DYNAMIC PLANAR *POINT LOCATION*
WITH
SUB-LOGARITHMIC *LOCAL UPDATES*

Maarten Löffler *Utrecht University*

Joe Simons *University of California, Irvine*

Darren Strash *Intel Oregon*

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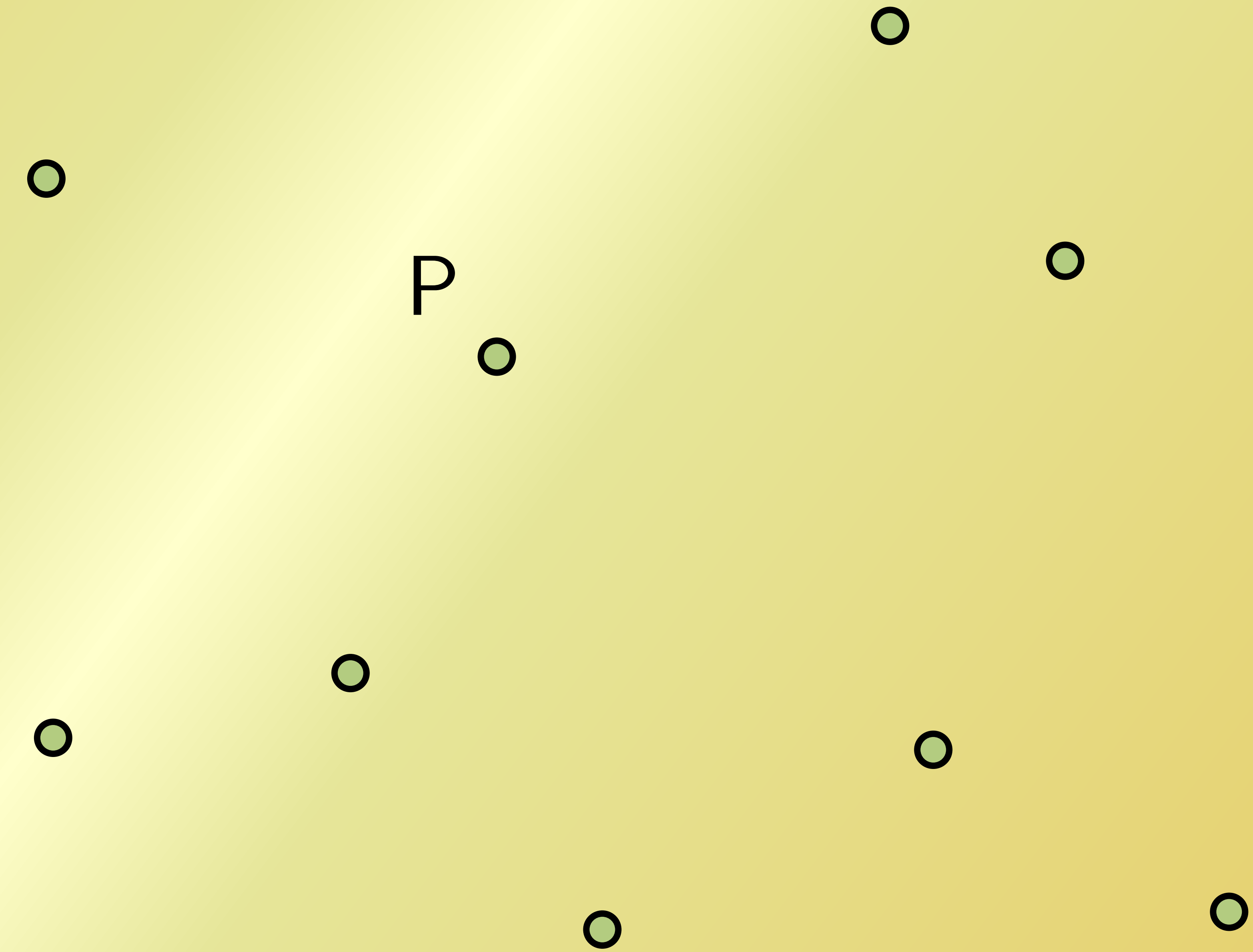

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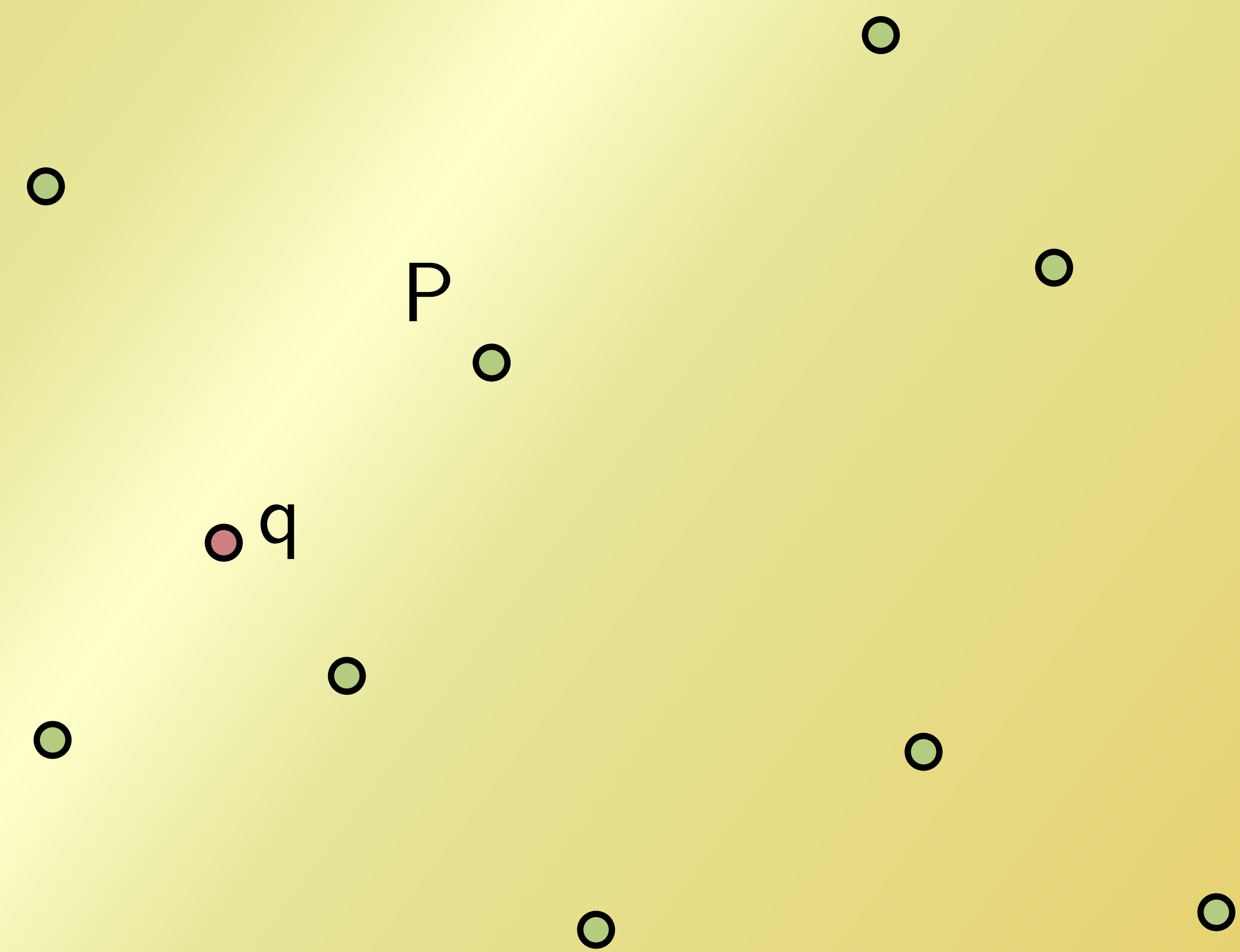

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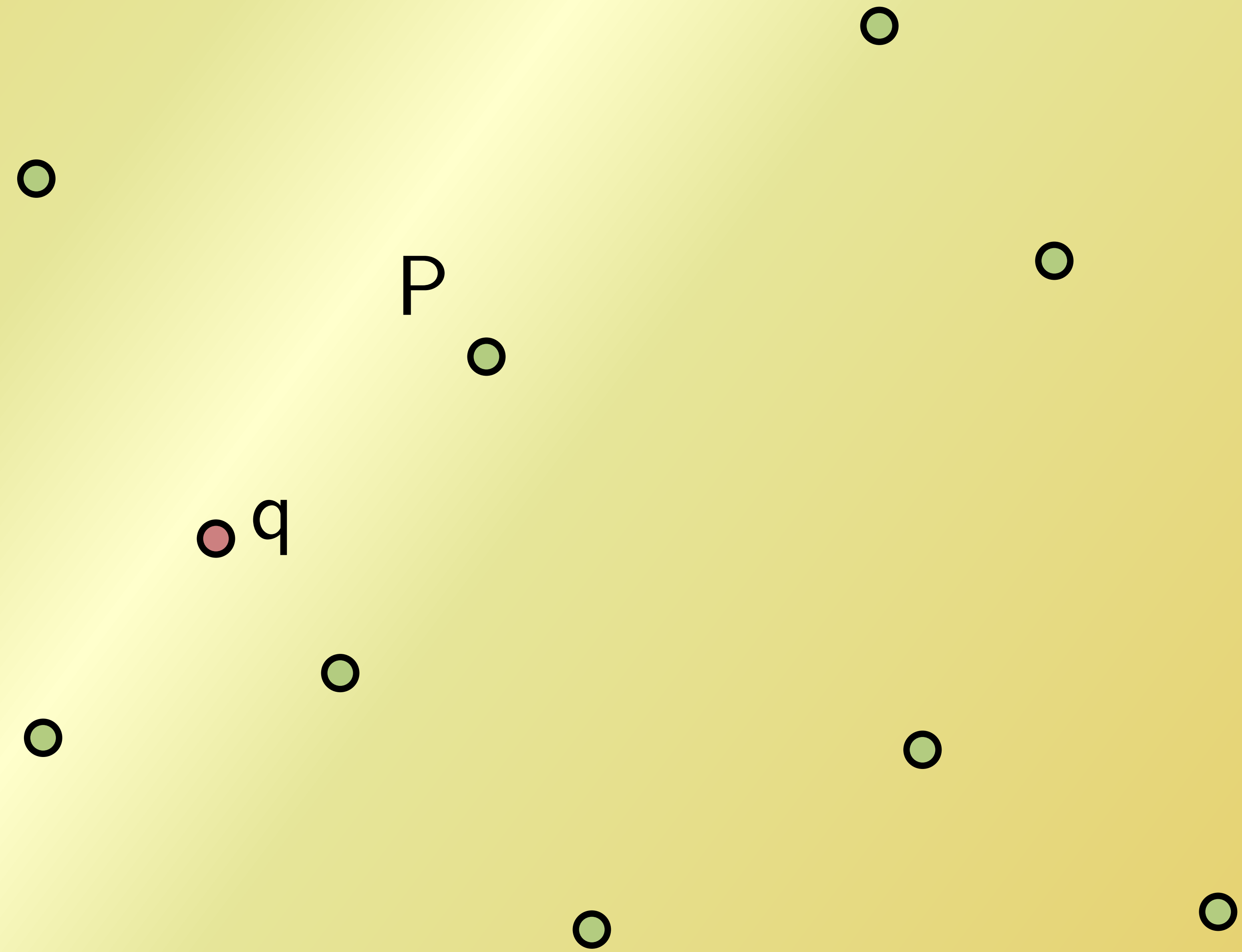

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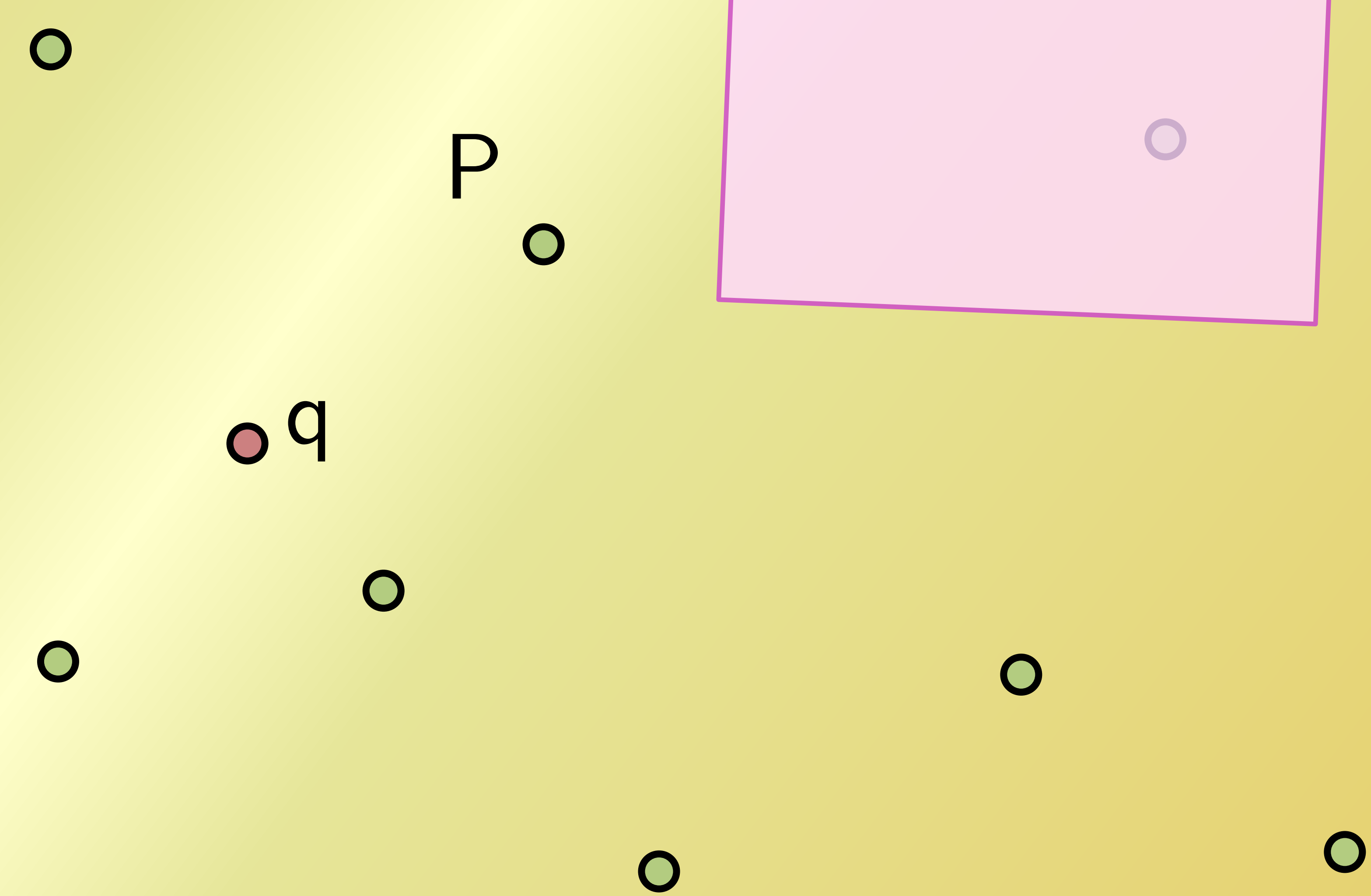

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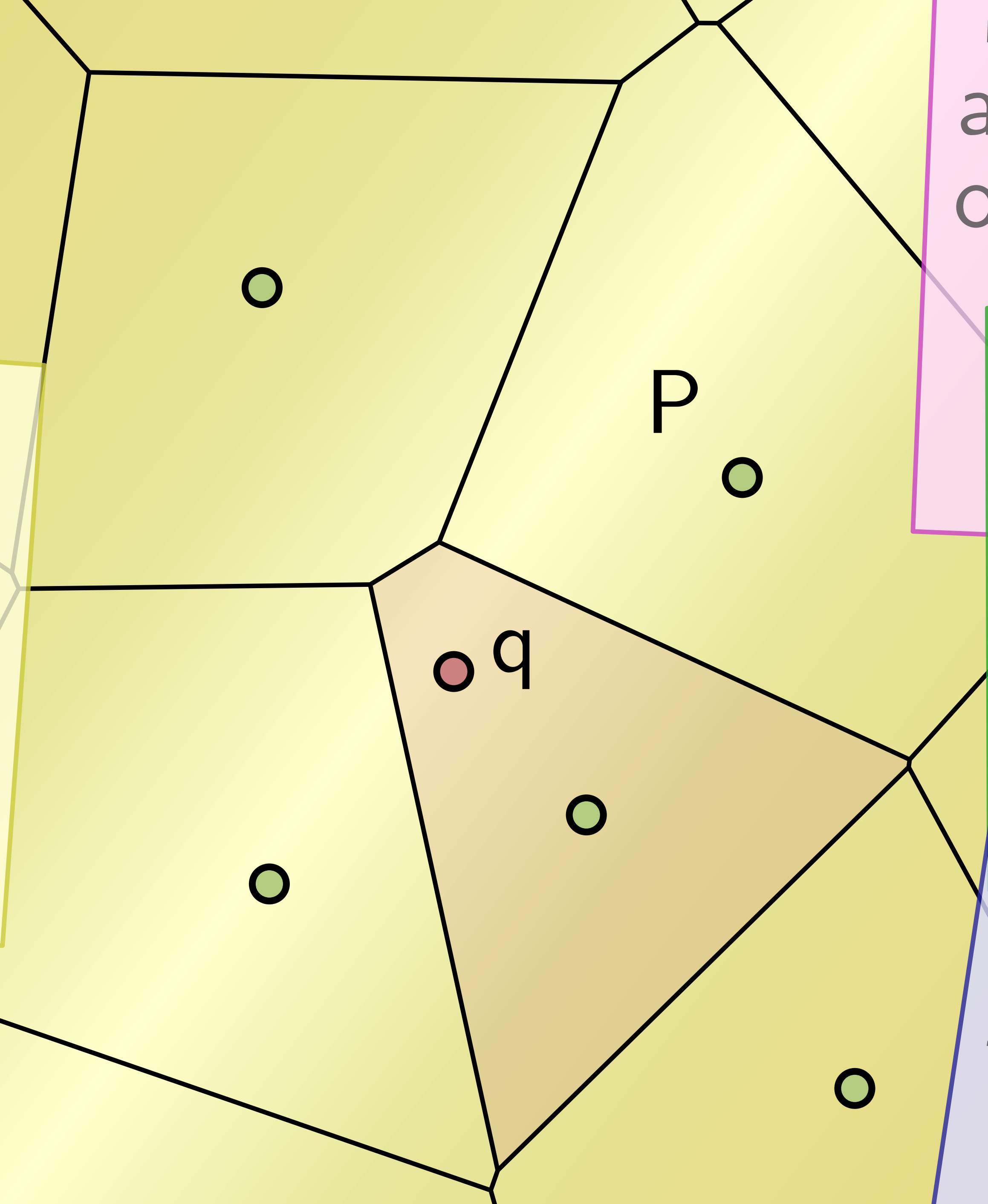

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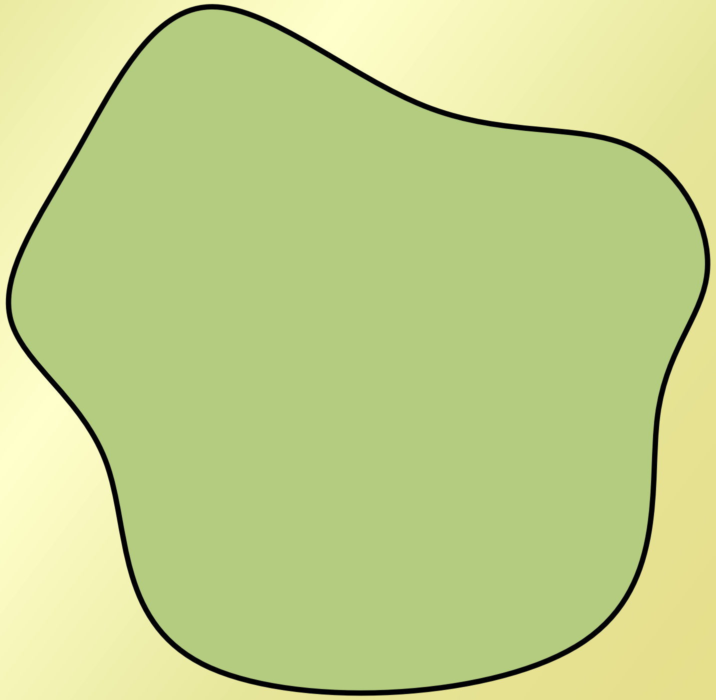

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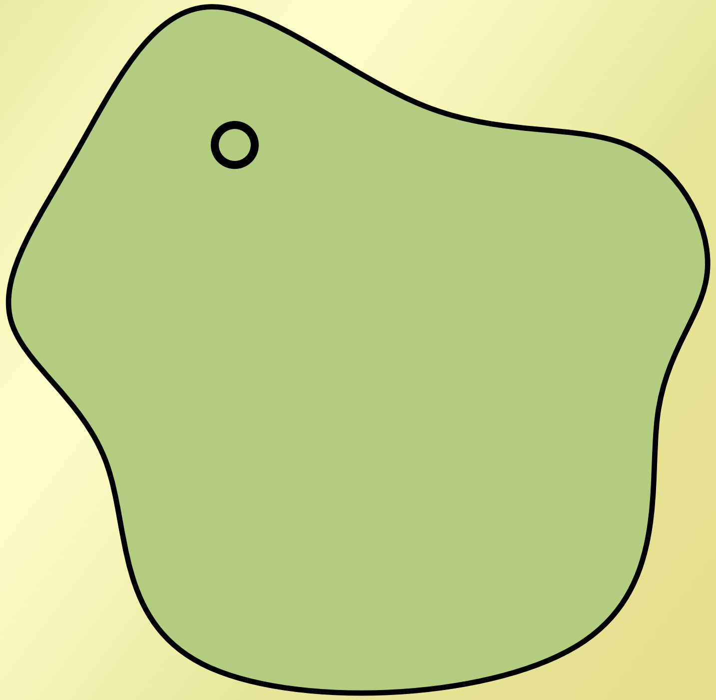

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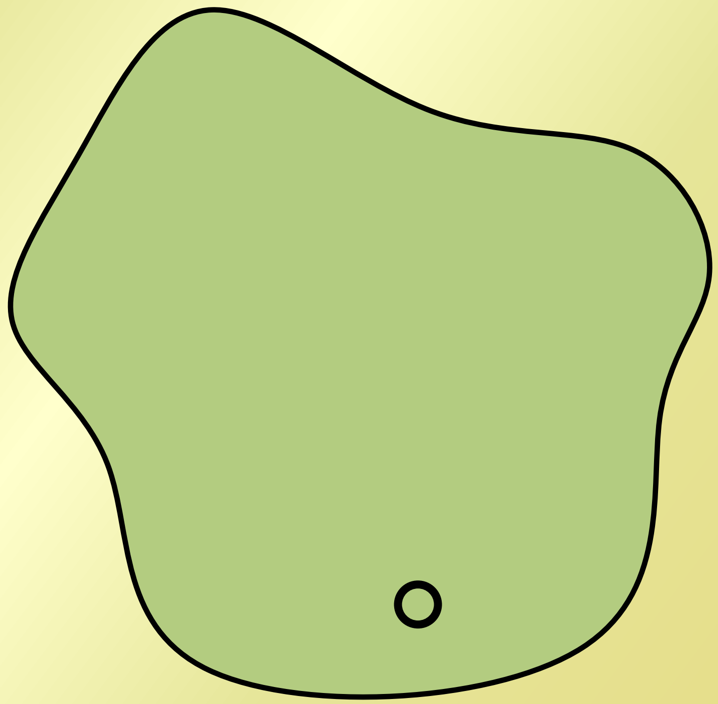

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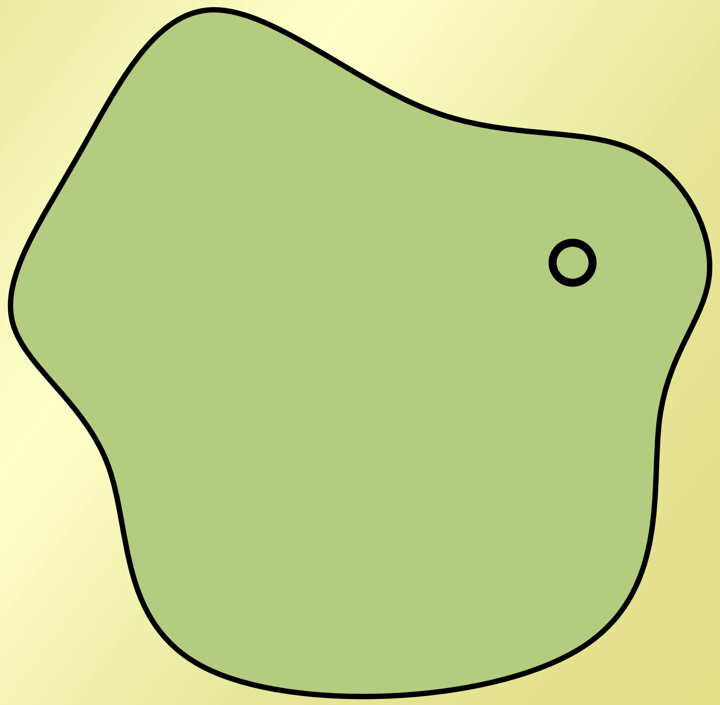

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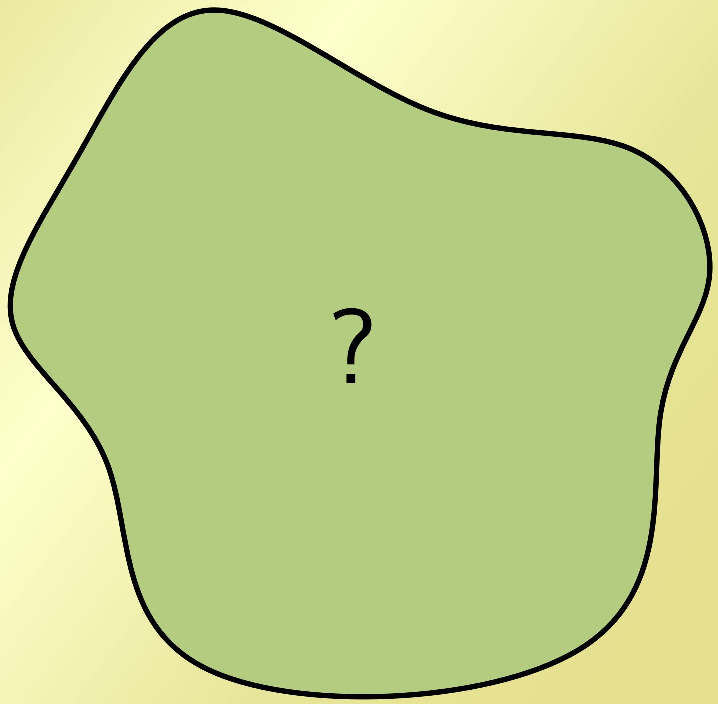

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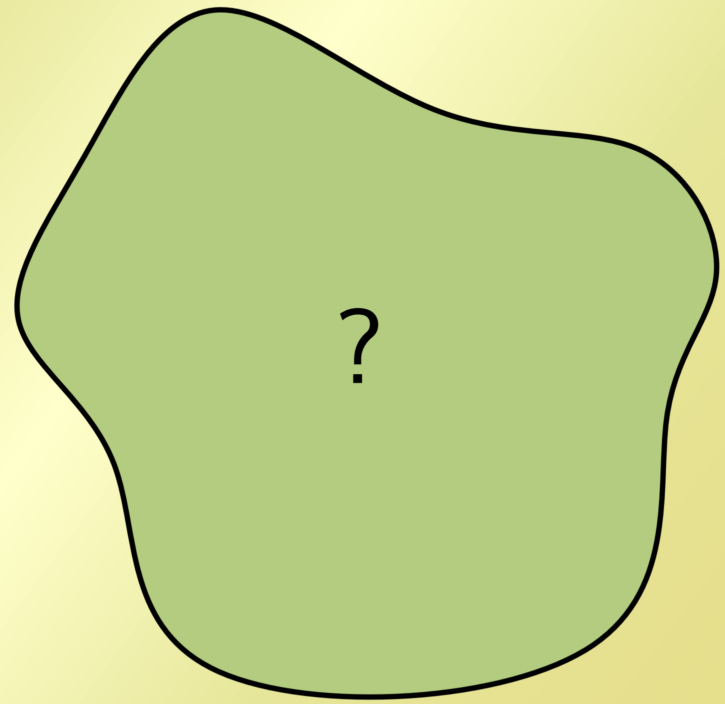

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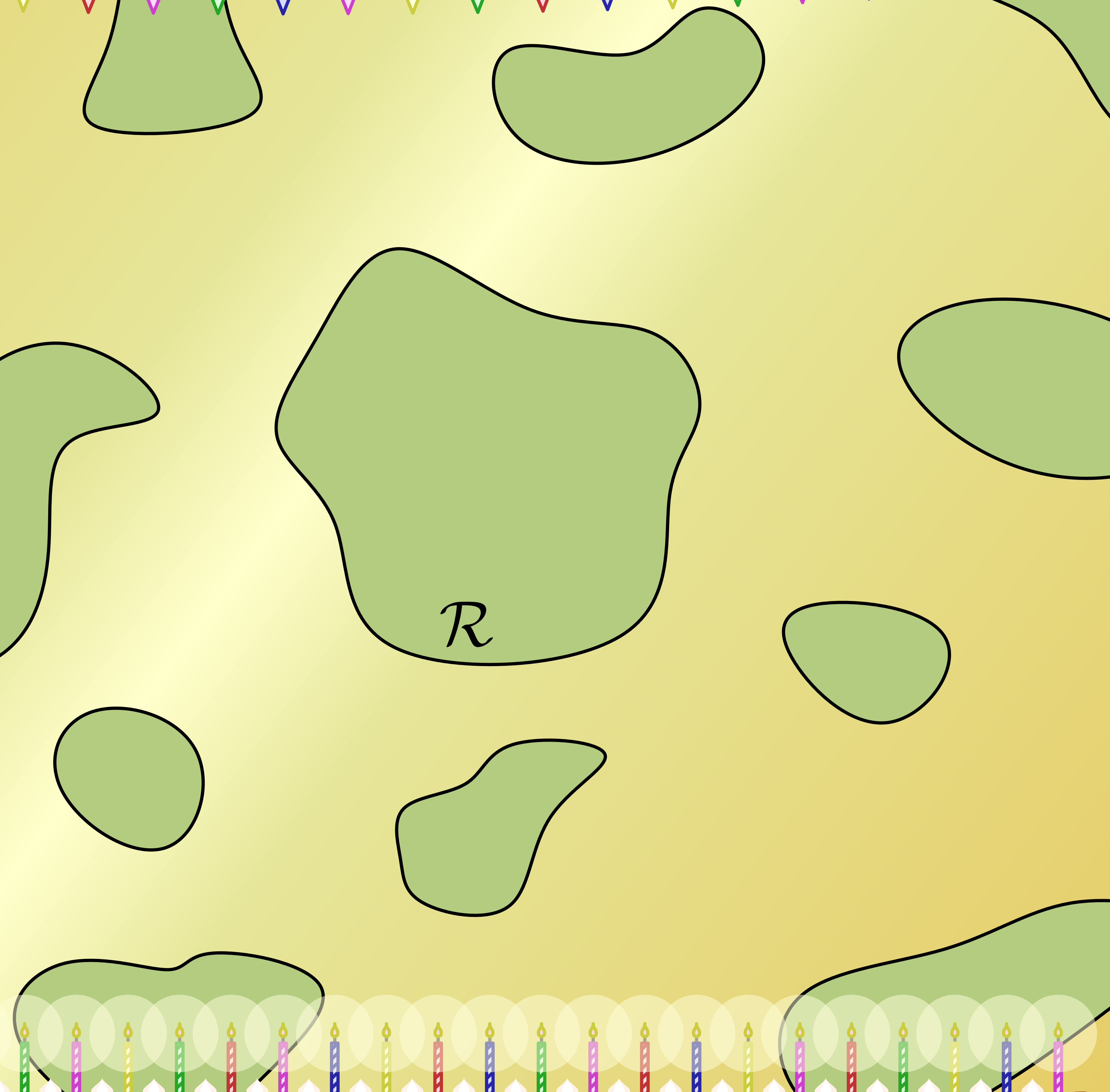

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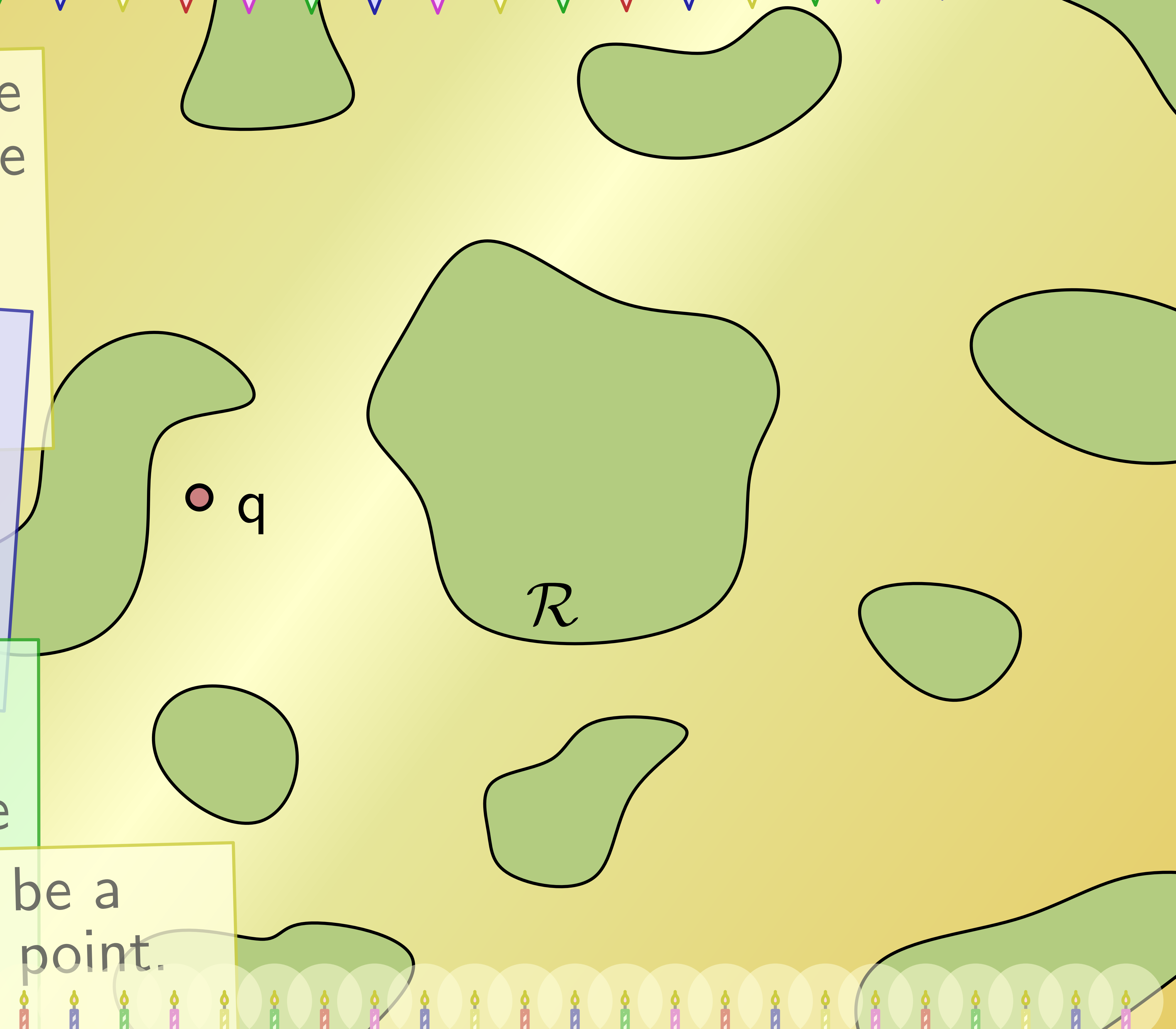

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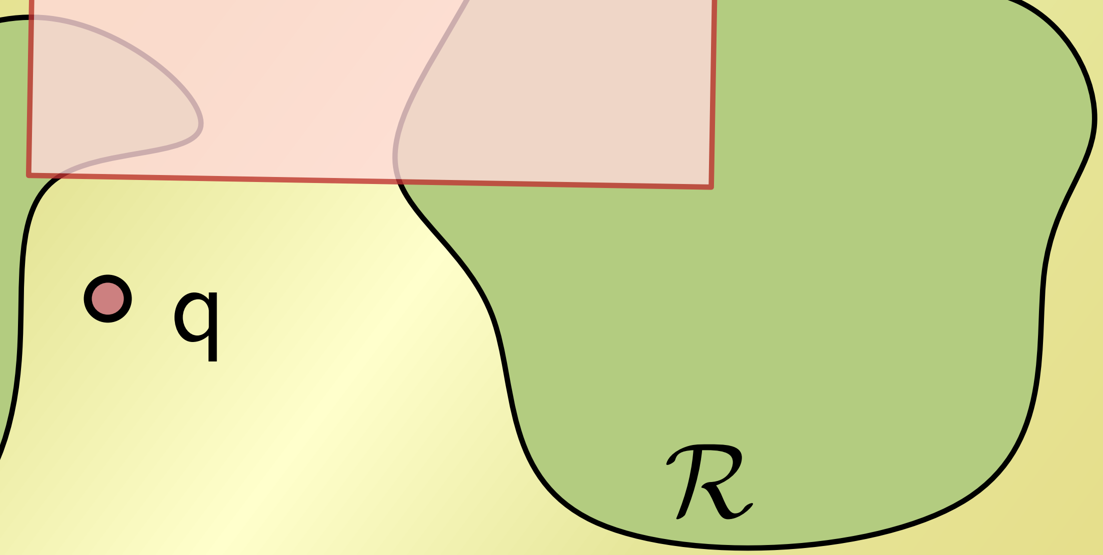

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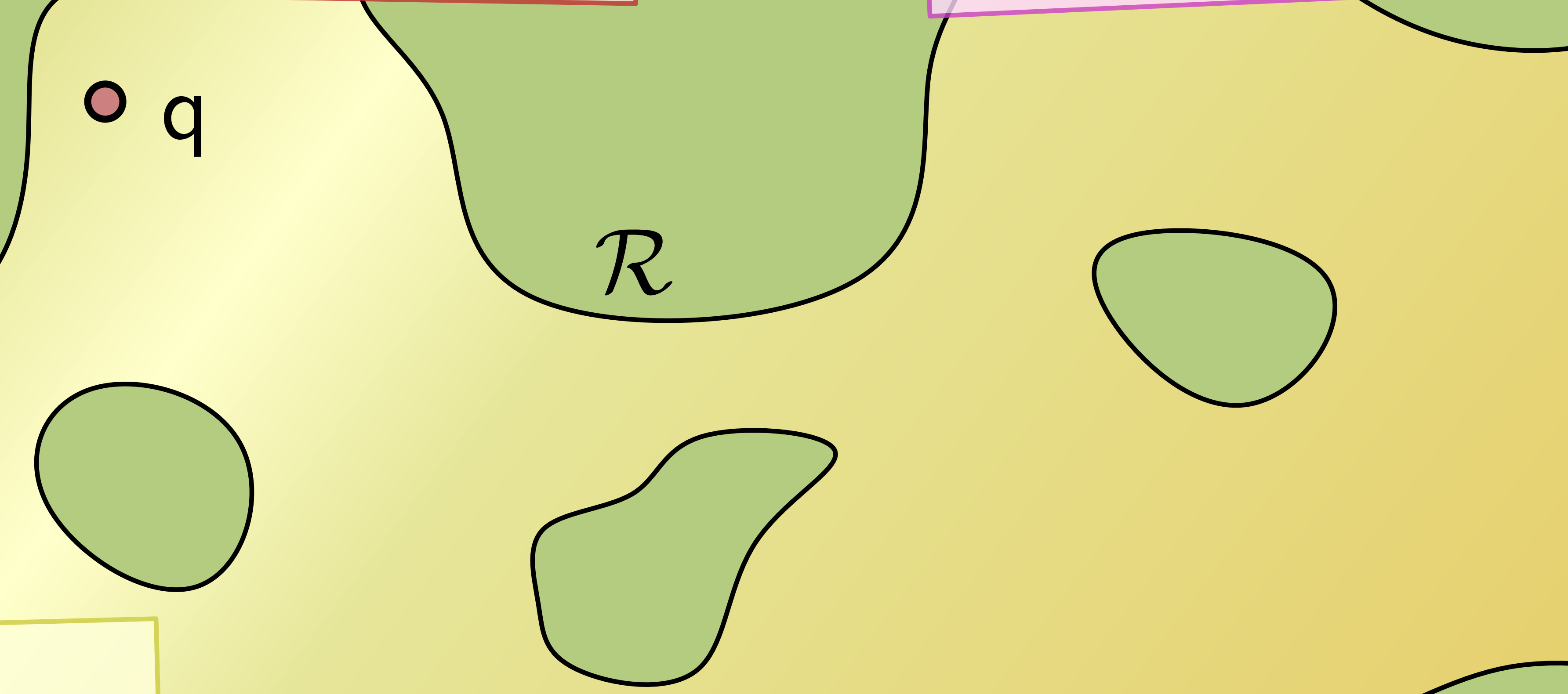

