TRACKING MOVING OBJECTS WITH FEW HANDOVERS

David Eppstein
Michael Goodrich
Maarten Löffler
University of California, Irvine

INGREDIENTS

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- 1 Plane

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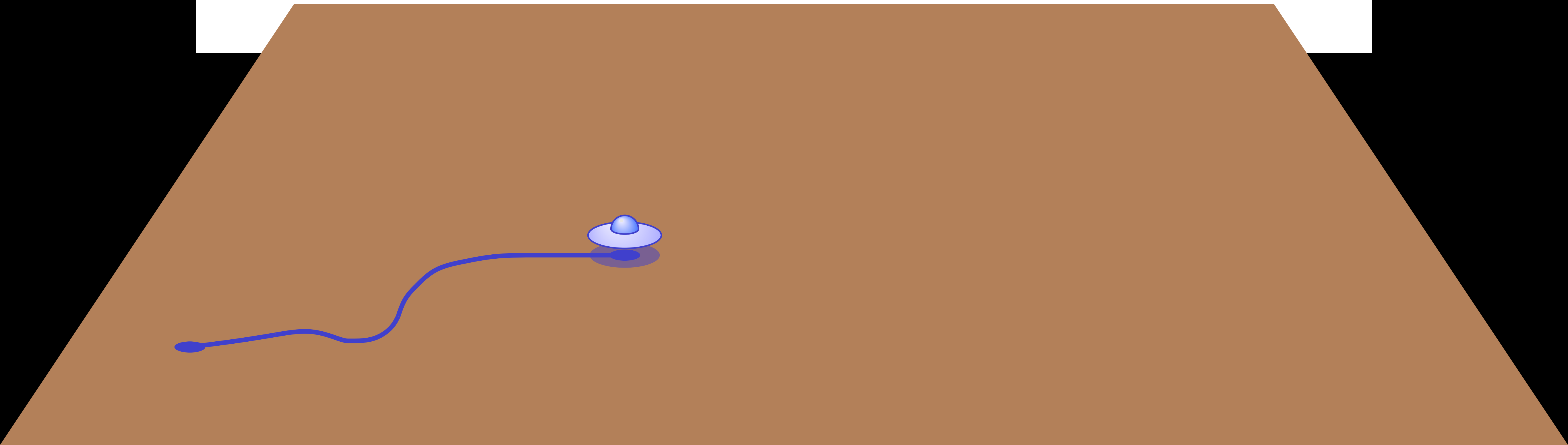

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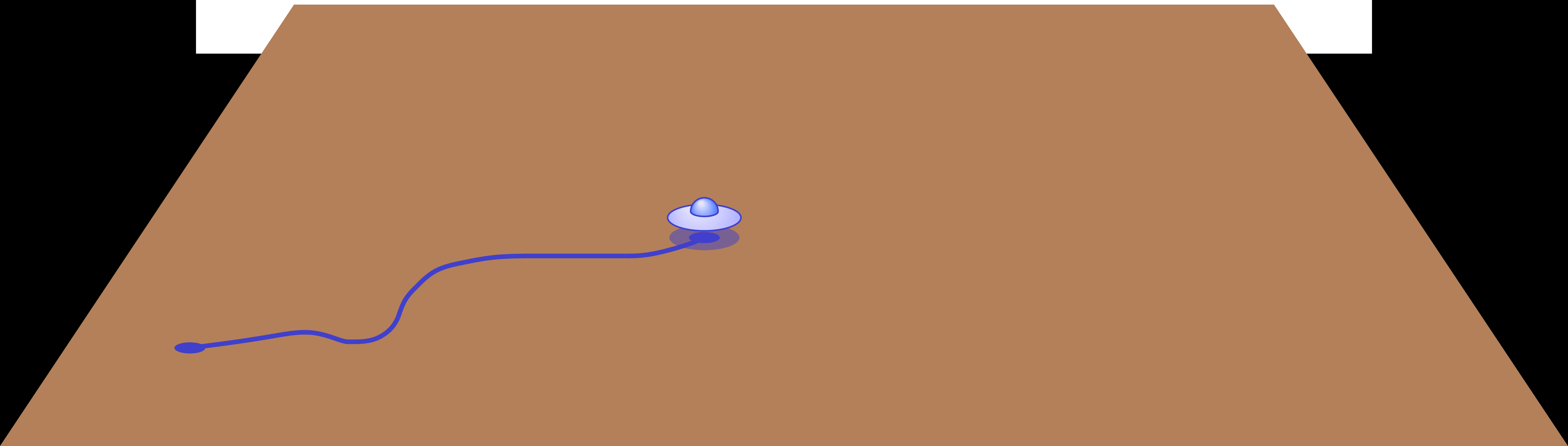

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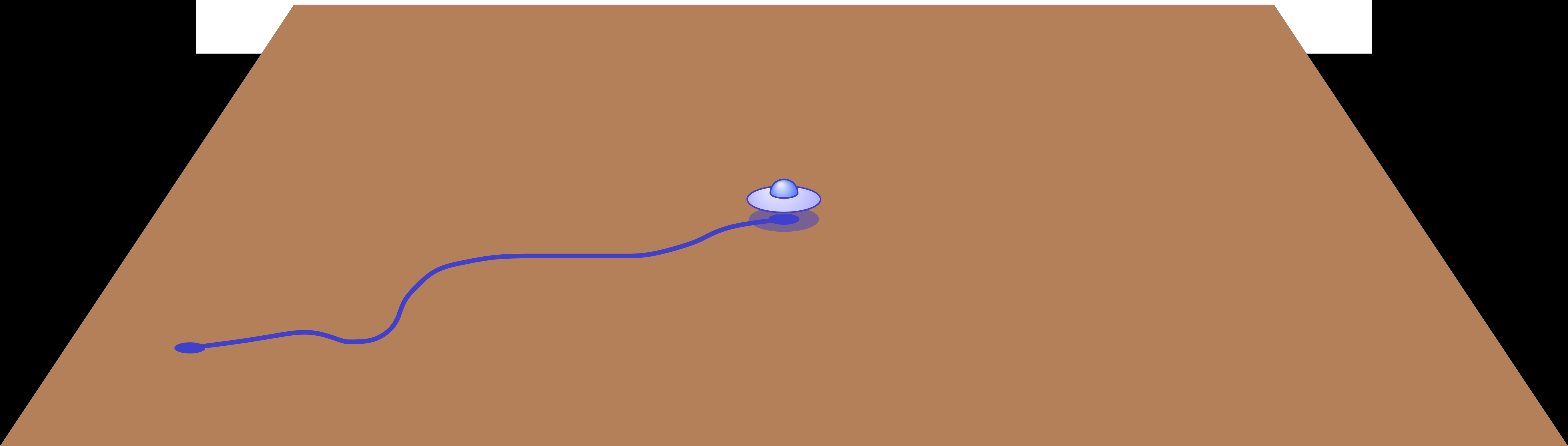

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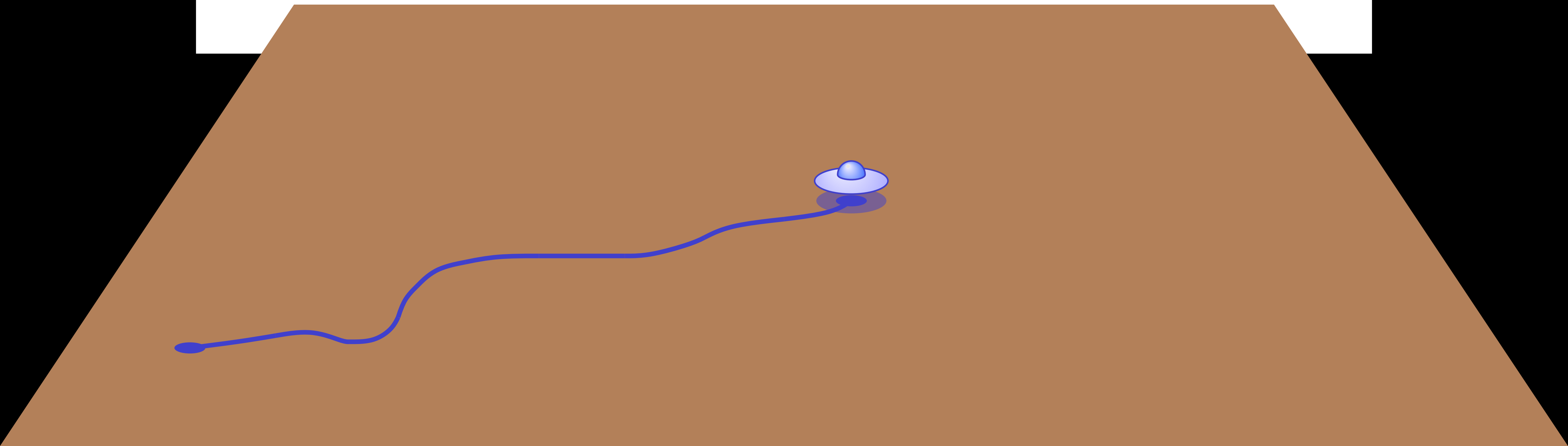

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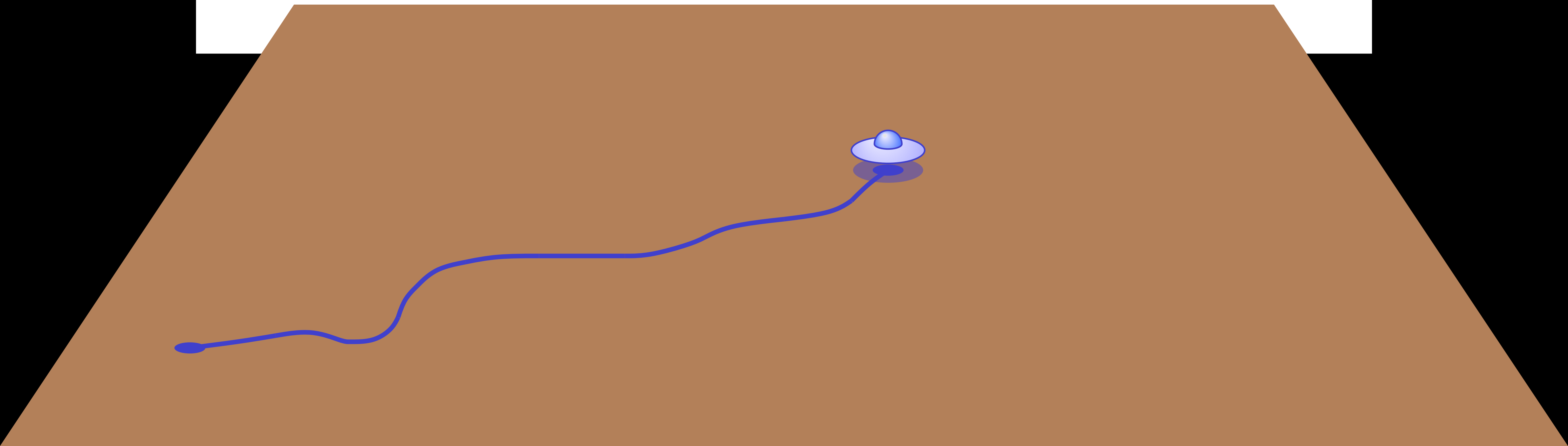

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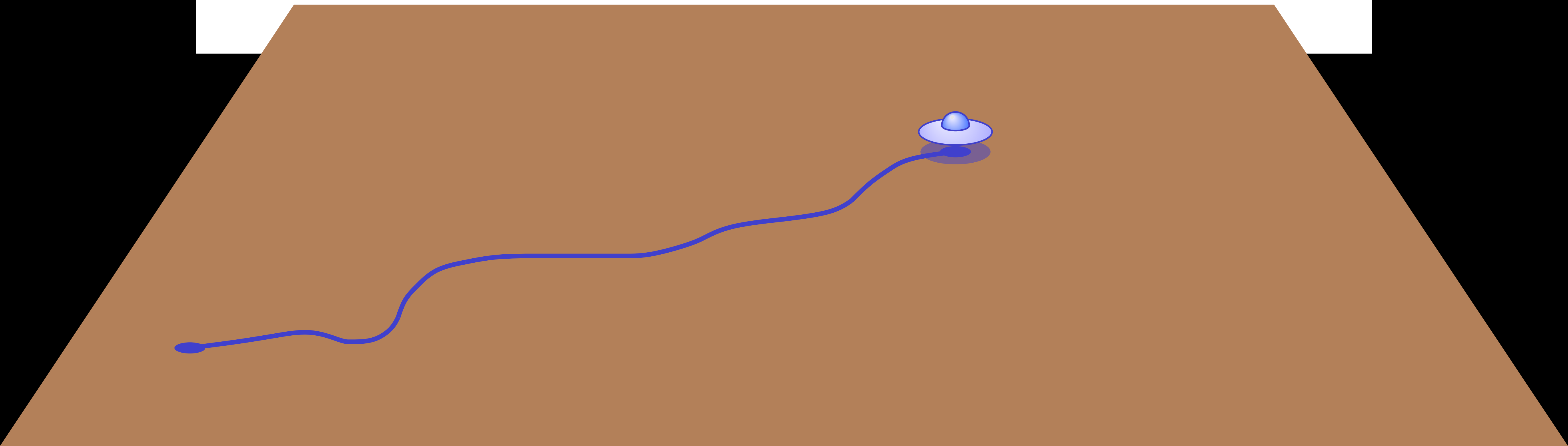

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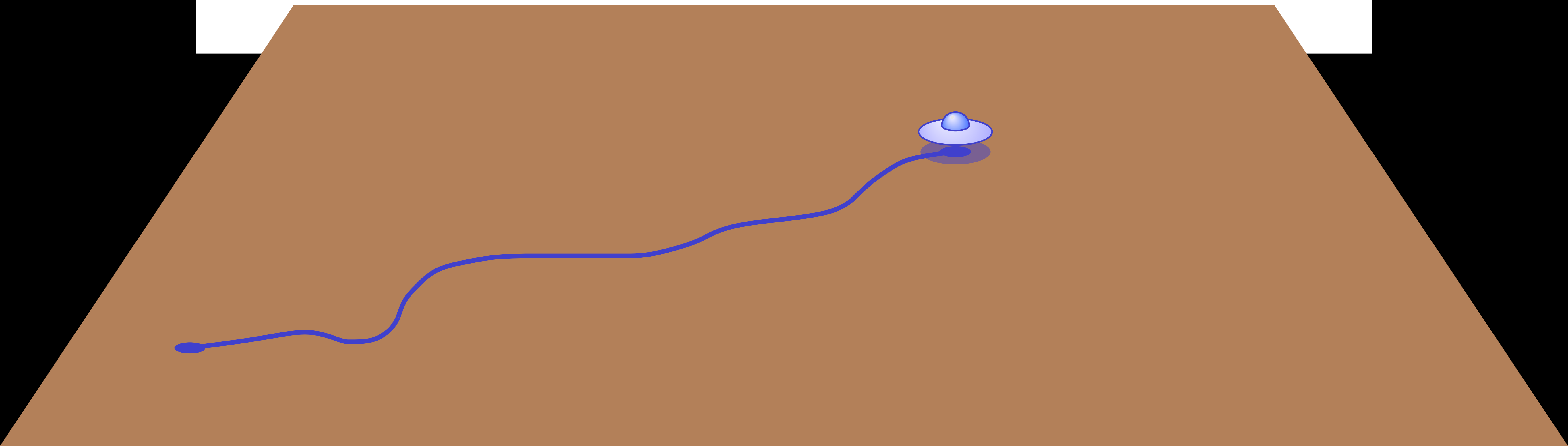

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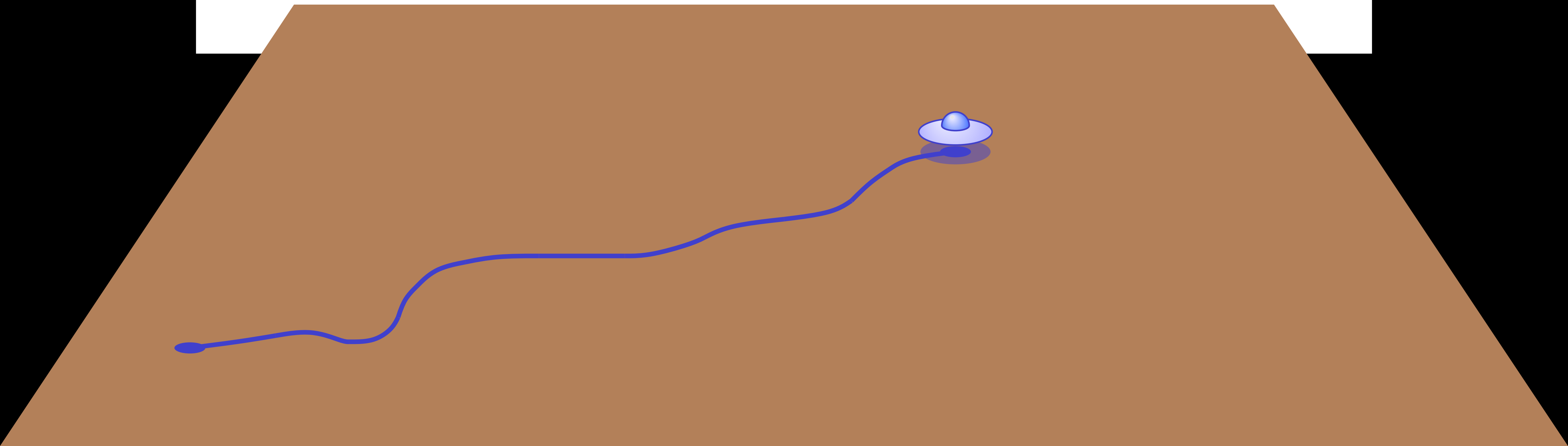

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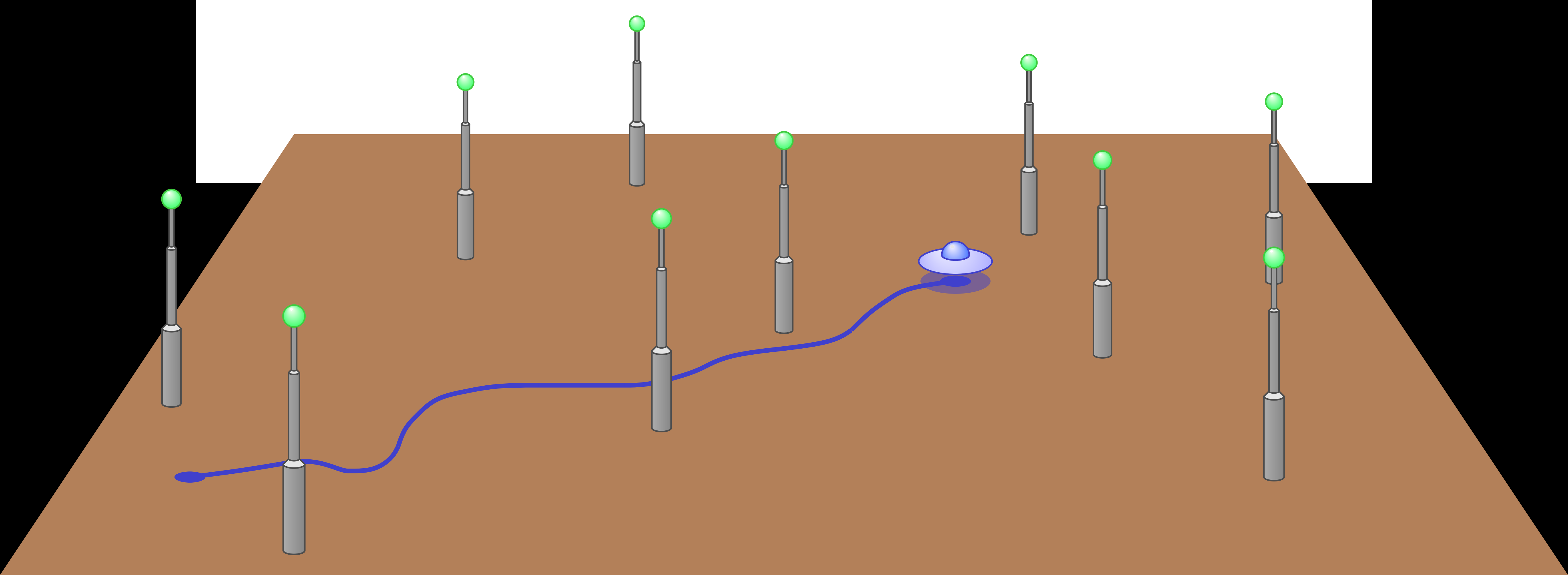

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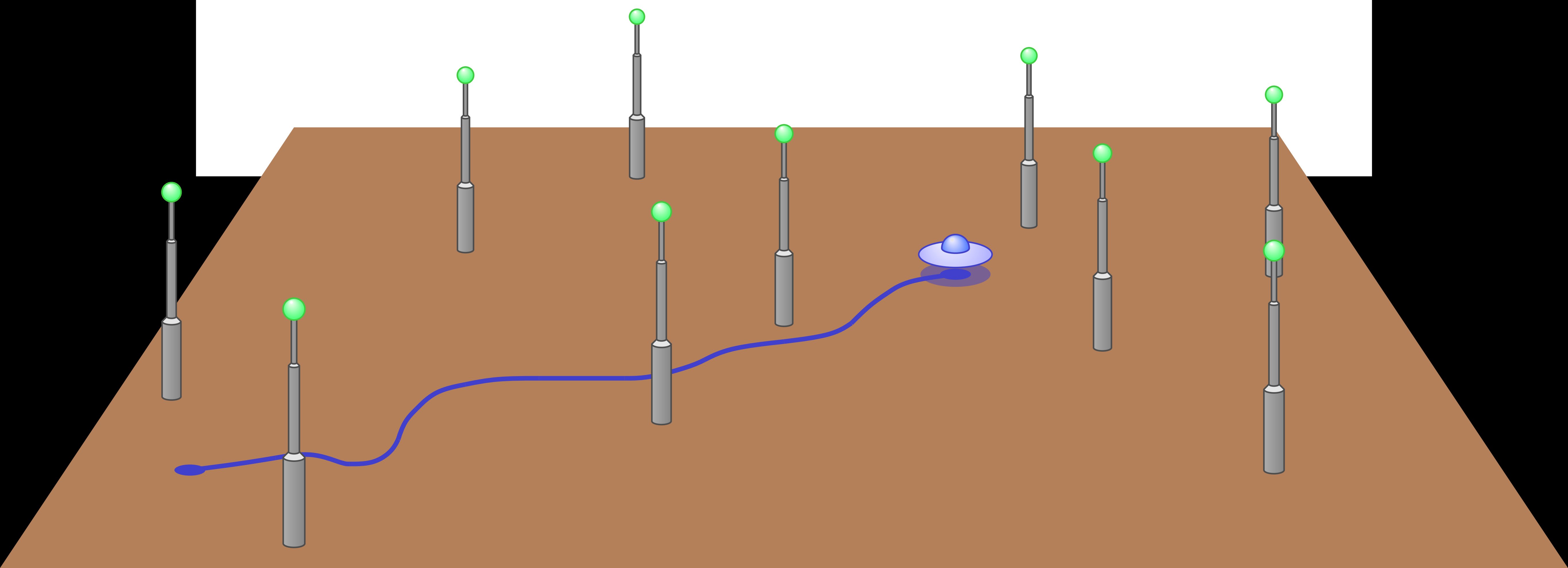

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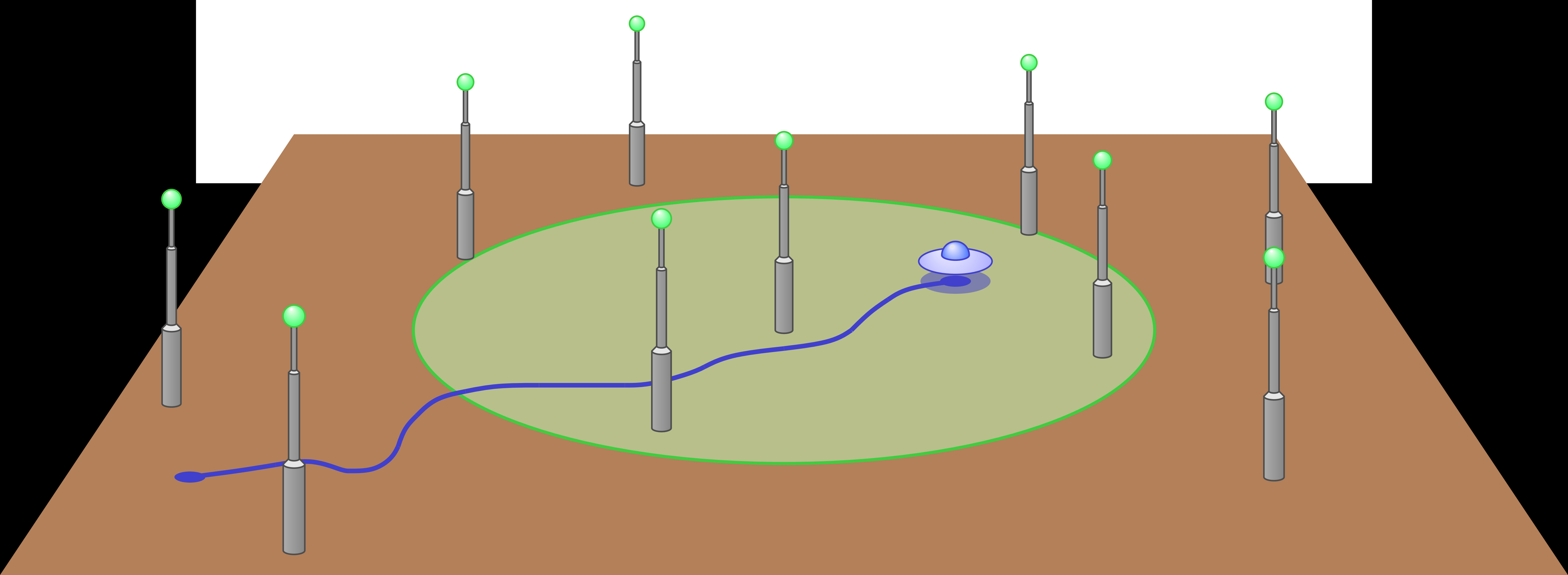

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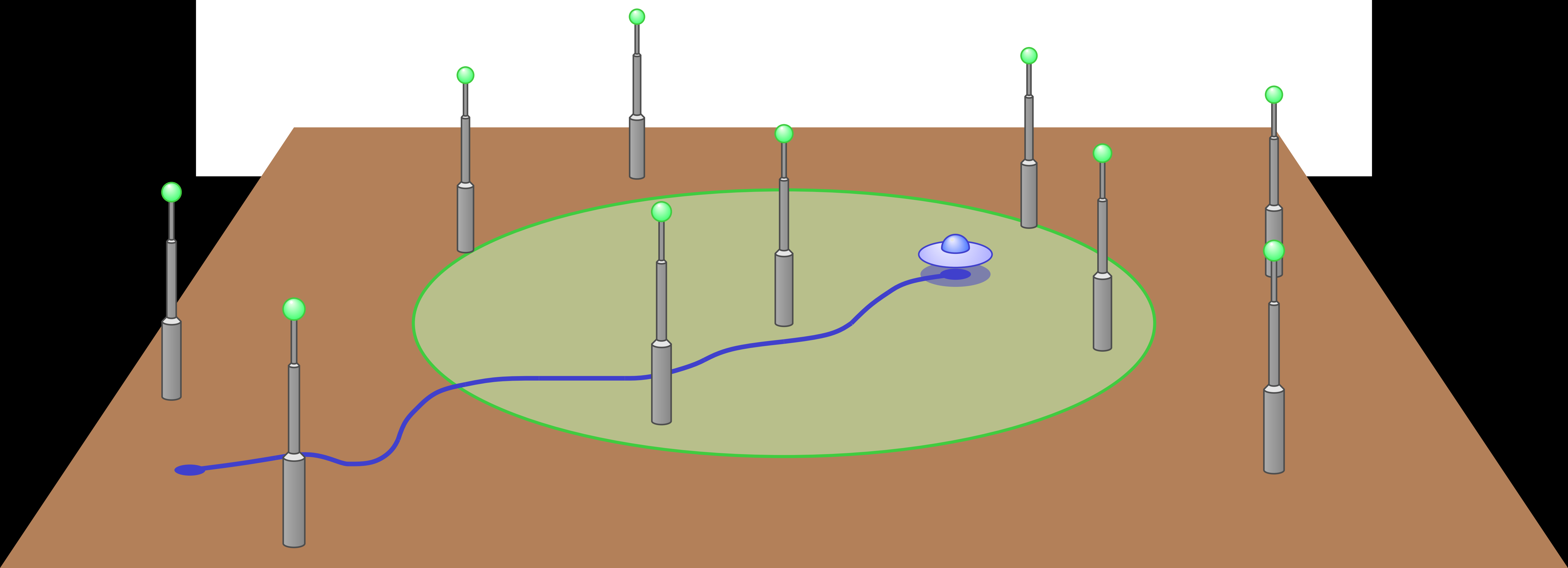

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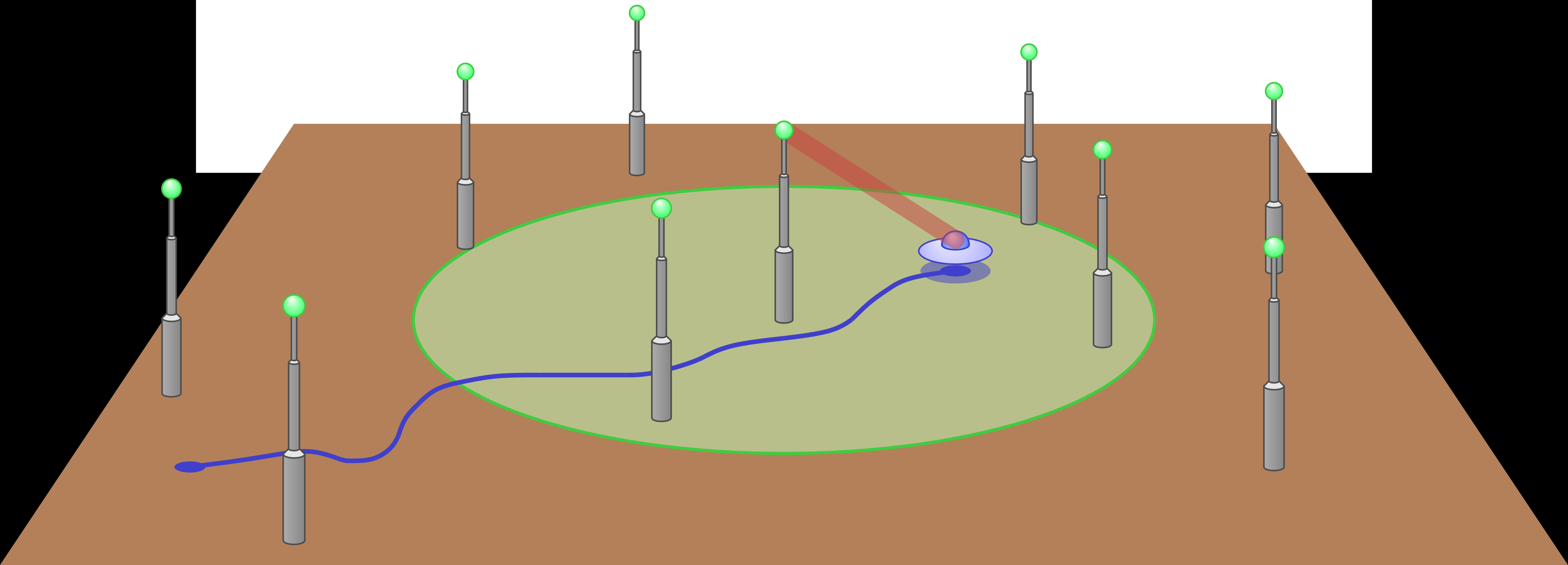

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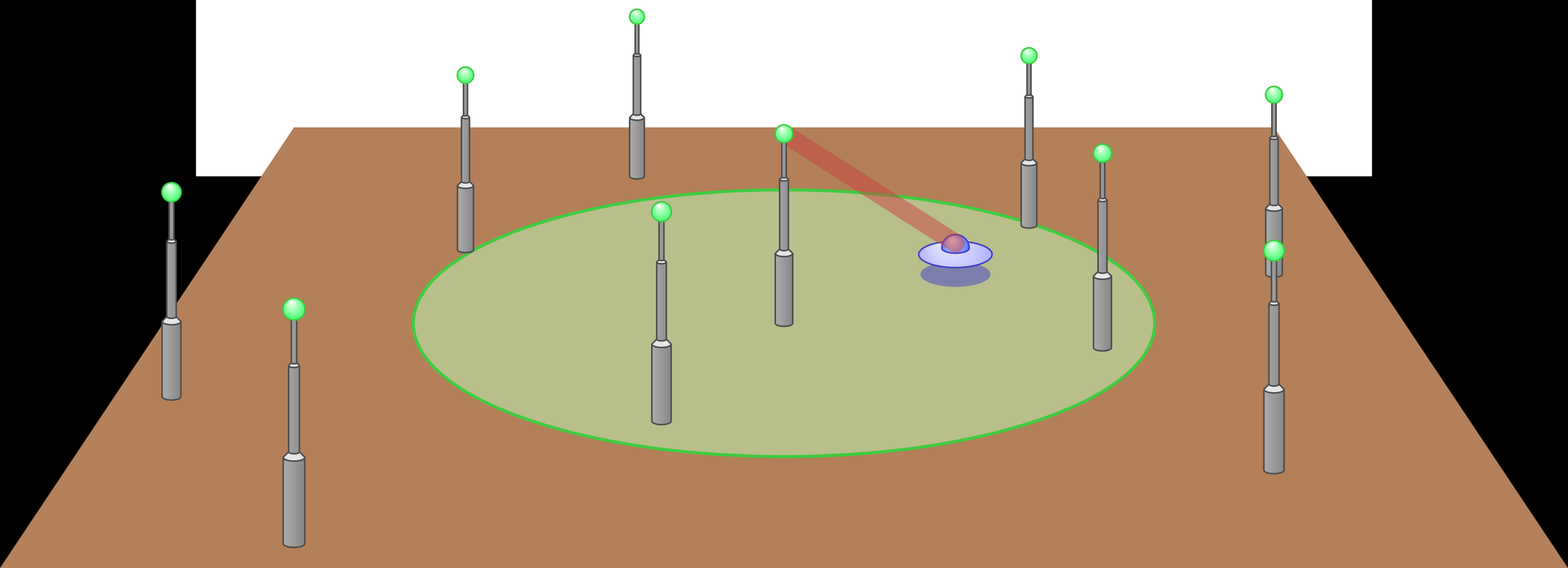

OBJECTIVES

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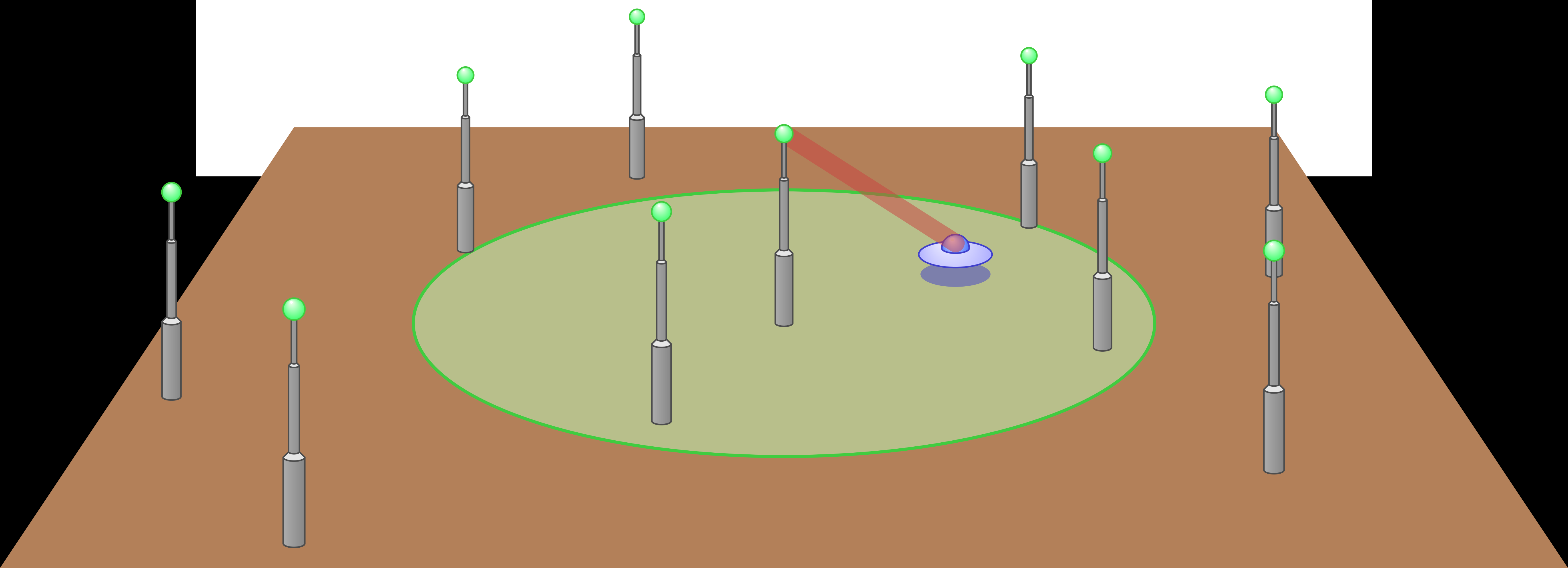

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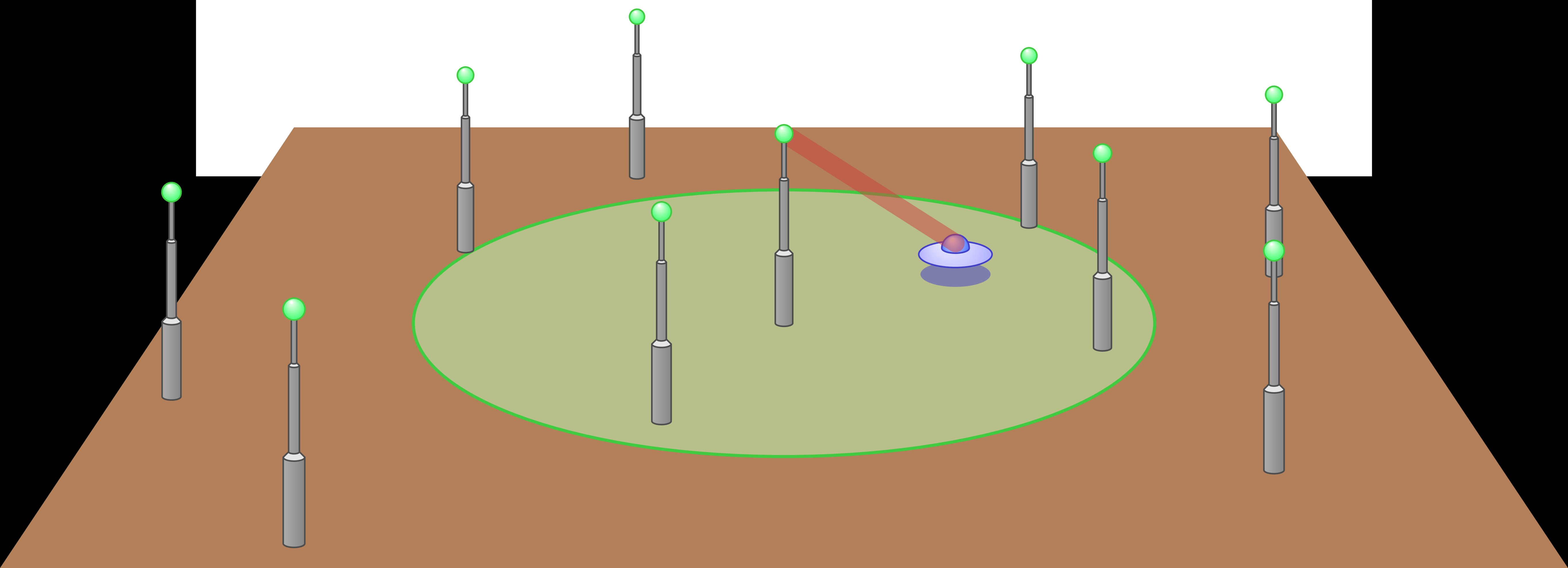
  - UMO must be tracked by 3 stations at every point in time



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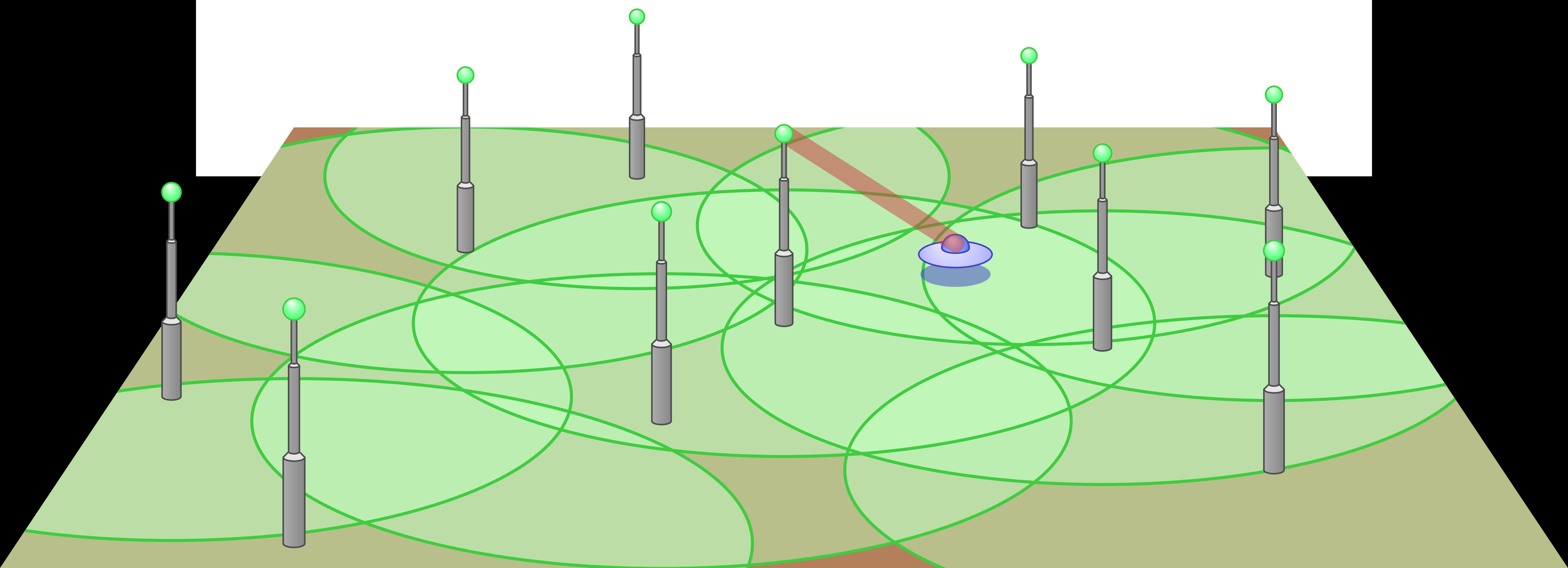

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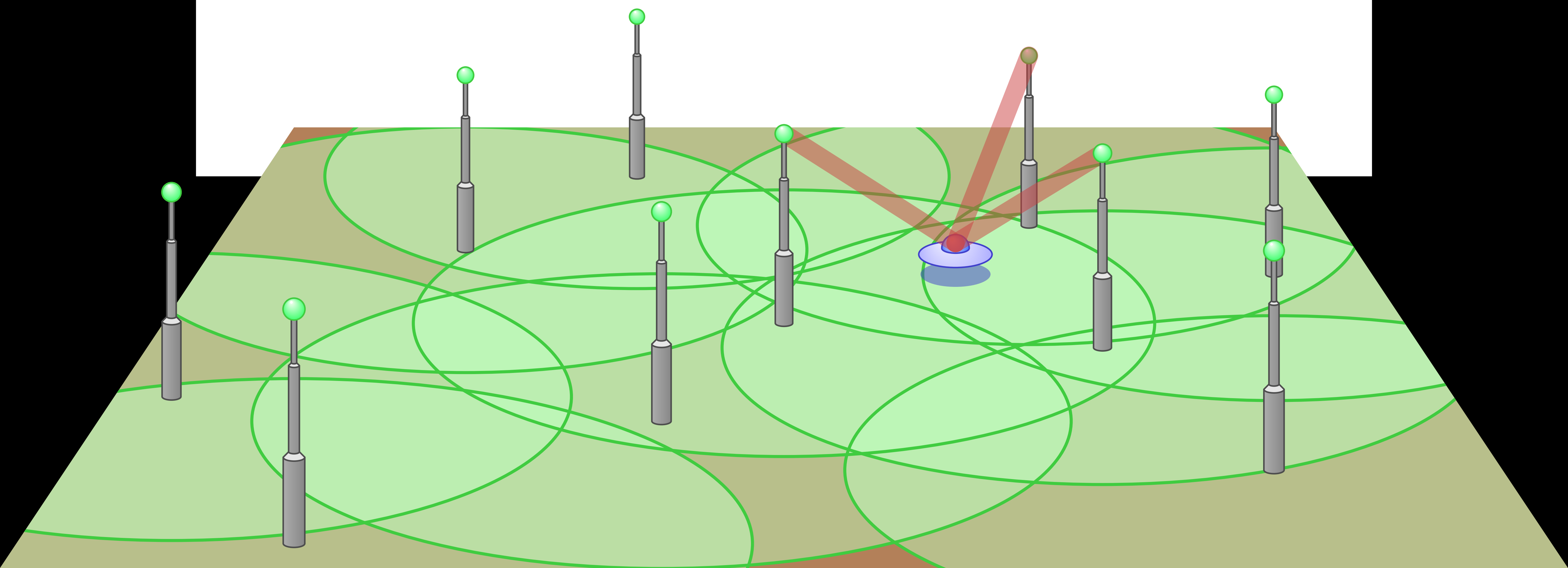

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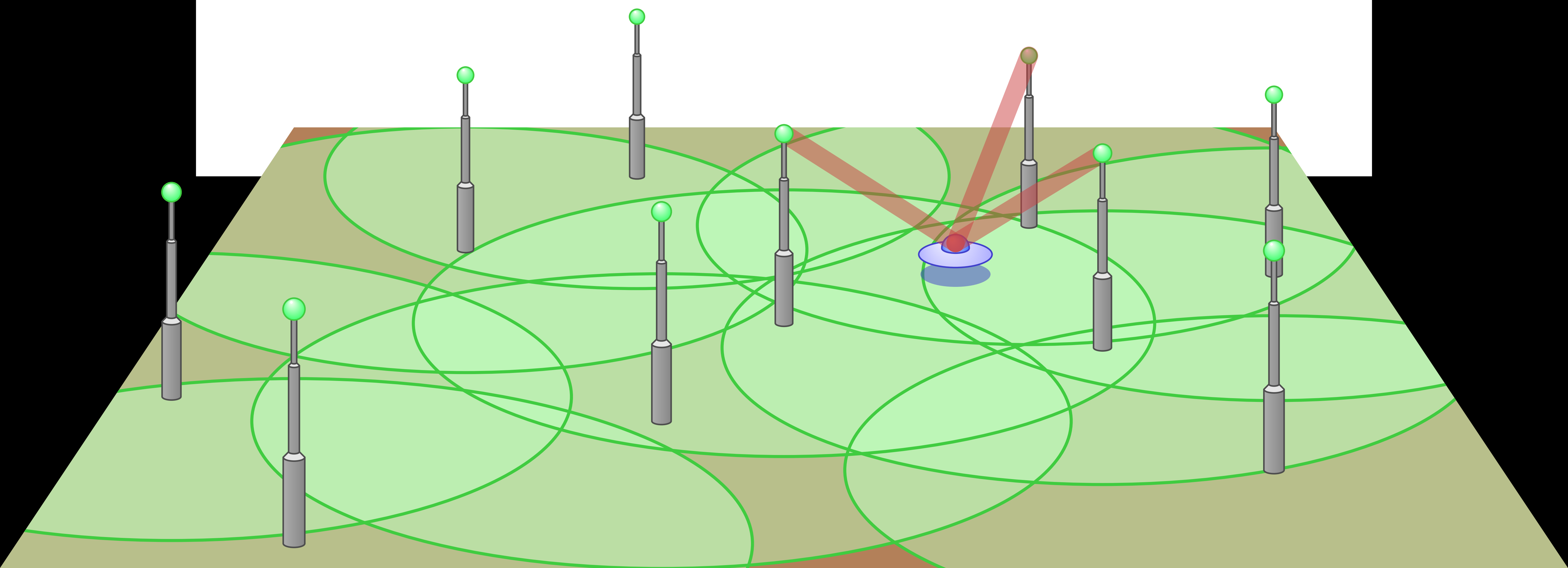

