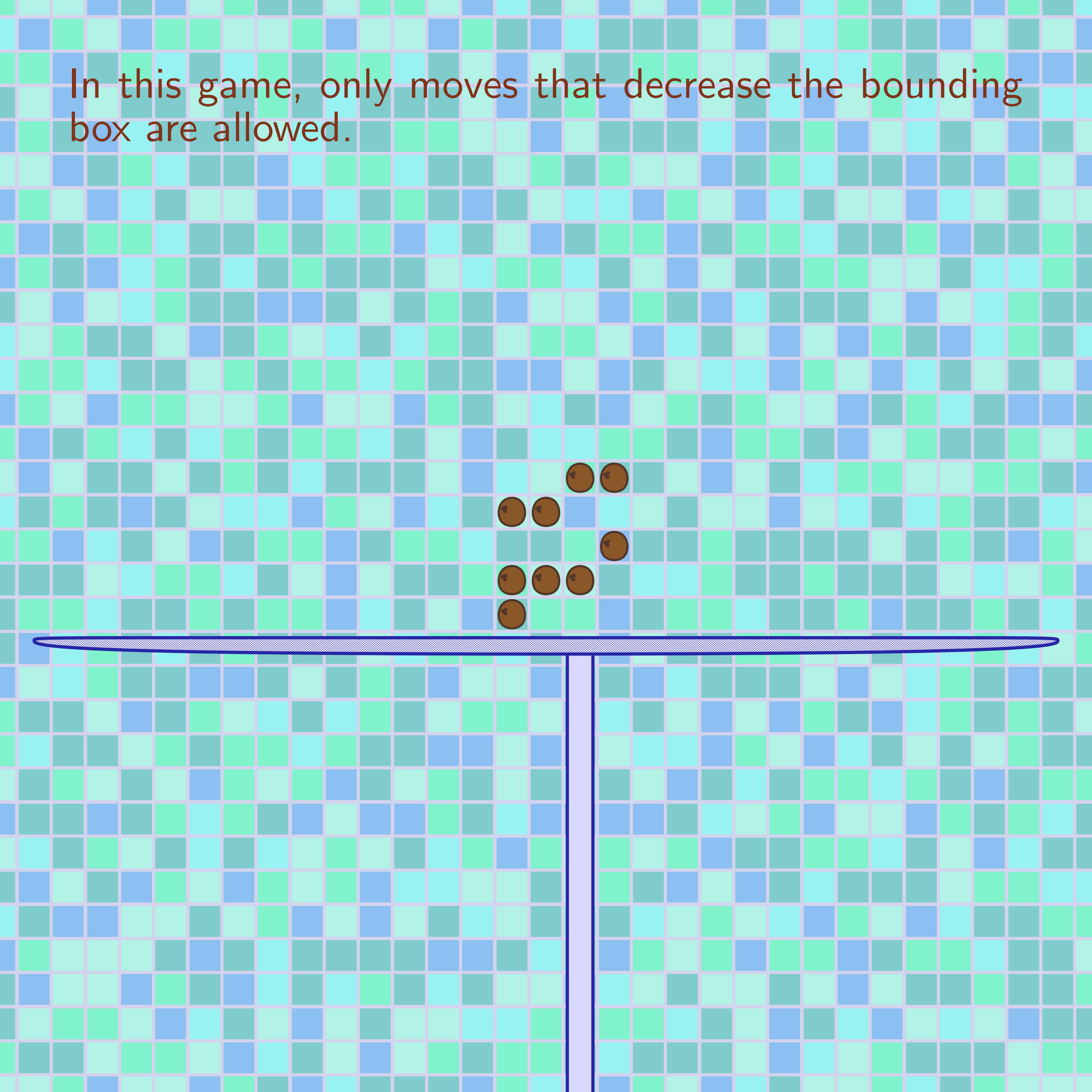

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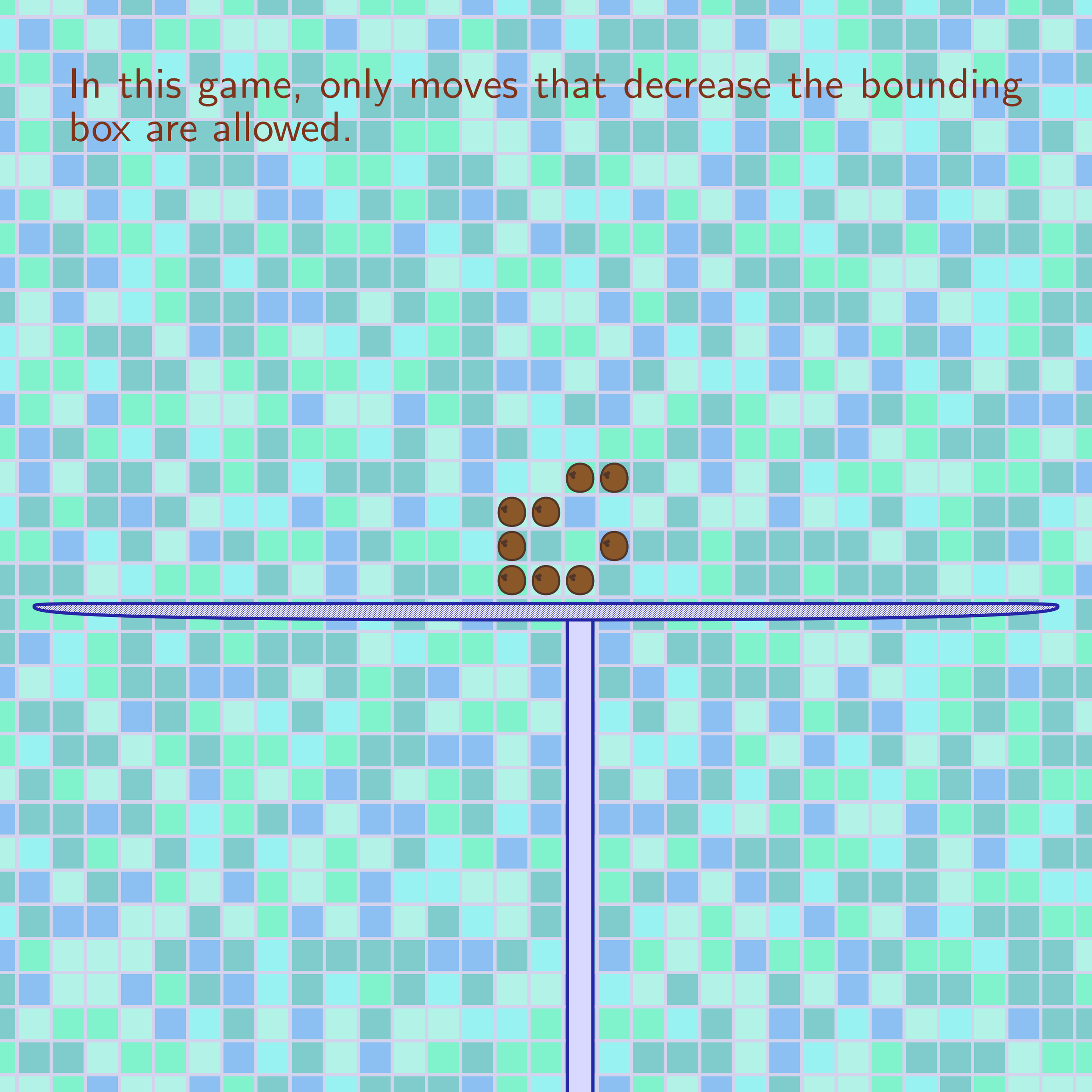

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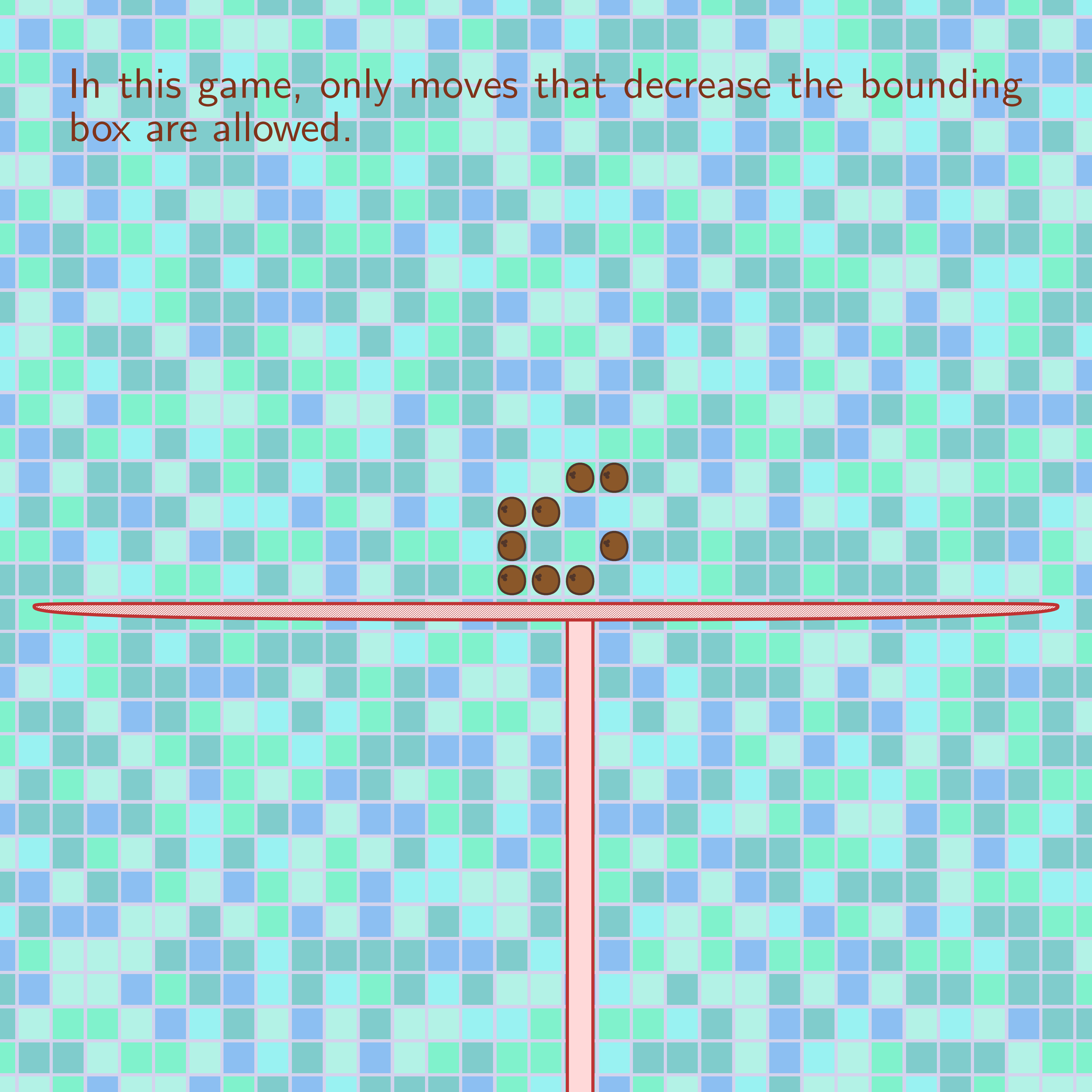

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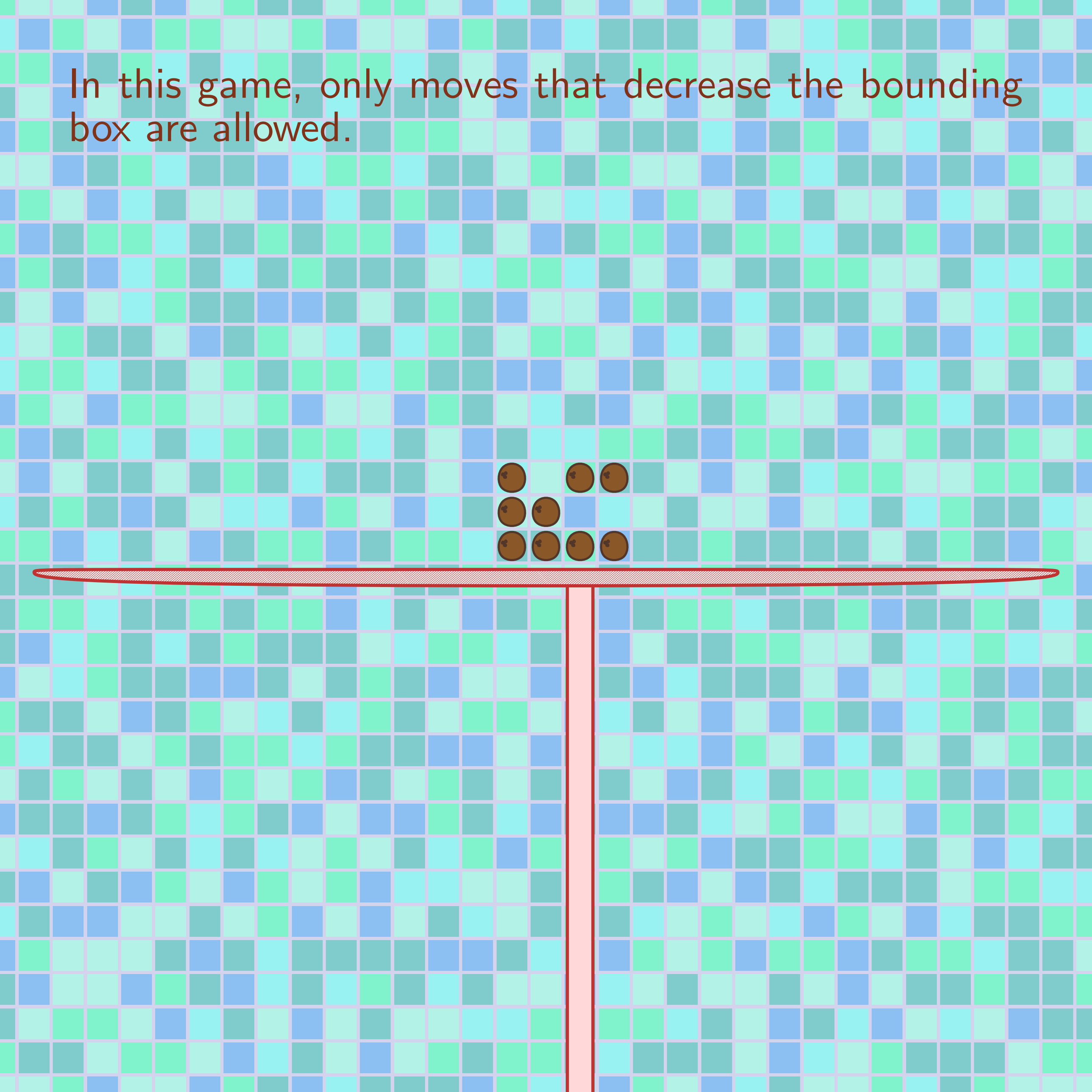

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RED WINS!

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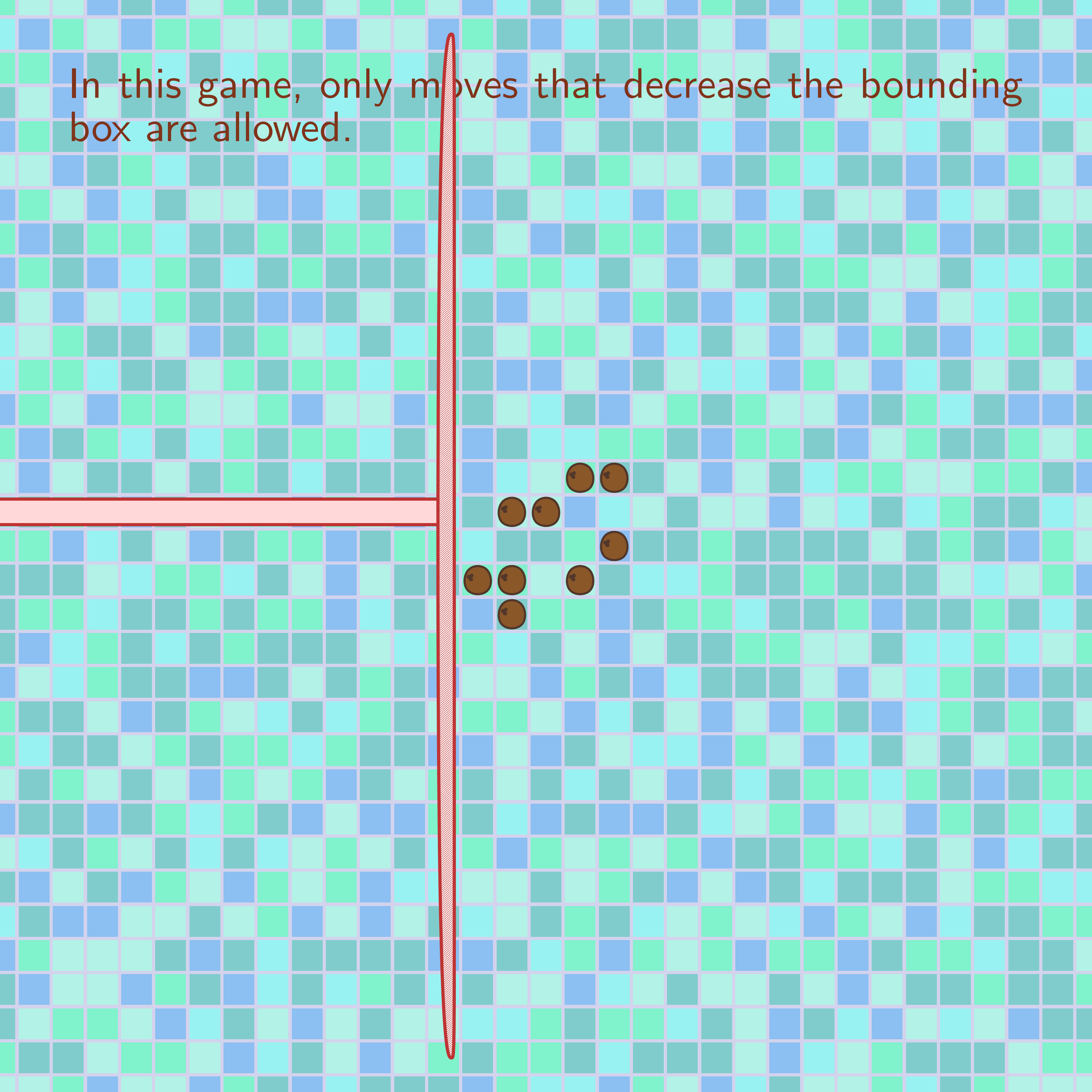

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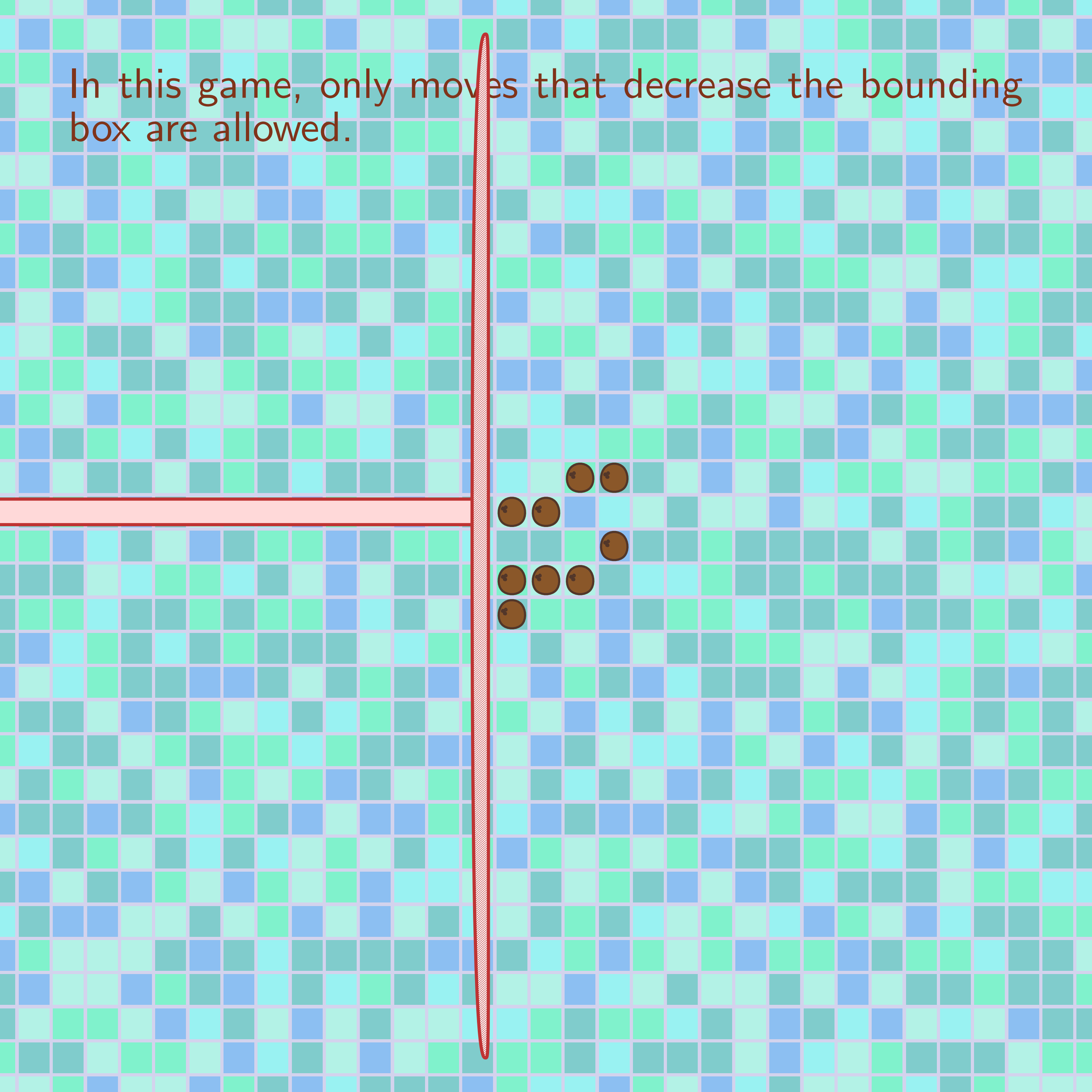