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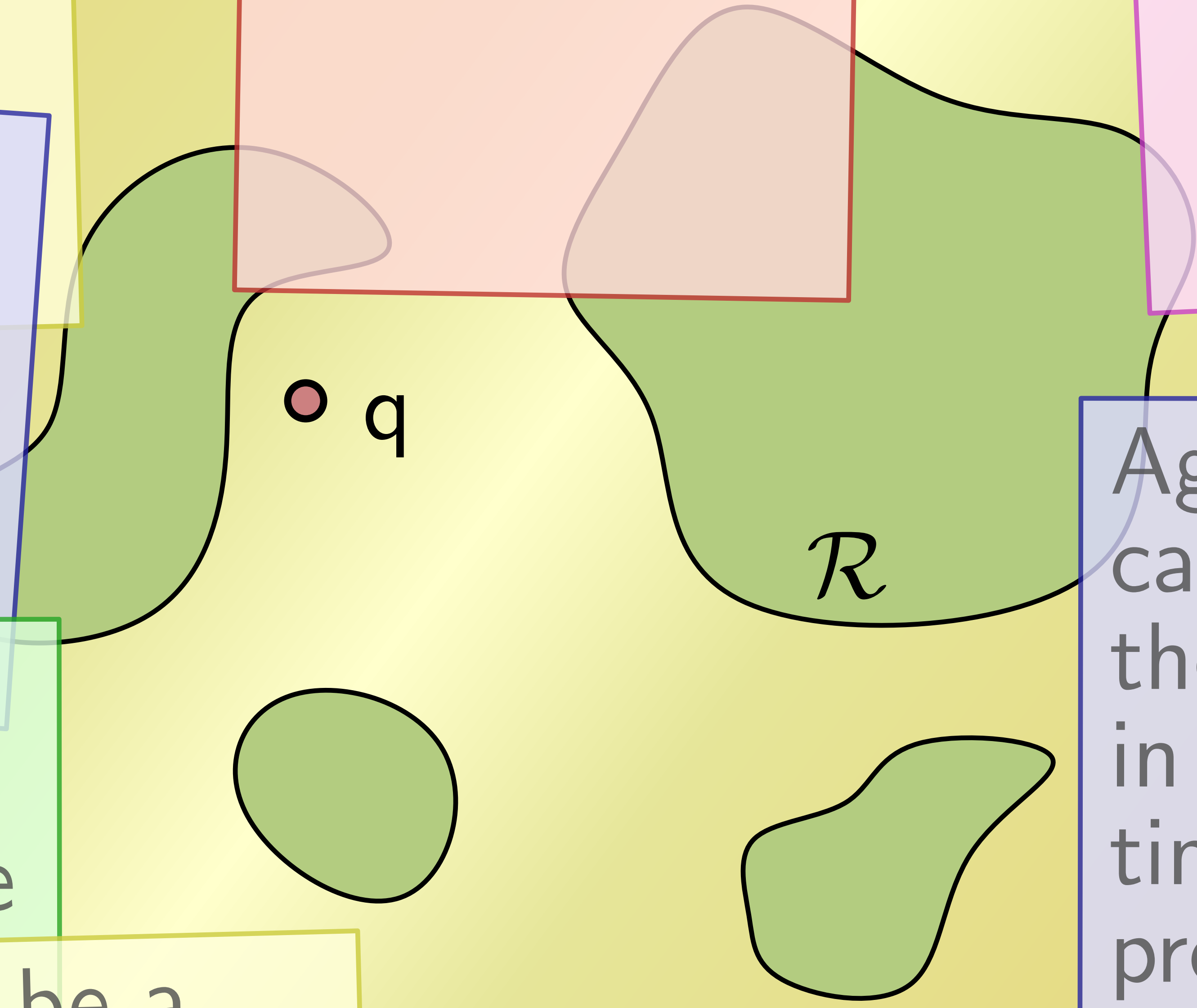

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Again, we can answer the question in logarithmic time after preprocessing

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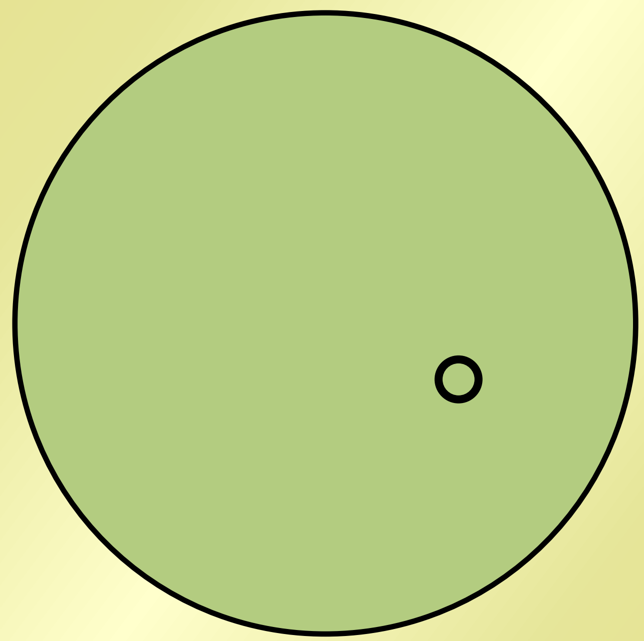

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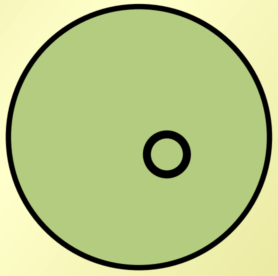

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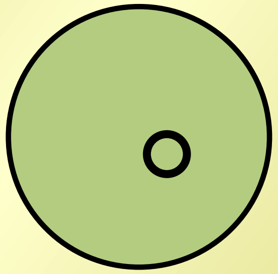

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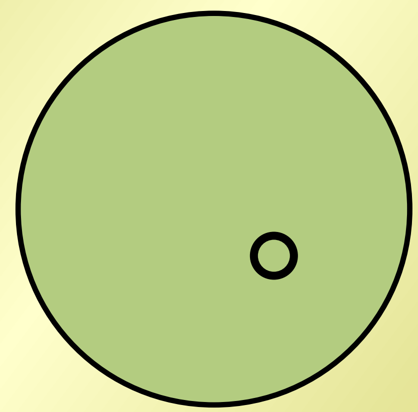

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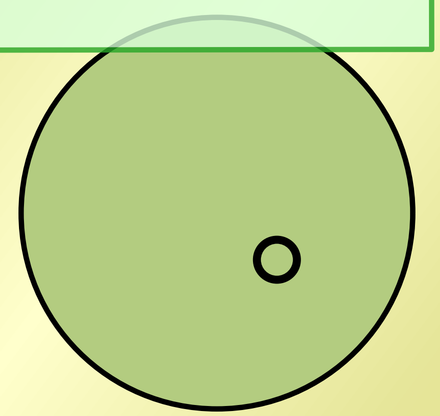

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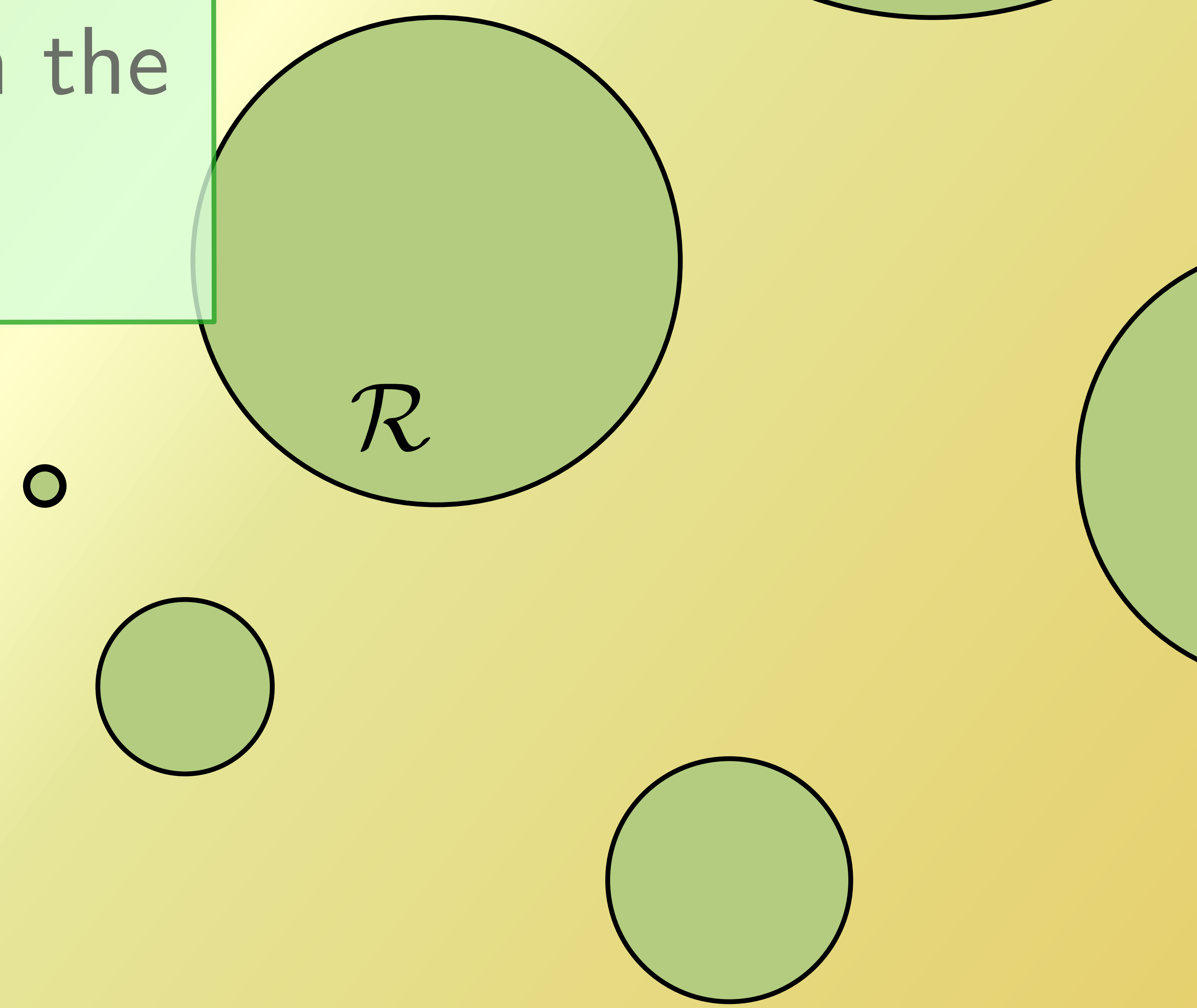

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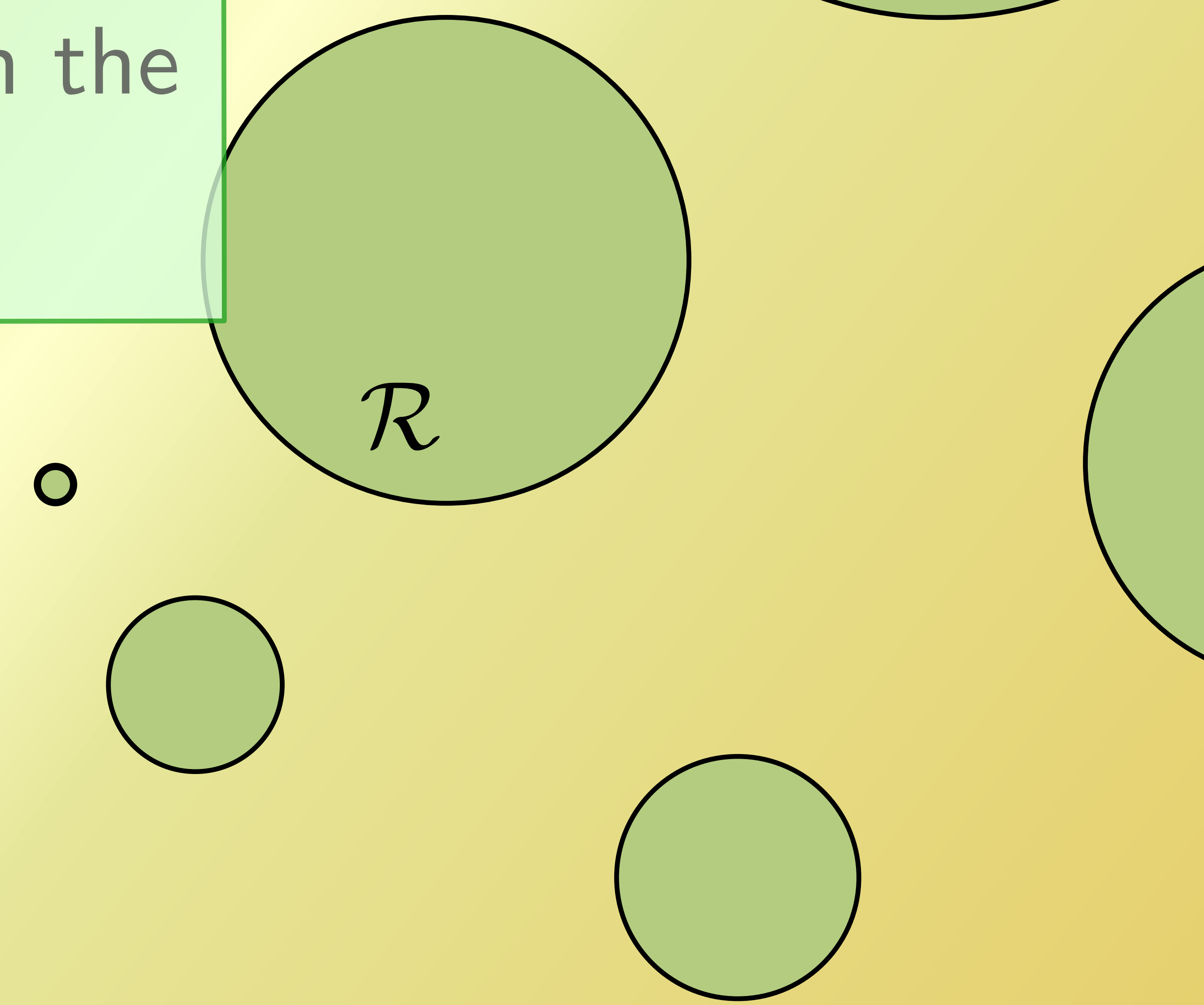

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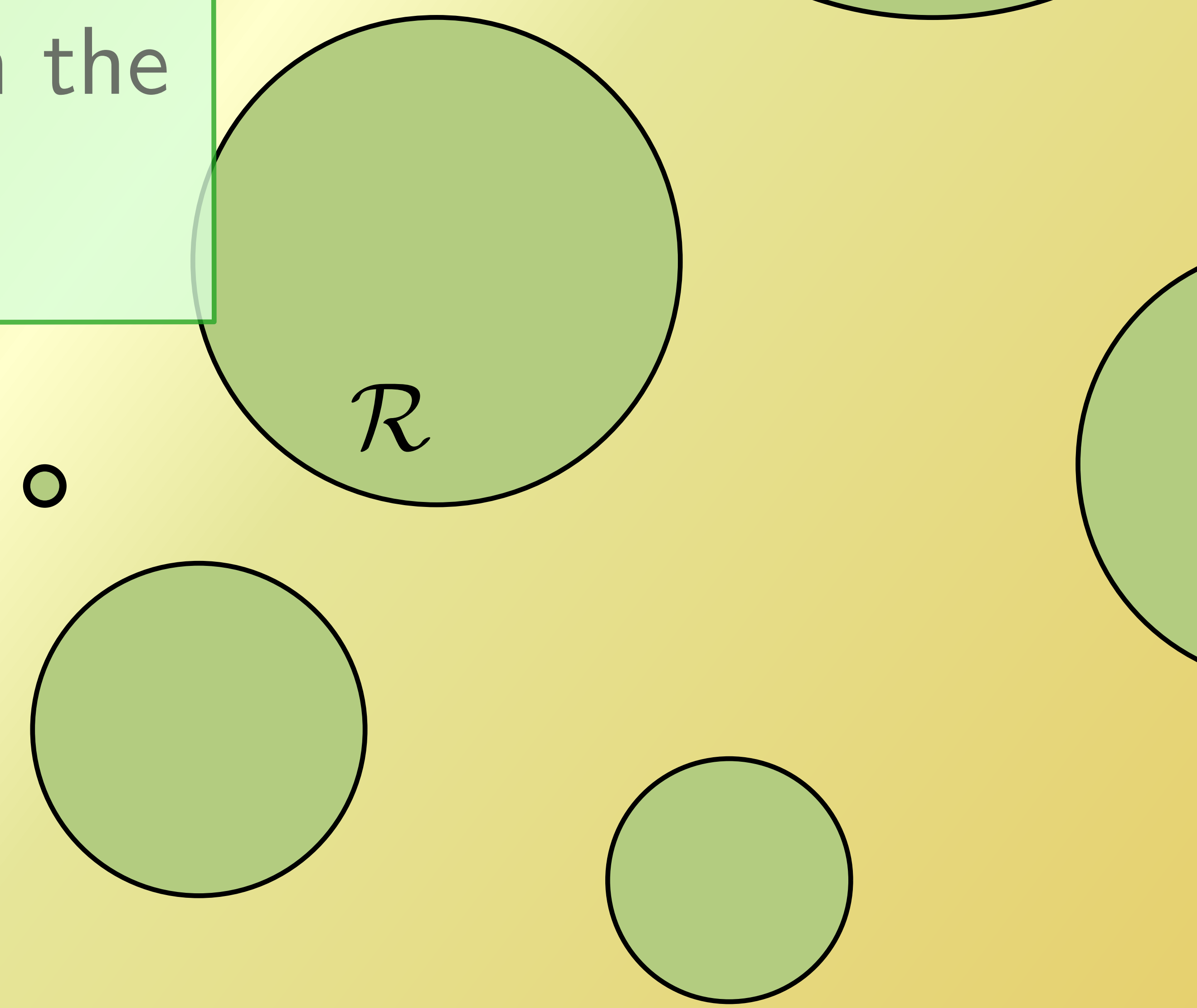

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We want to also handle *updates* efficiently.

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$O(\log^2 n)$ queries with $O(\log n)$ updates.

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$O(\log^2 n)$ queries with $O(\log n)$ updates.

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What is known about dynamic planar point location?

. . . or in *rectilinear* subdivisions.

[Blelloch, 2008]
[Giora & Kaplan, 2009]

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Is is possible to break the $\log n$ barrier in this case?

PROBLEM STATEMENT & RESULTS

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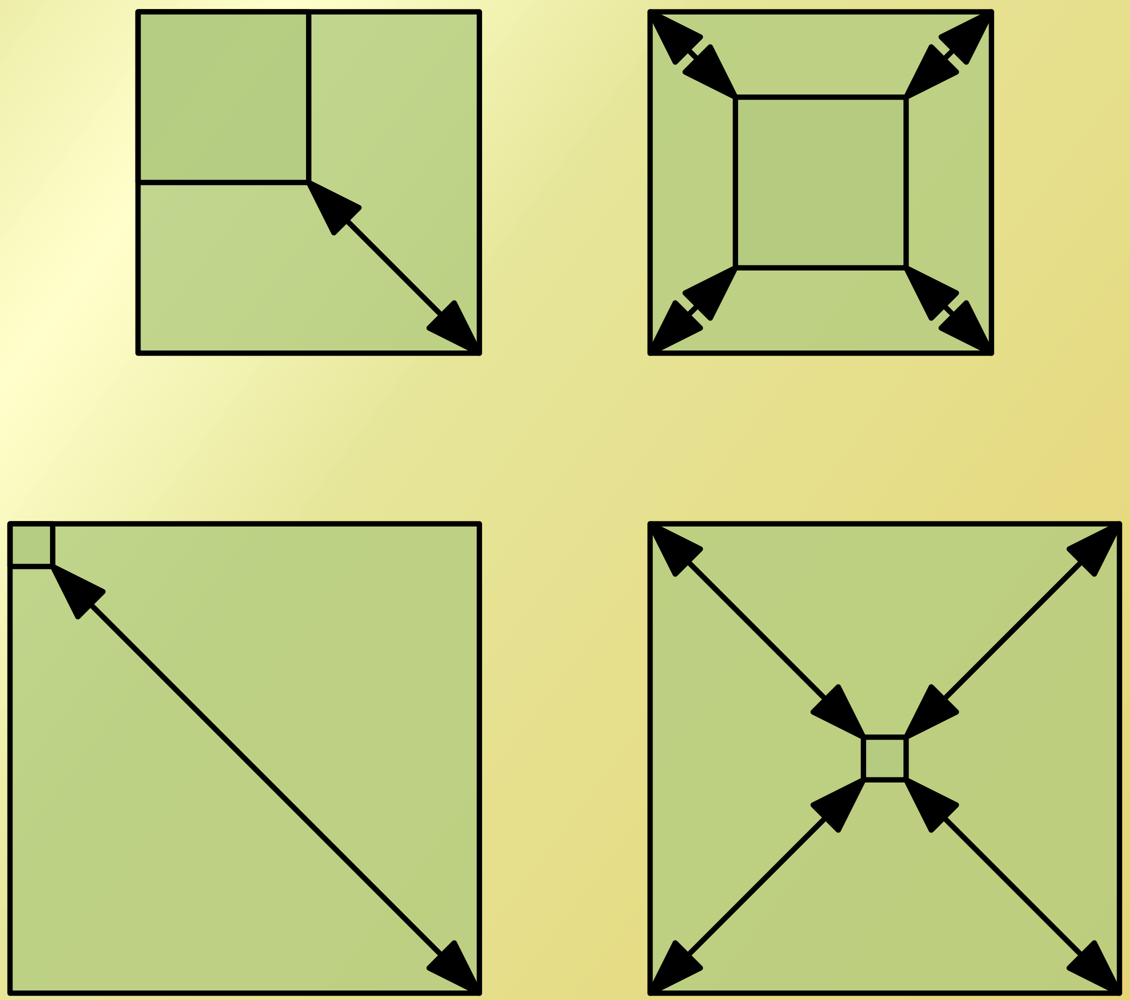

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When is an update *local*?

Regions can grow or shrink by at most a constant factor.

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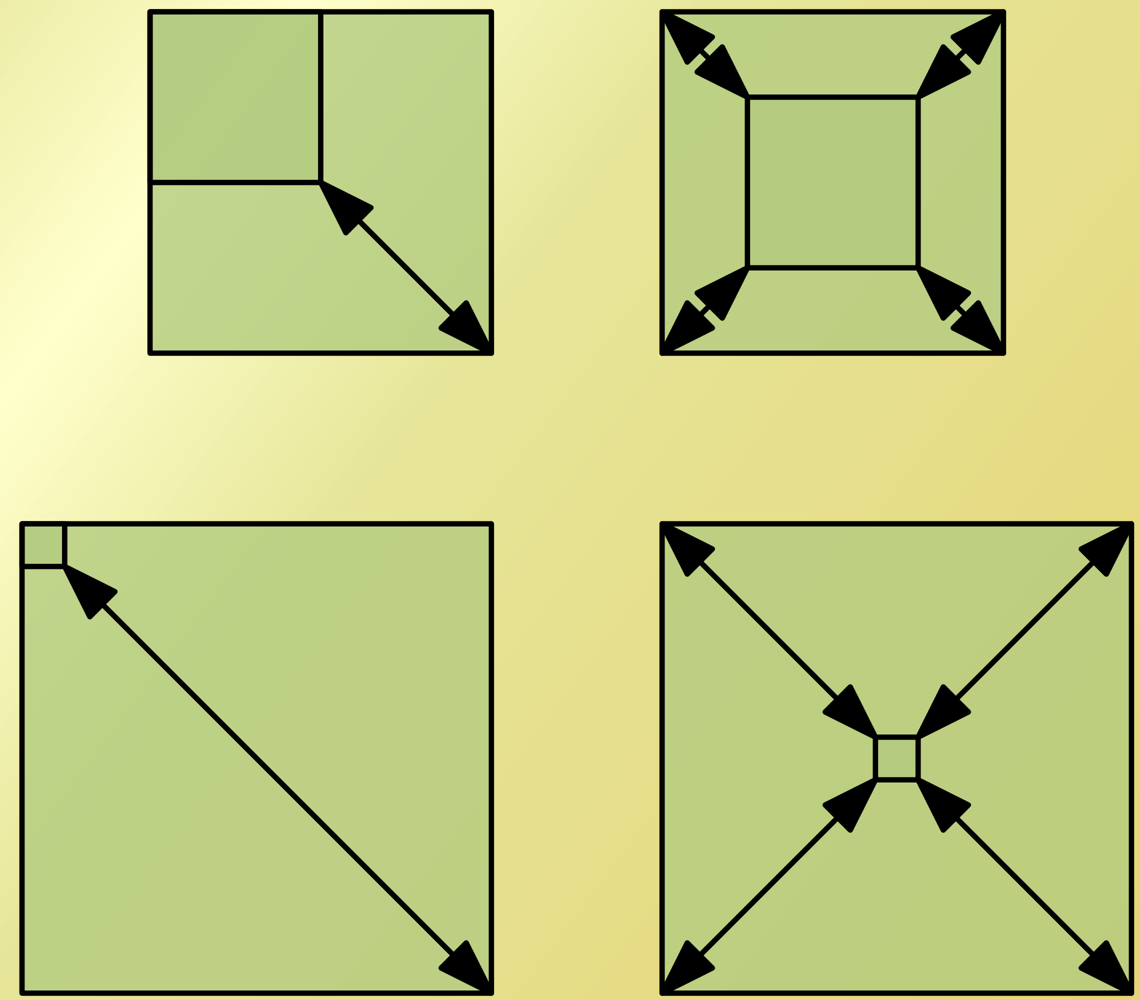

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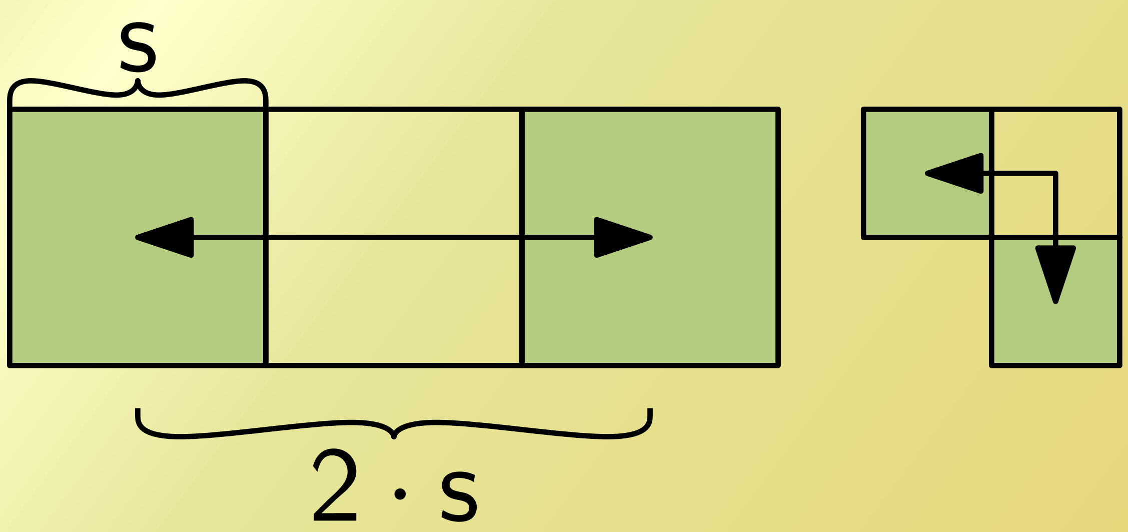

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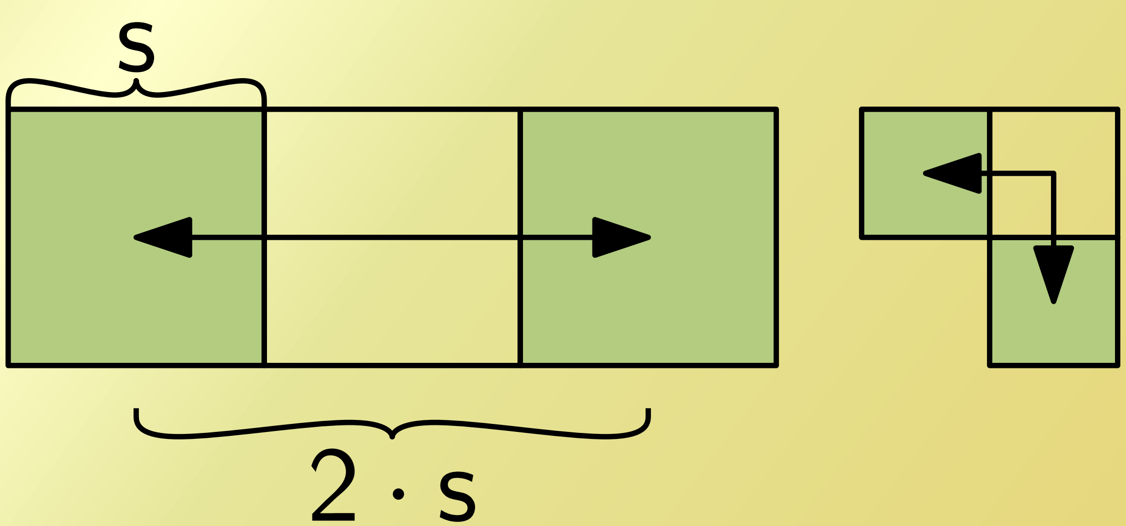

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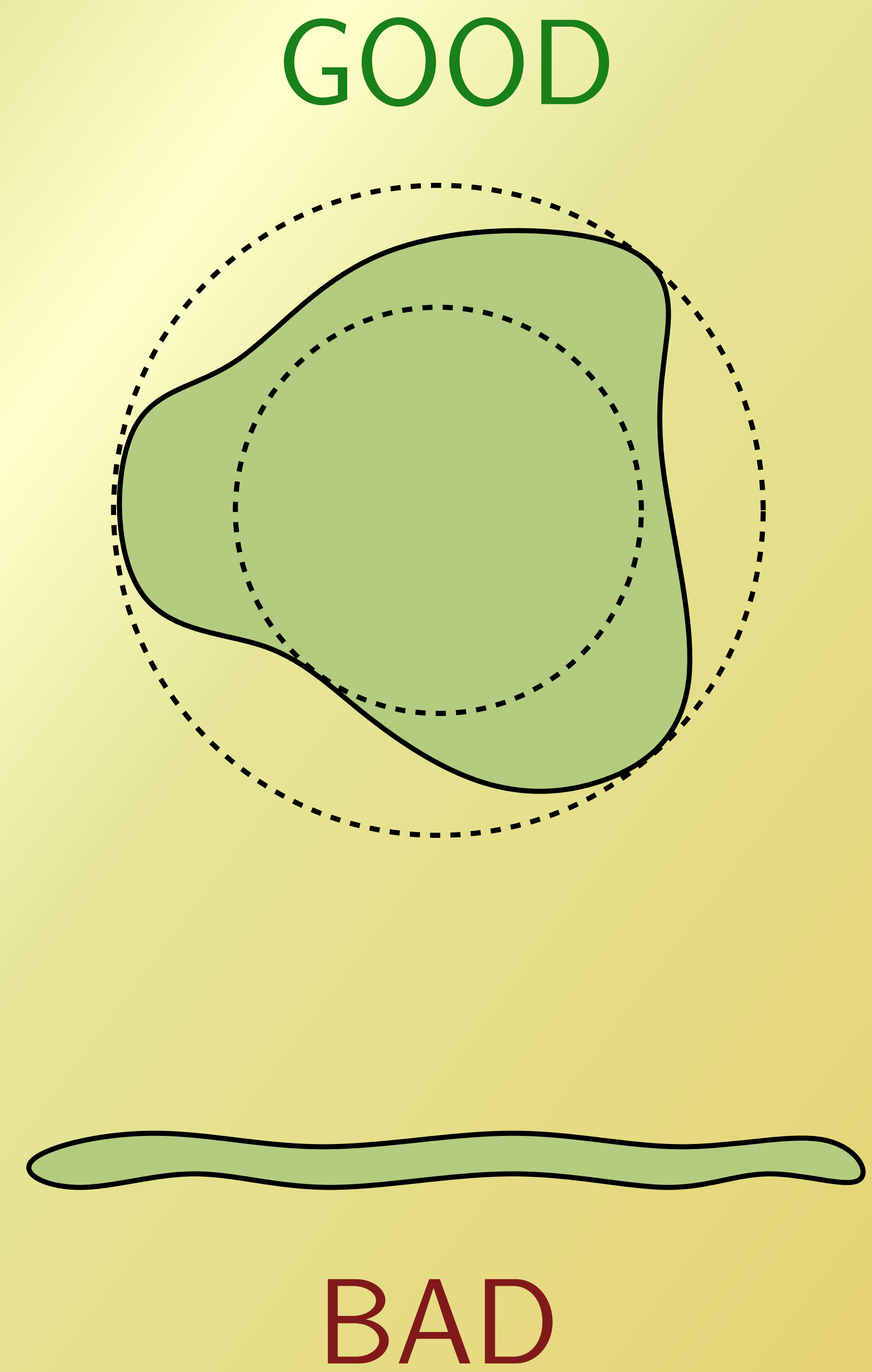

One more assumption: the regions are and stay disjoint!

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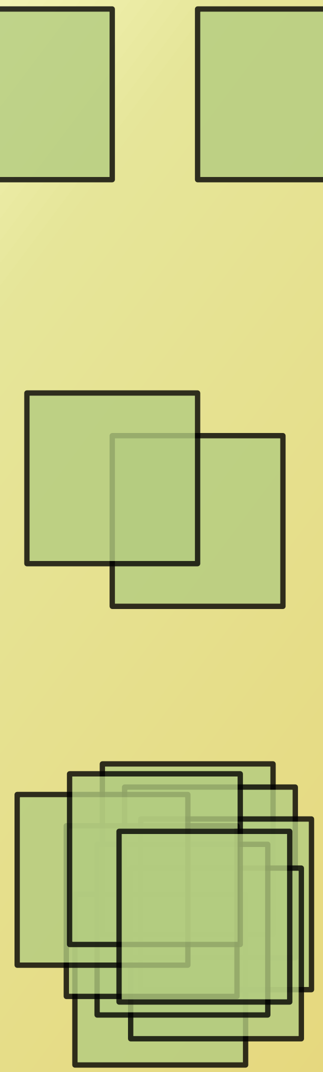
UGLY



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	Local updates:	$O(\log n / \log \log n)$ time

TECHNICAL DETAILS:
1 DIMENSION

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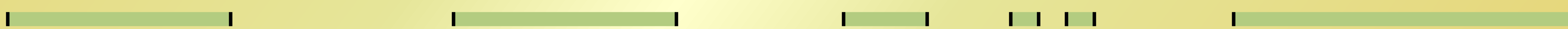

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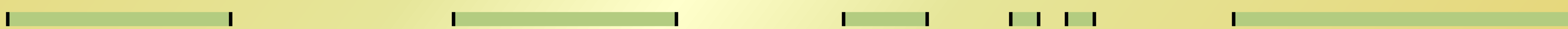
They move around on a line: big intervals are fast, small ones are slow.



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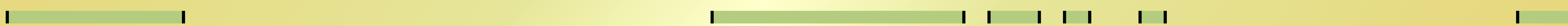

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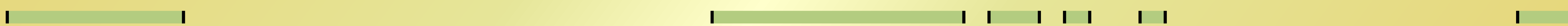

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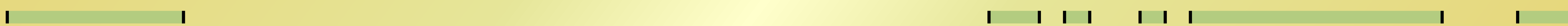

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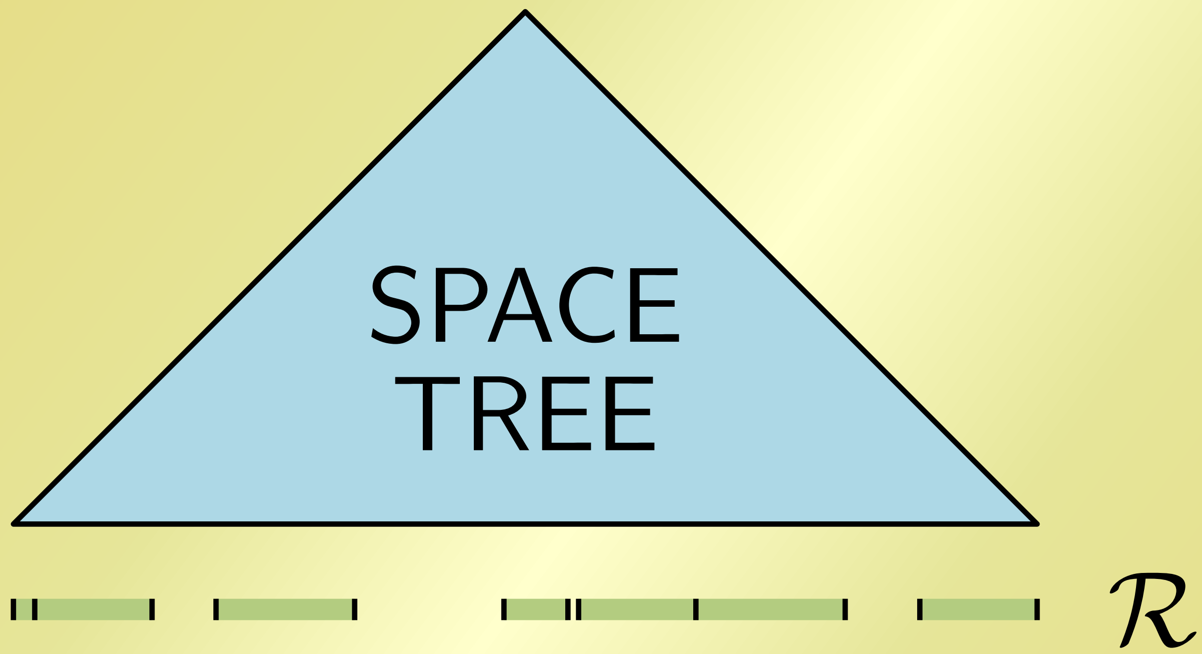

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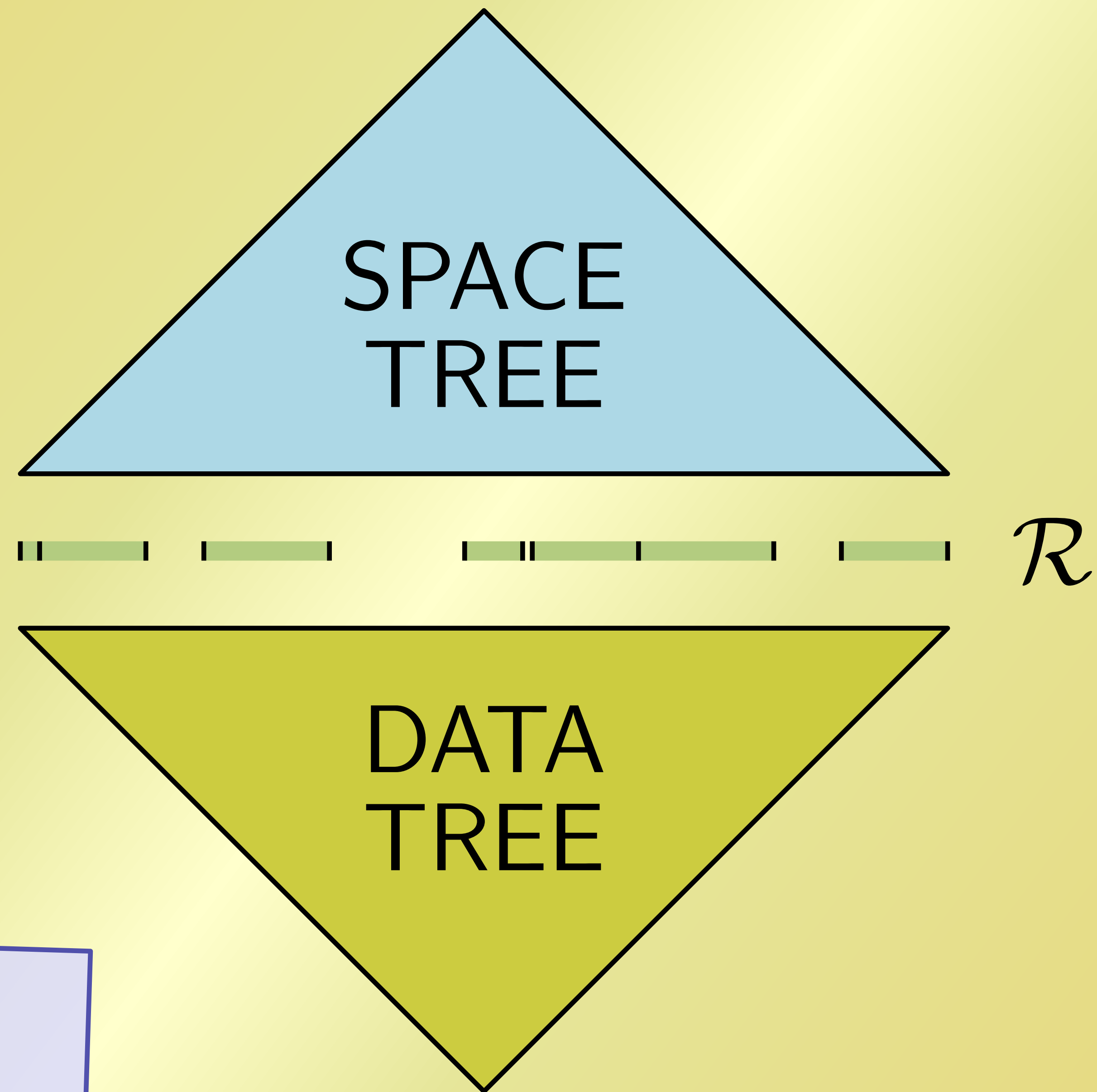

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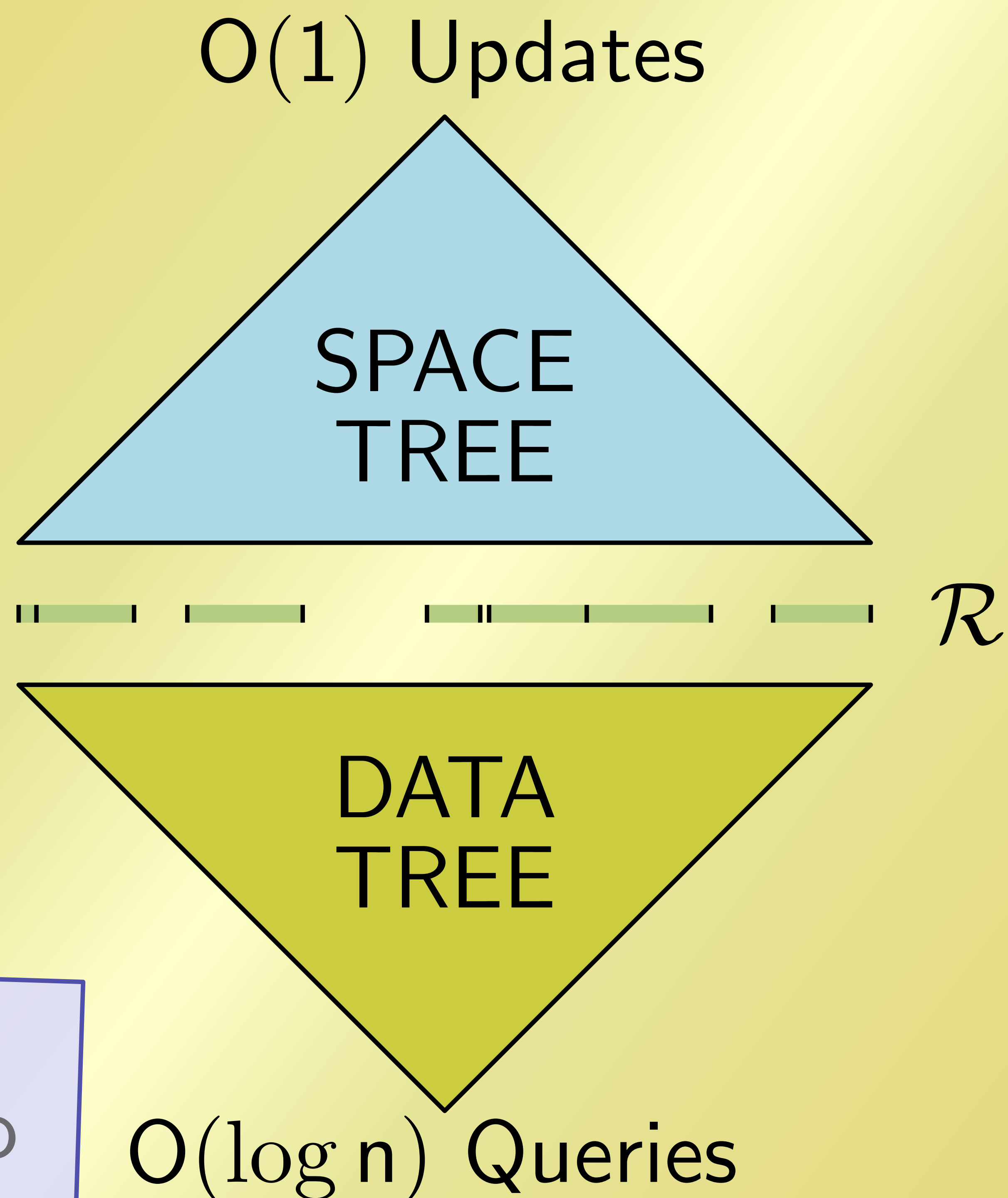
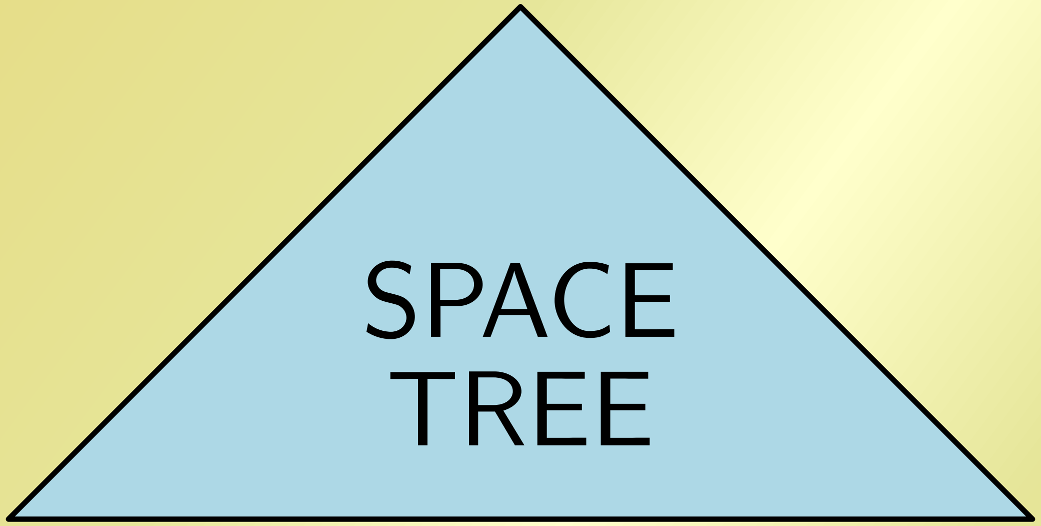


SPACE TREE

For the space tree we use a *quadtree*.