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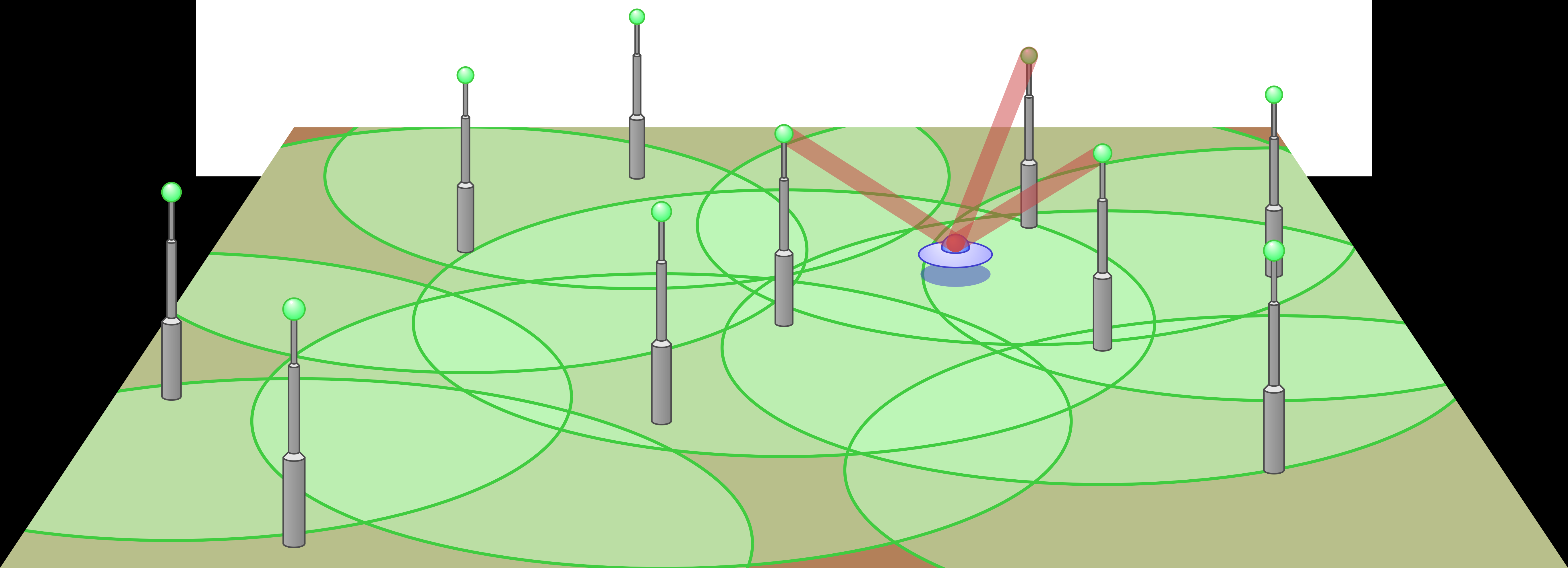

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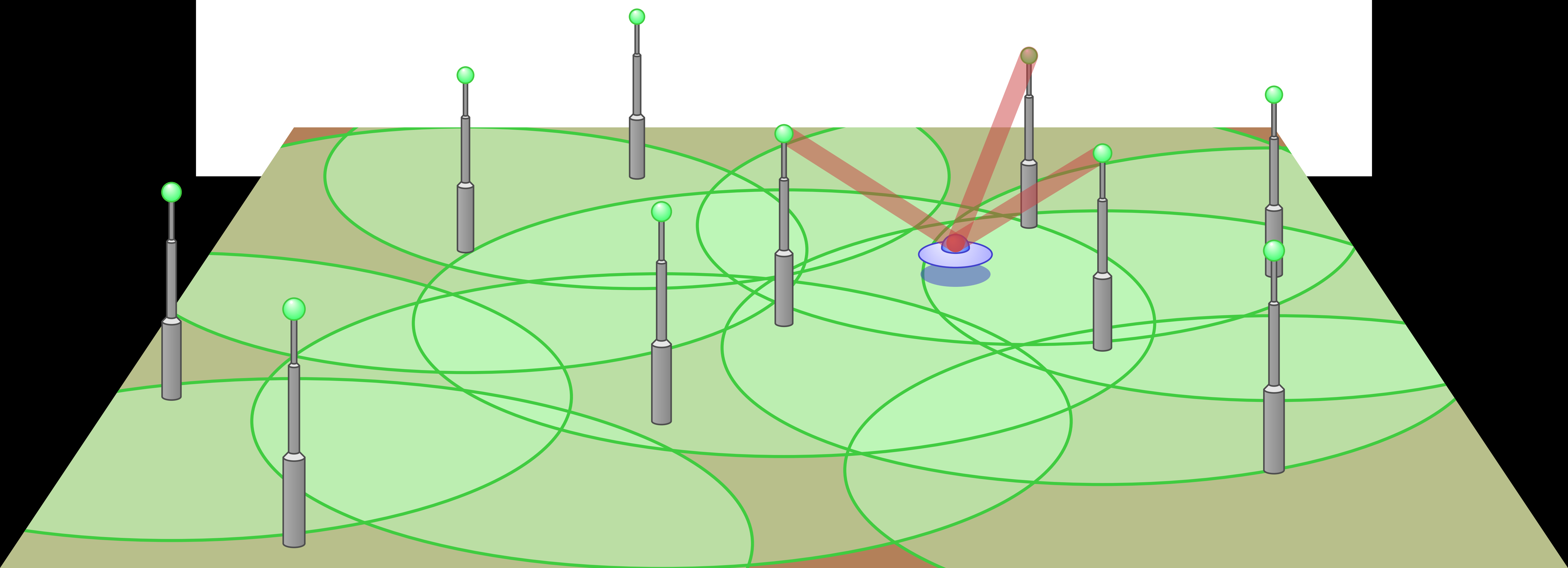

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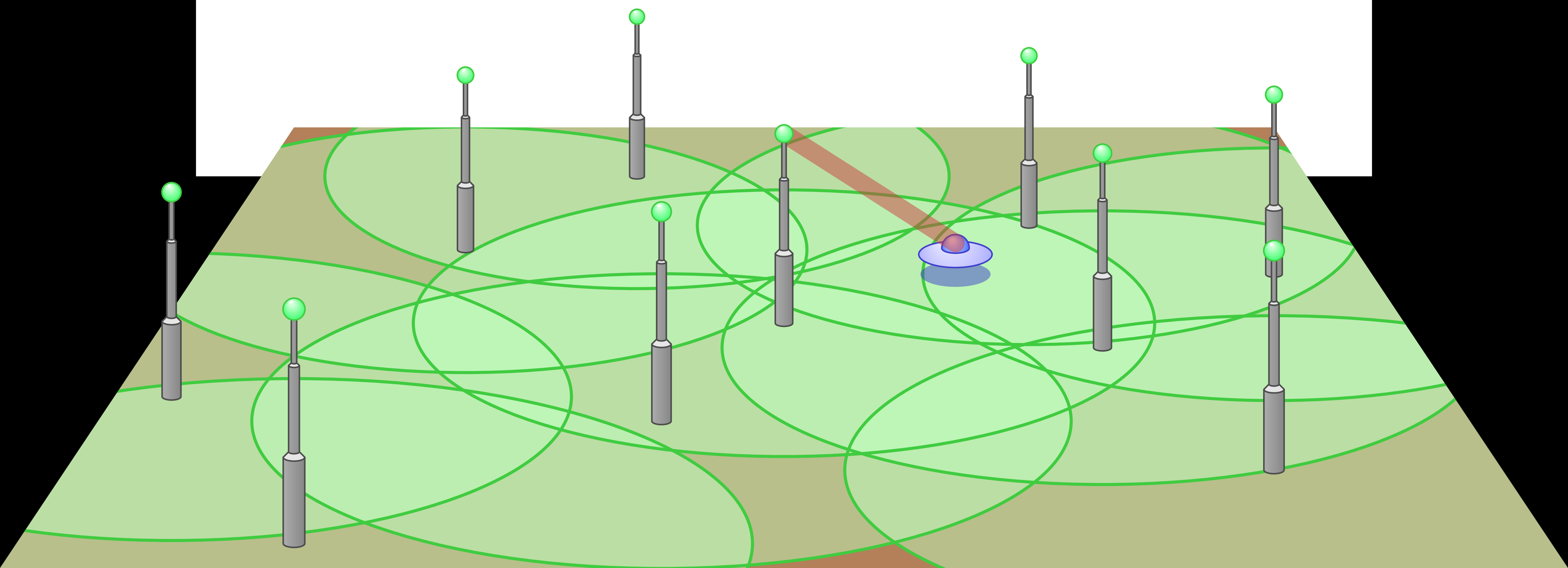

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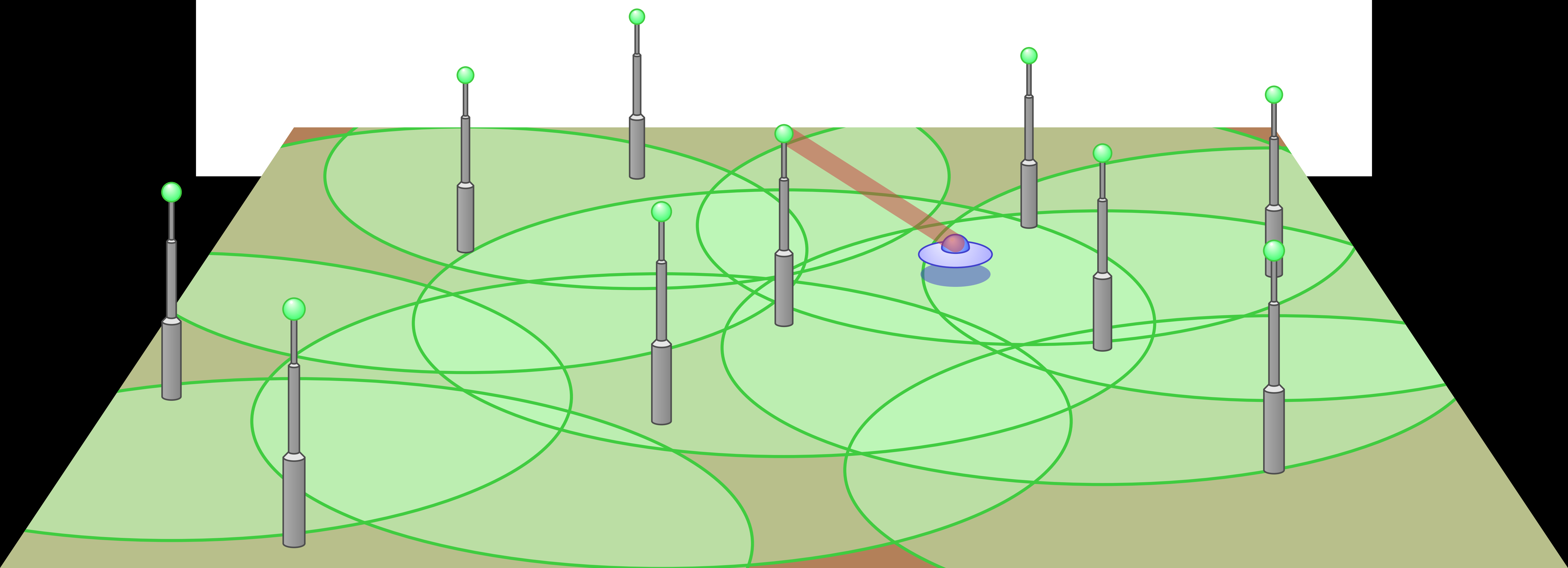

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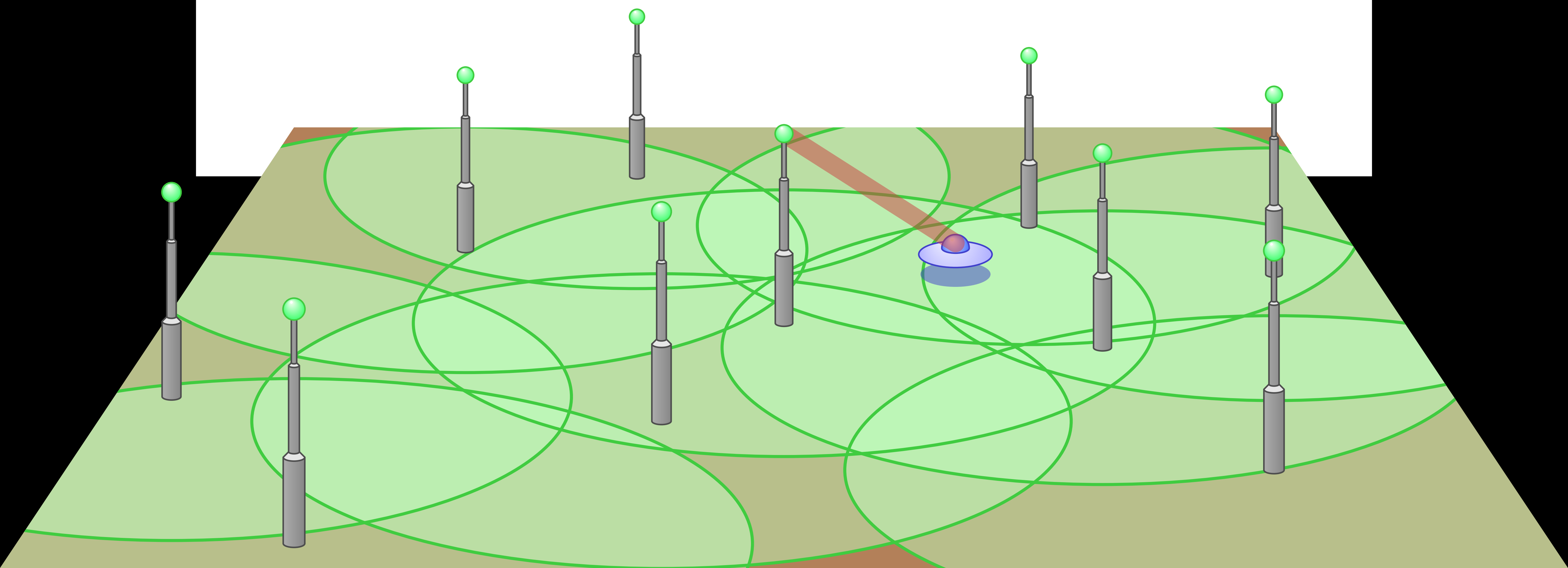

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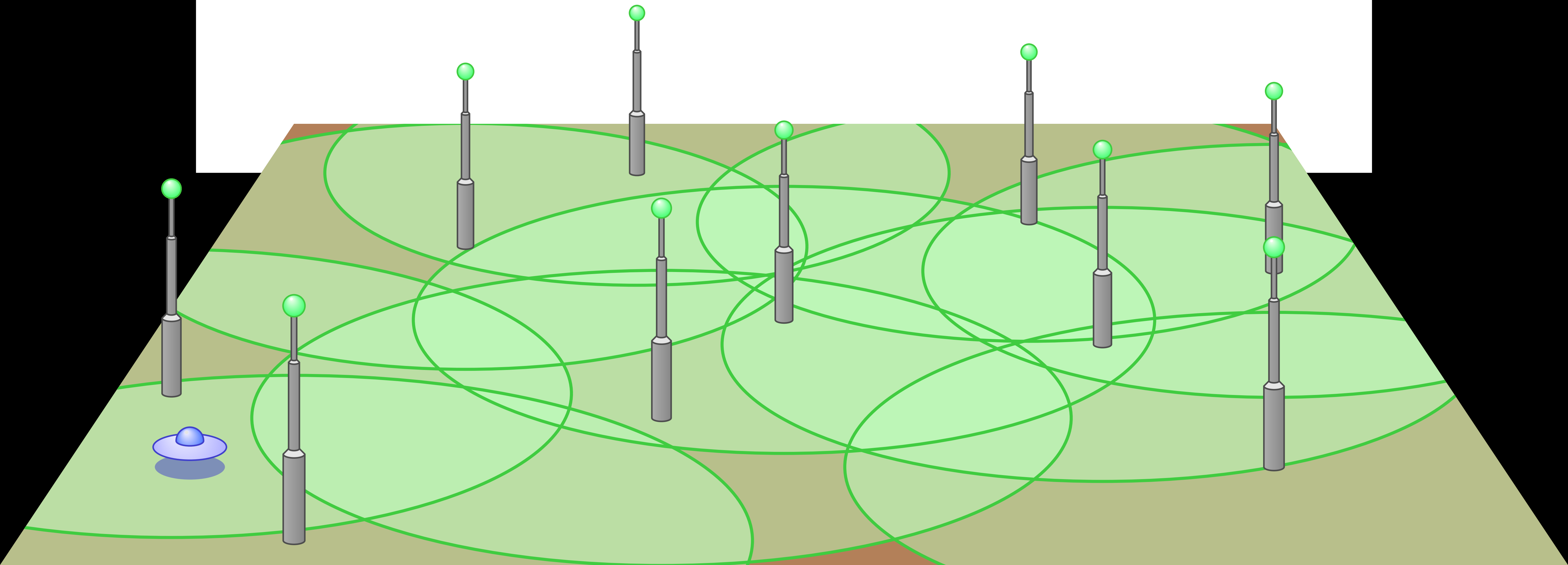

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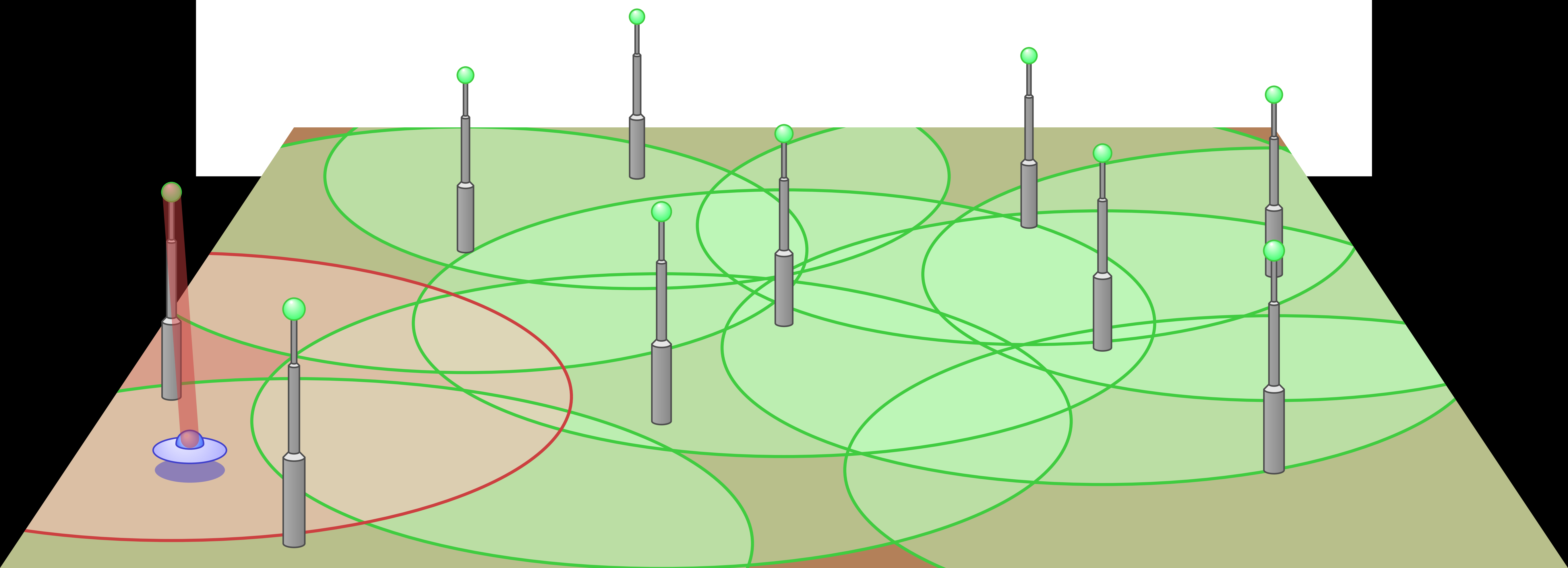

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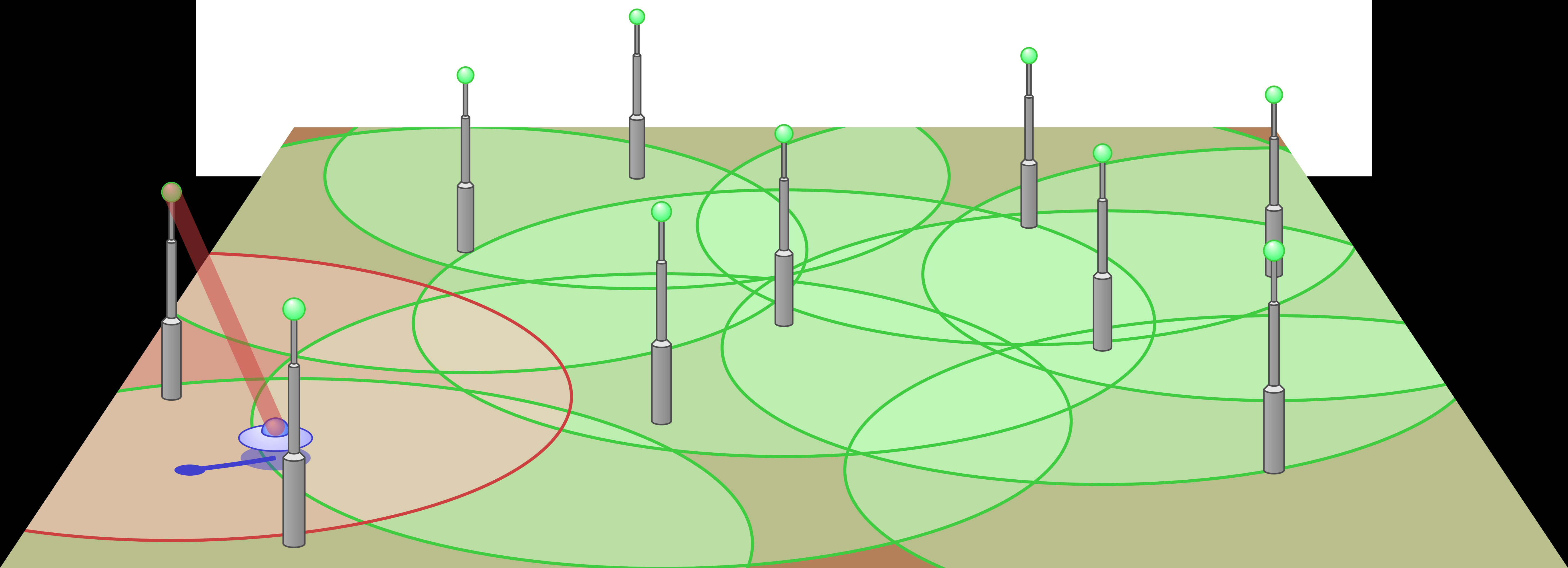

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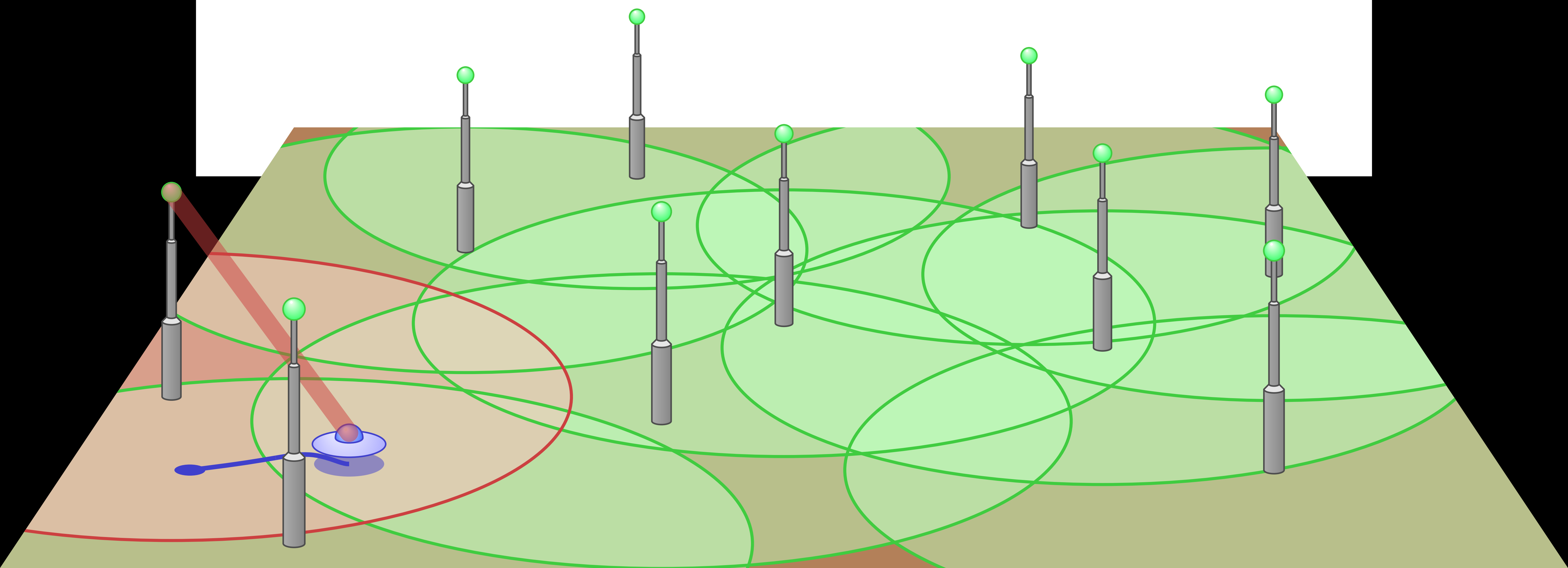

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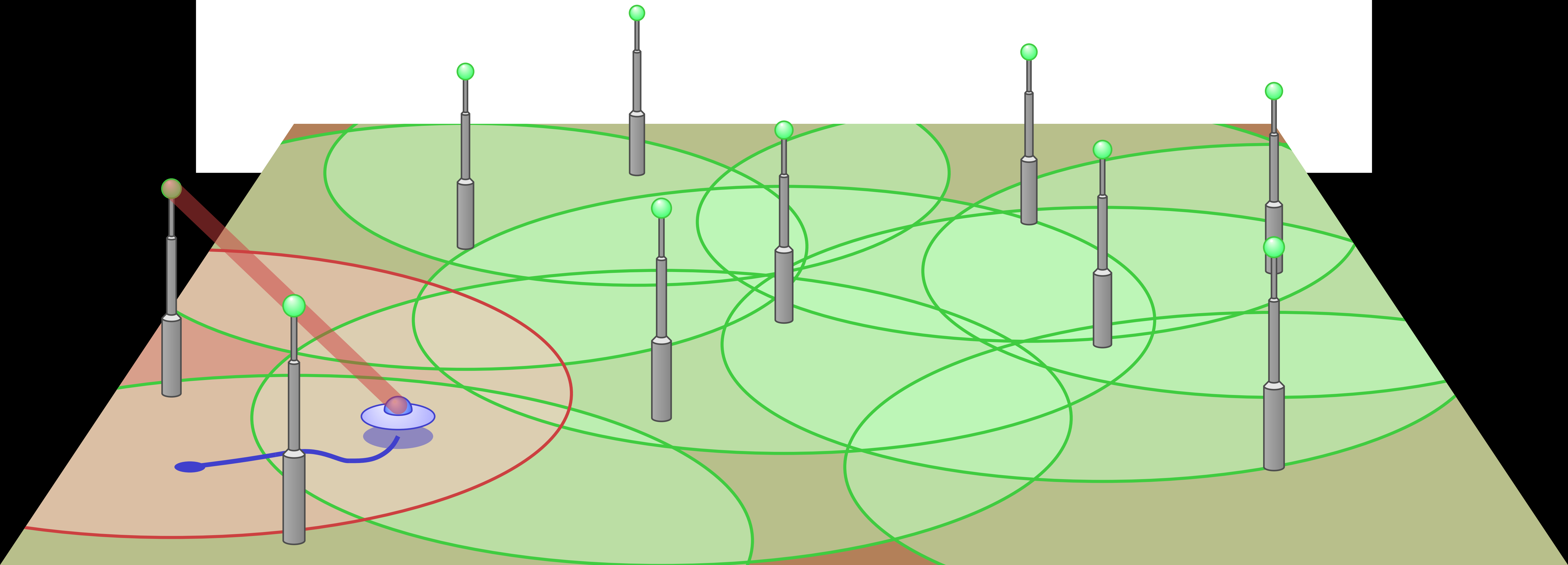

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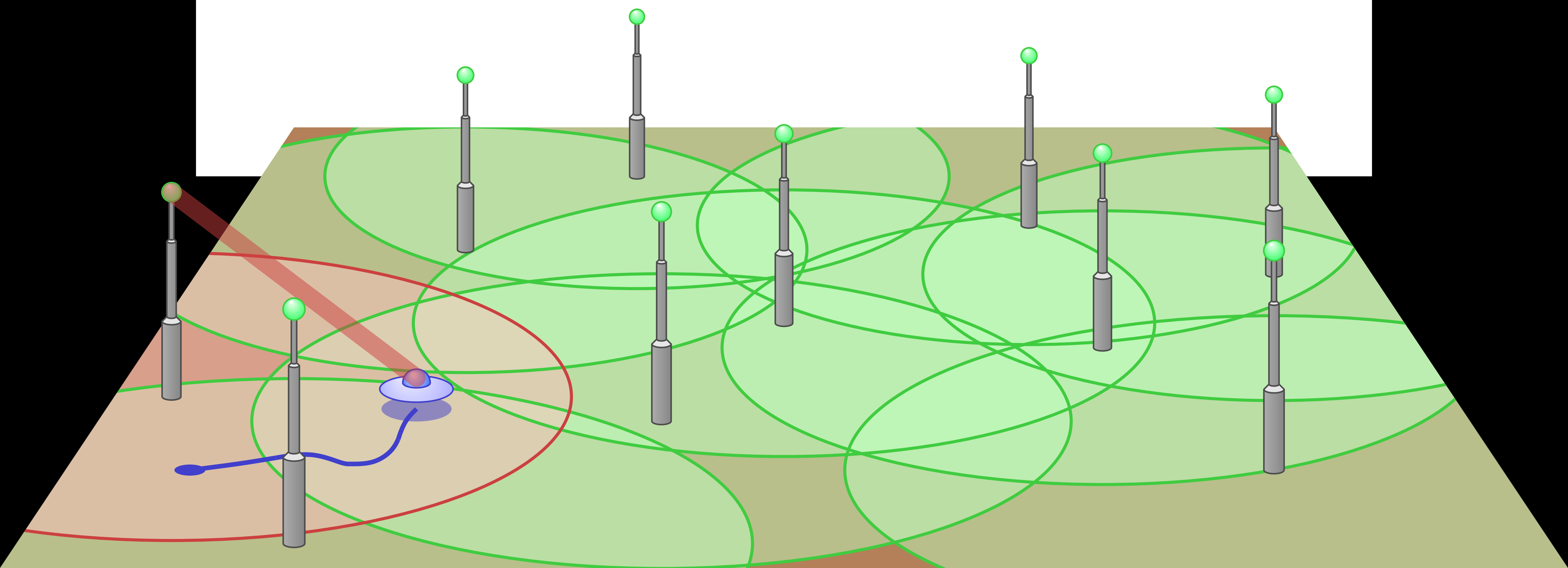

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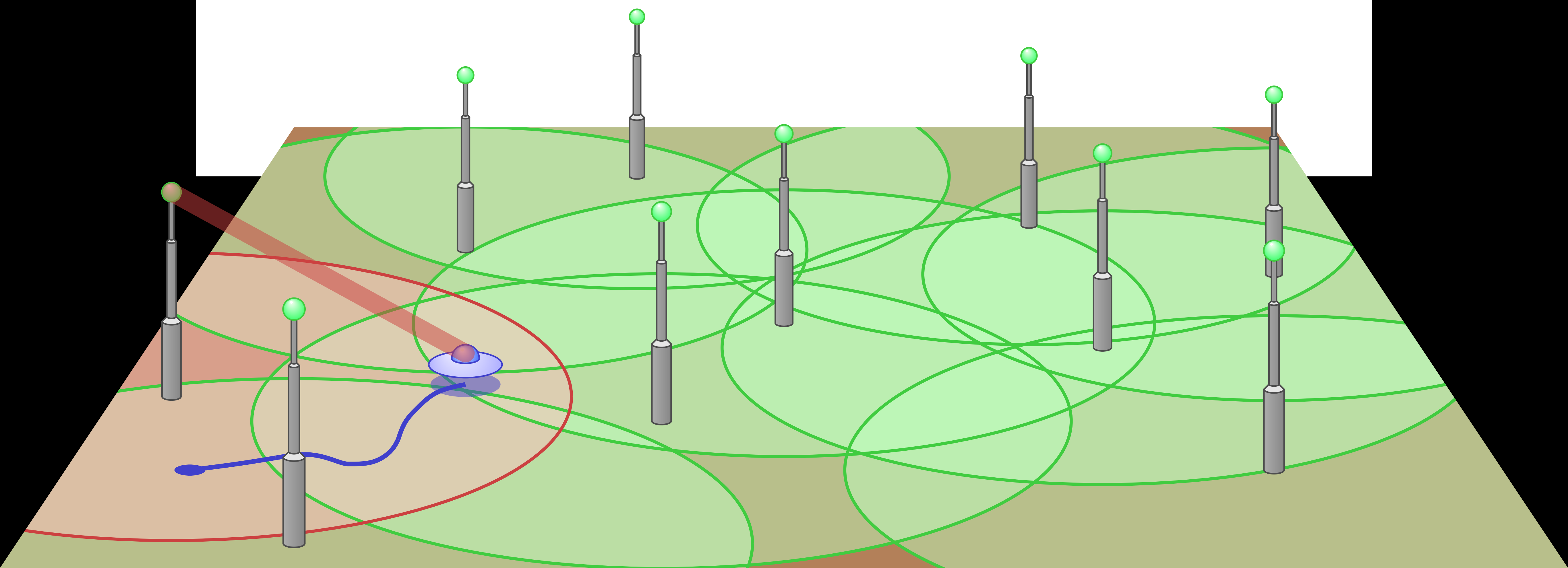

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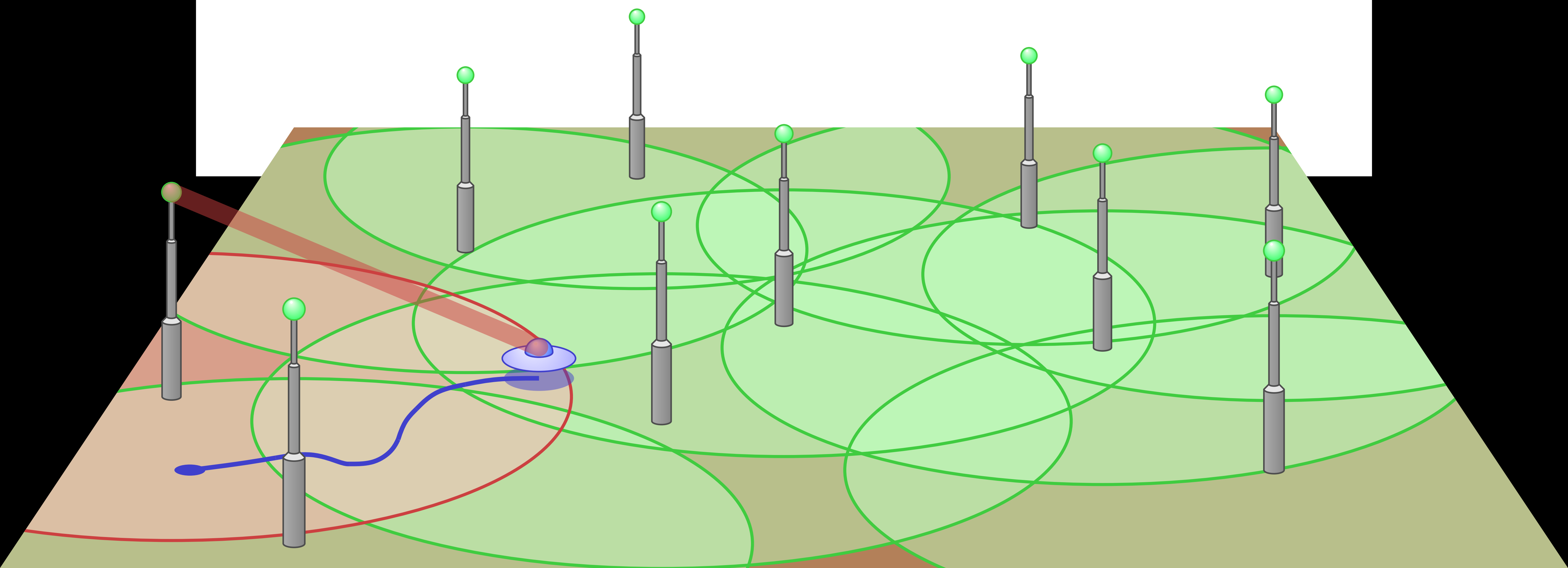

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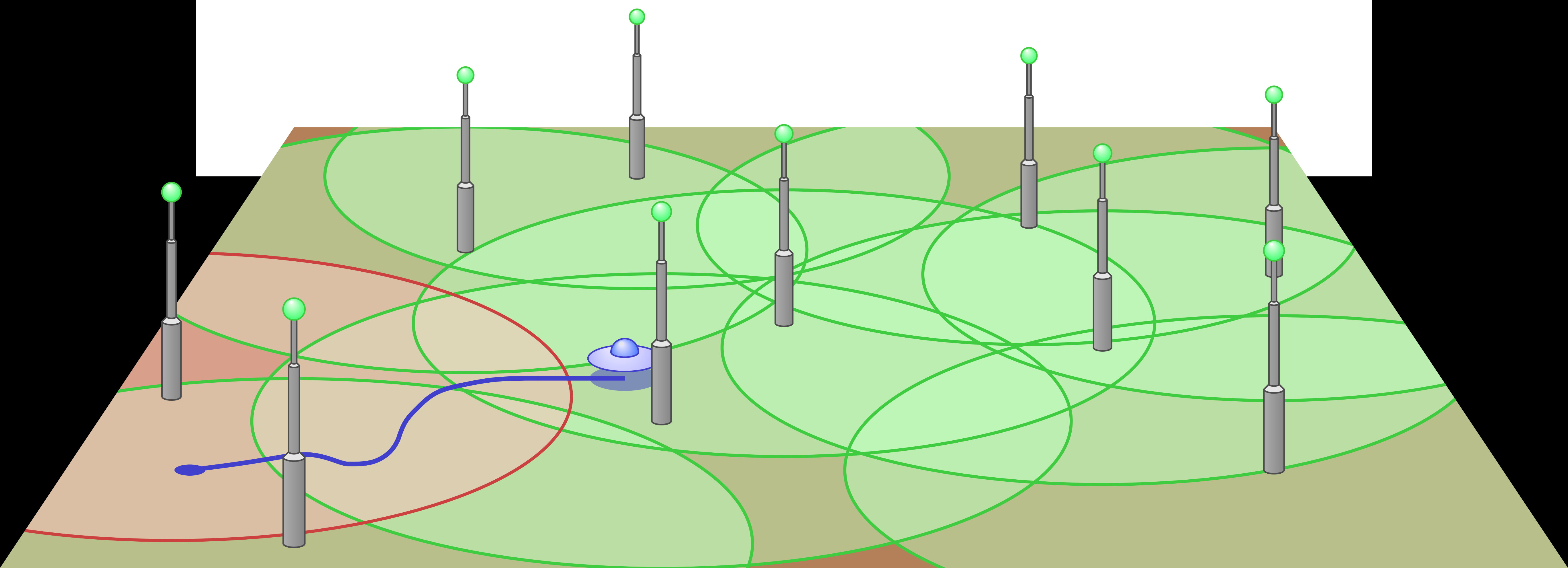

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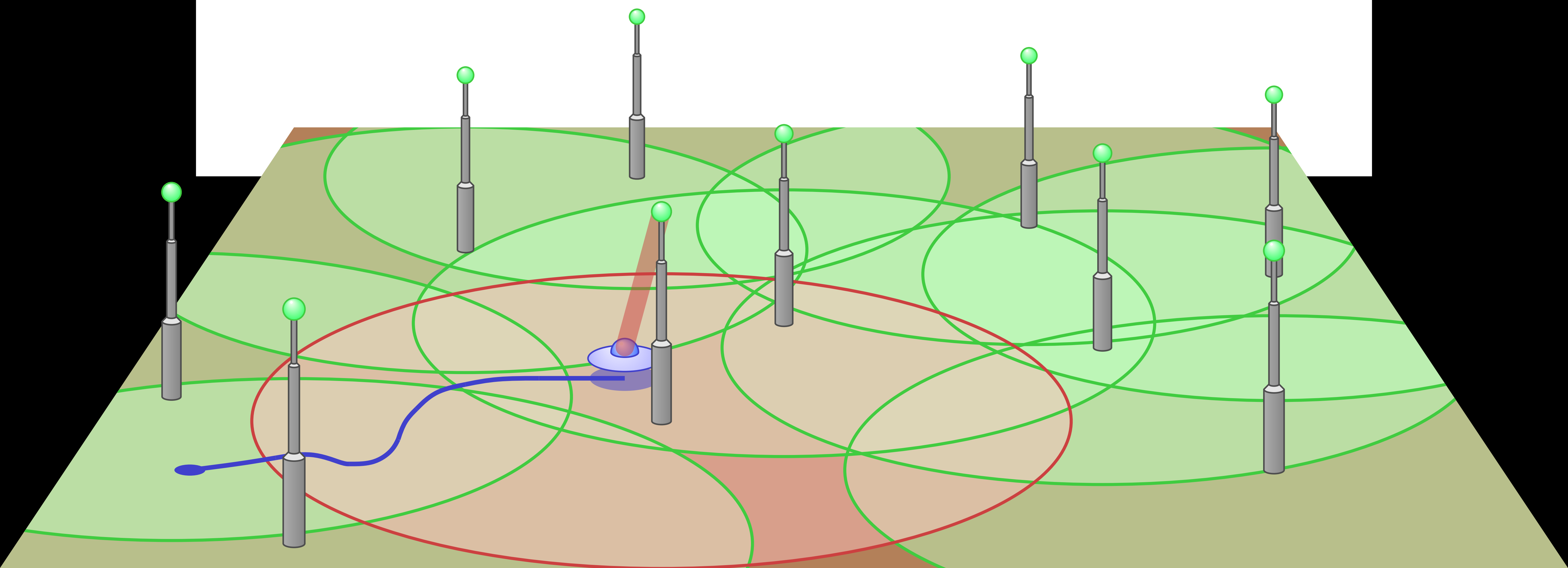

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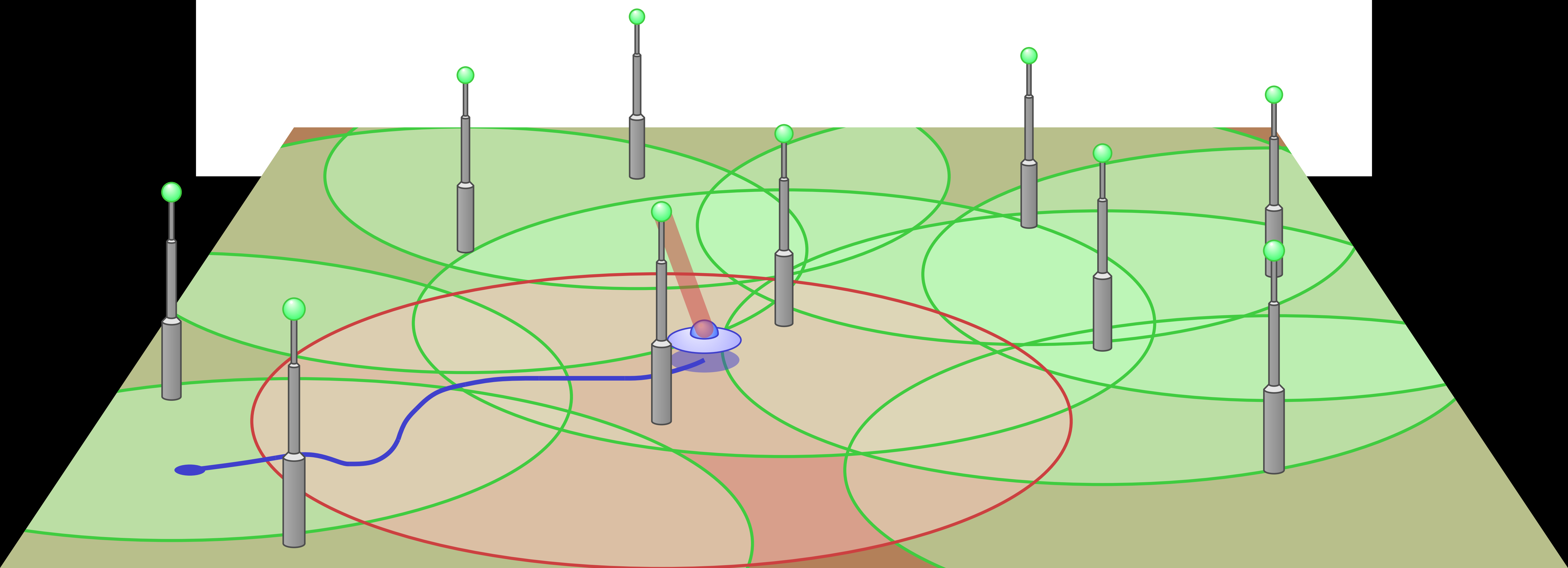

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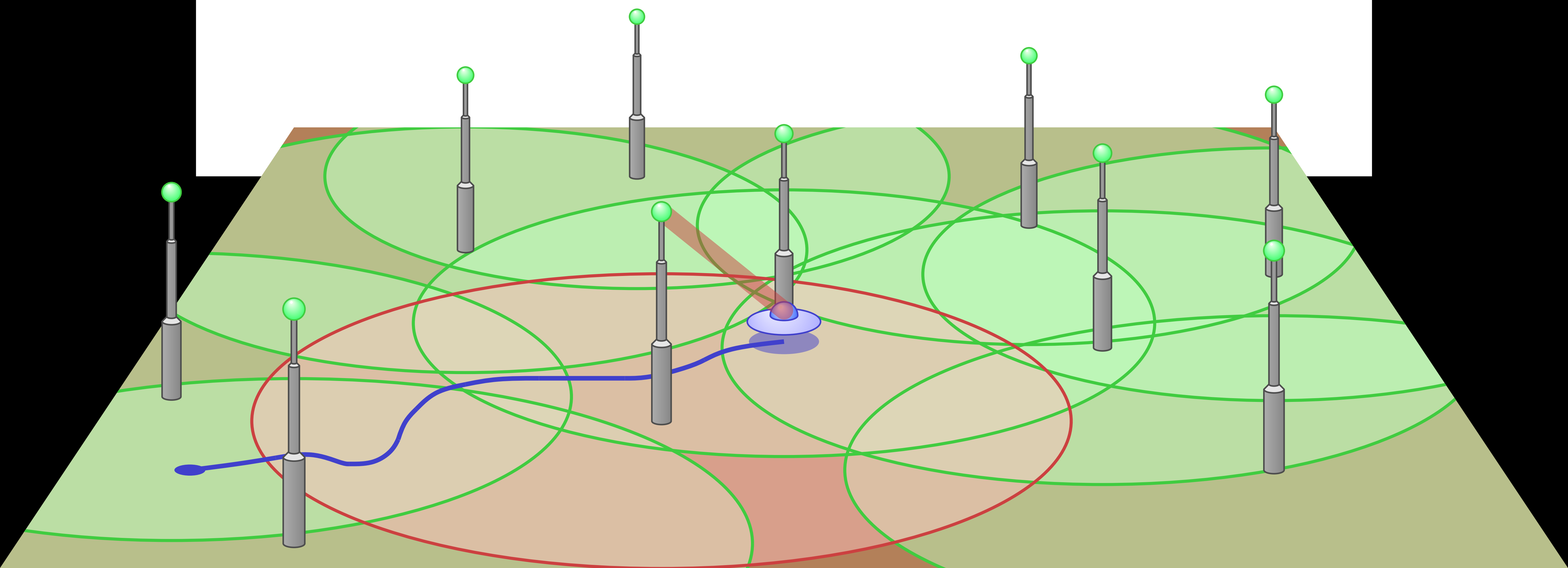