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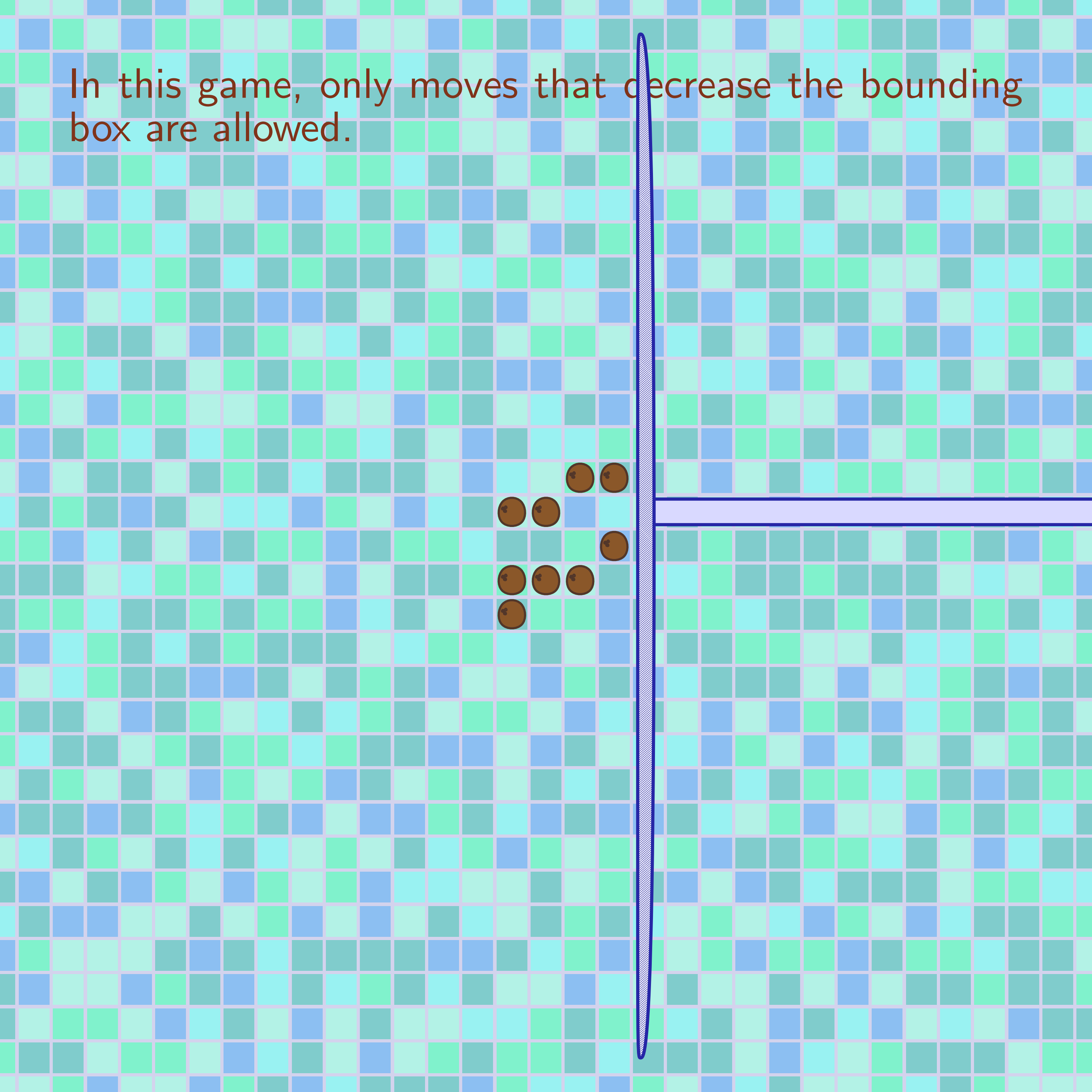

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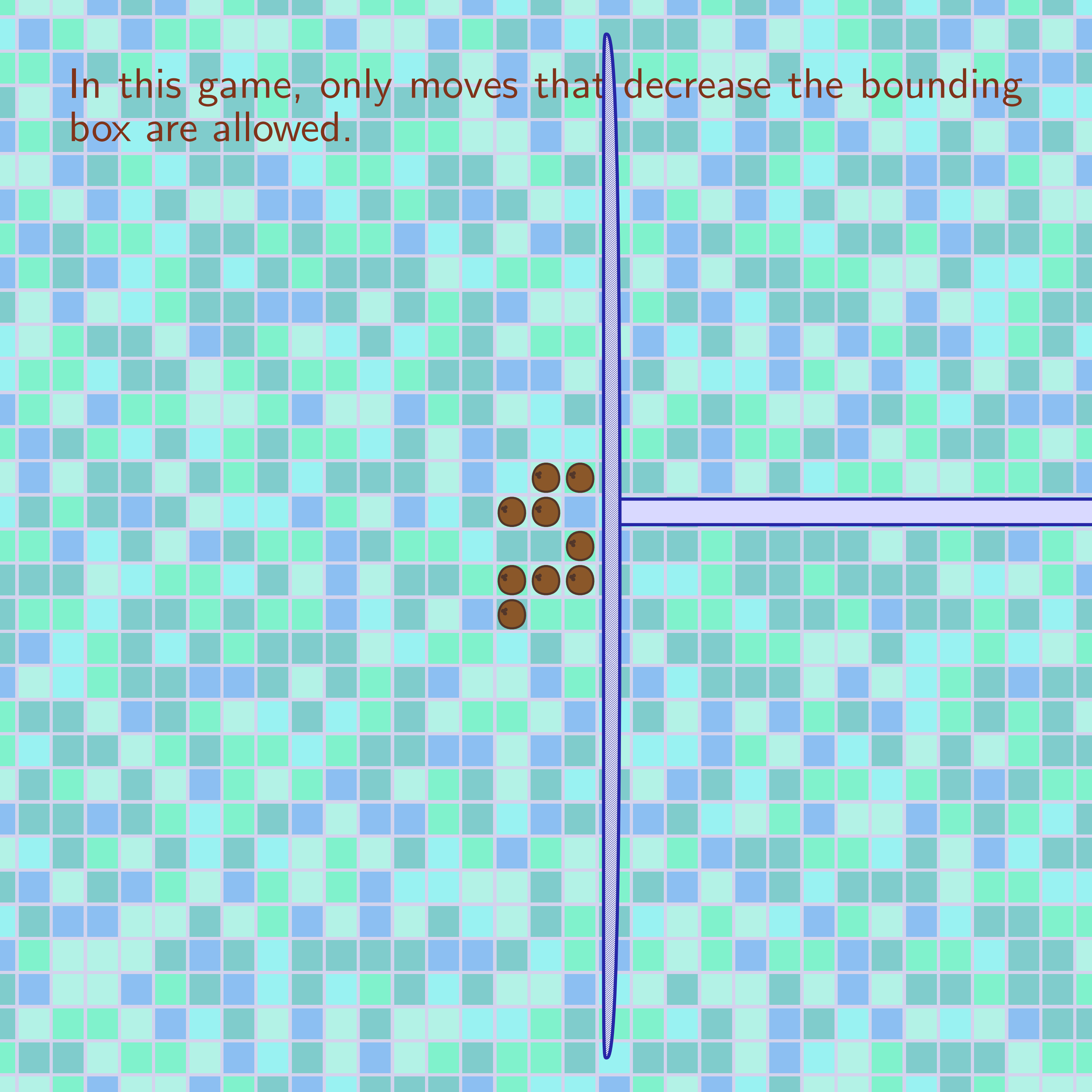

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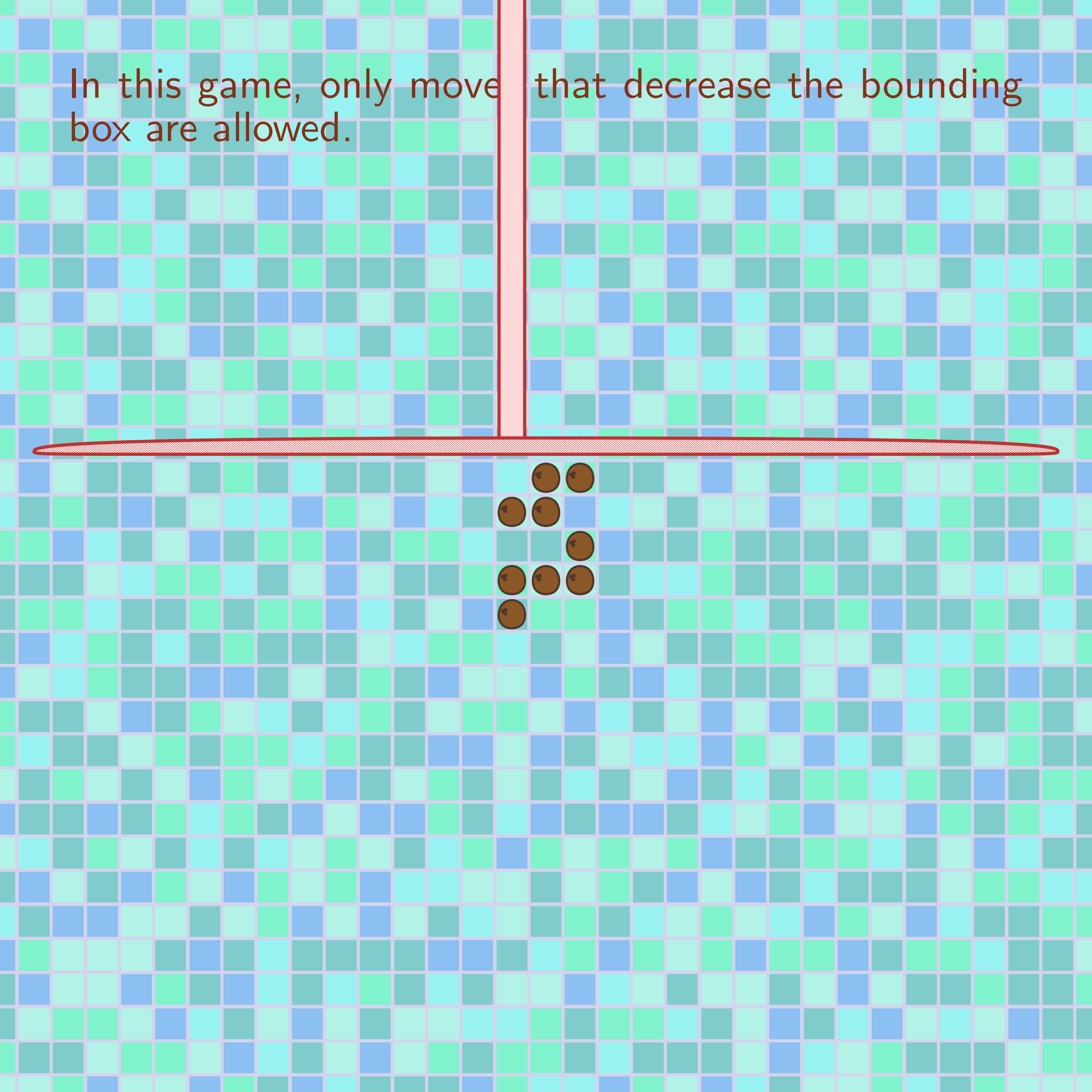

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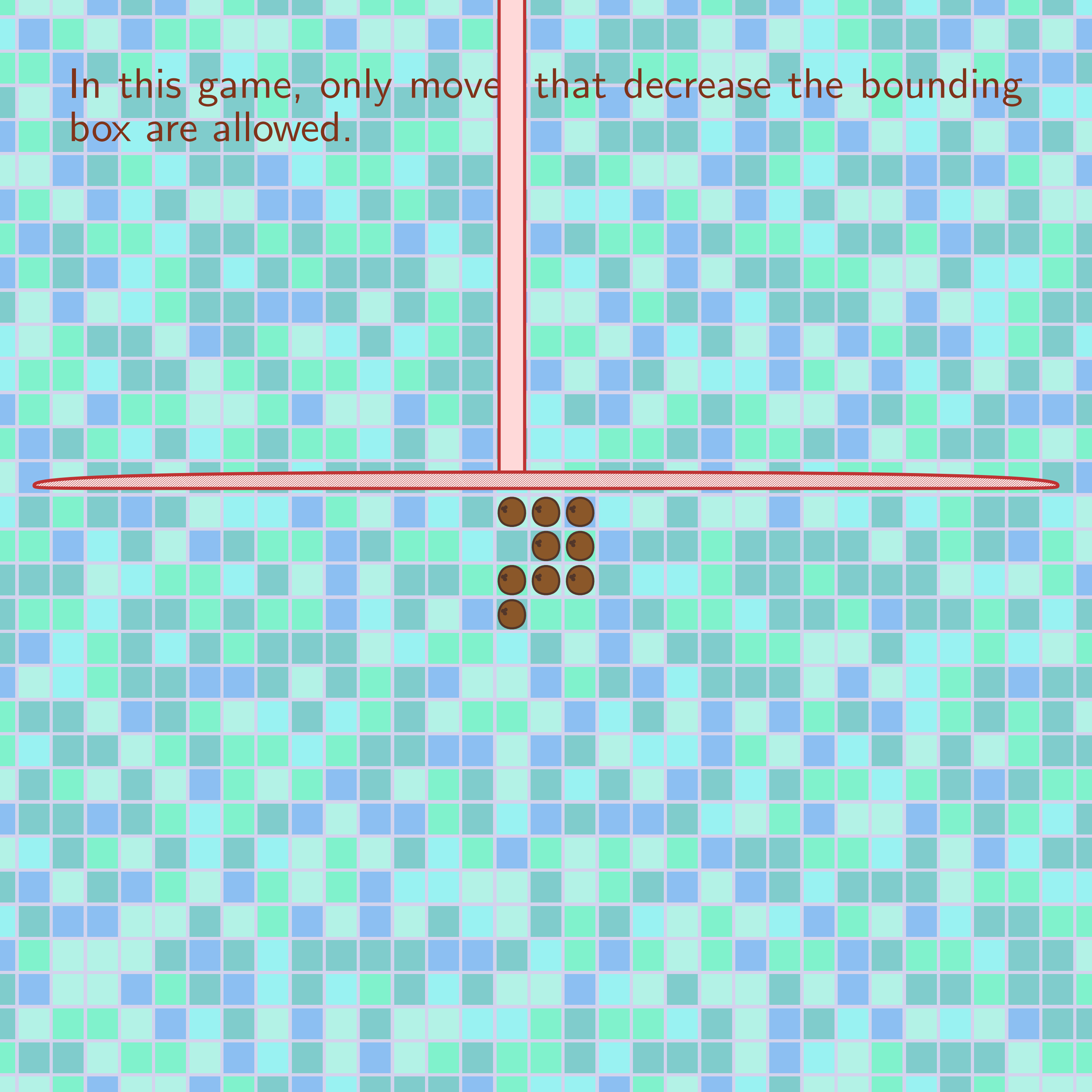

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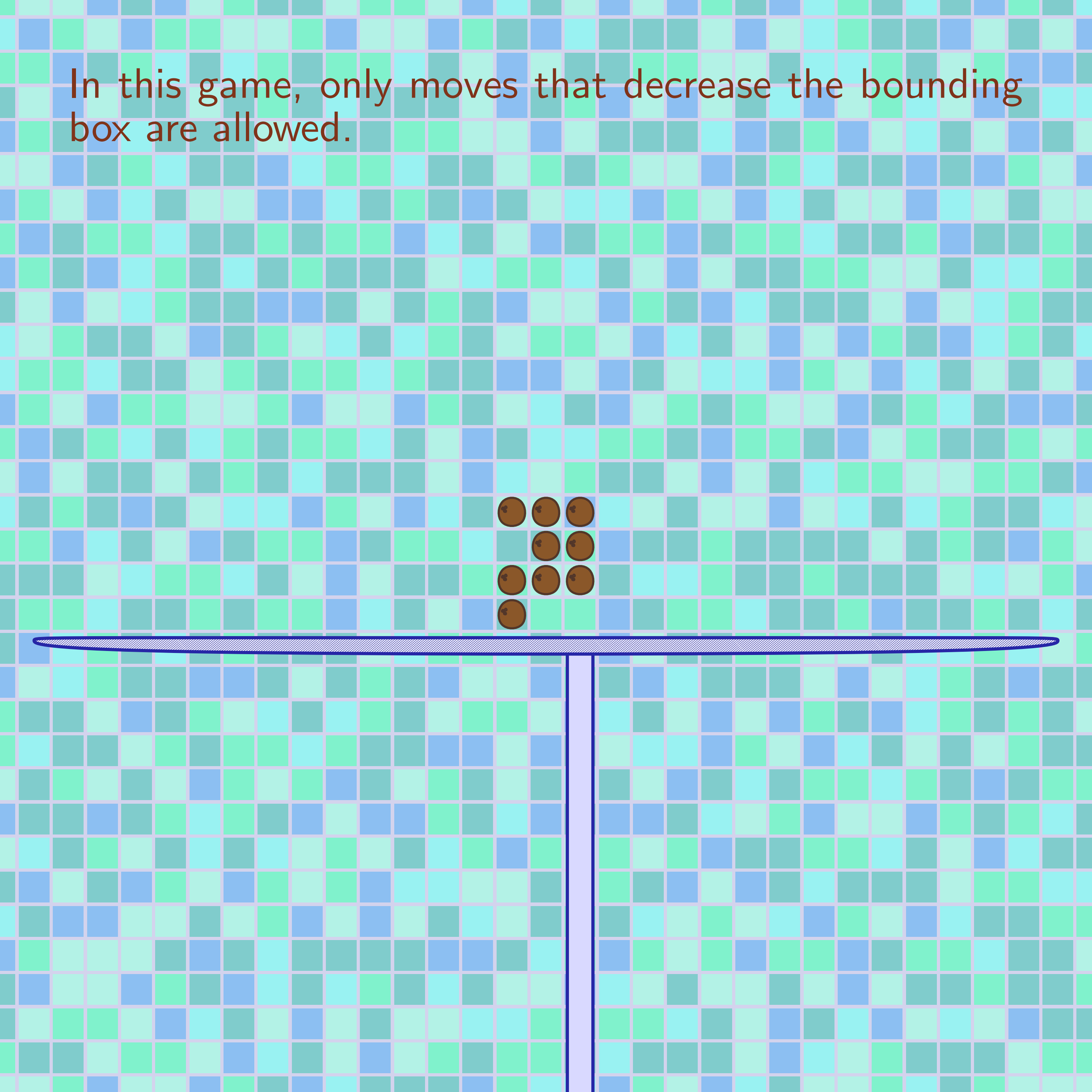

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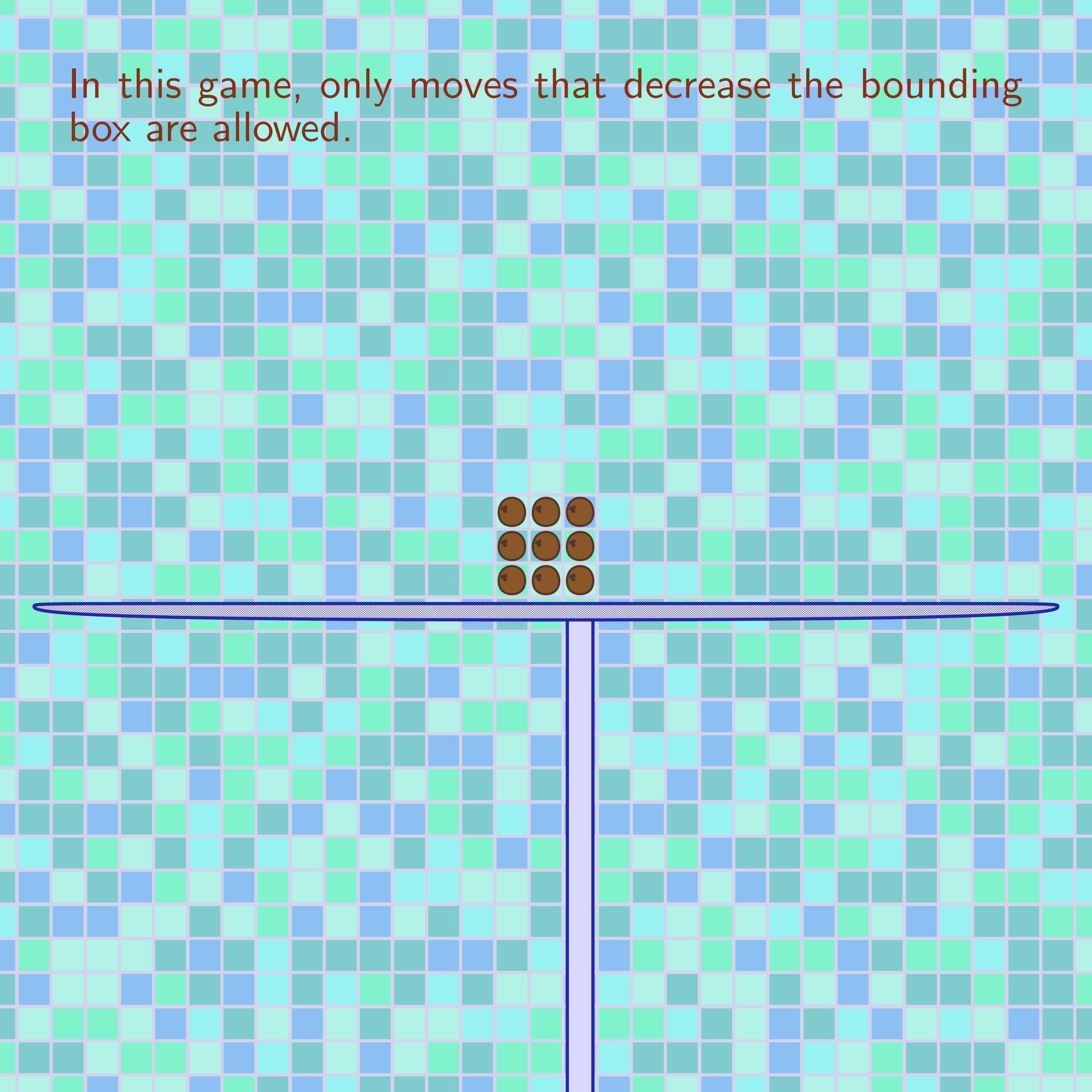