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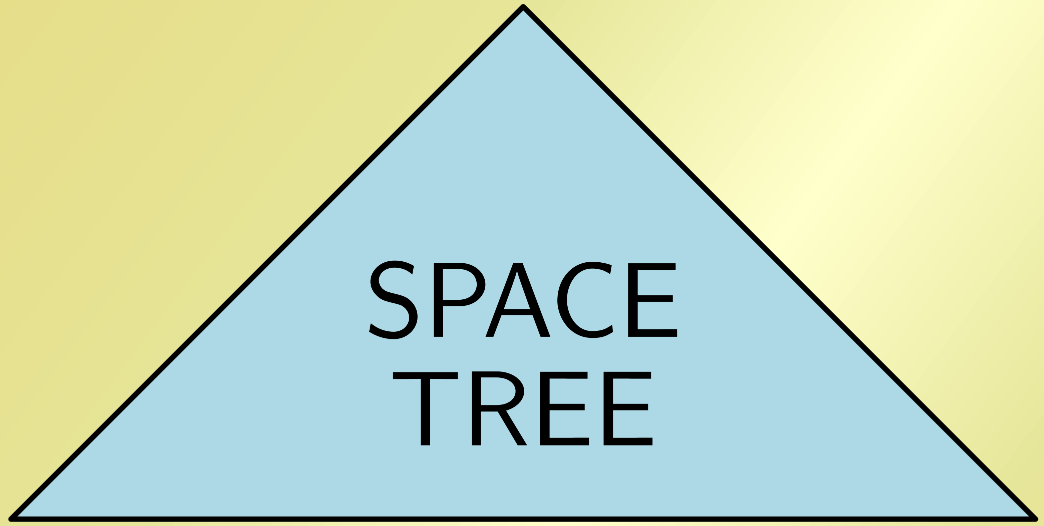

Consider the set P of midpoints of the intervals.

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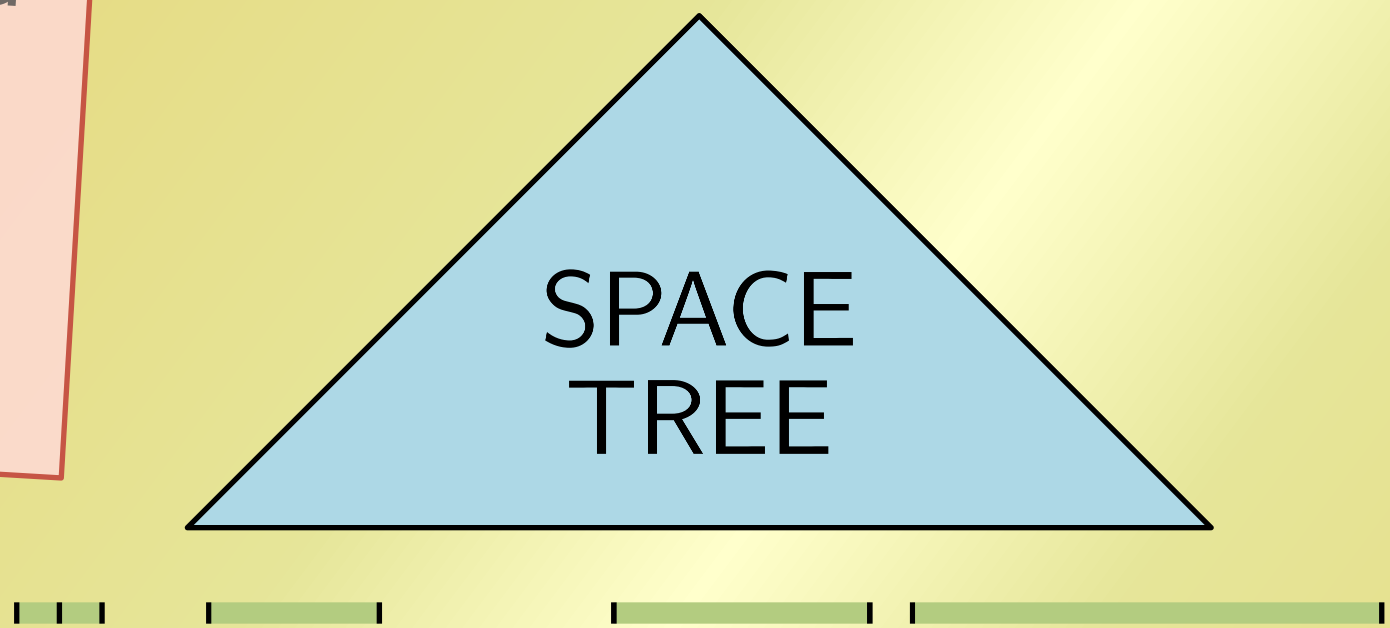

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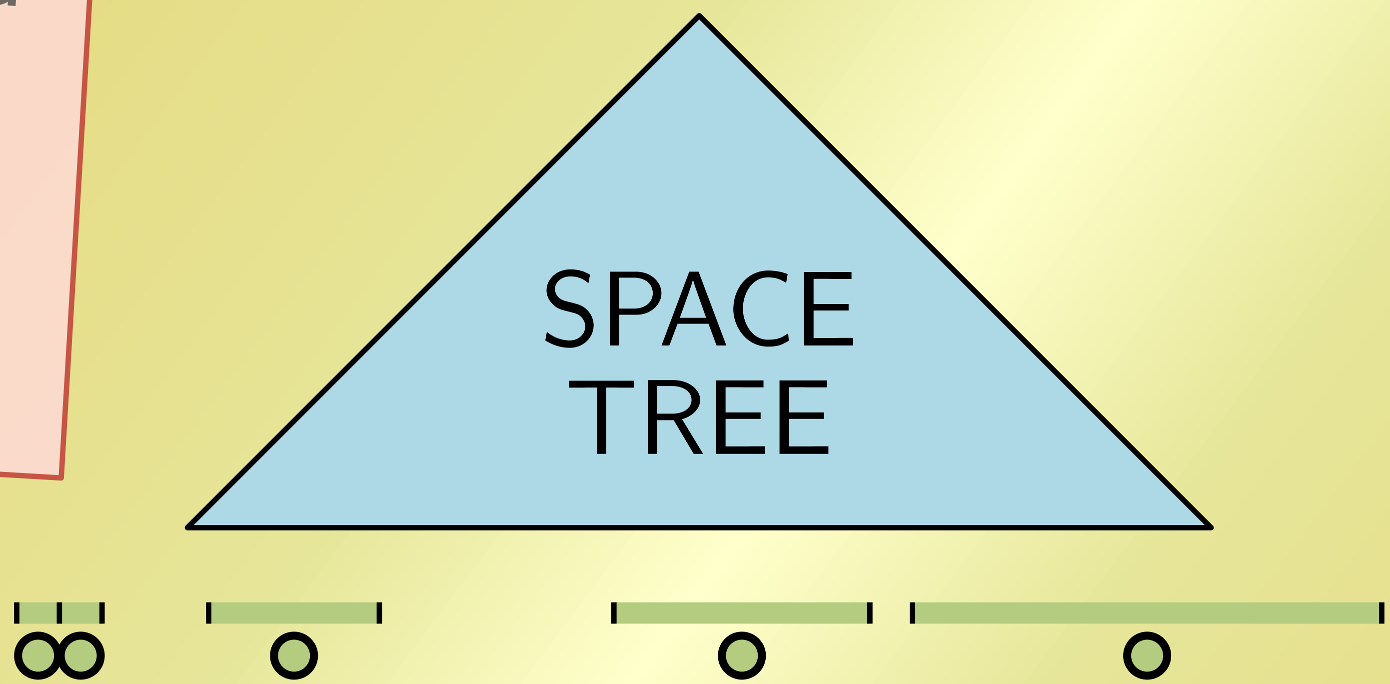

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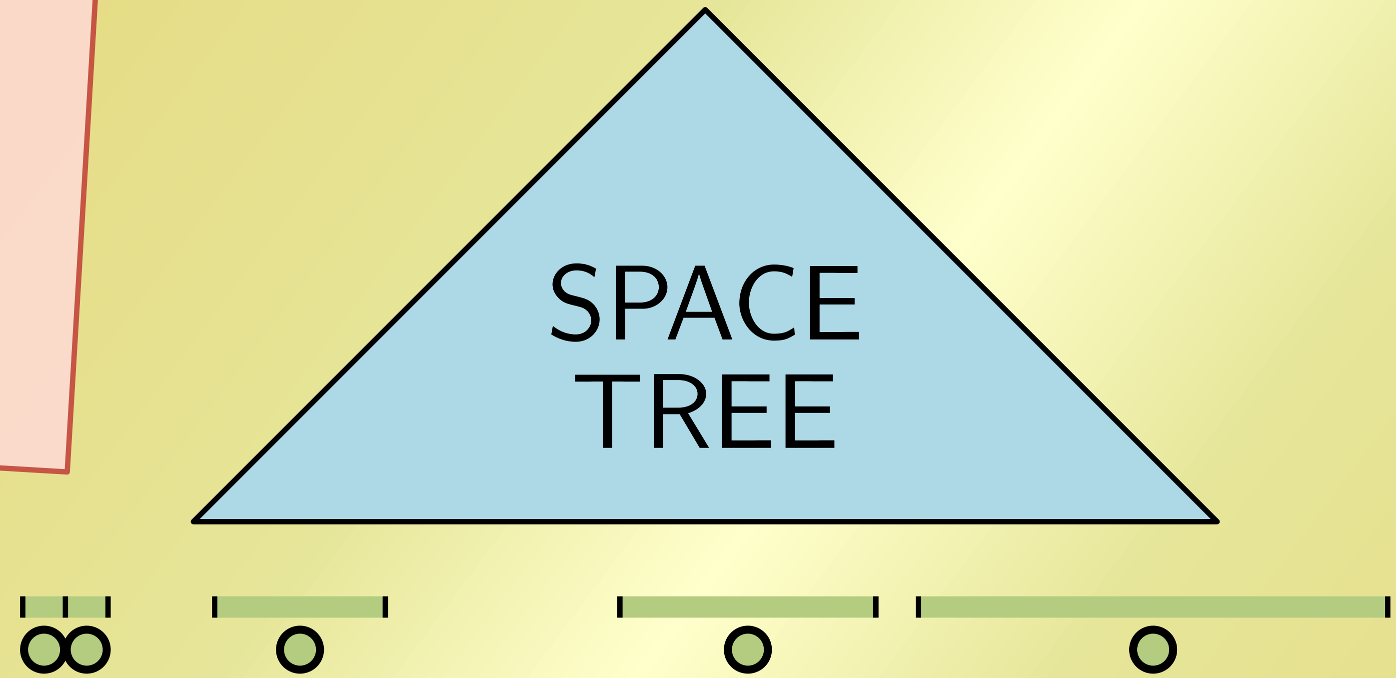

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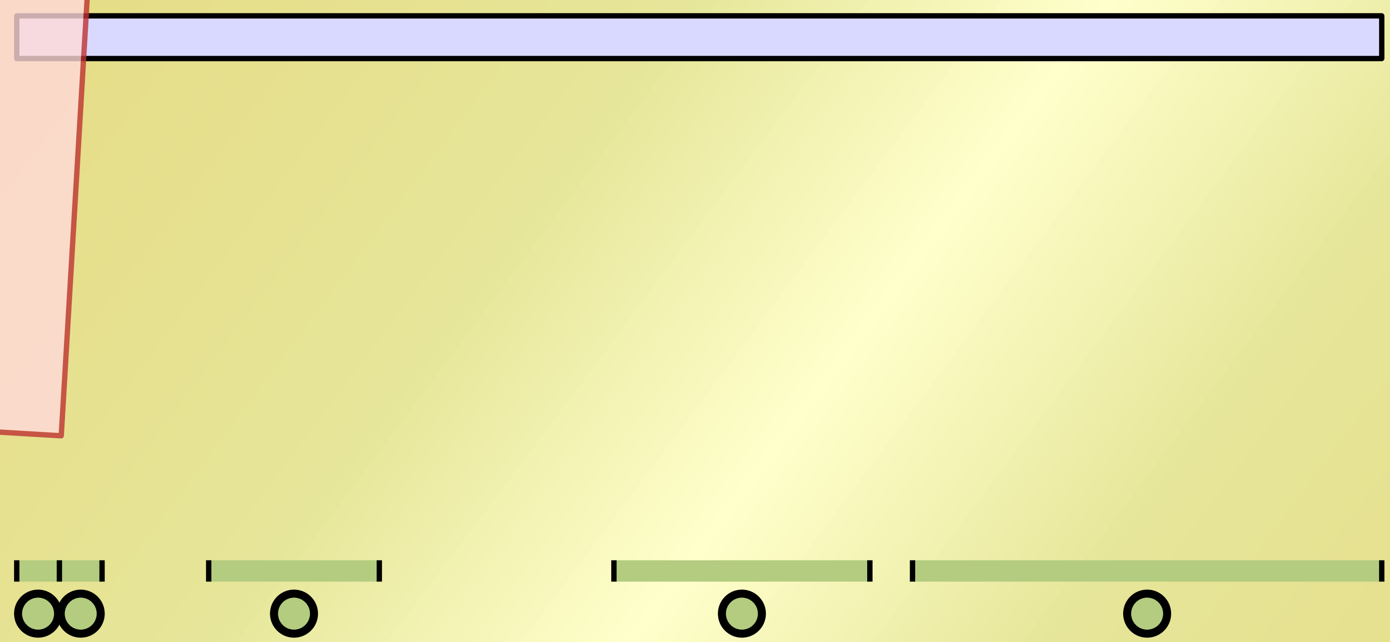

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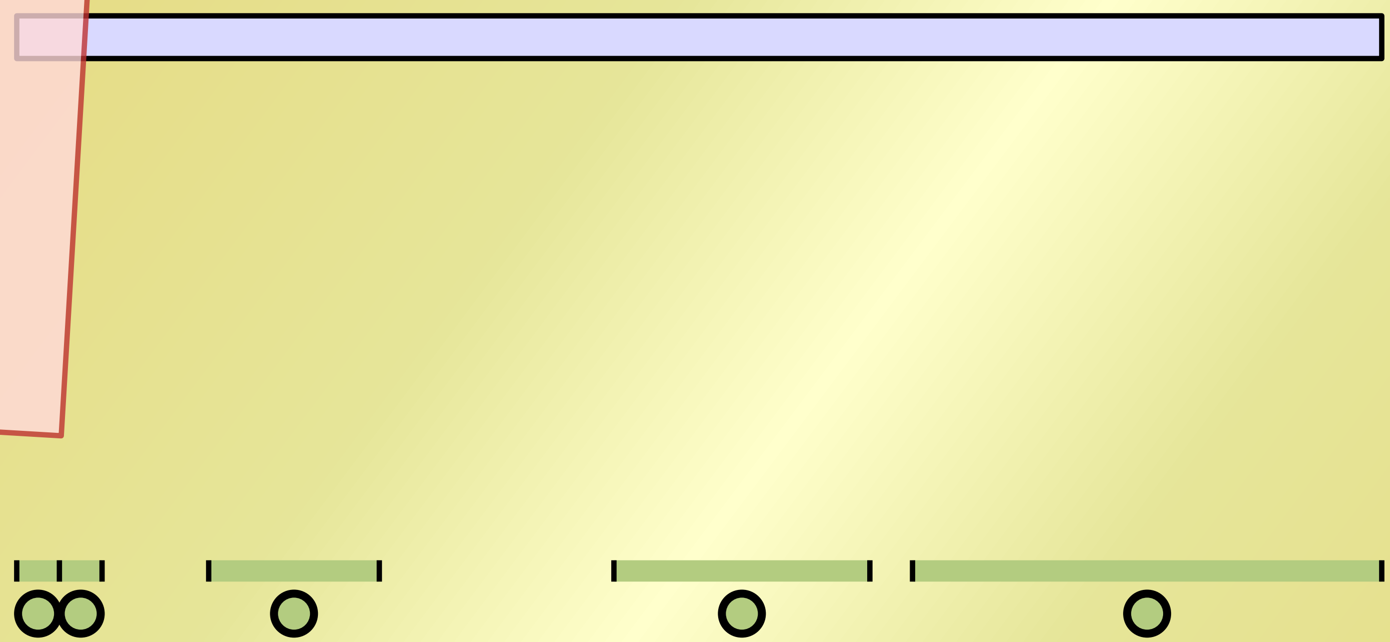

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Recursively split boxes that contain at least 2 points.

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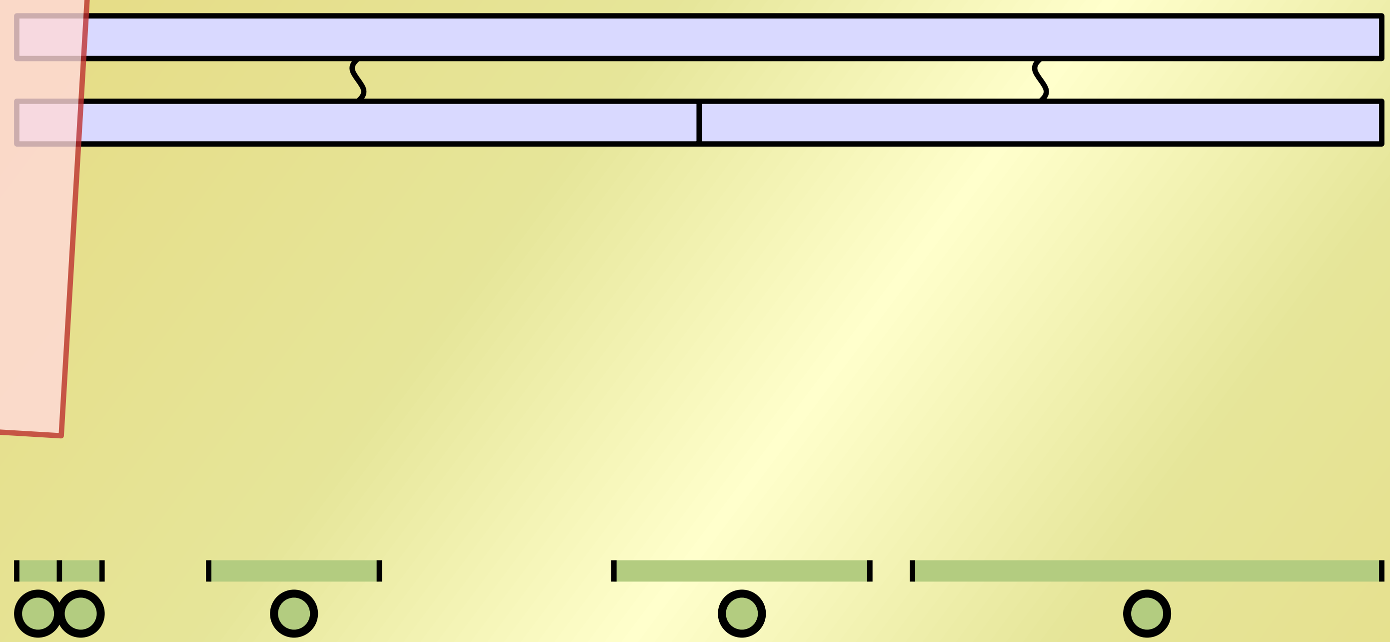

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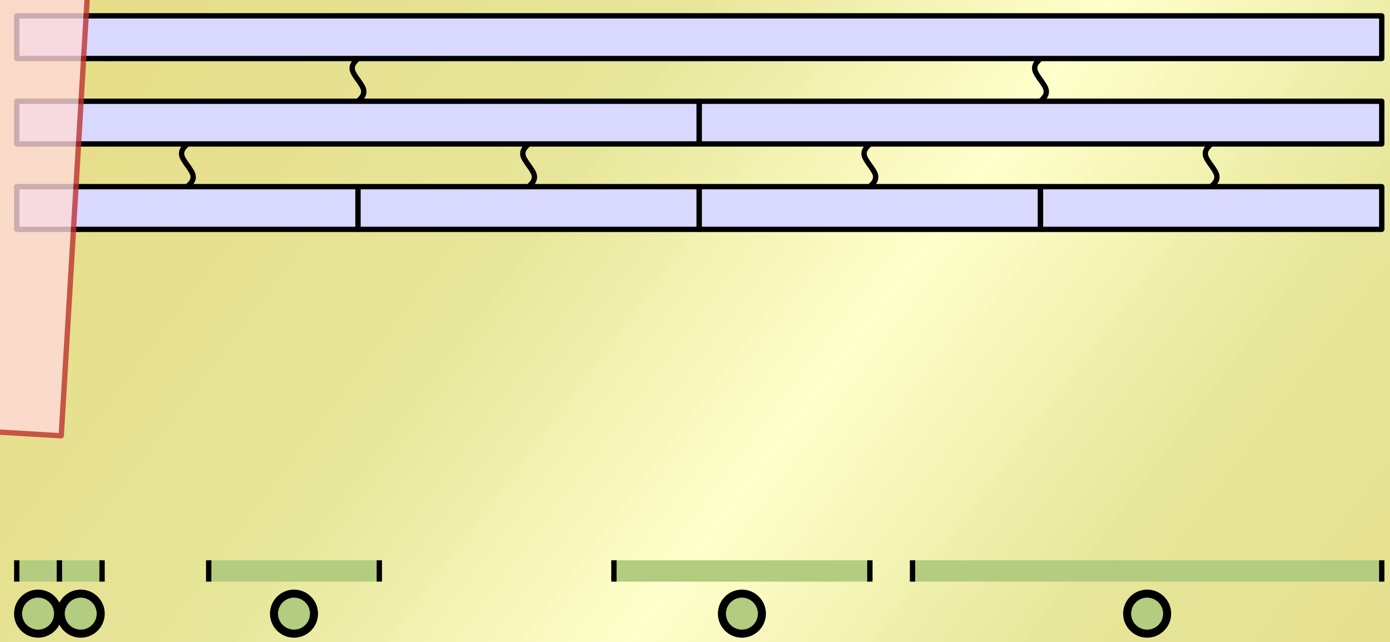

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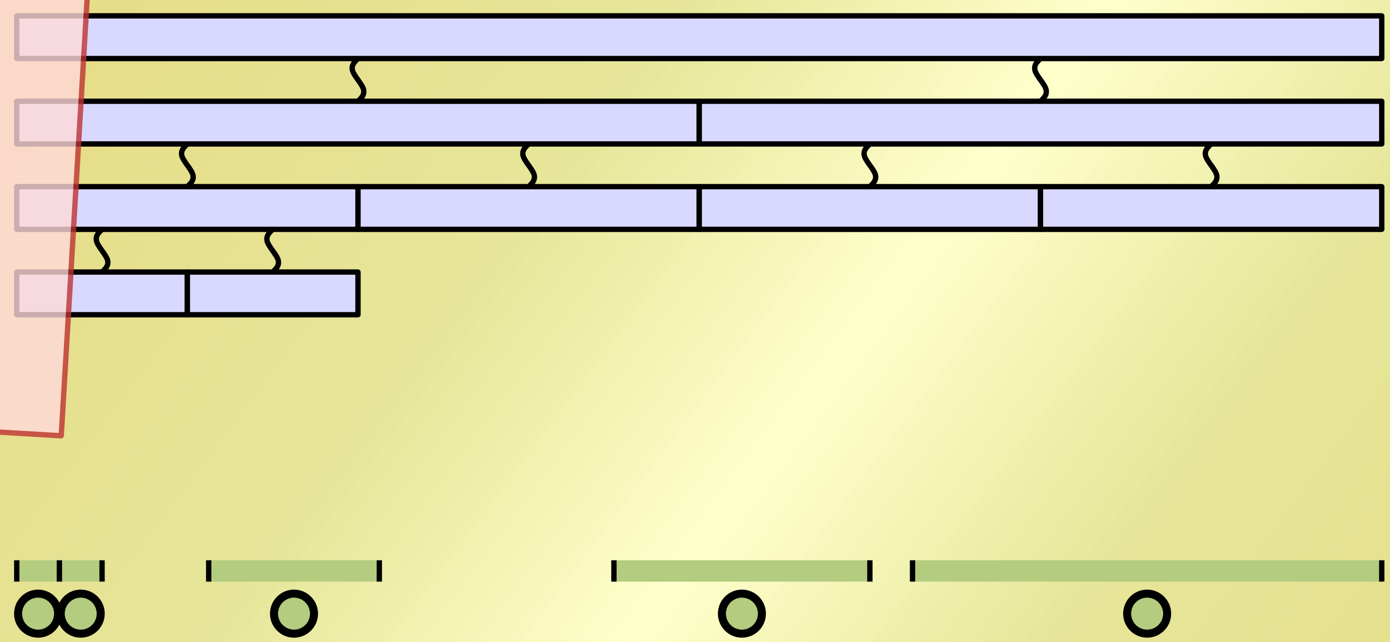

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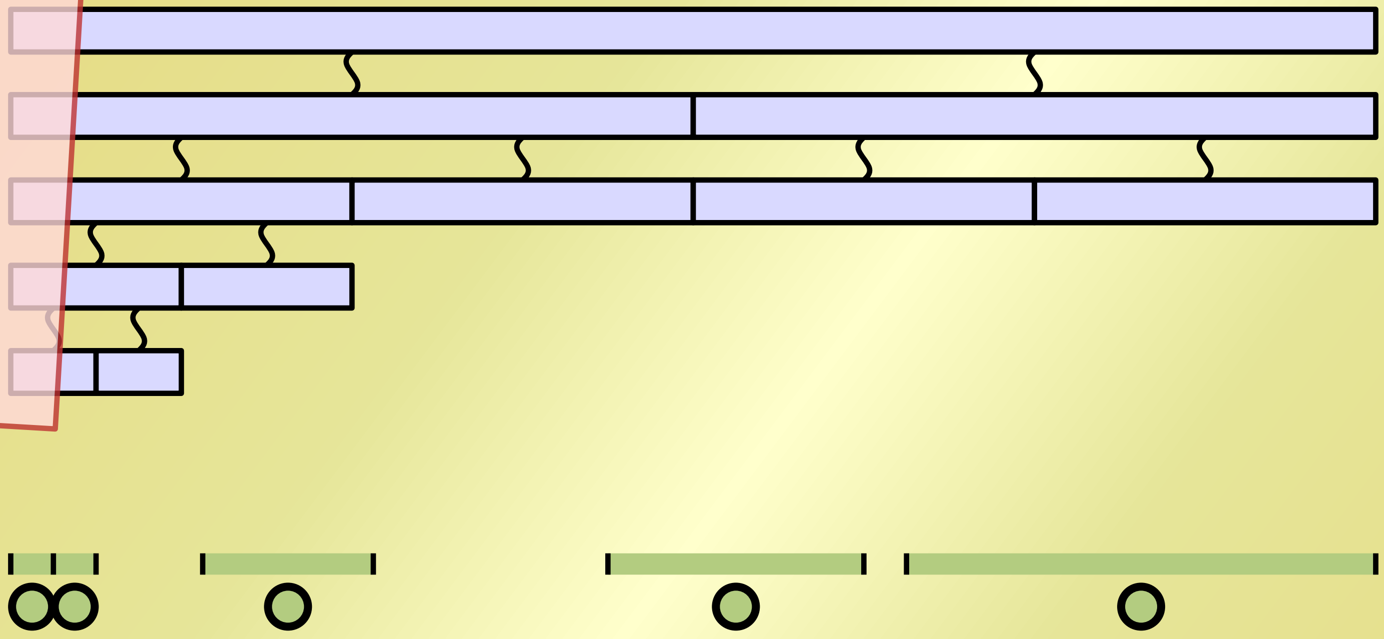

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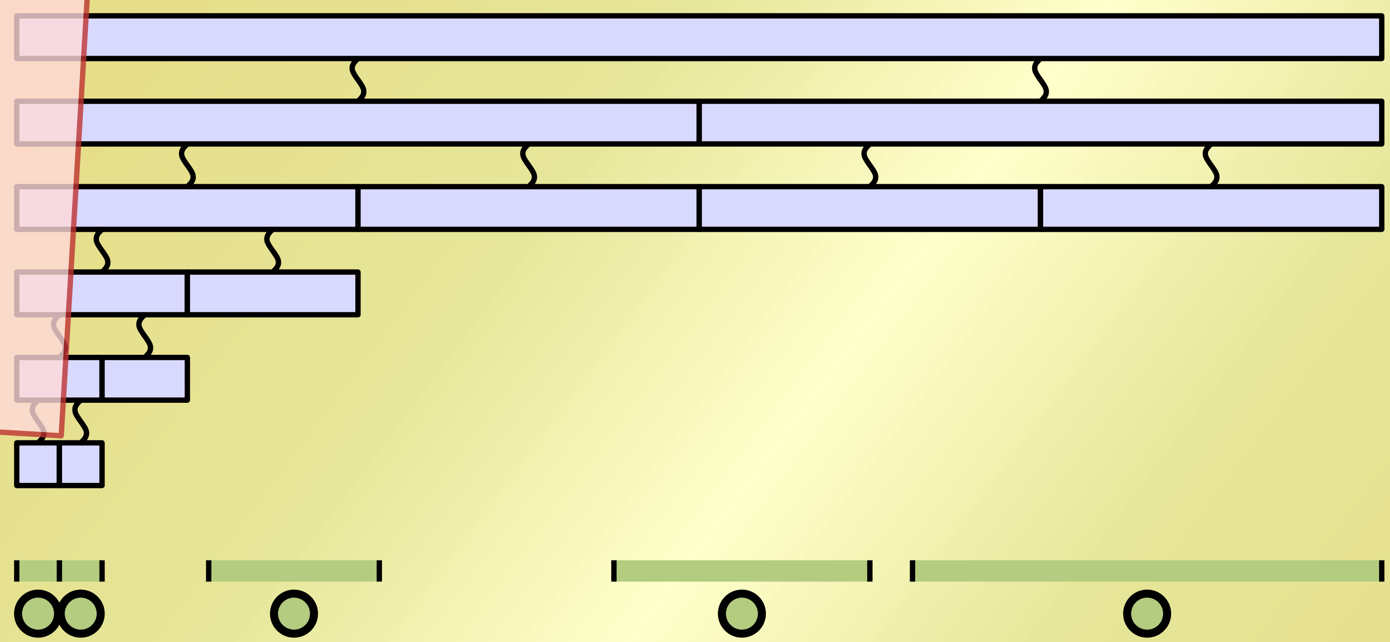

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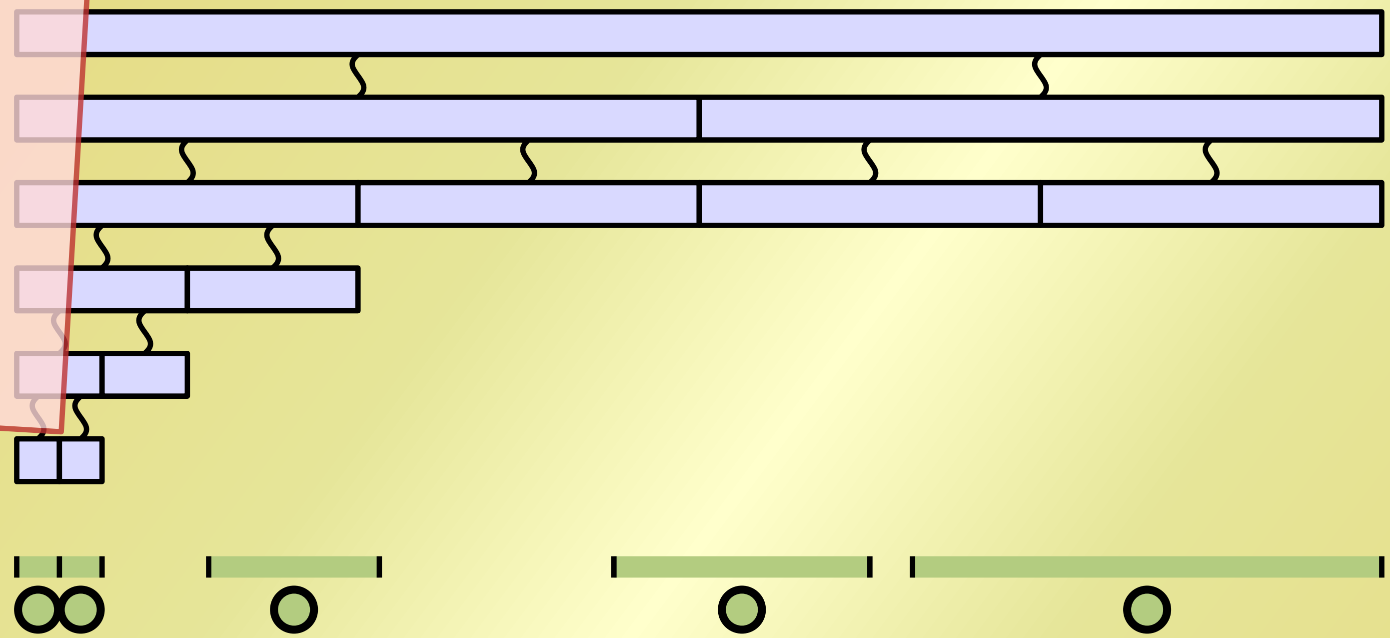

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Compress the tree by deleting long empty paths.

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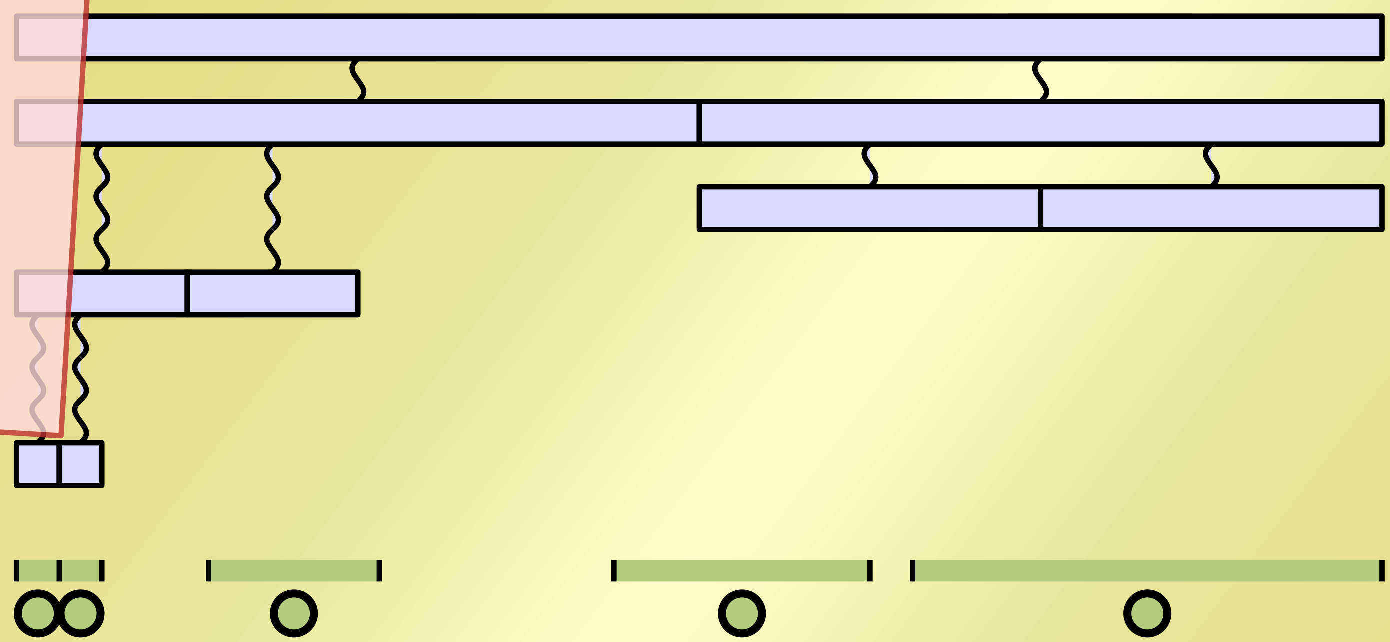

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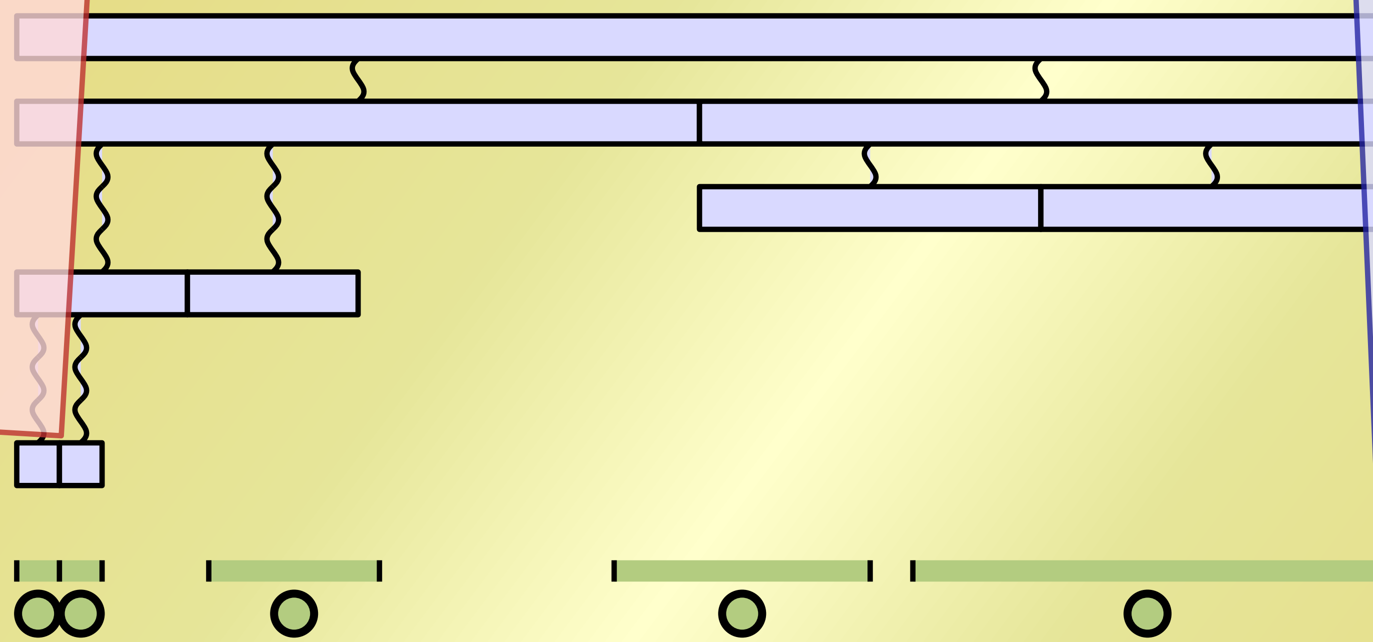

Finally, add pointers between neighbouring boxes of the same size.