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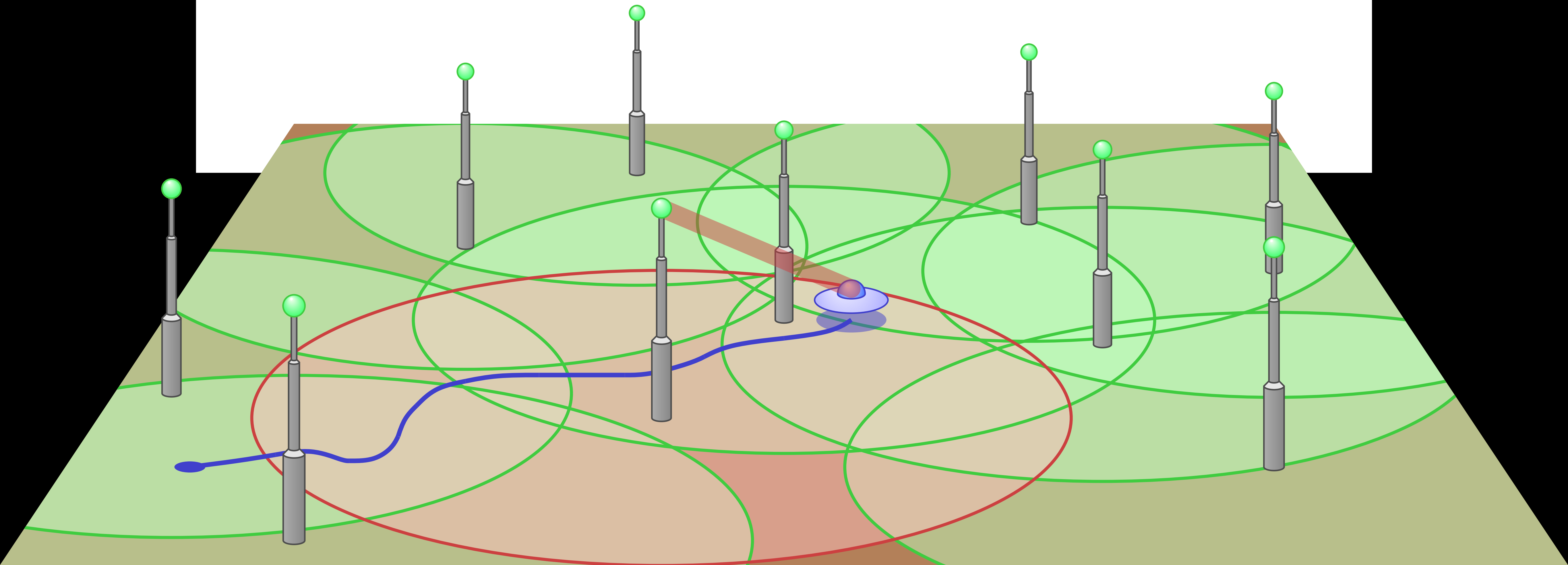

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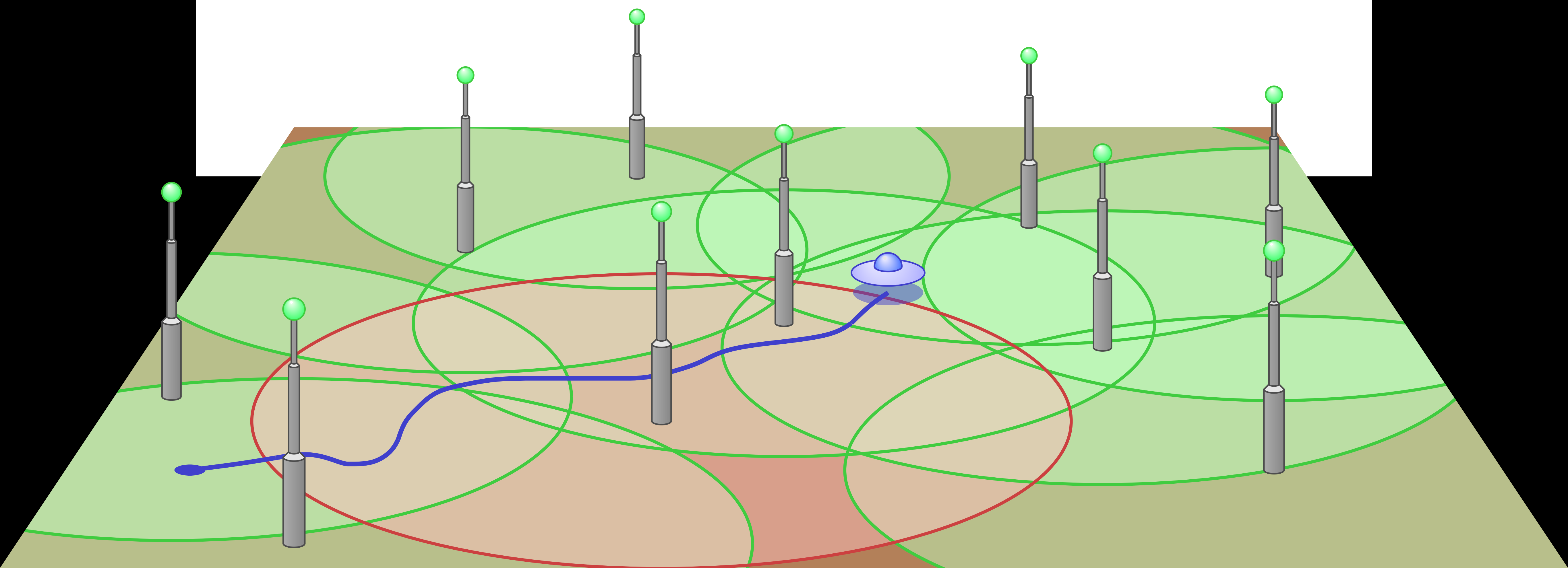

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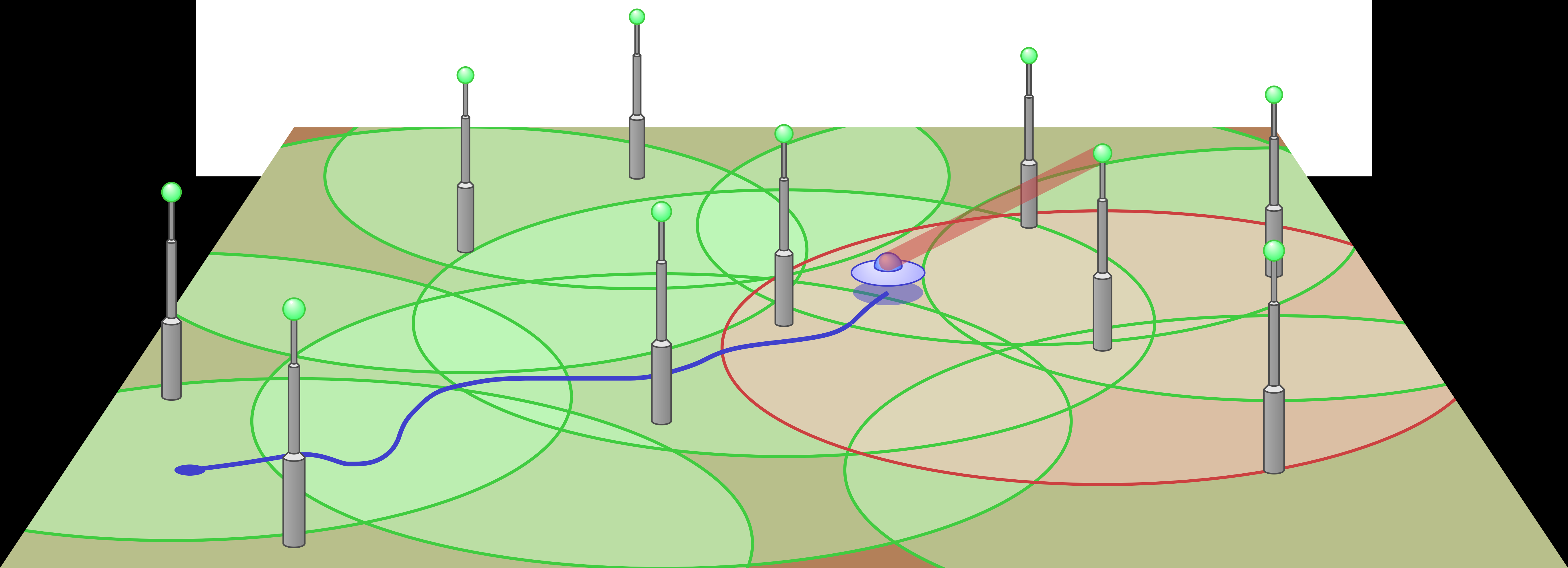

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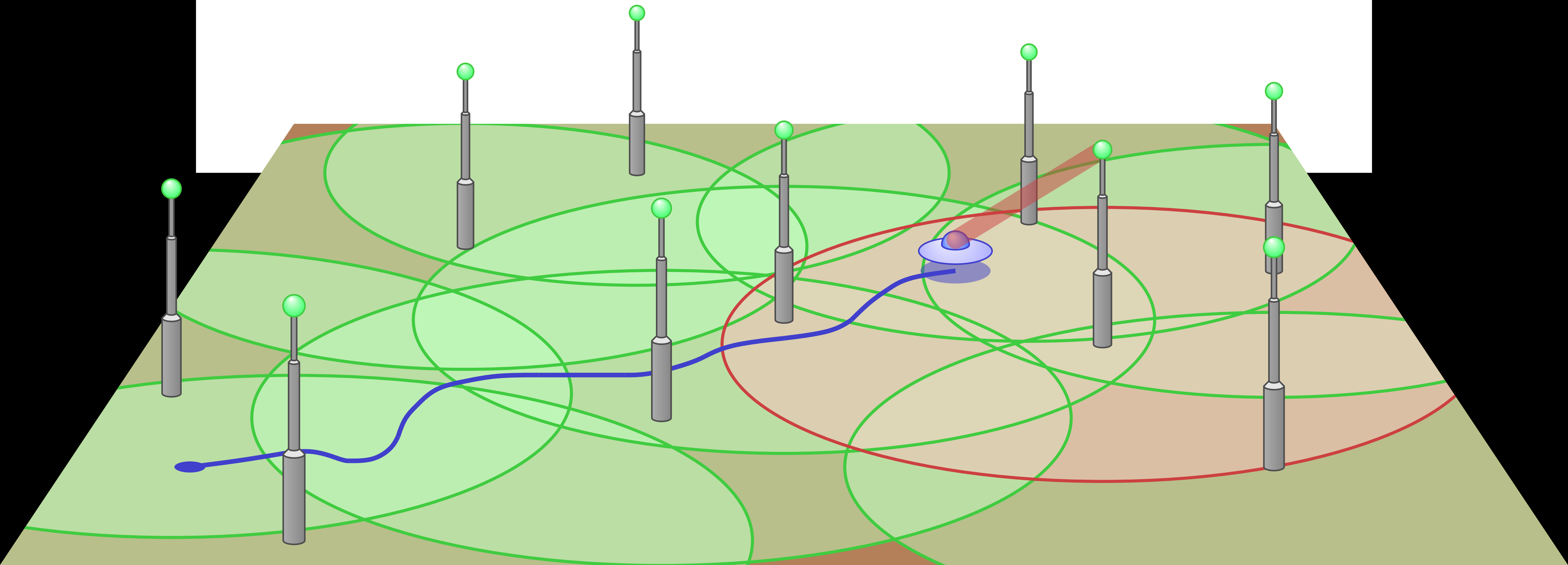

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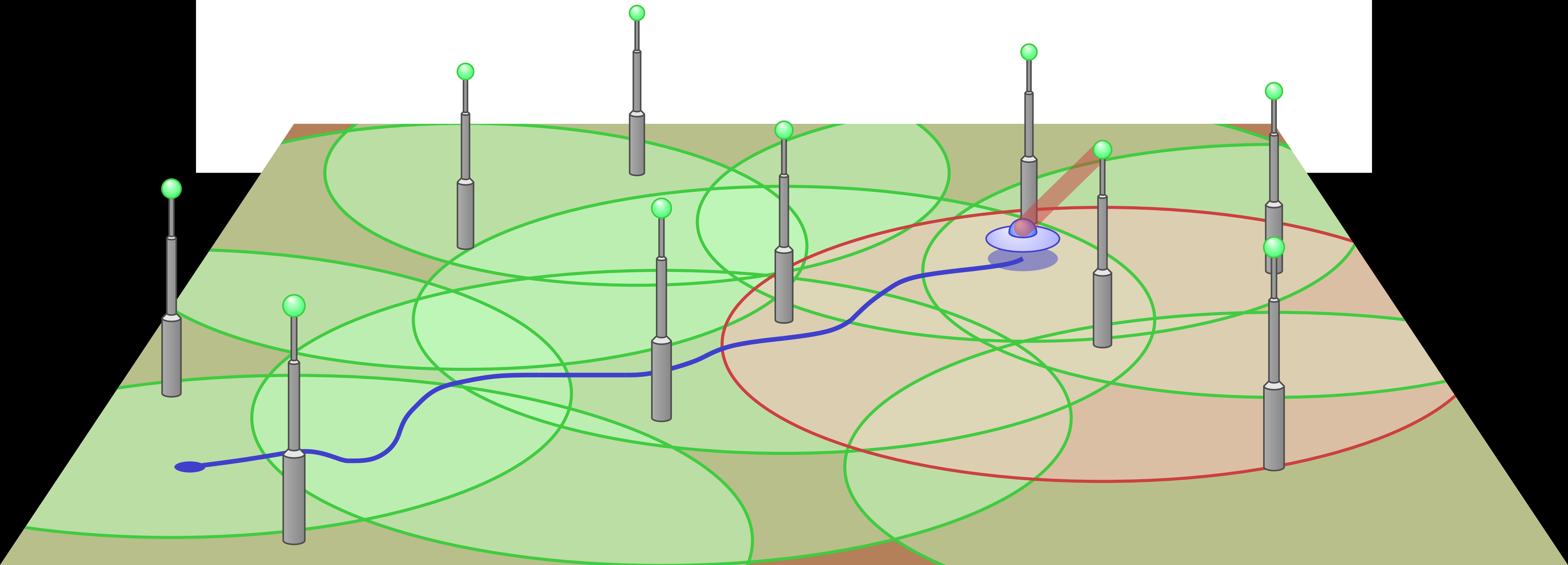

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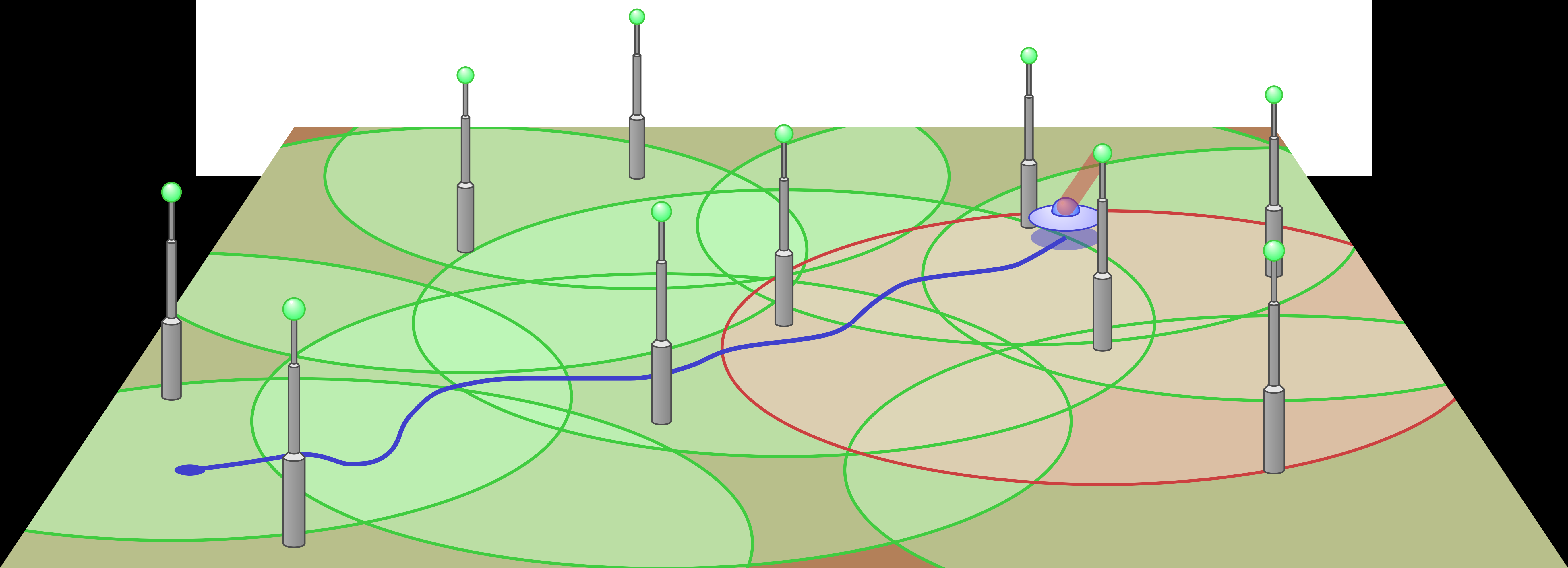

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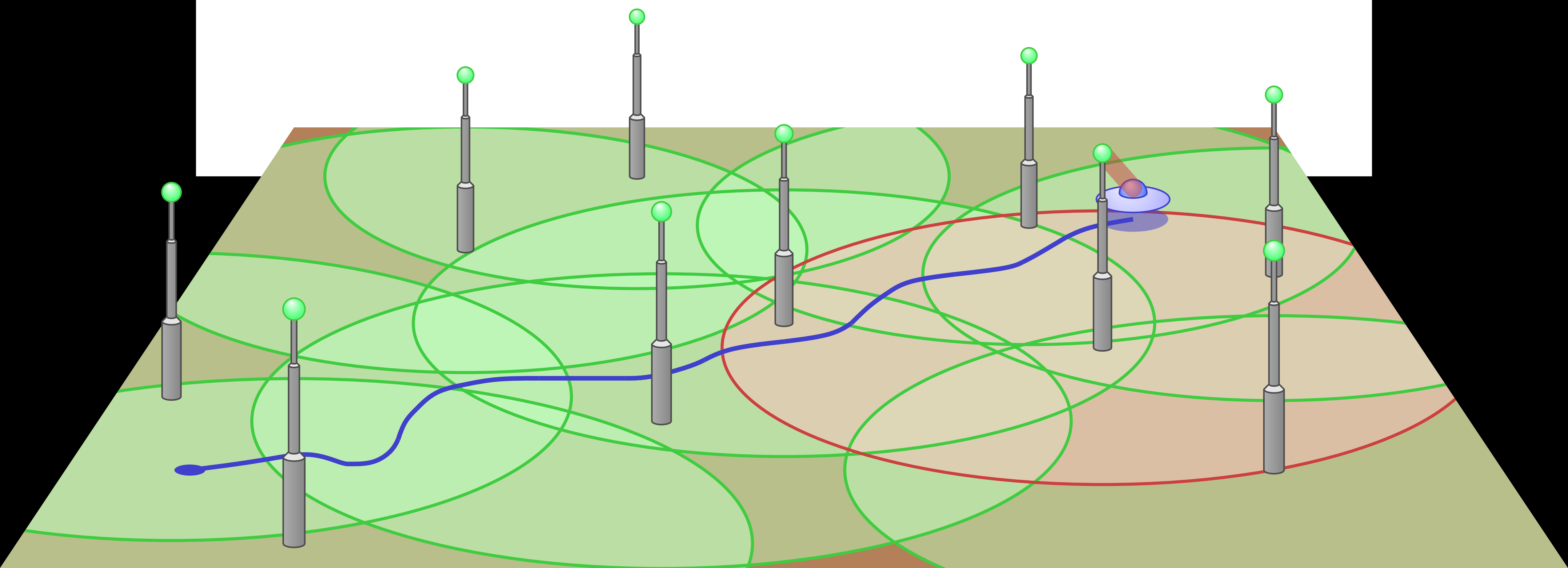

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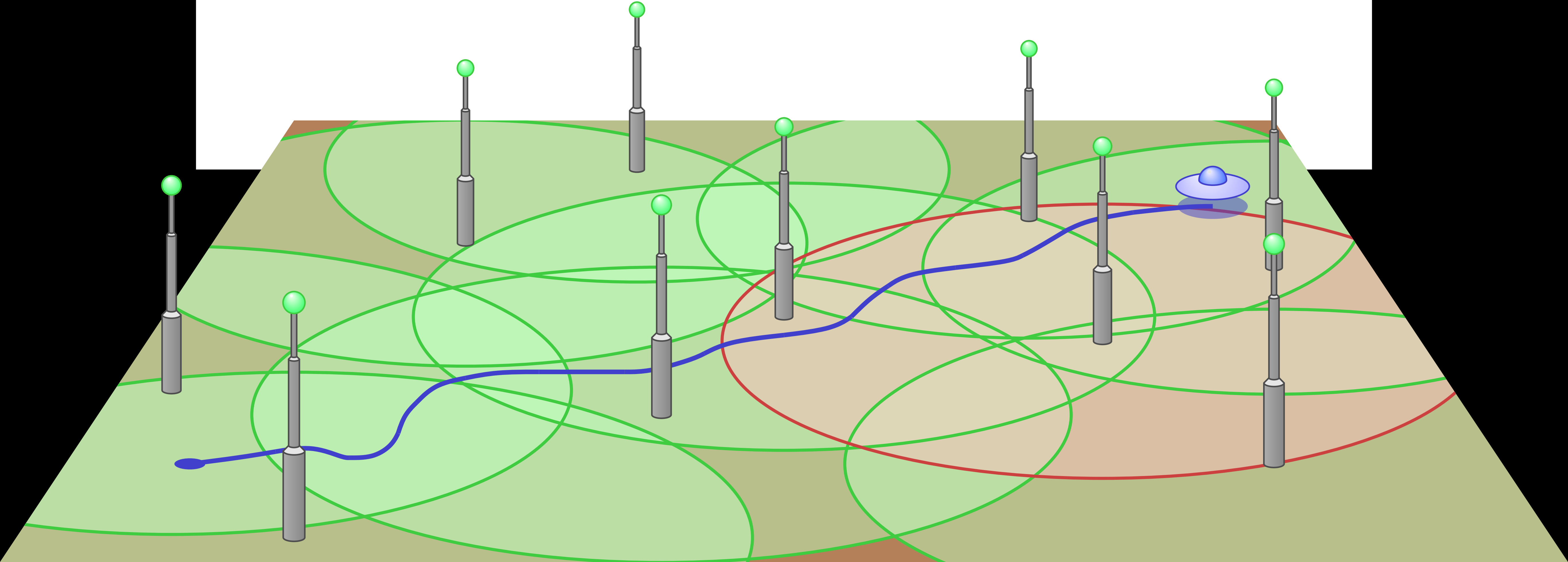

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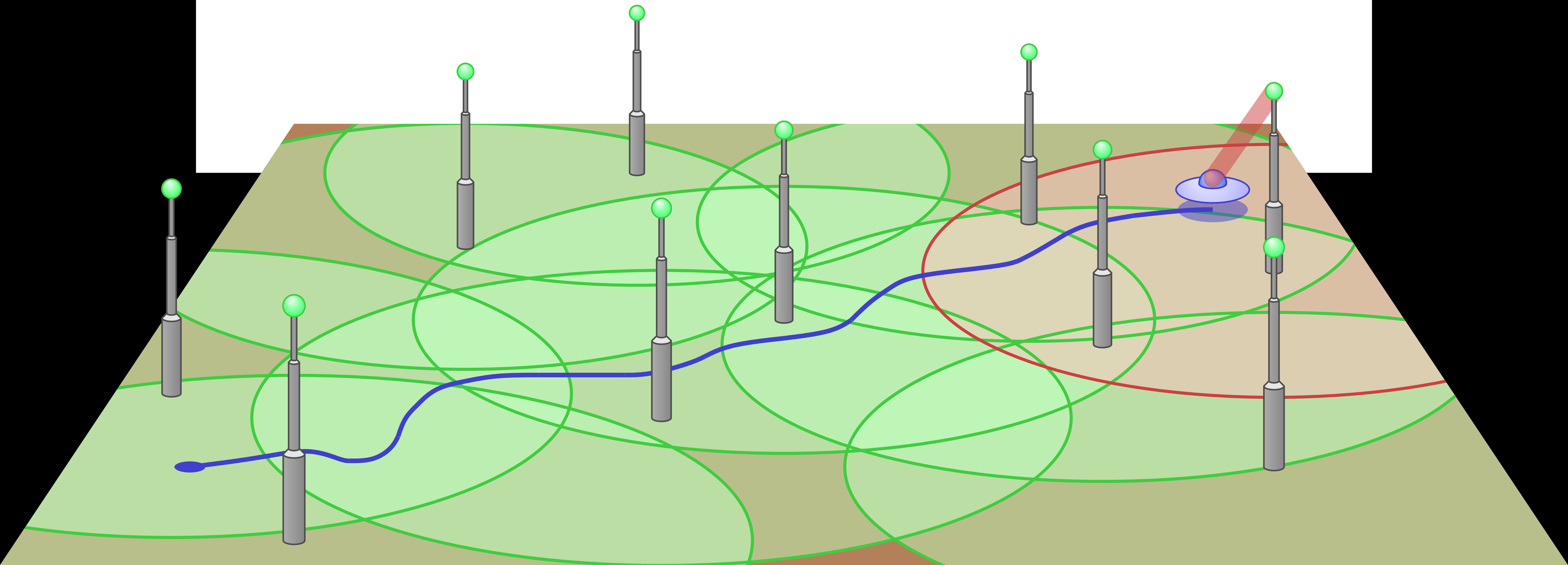

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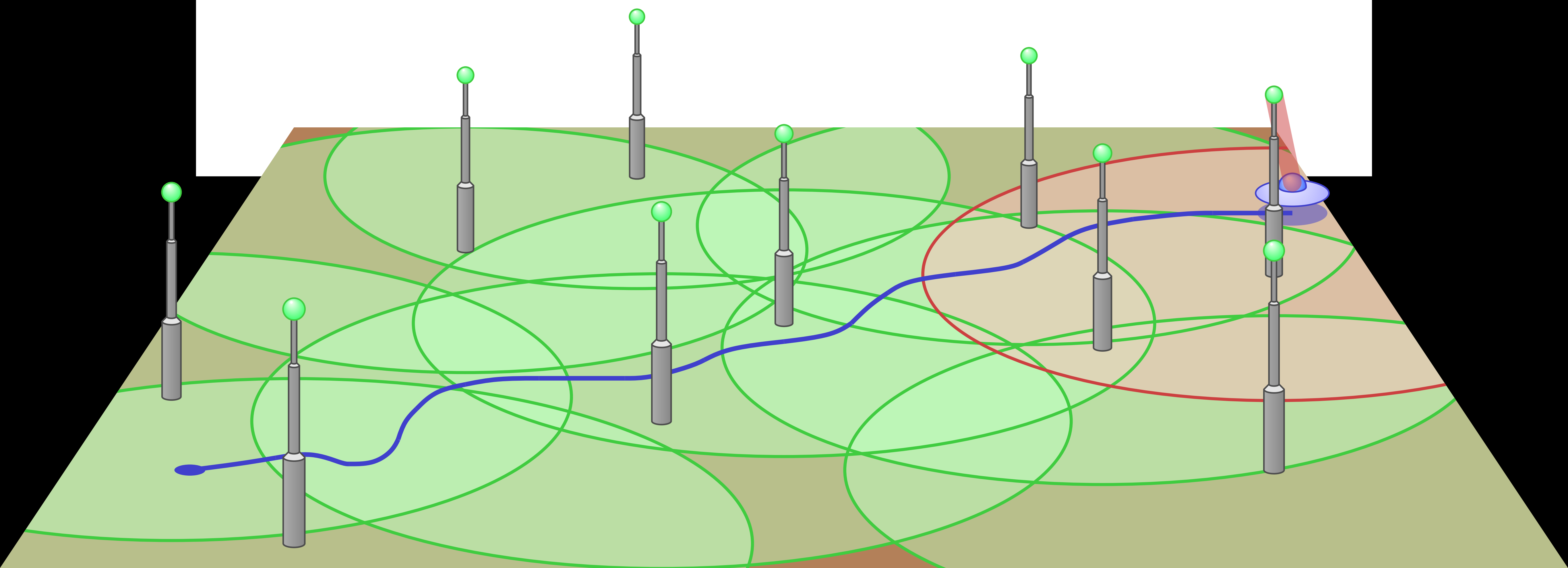

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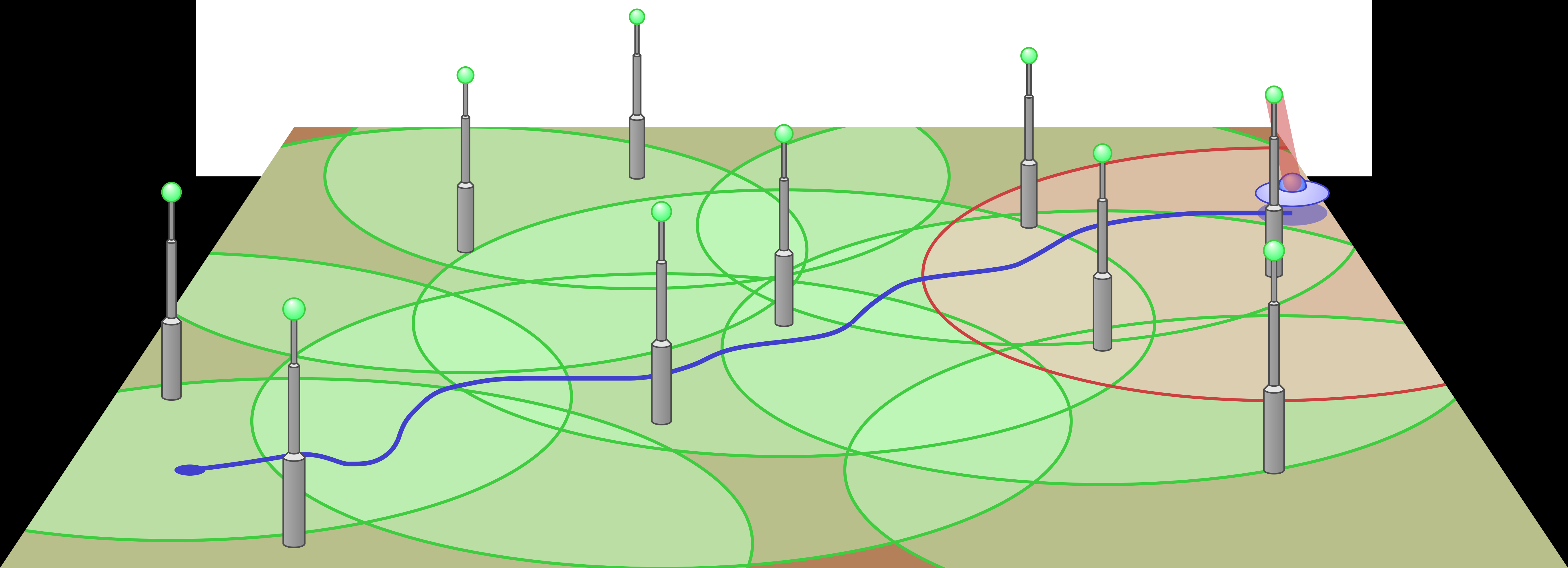

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  - When UMO leaves sensor region, we need to assign it to a different sensor
  - Handovers are costly
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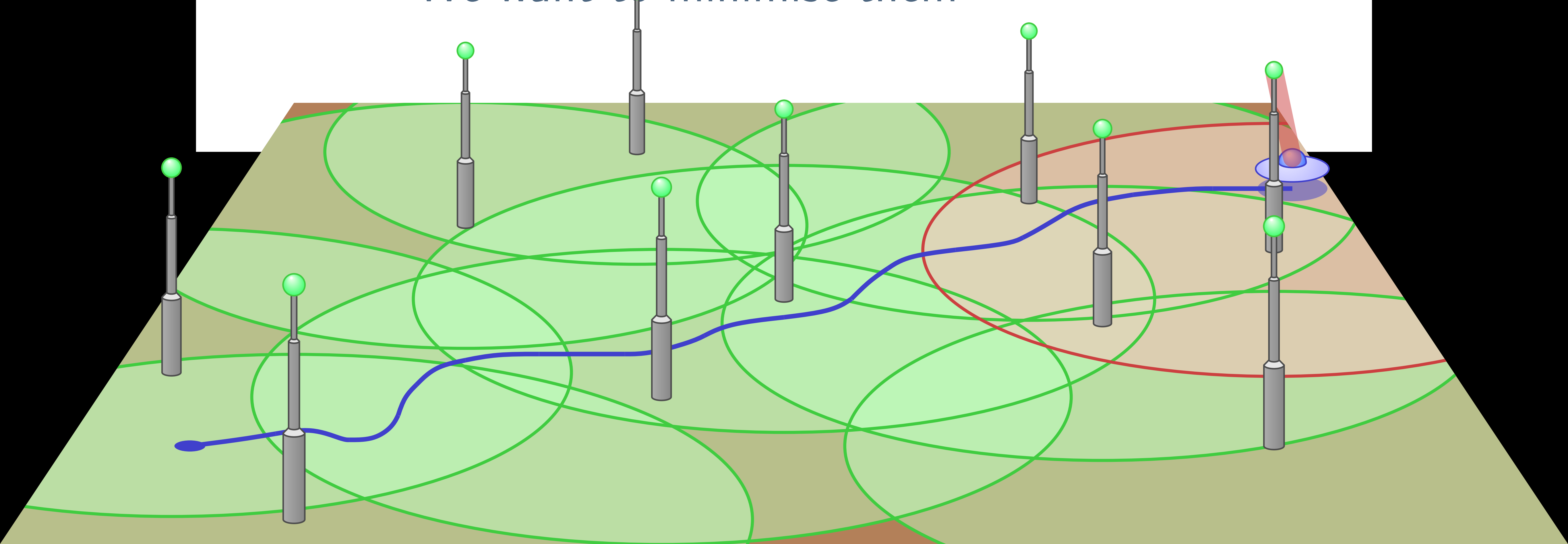

OBJECTIVES

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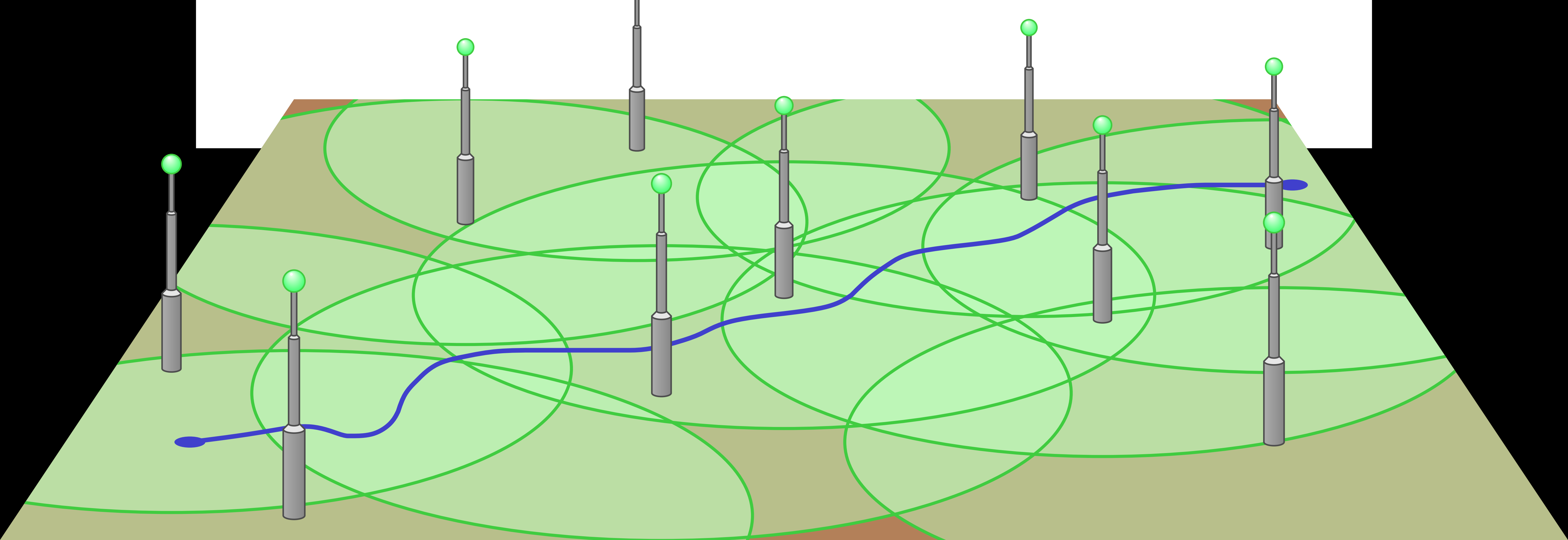

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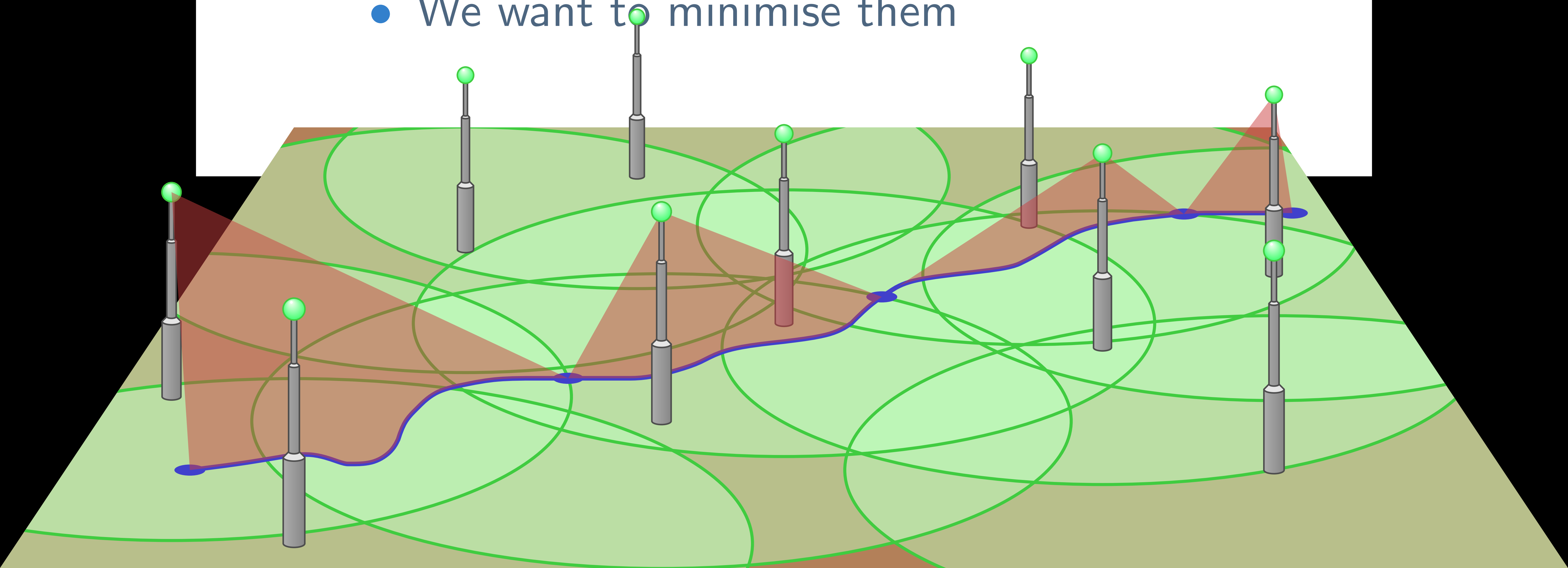

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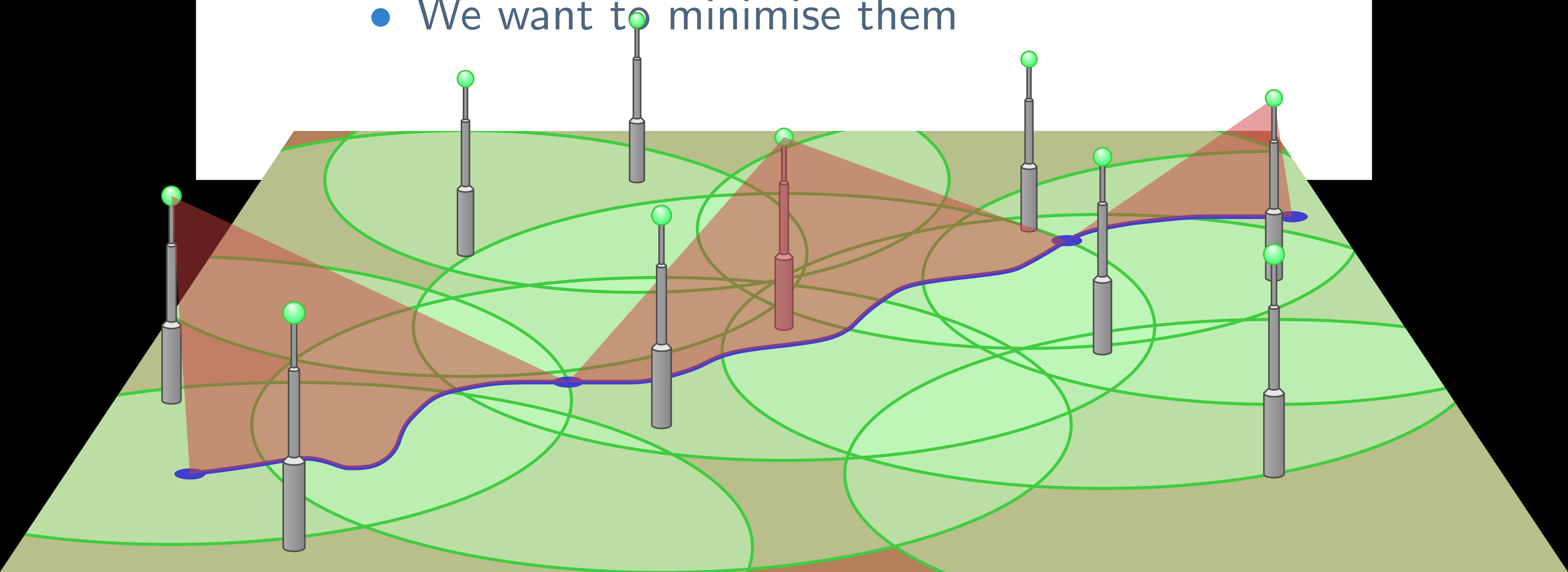

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- Performance measure
  - *Competitive ratio*: cost of online algorithm divided by cost of optimal offline algorithm

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- Randomised algorithms
  - Algorithm knows all past events
  - Adversary knows everything except the random choices made by the algorithm

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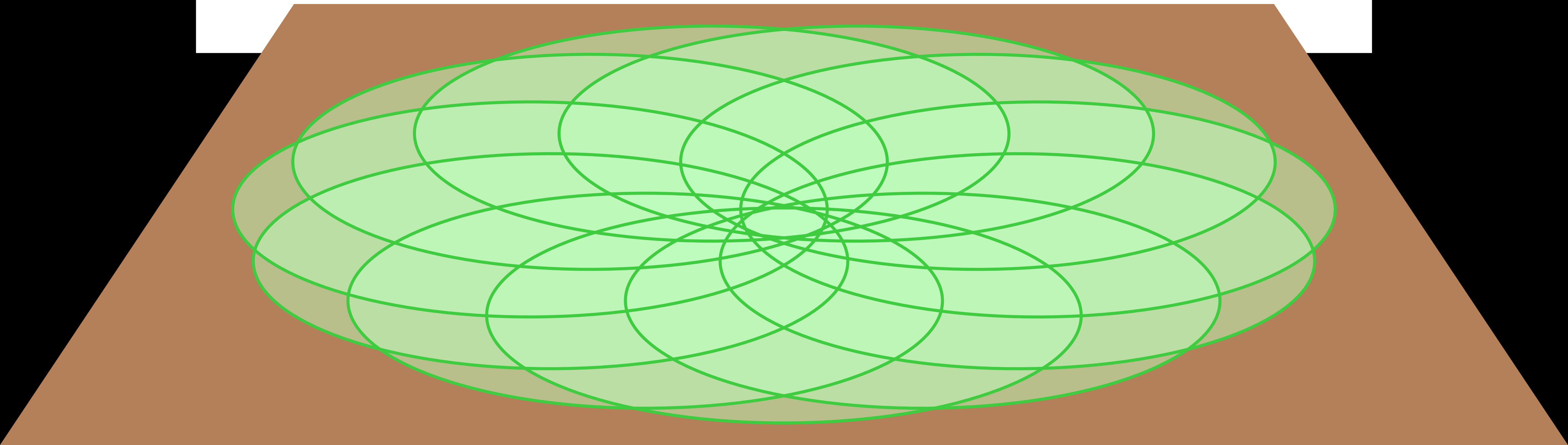

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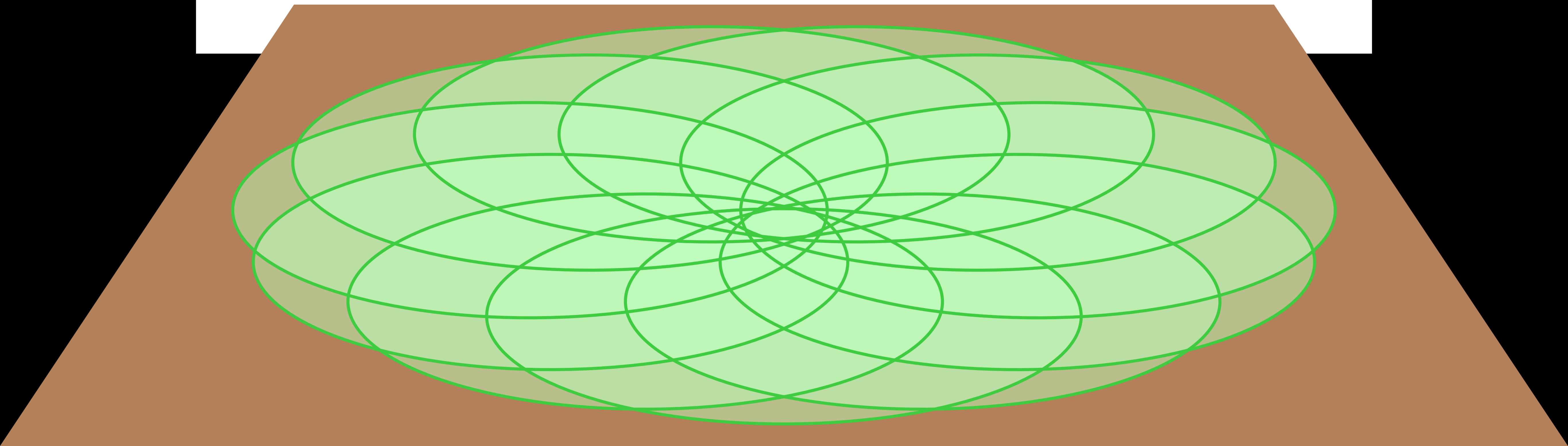

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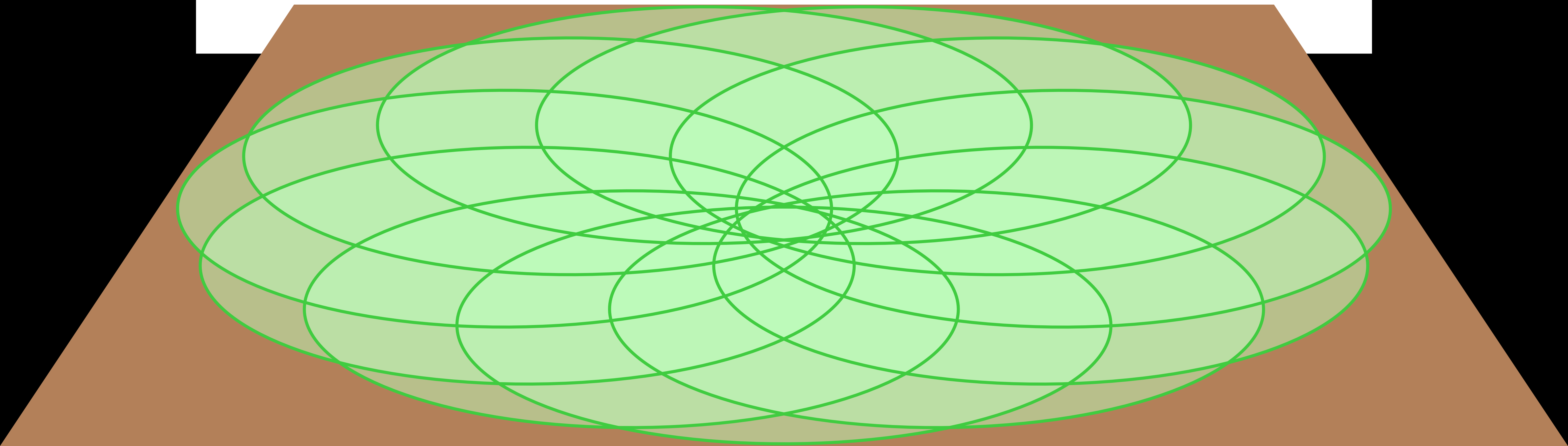