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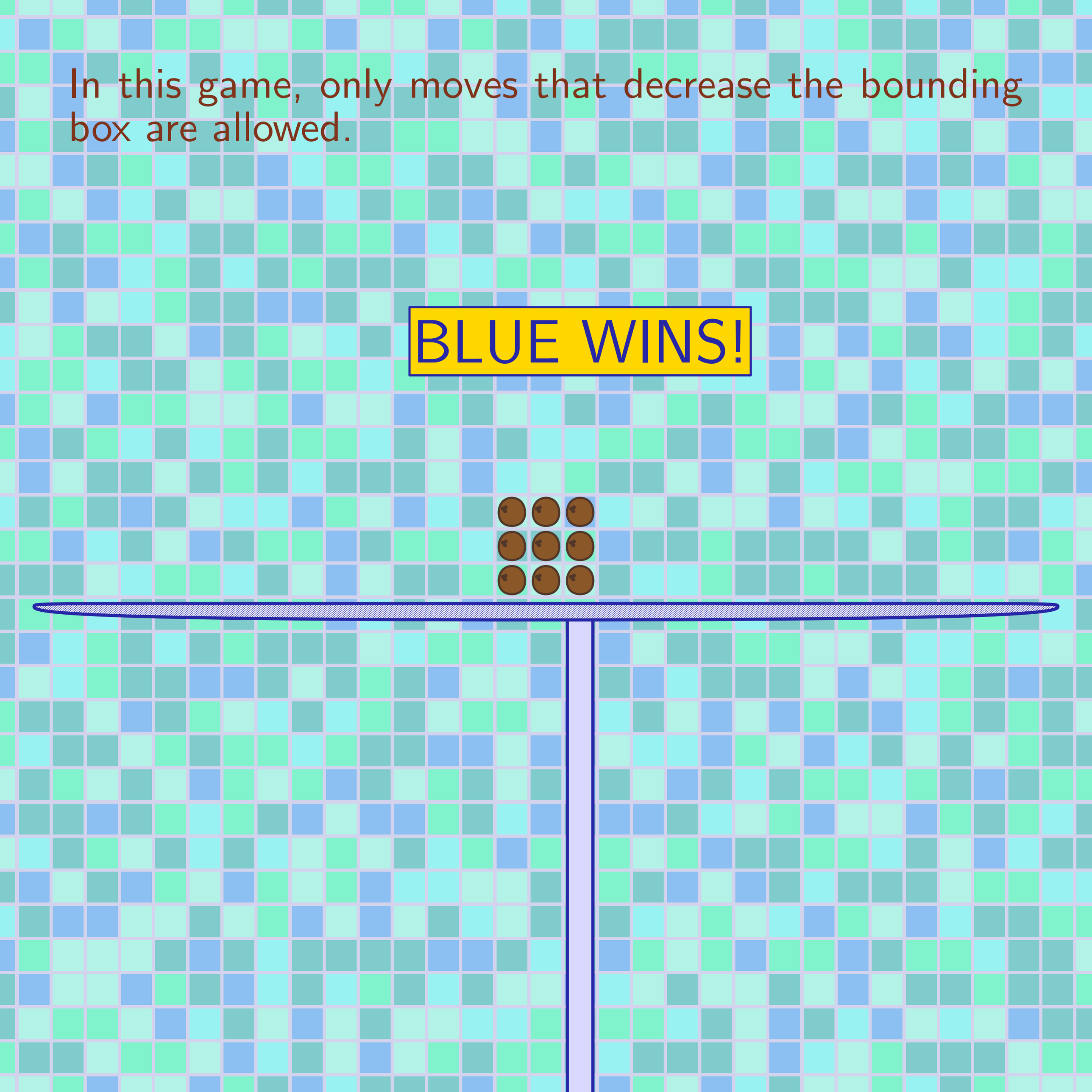

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We define the *potential* of a configuration as the minimum number of empty cells in any row plus the minimum number of empty cells in any column.

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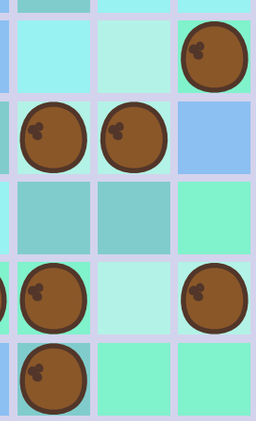

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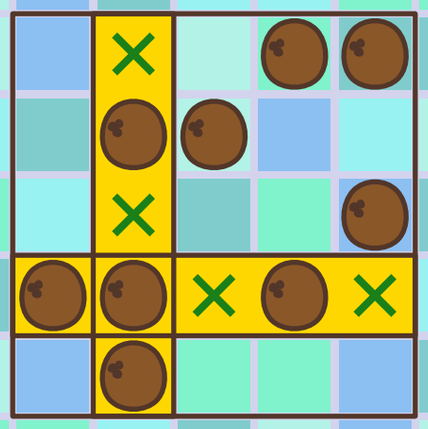

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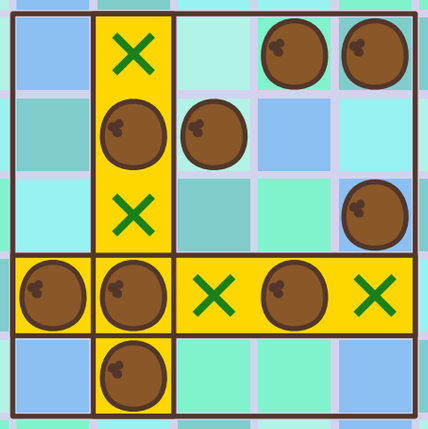

Observation
Every move decreases the potential by at least 1.

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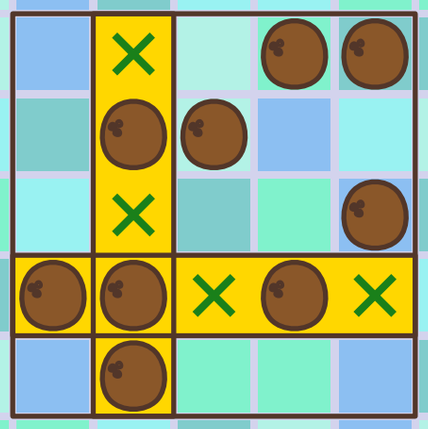

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Every move decreases the potential by at least 1.

But it can decrease arbitrarily!

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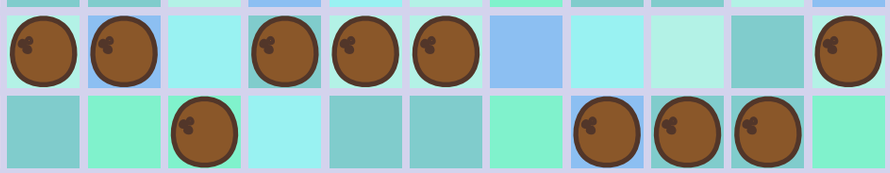

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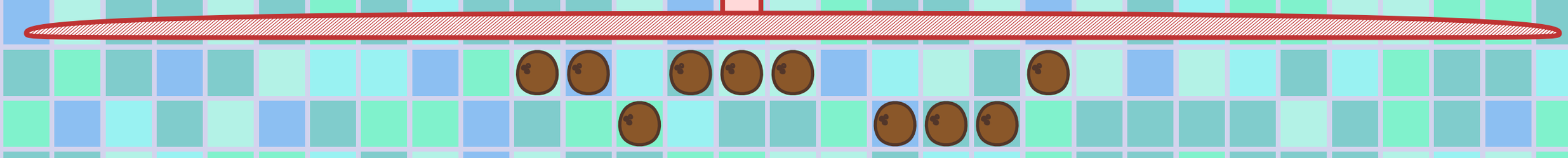

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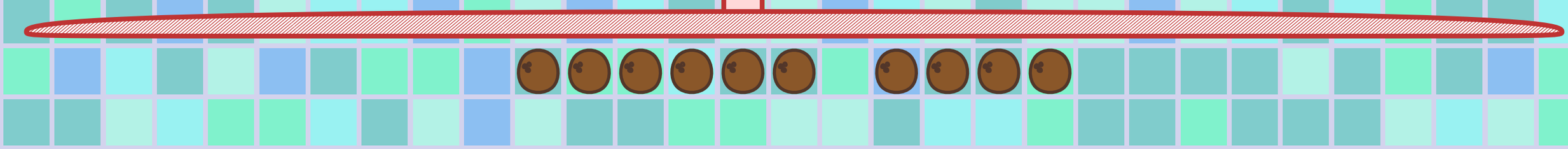

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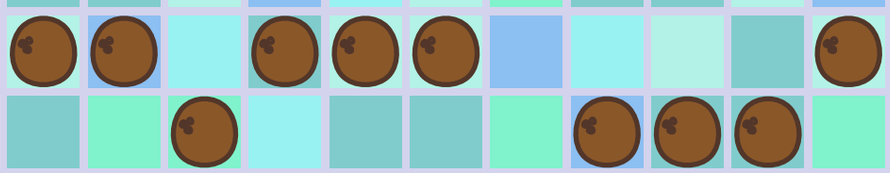

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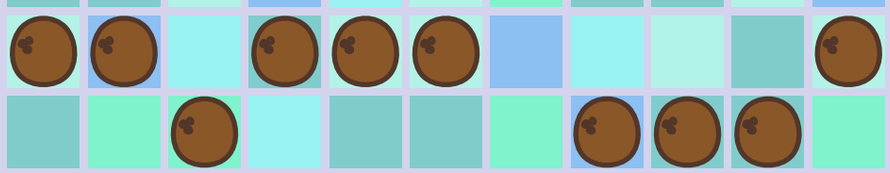

Observation
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But it can decrease arbitrarily!

Controlling the potential is non-trivial.

Idea

The winning condition is reaching a configuration with potential 0. Maybe it is a good idea to make the potential *even* when possible?

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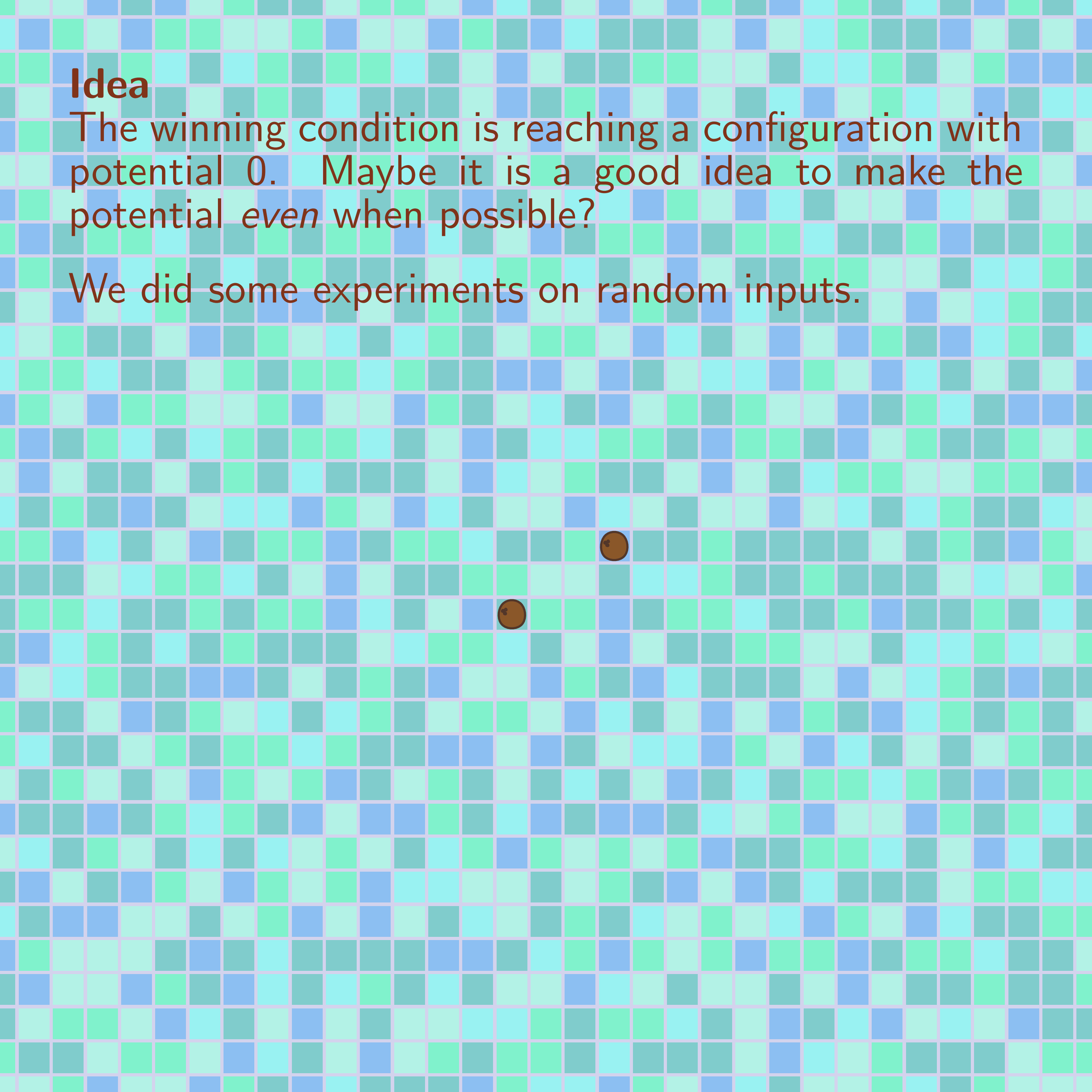

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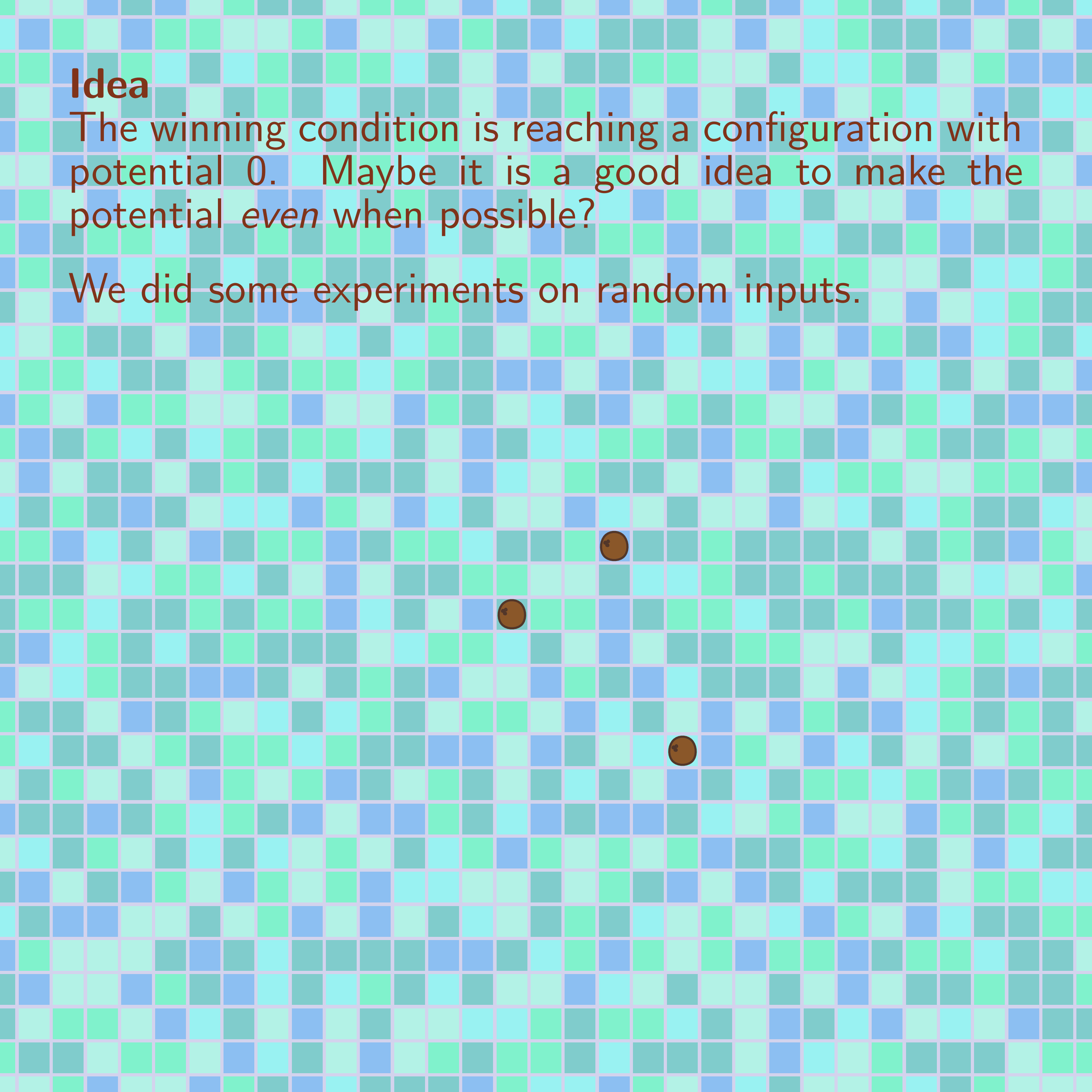

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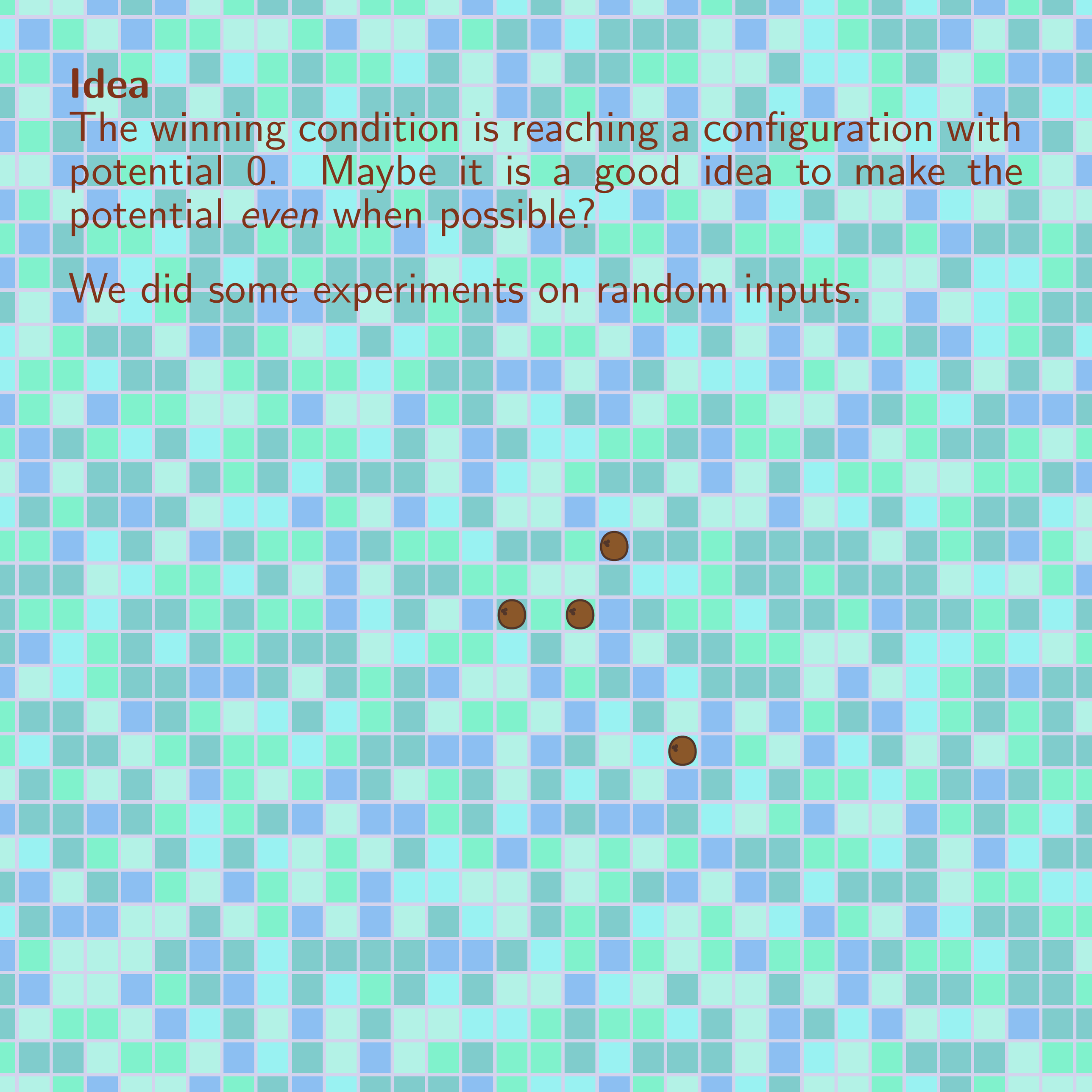

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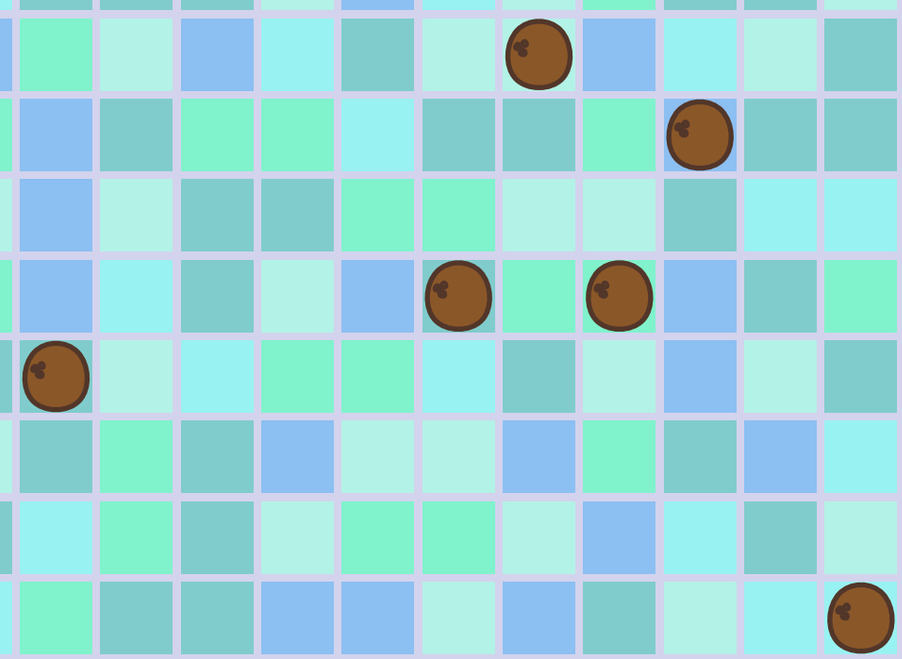