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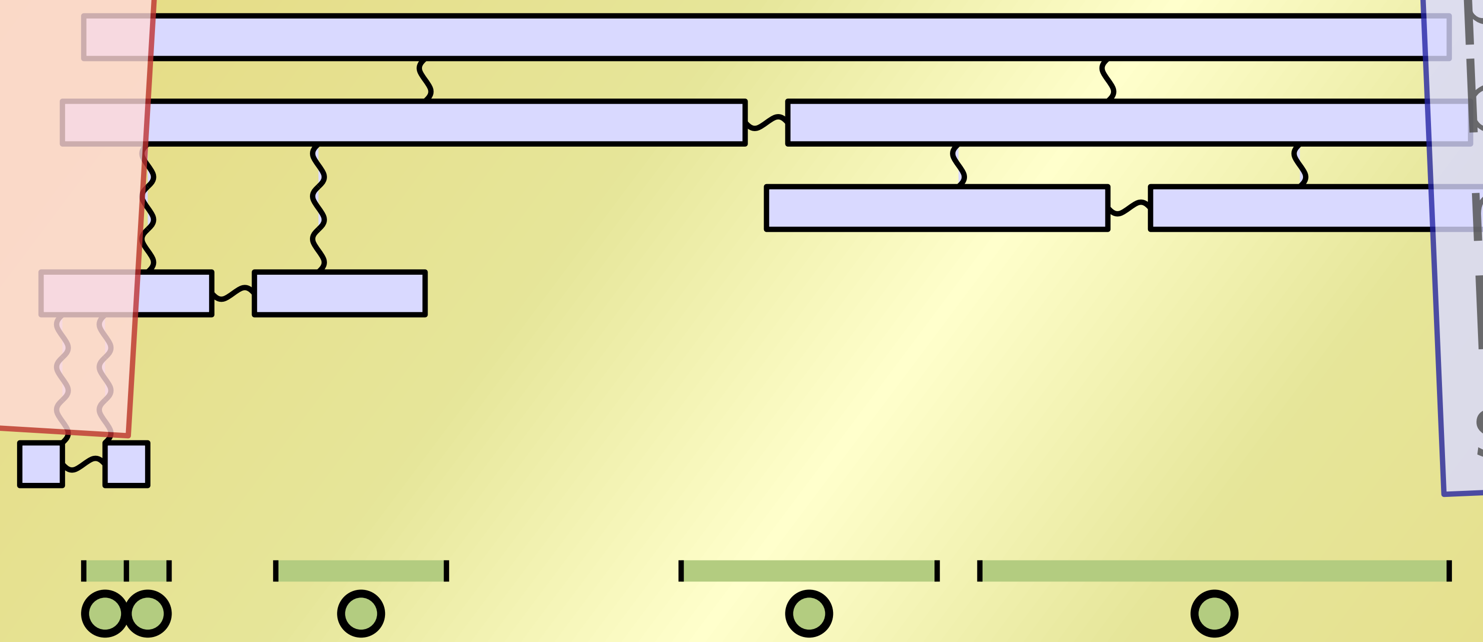

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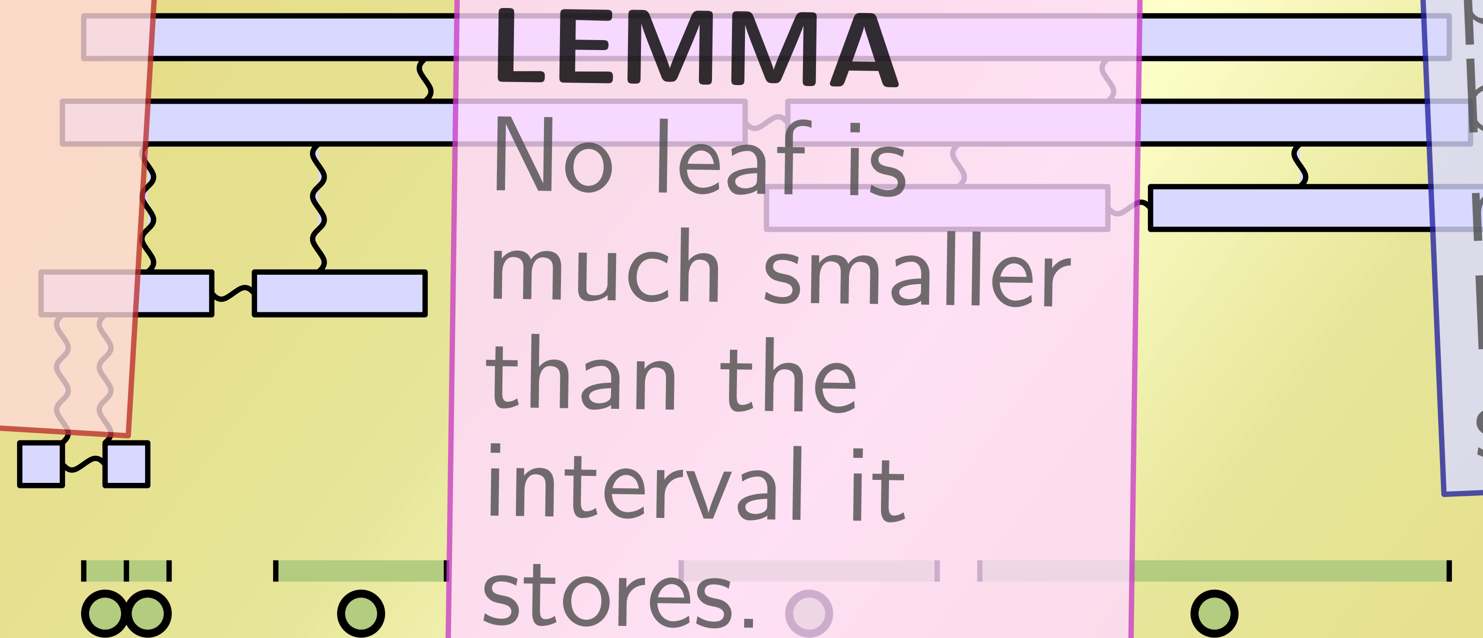

LEMMA
No leaf is much smaller than the interval it stores.

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DATA TREE

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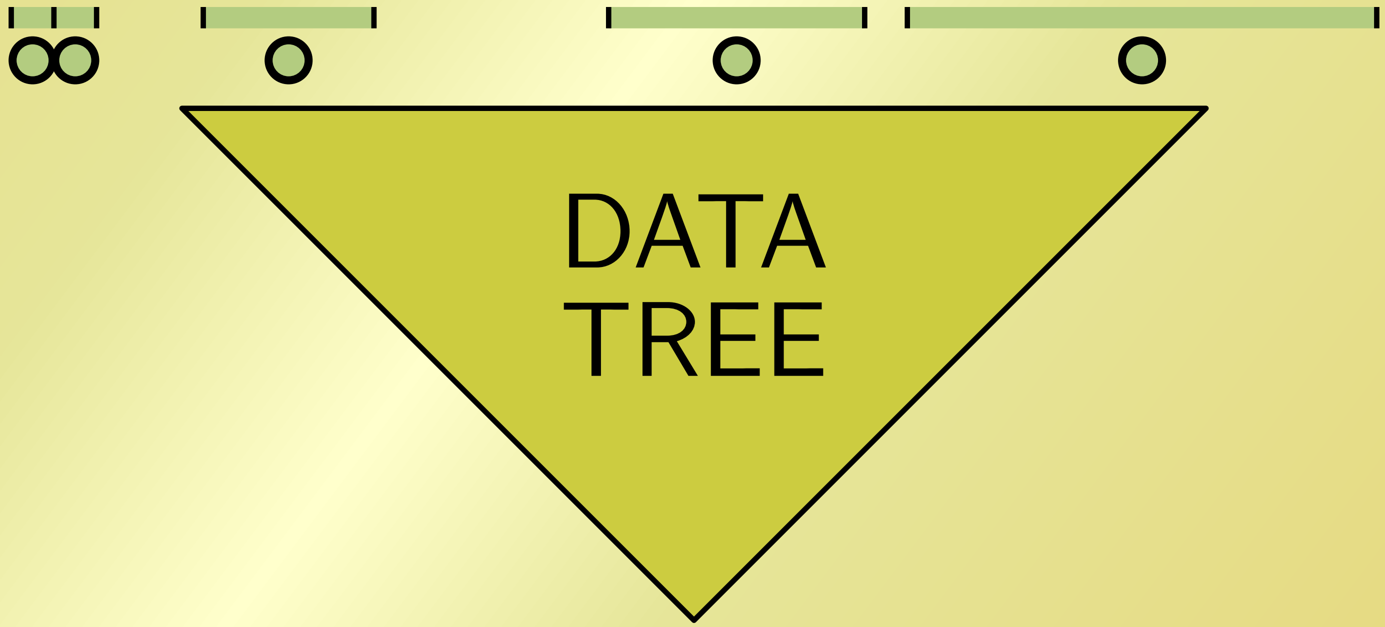

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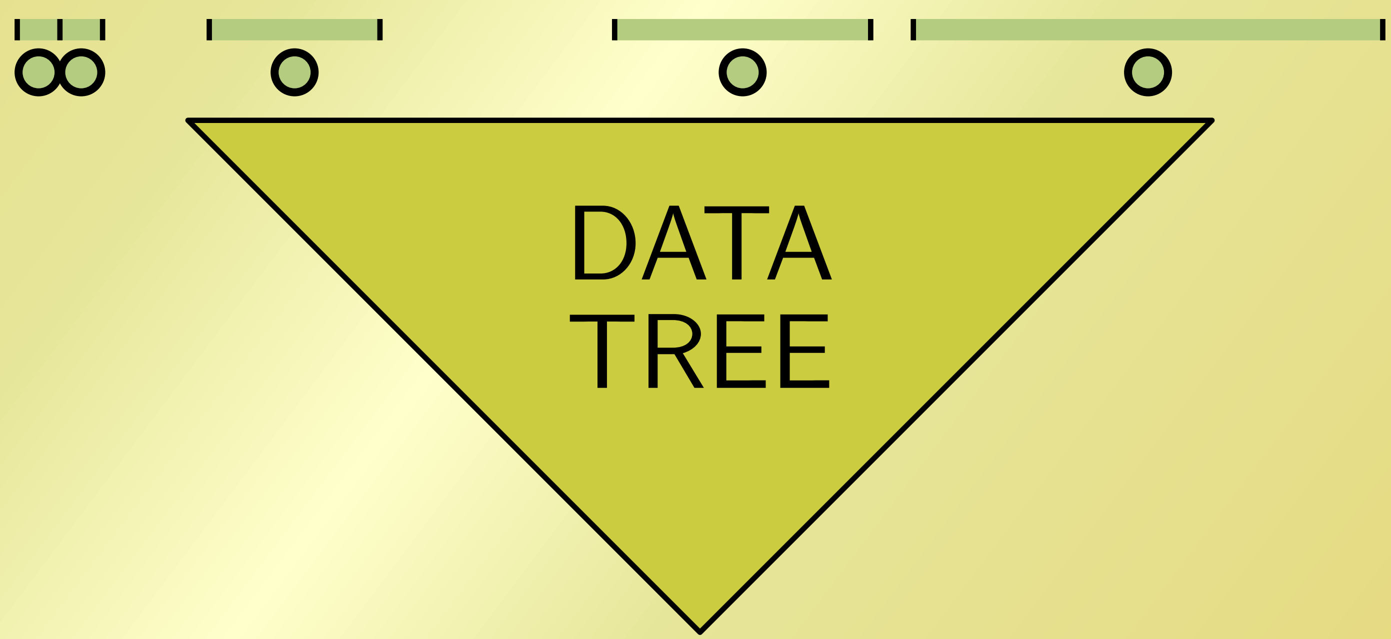
Again, consider the midpoints of the intervals.

We now only care about their order, and build a *balanced* tree.



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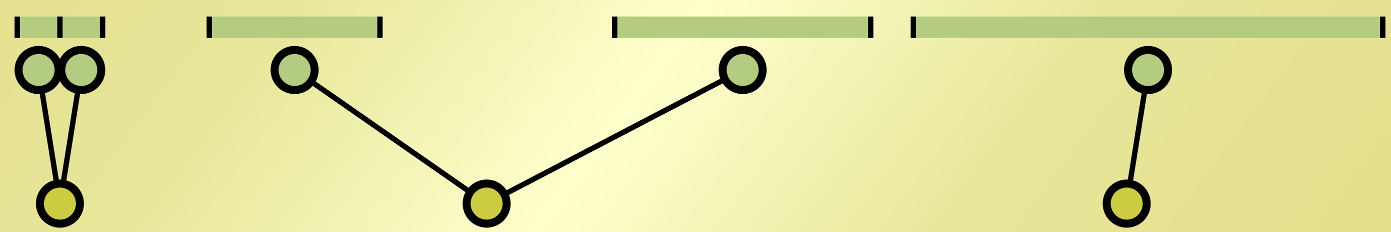

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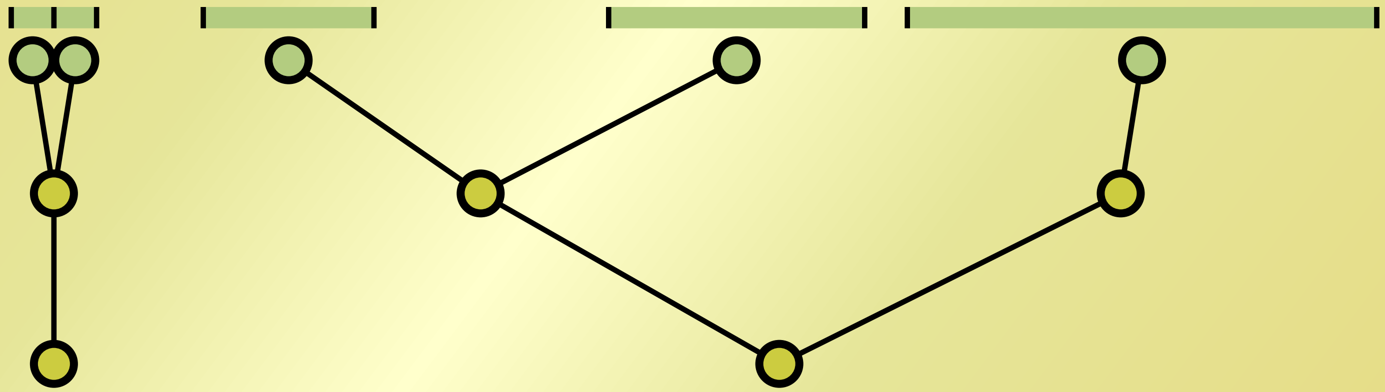

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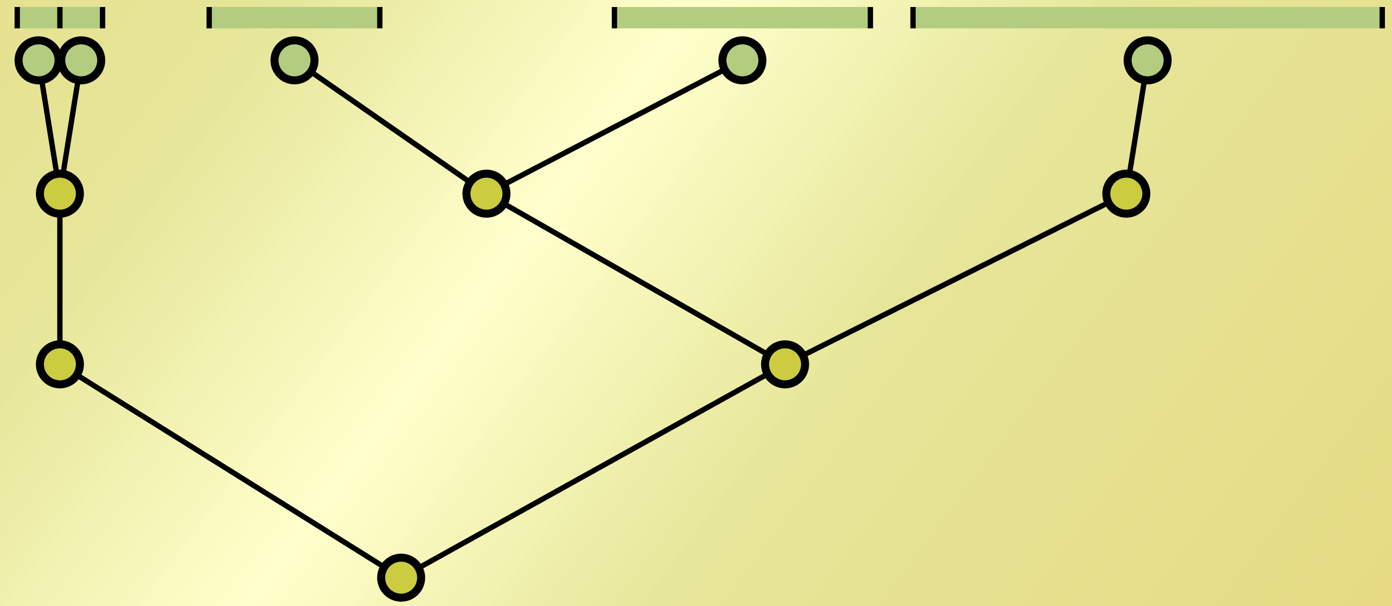

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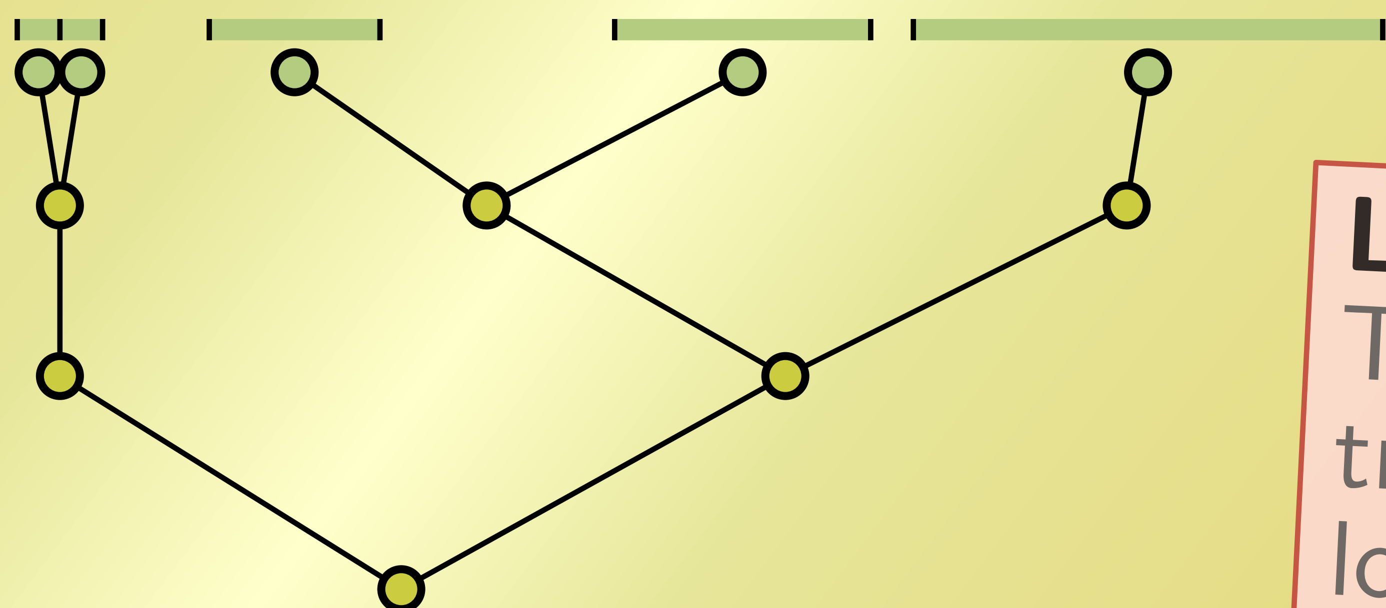
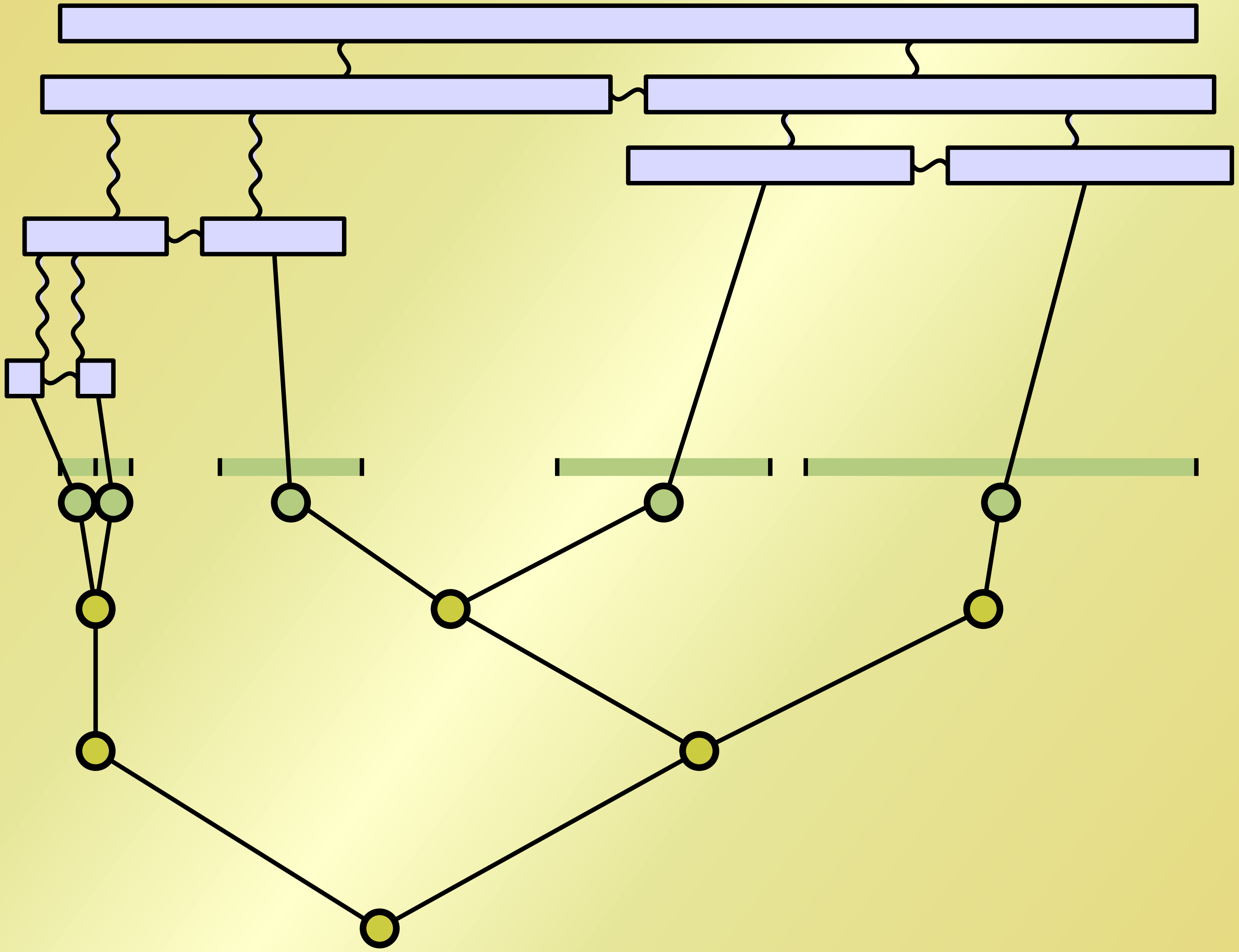
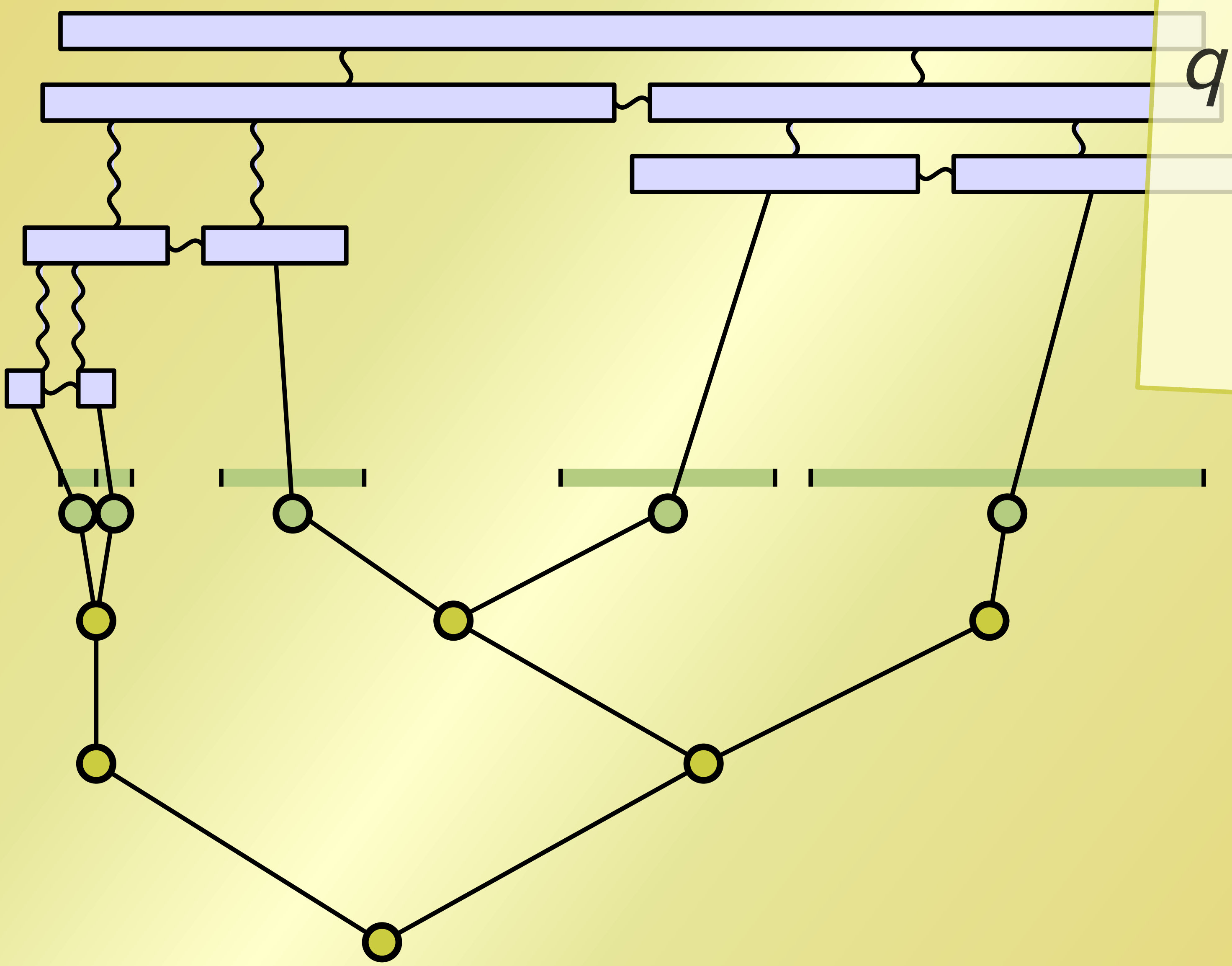
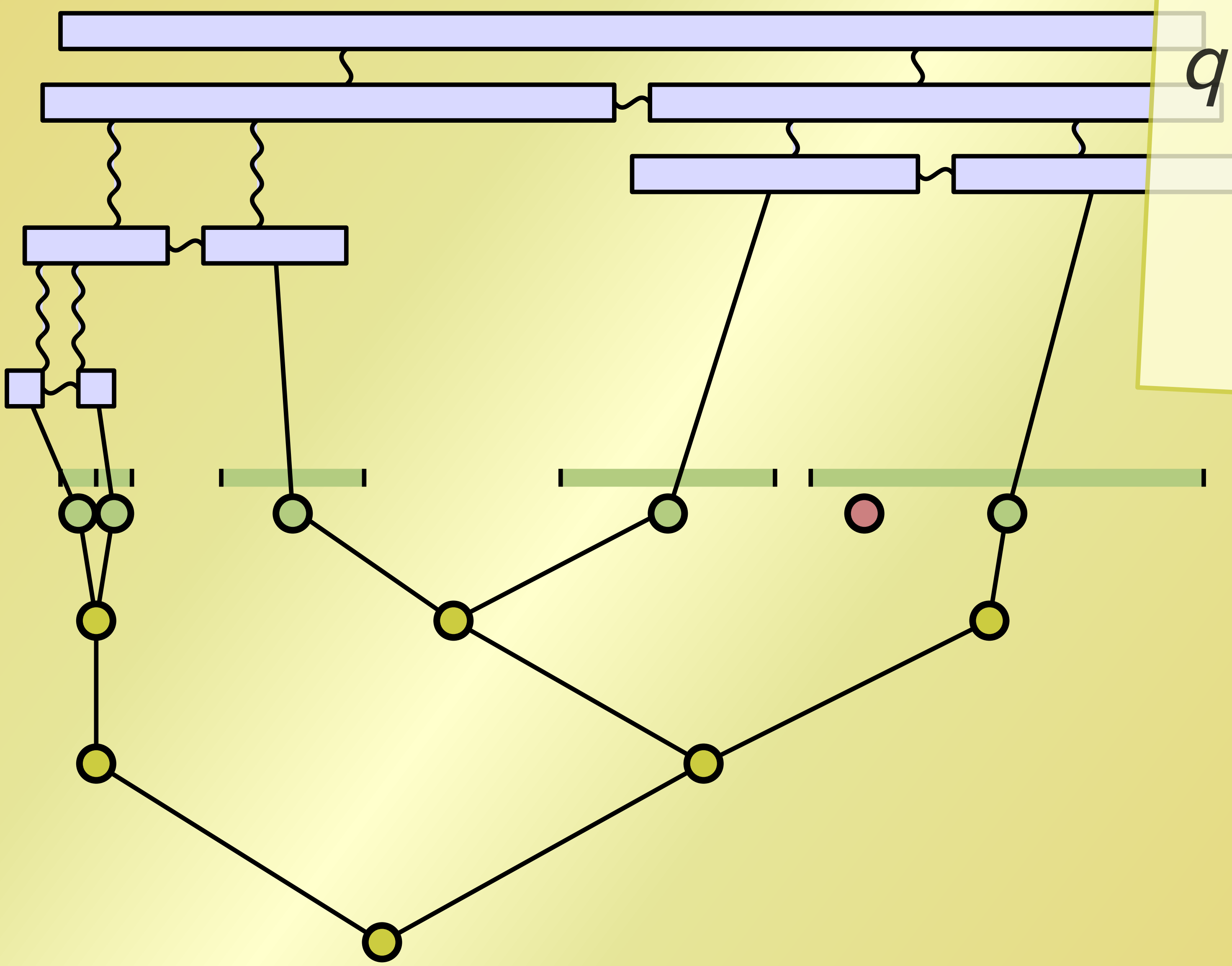


LEMMA
The search tree has logarithmic depth.





How do we handle a *query*?



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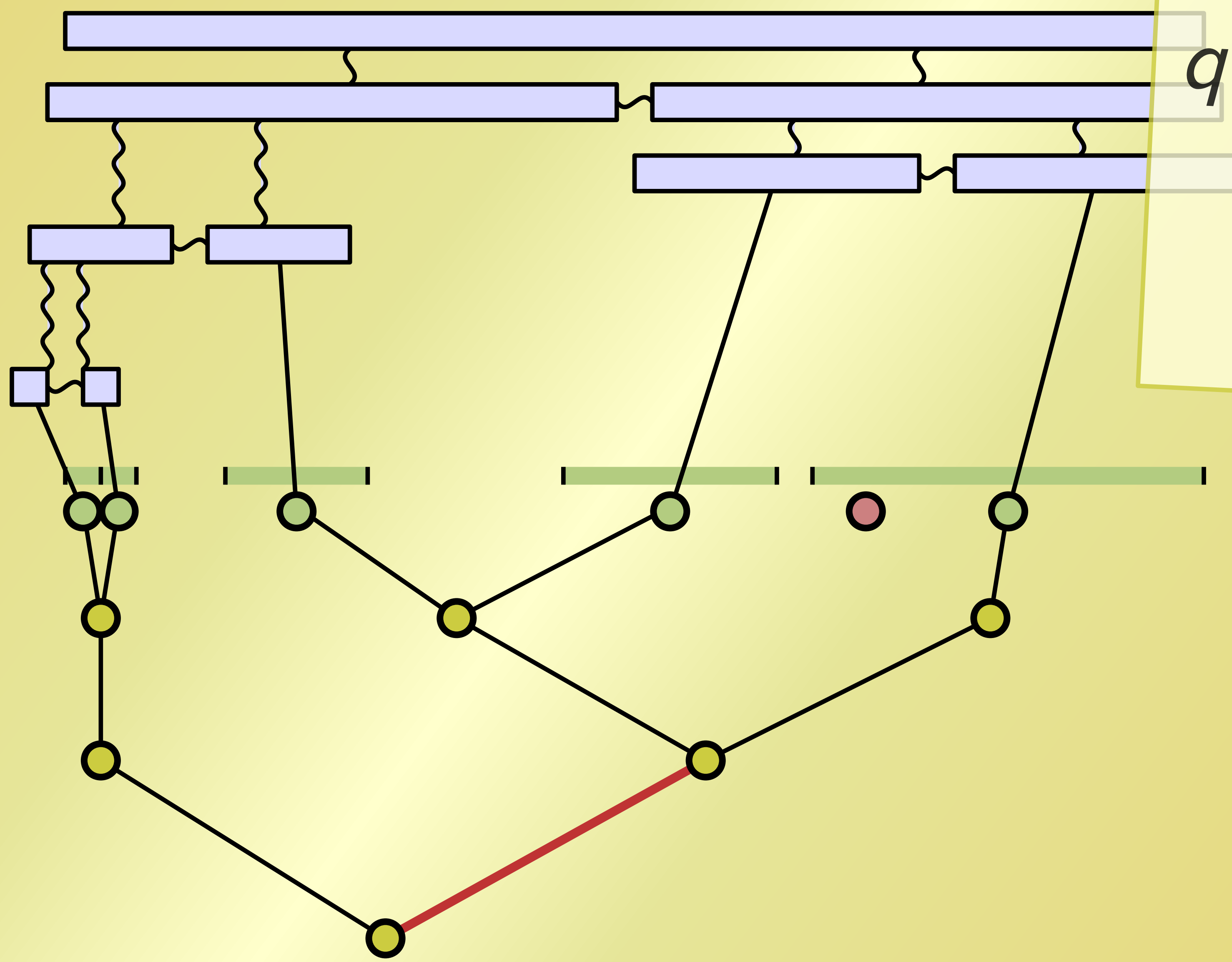

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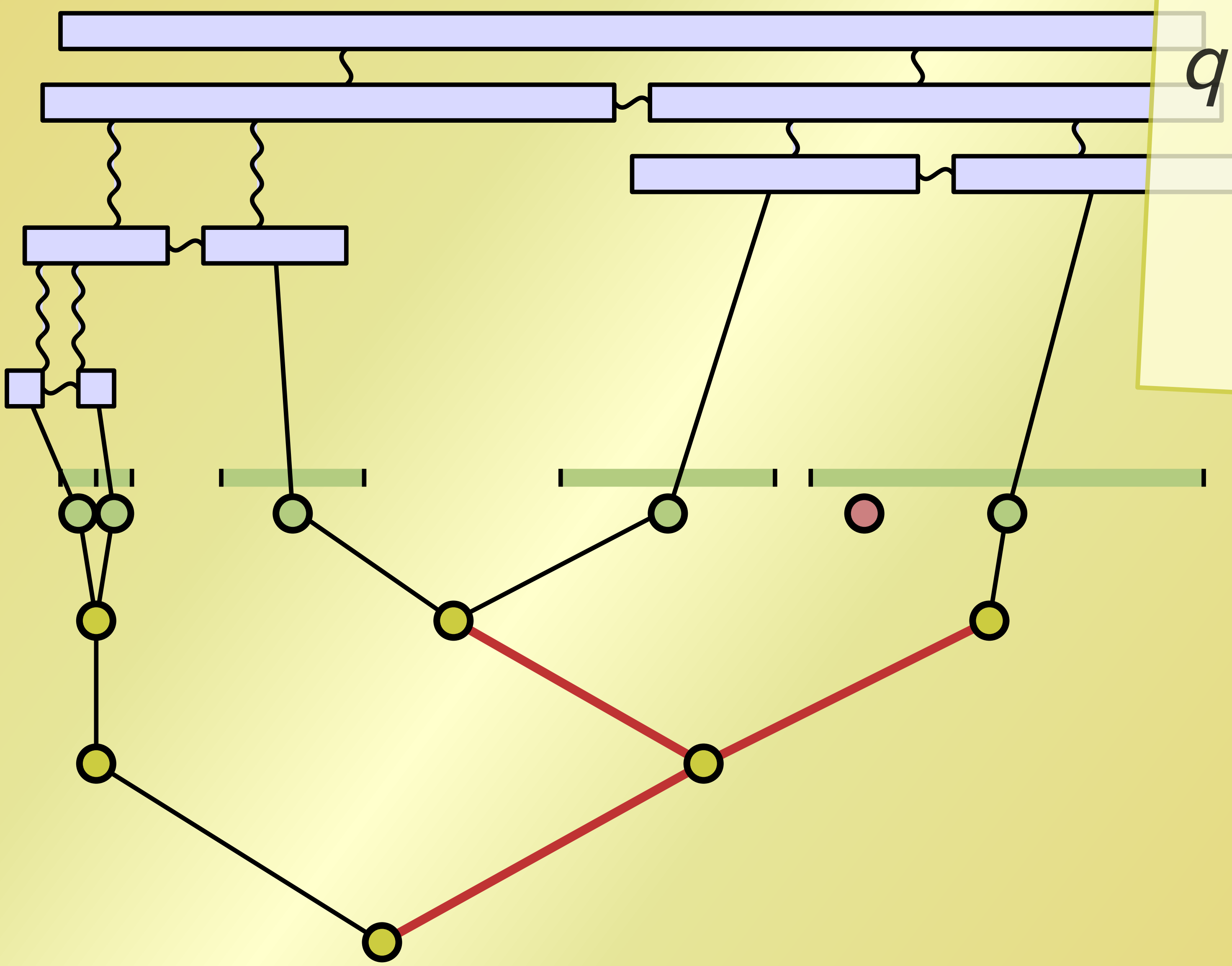

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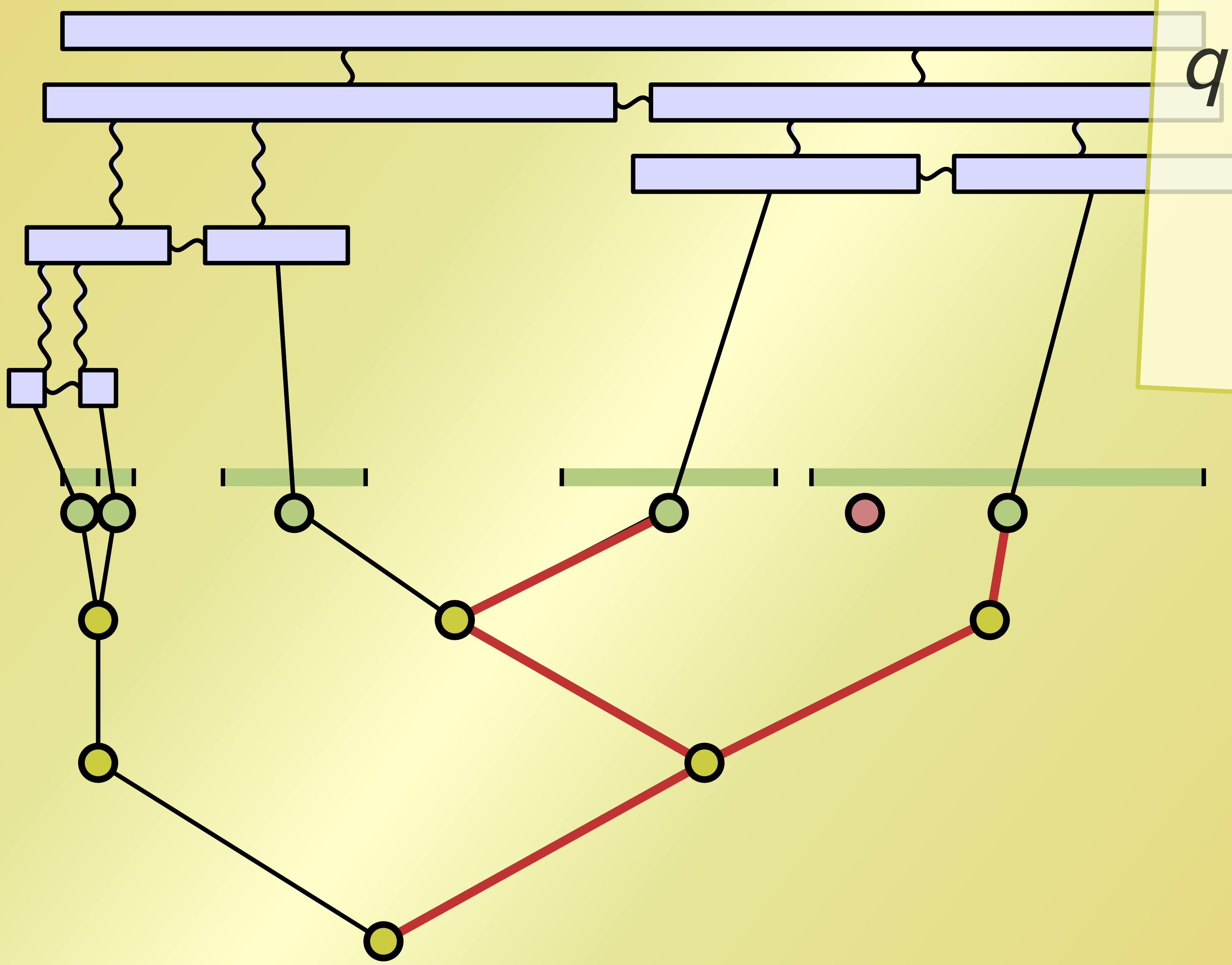