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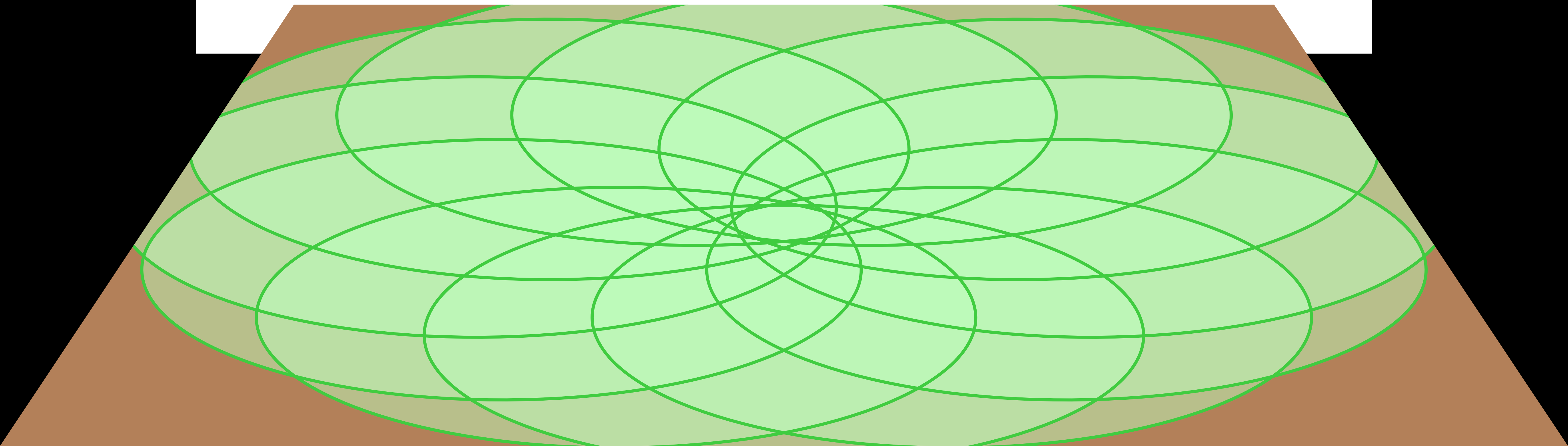

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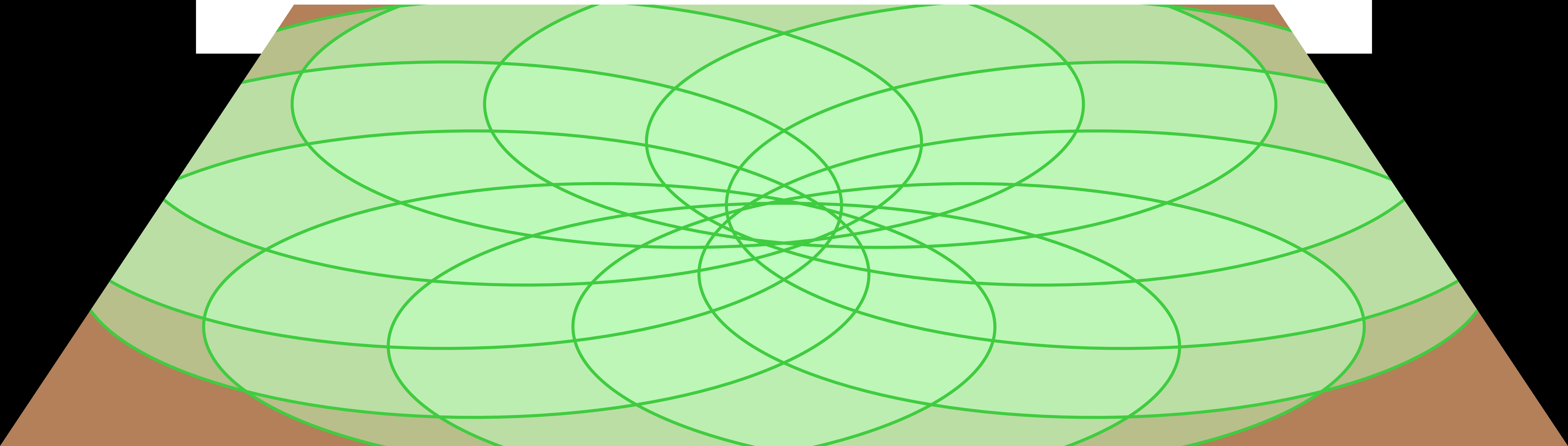

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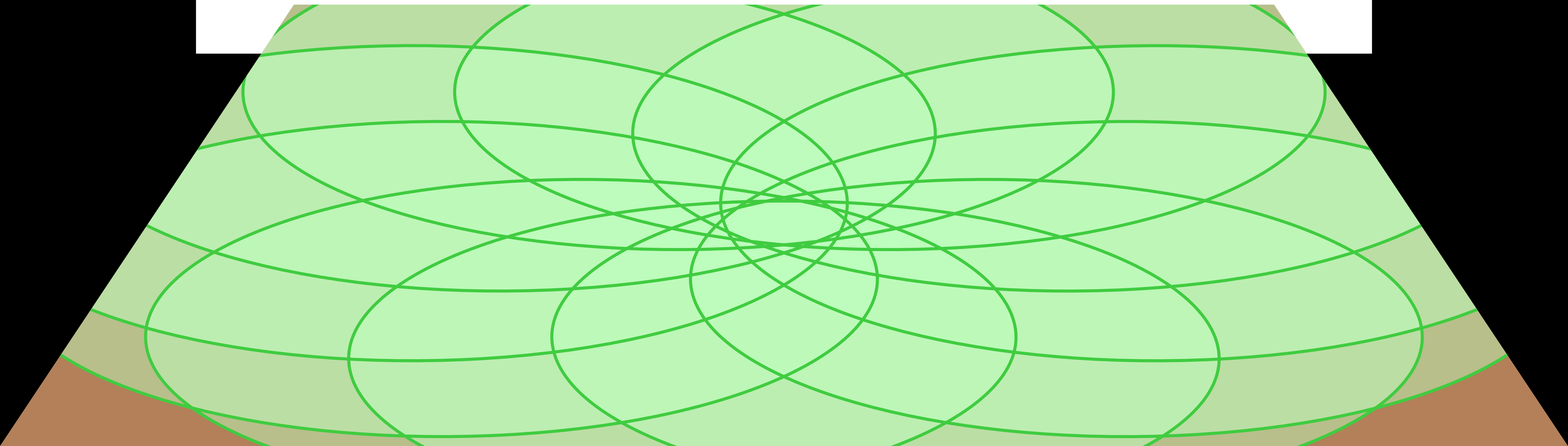

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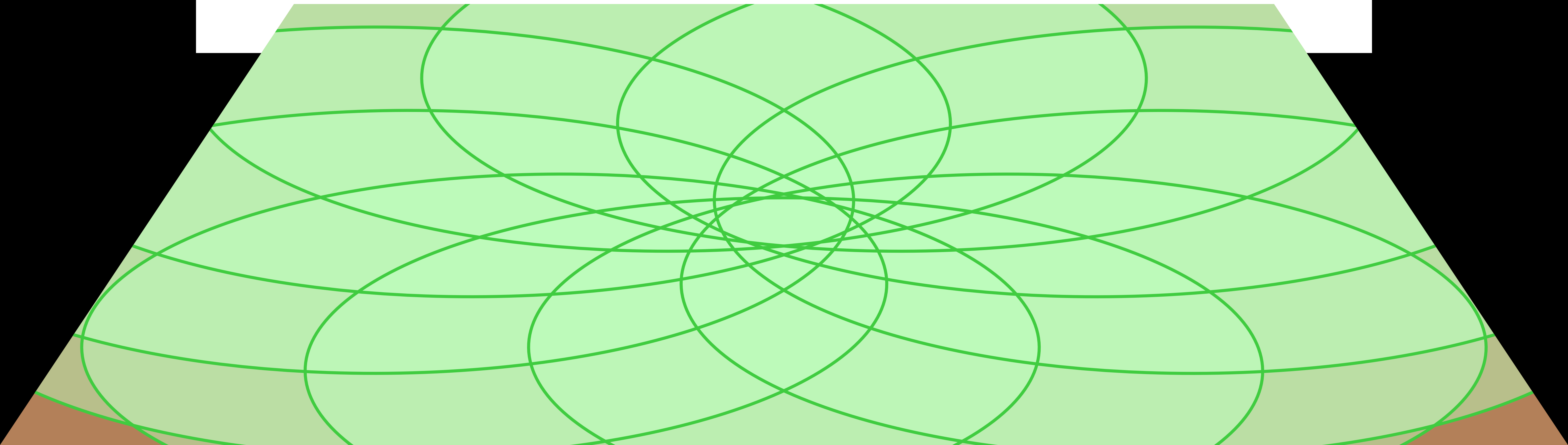

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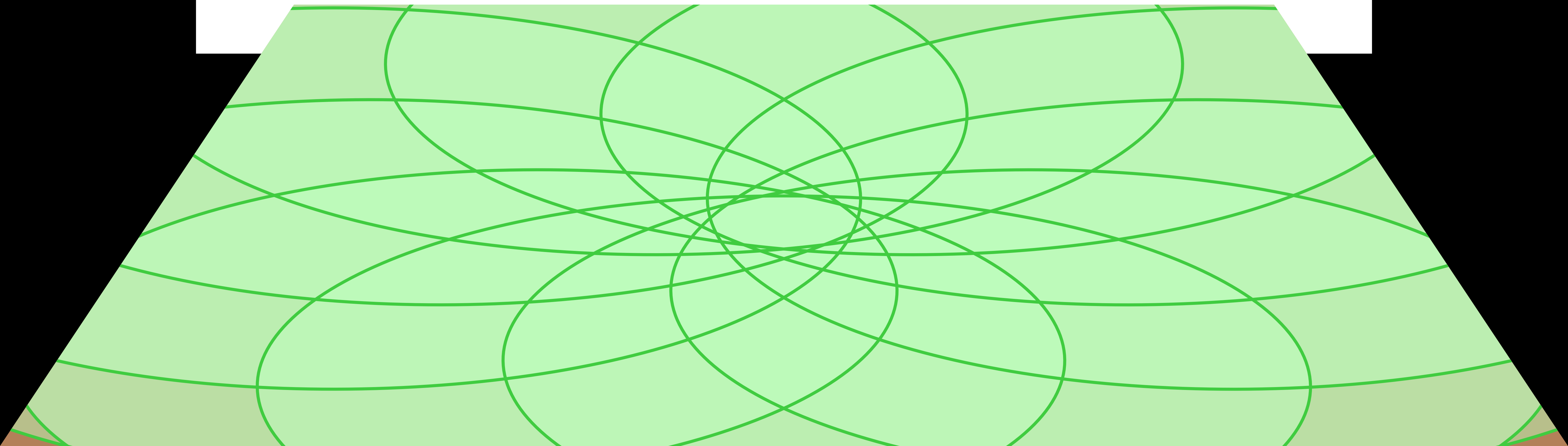

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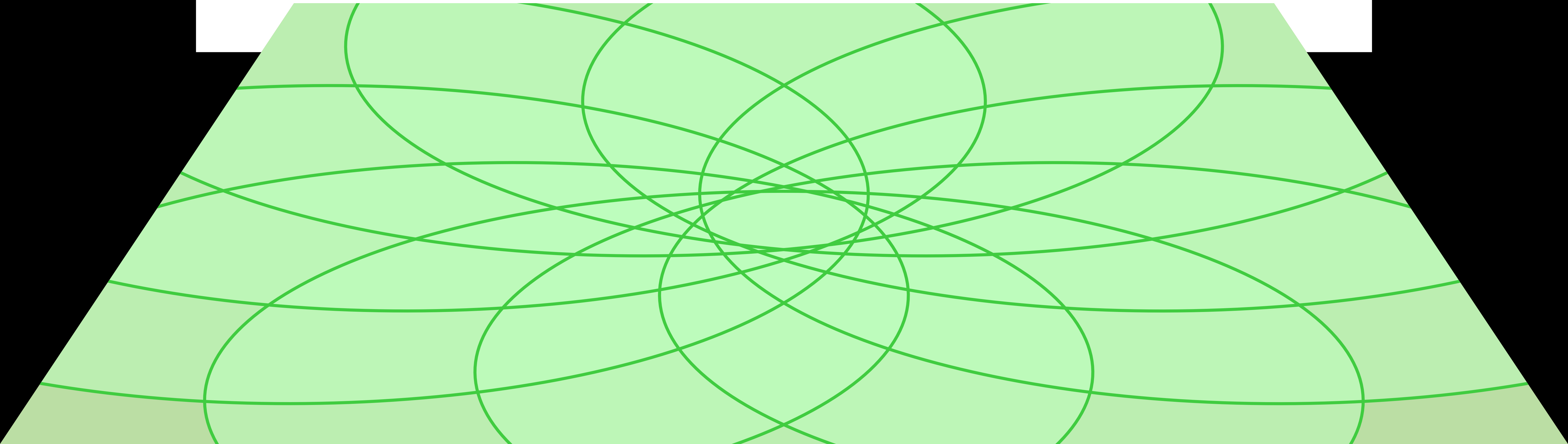

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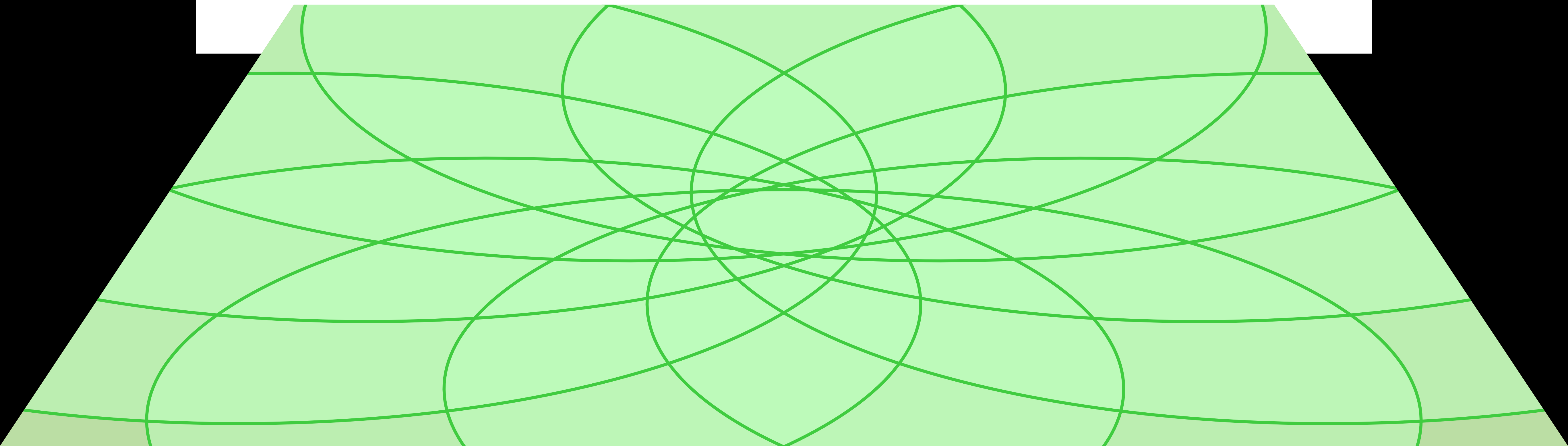

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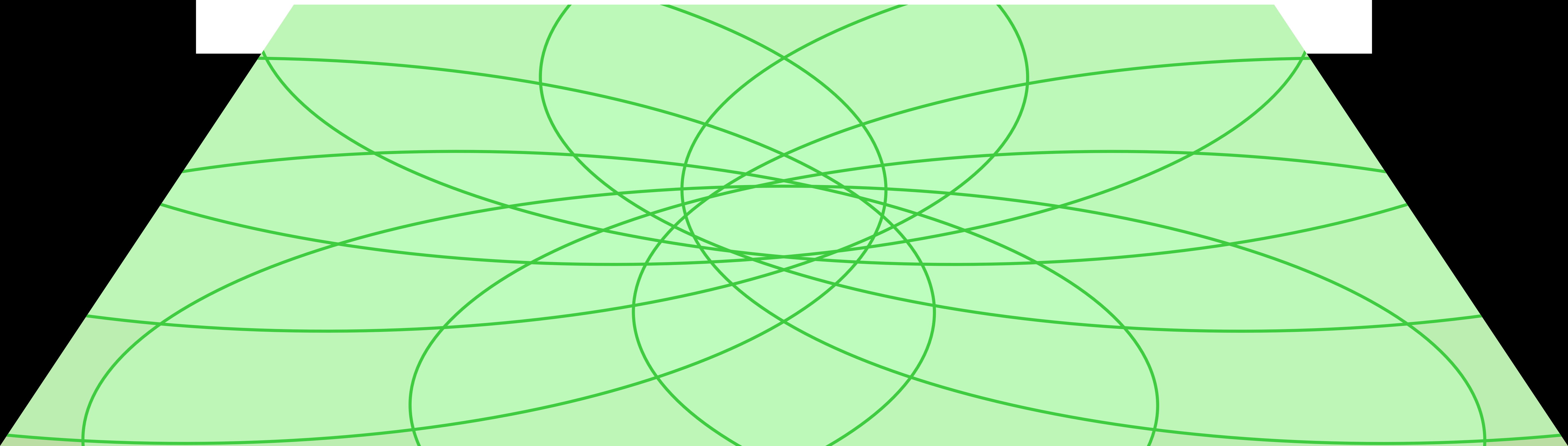

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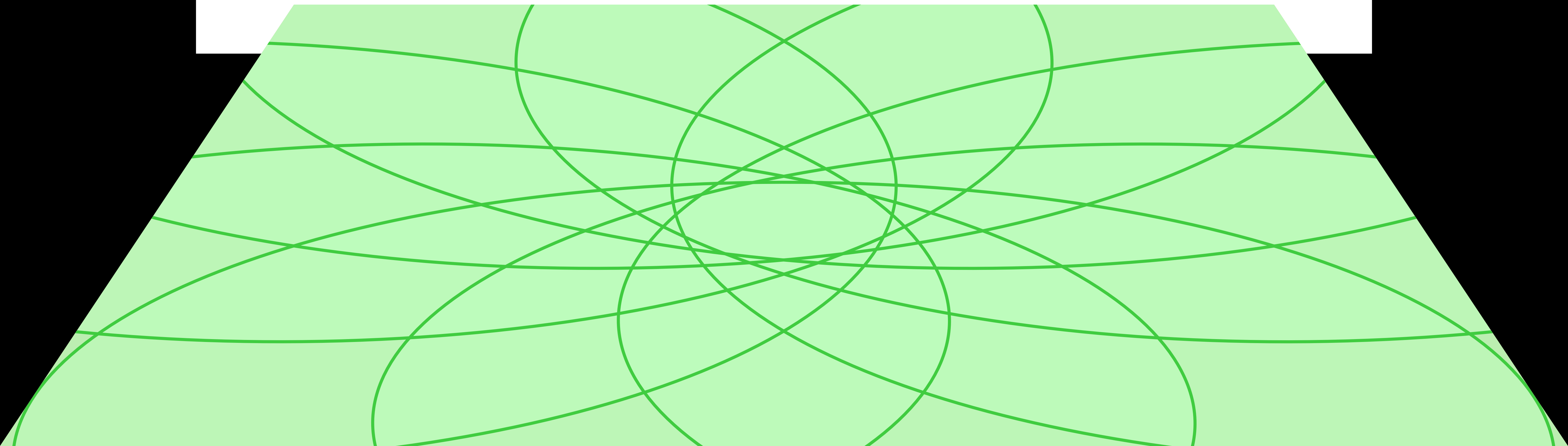

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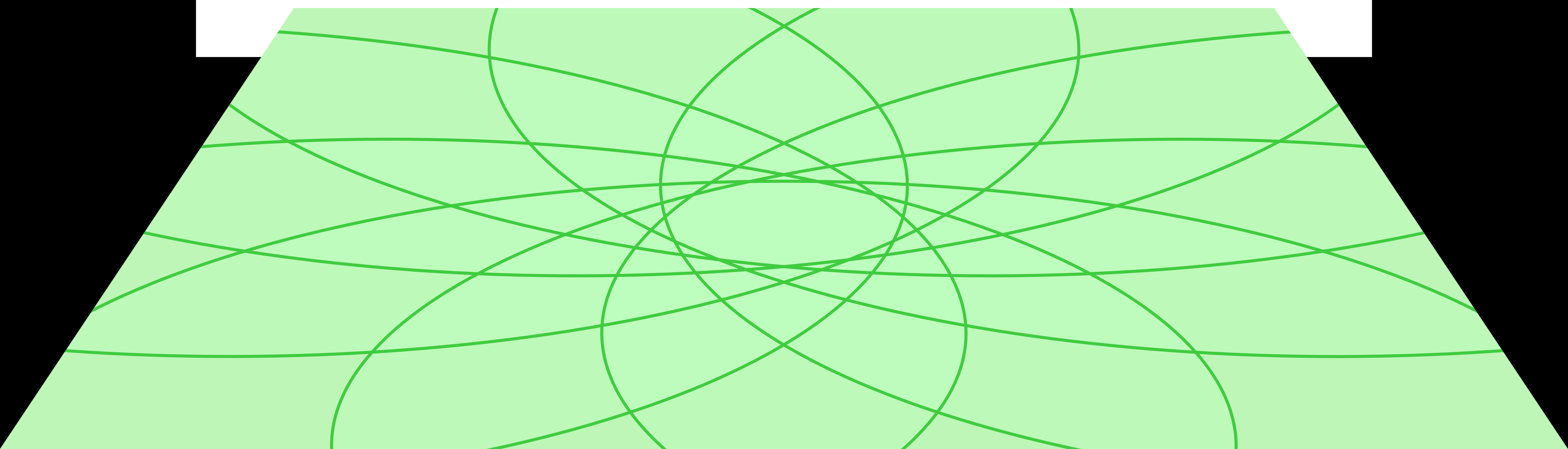

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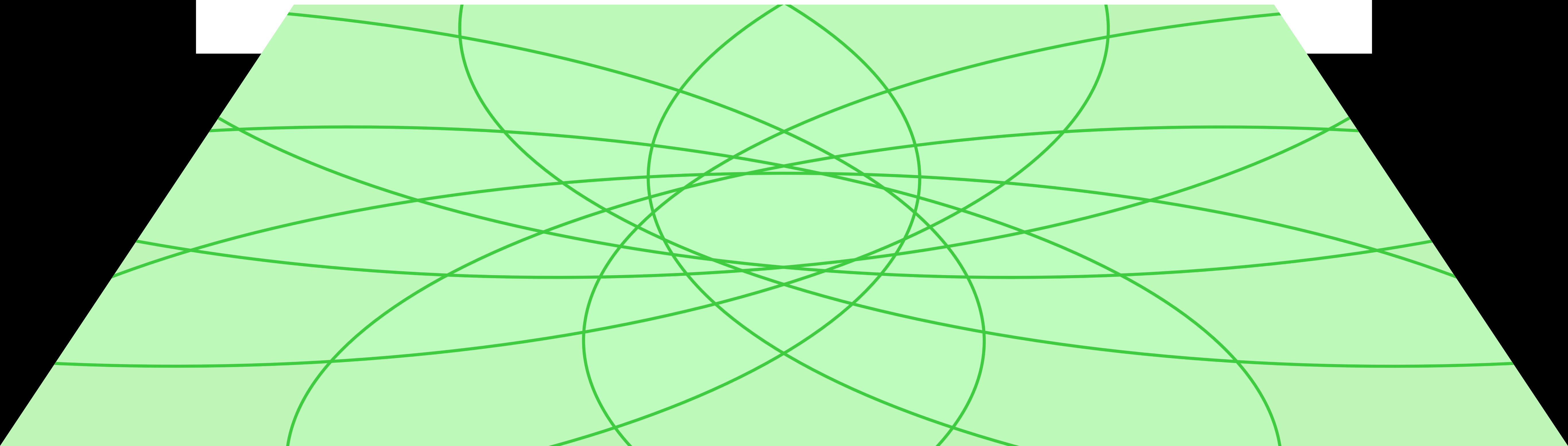

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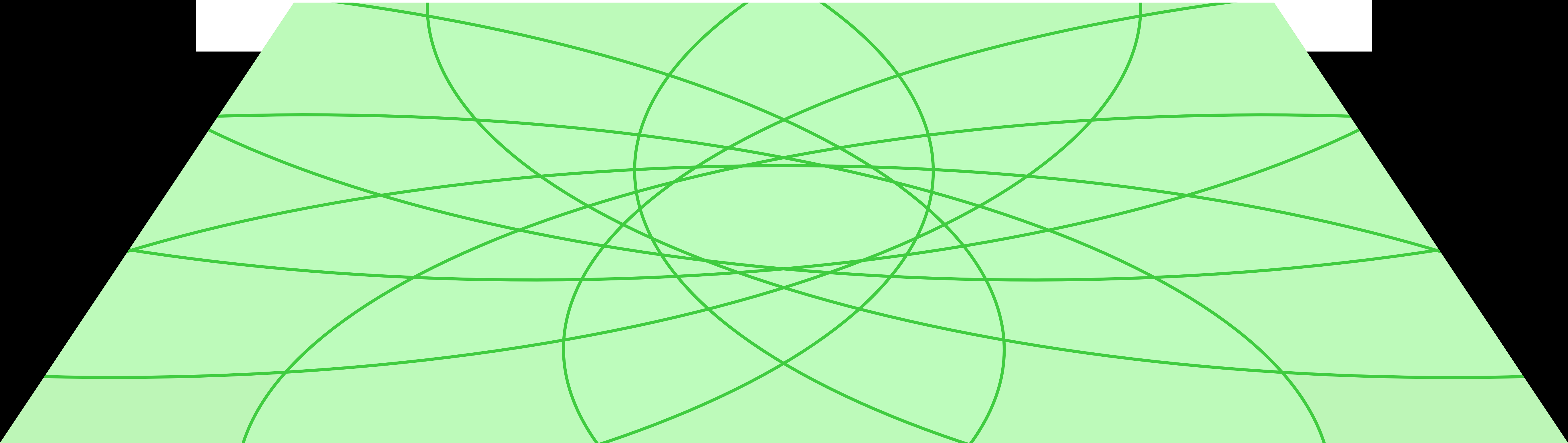

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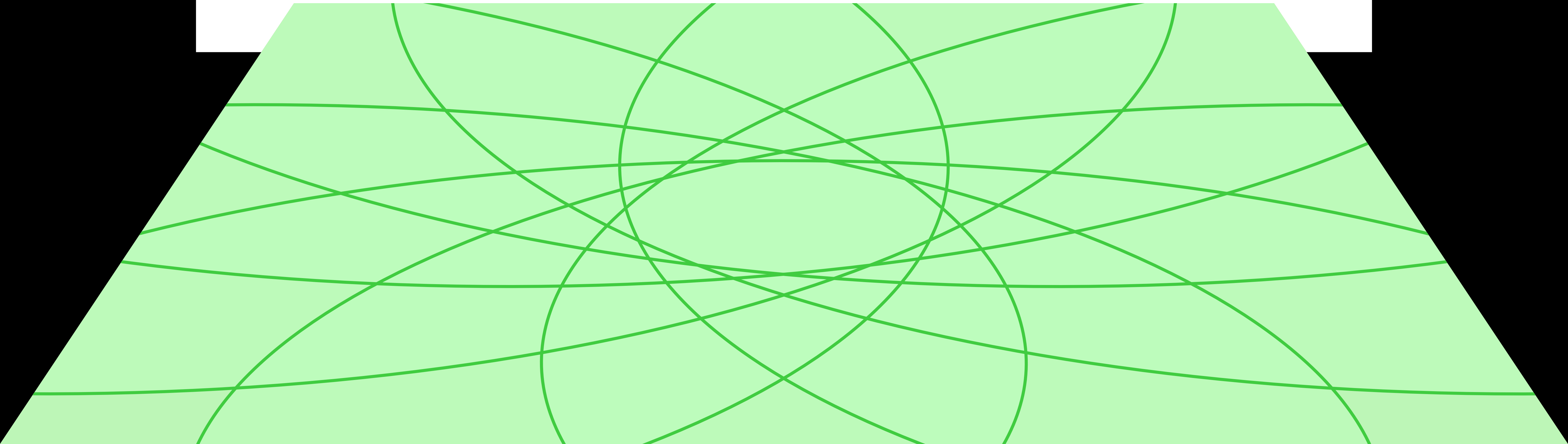

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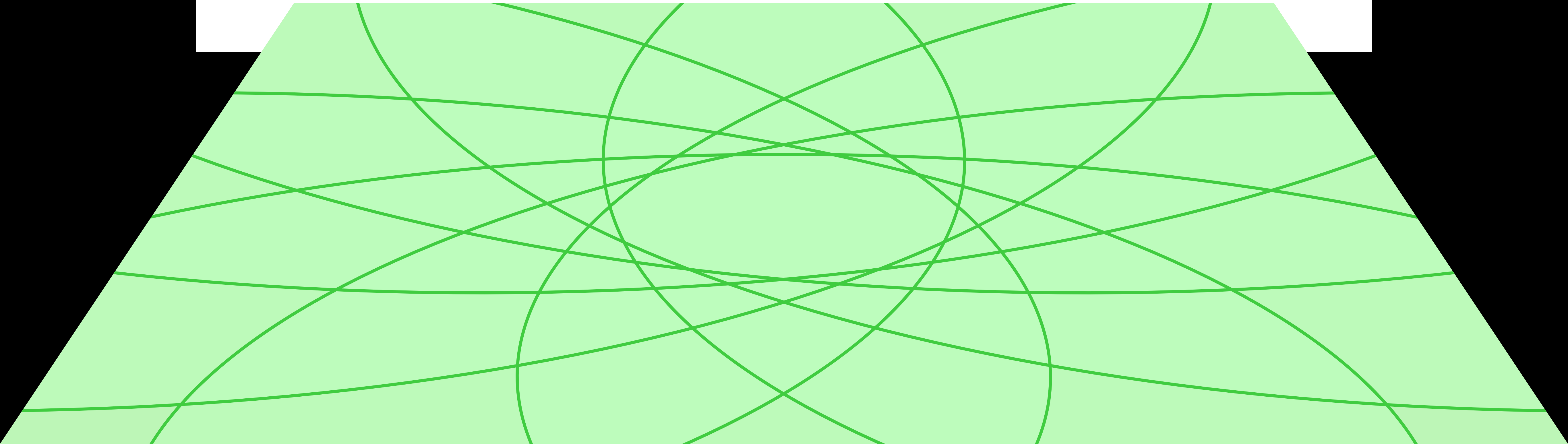

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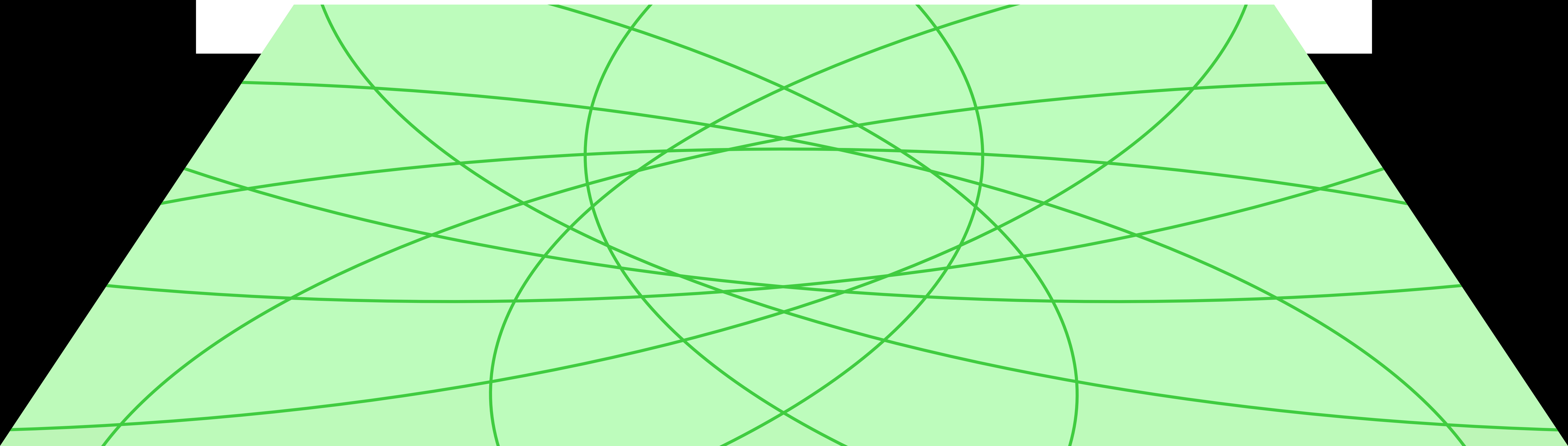

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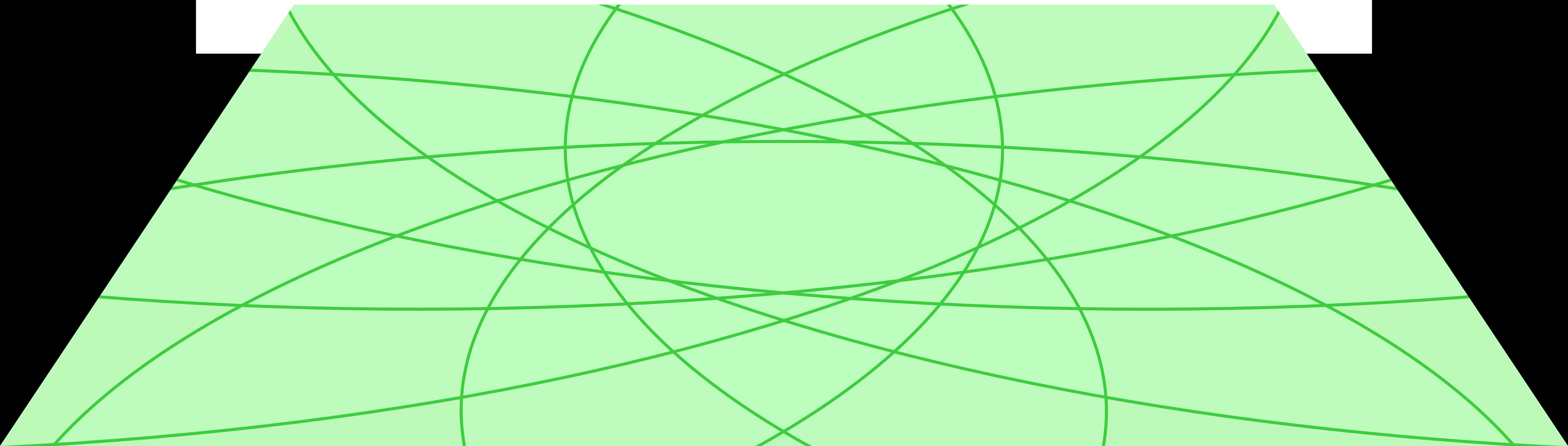

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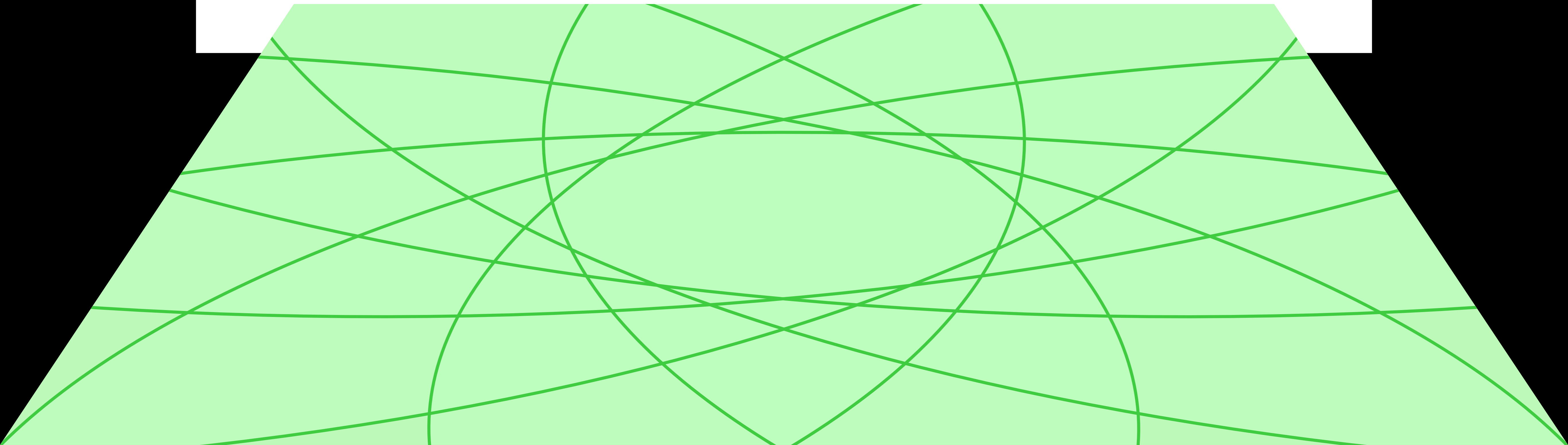

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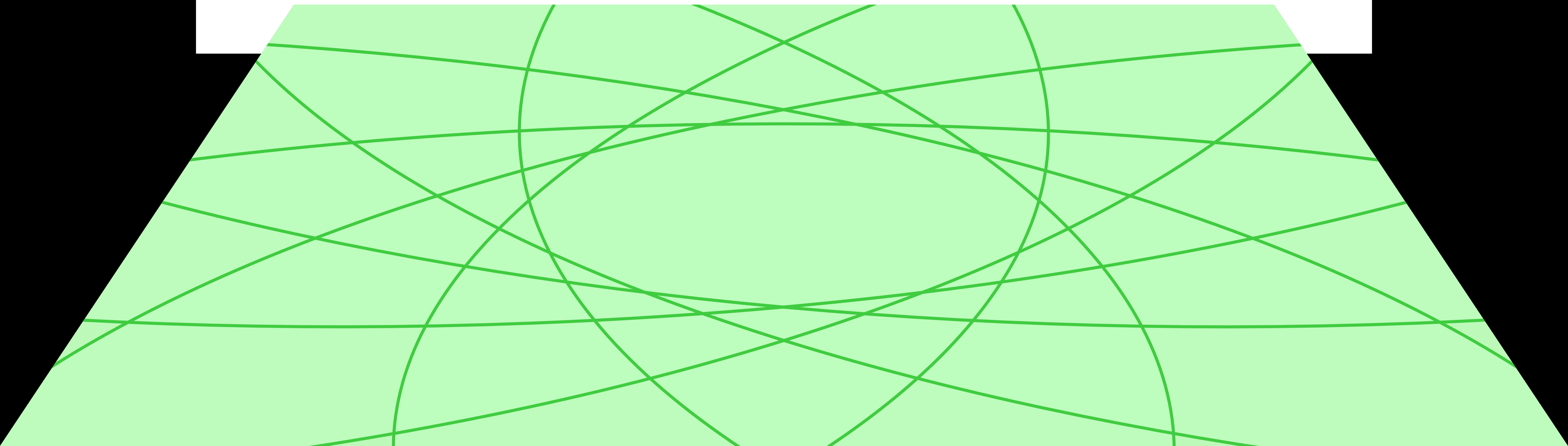

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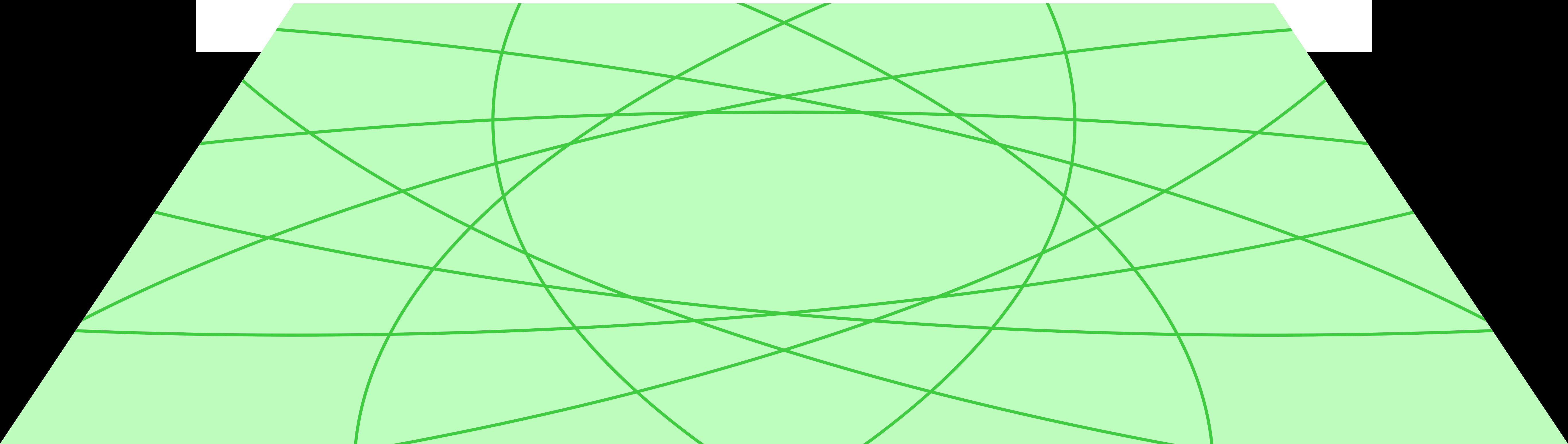

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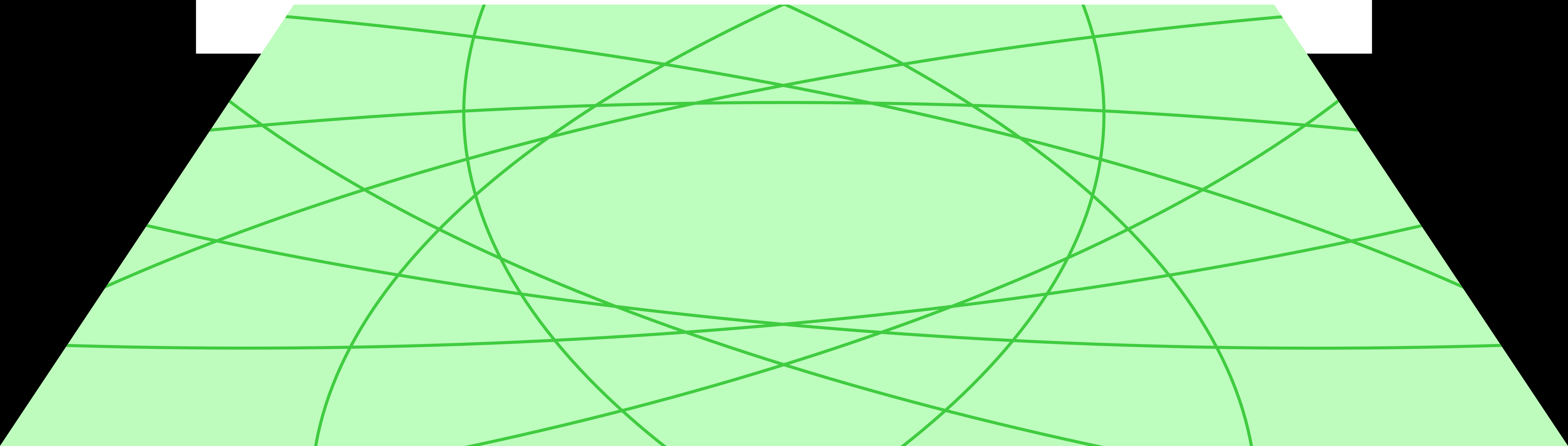

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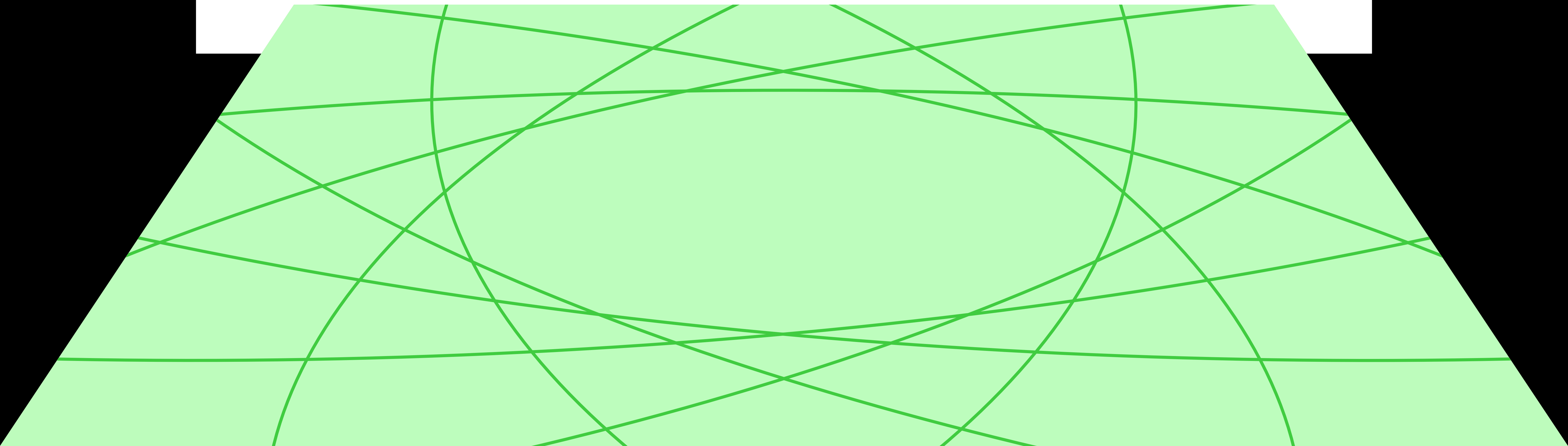

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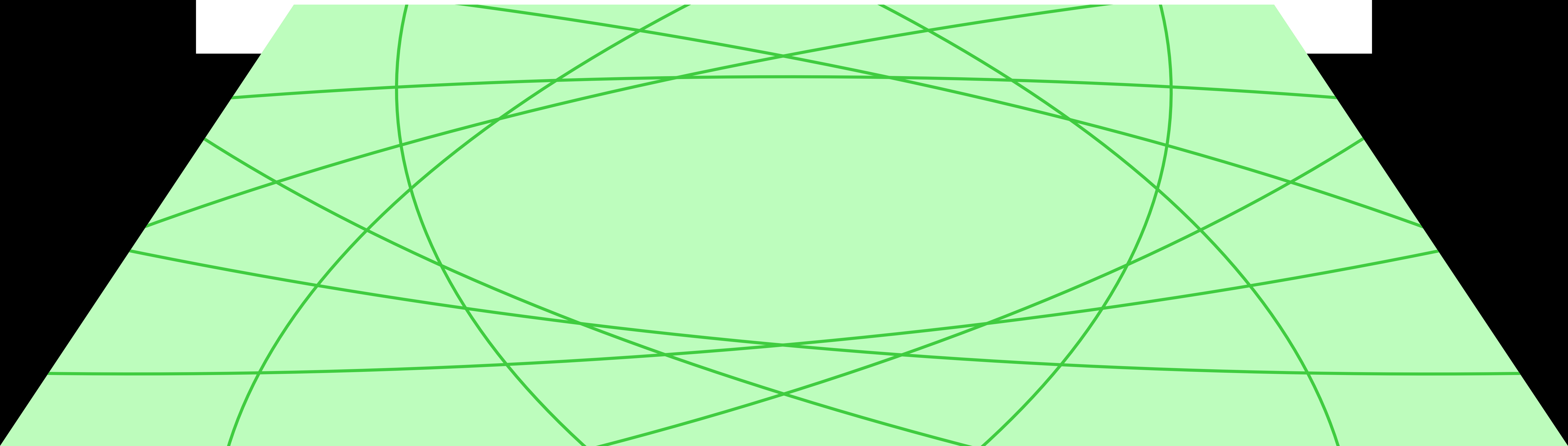