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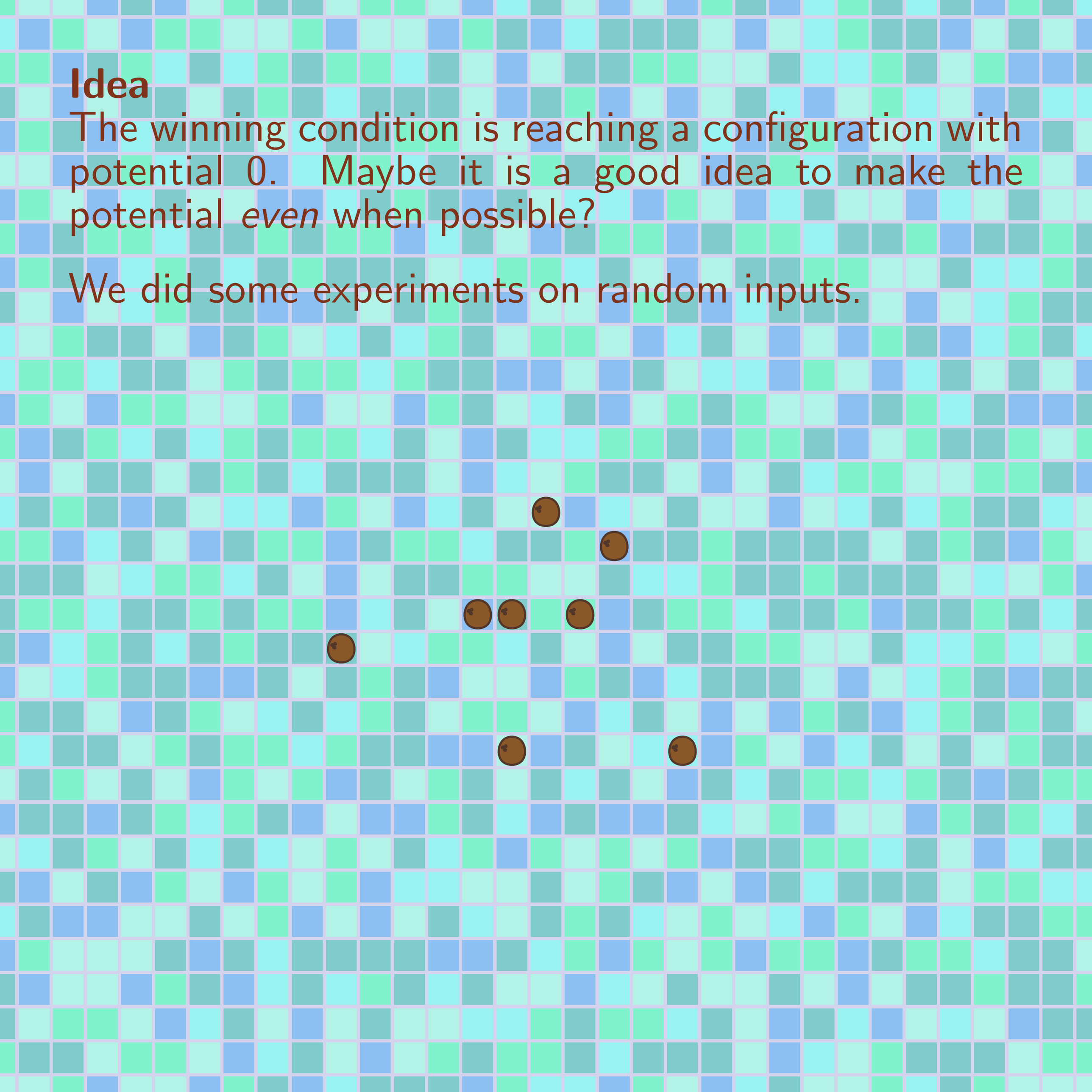

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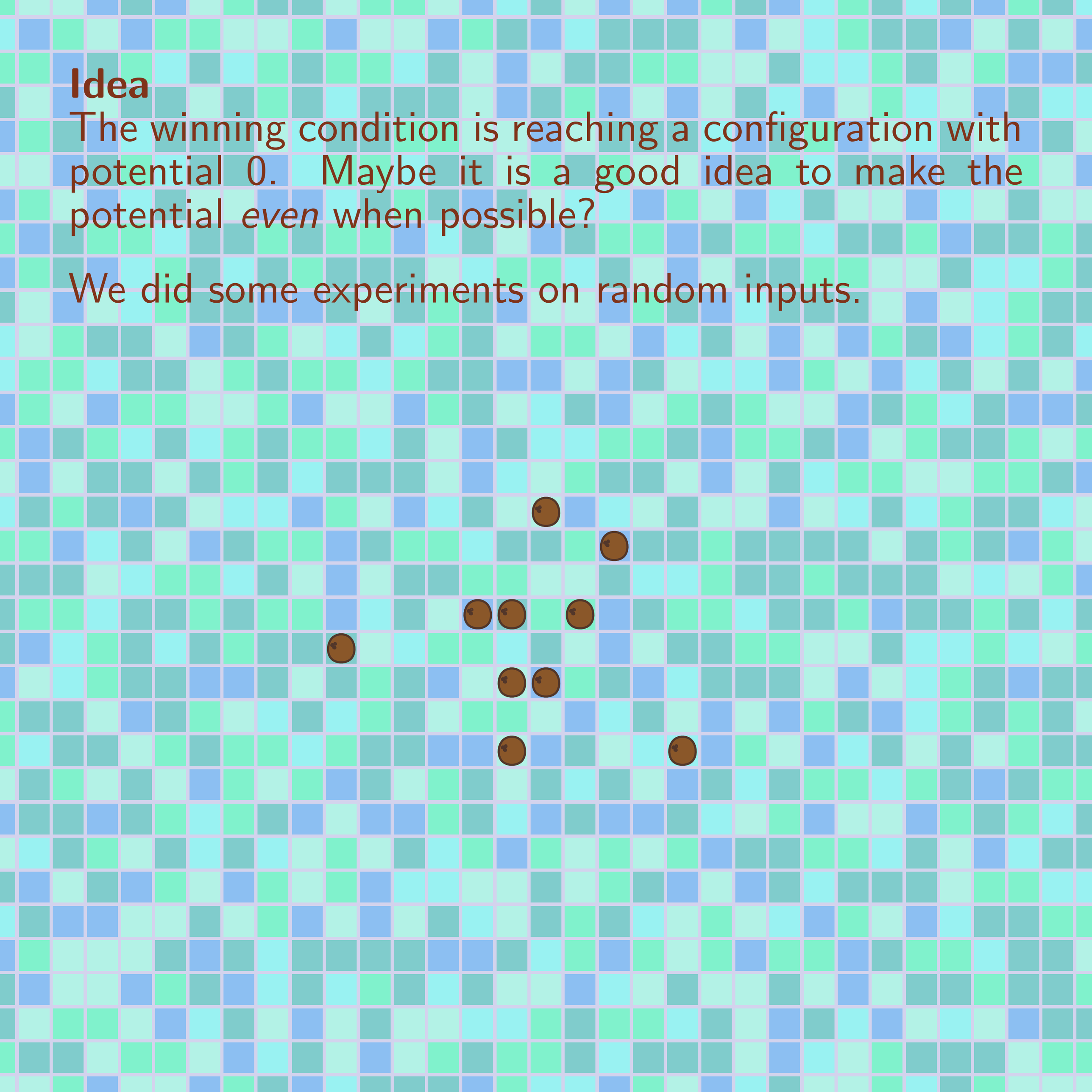

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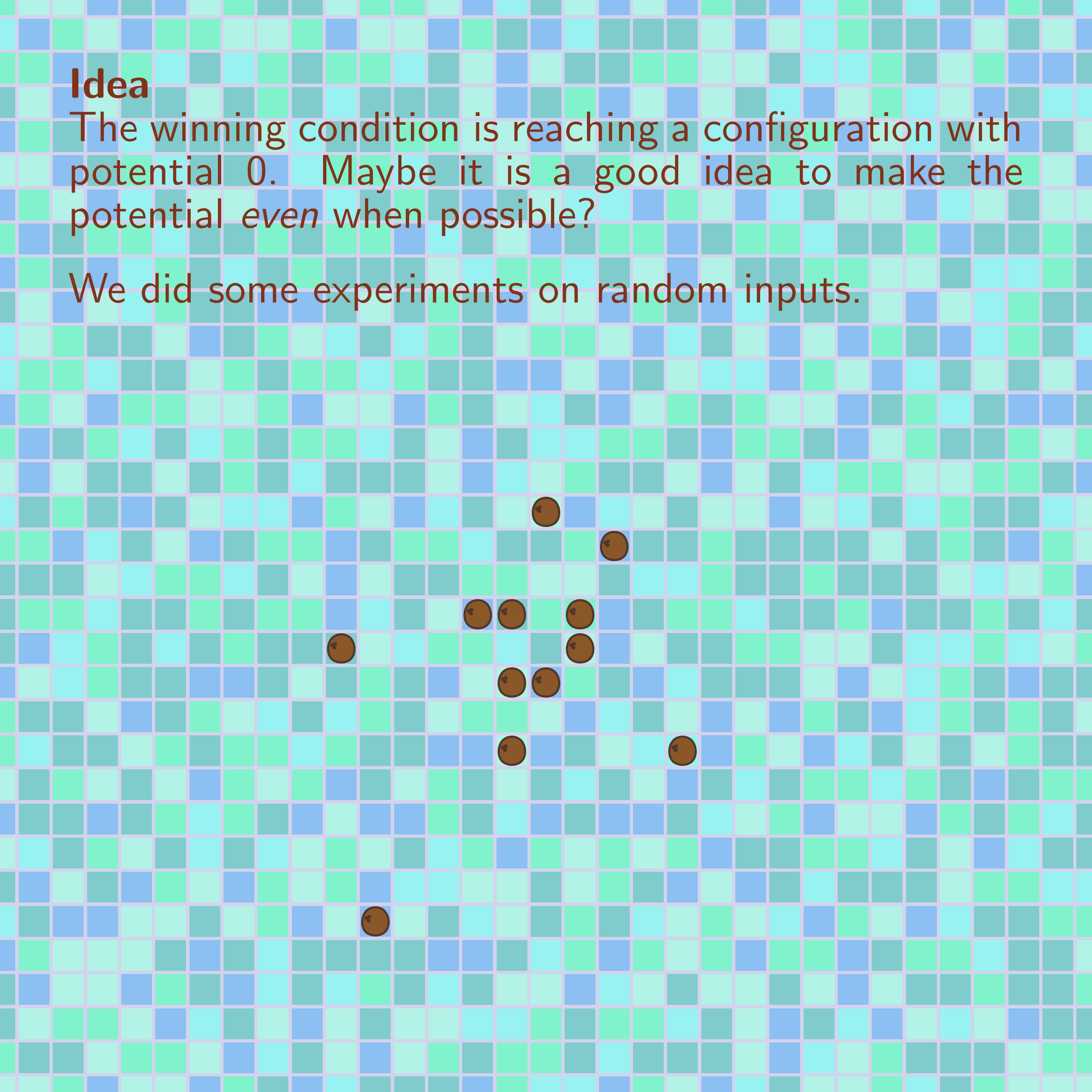

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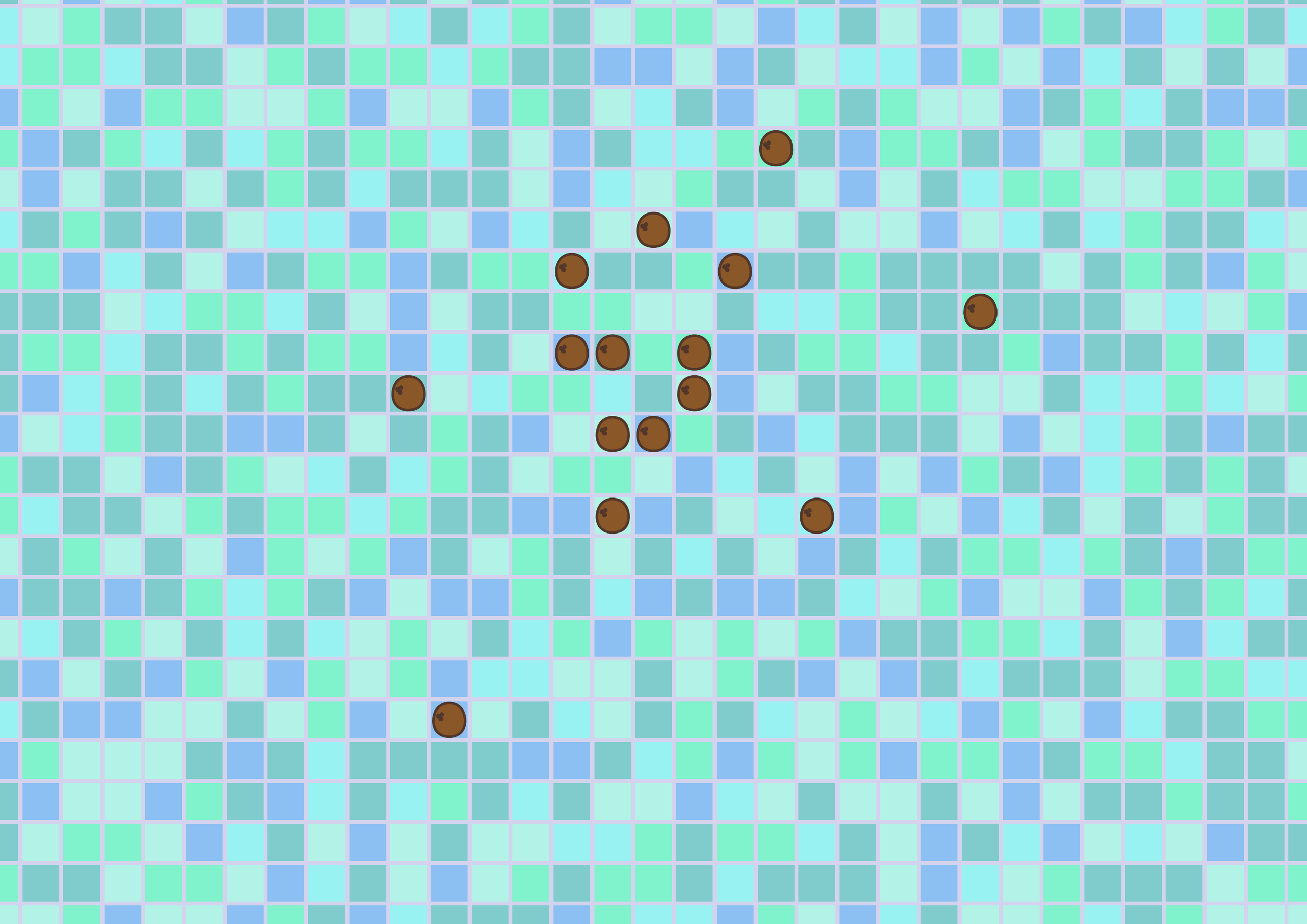

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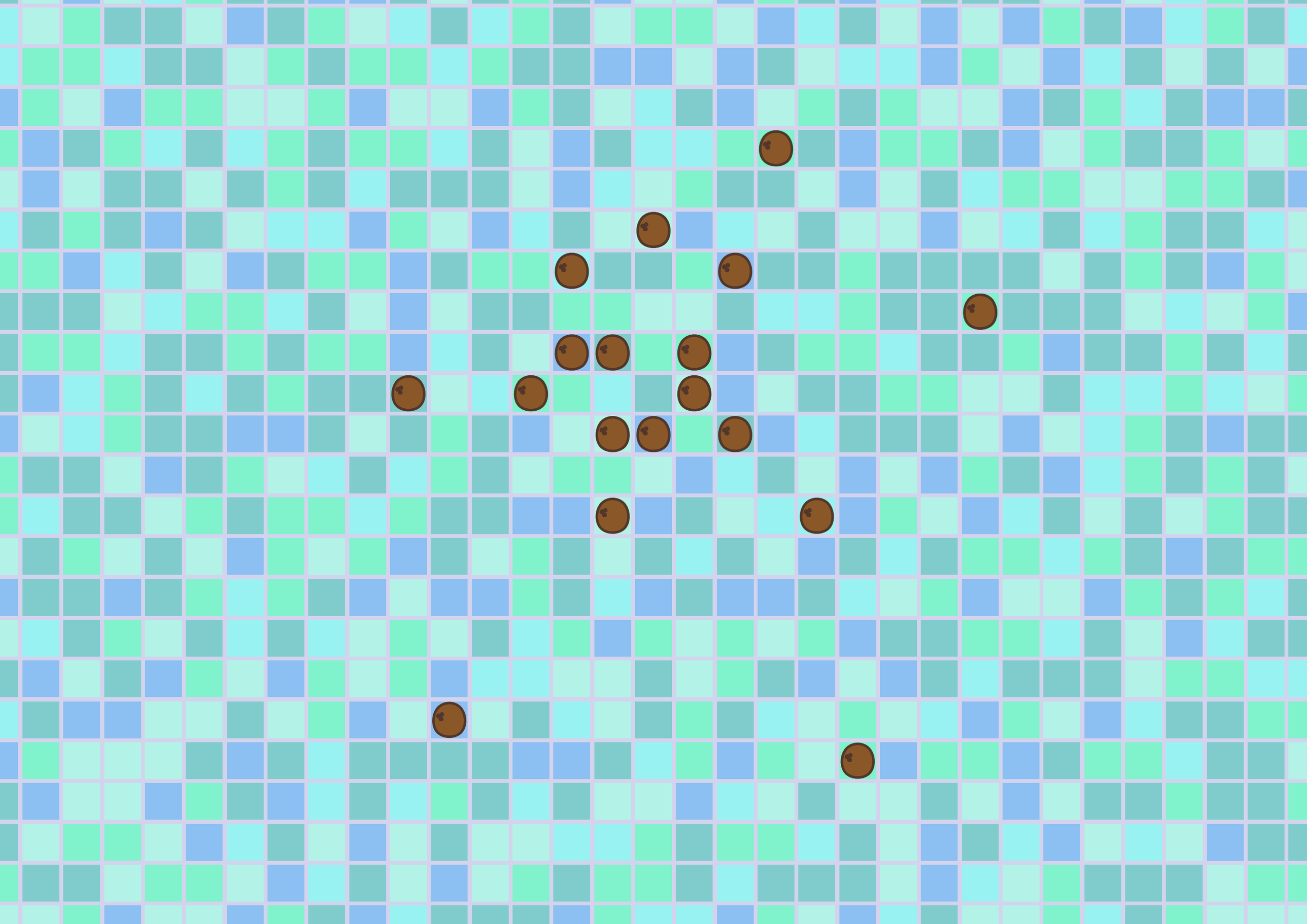

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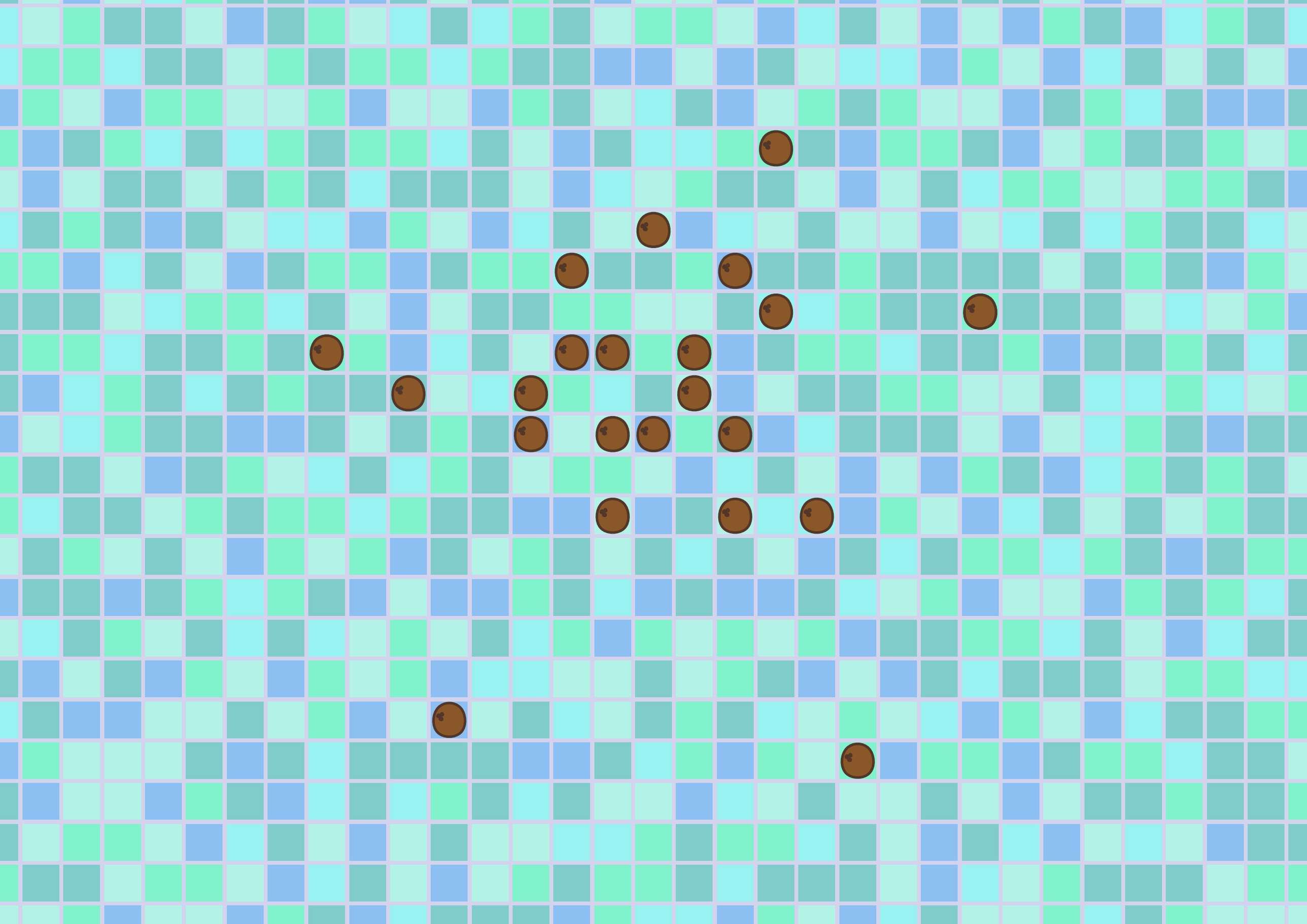