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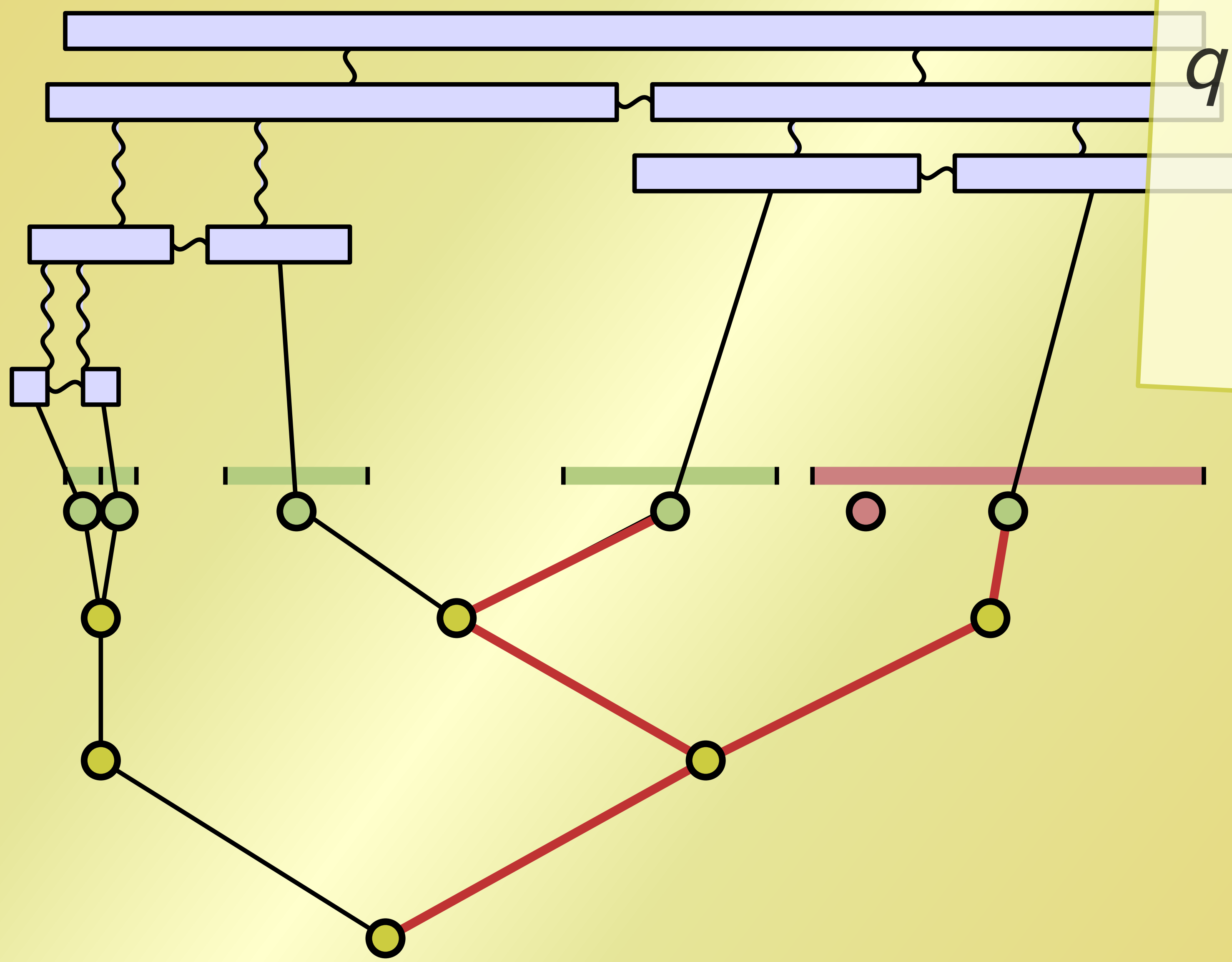

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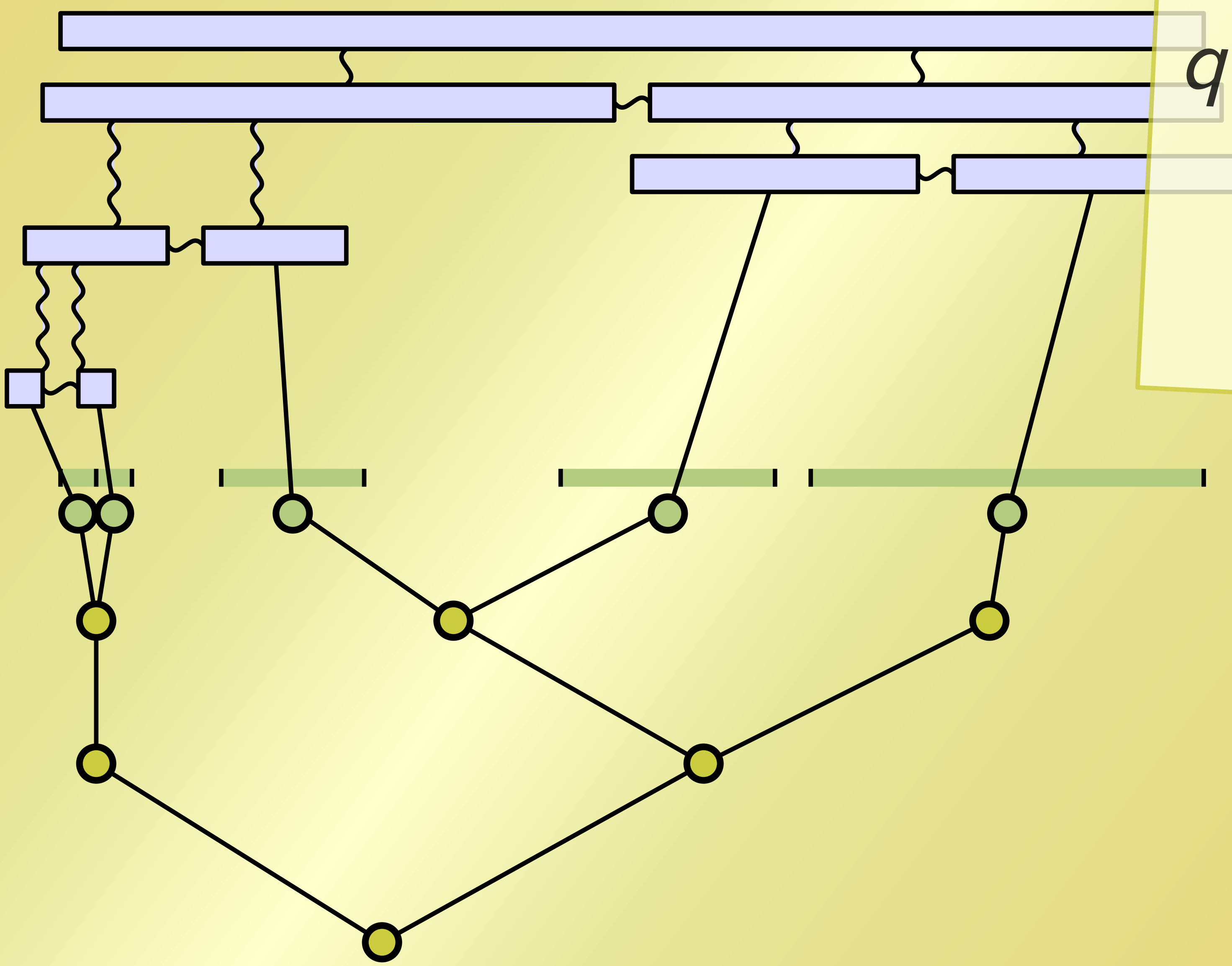

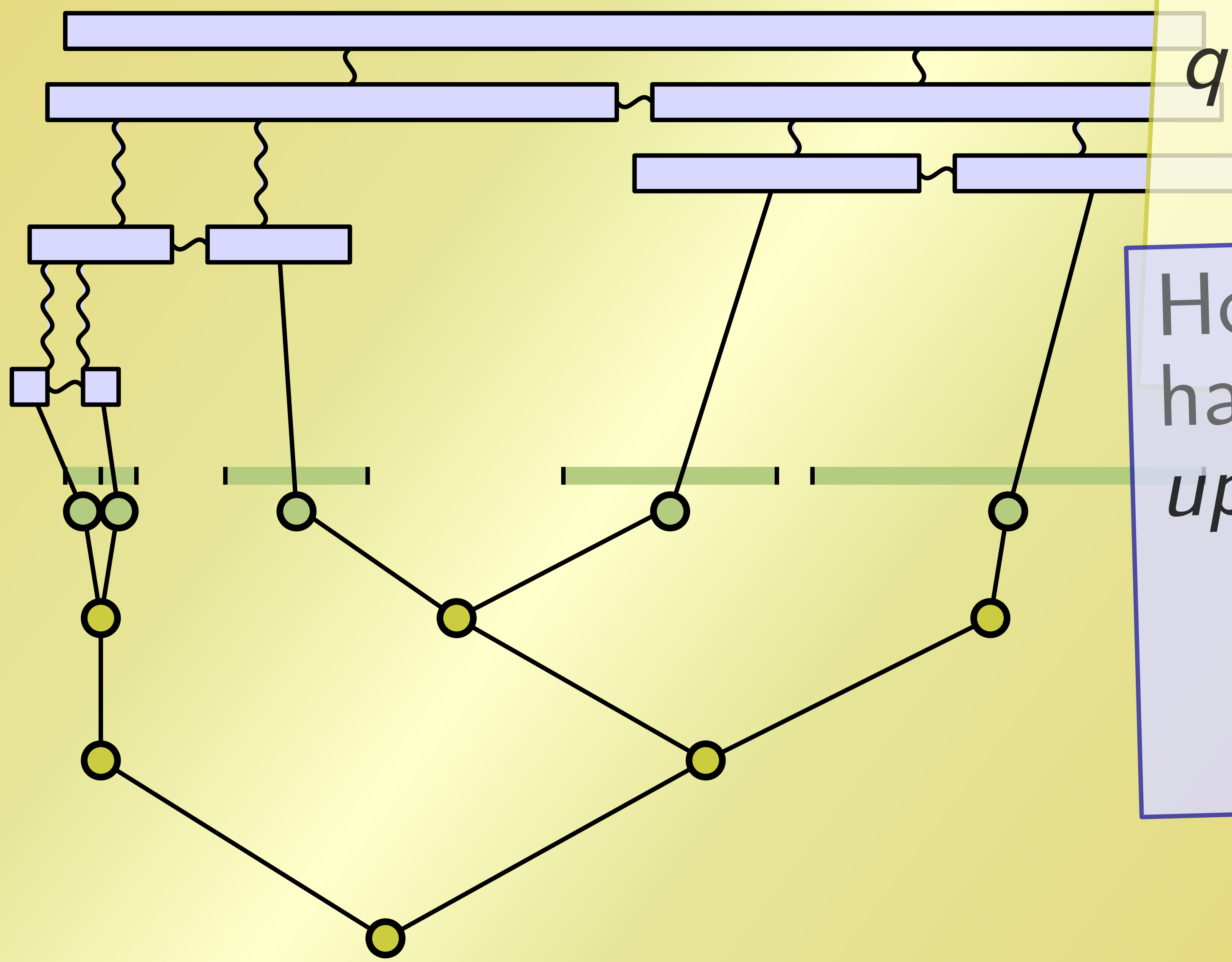

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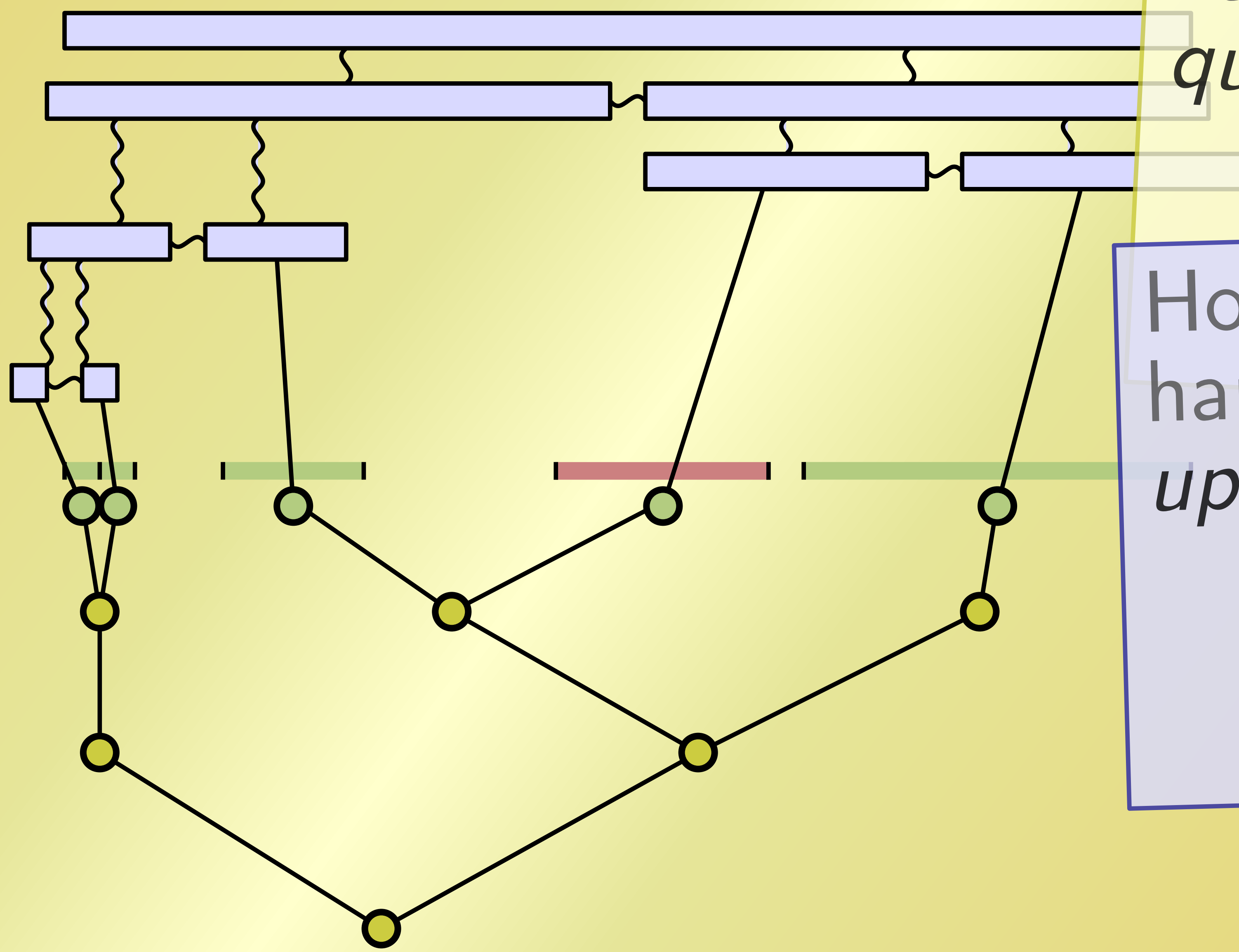

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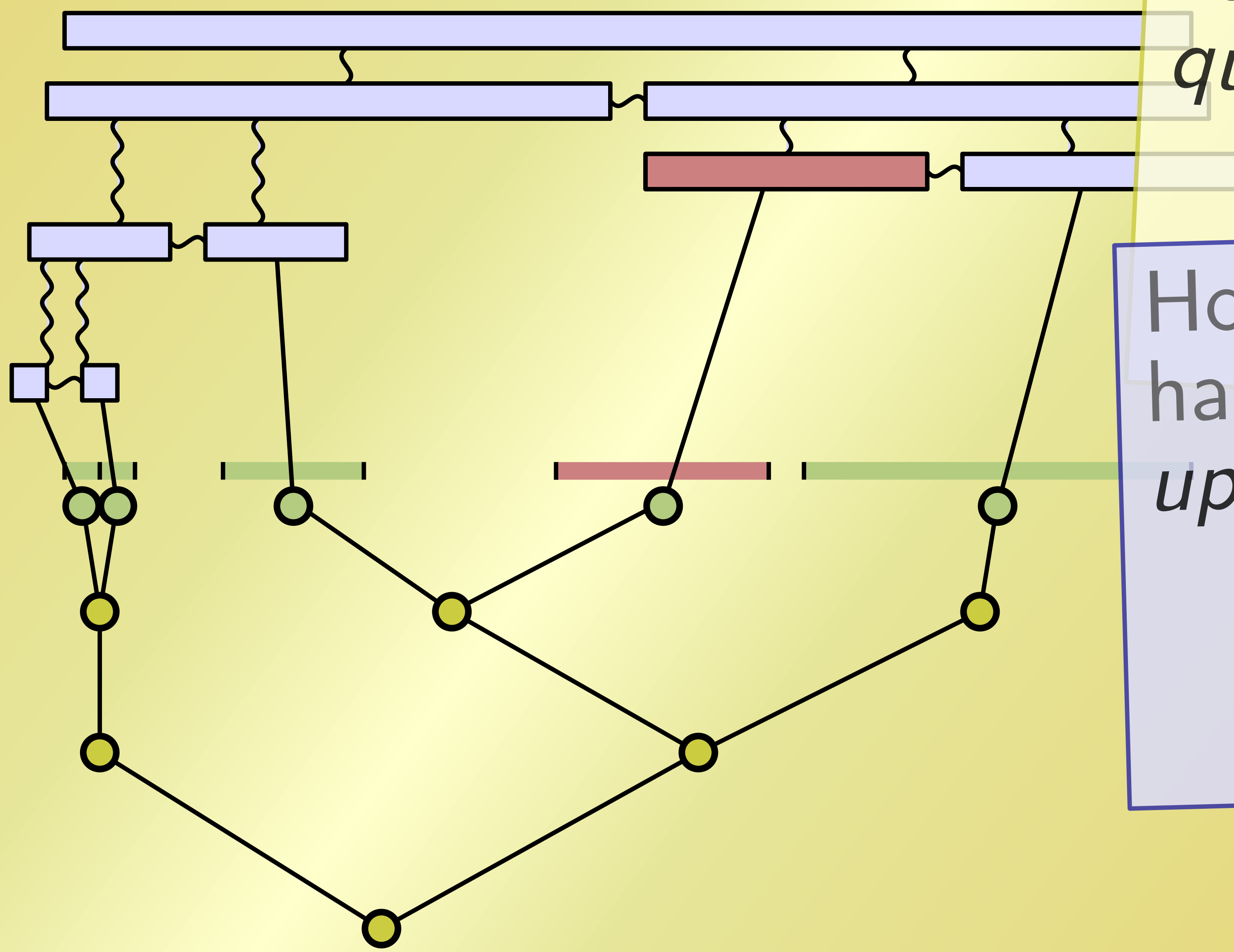

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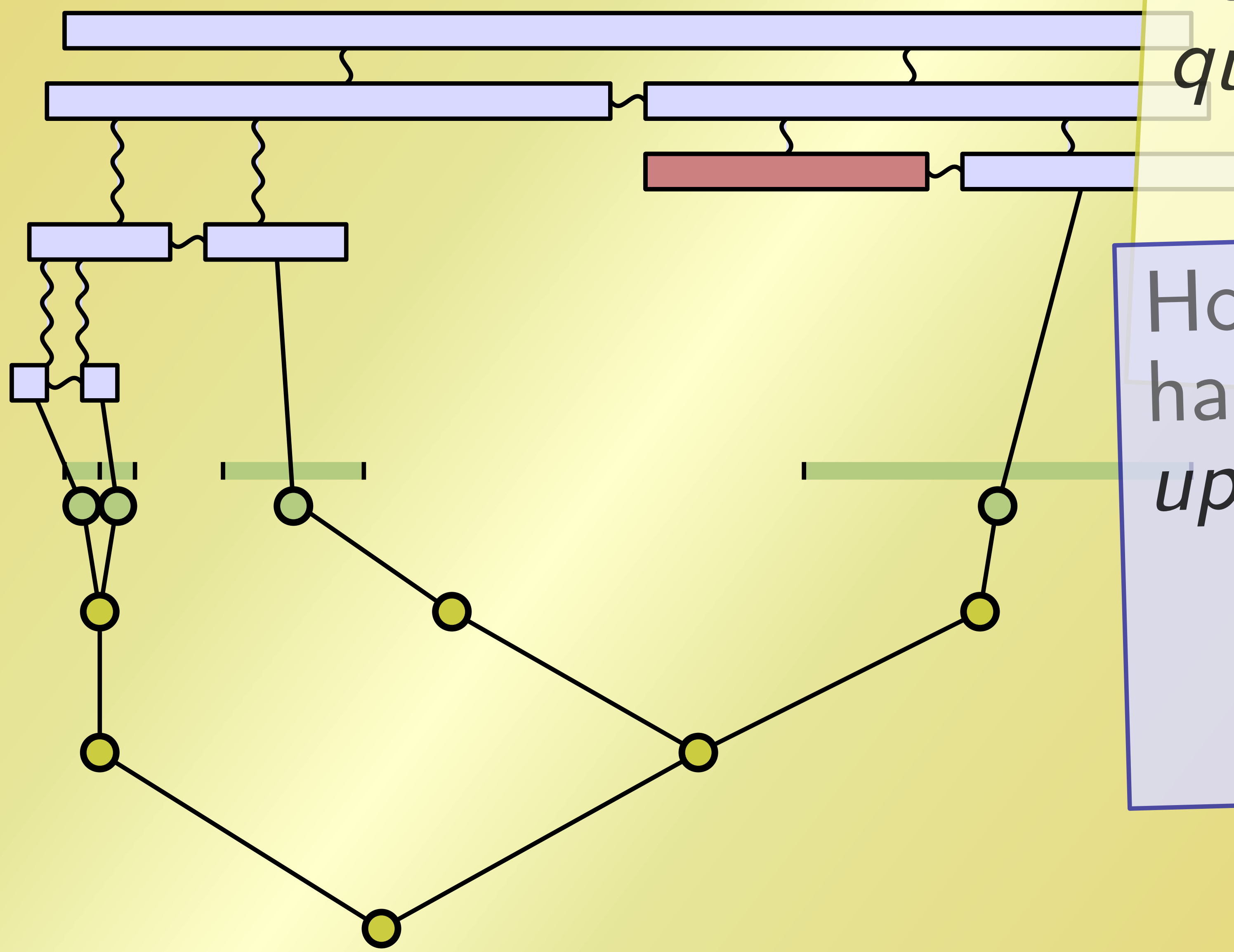

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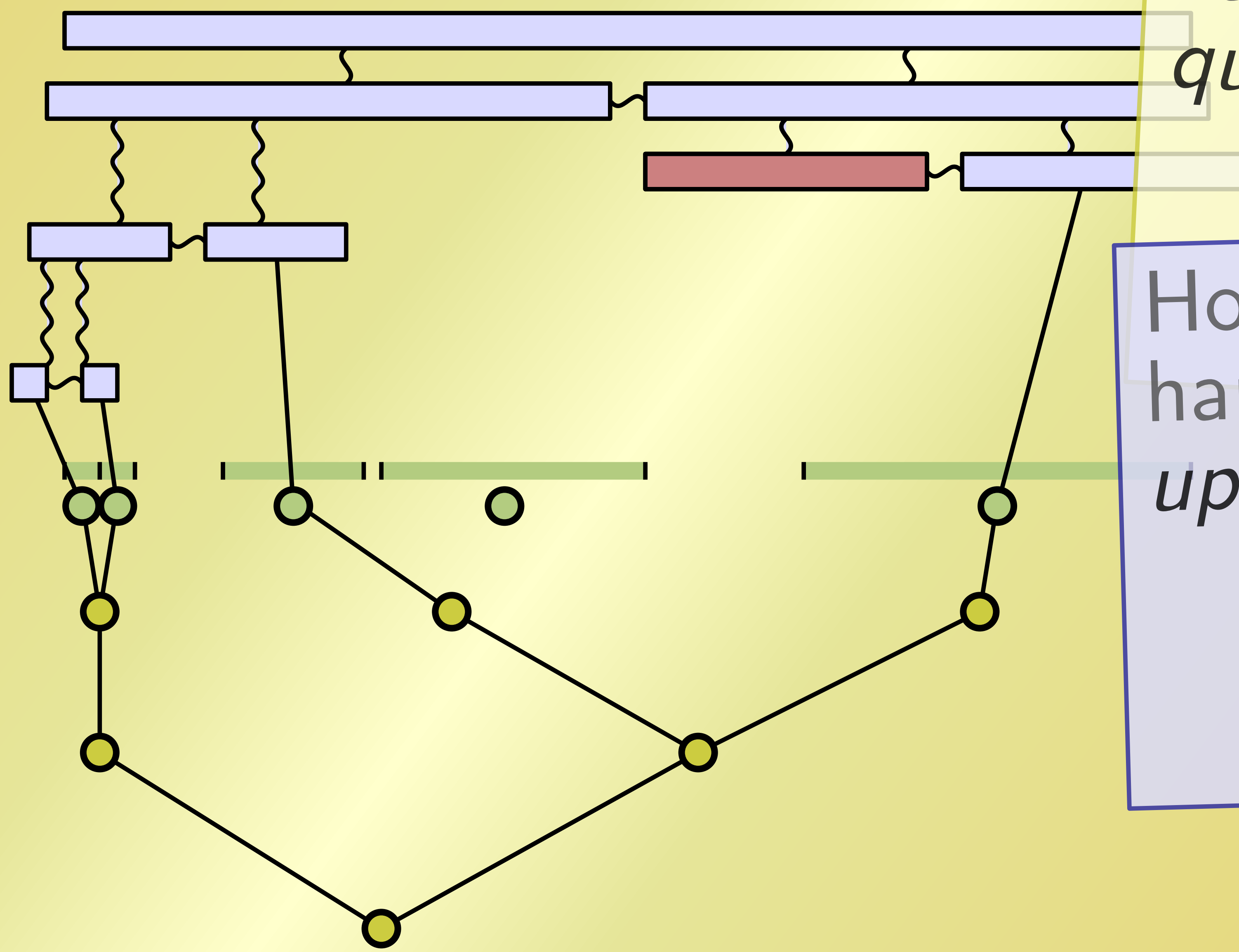

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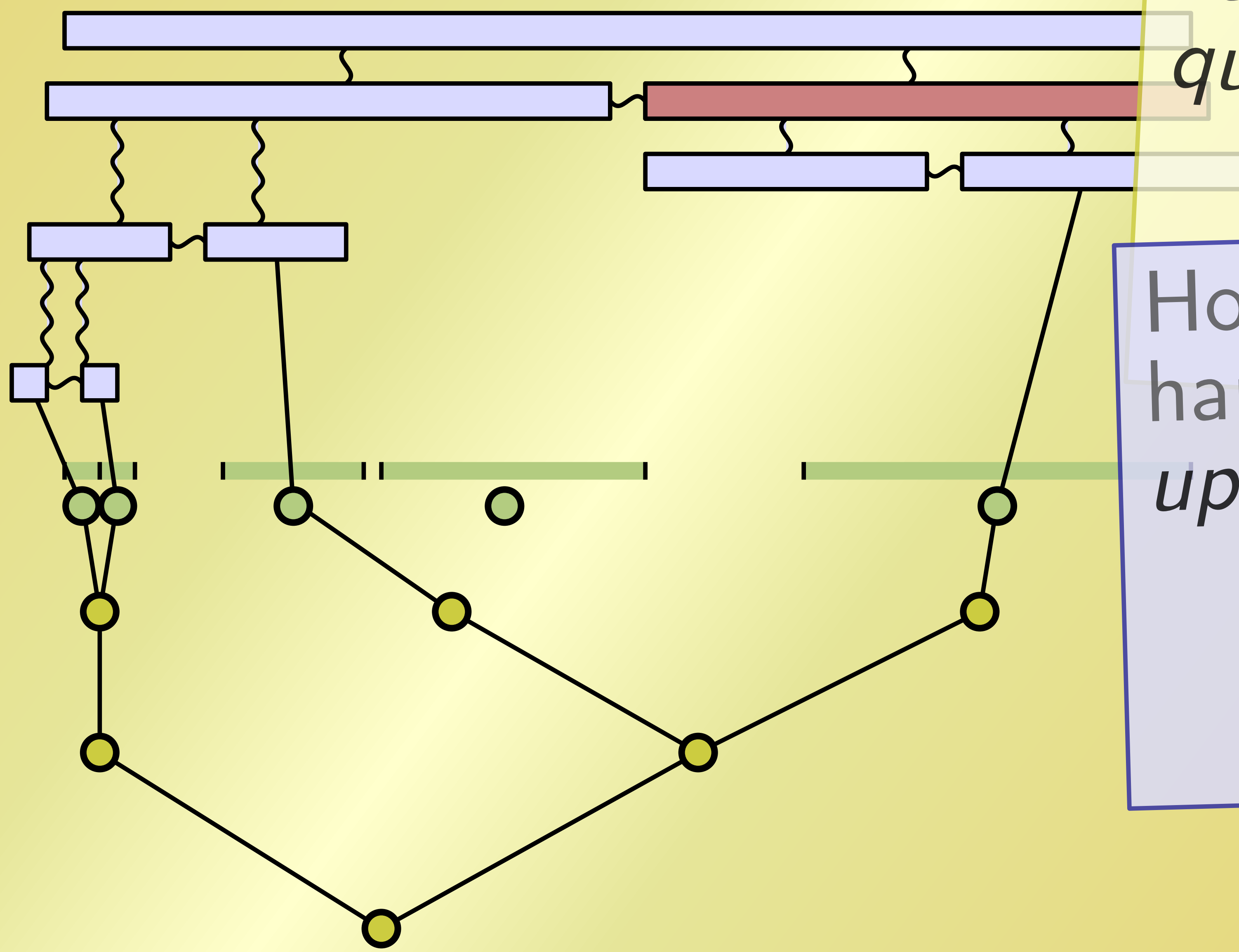

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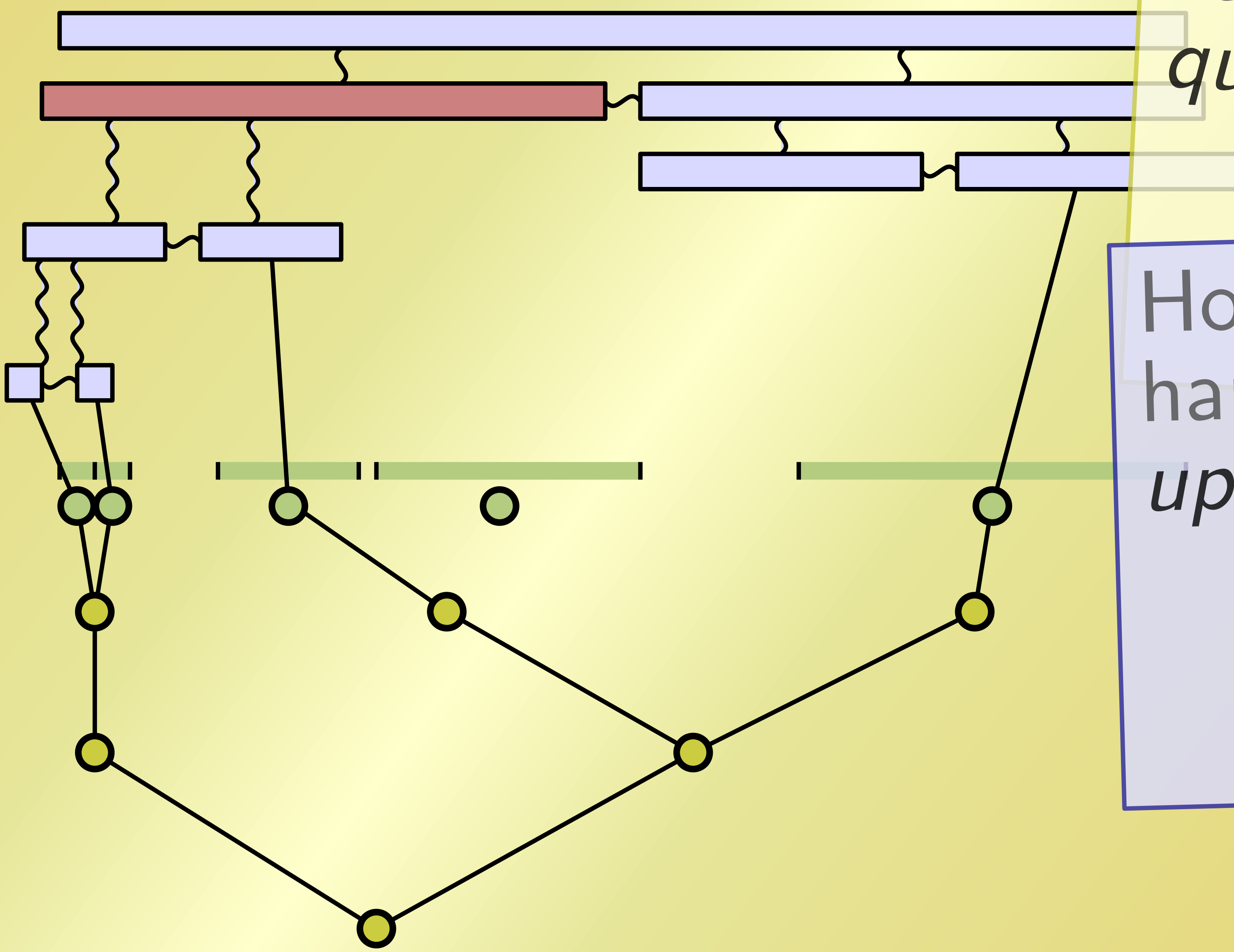

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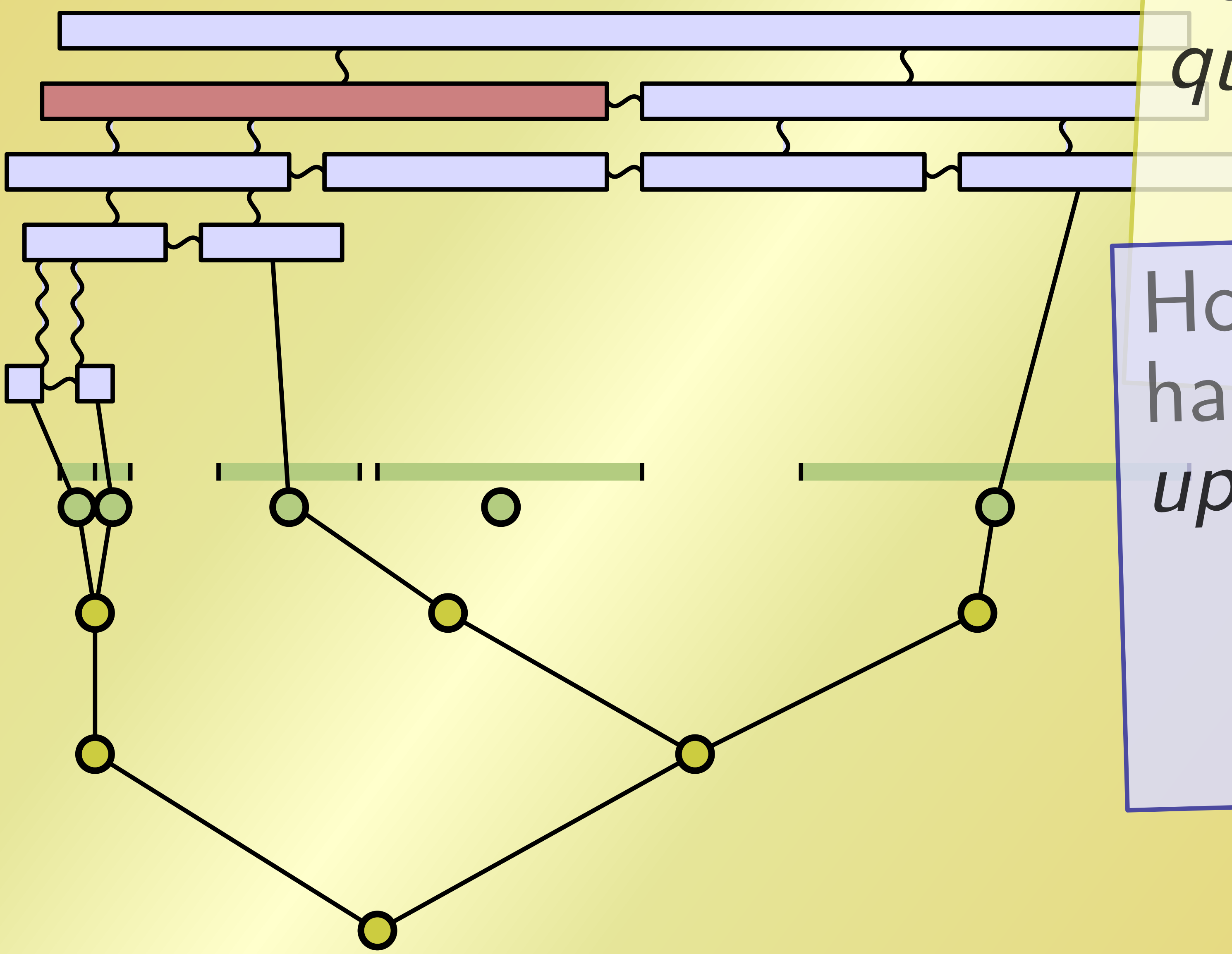

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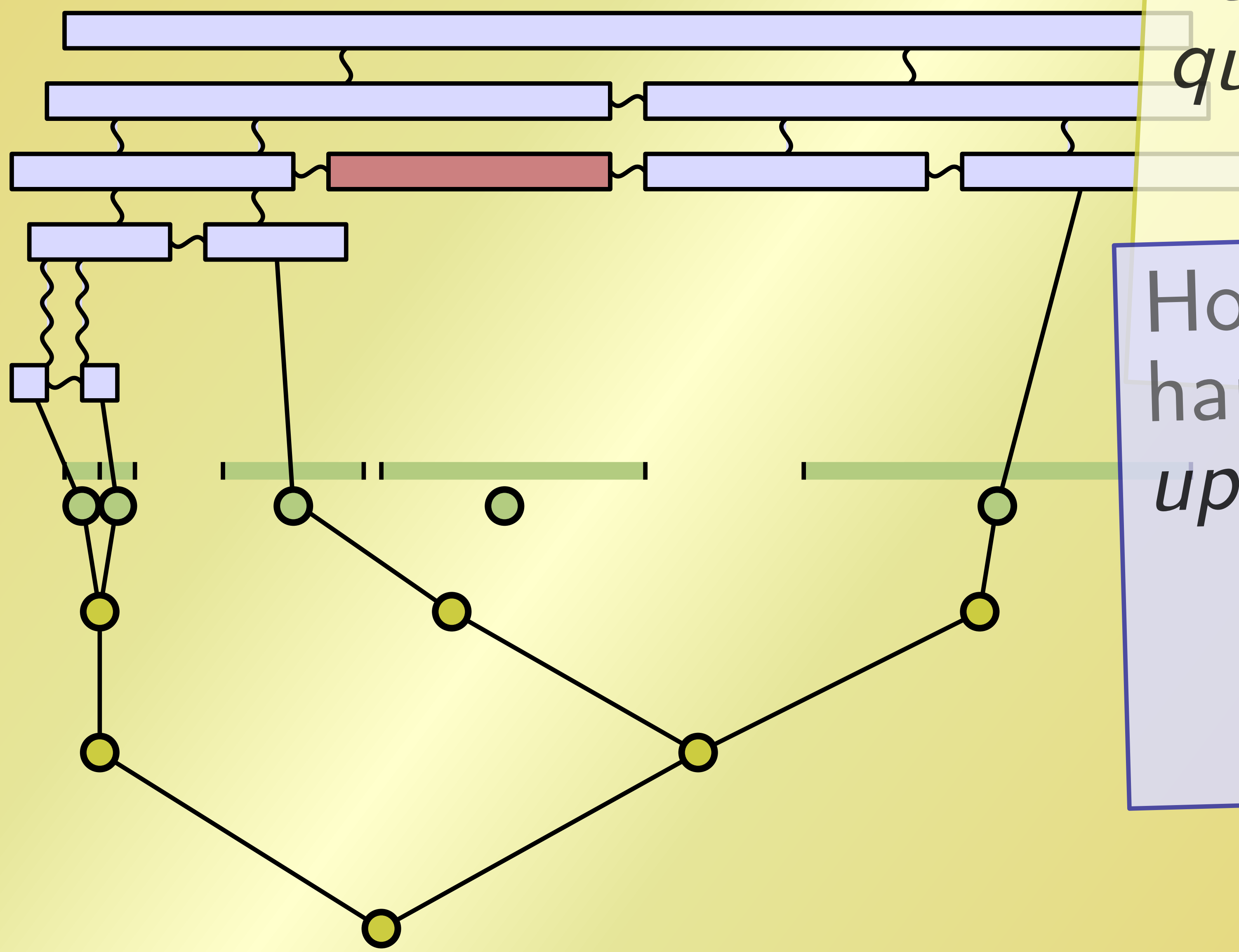

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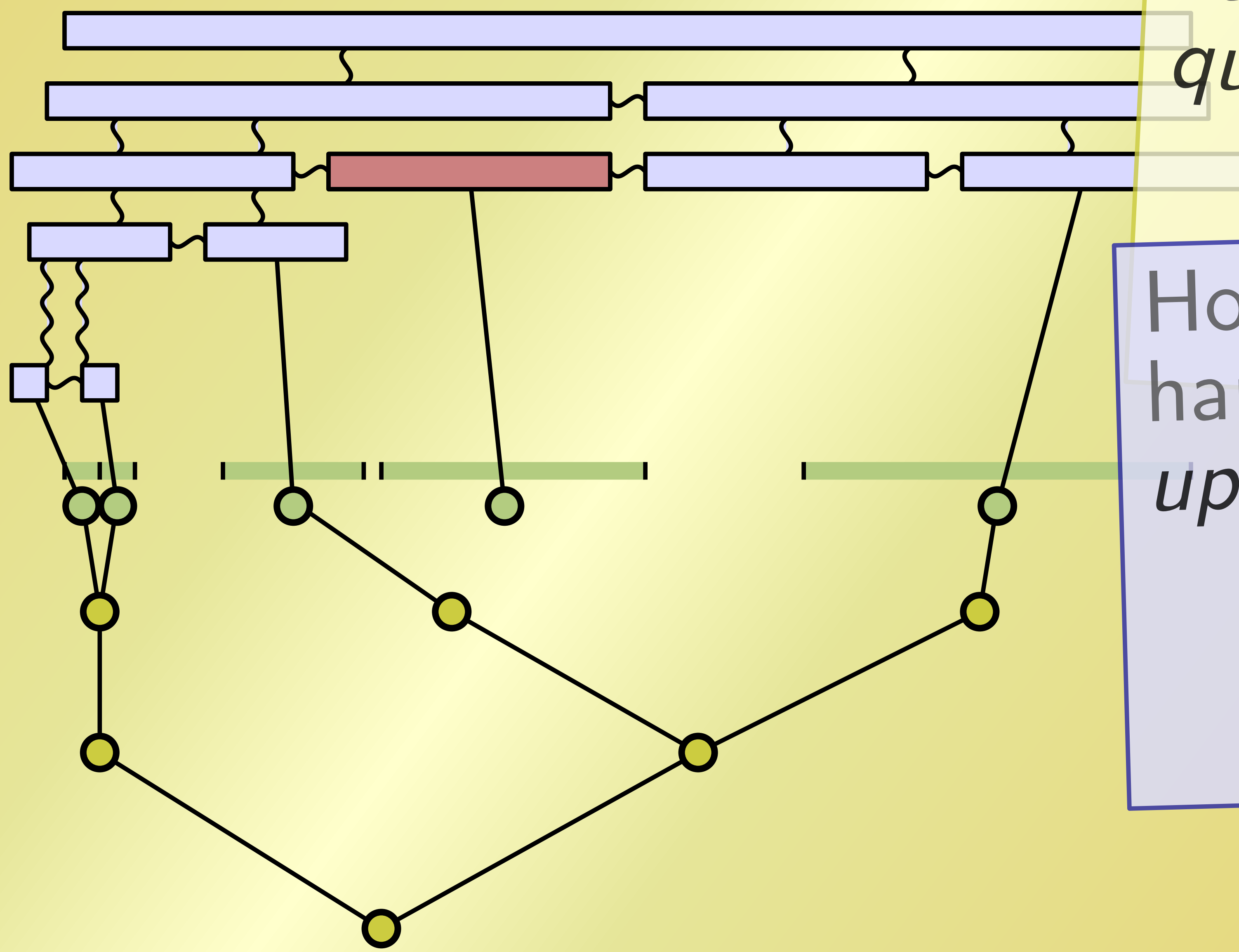