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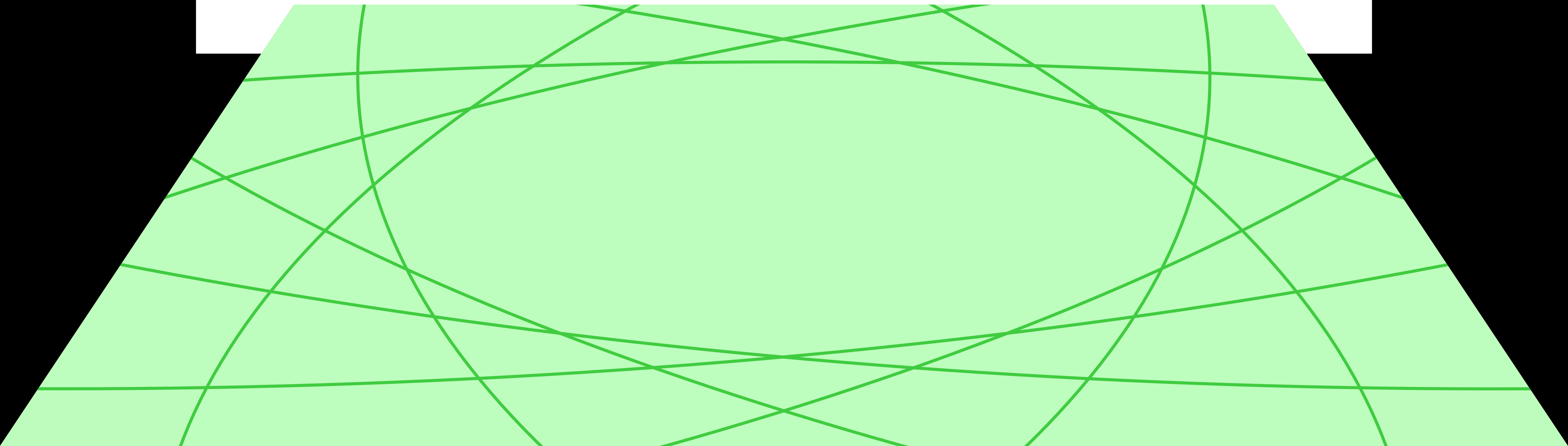

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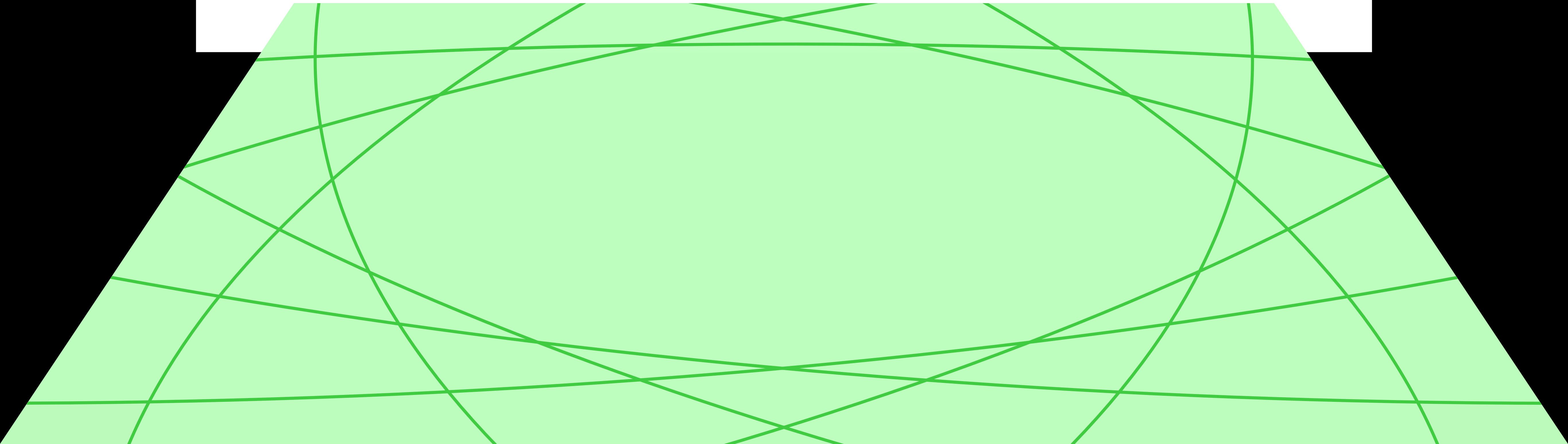

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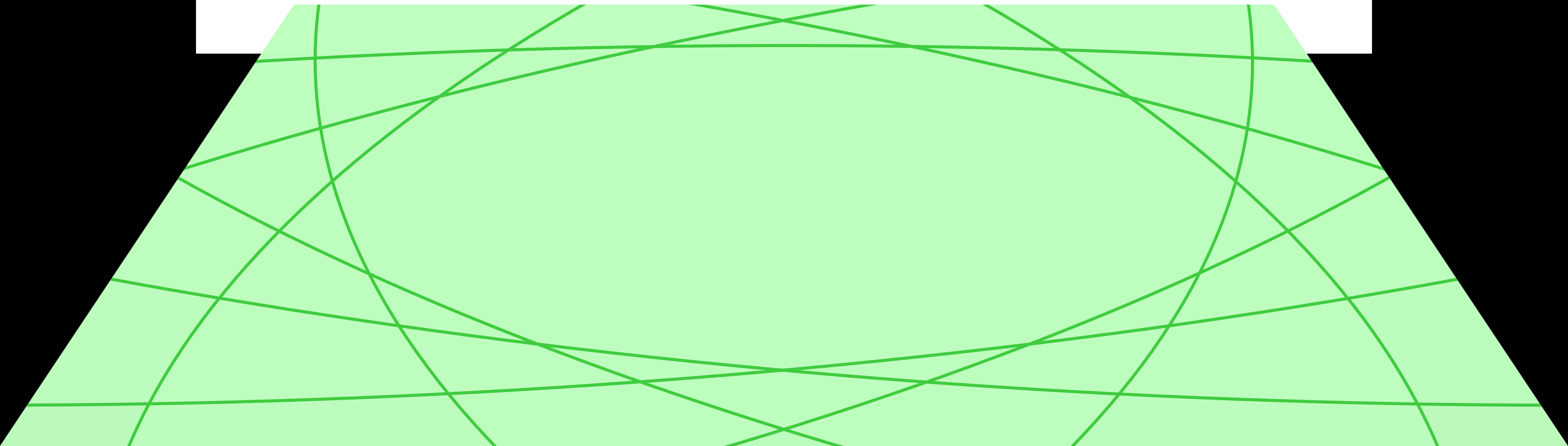

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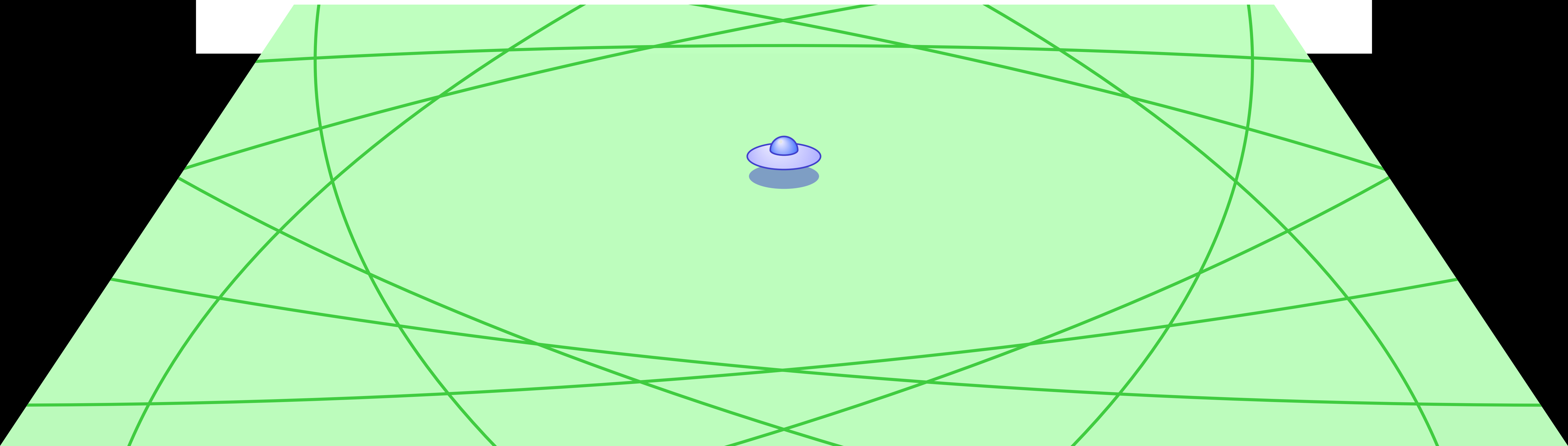

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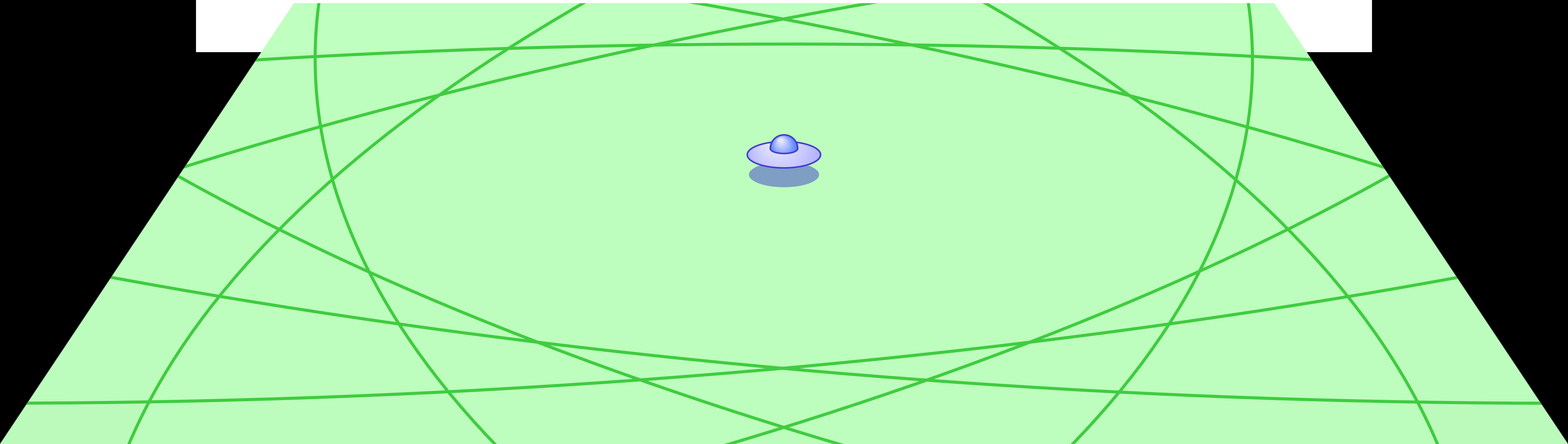

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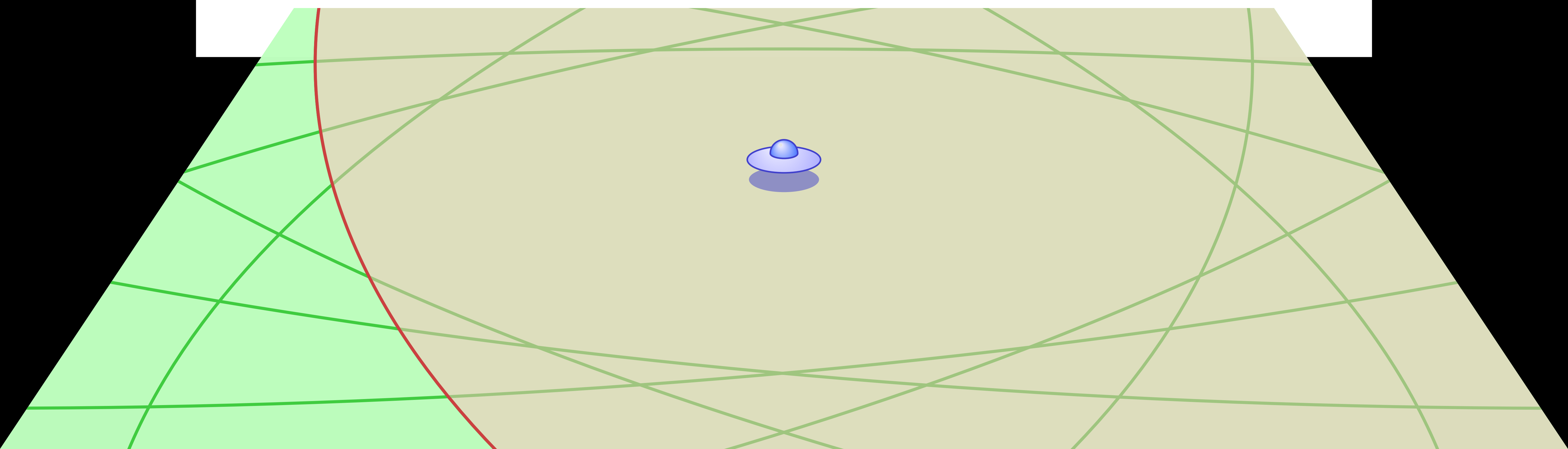

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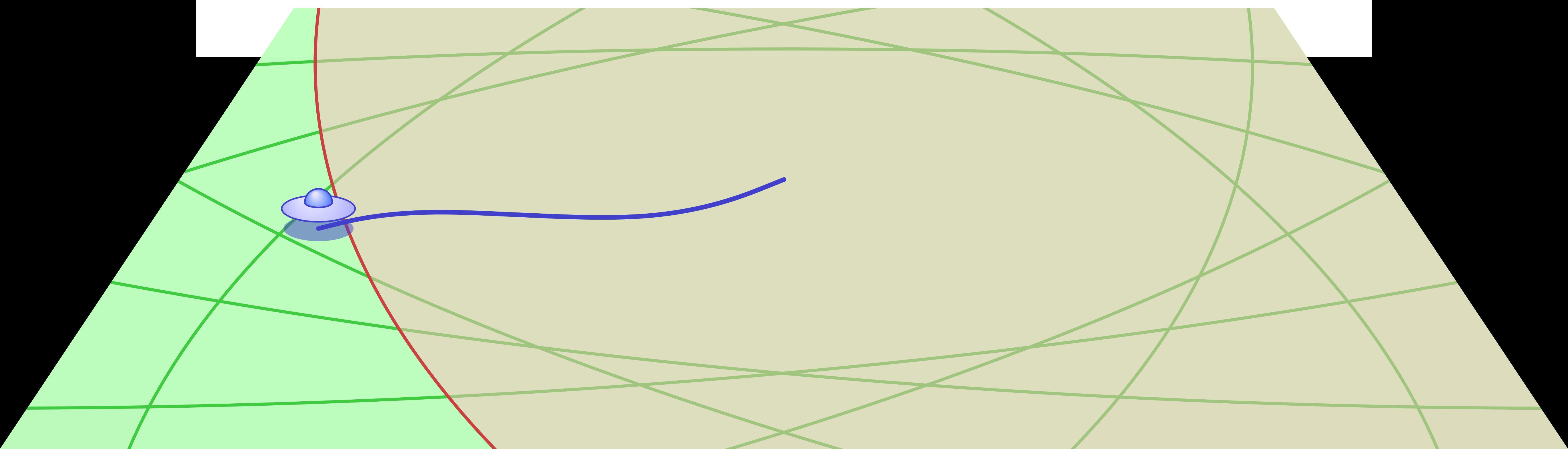

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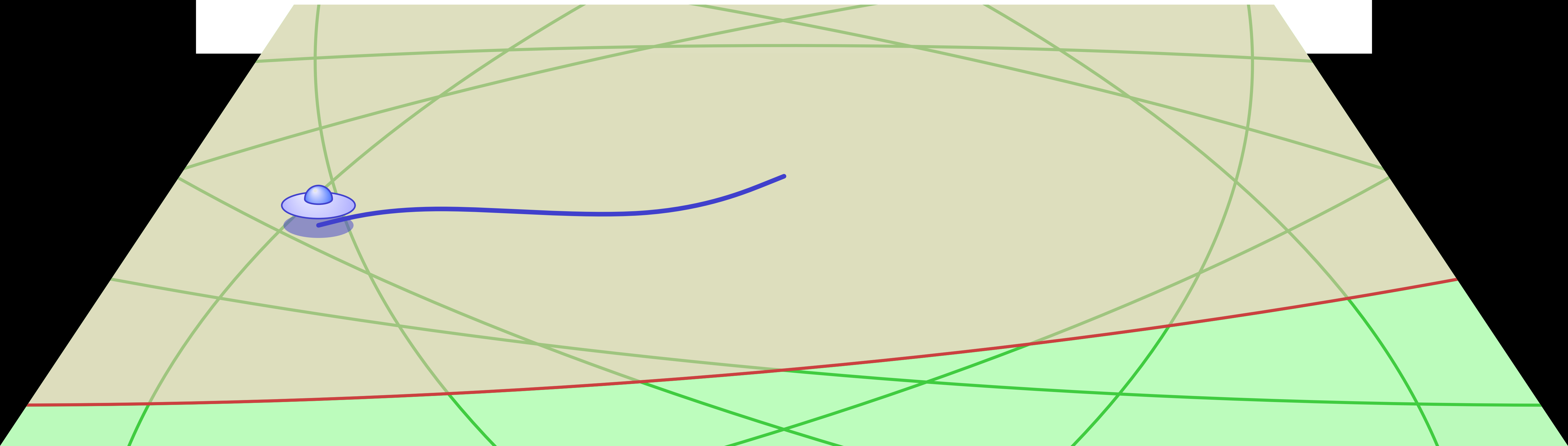

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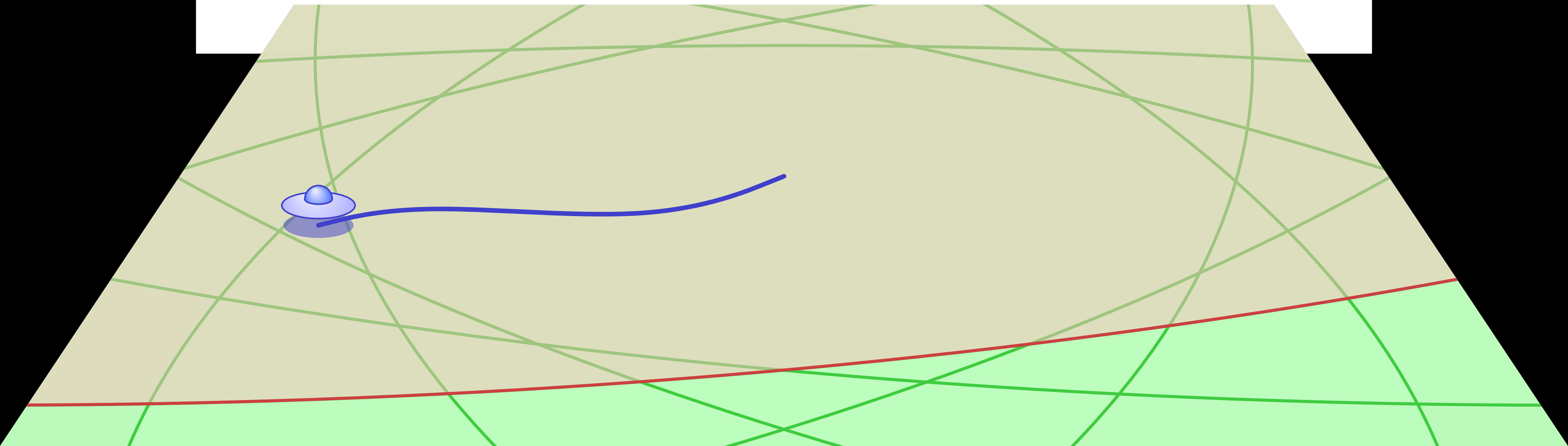

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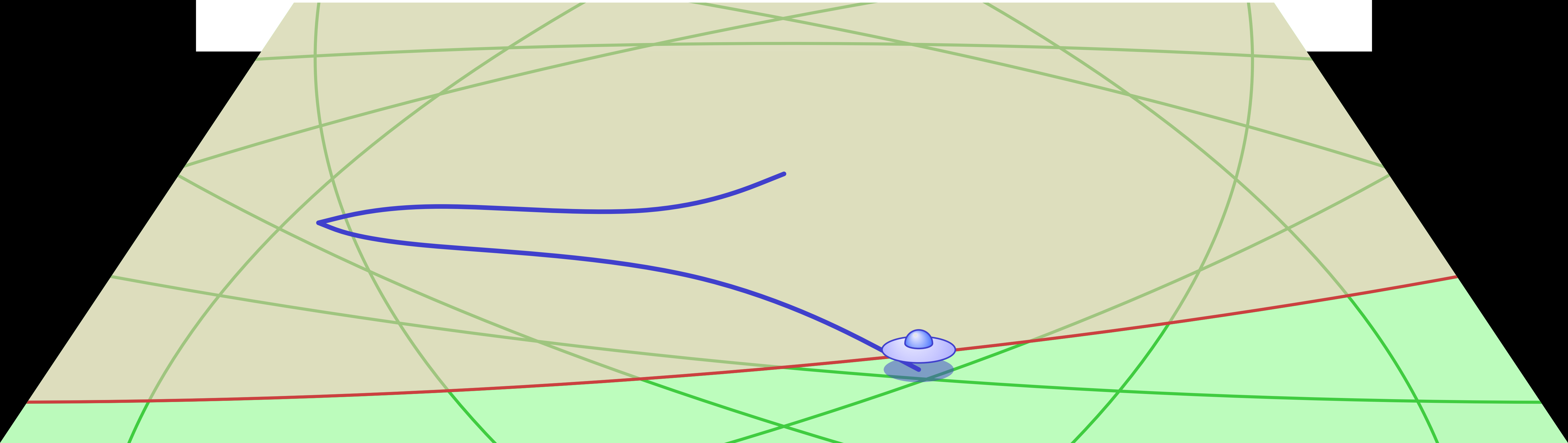

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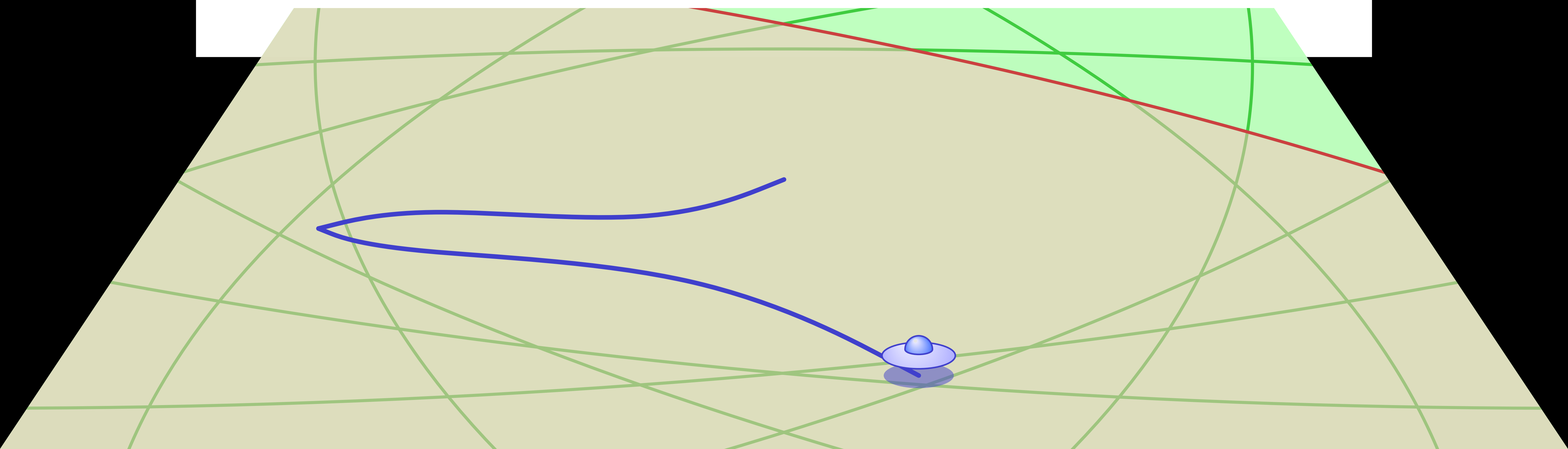

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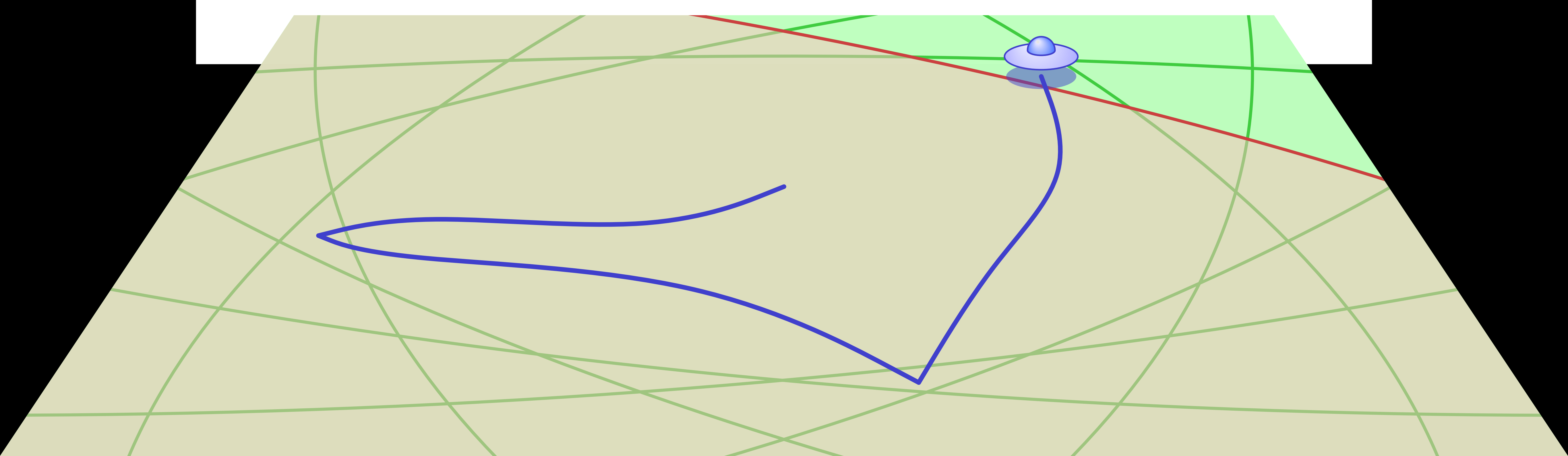

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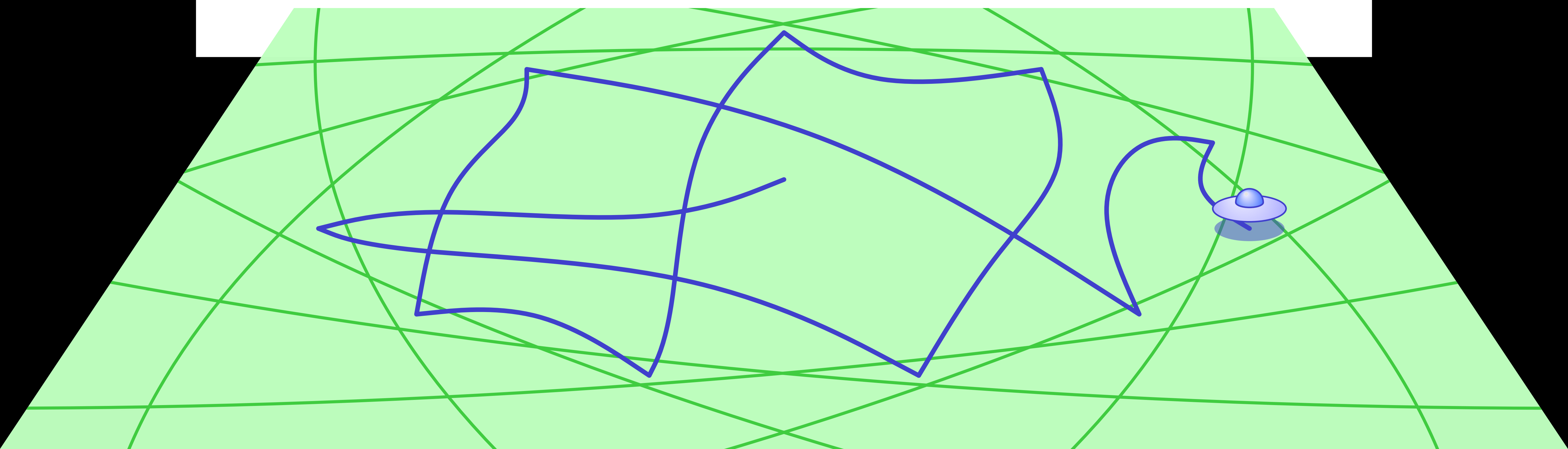

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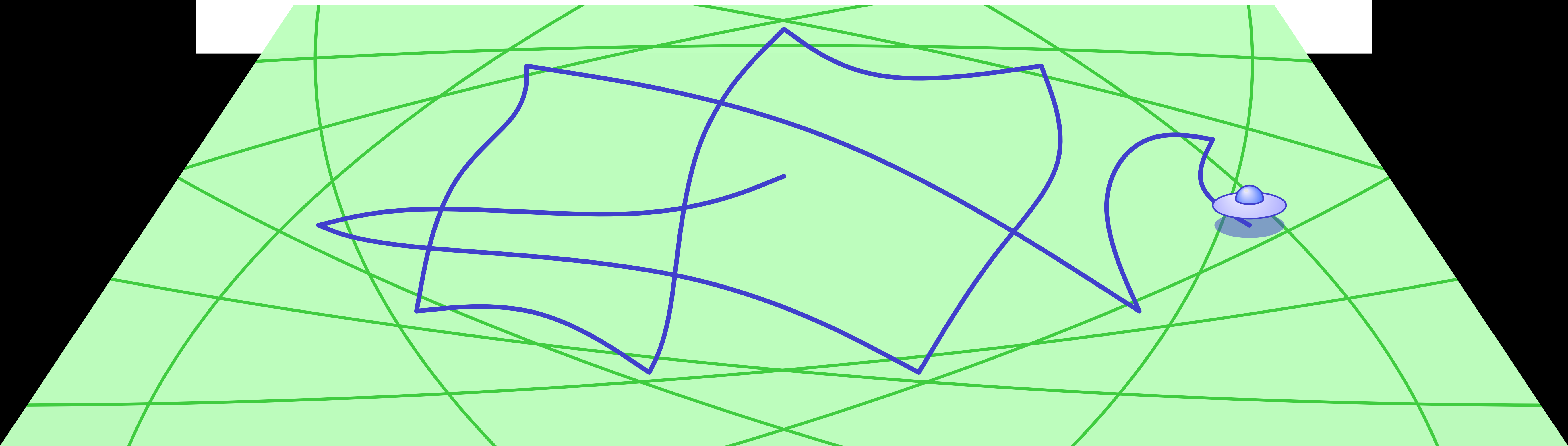

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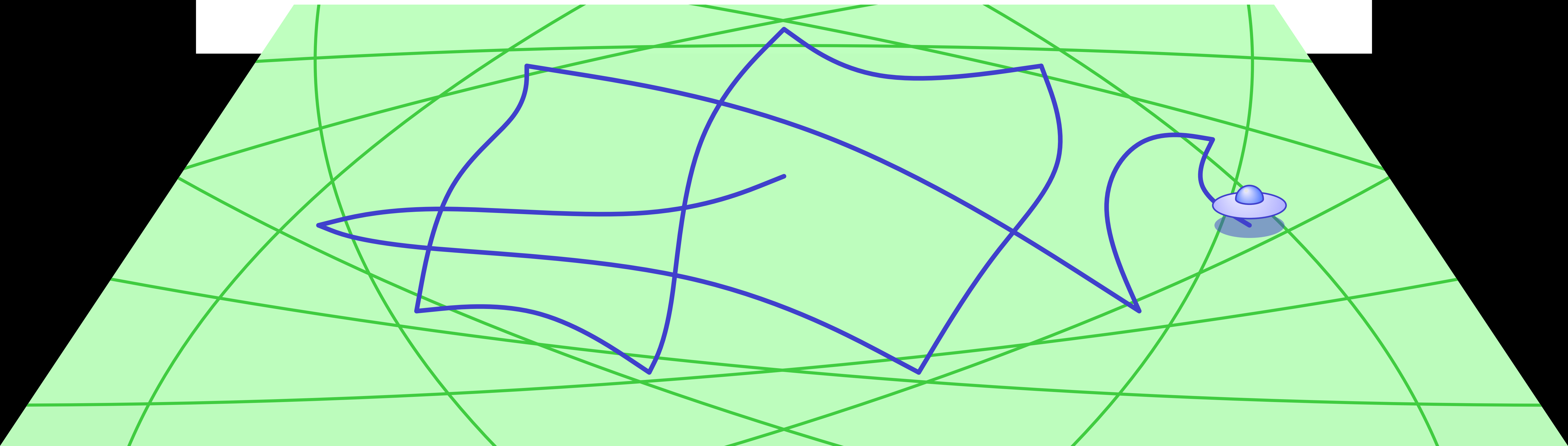
  - Just take the one remaining disk



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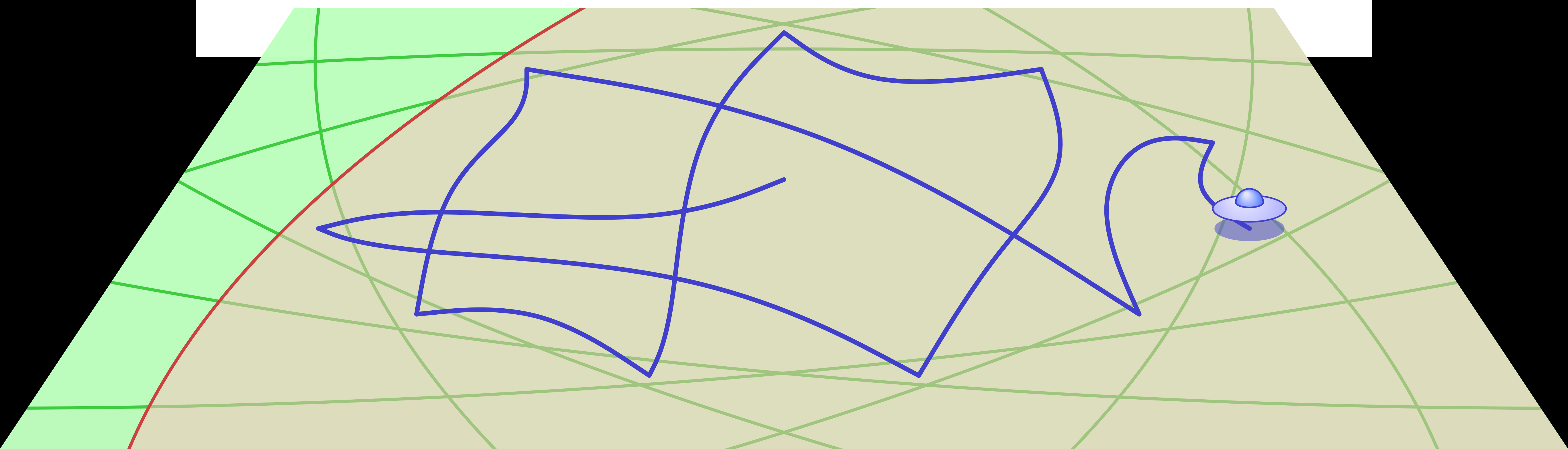

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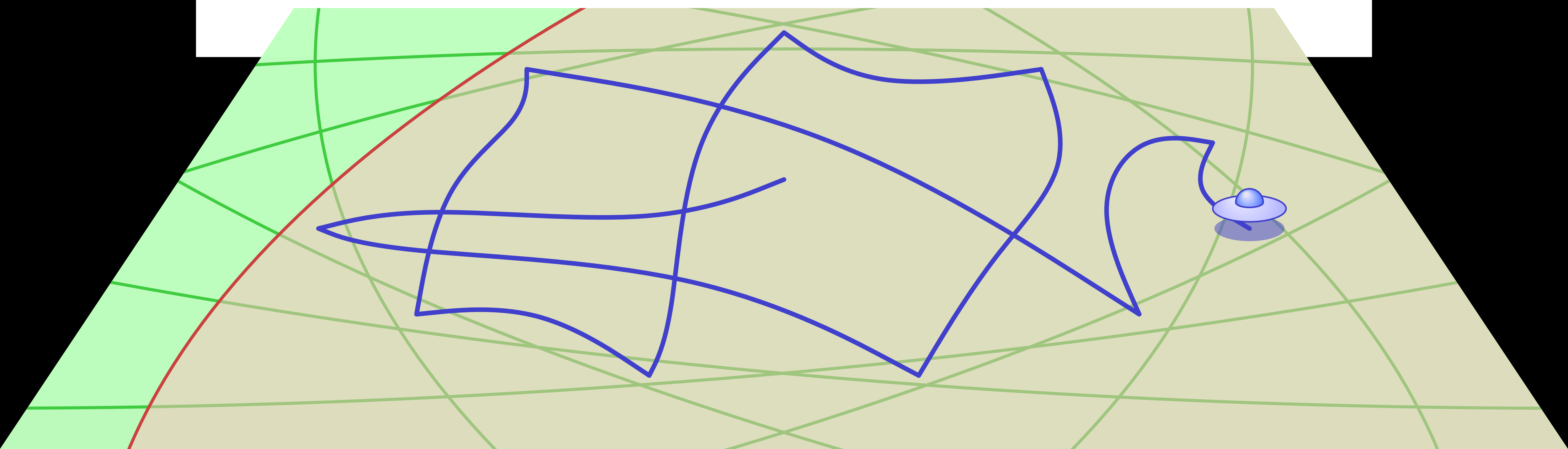
  - Just take the one remaining disk
  - Competitive ratio is $n - 1$



PLY

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- Lower bound requires many overlapping disks

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- Ply of a point

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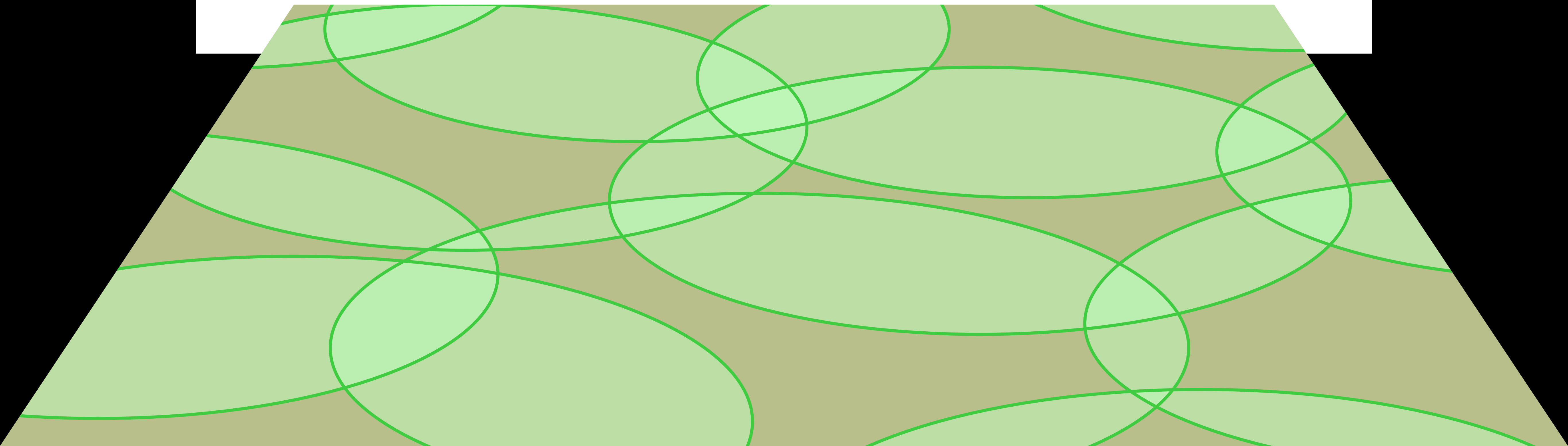

PLY

- Lower bound requires many overlapping disks
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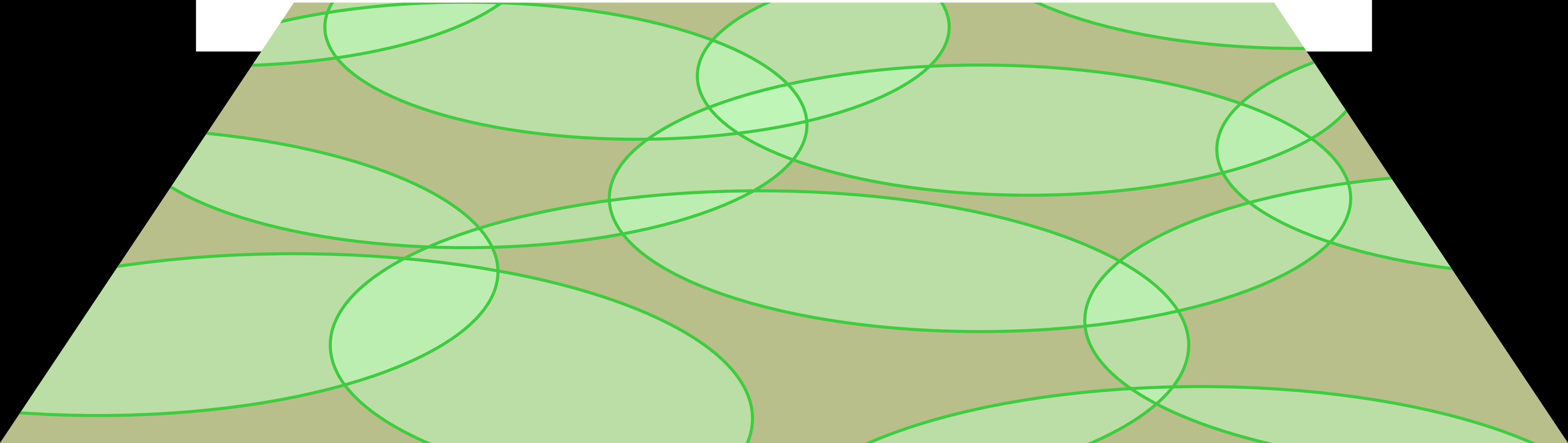
  - $\delta(p) = |\{R : p \in R\}|$



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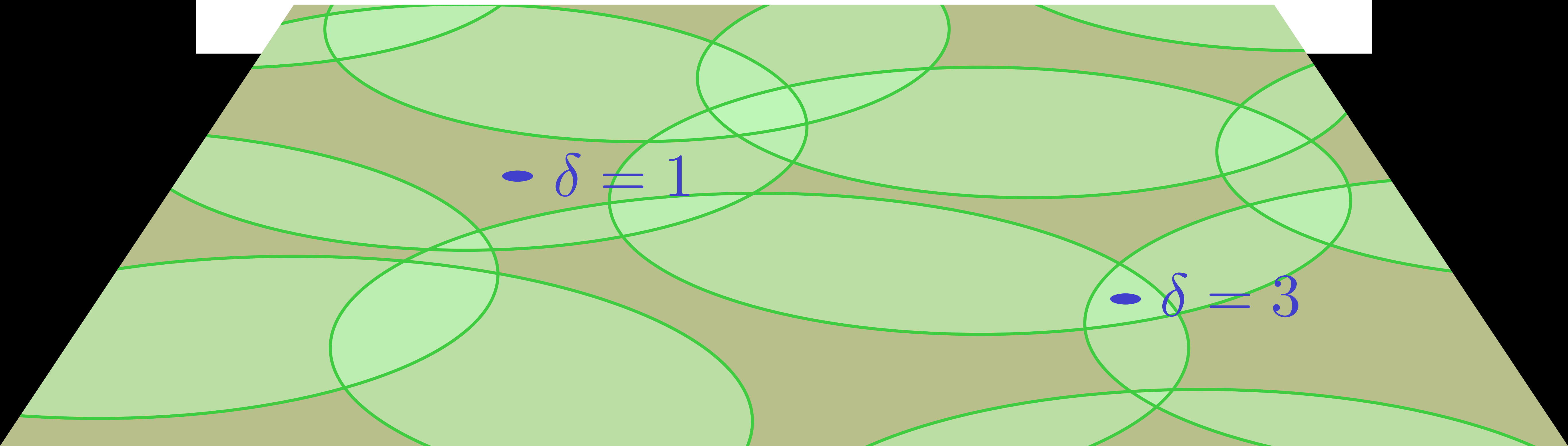

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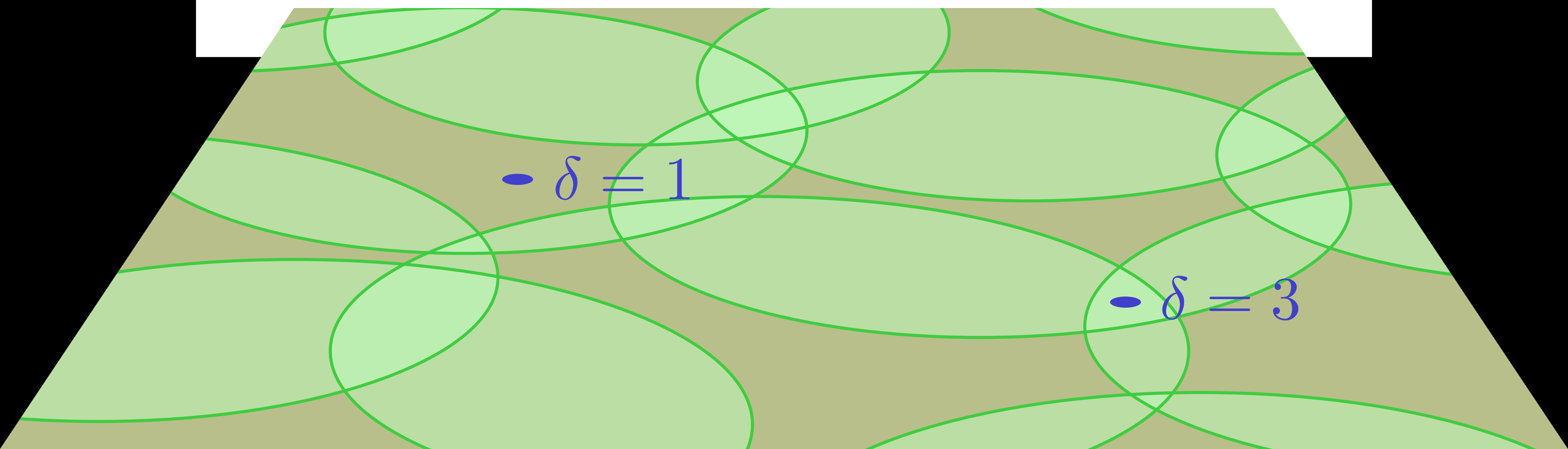

PLY

- Lower bound requires many overlapping disks
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  - $\delta(p) = |\{R : p \in R\}|$
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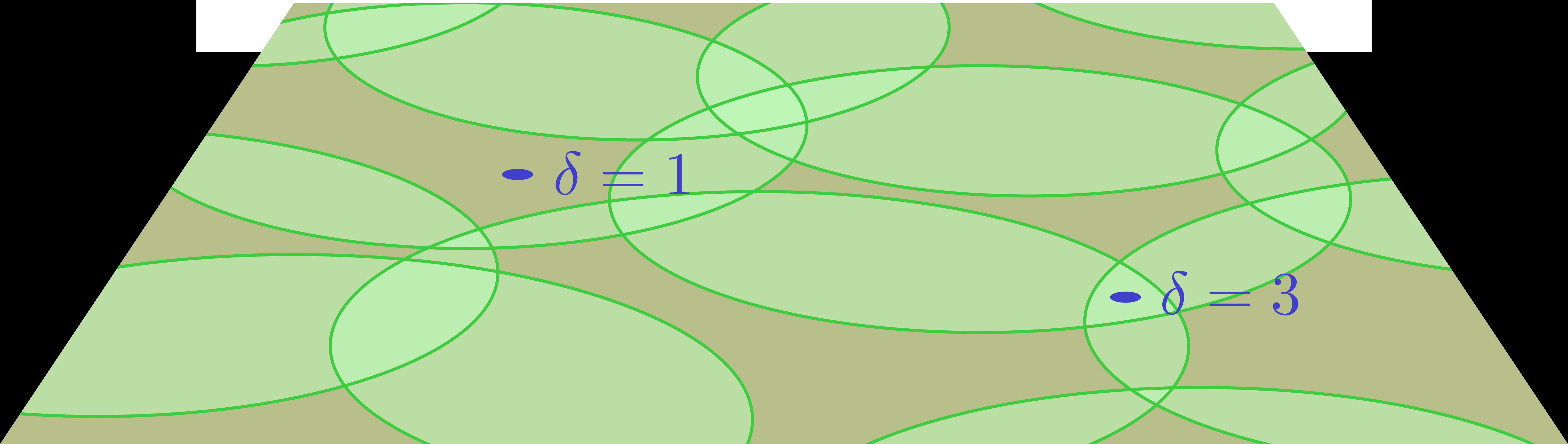
  - Maximum ply of all points