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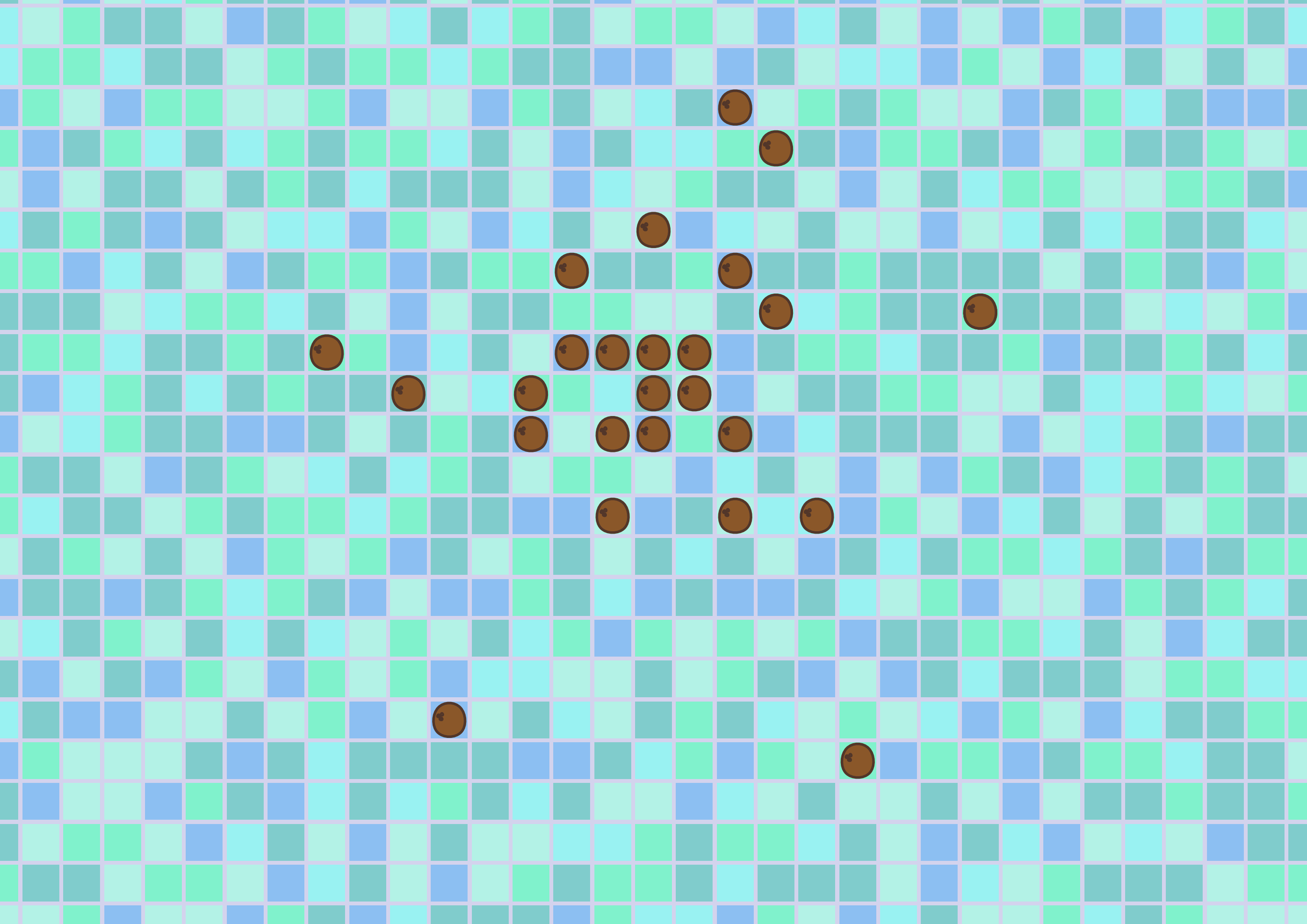

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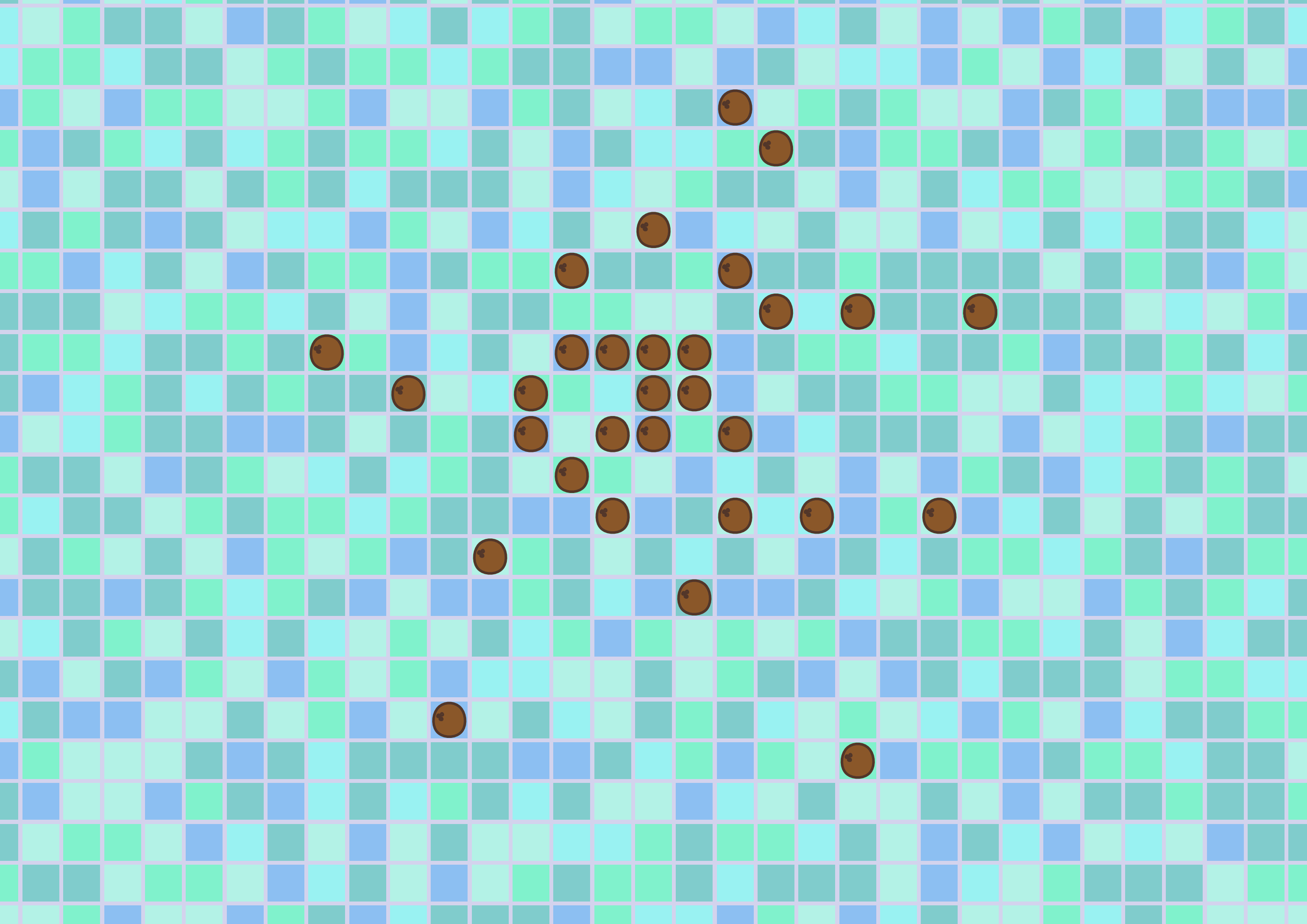

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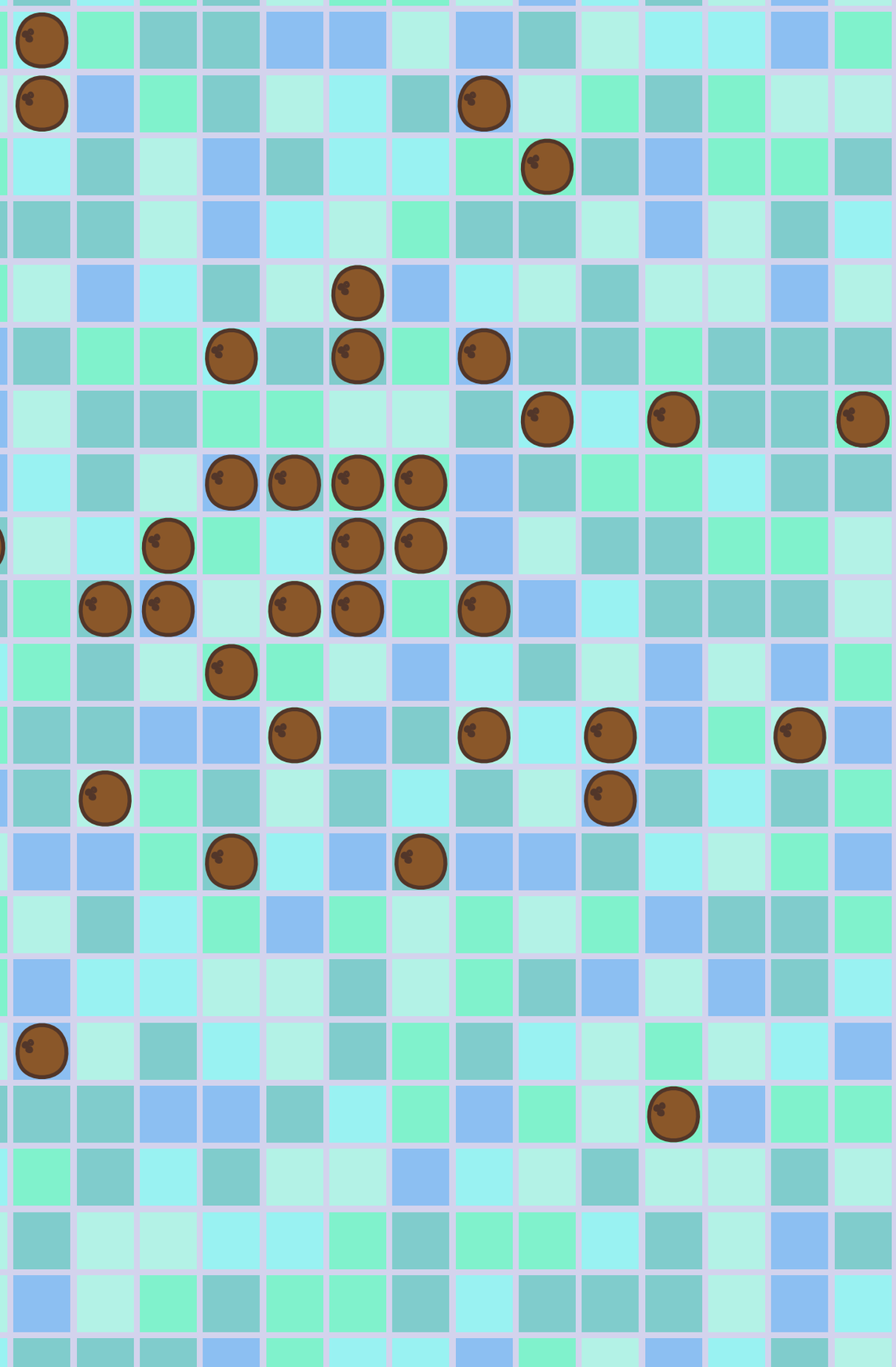

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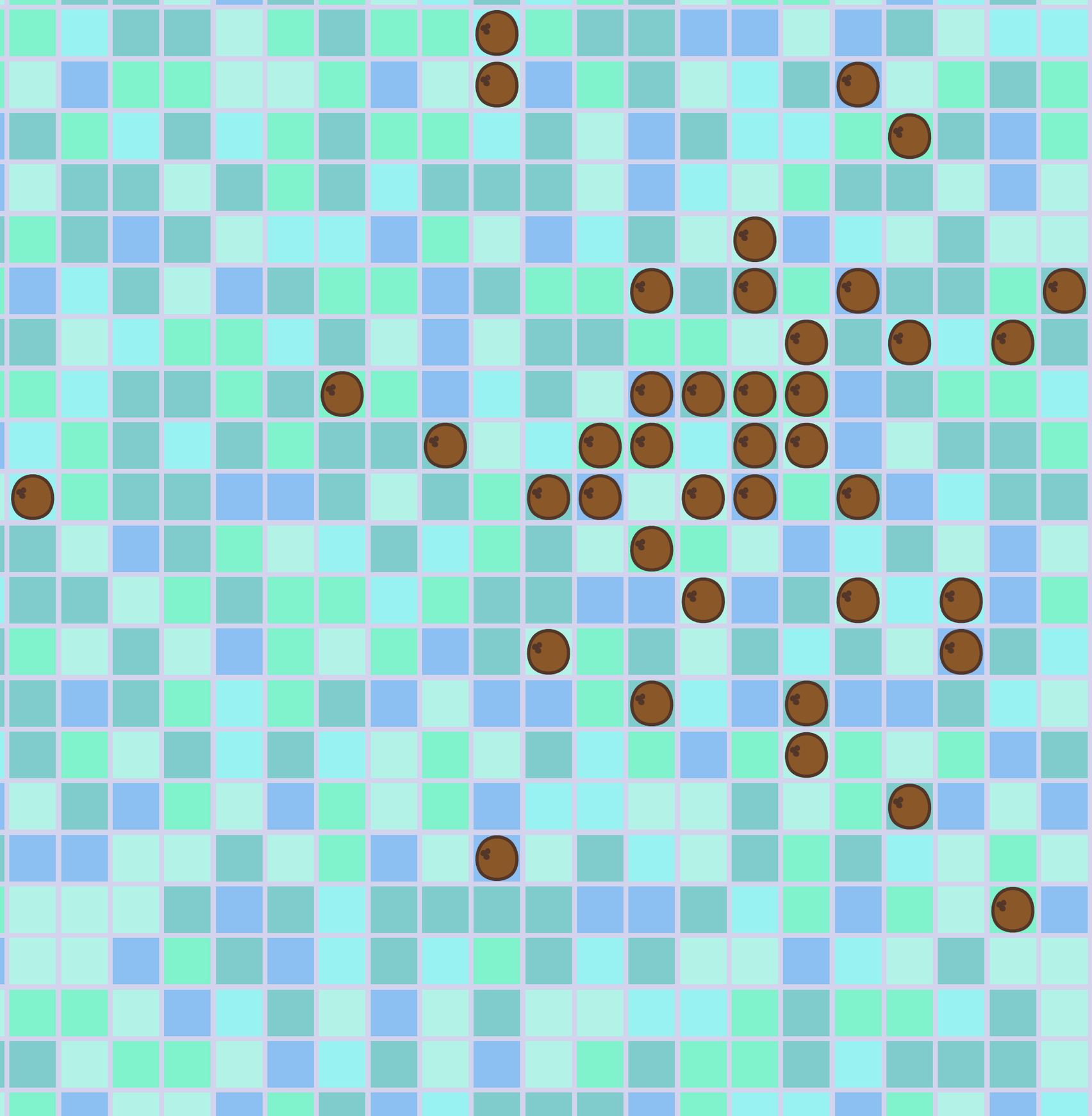

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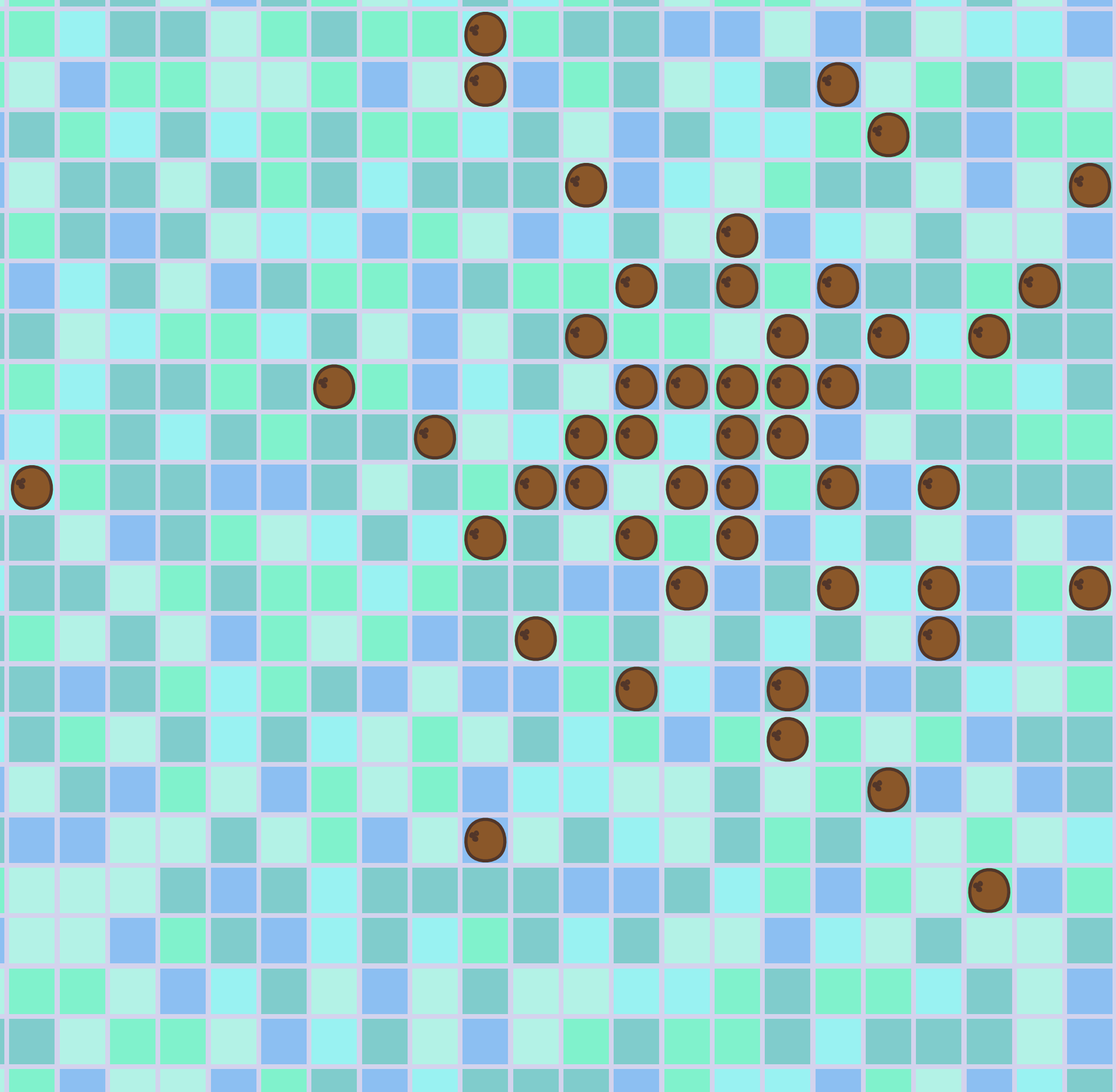

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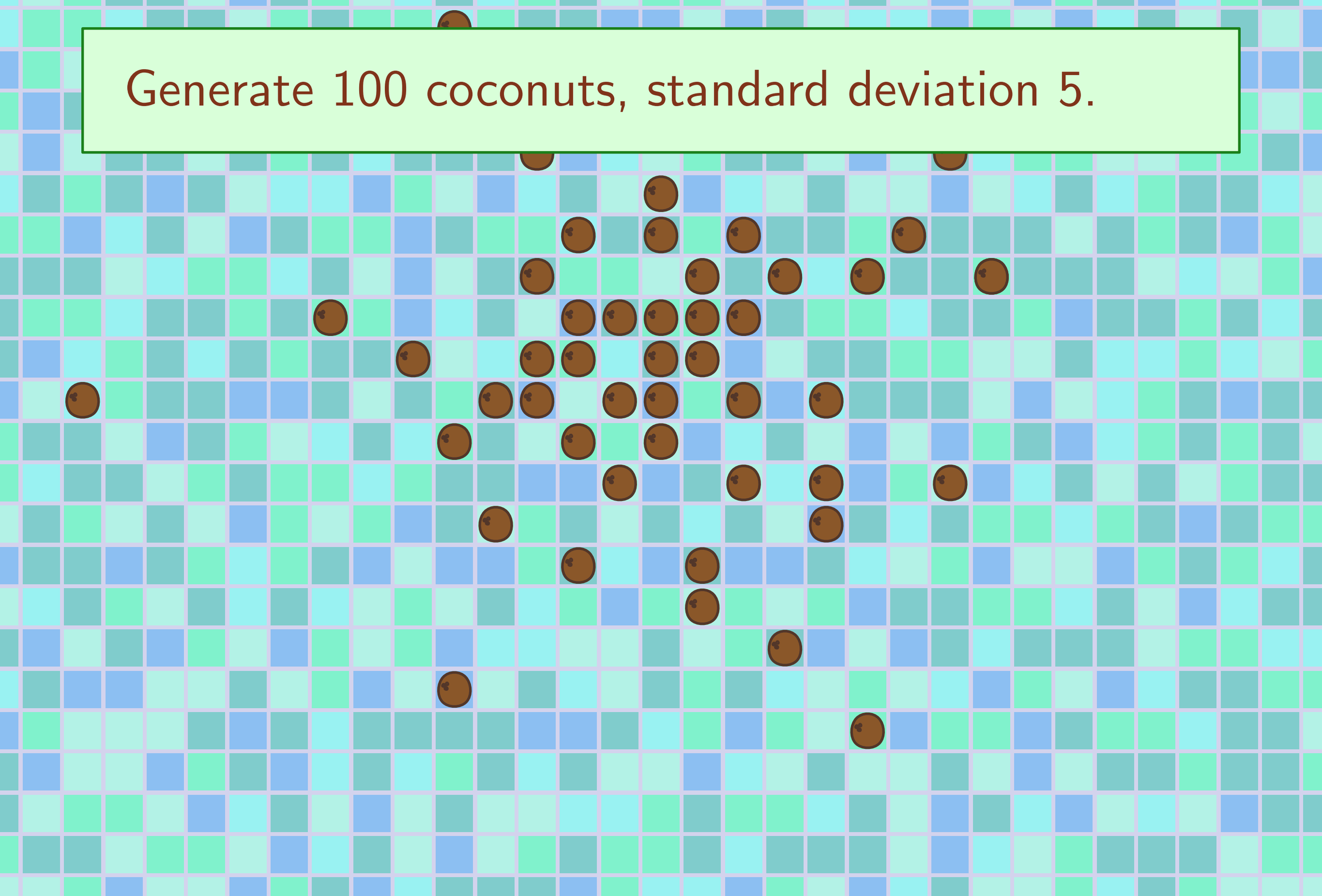
The winning condition is reaching a configuration with potential 0. Maybe it is a good idea to make the potential *even* when possible?