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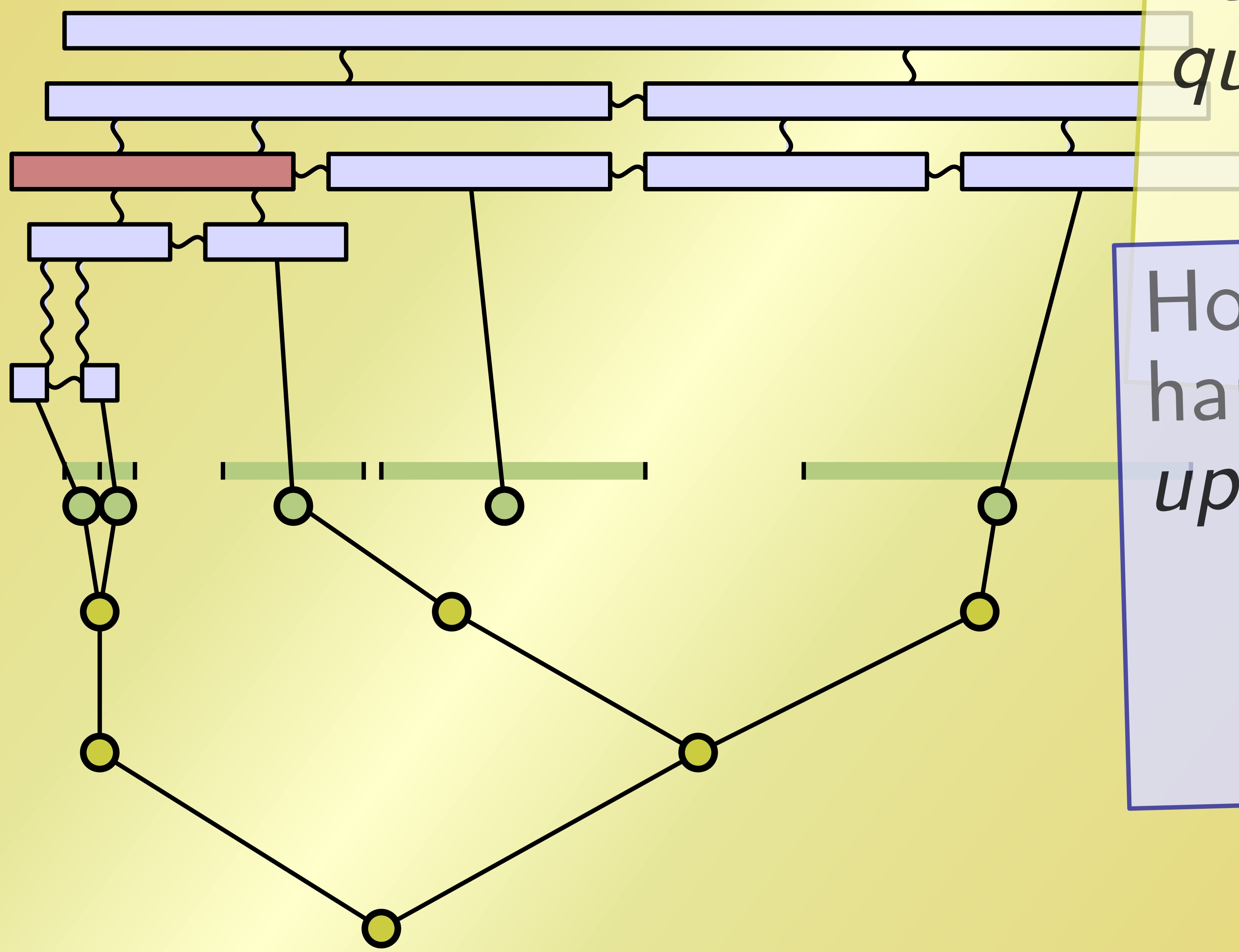

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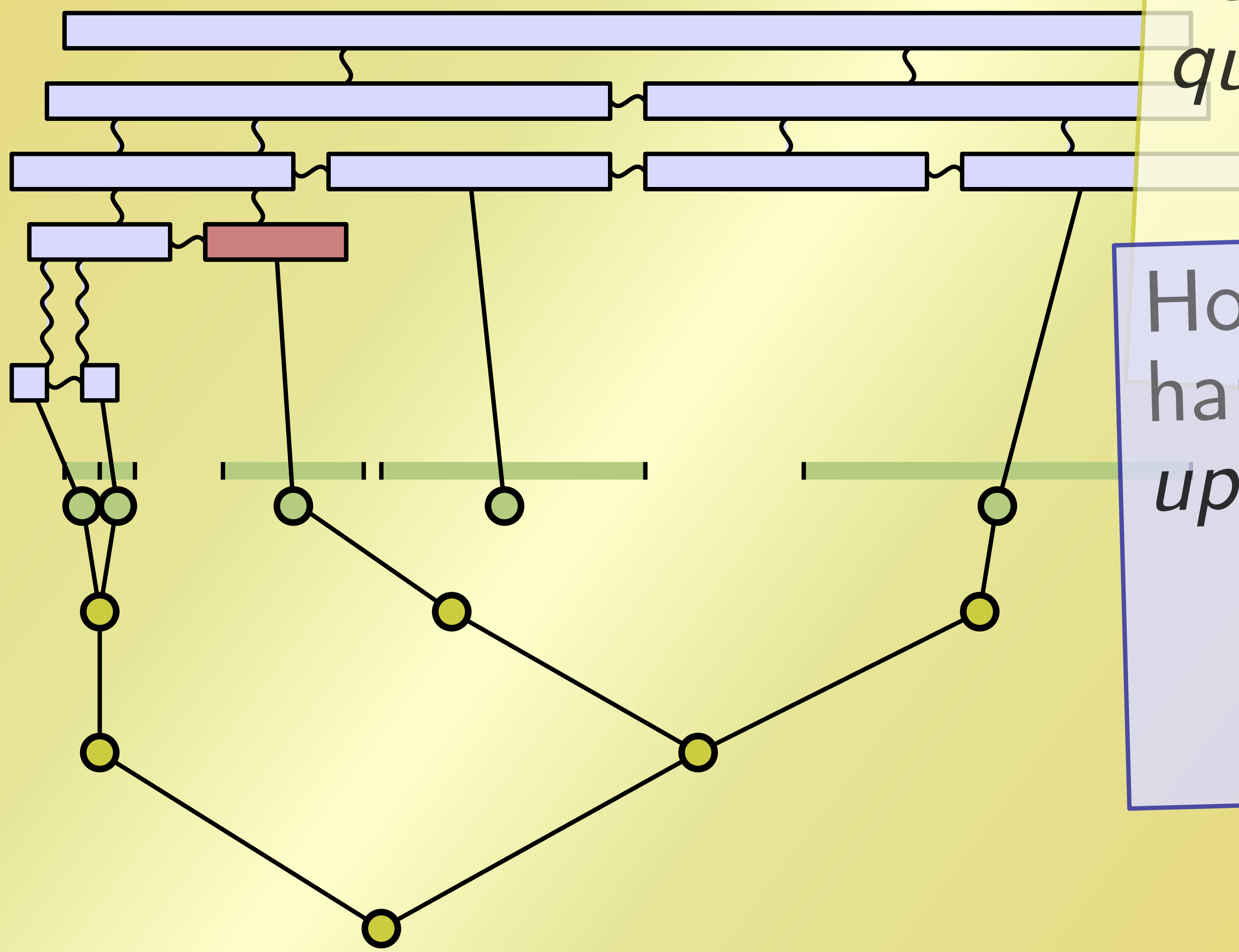

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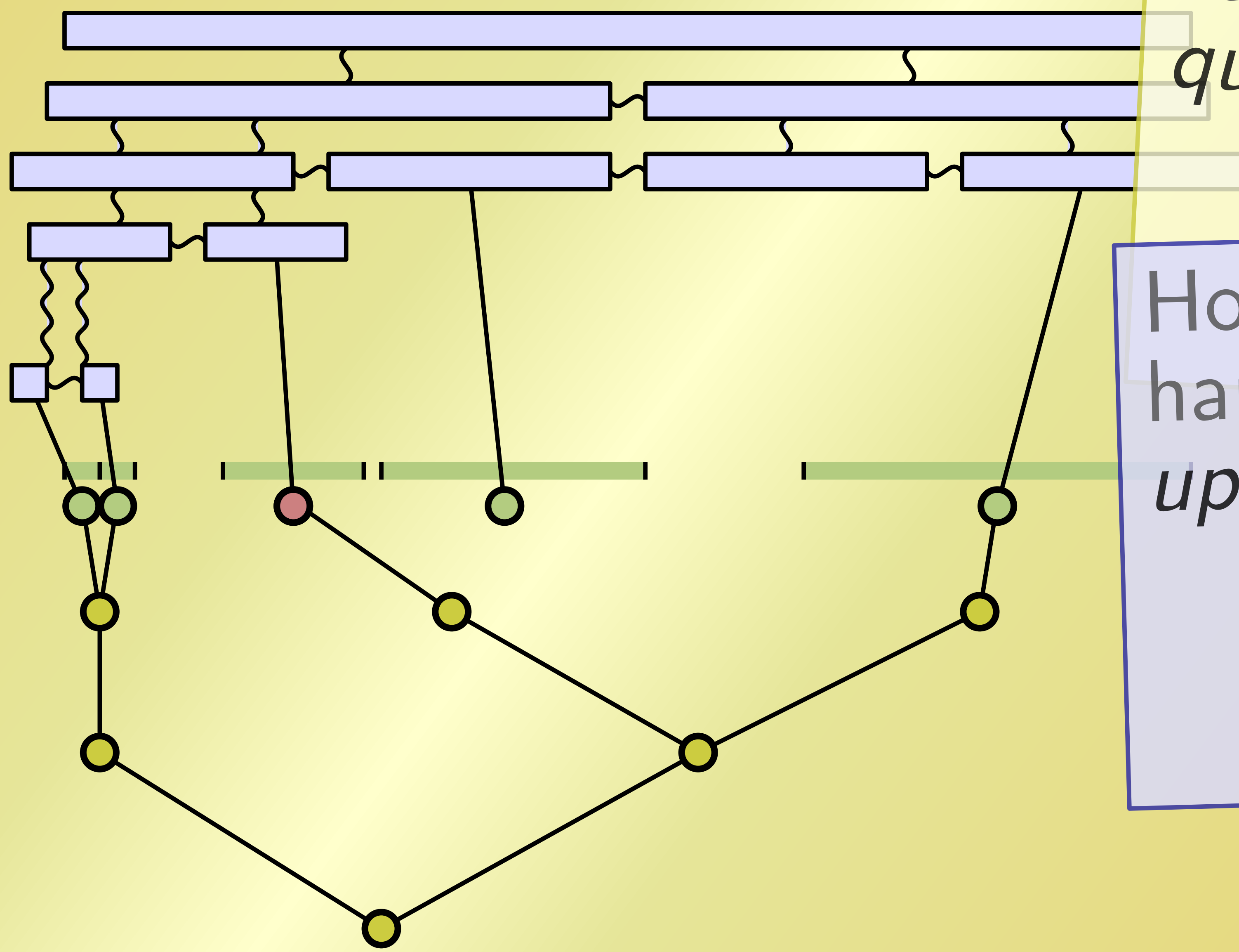

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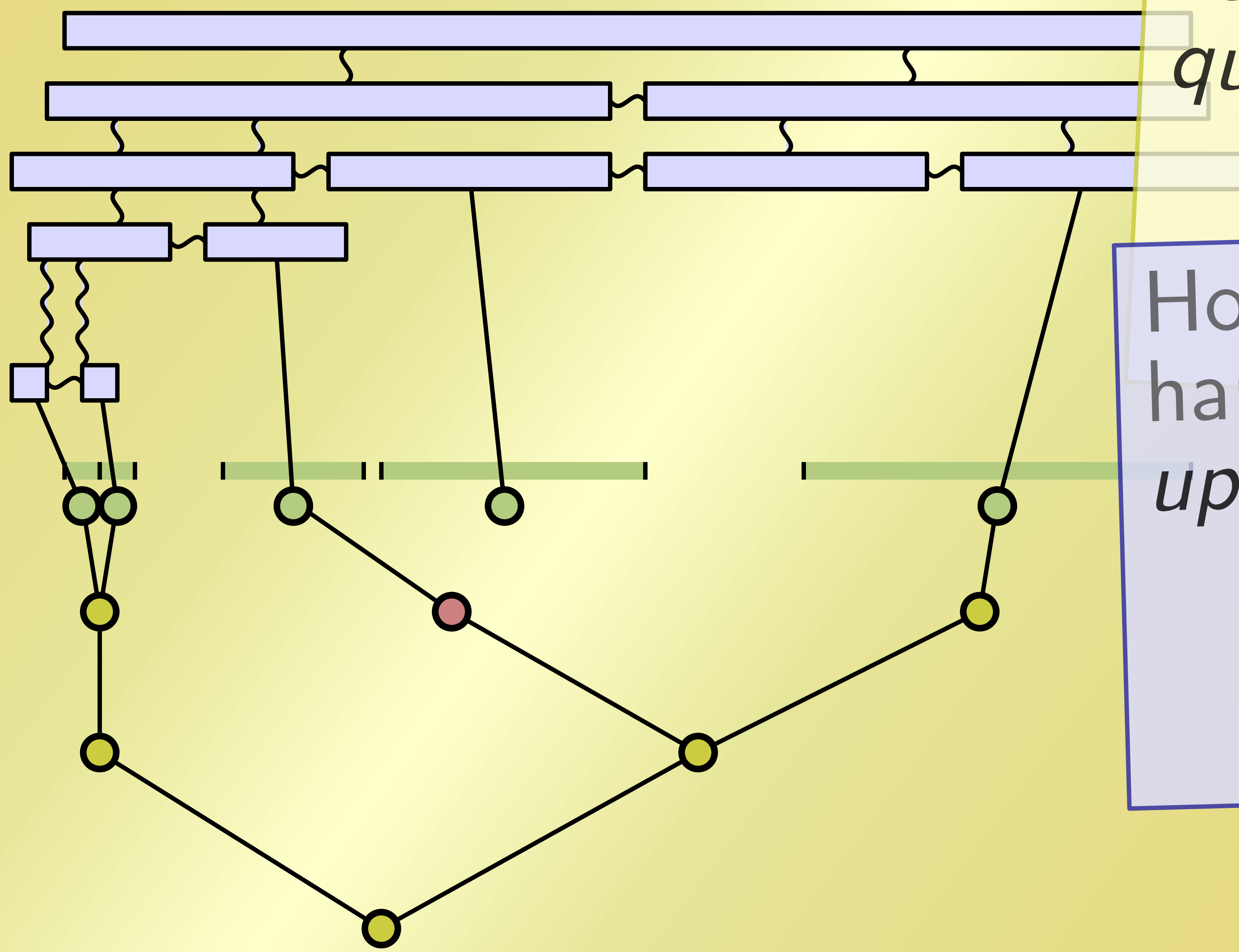

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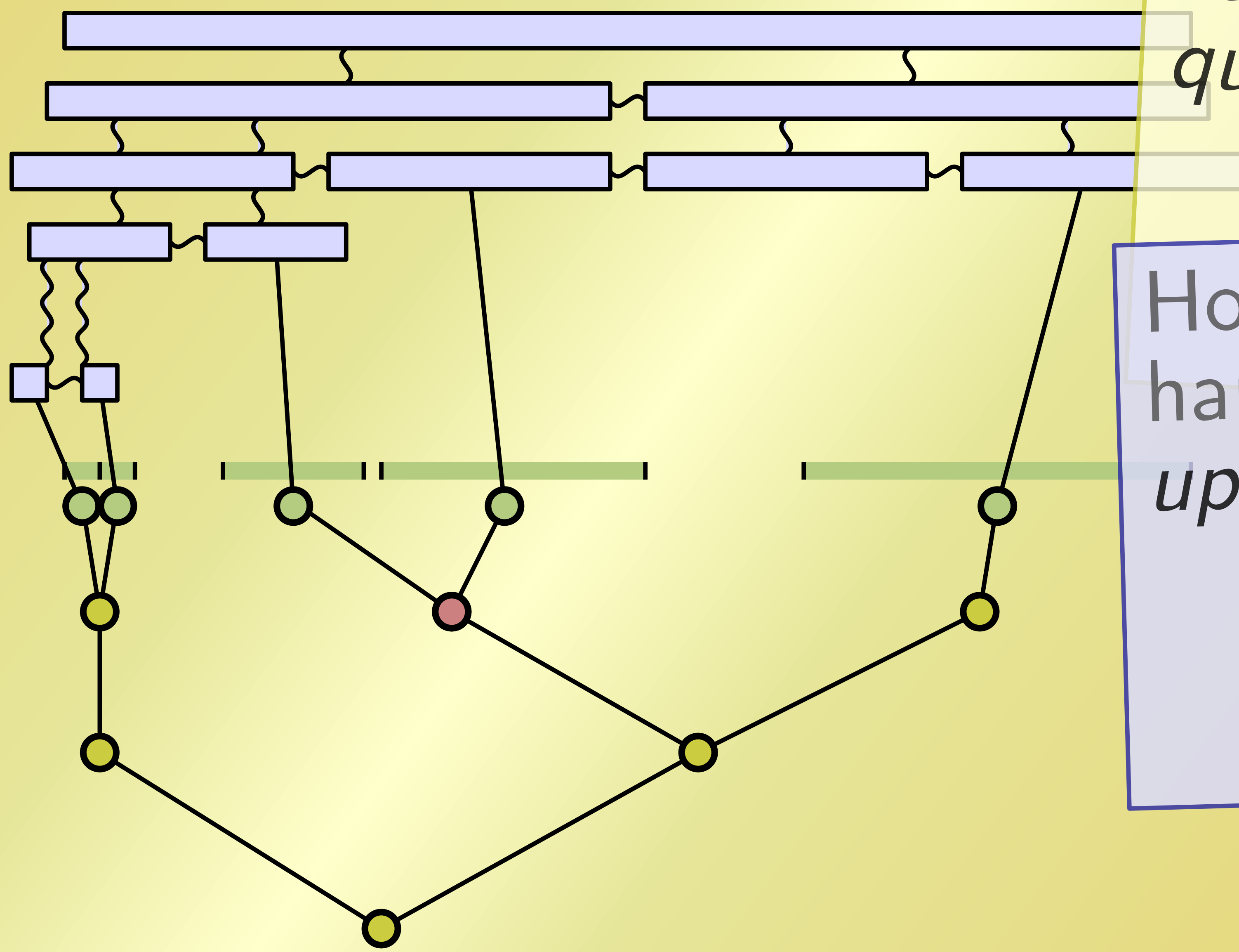

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TECHNICAL DETAILS:
2 DIMENSIONS

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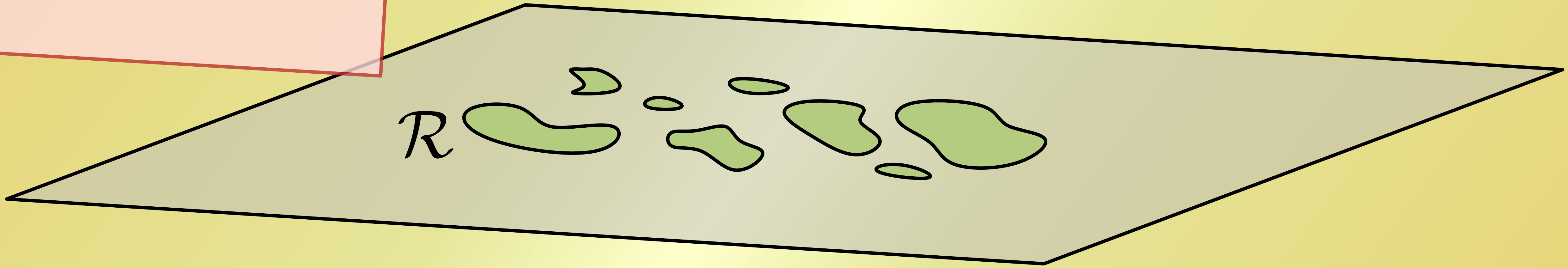

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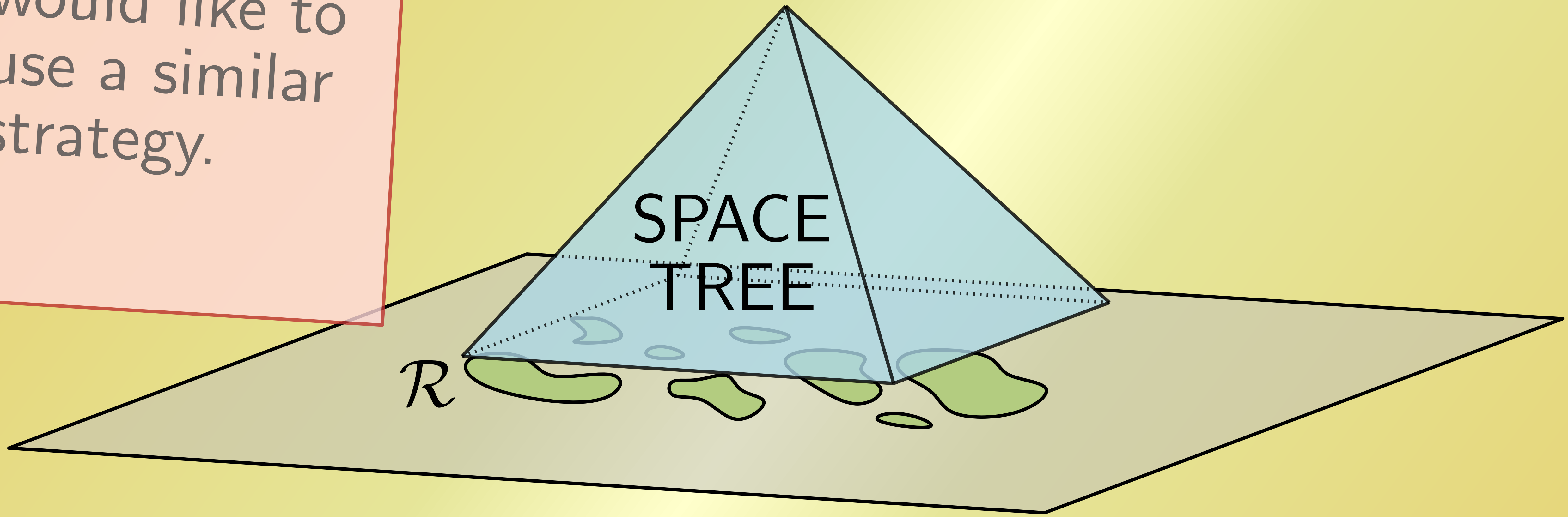

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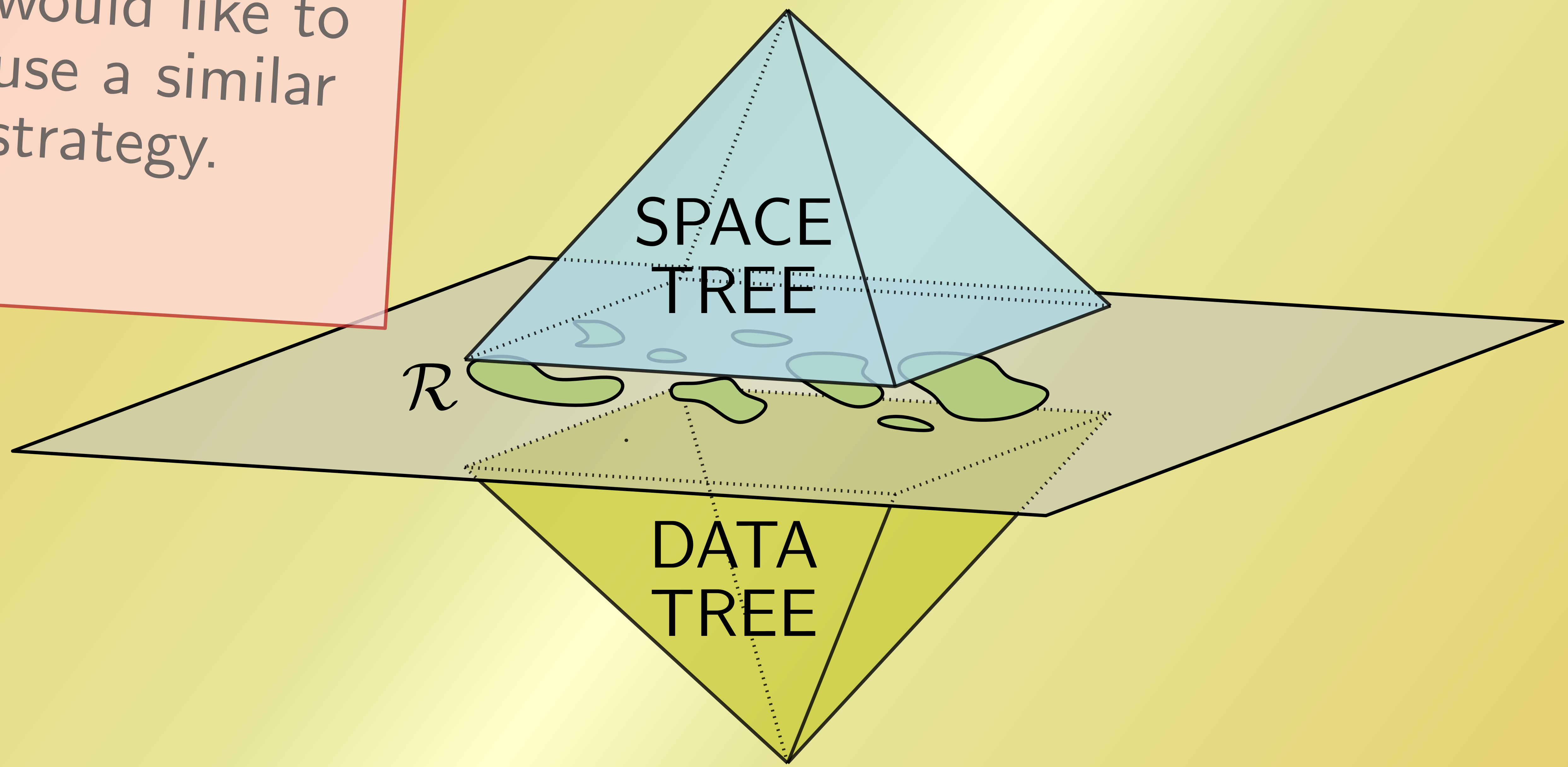

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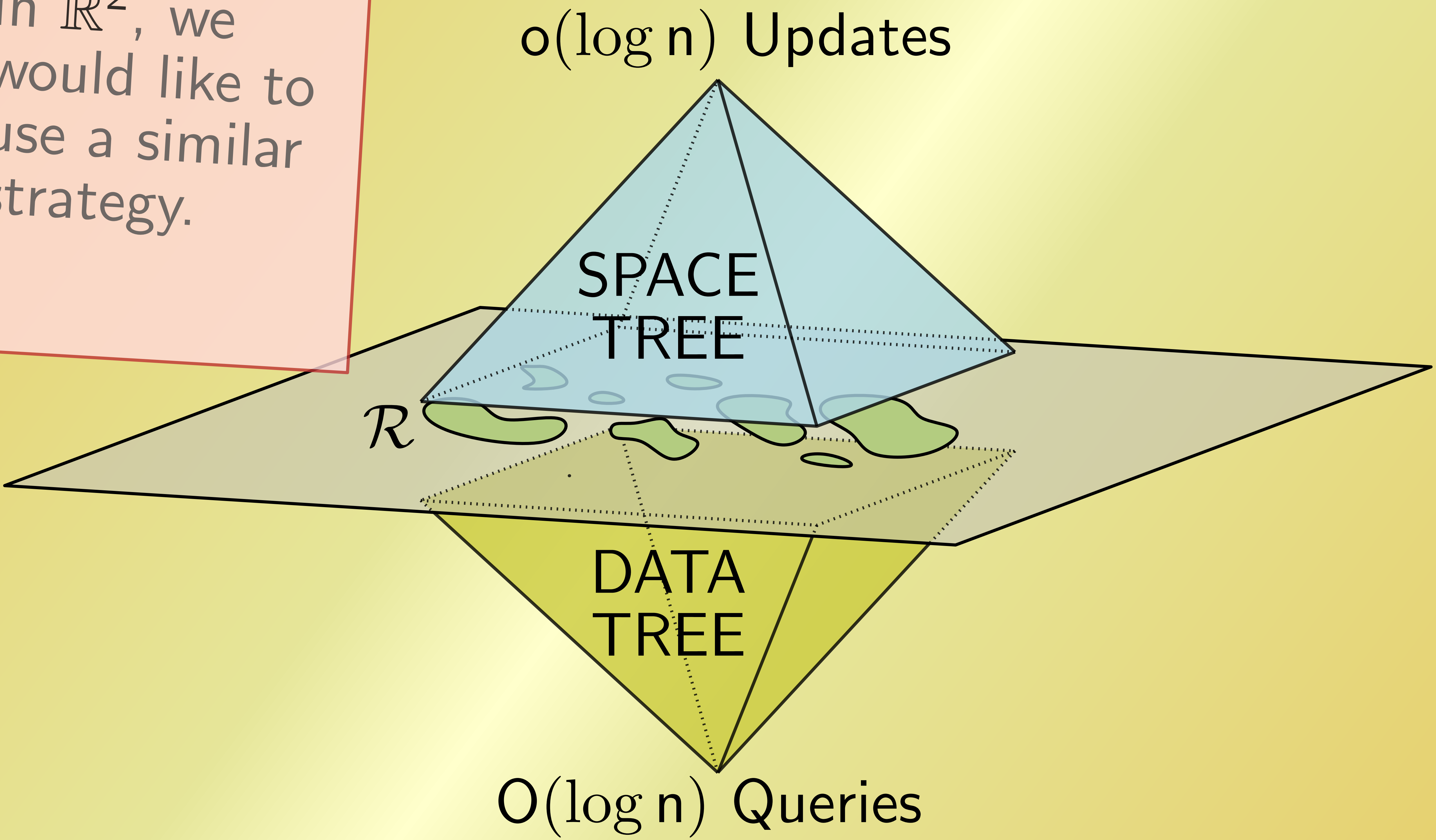

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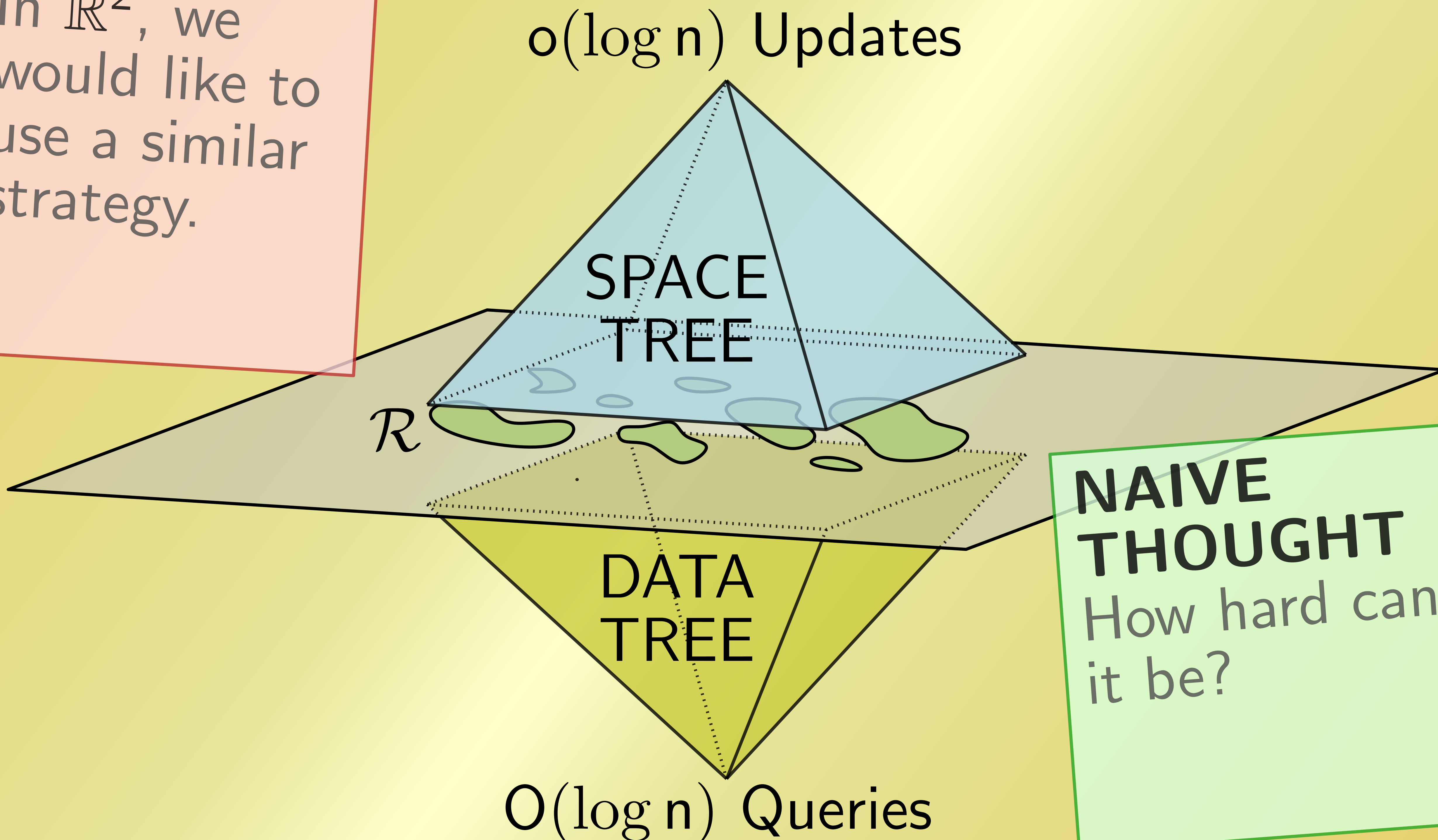
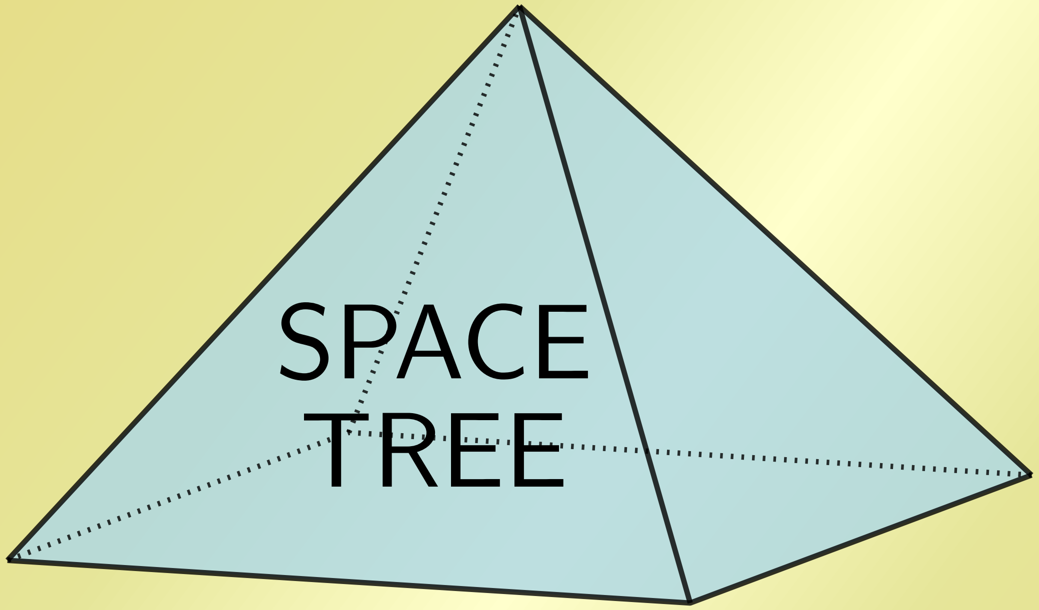
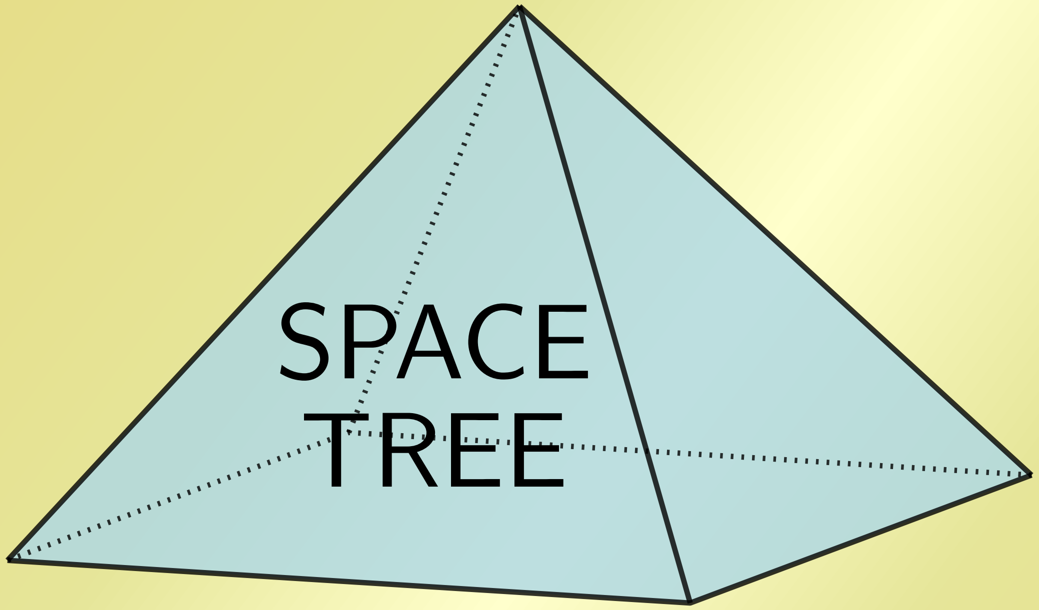


NAIVE THOUGHT

How hard can it be?


SPACE
TREE

Quadtrees also exist in 2 dimensions!