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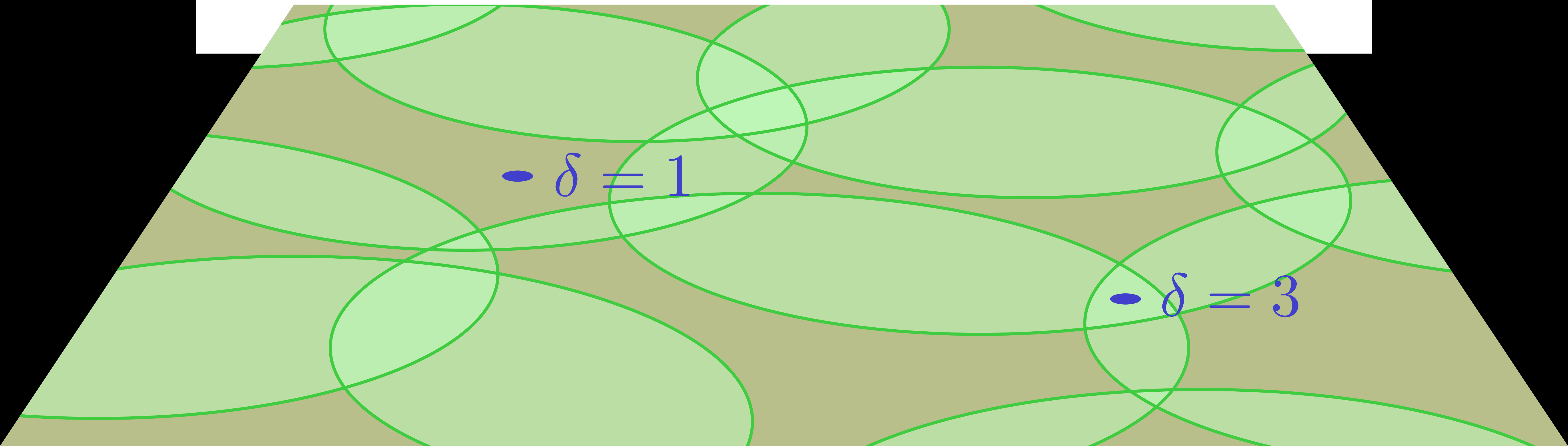

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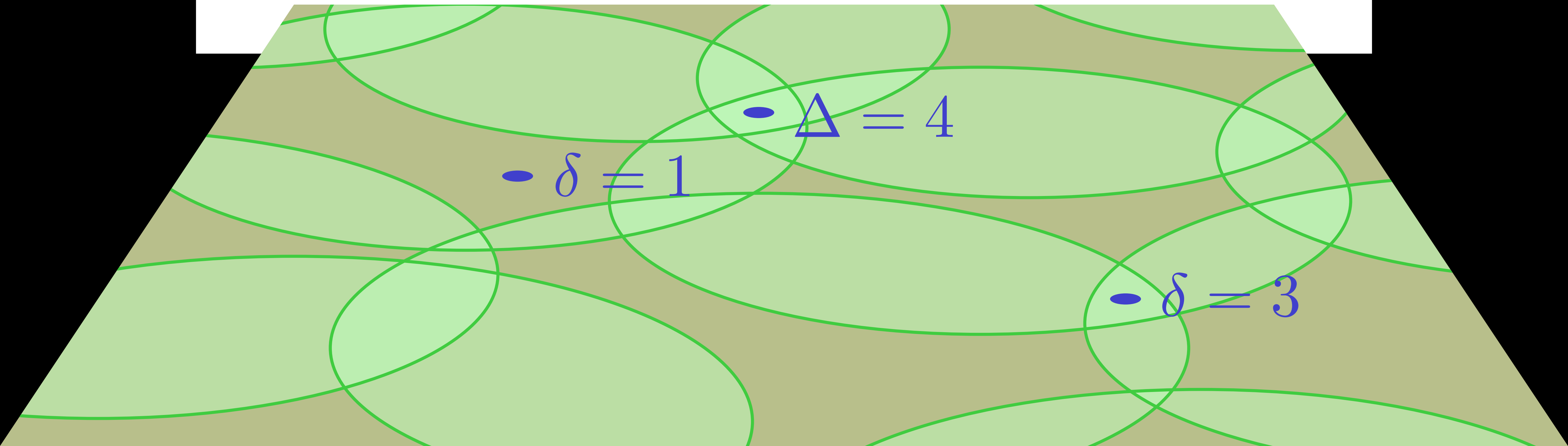

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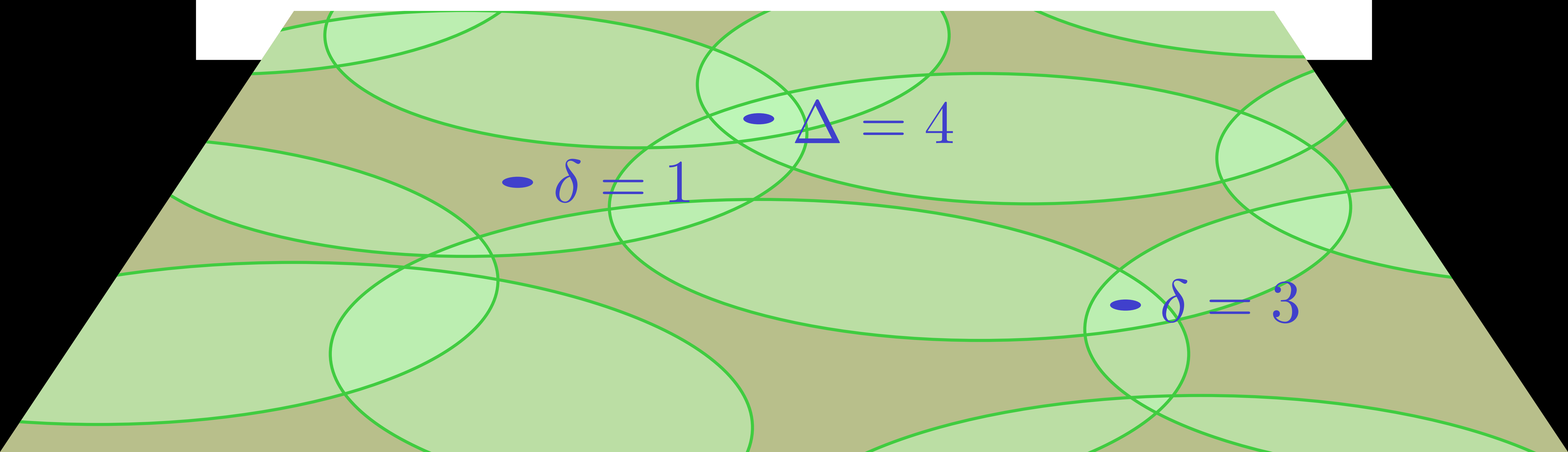
  - $\Delta = \max_p \delta(p)$
- In practice, $\Delta \ll n$



RESULTS

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- Bounds on competitive ratio

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  - Stateless unbounded

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OFFLINE

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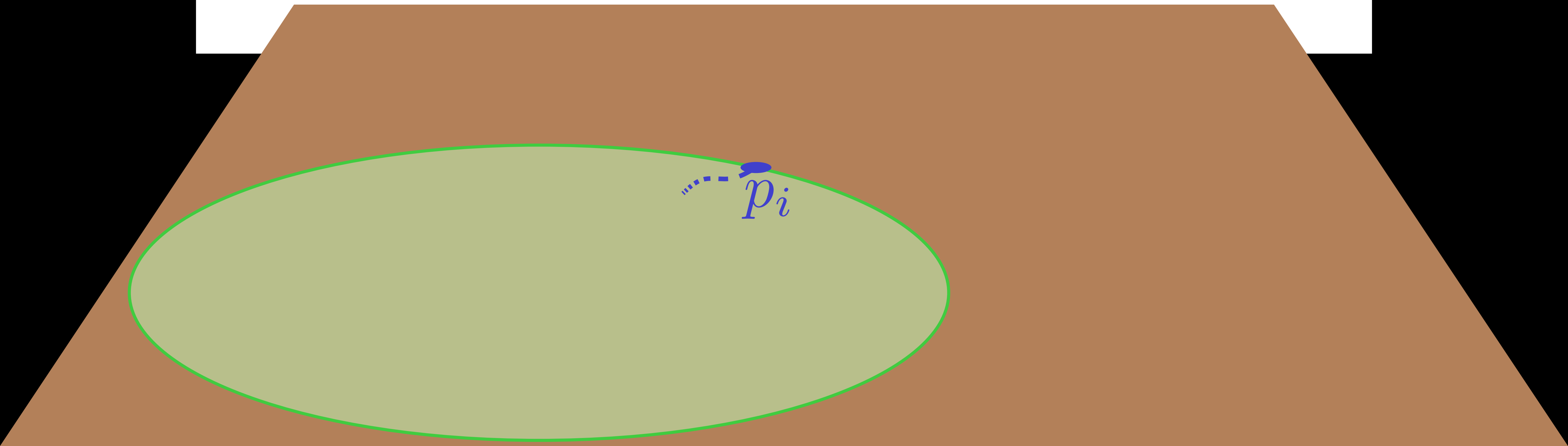

OFFLINE

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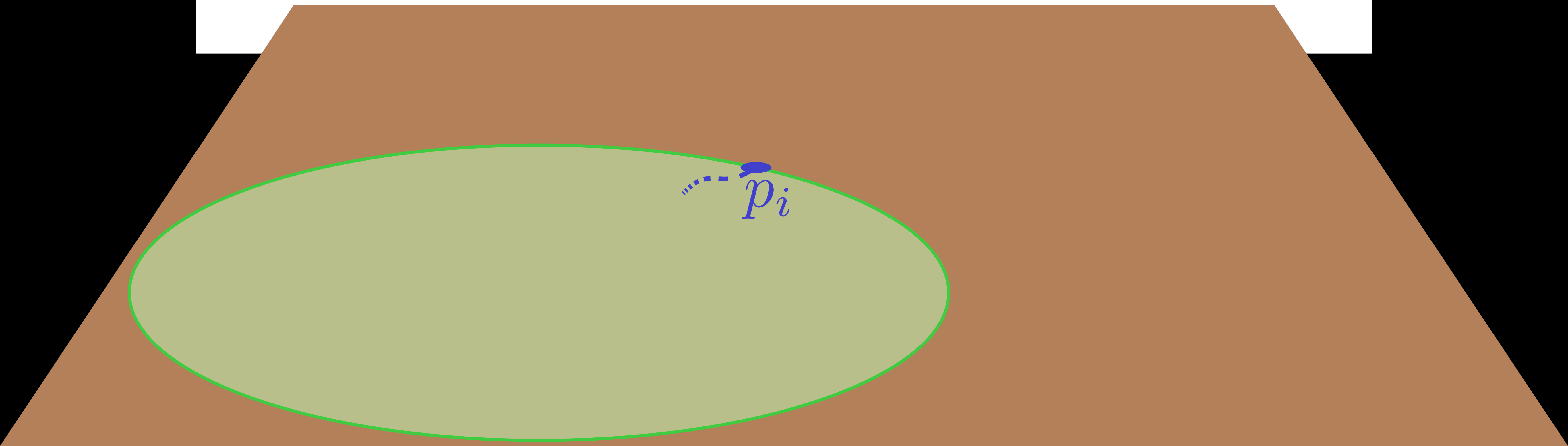

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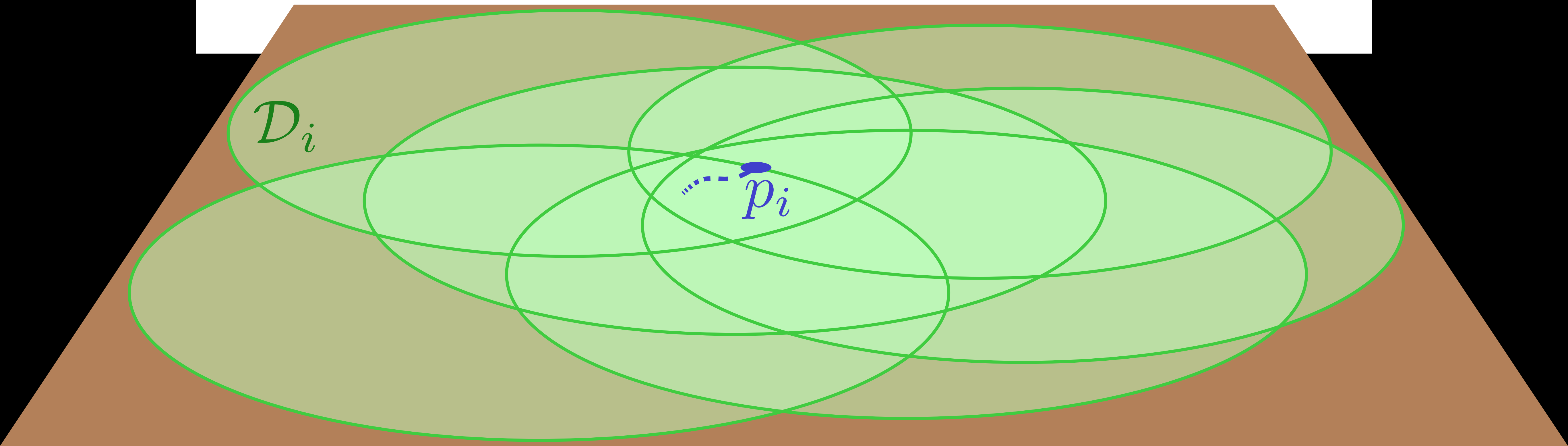

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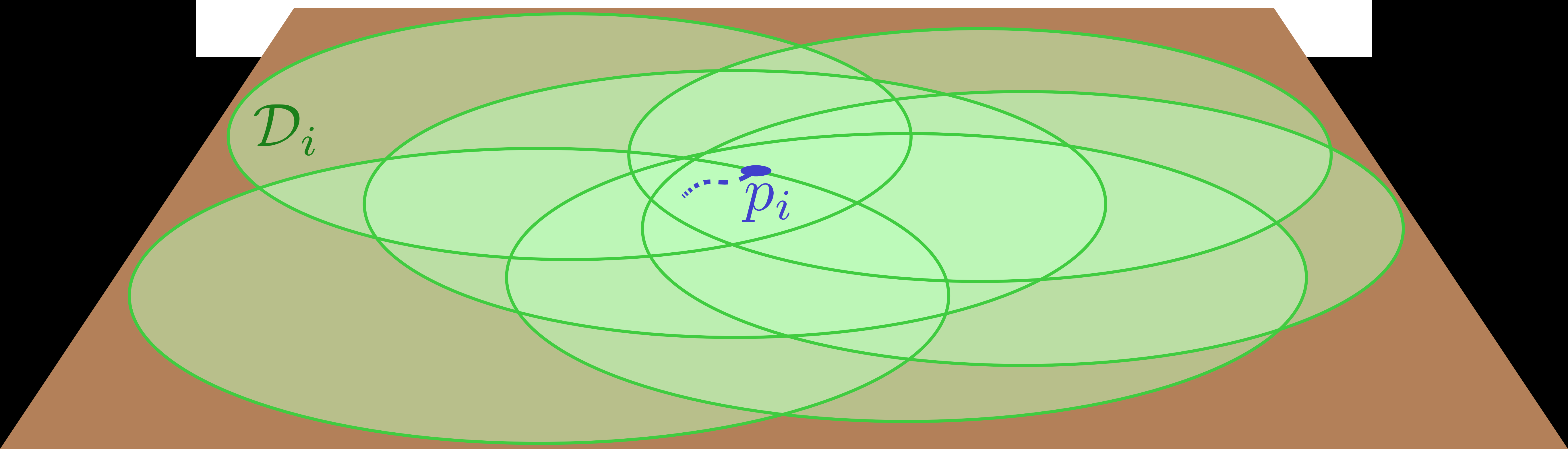

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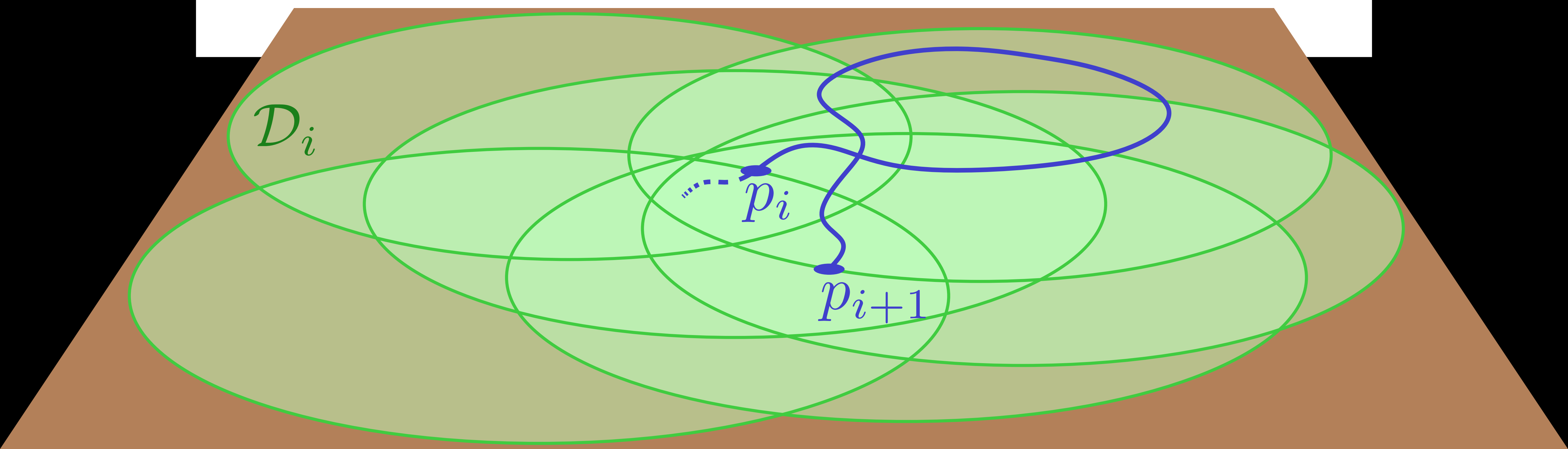

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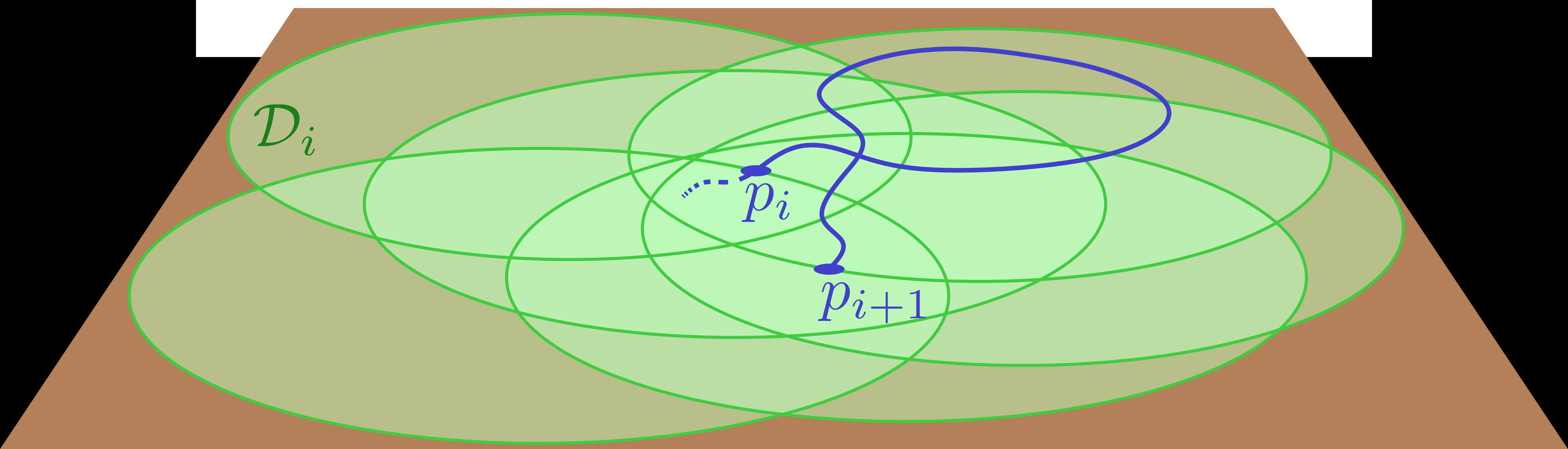

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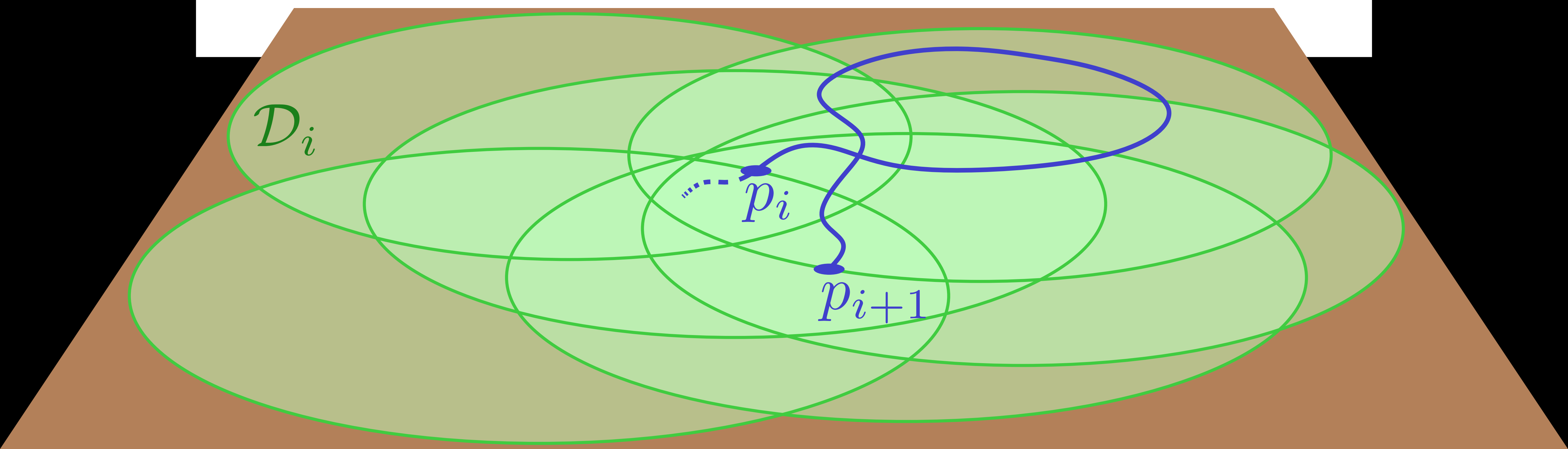

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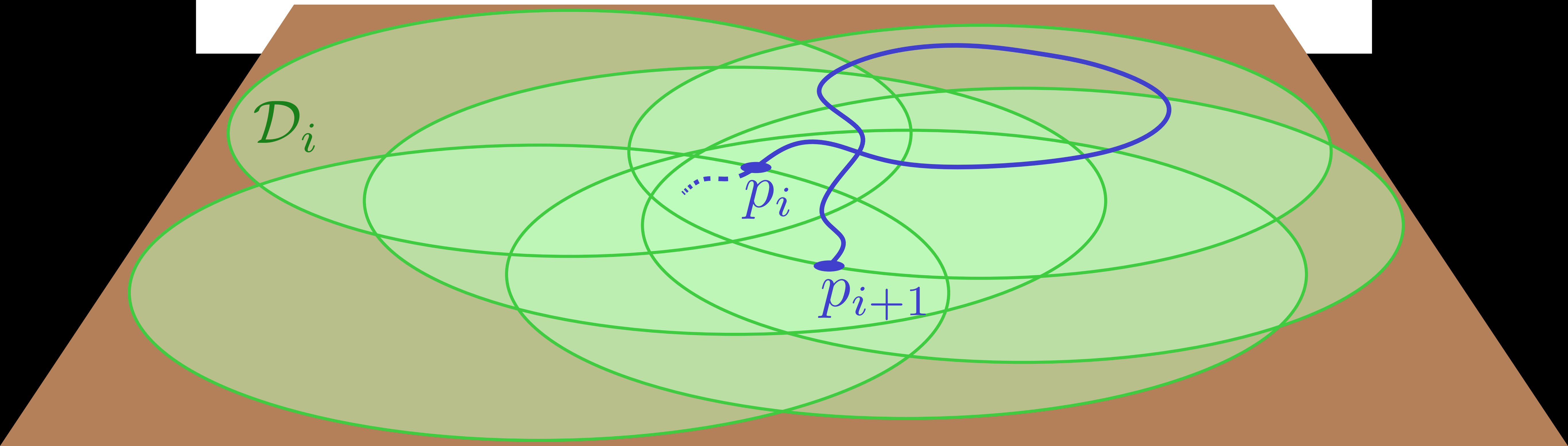
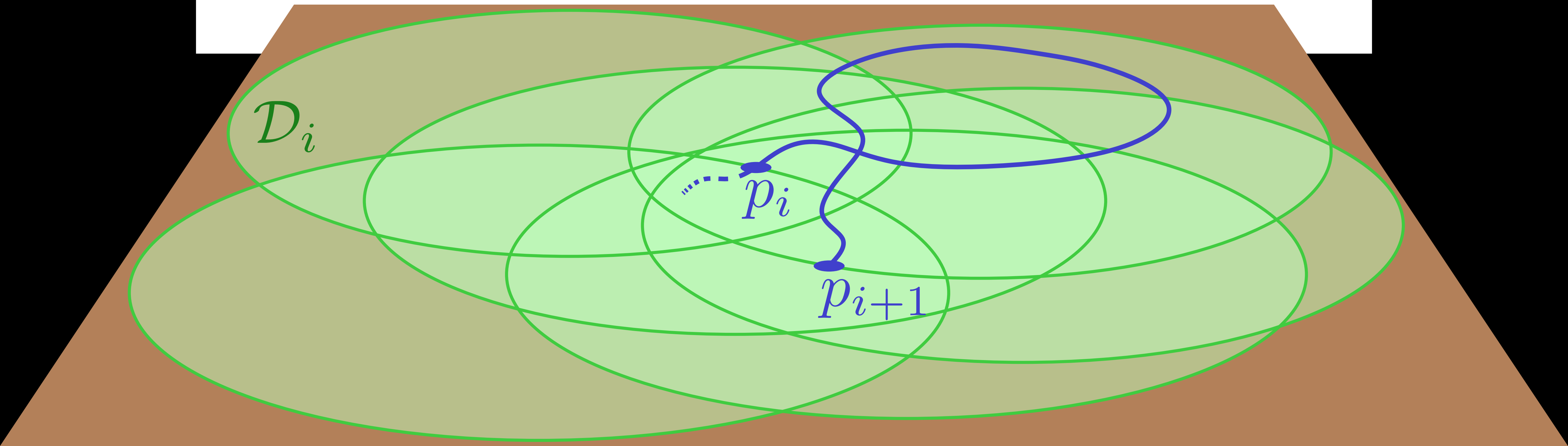
  - Optimal cost up to p_k is k



DETERMINISTIC ONLINE



DETERMINISTIC ONLINE

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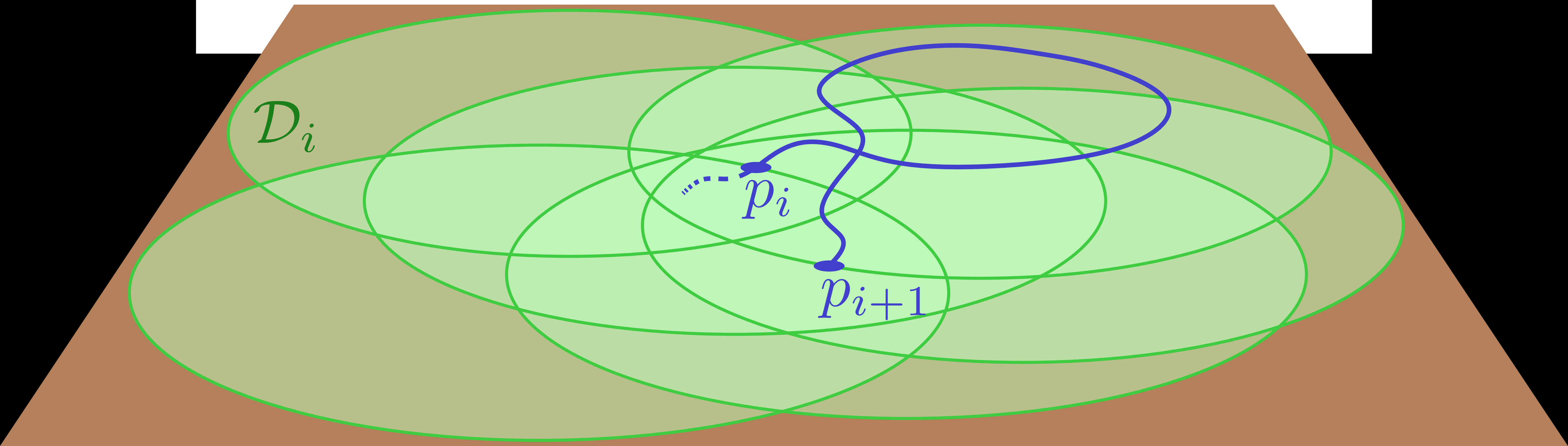

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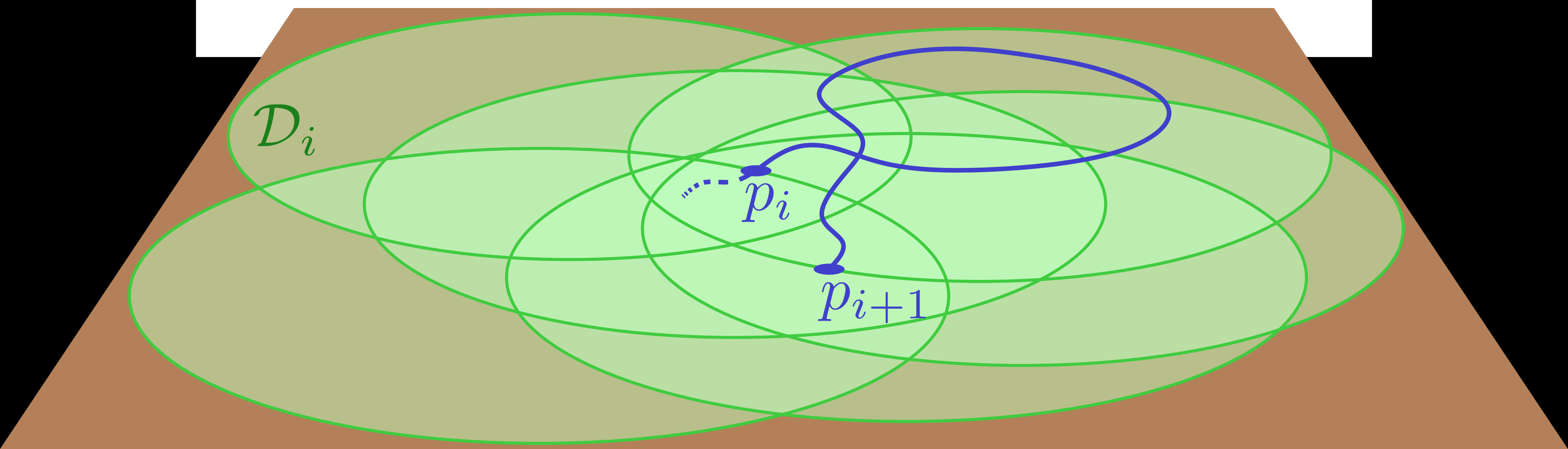

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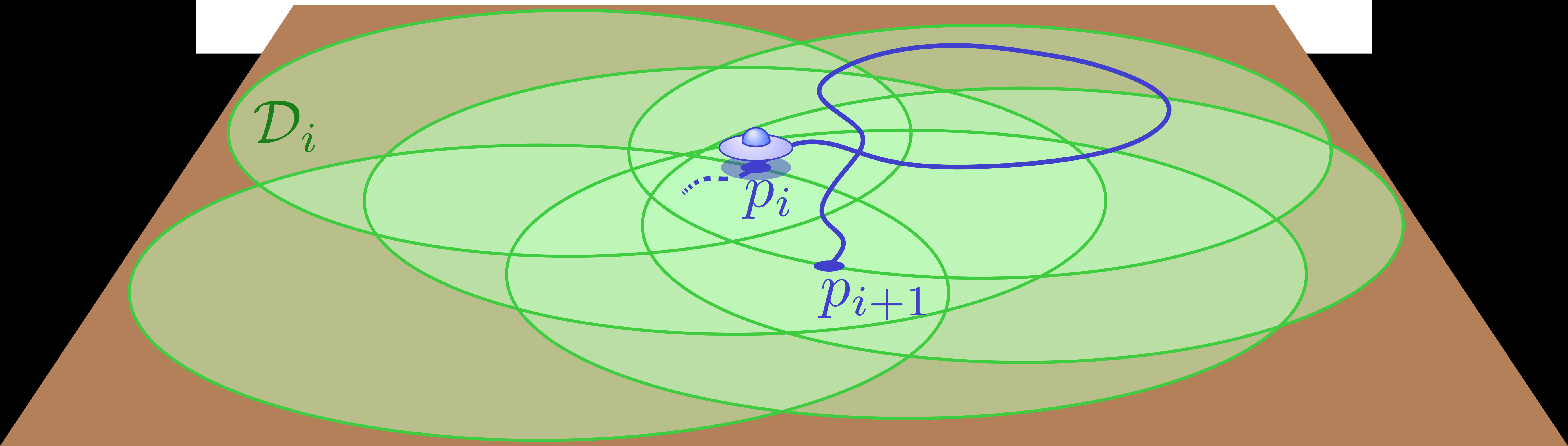

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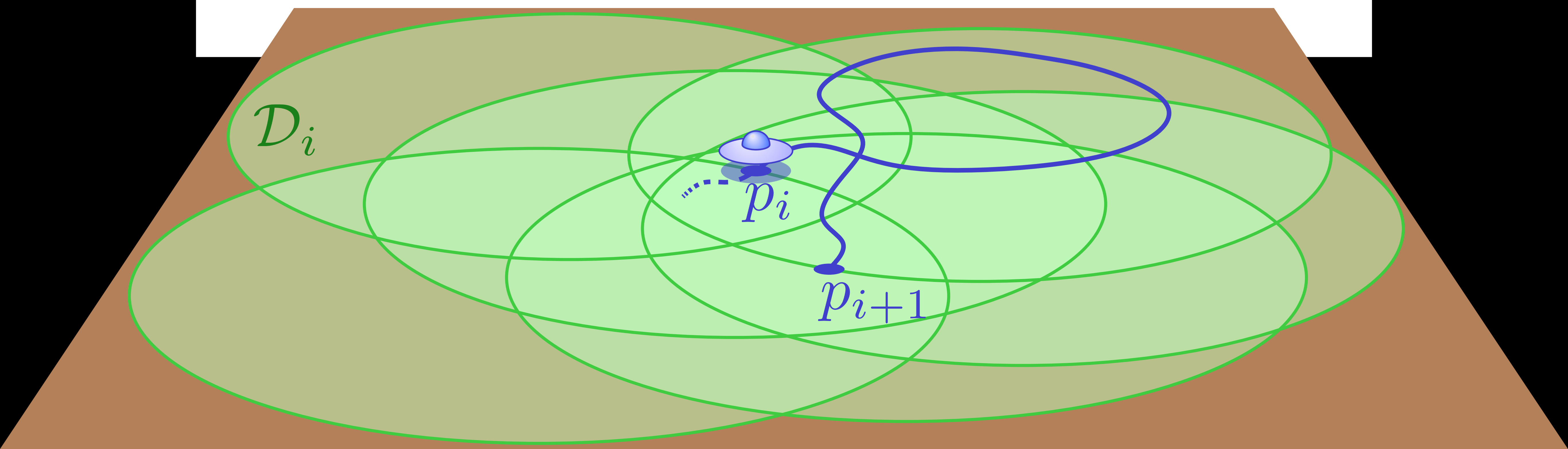

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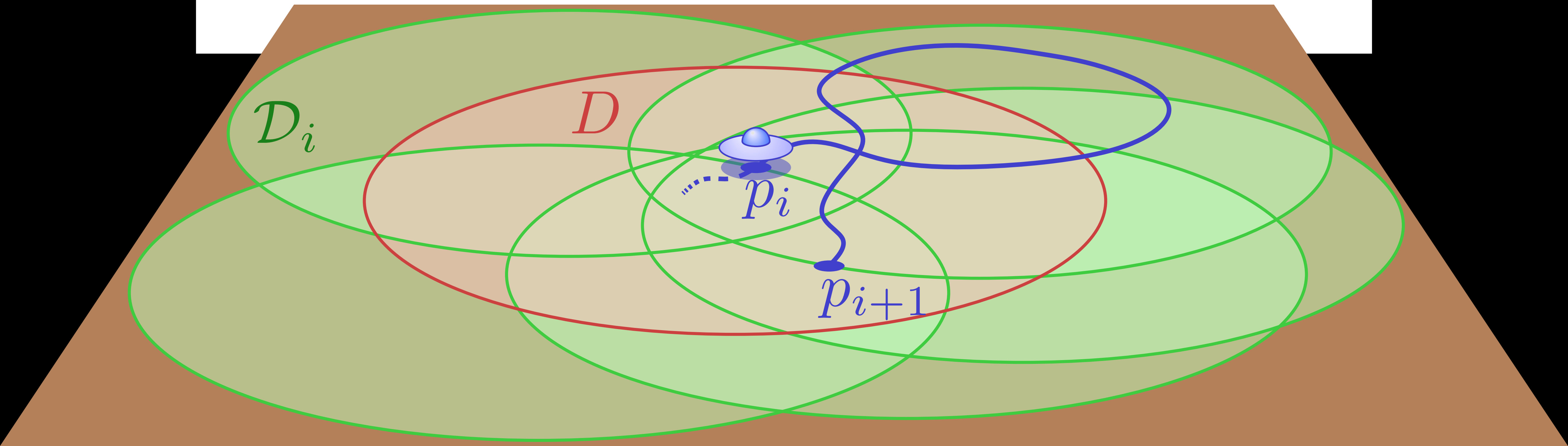

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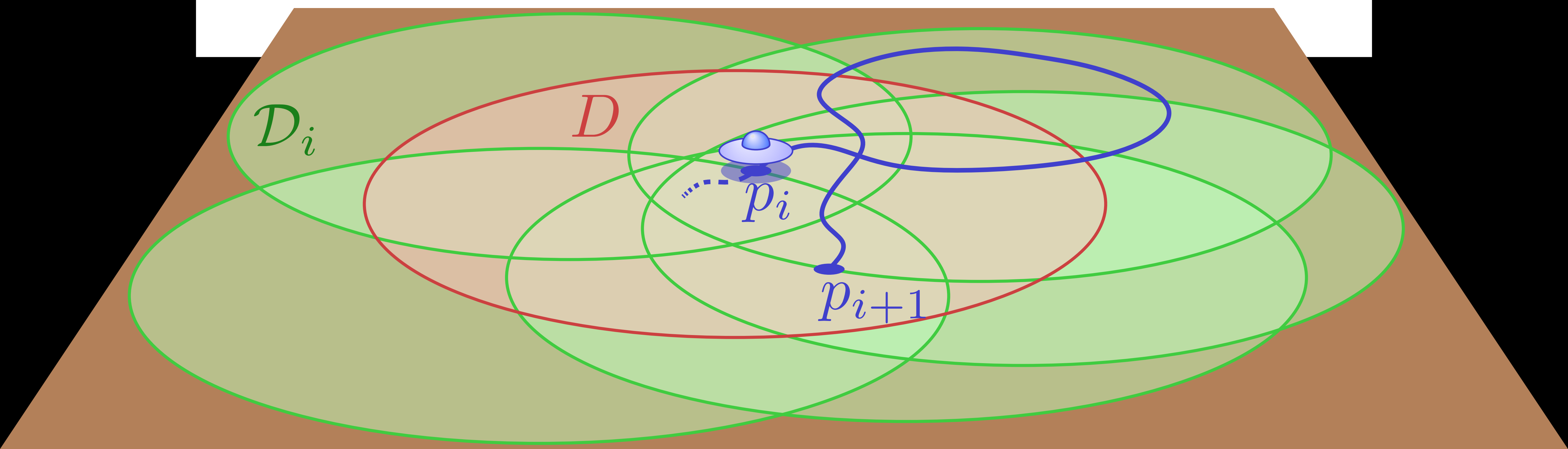

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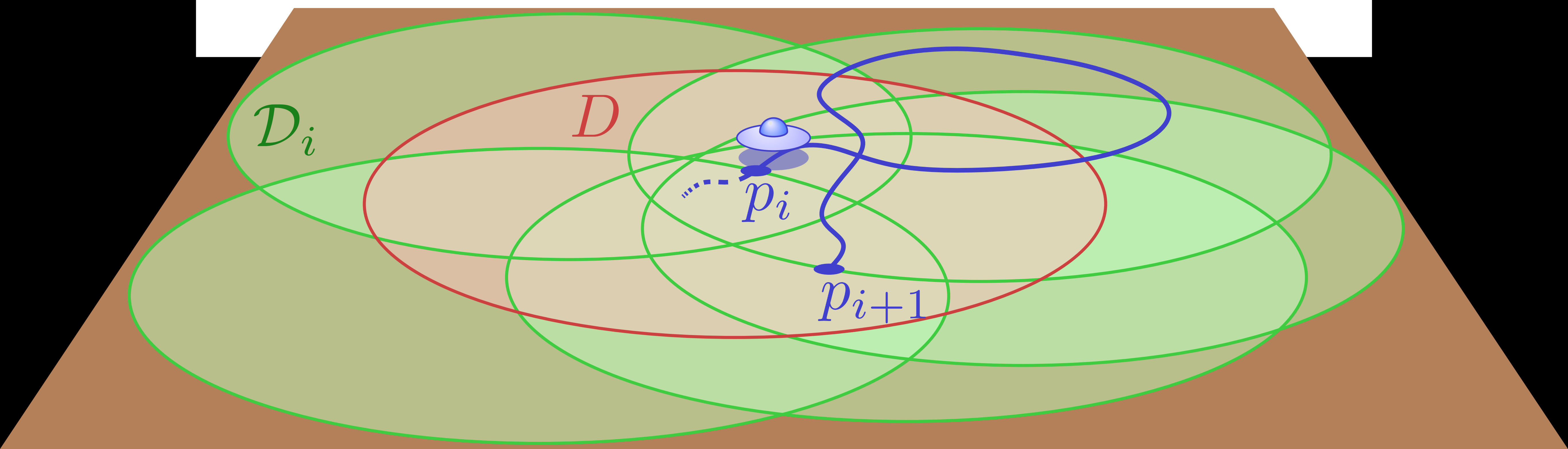

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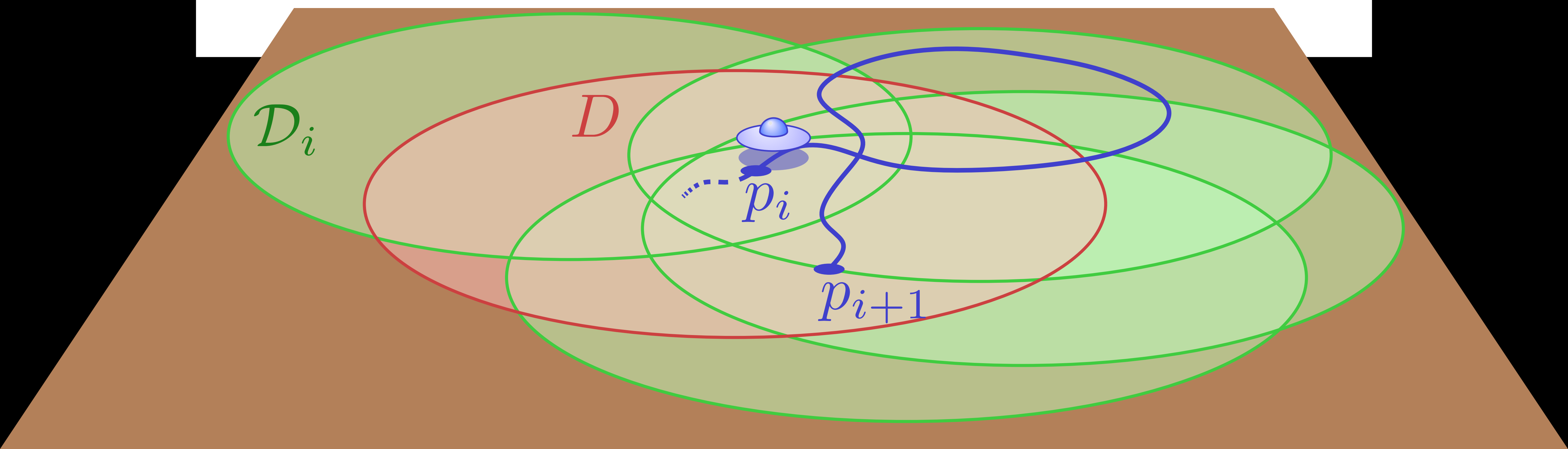

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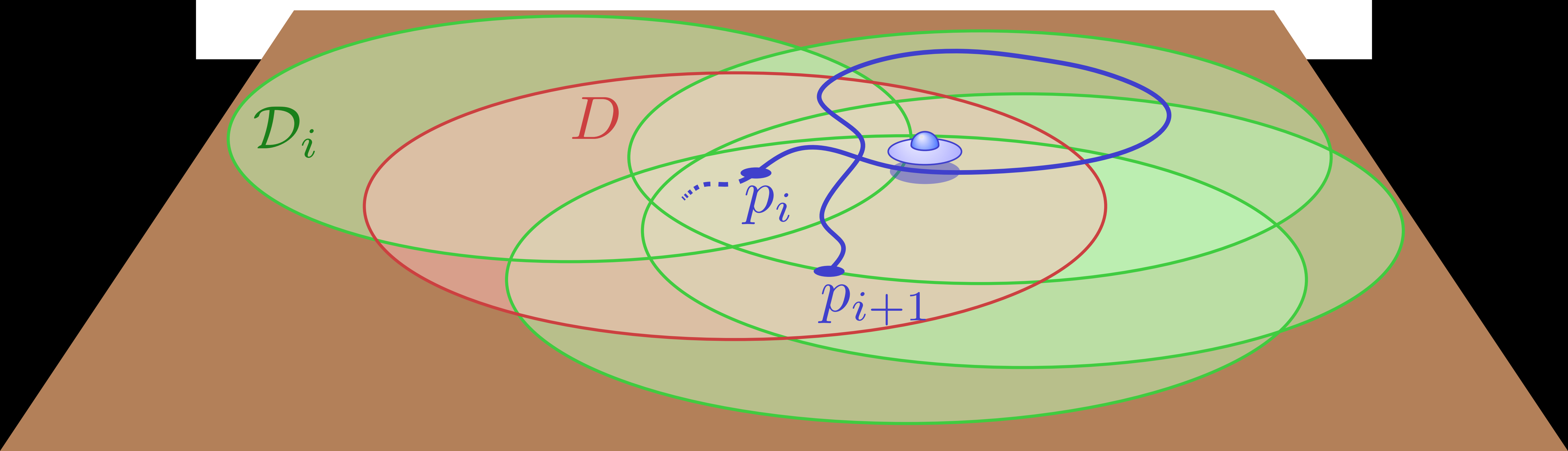