We did some experiments on random inputs.

Generate 100 coconuts, standard deviation 5.



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Just play randomly.

Player 1 wins $\approx 70\%$ of the games!

Open Question

Is there a better measure of the state of the game than the potential?

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Open Question
Is LAST MOVE WINS *hard?*

GAME 2

K IN A ROW

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In this game, the coconuts are also red or blue.

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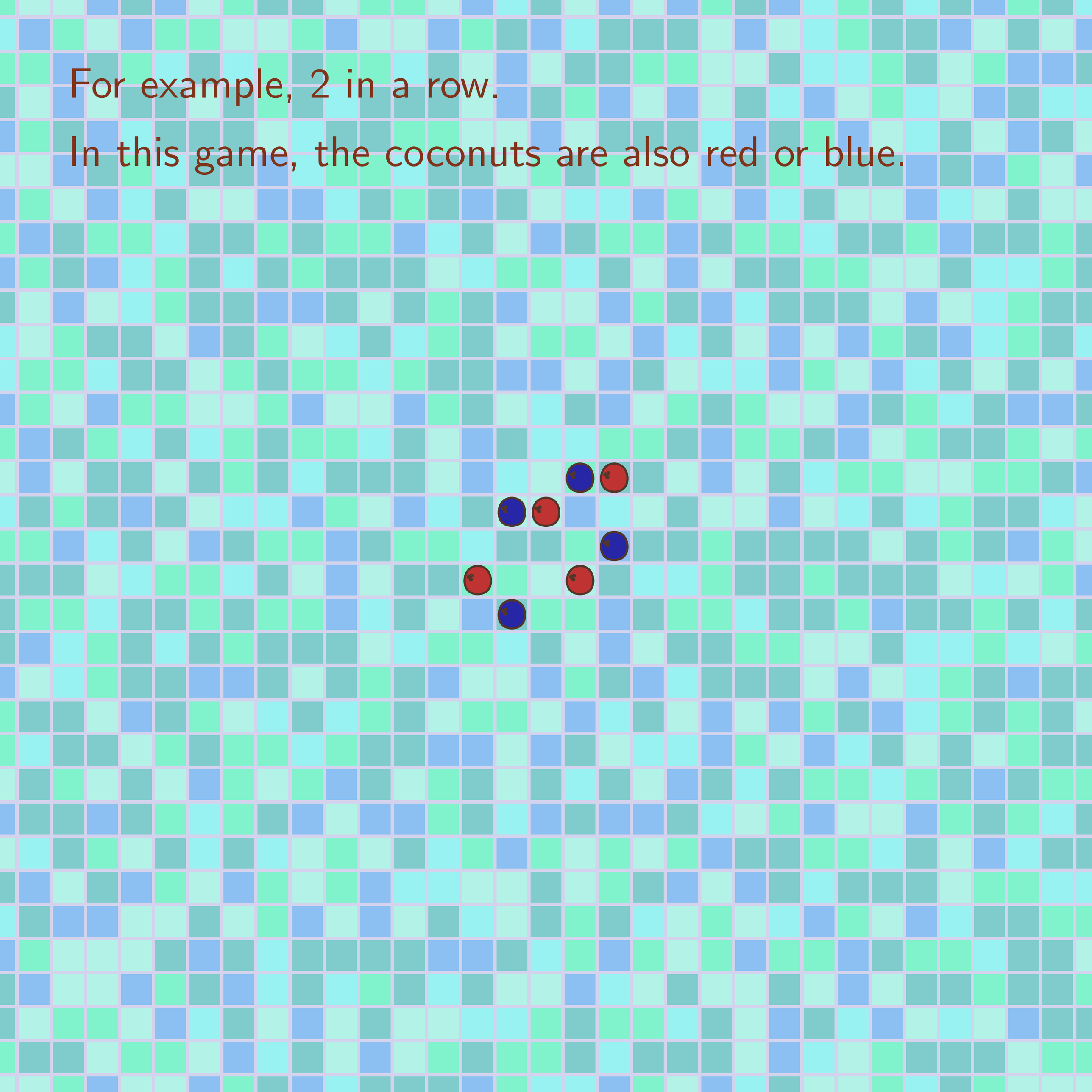

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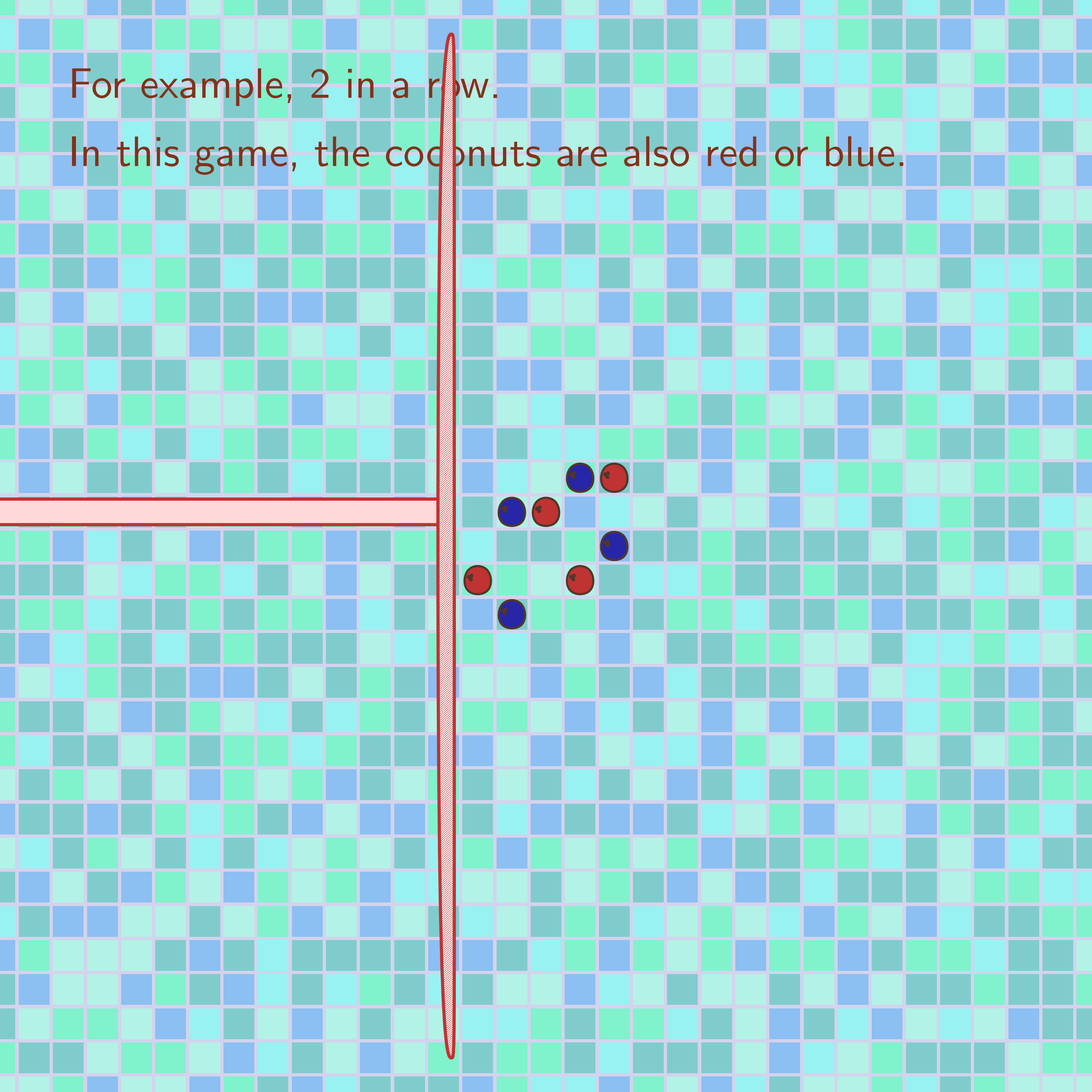

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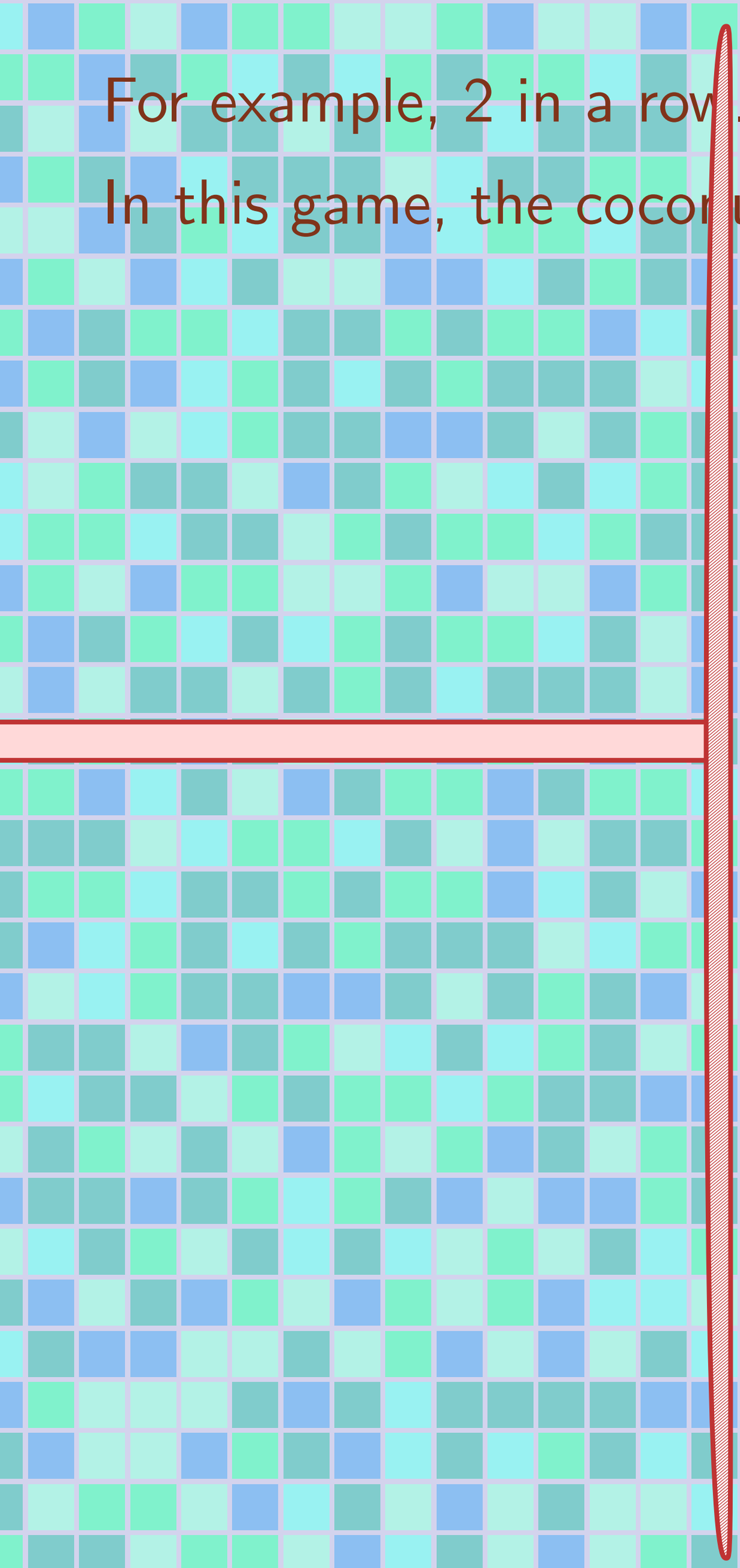

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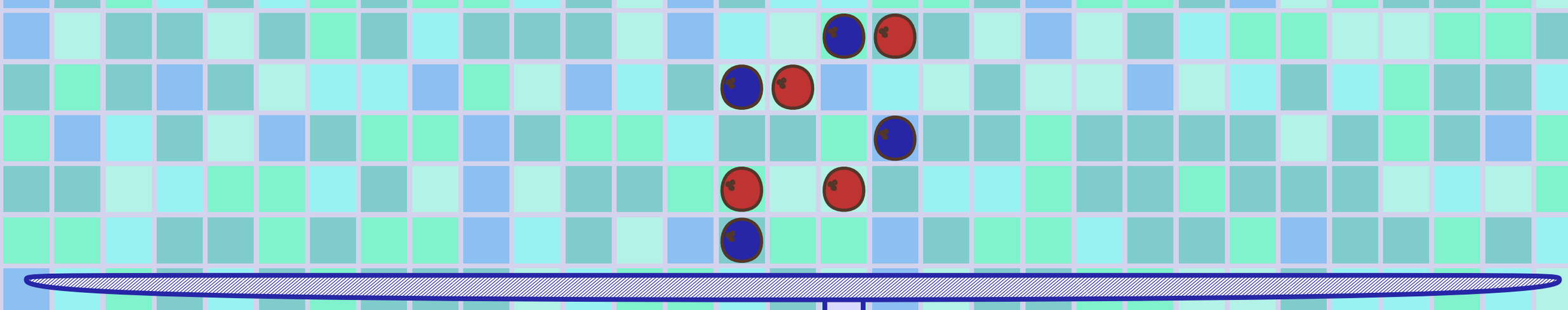

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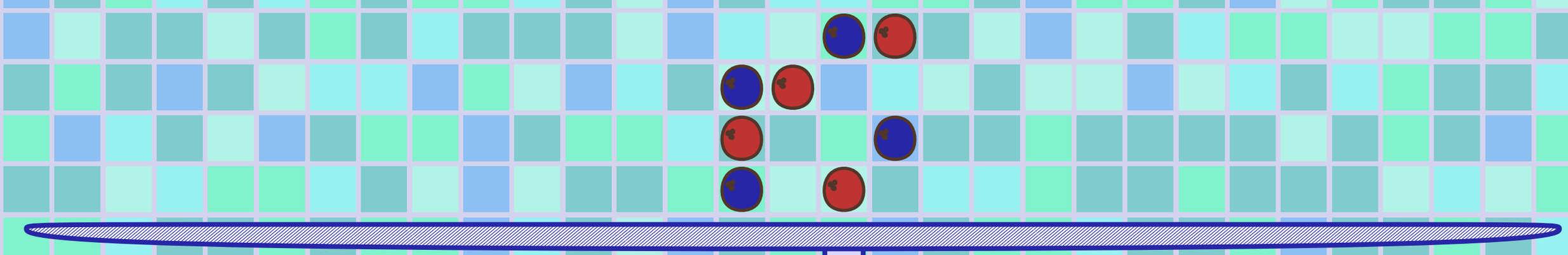

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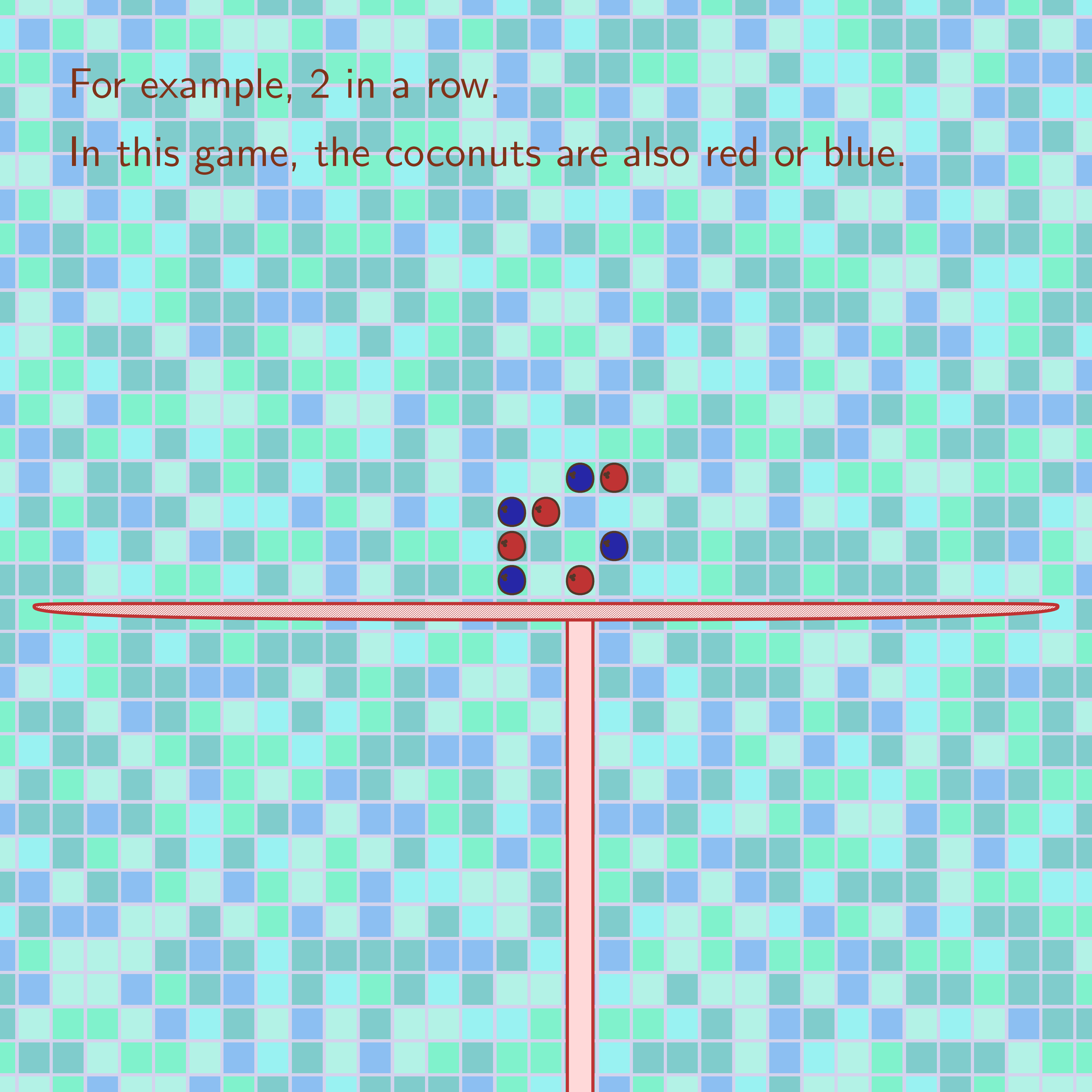

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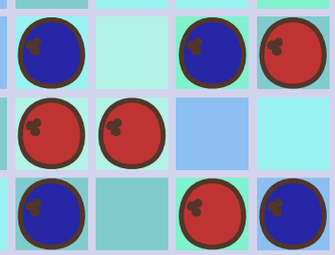

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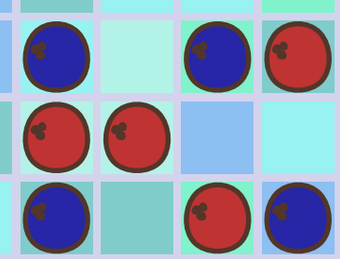
RED WINS!



