

Hugo Akitaya
Maarten Löffler Utrecht University
Anika Rounds Amazon
Giovanni Viglietta JAIST


Trash Cubes designed by Mary Bogdan, 2008. Brooklyn Art Project












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# KNOWN RESULTS 

## Question

Can we always push our coconuts into tidy rectangles?

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... and some aspect ratios are not even possible if we start with at most one coconut per row and column.

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[Akitaya, Aloupis, Löffler, Rounds, 2016]

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Controlling the potential is non-trivial.

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The winning condition is reaching a configuration with potential 0. Maybe it is a good idea to make the potential even when possible?

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Make potential even when possible.
Otherwise random.

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Otherwise random.

Player 2 Just play randomly.

Player 1 wins $\approx 70 \%$ of the games!

## Open Question

Is there a better measure of the state of the game than the potential?

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Open Question Is LAST MOVE WINS hard?

$$
\begin{aligned}
& \text { GRME } 2 \\
& \text { K IN A ROW }
\end{aligned}
$$

For example, 2 in a row.

For example, 2 in a row.
In this game, the coconuts are also red or blue.

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Open Question
Is 2-IN-A-ROW hard?

## QUESTIONS?


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