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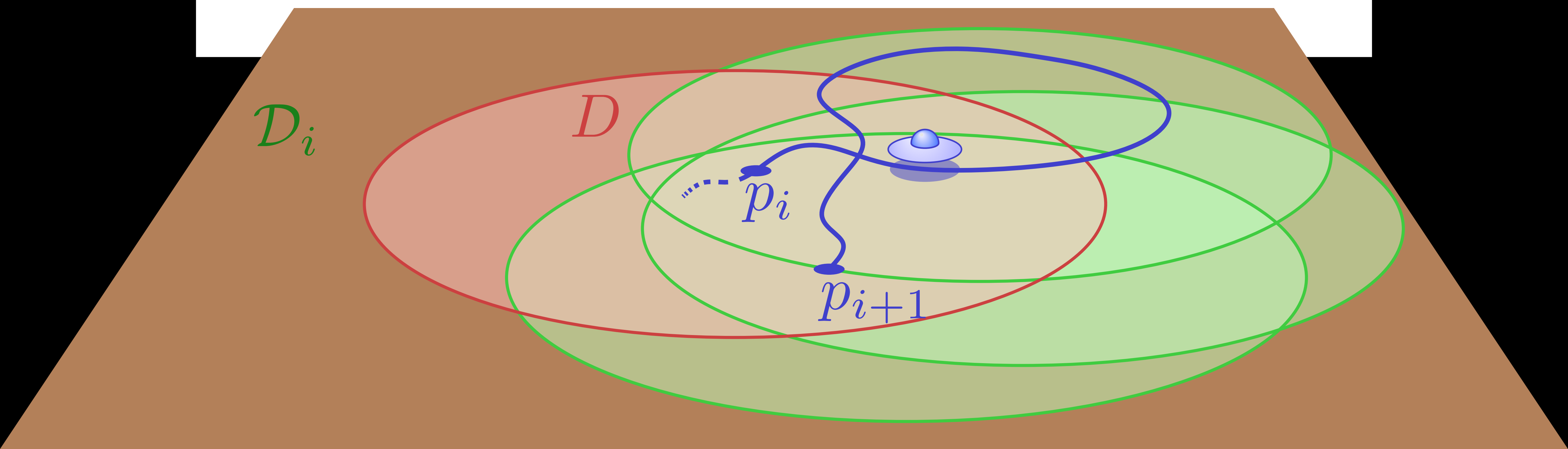

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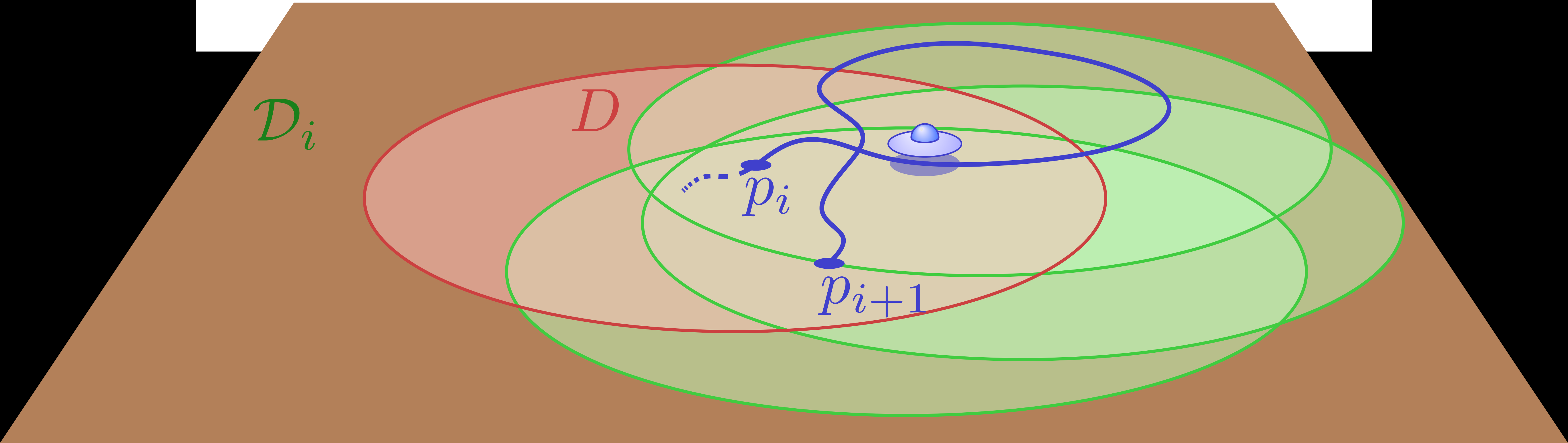

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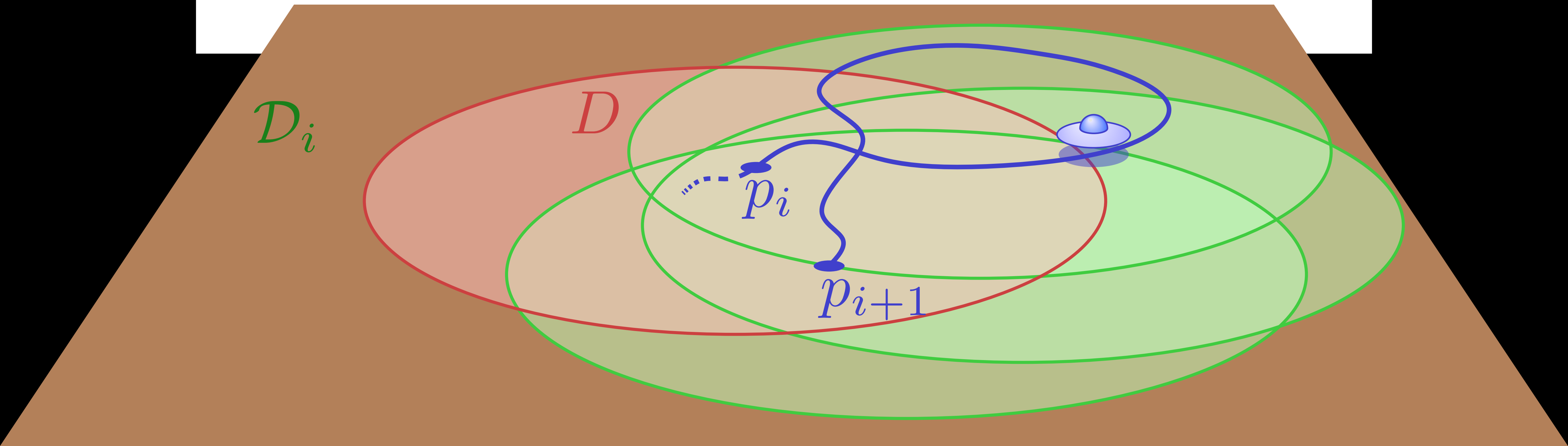

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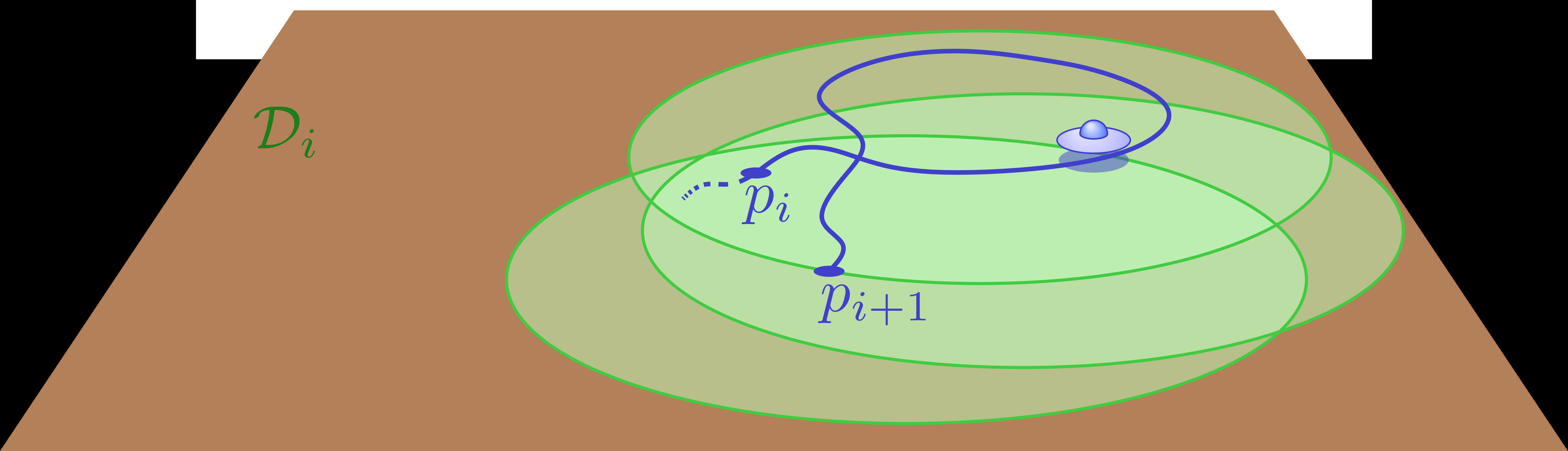

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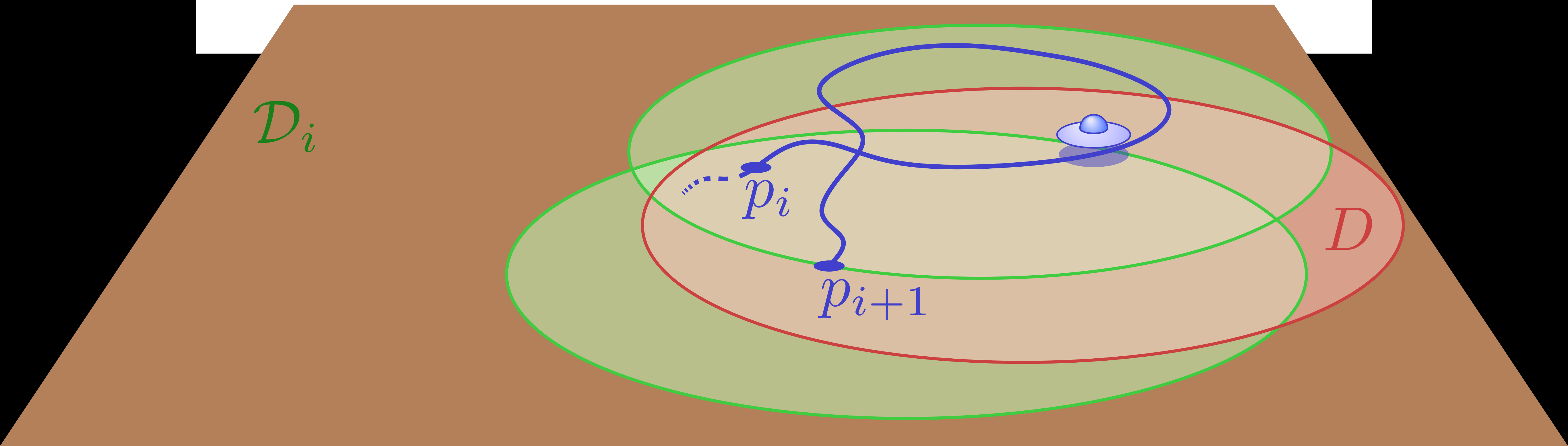

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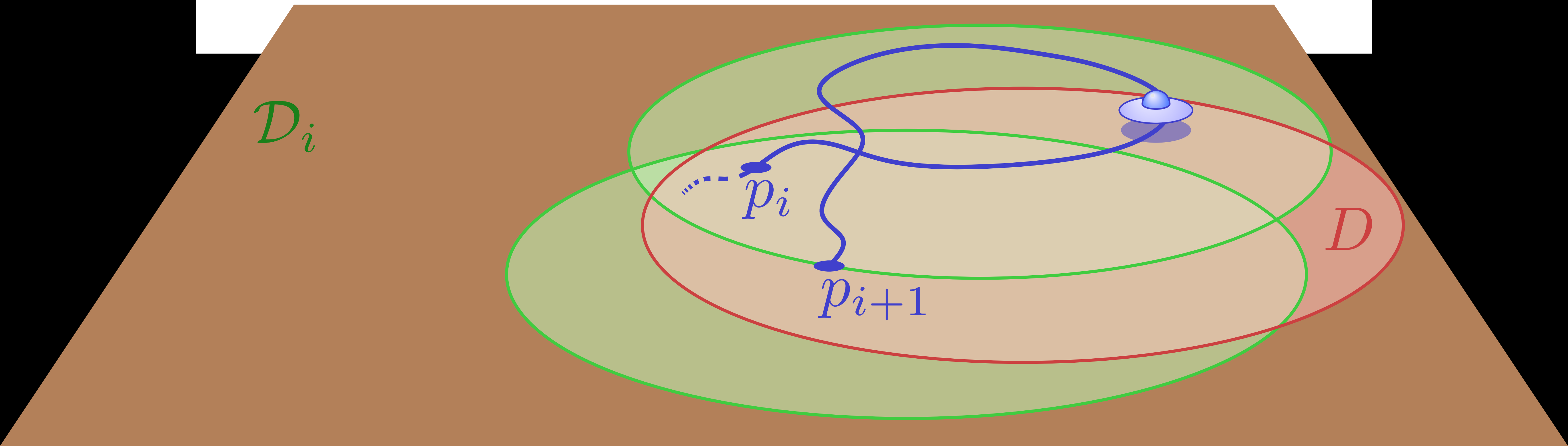

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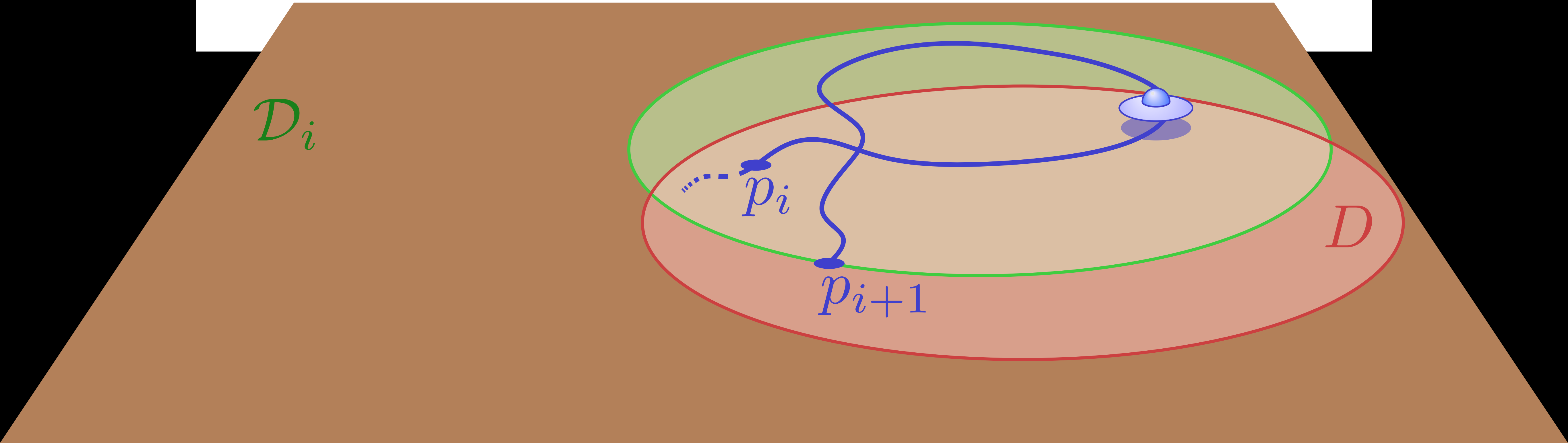

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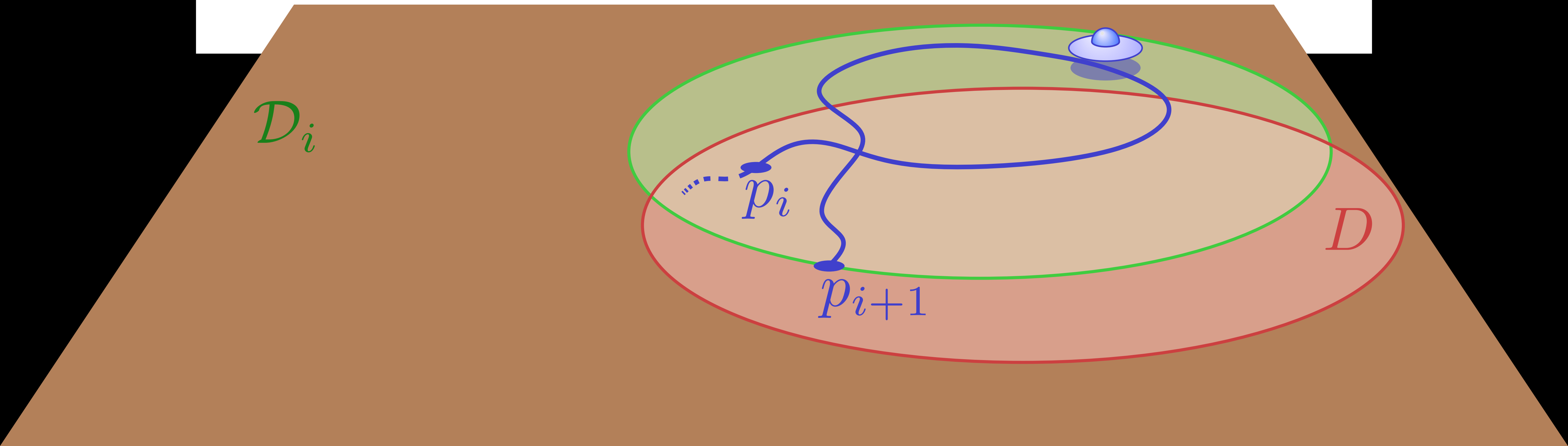

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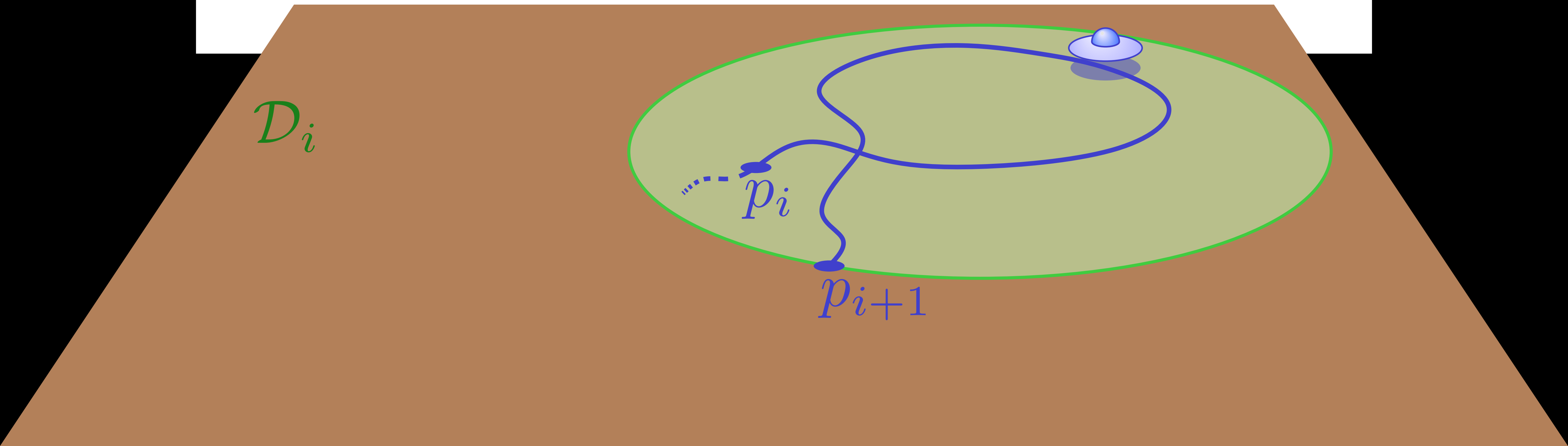

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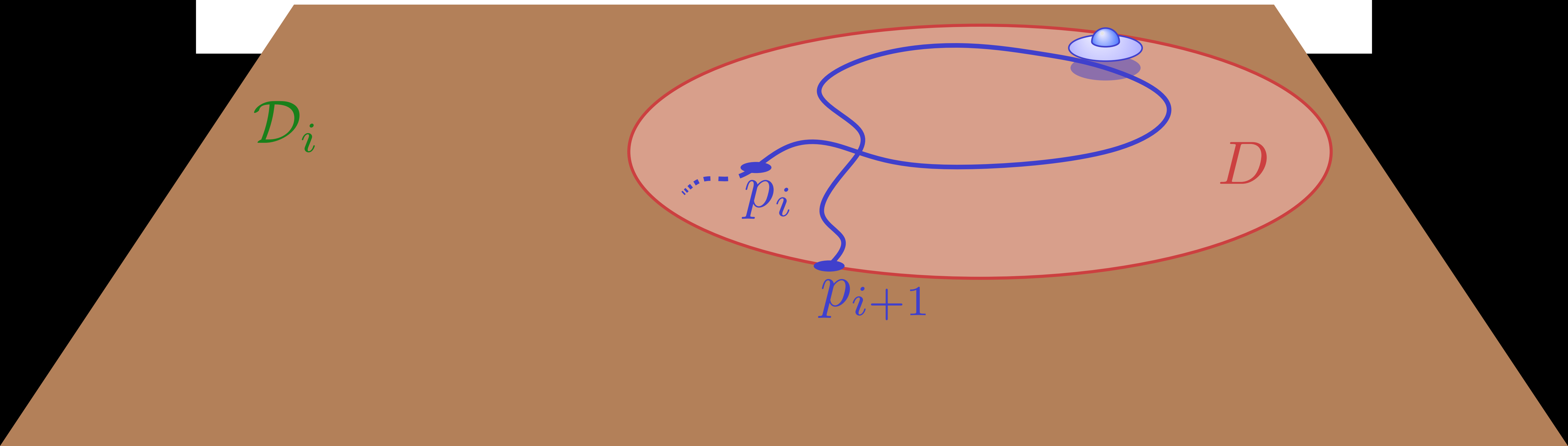

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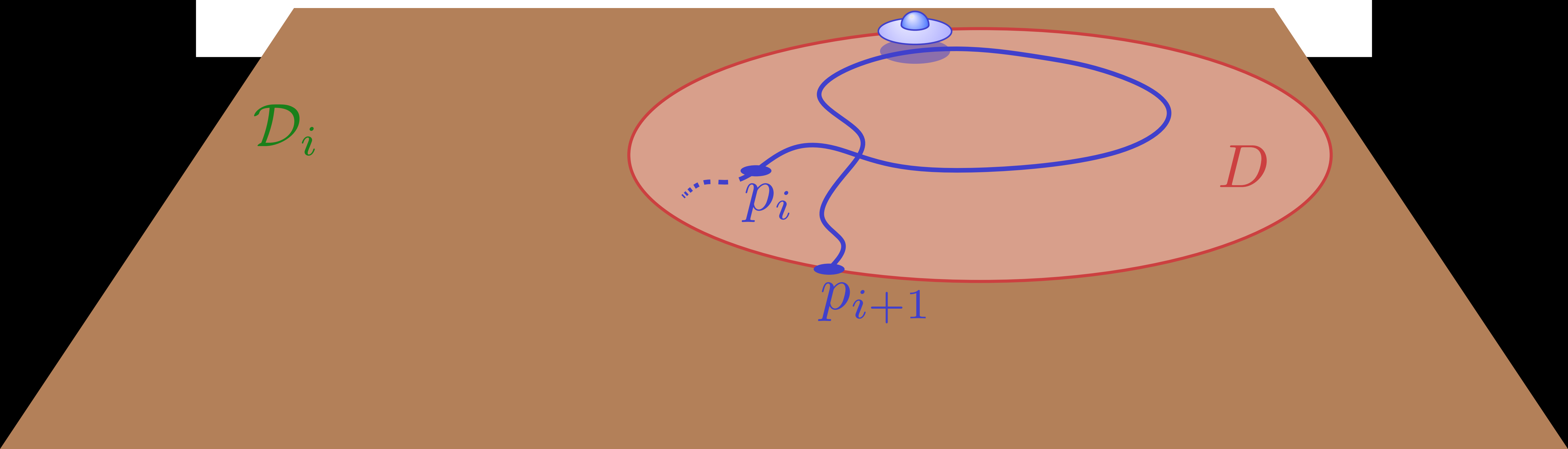

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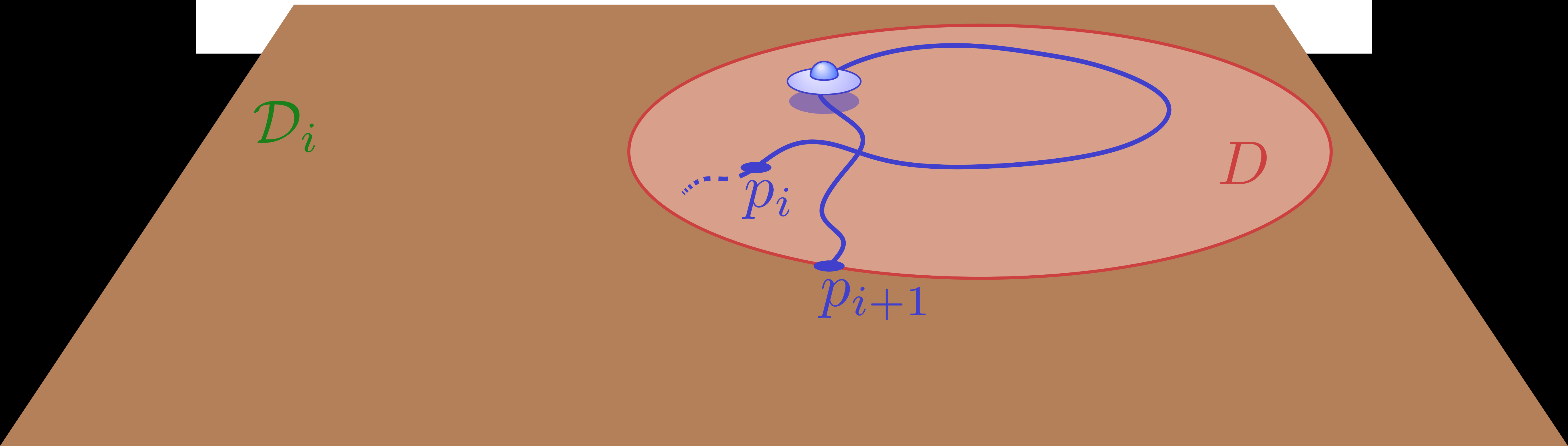

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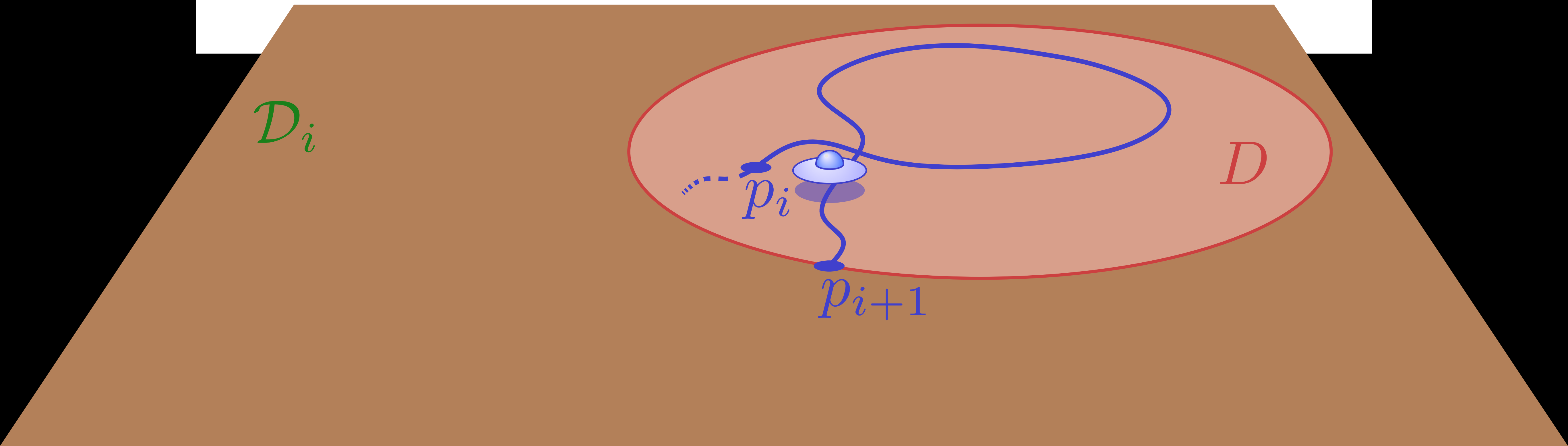

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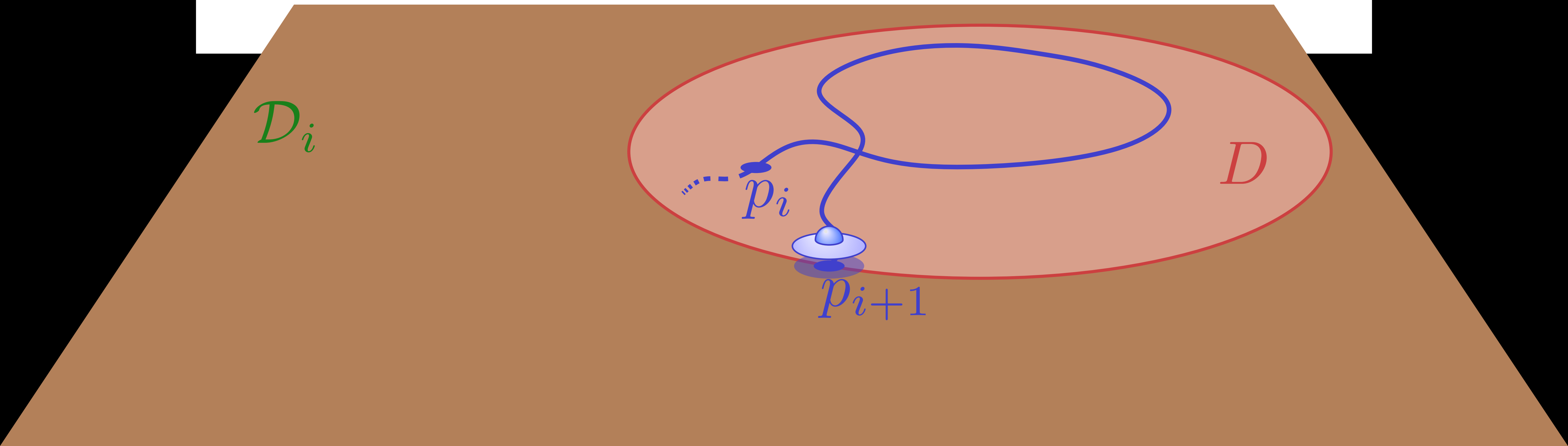

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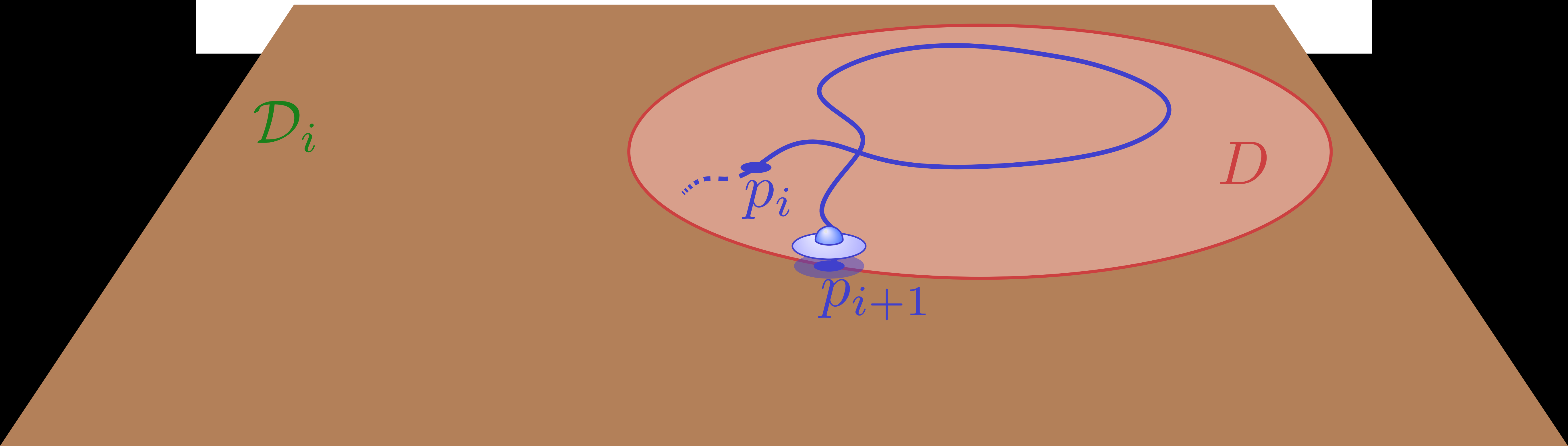

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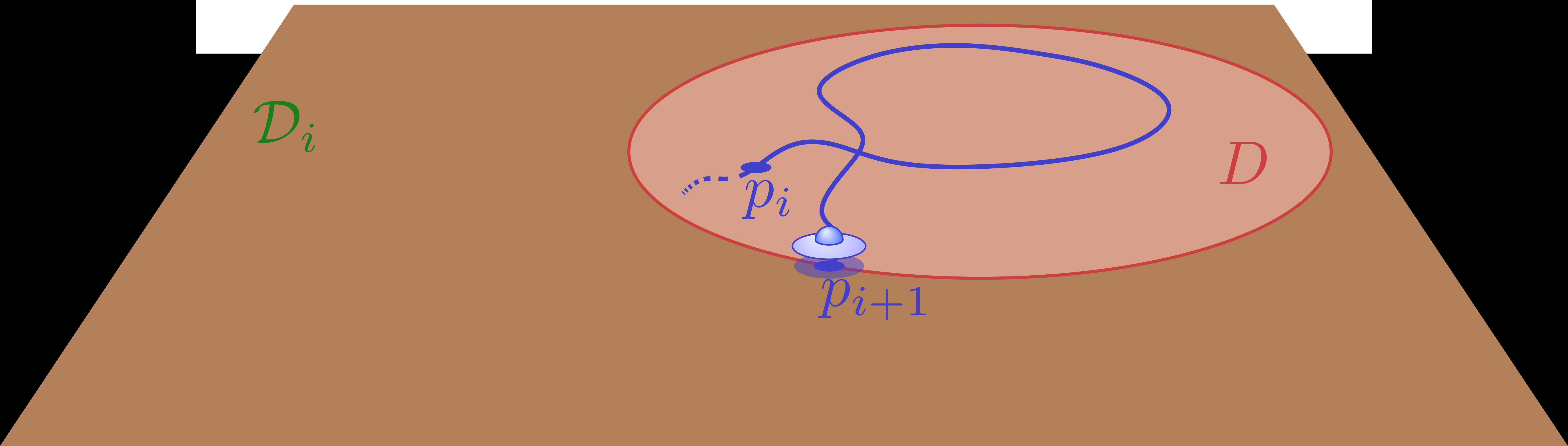
  - When $\mathcal{D}_i$ is empty, we found p_{i+1}
- Analysis



DETERMINISTIC ONLINE

- Algorithm
  - When at p_i, compute $\mathcal{D}_i$
  - Let D be "first" region of $\mathcal{D}_i$
  - When leaving a region, remove it from $\mathcal{D}_i$
  - When leaving D and $\mathcal{D}_i$ is not empty yet, pick "next" region *from* $\mathcal{D}_i$
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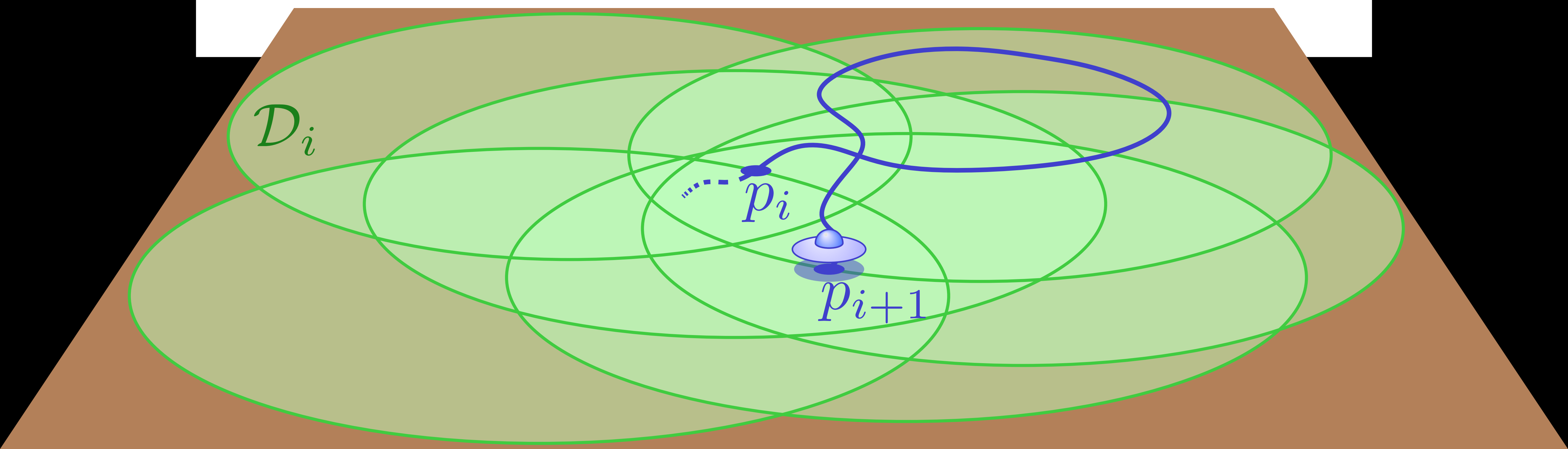

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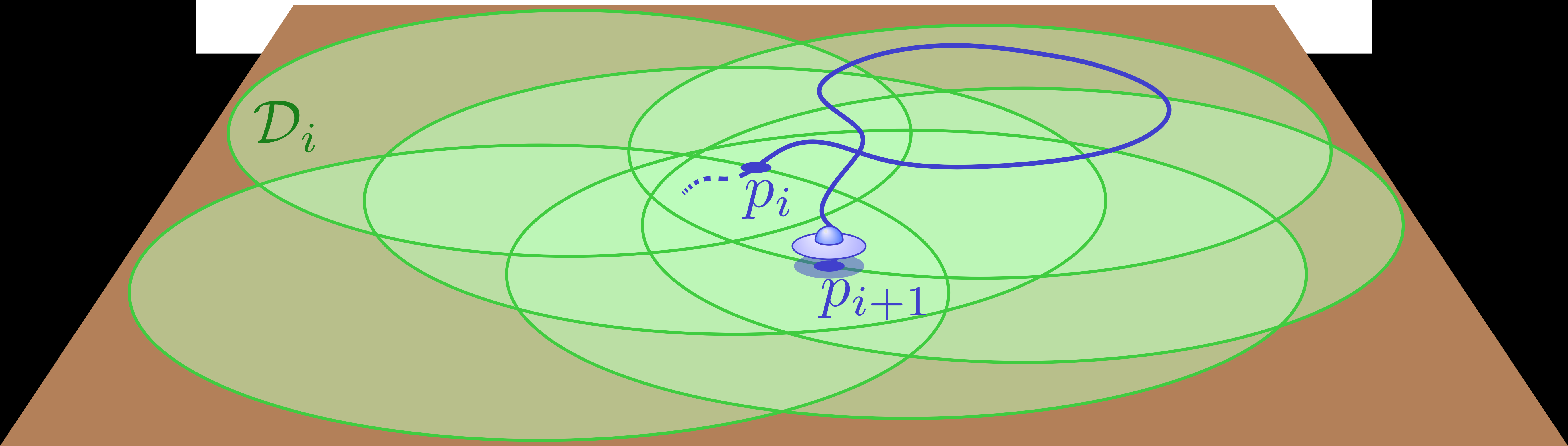

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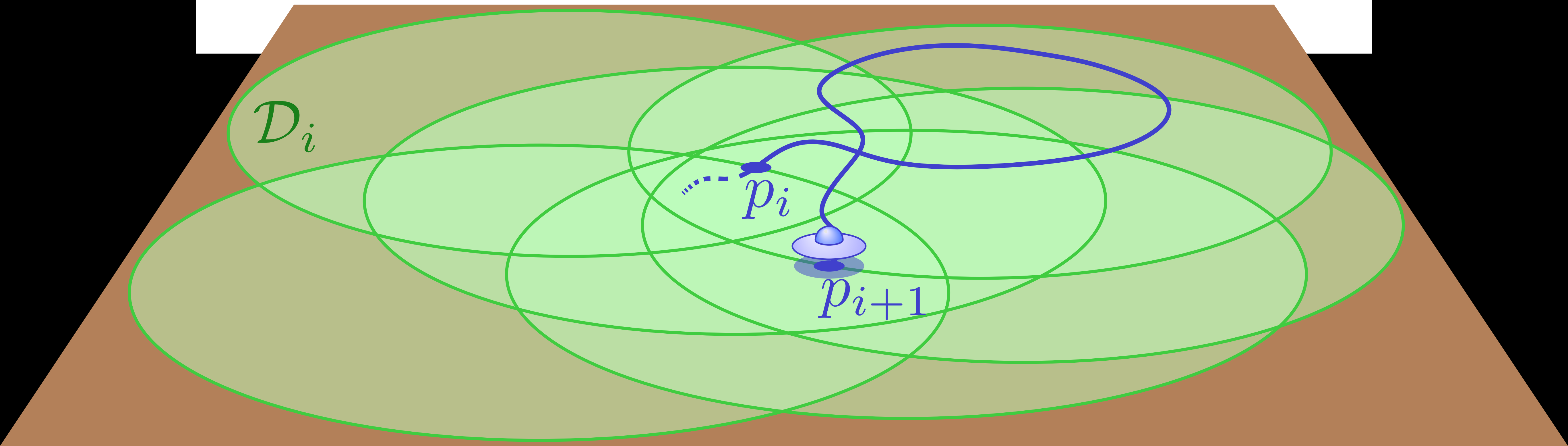
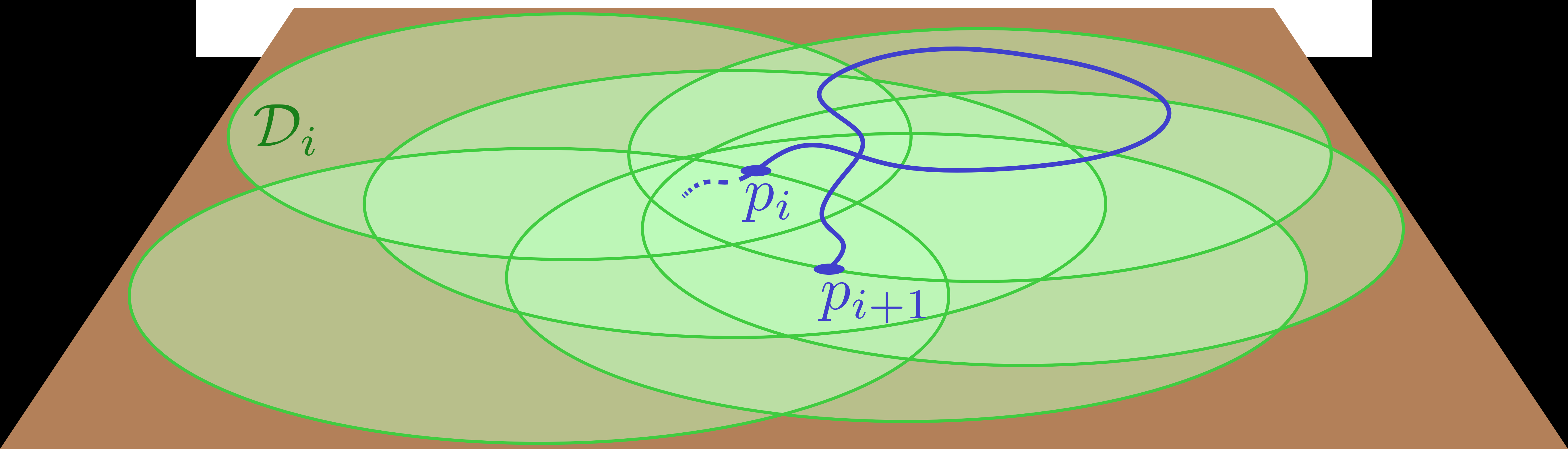
  - At most $|\mathcal{D}_i| \leq \Delta$ handovers
  - Competitive ratio is Δ

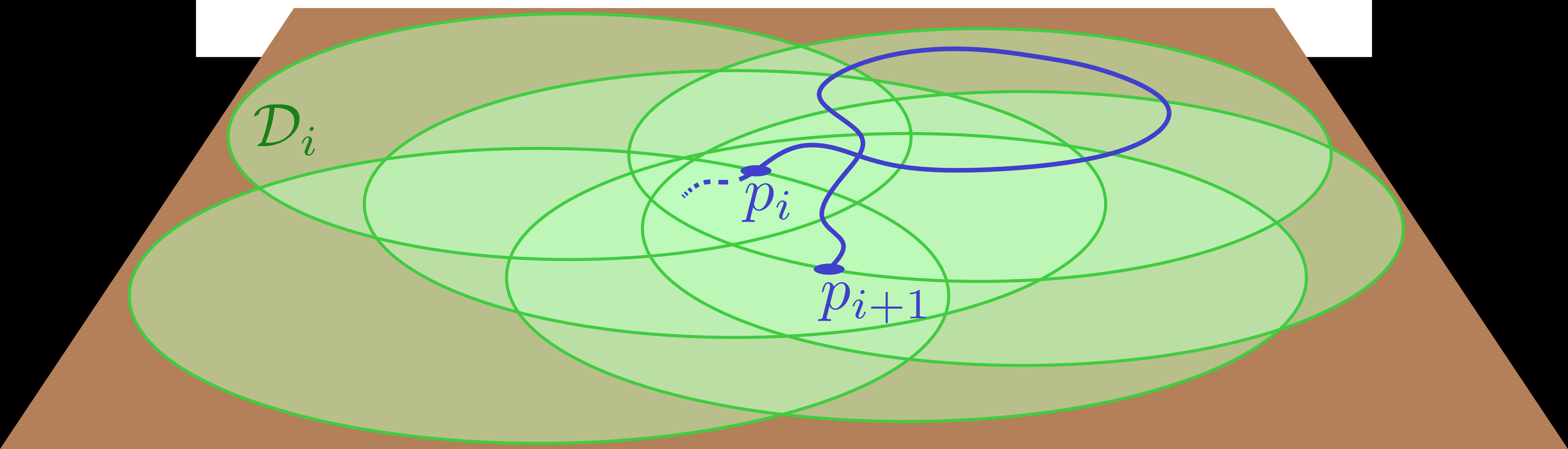


RANDOMISED ONLINE



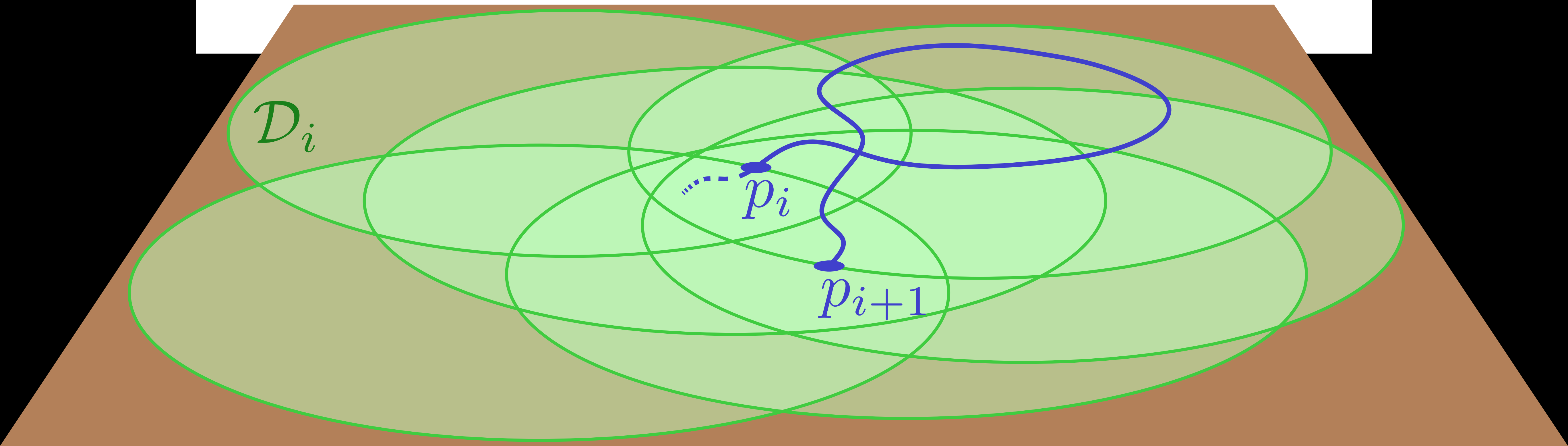
RANDOMISED ONLINE

- Algorithm



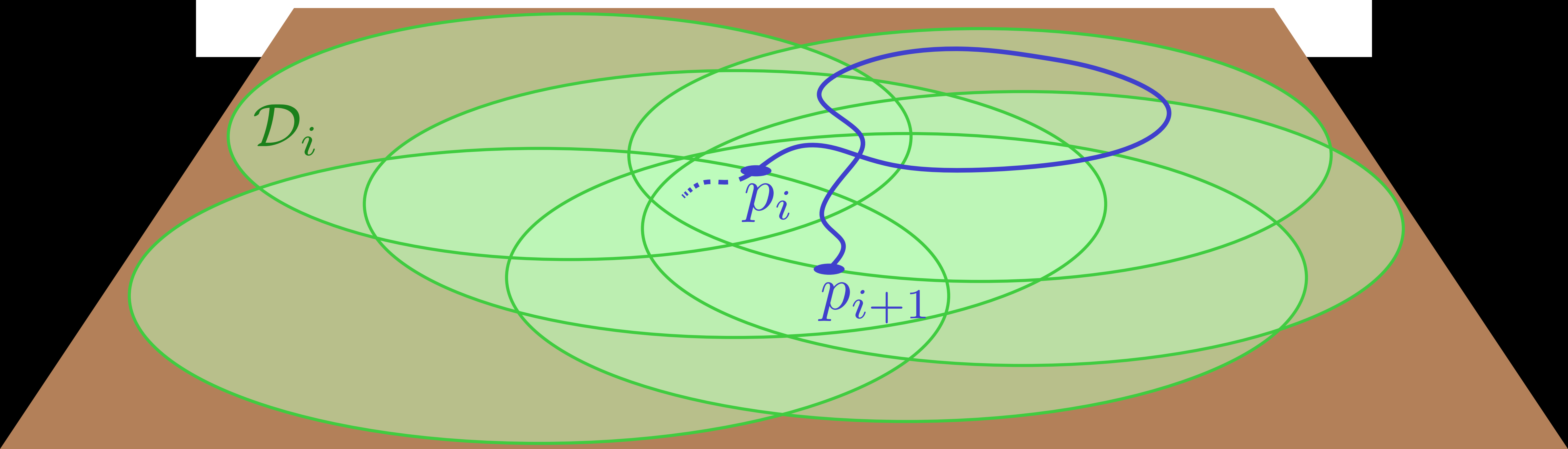
RANDOMISED ONLINE

- Algorithm
  - Almost the same as deterministic



RANDOMISED ONLINE

- Algorithm
  - Almost the same as deterministic
  - Just choose $D \in \mathcal{D}_i$ randomly