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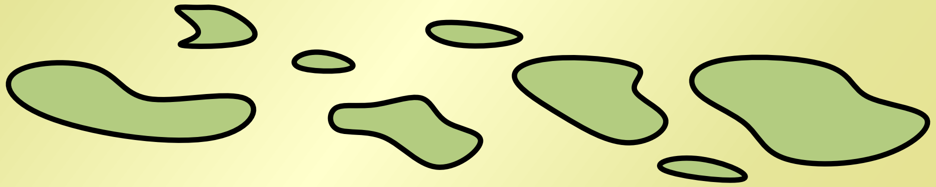

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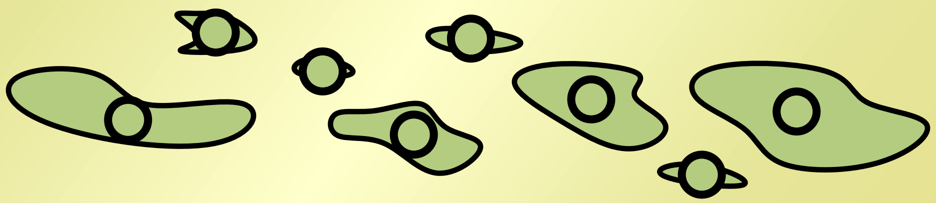

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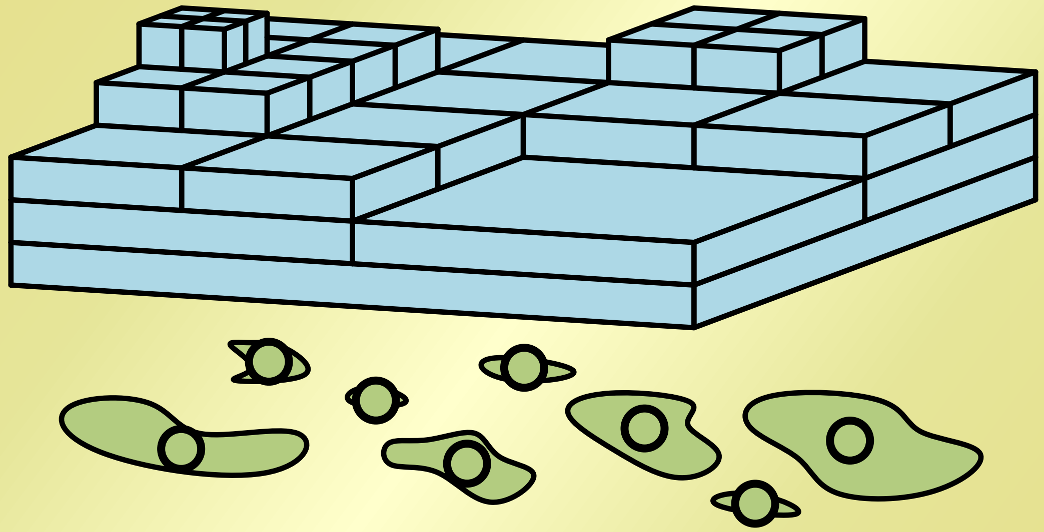

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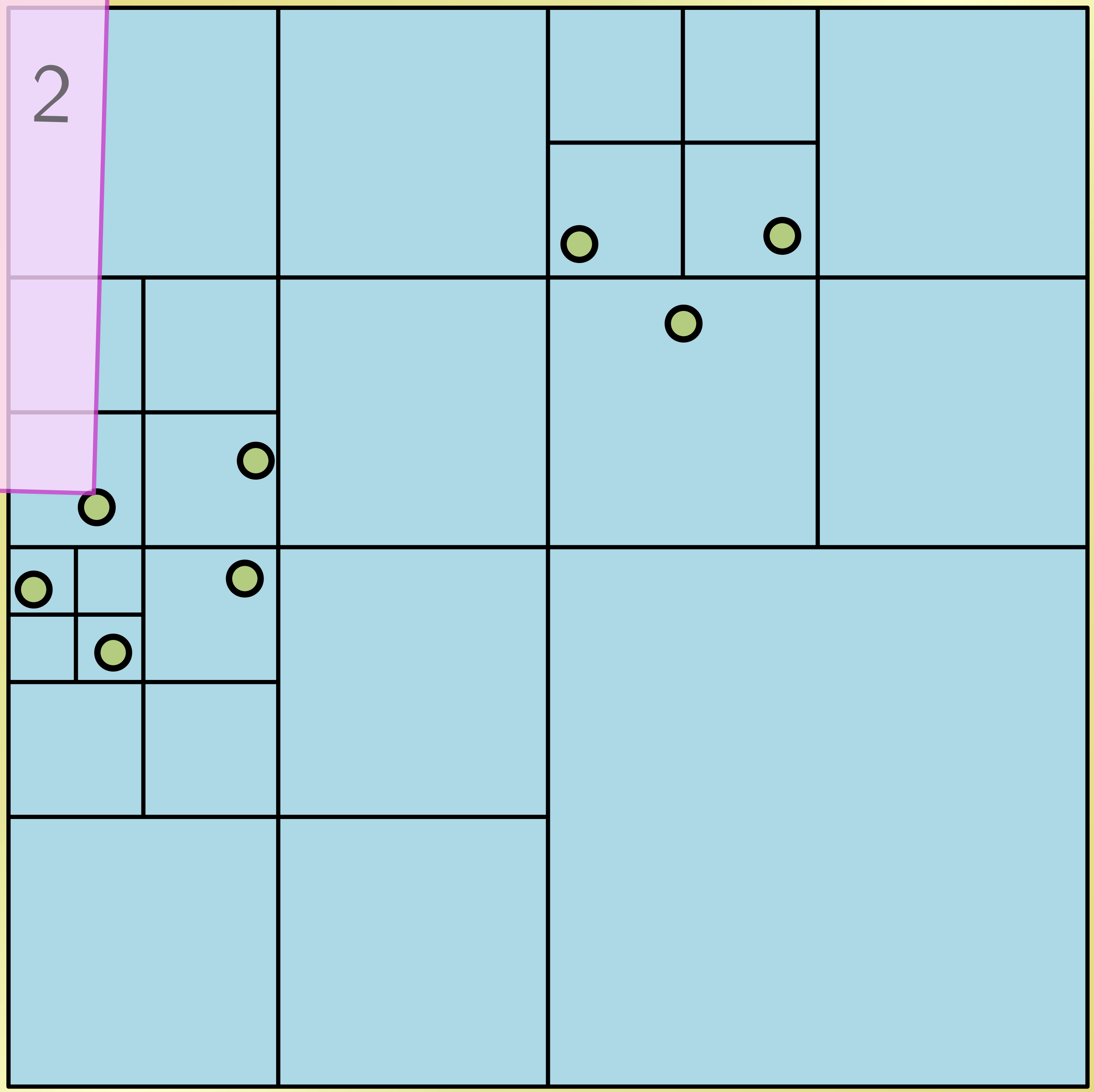

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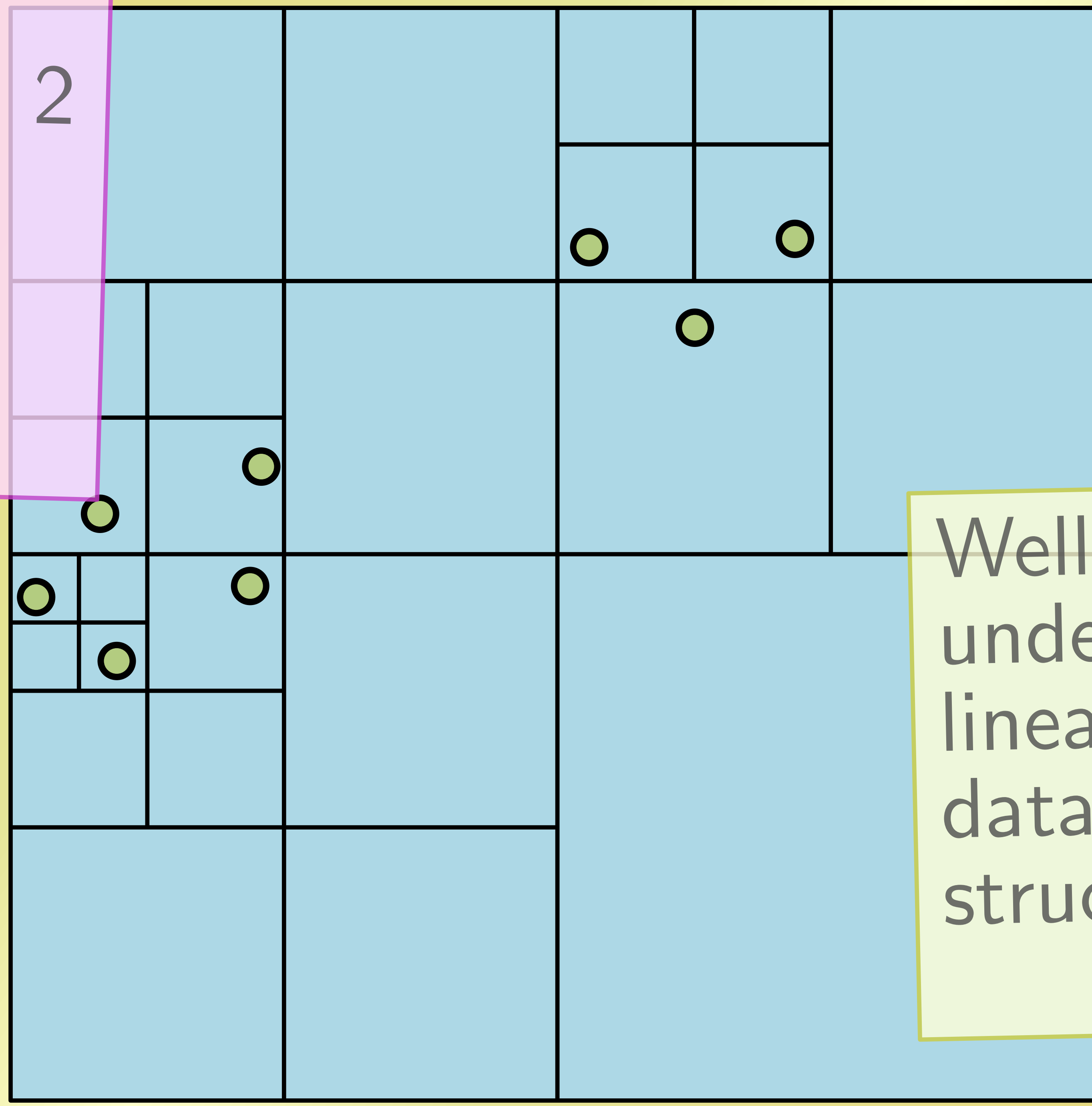
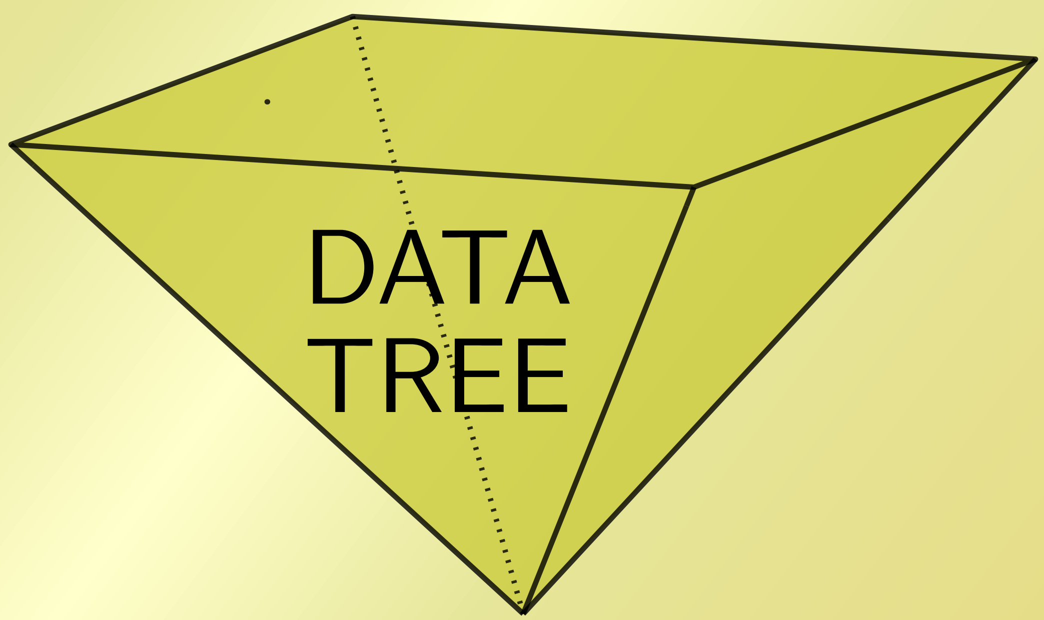
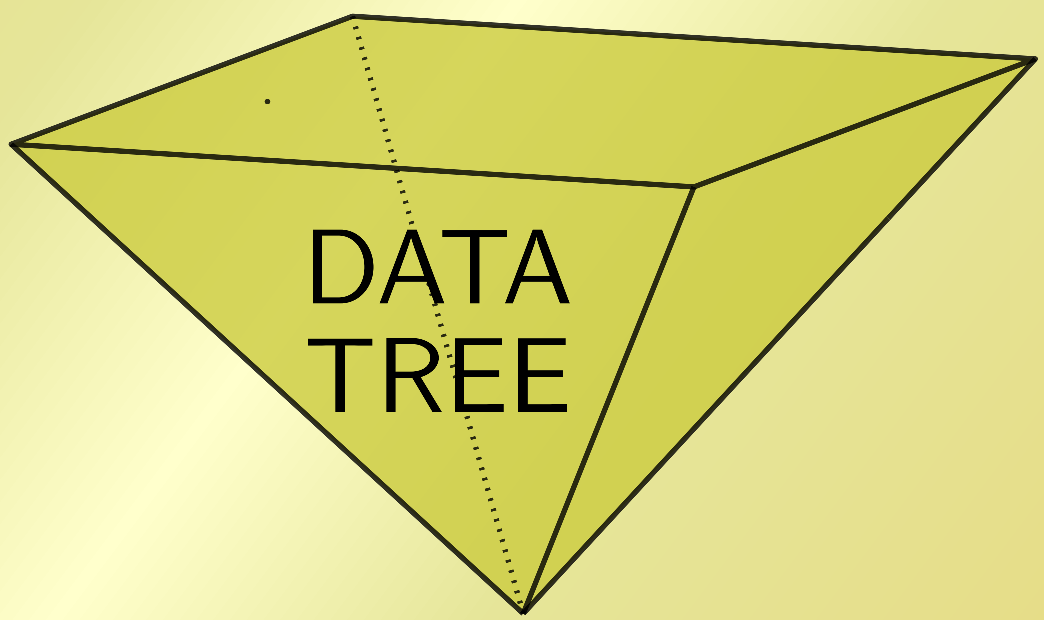


Well understood, linear size data structure.


DATA
TREE

We still need something for the actual point location.



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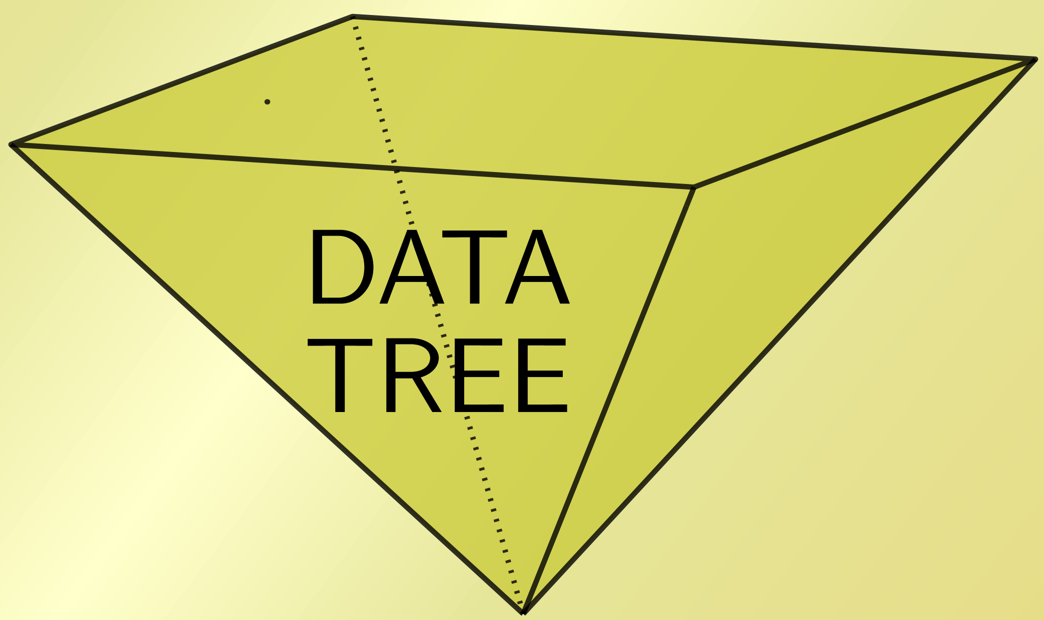
Build existing structure on regions, and use cross pointers as before?



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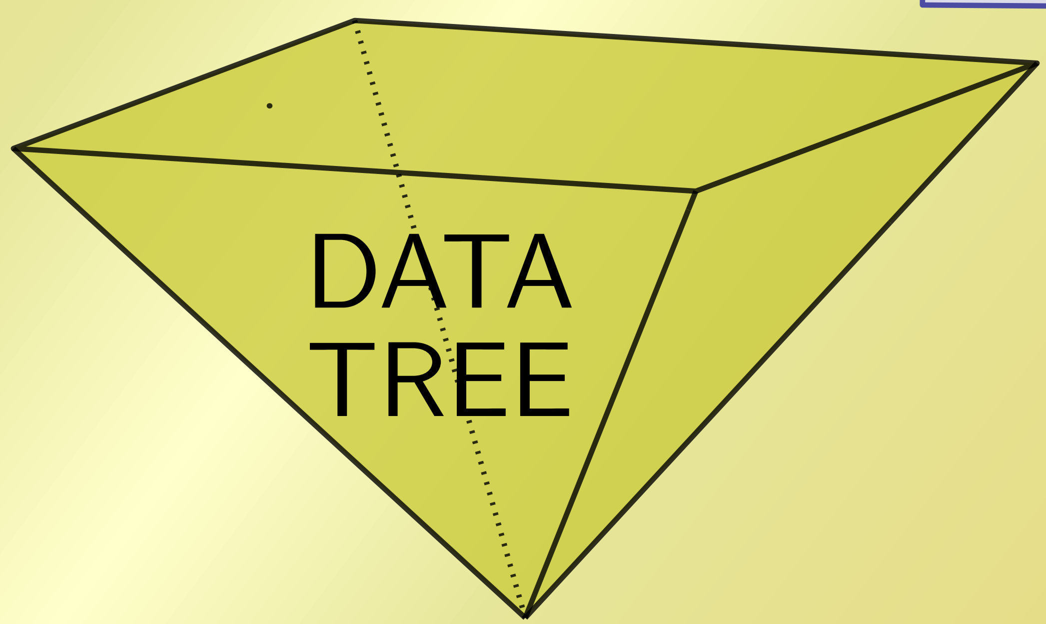
Build existing structure on regions, and use cross pointers as before?

How do regions relate to the quadtree?



PROBLEM

We can't just use any search tree anymore.

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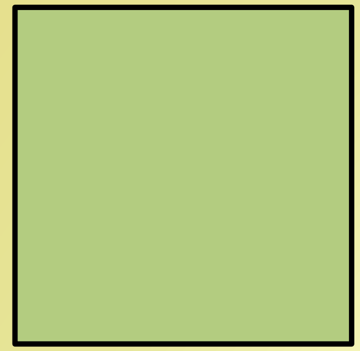

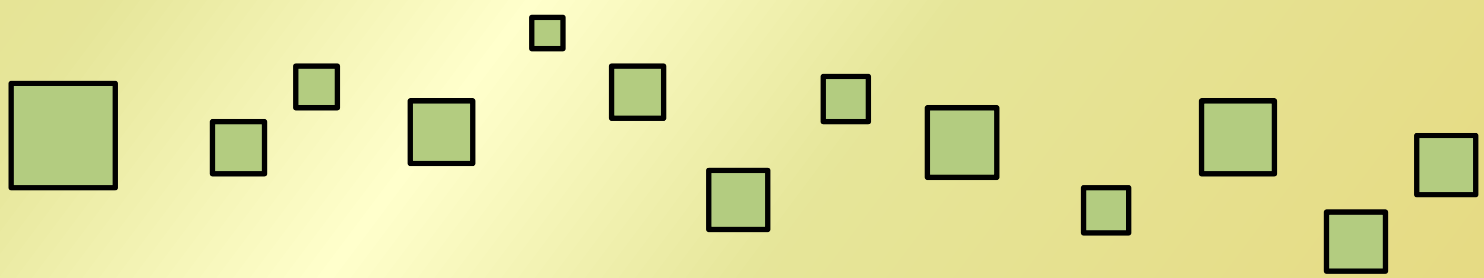

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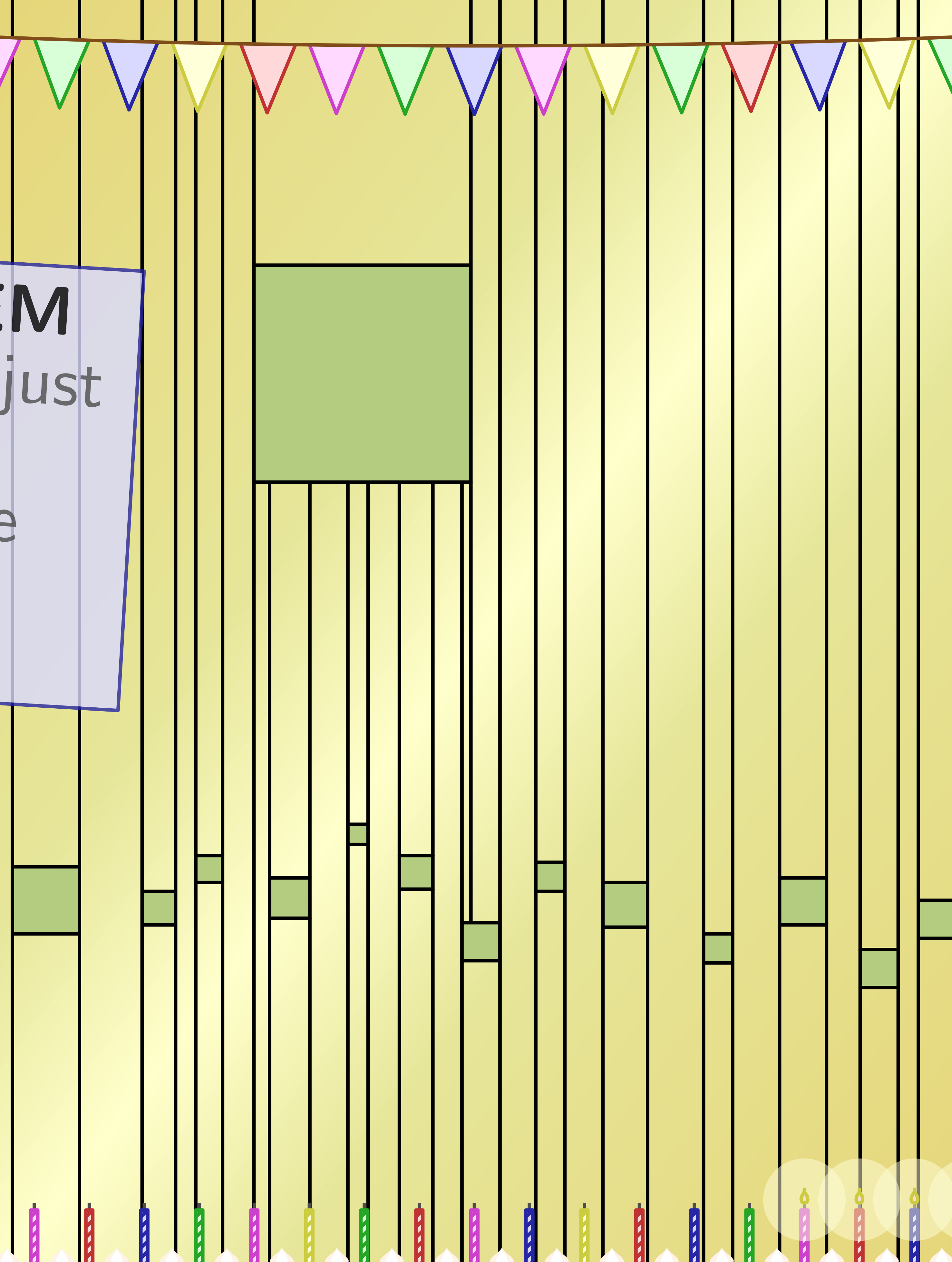

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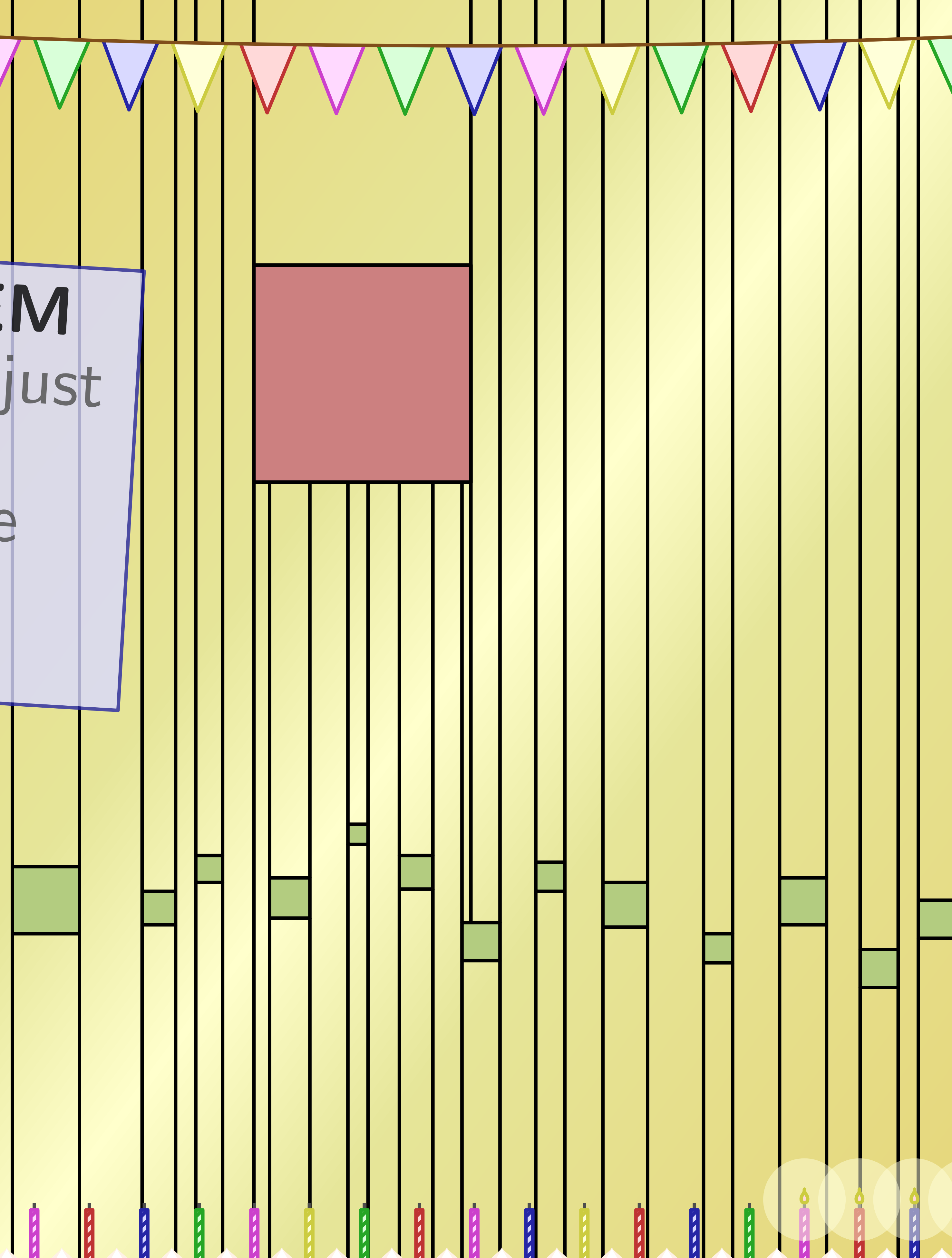

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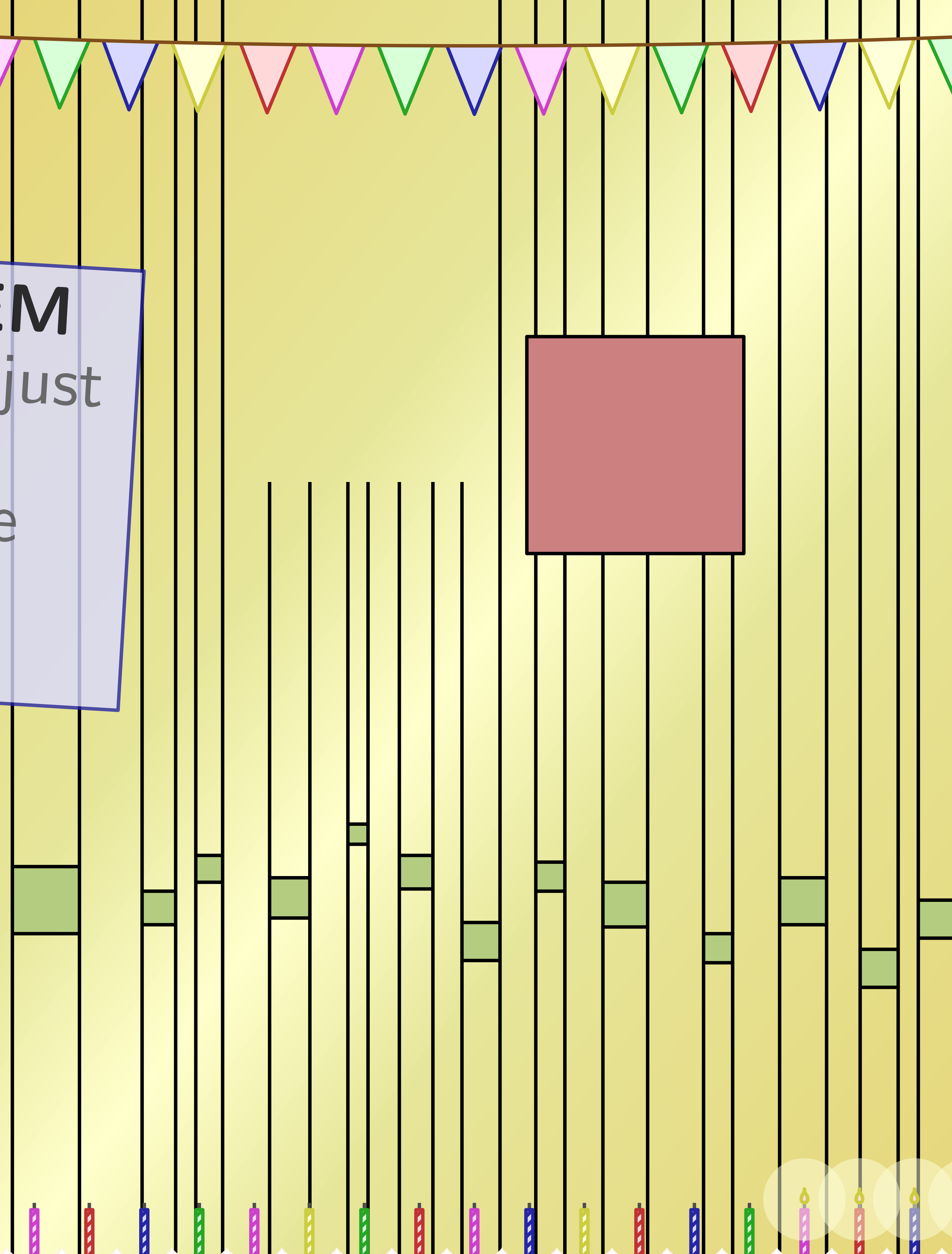

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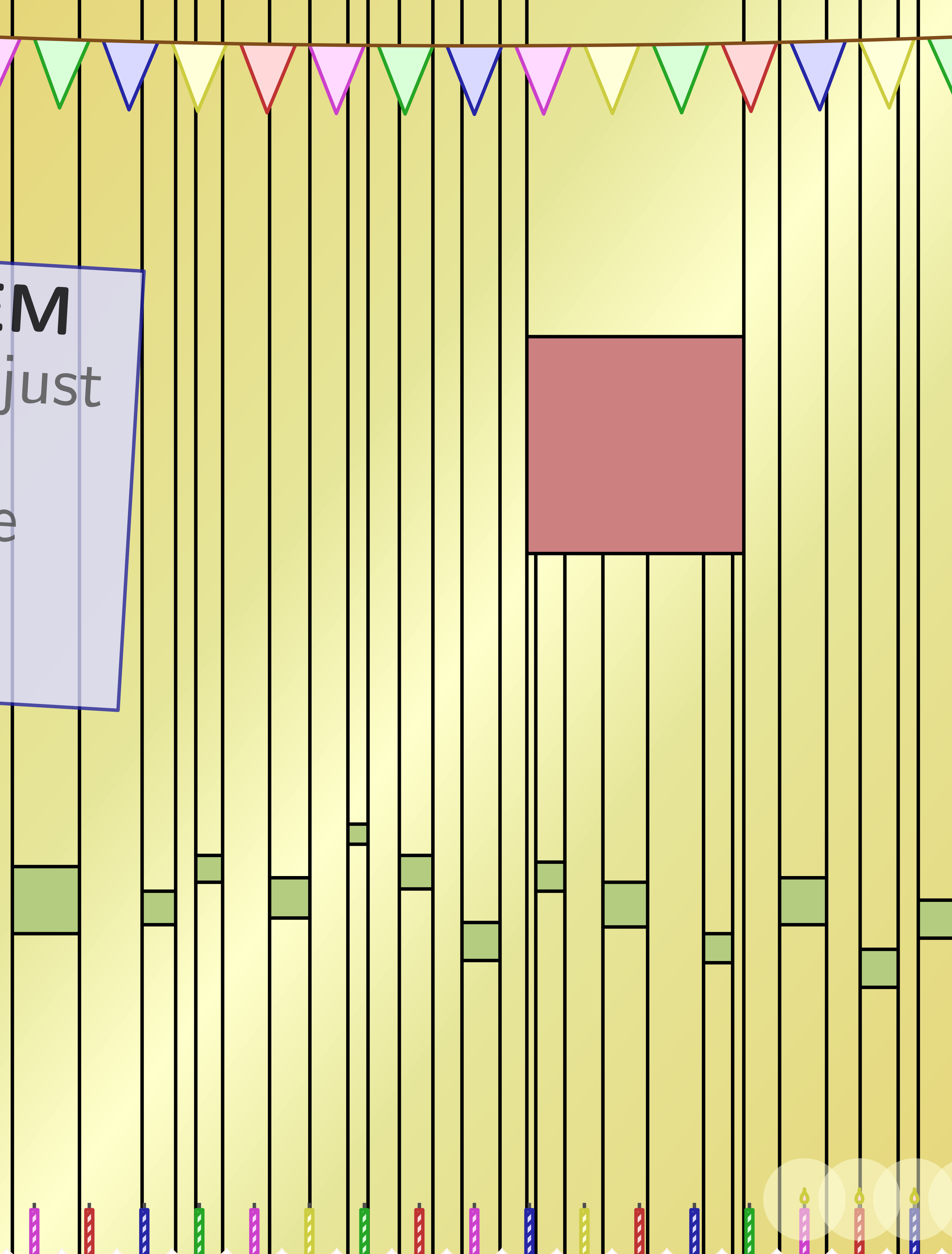

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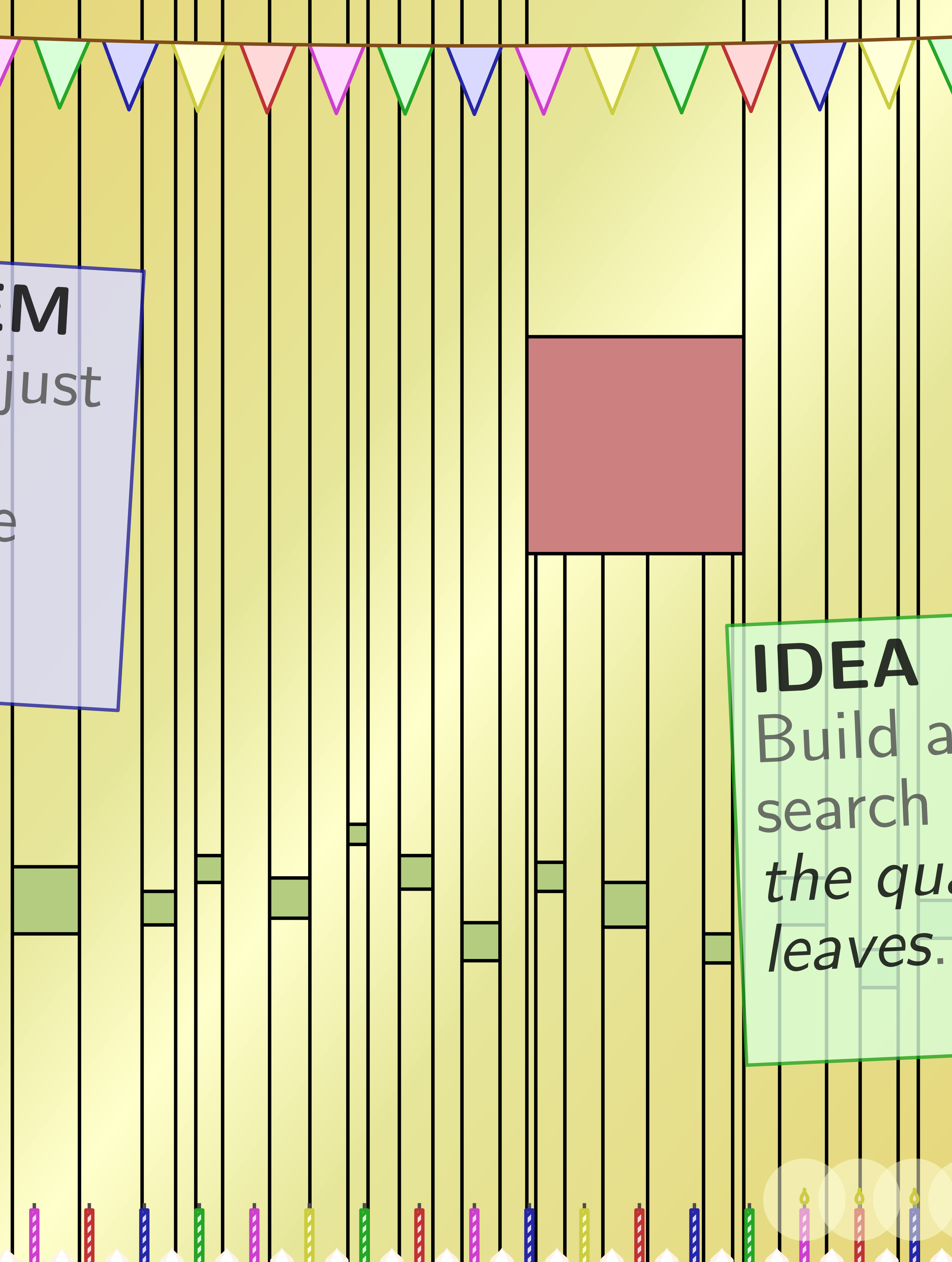

IDEA

Build a search tree *on the quadtree leaves*.

Take another look at a quadtree.

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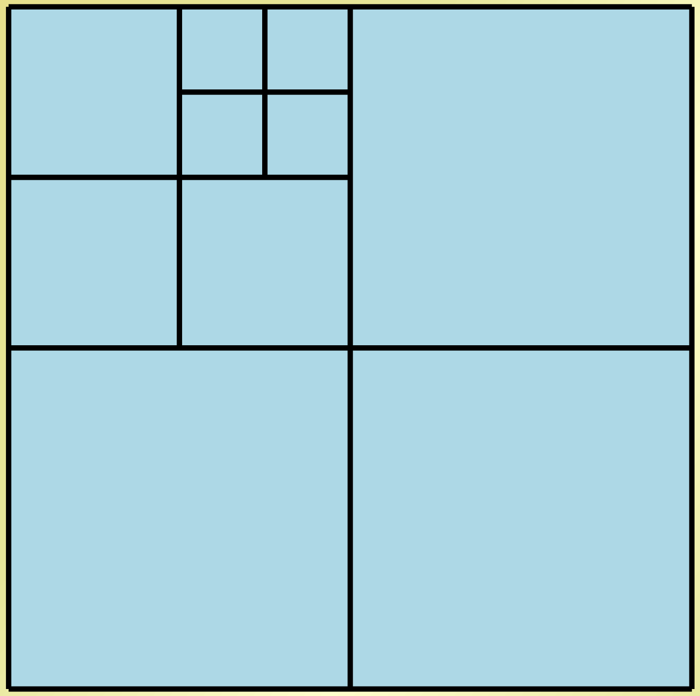
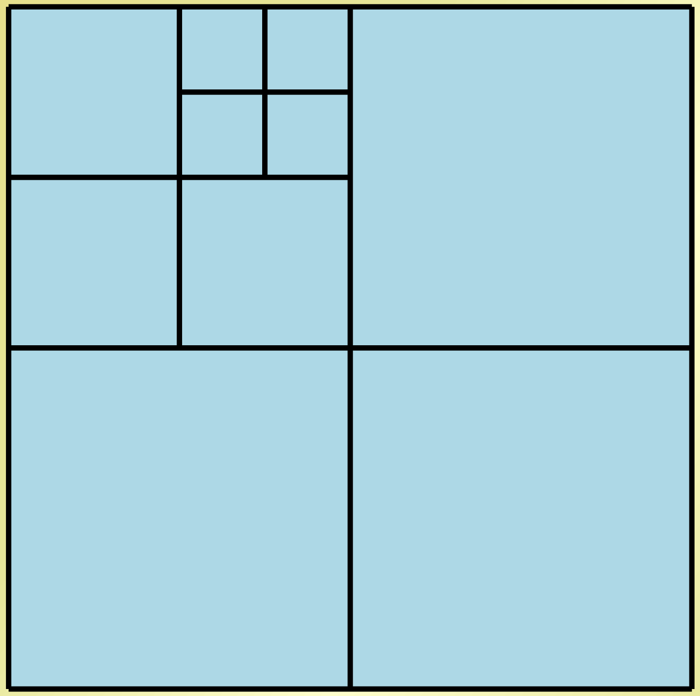


Take another look at a quadtree.

It's a degree 4 tree of potentially linear height.



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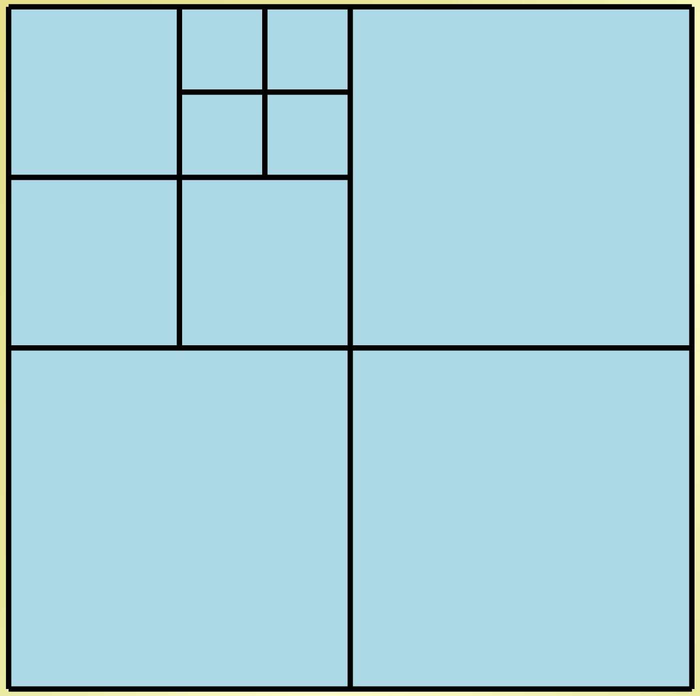

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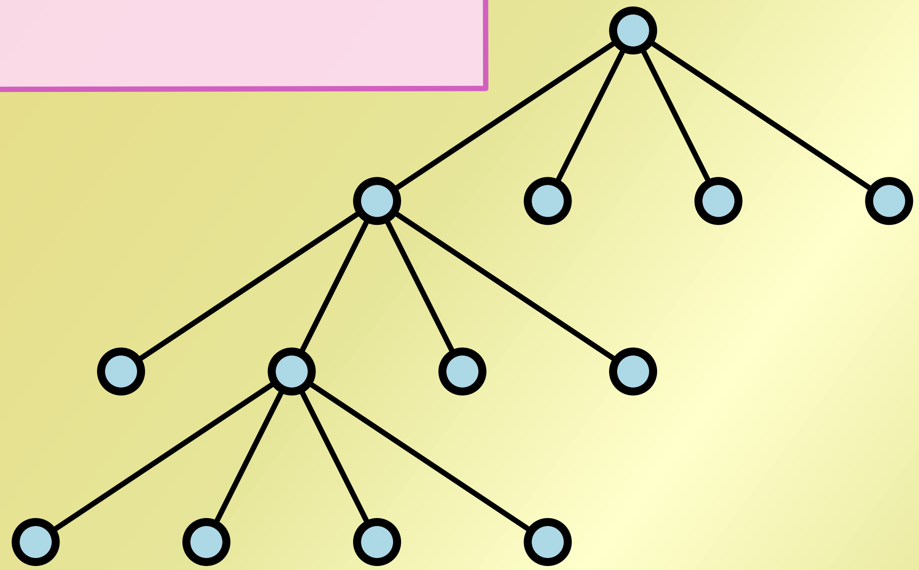

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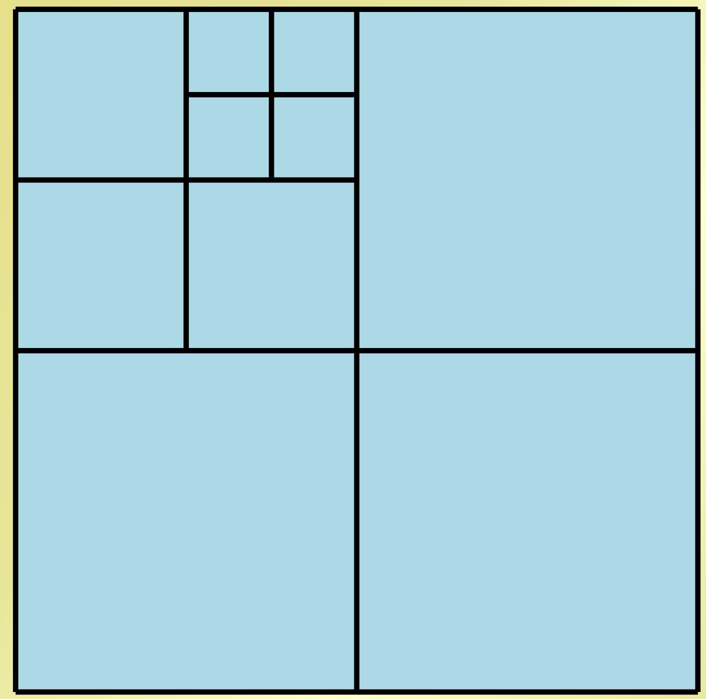

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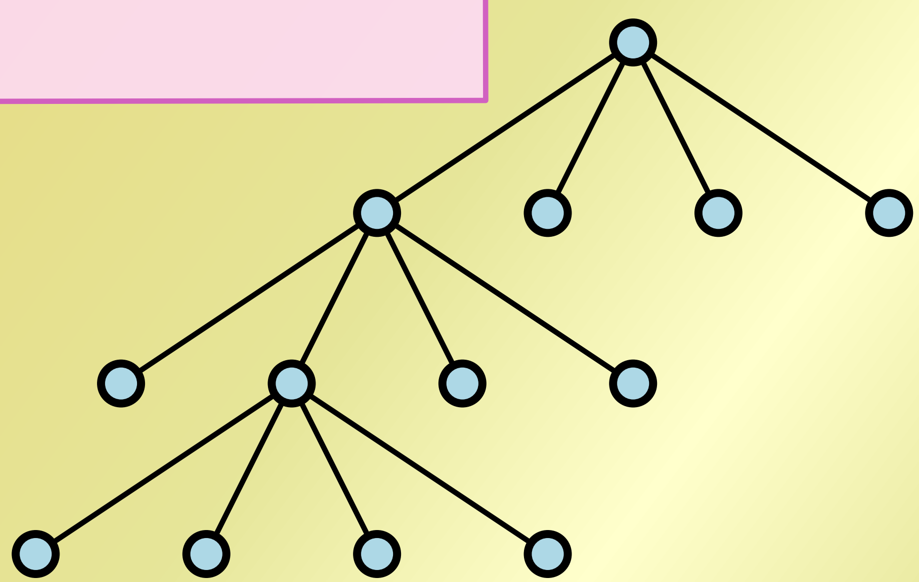

We build another tree, with a vertex for each edge of the quadtree.