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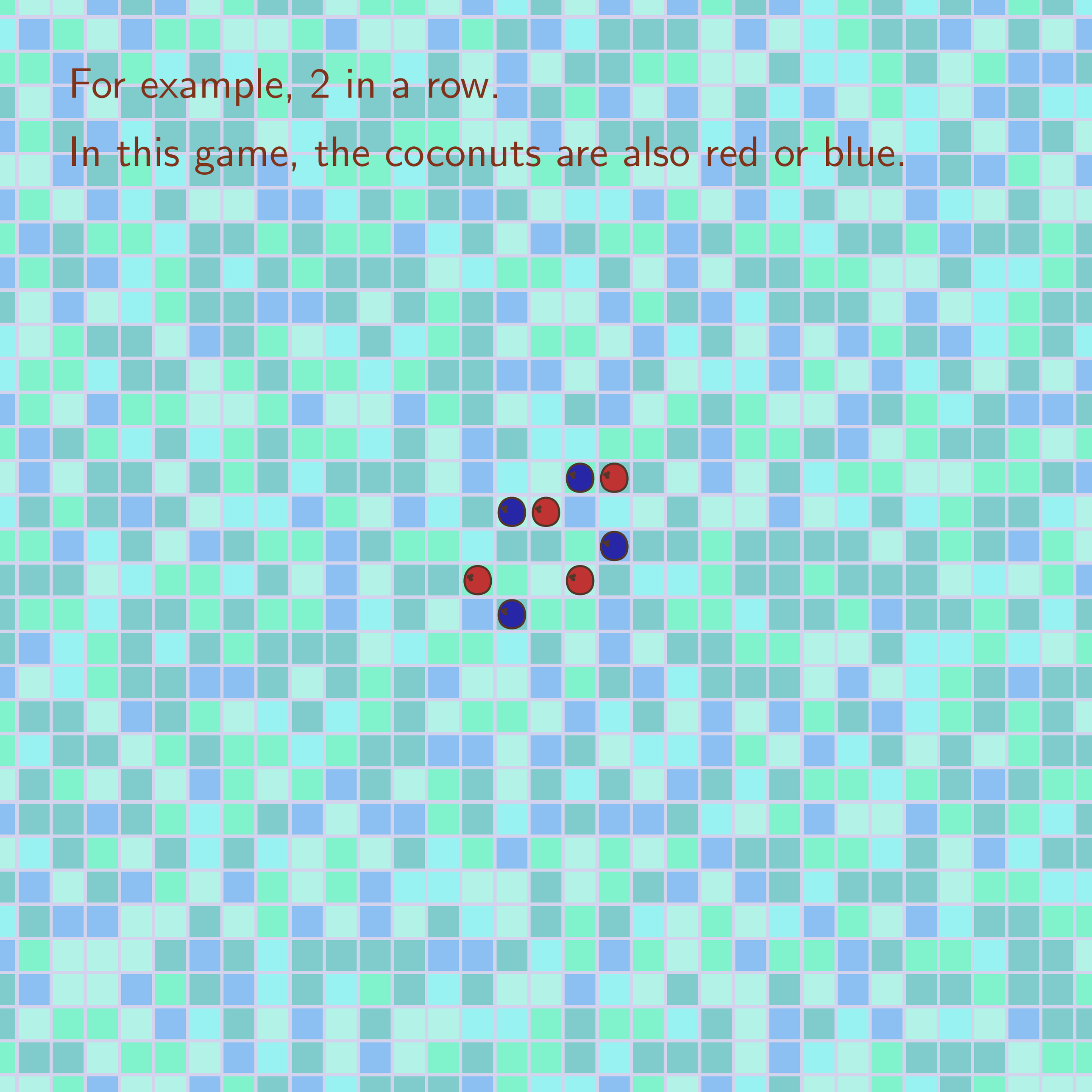

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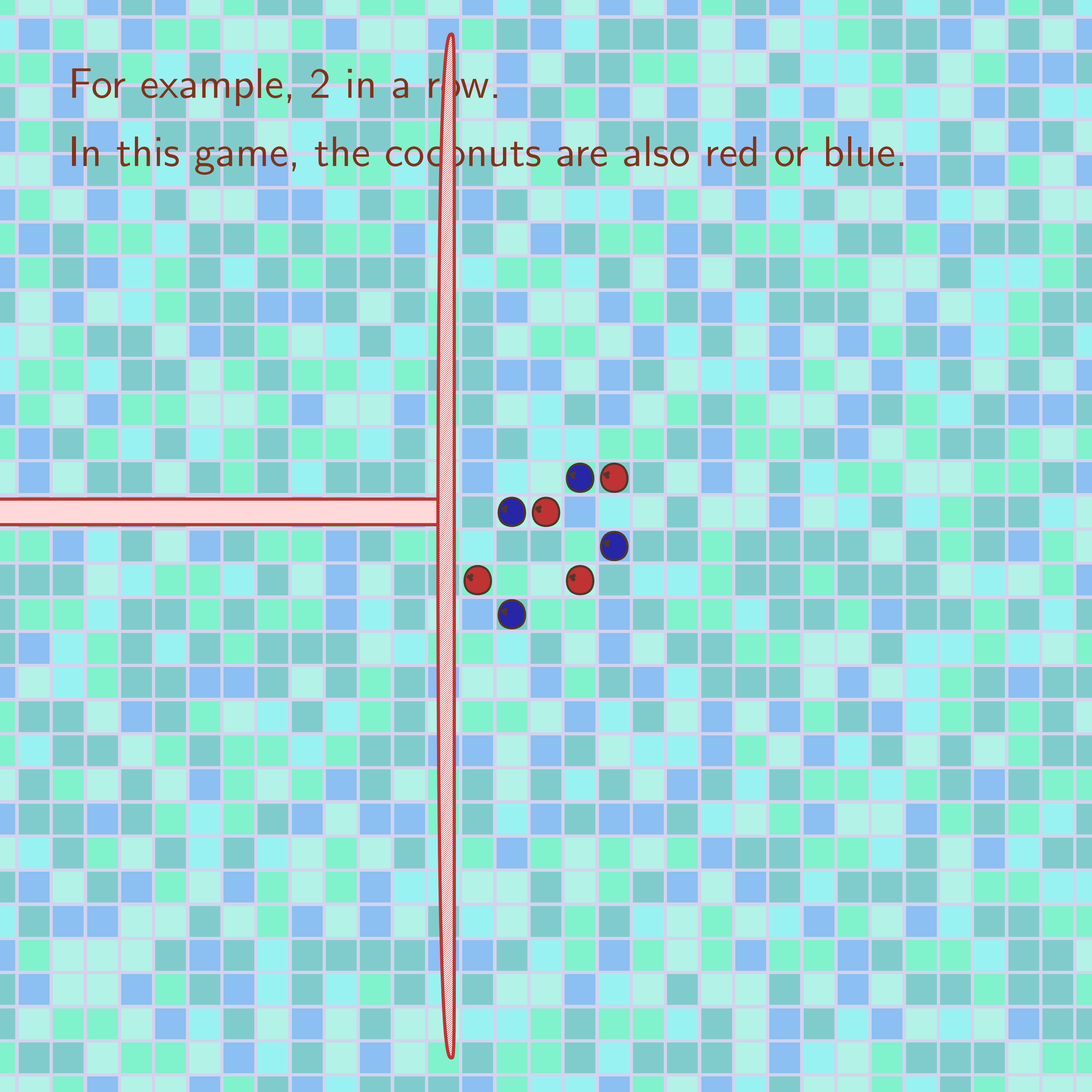

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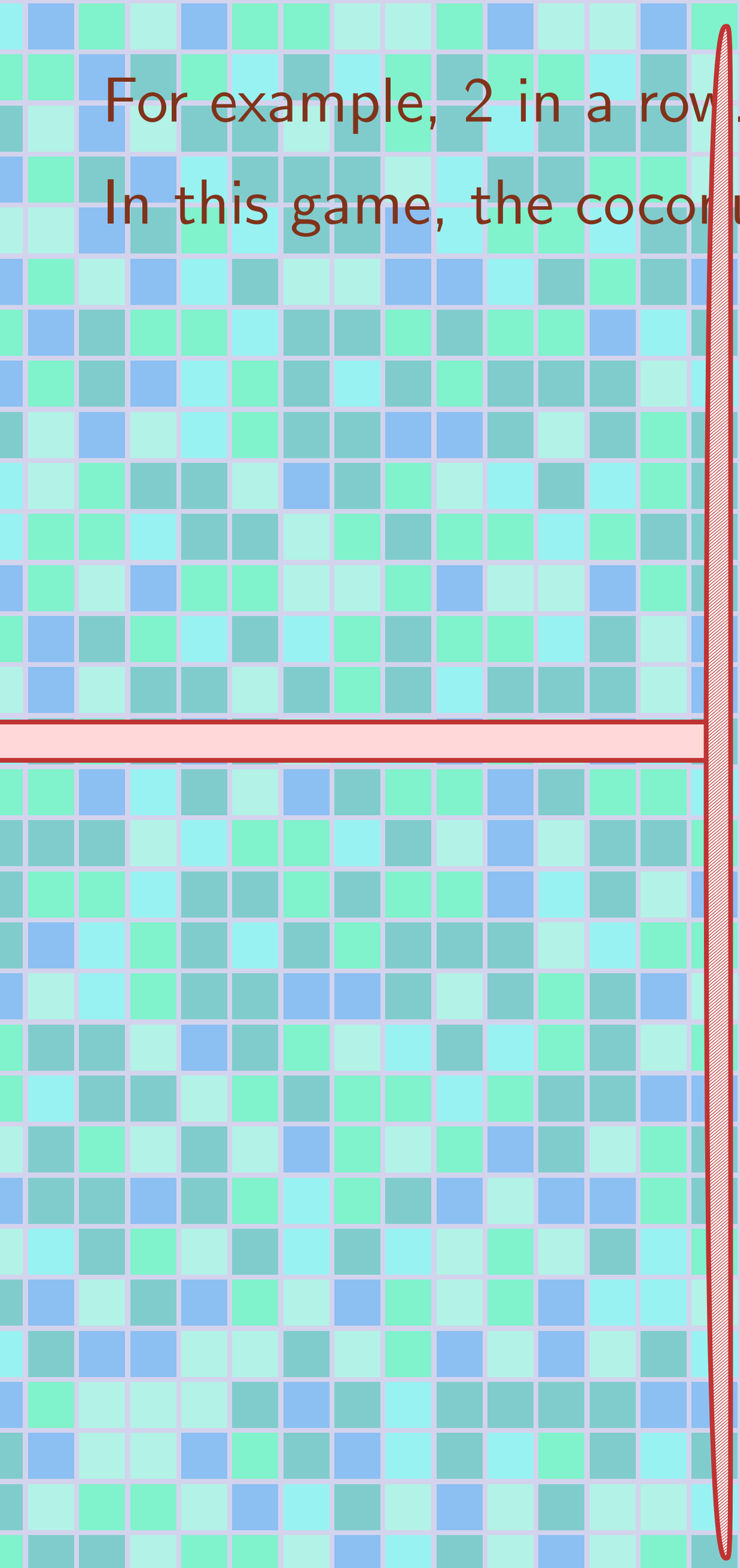

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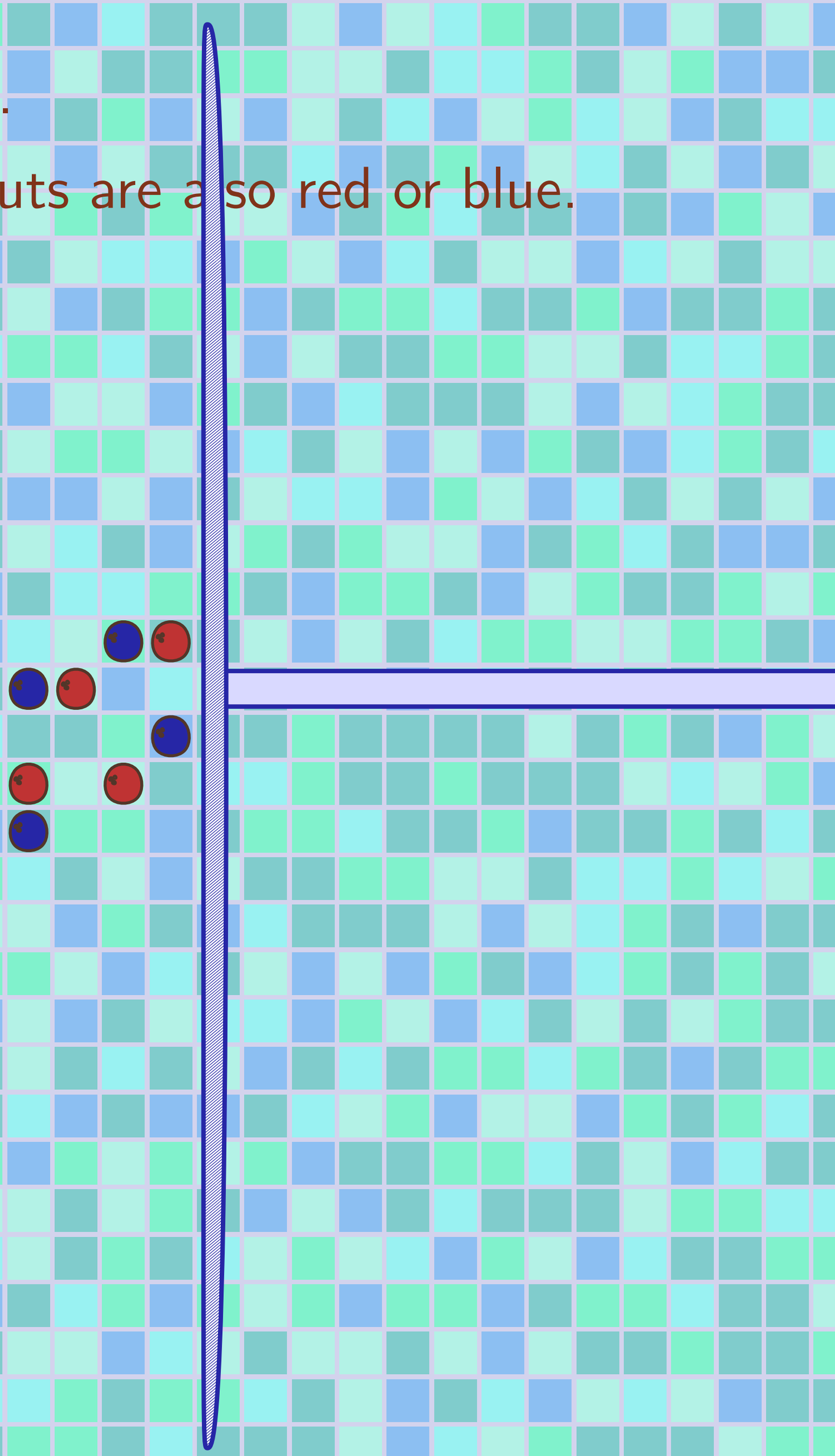

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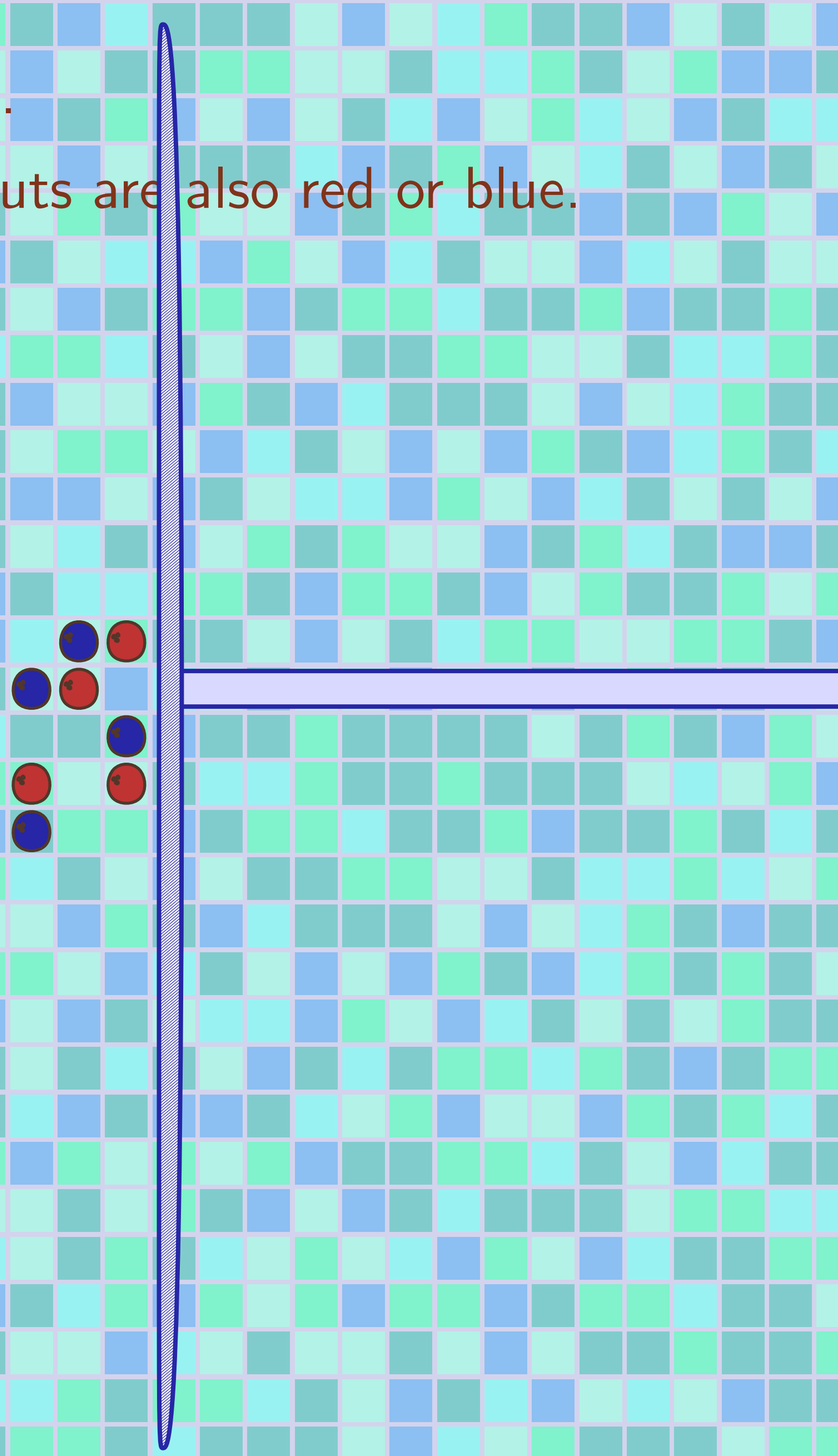

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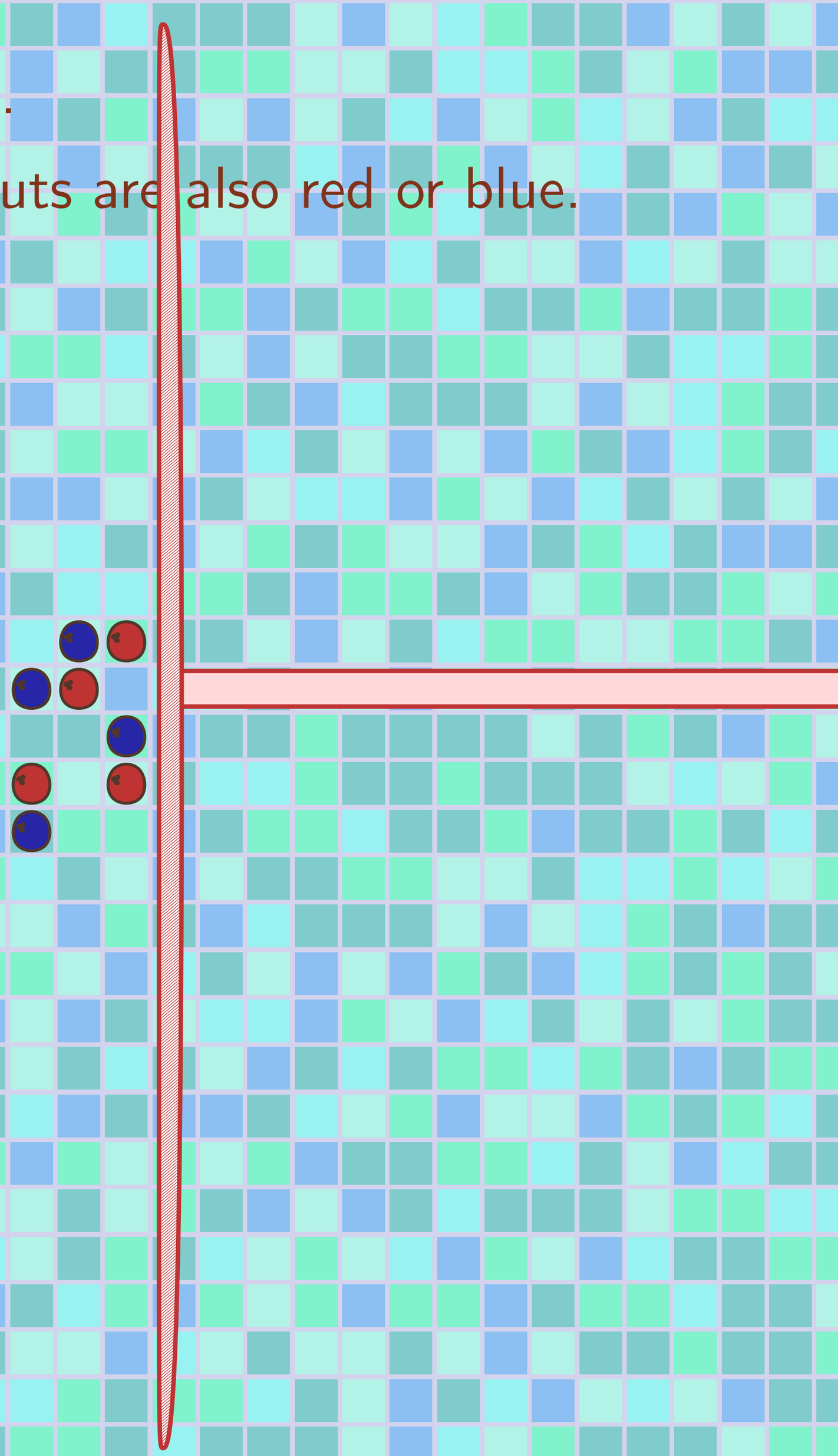