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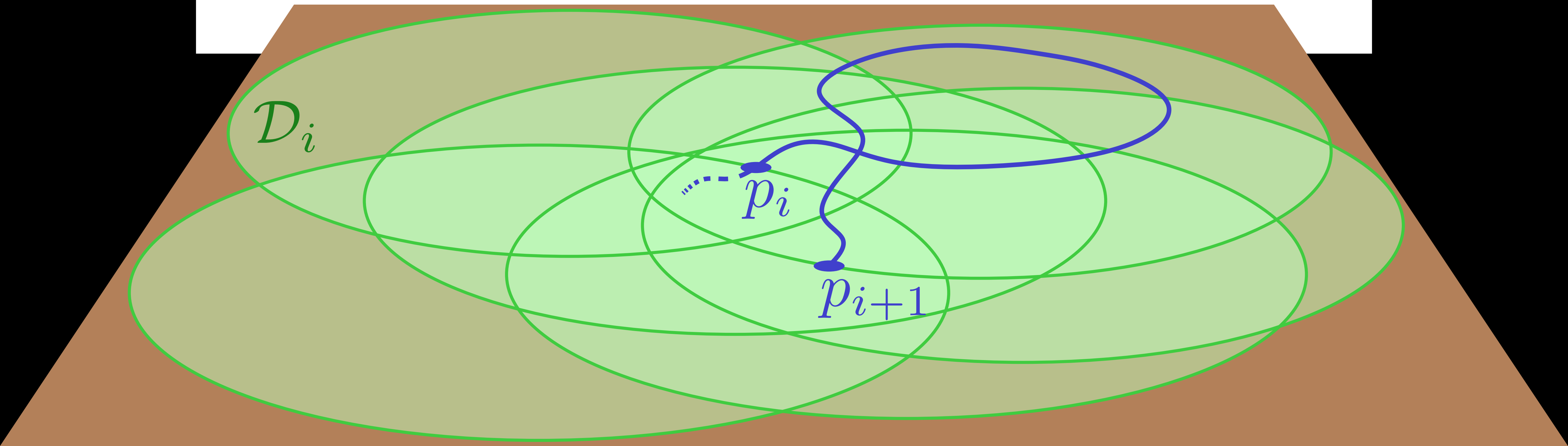

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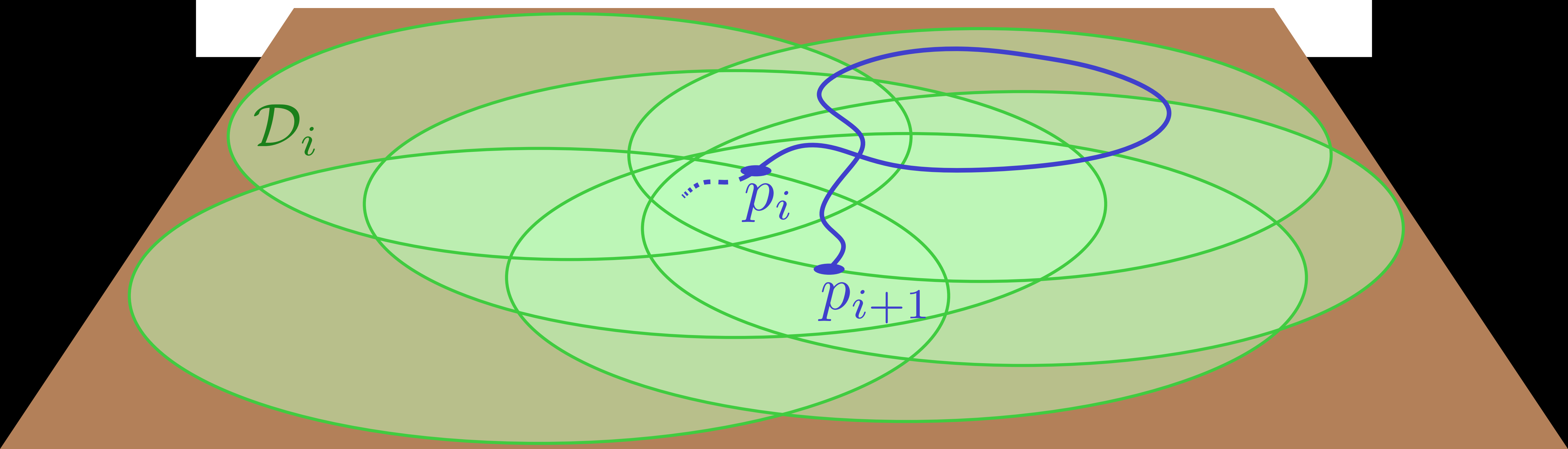
  - Sort $\mathcal{D}_i = D_1, \ldots, D_k$ by time object leaves them



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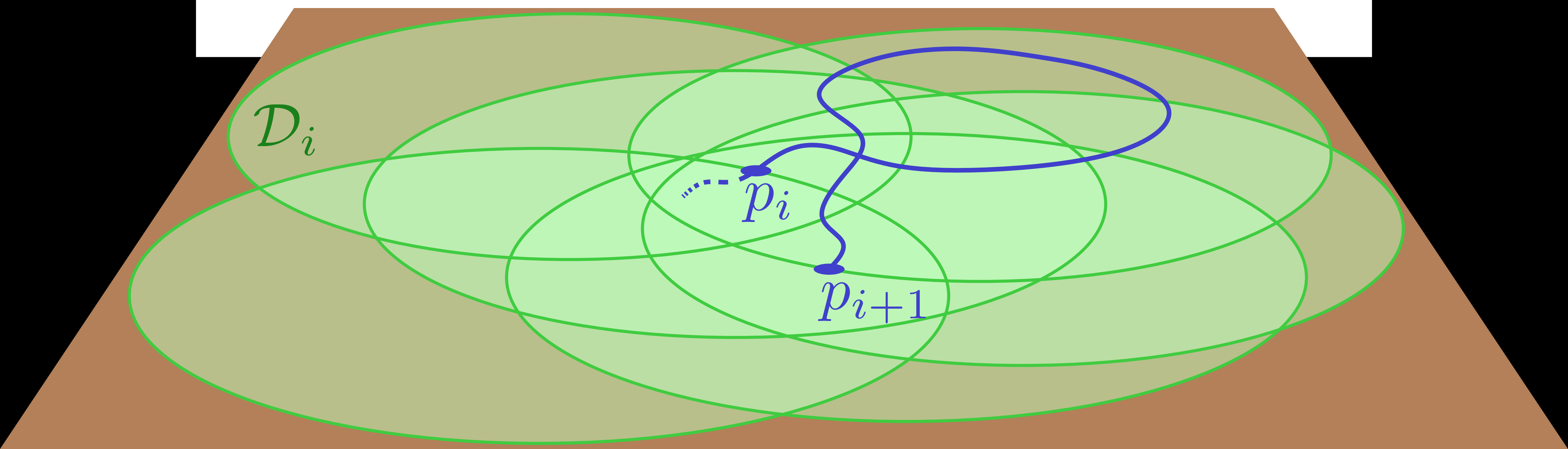
  - Sort $\mathcal{D}_i = D_1, \ldots, D_k$ by time object leaves them
  - Take random increasing sequence of indices



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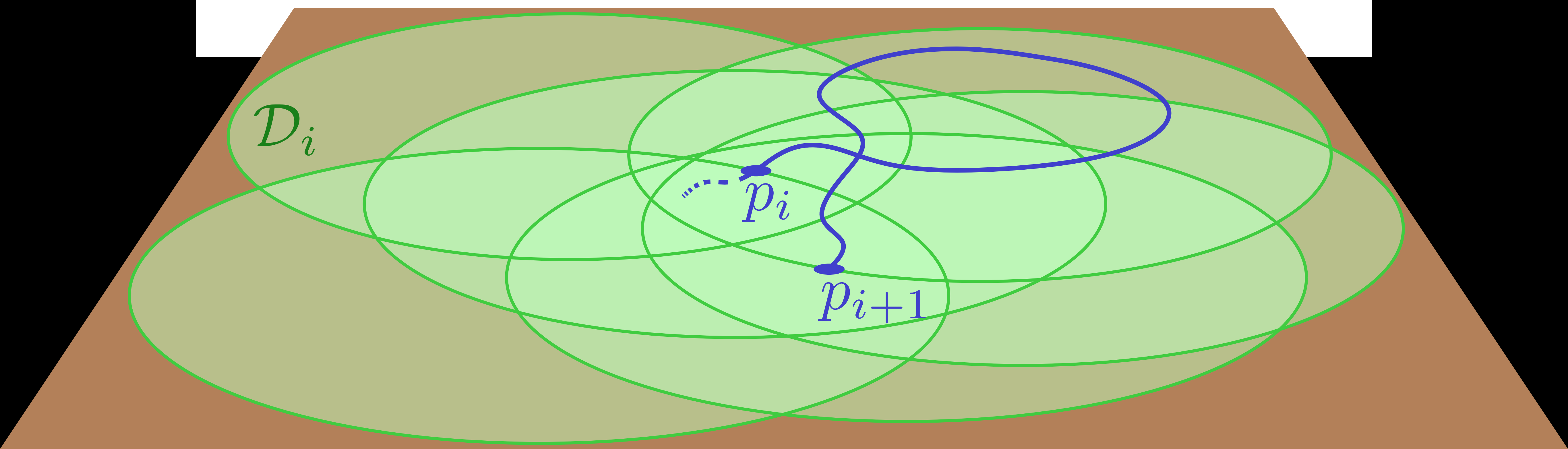
  - Sort $\mathcal{D}_i = D_1, \ldots, D_k$ by time object leaves them
  - Take random increasing sequence of indices
  - Expected lenght $\log k \leq \log \Delta$



RANDOMISED ONLINE

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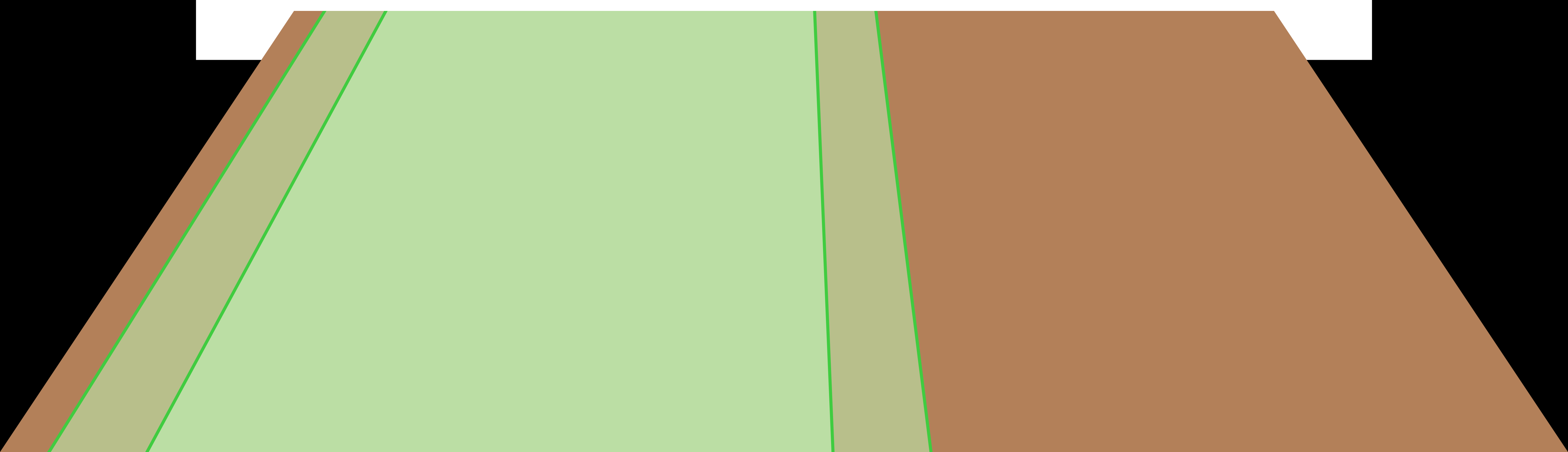

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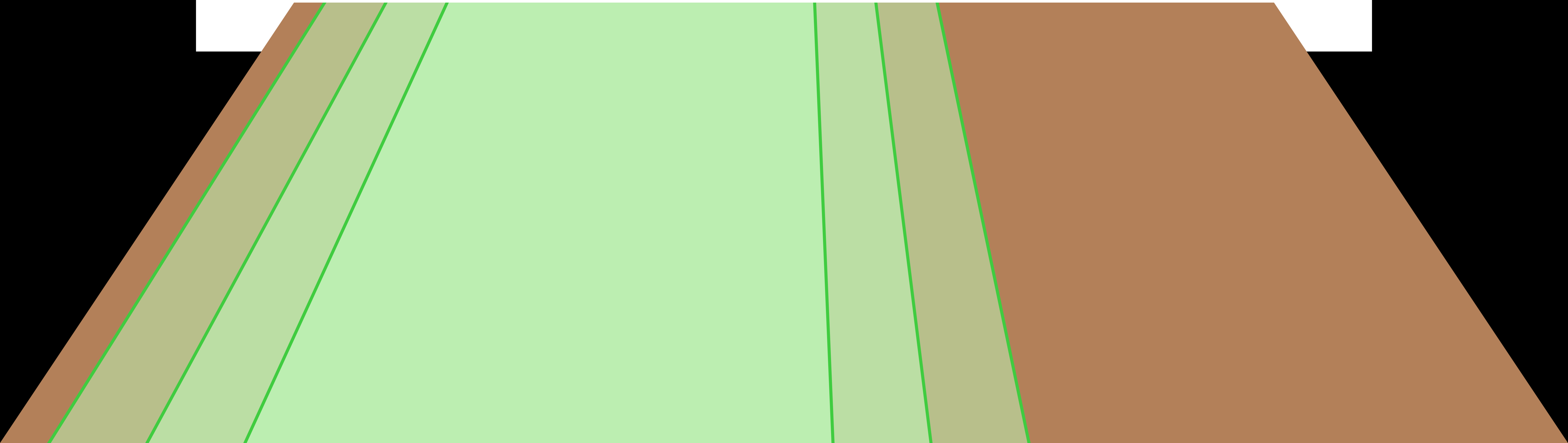

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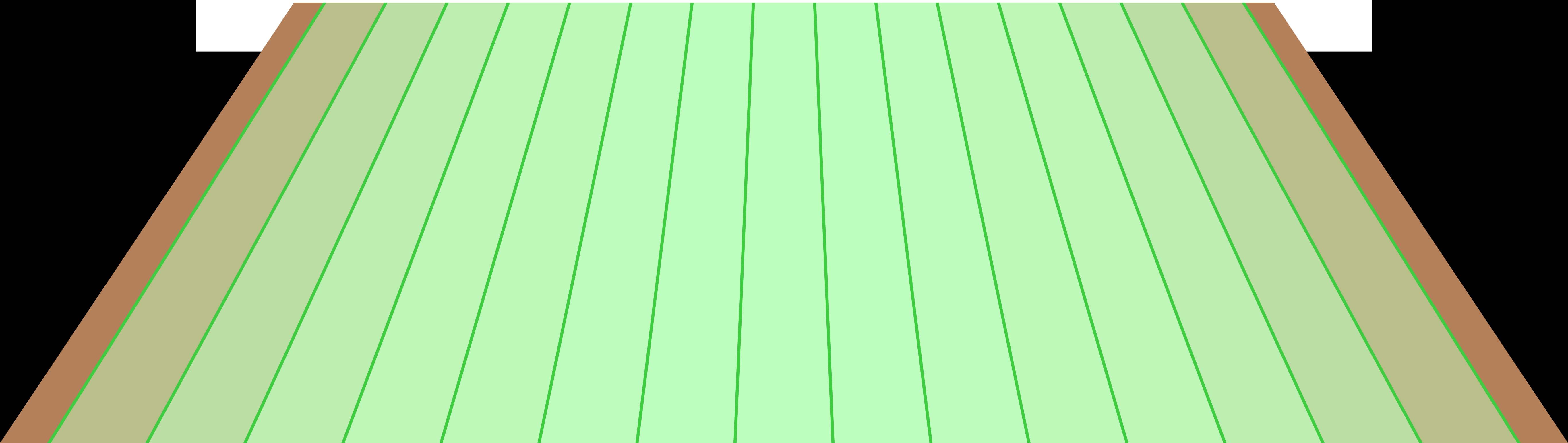

RANDOMISED ONLINE

- Lower bound construction
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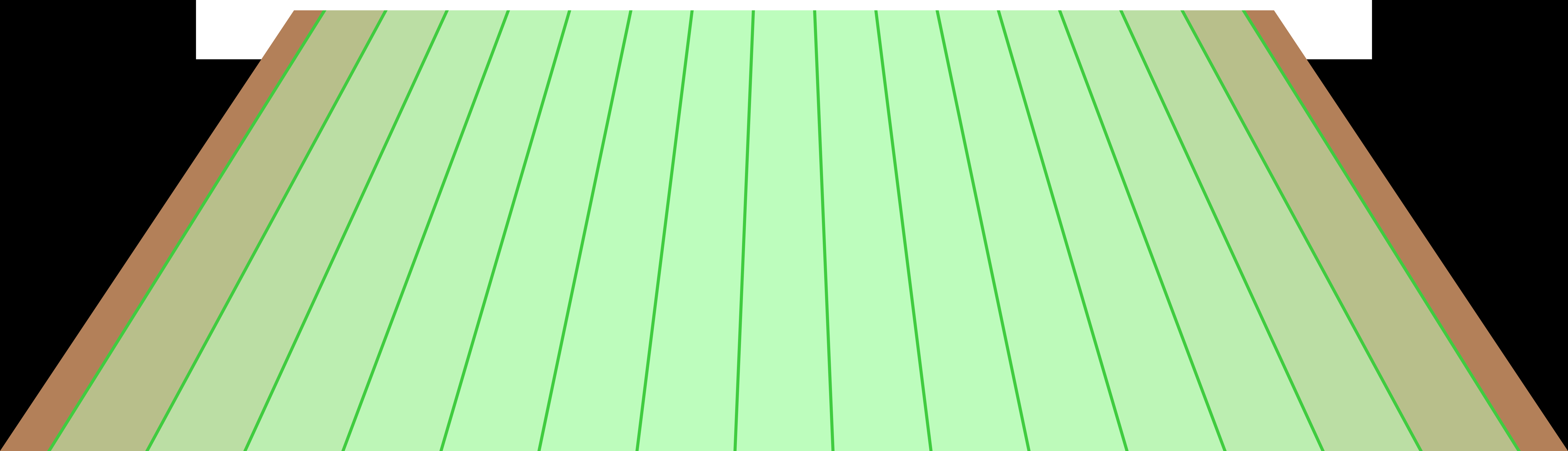
- Adversary strategy



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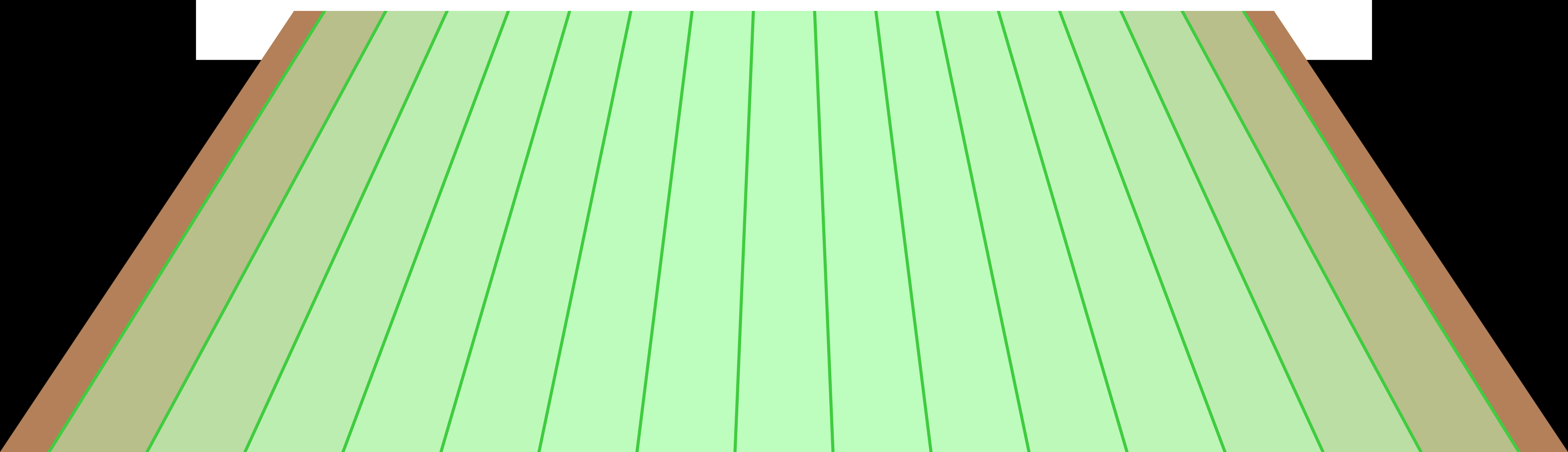

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- Lower bound construction
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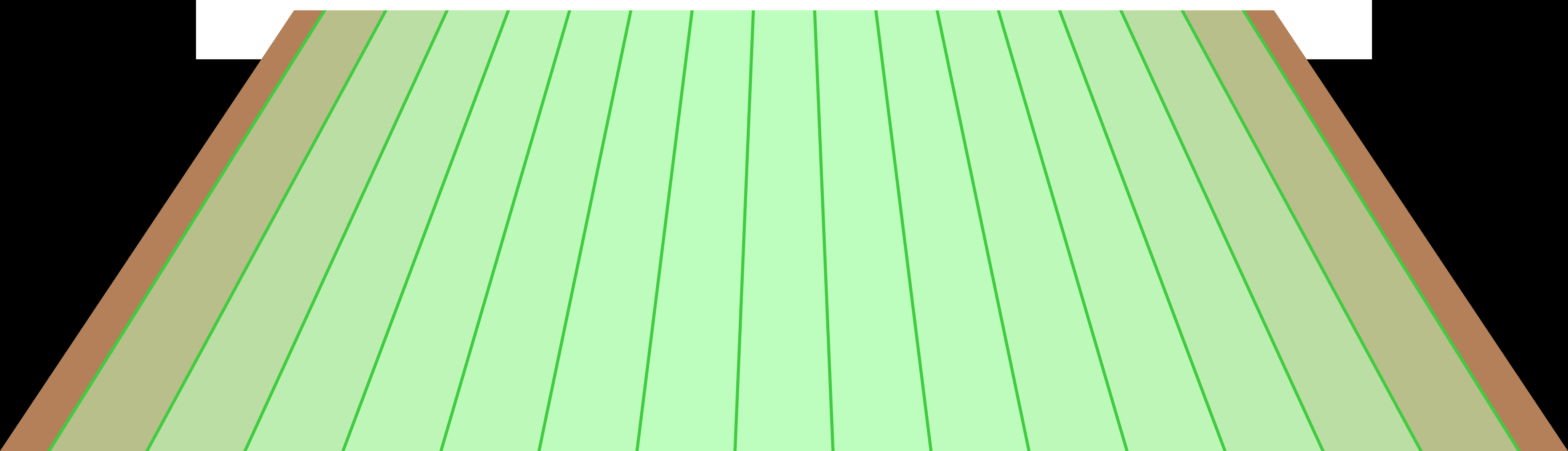
  - Doesn't know random choices of algorithm
  - Create tree of possible paths with high average number of handovers



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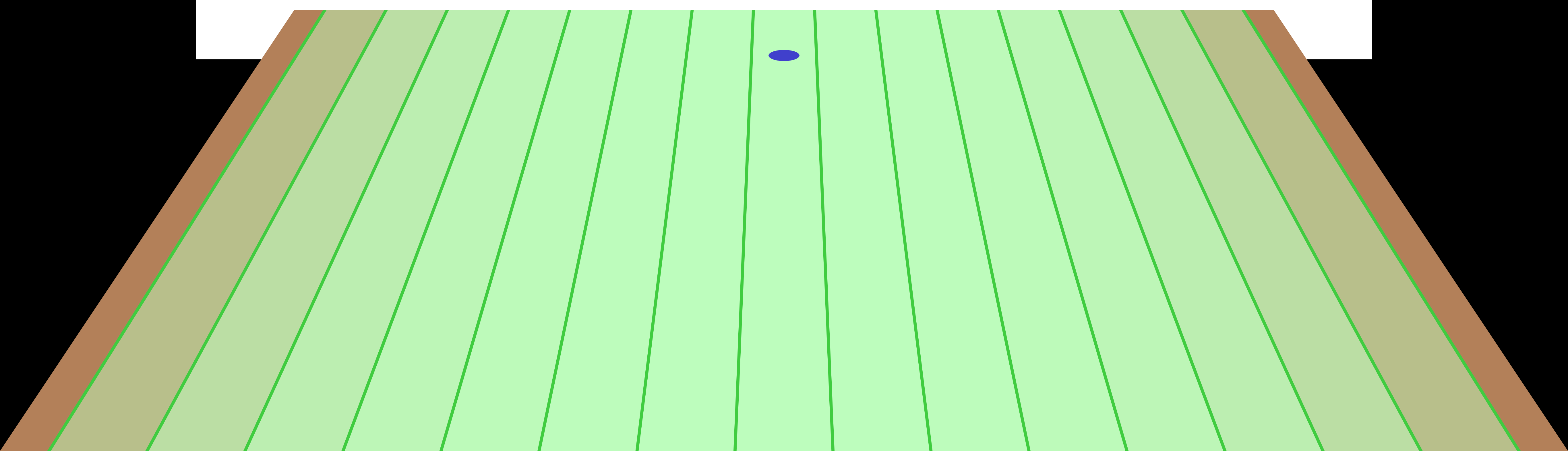

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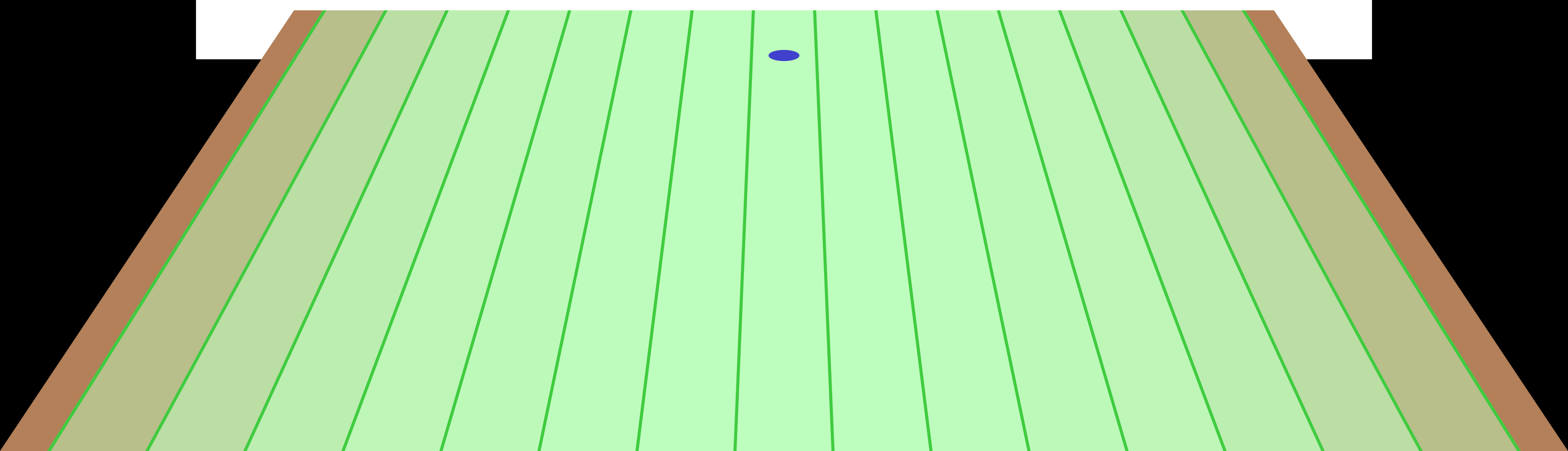
  - At each node, leave half of intervals



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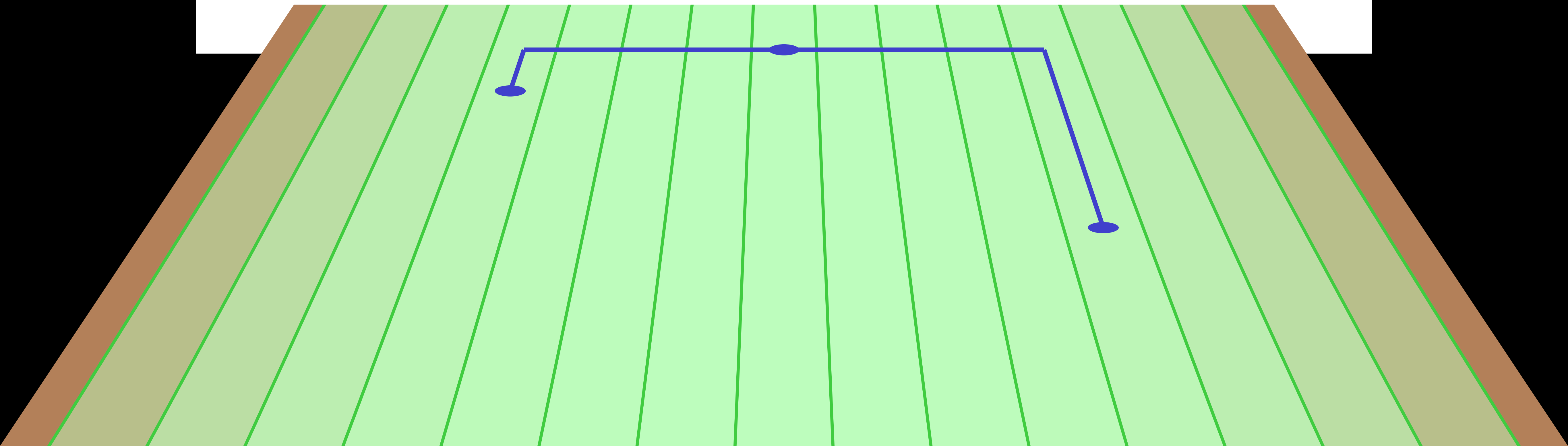

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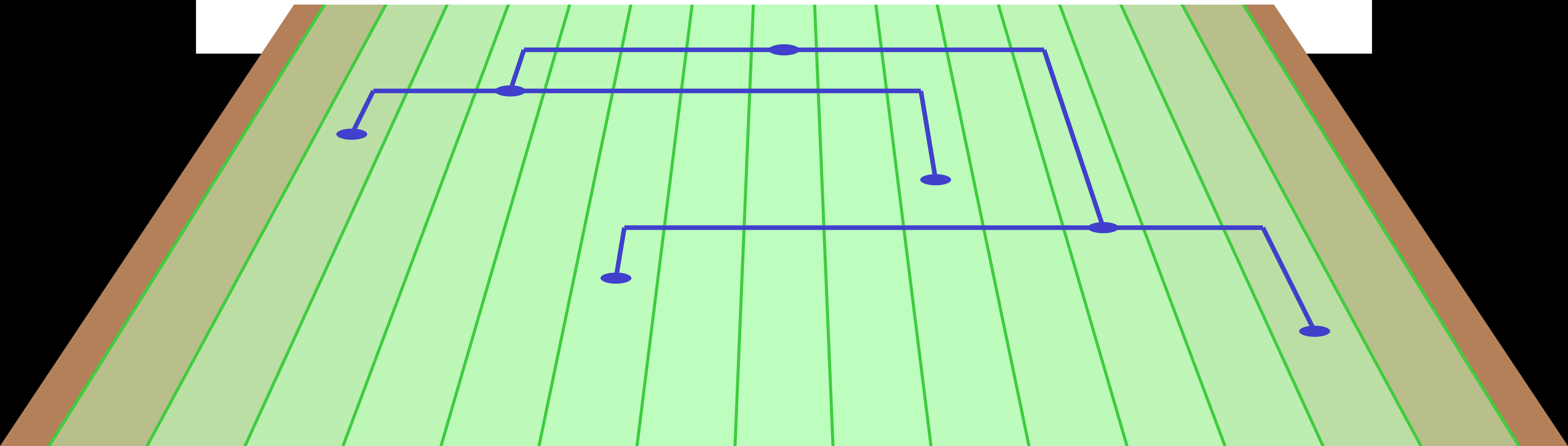

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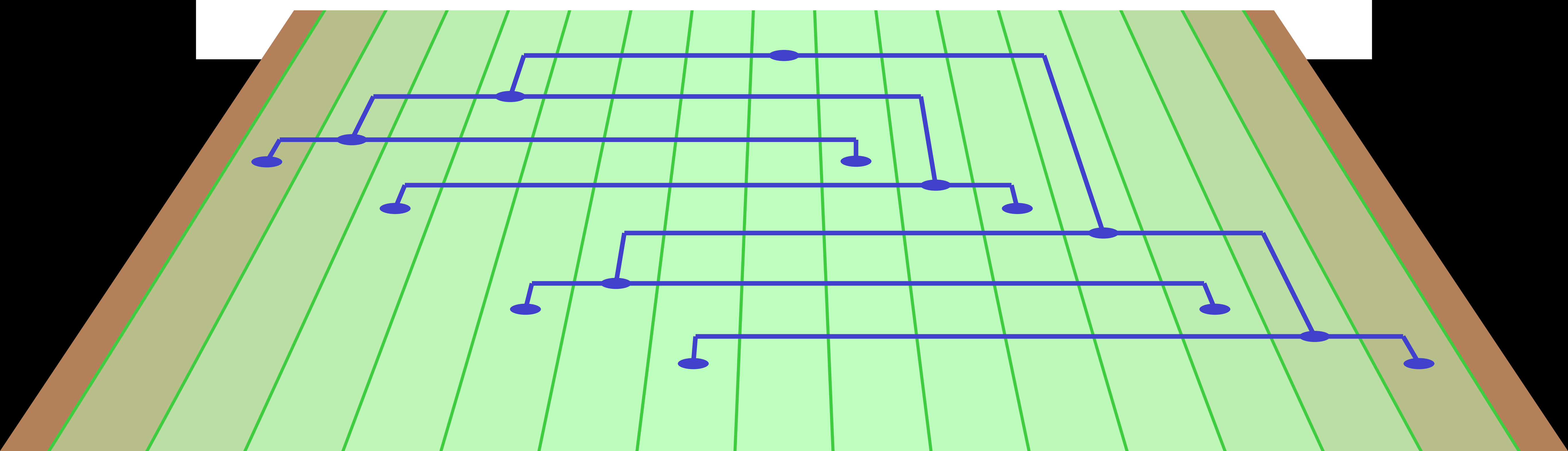

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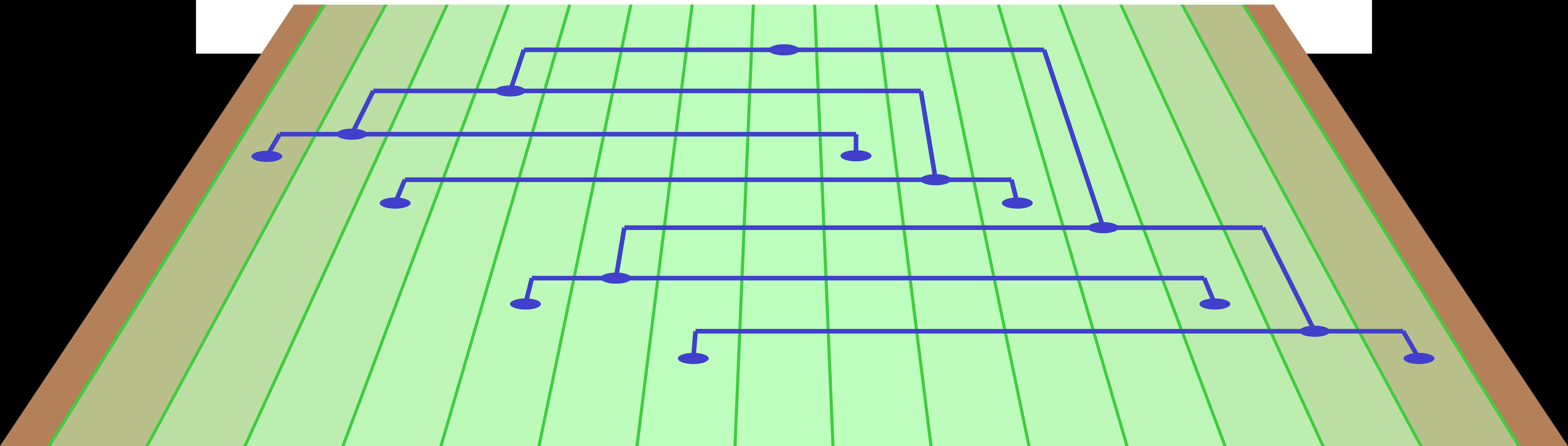

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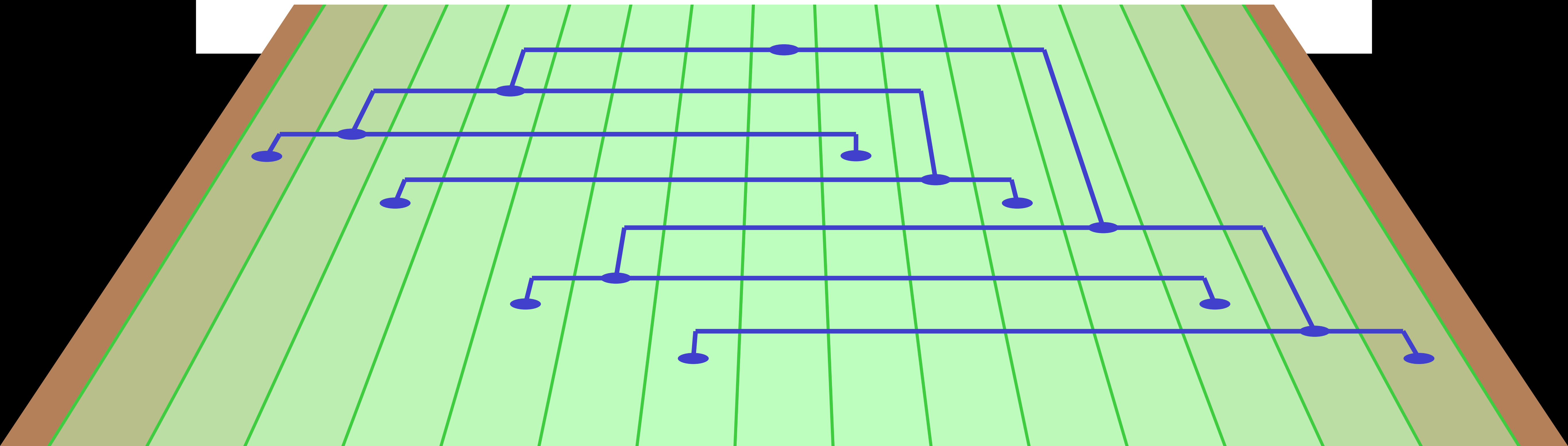

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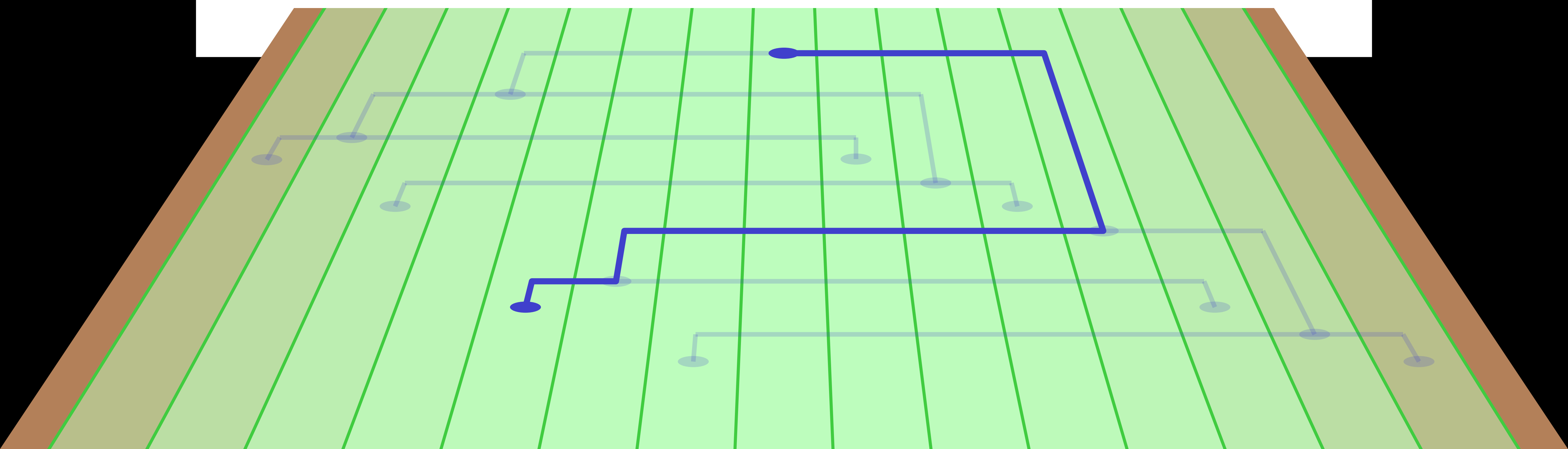

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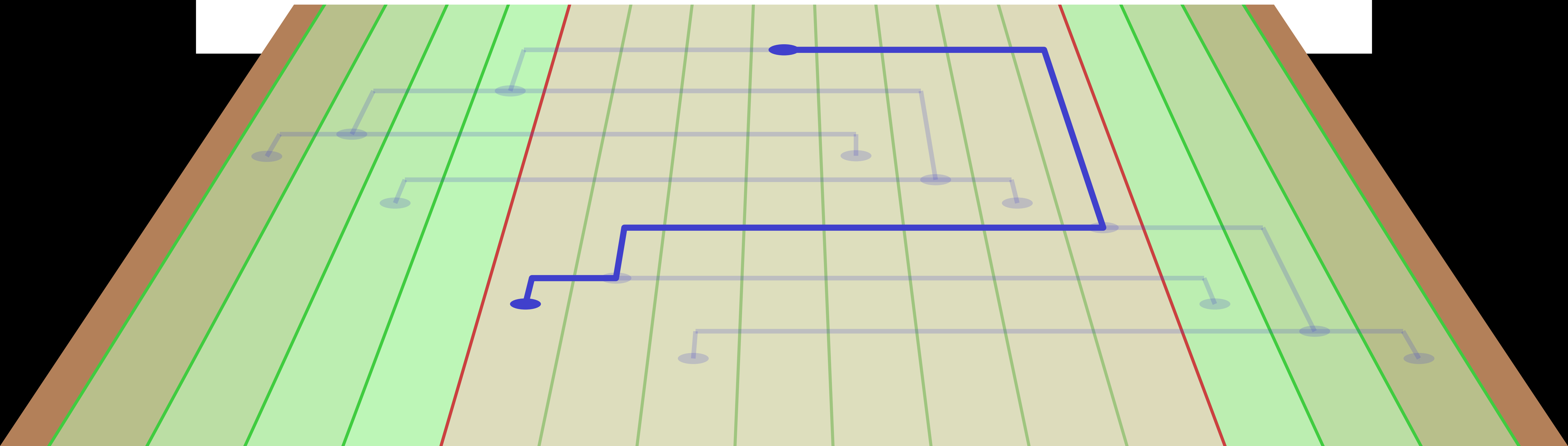

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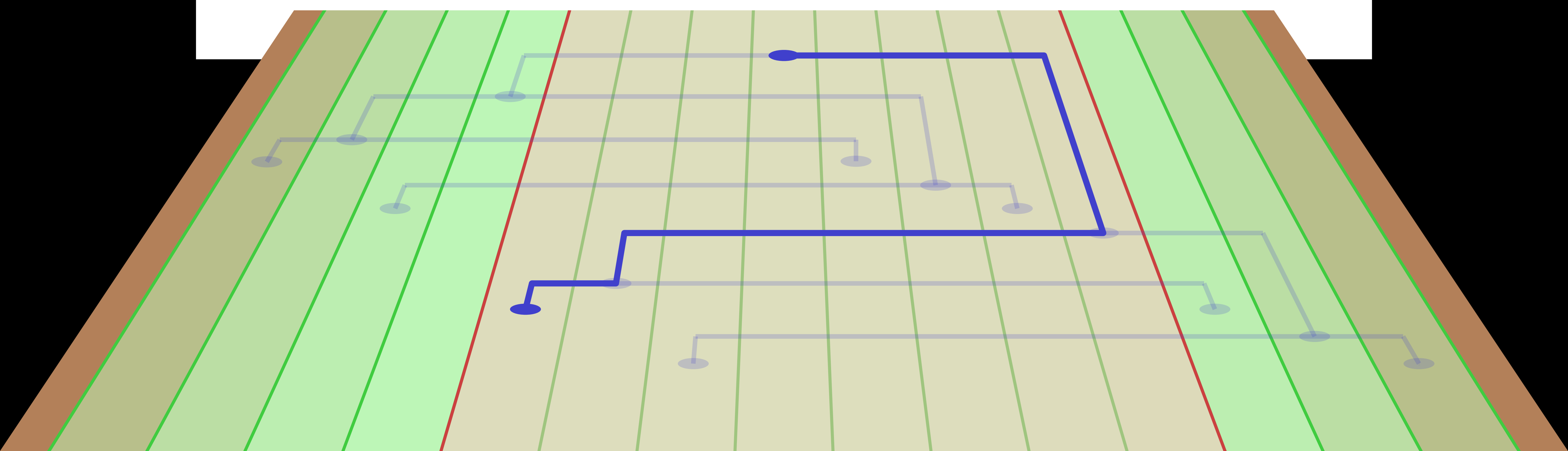

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THANK YOU!