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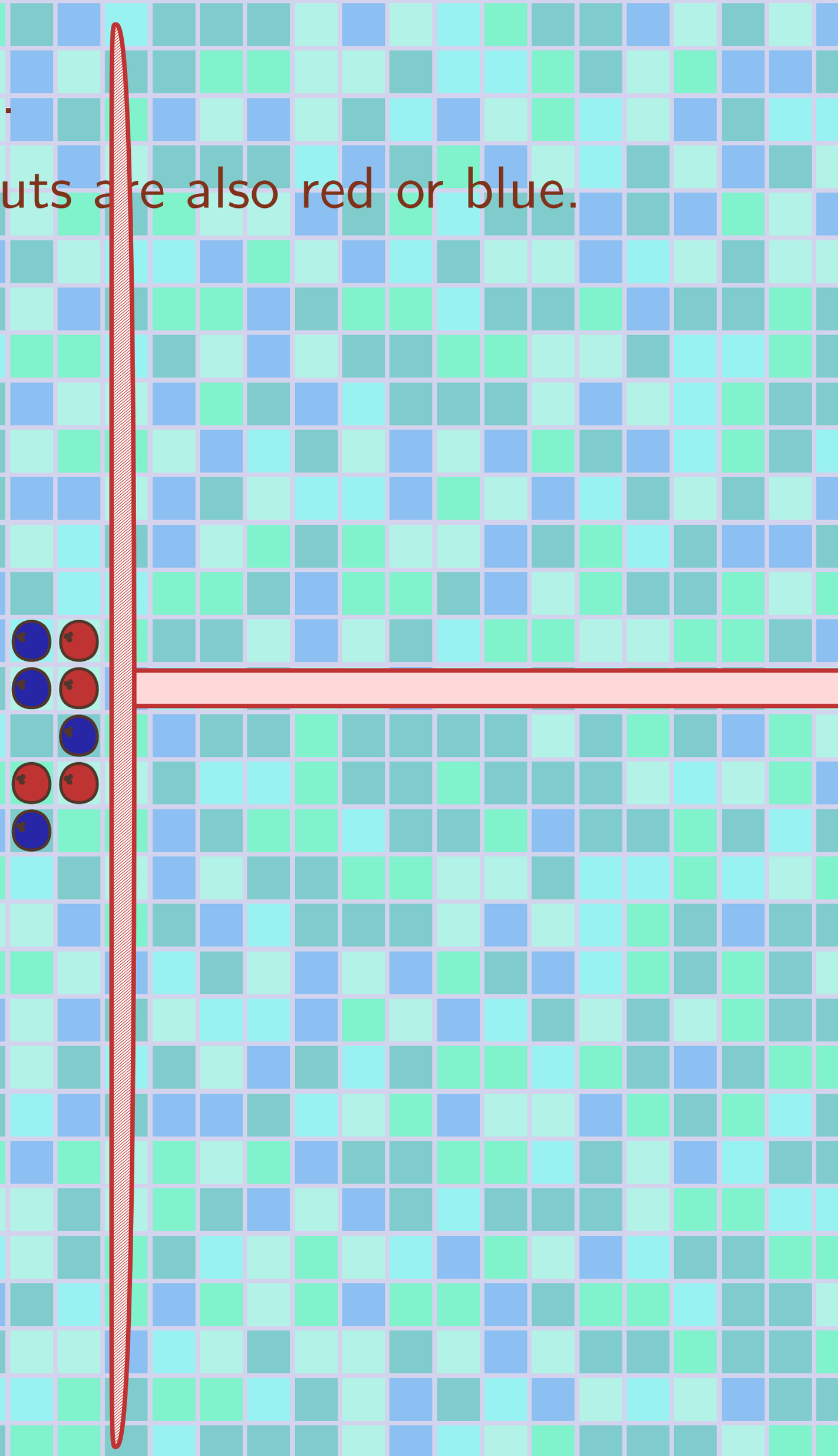

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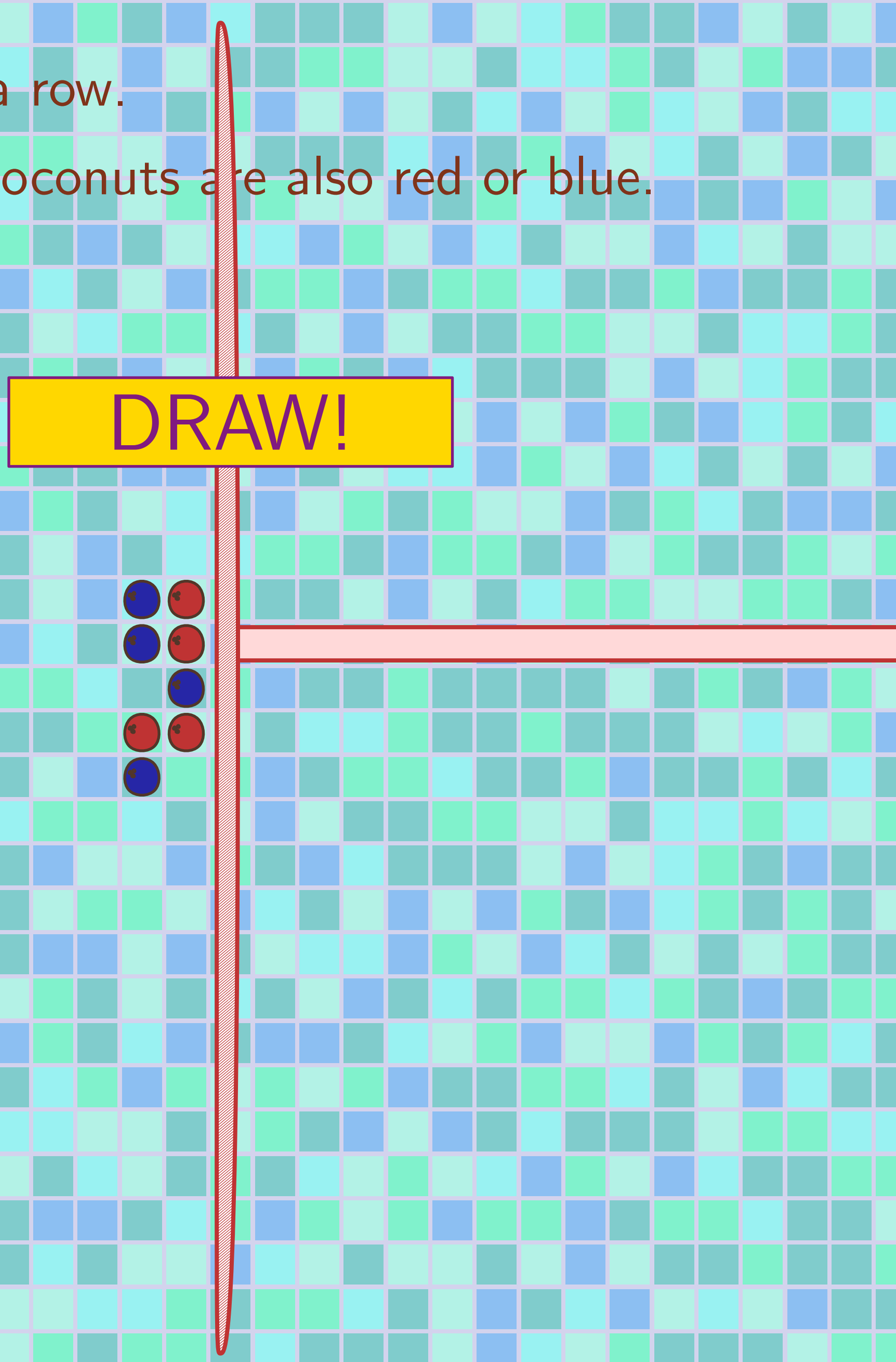

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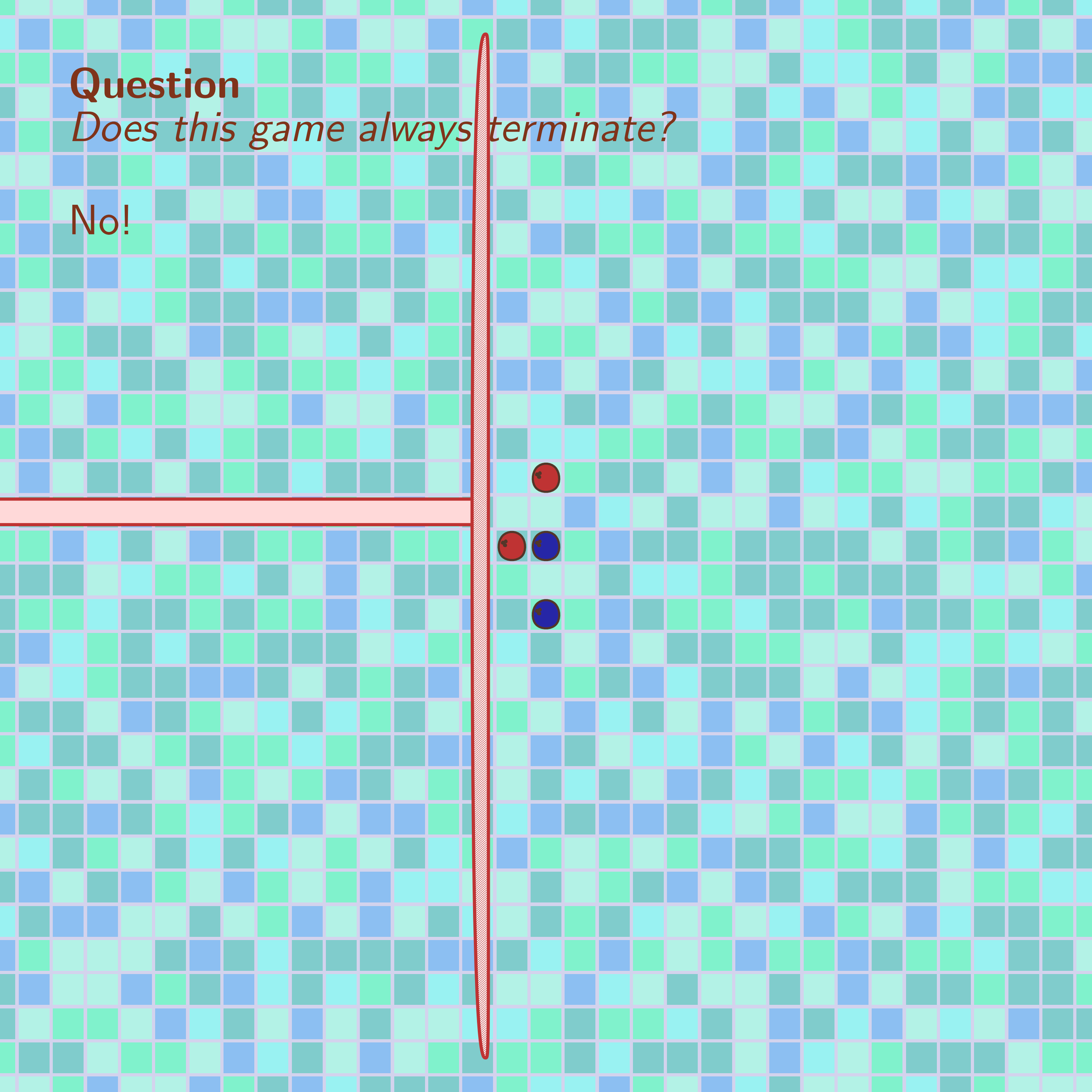

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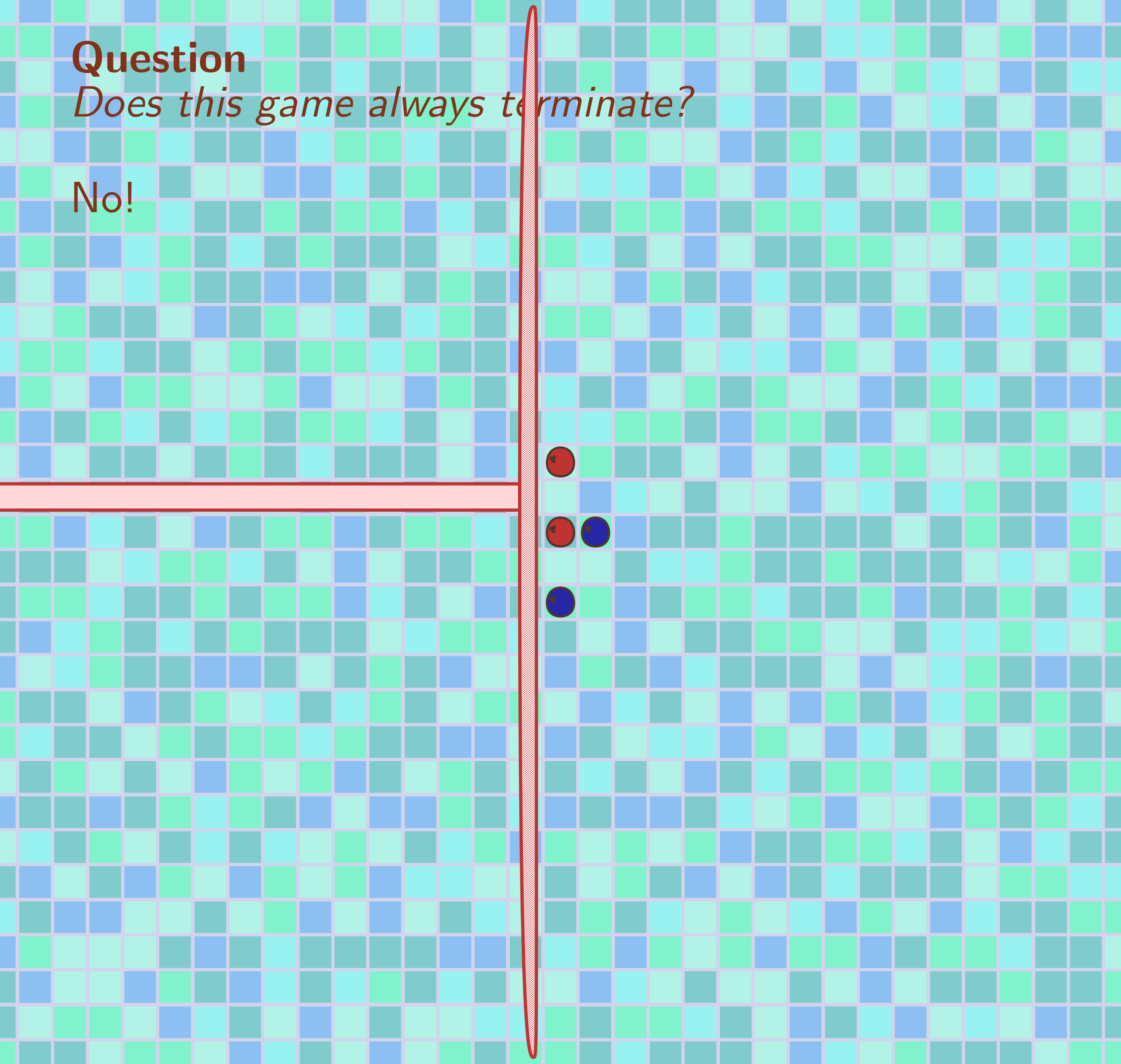

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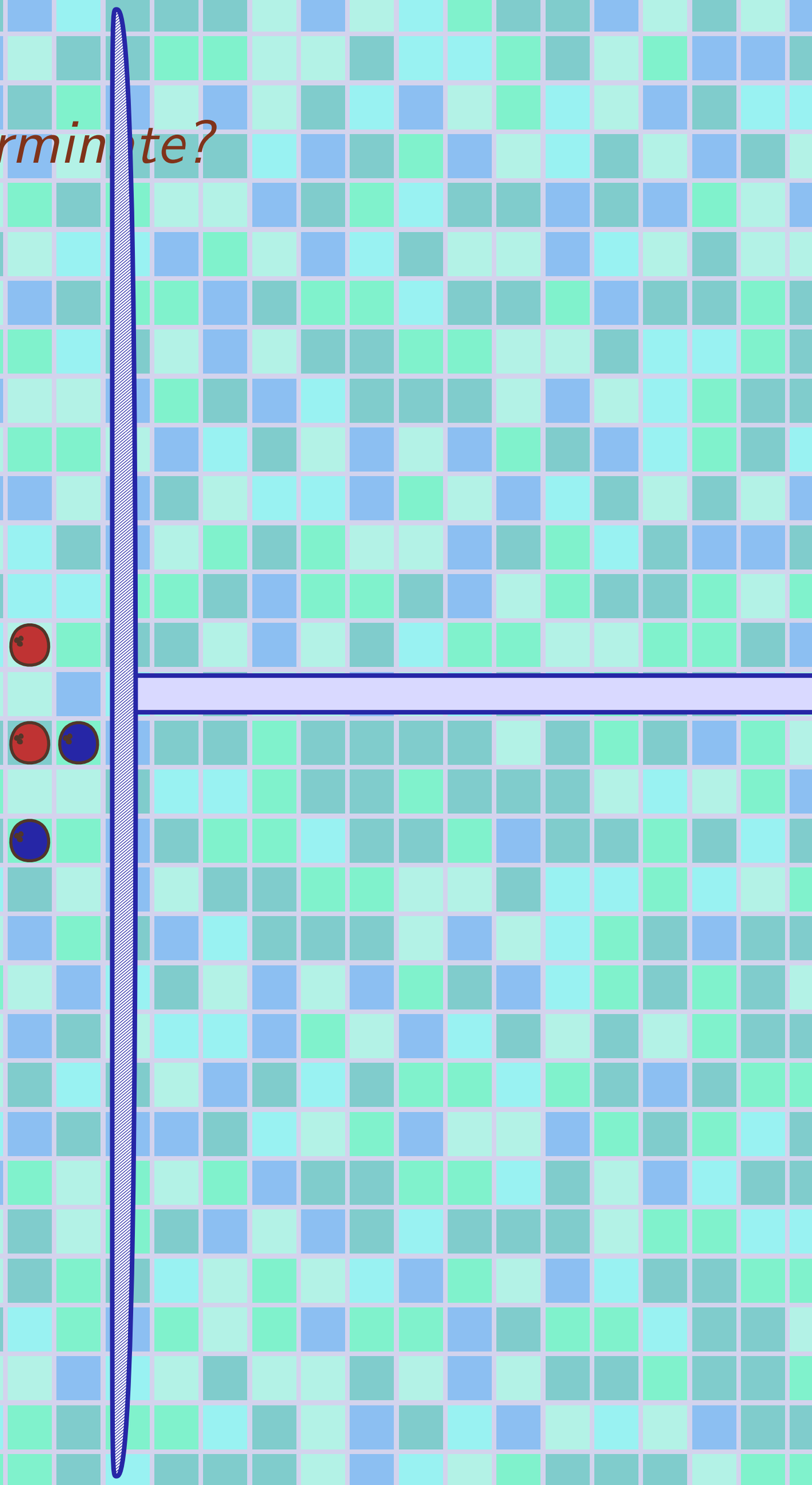

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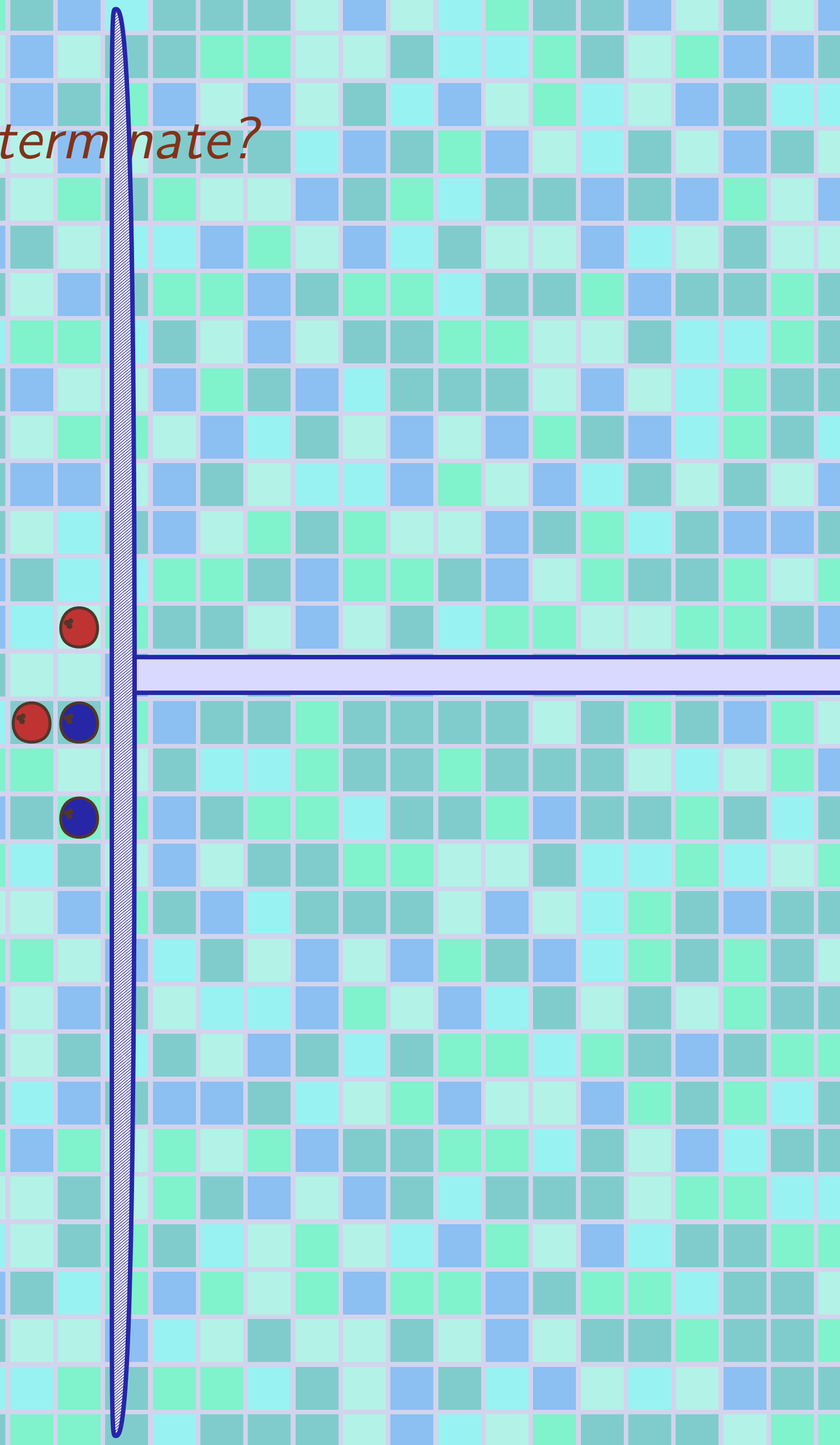

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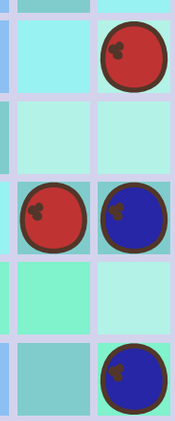

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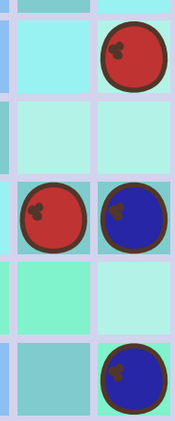

Open Question
Is 2-IN-A-ROW *hard?*

QUESTIONS?