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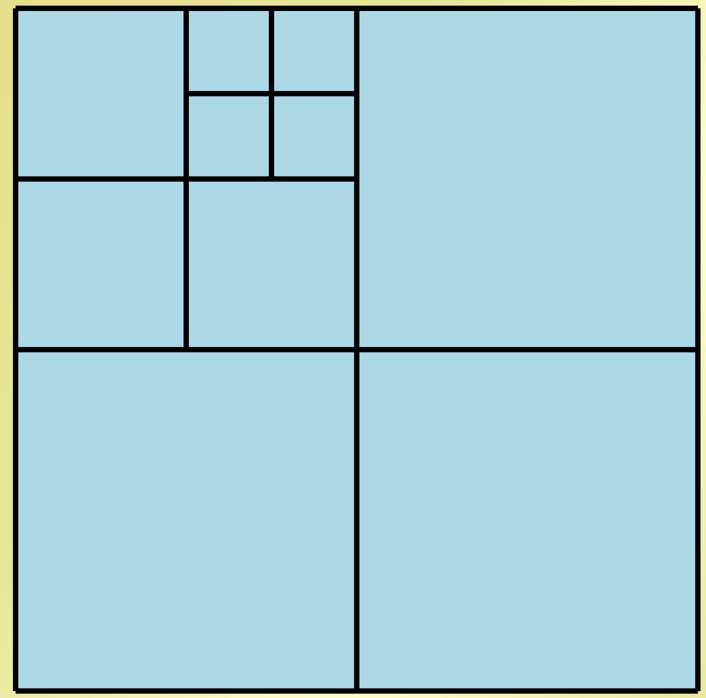

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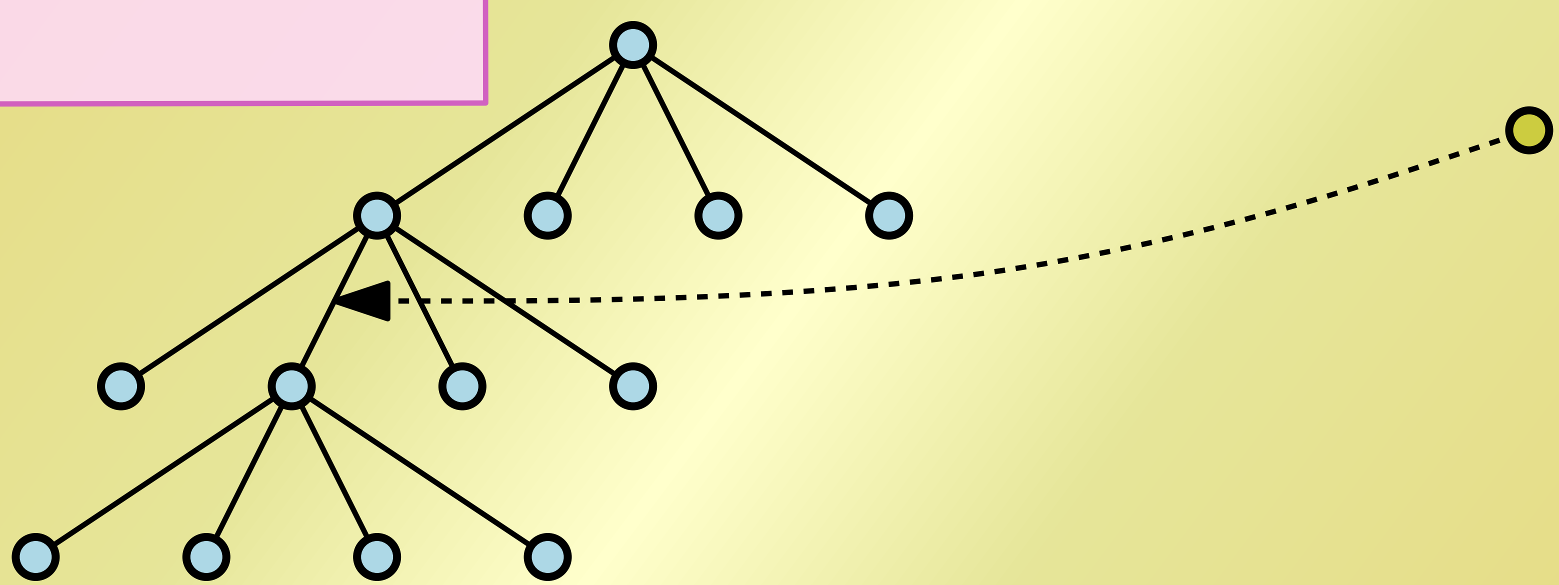

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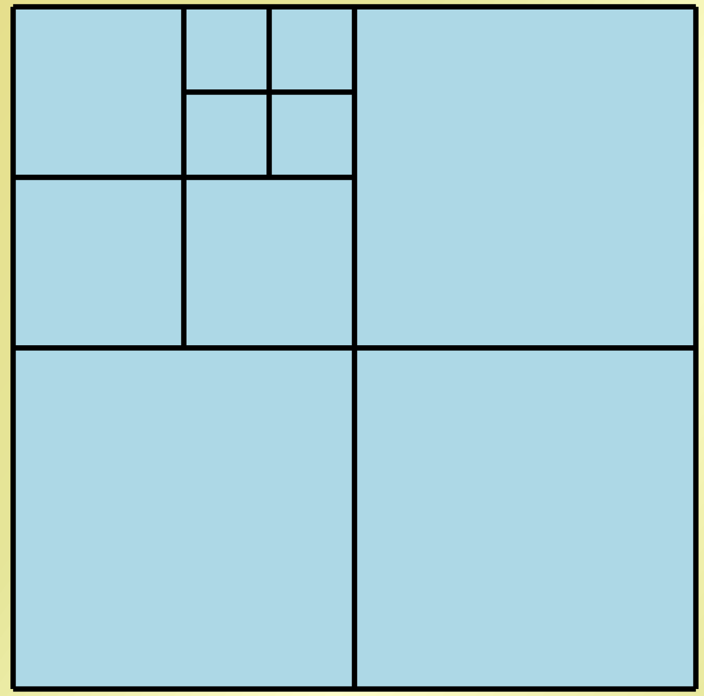

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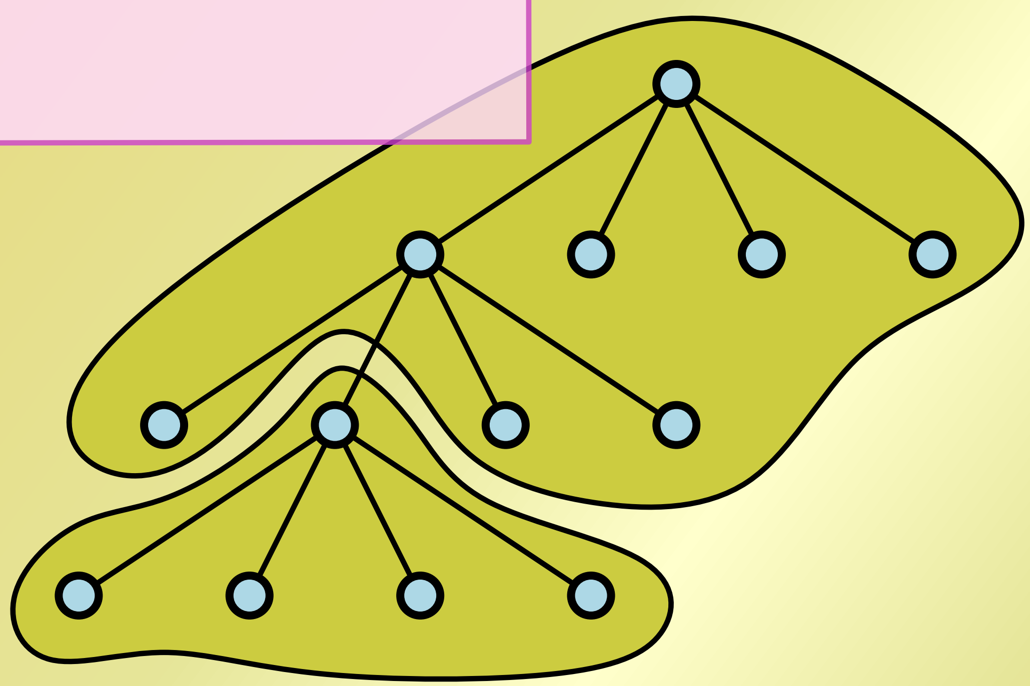

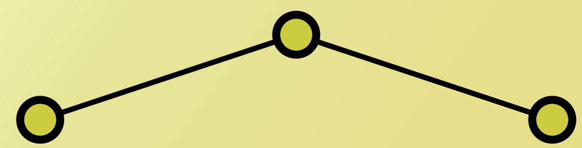

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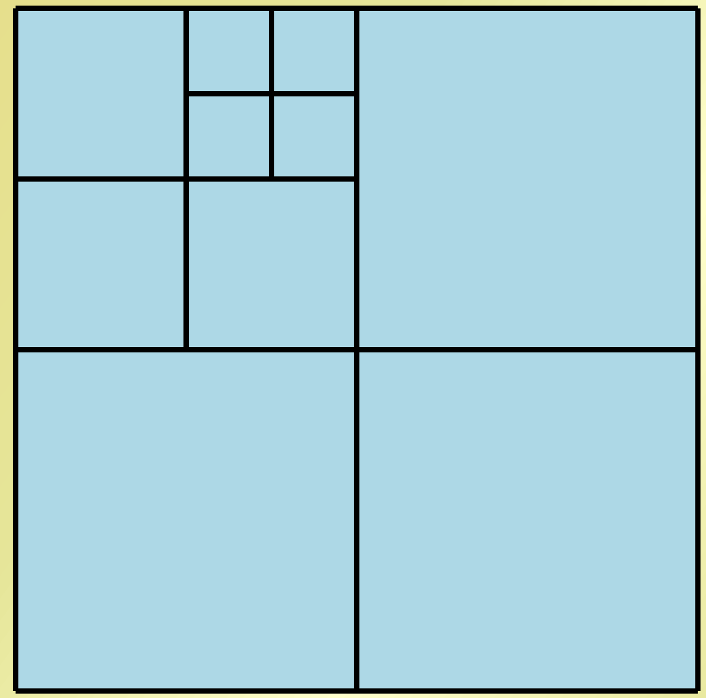

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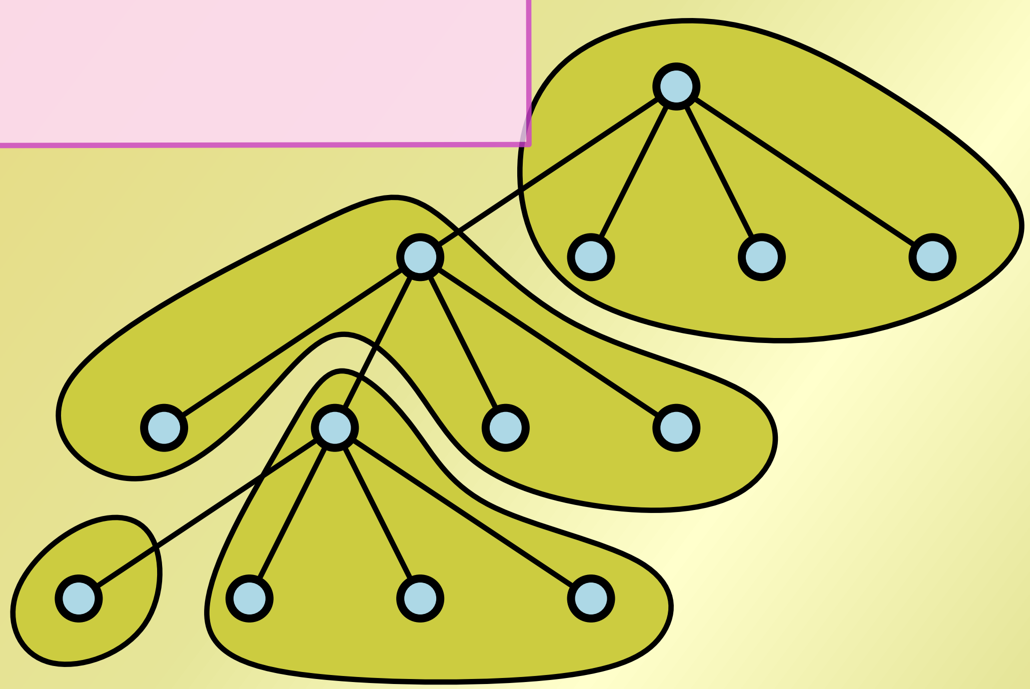

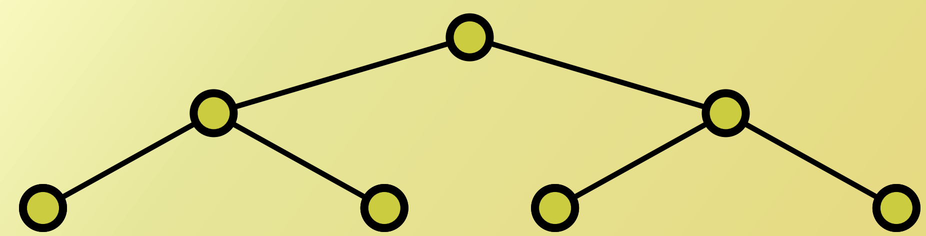

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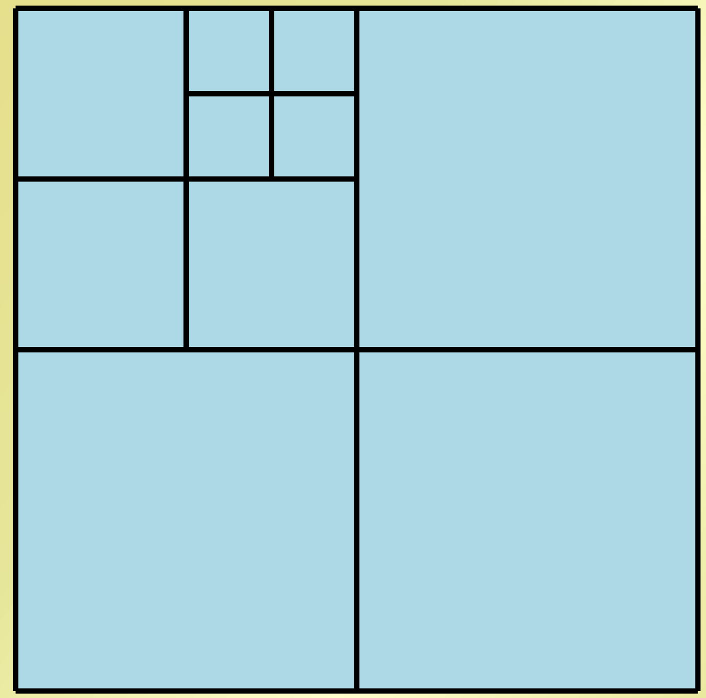

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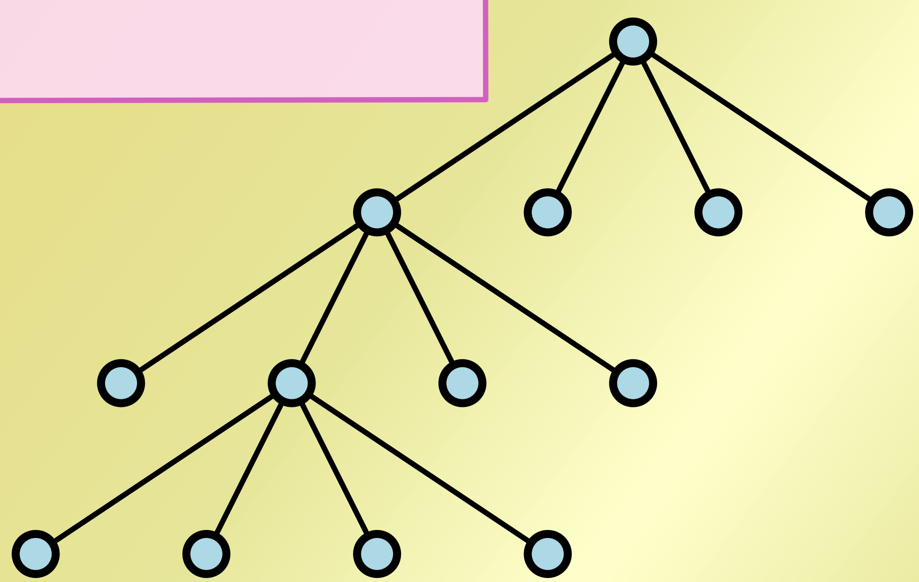

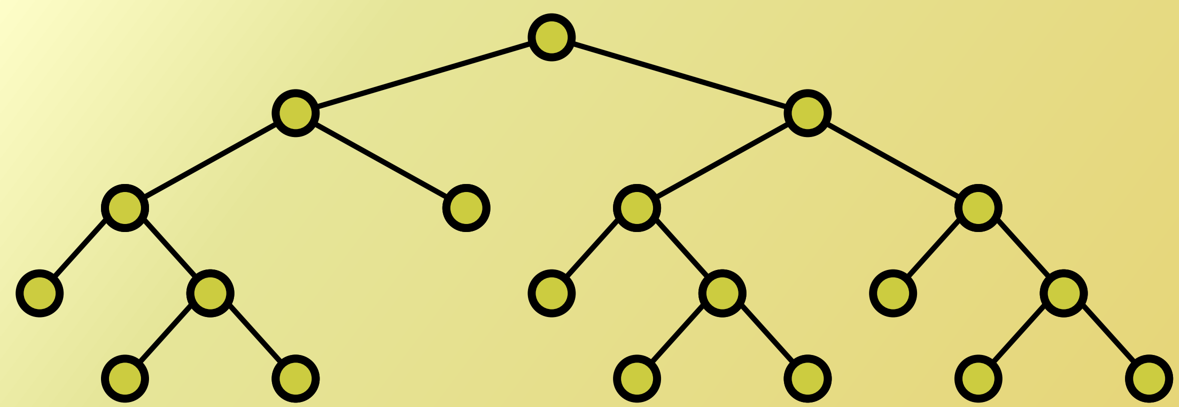

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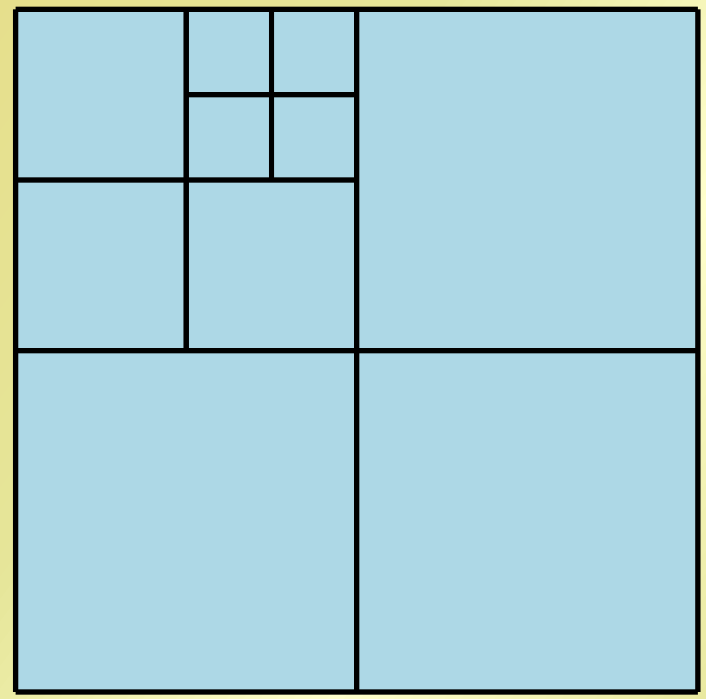

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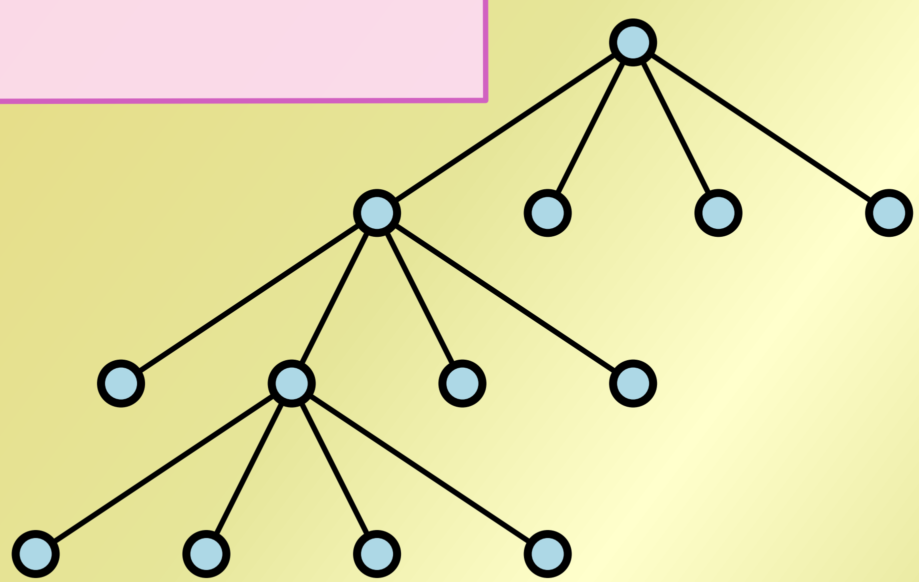
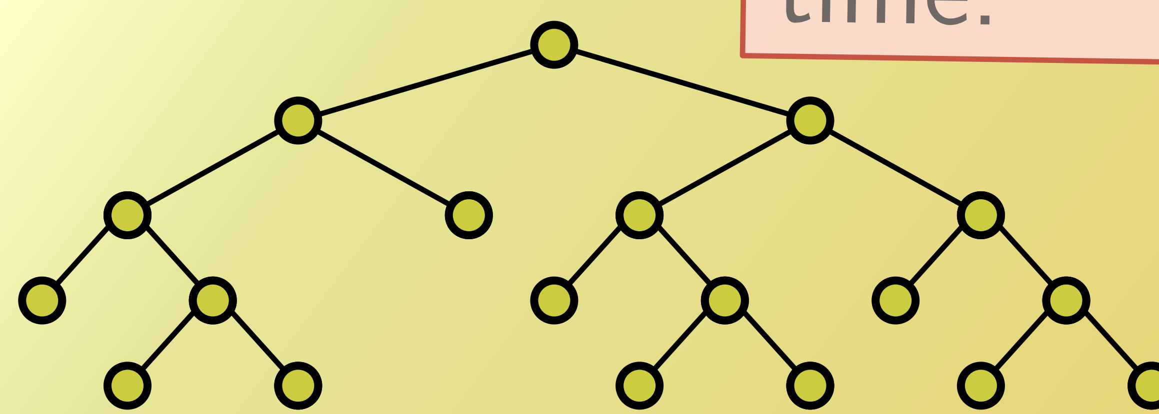
Now we can locate points in the quadtree in $O(\log n)$ time.





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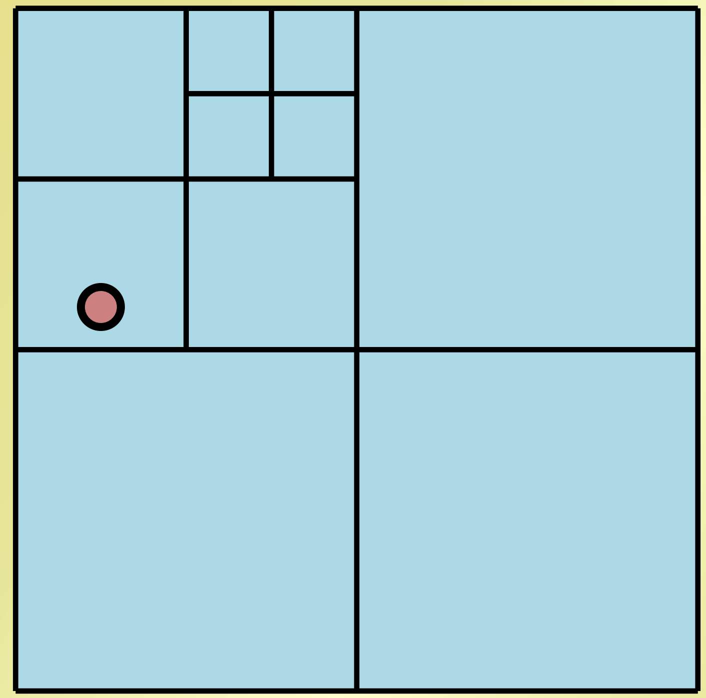

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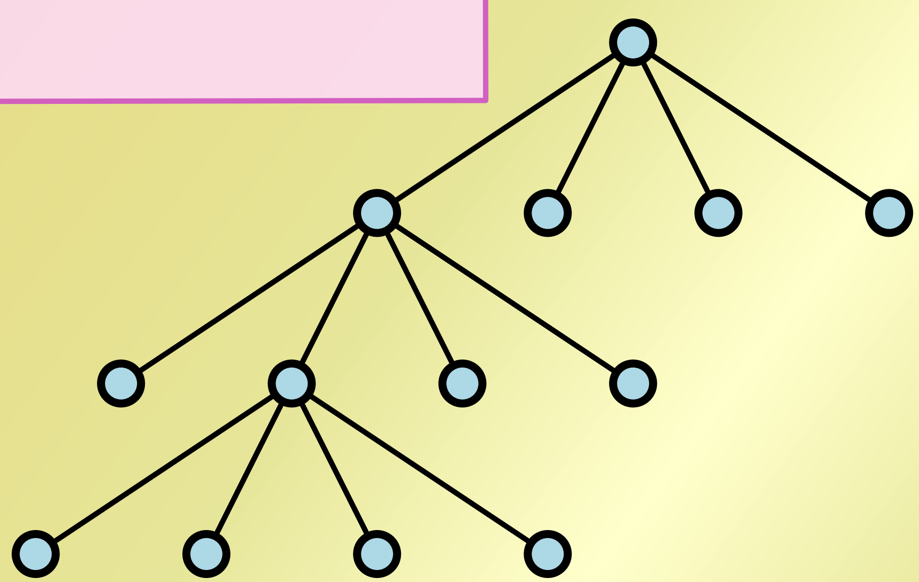

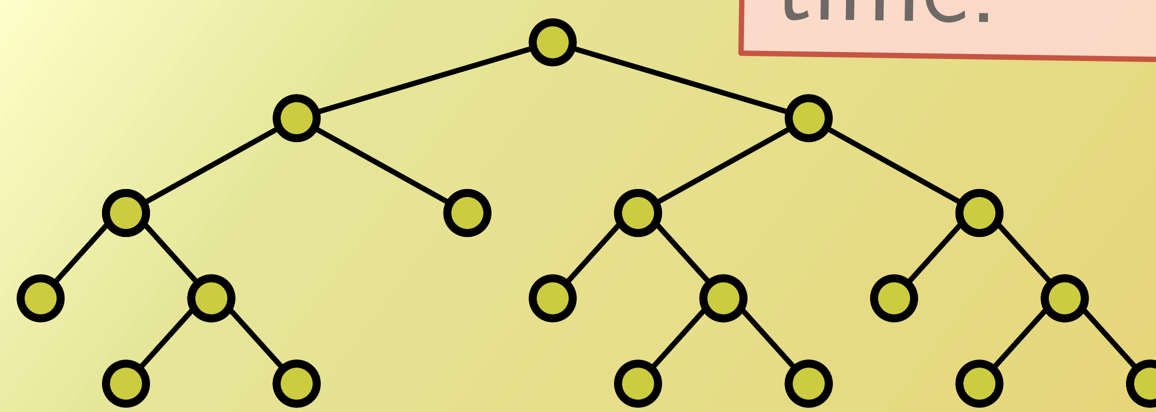

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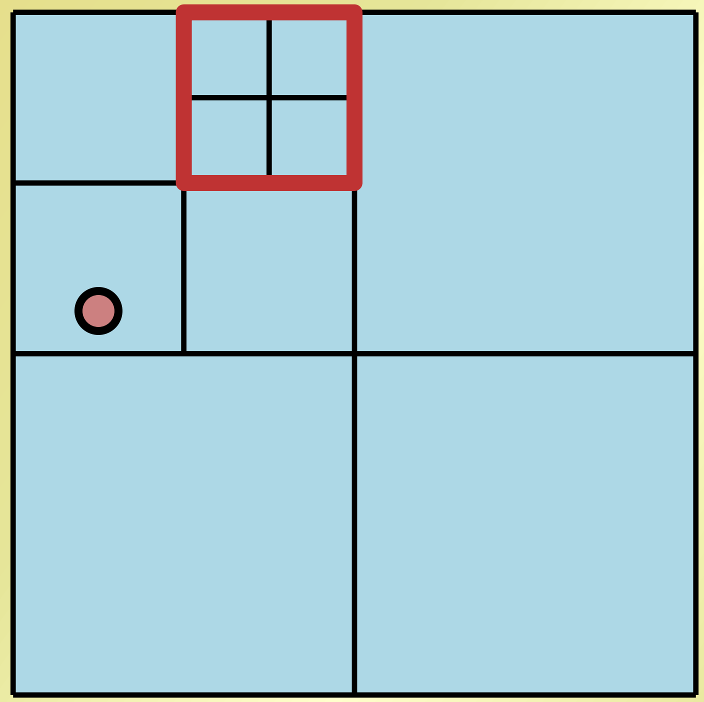

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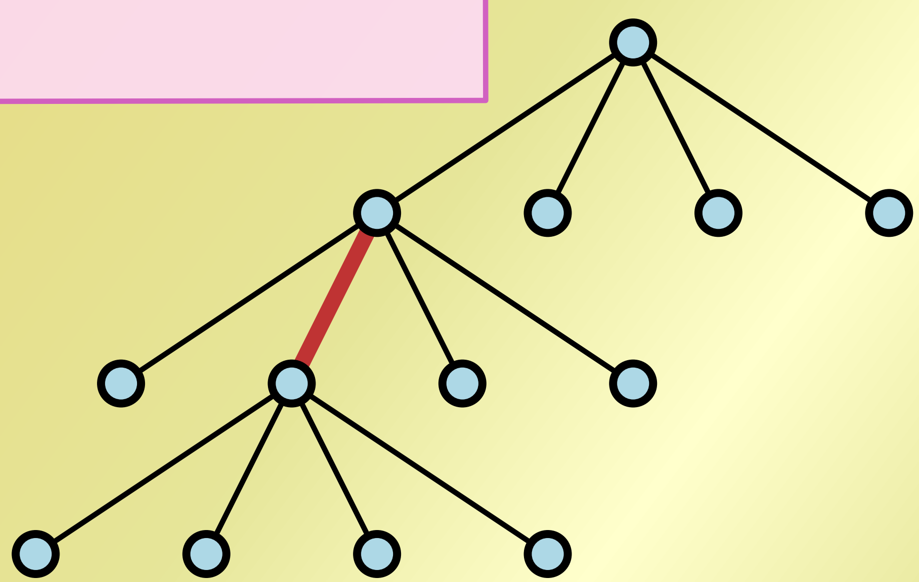

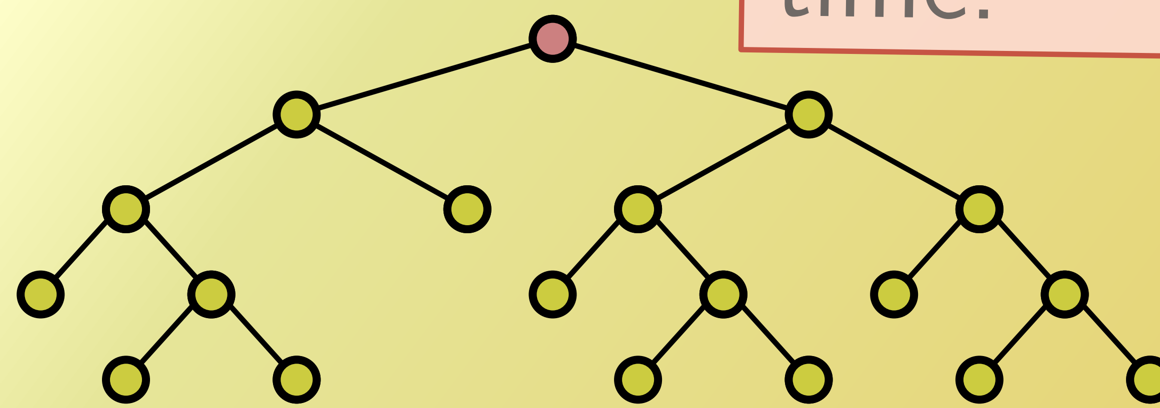

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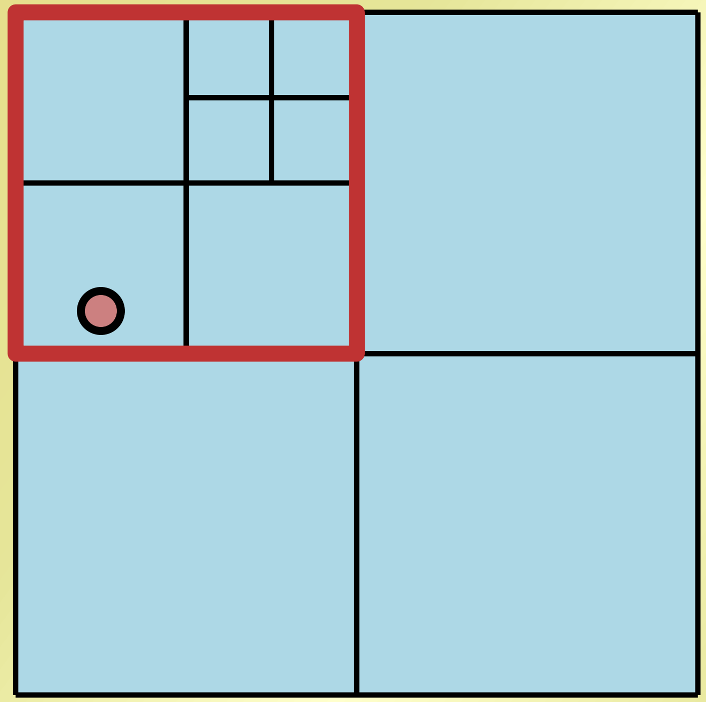

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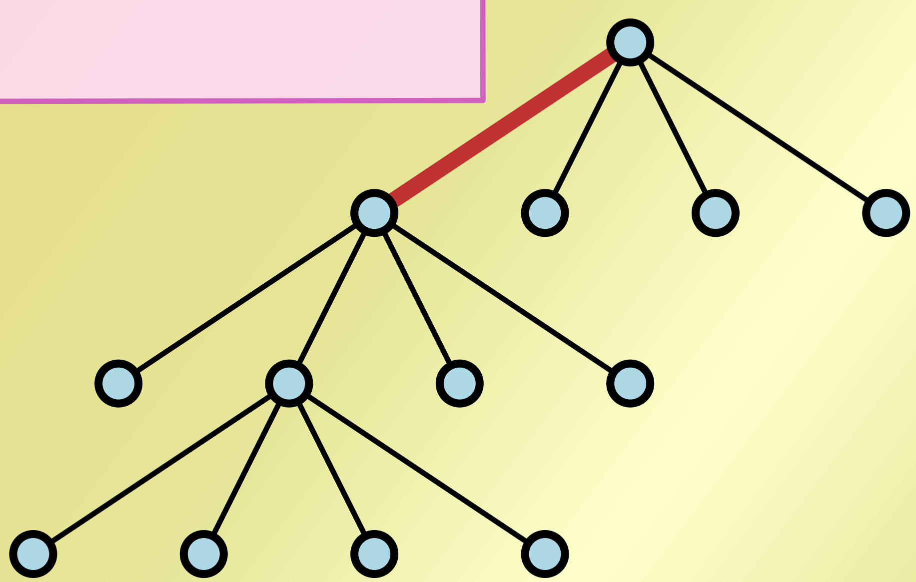

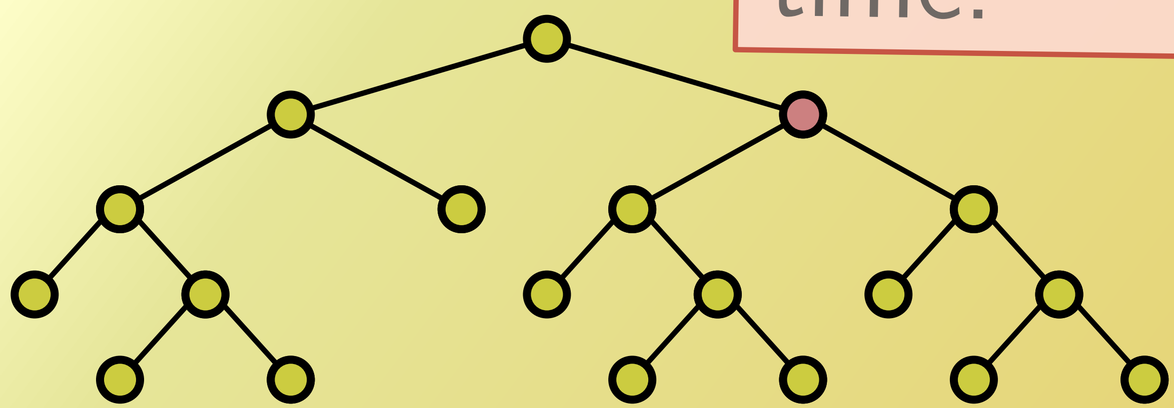

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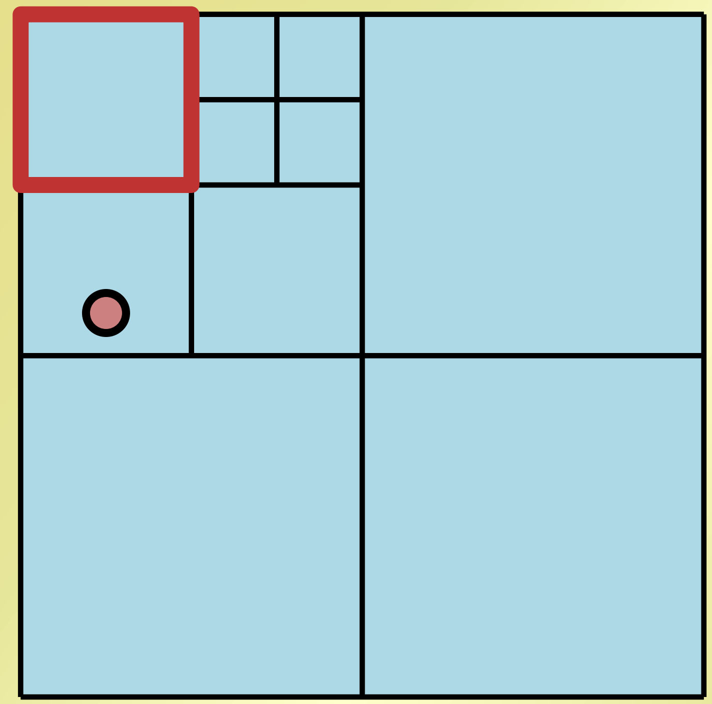

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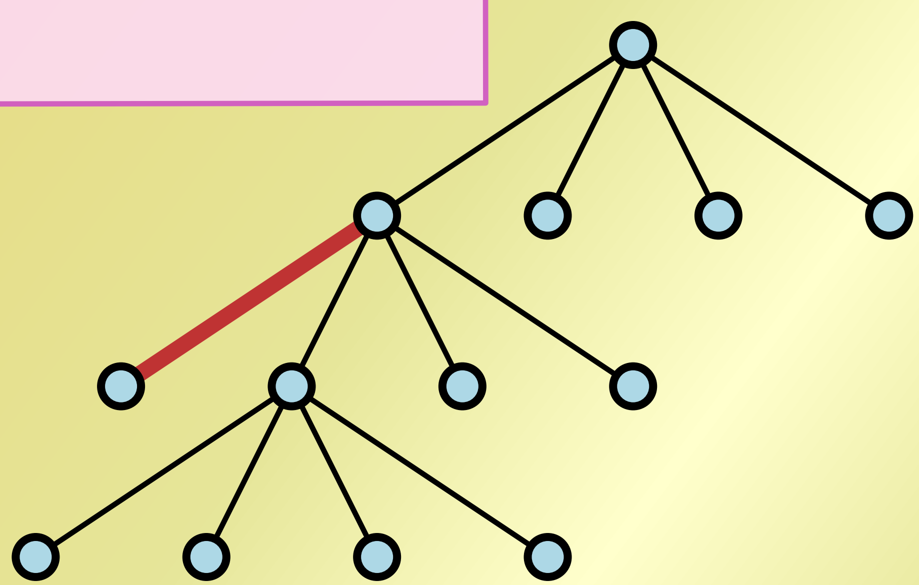

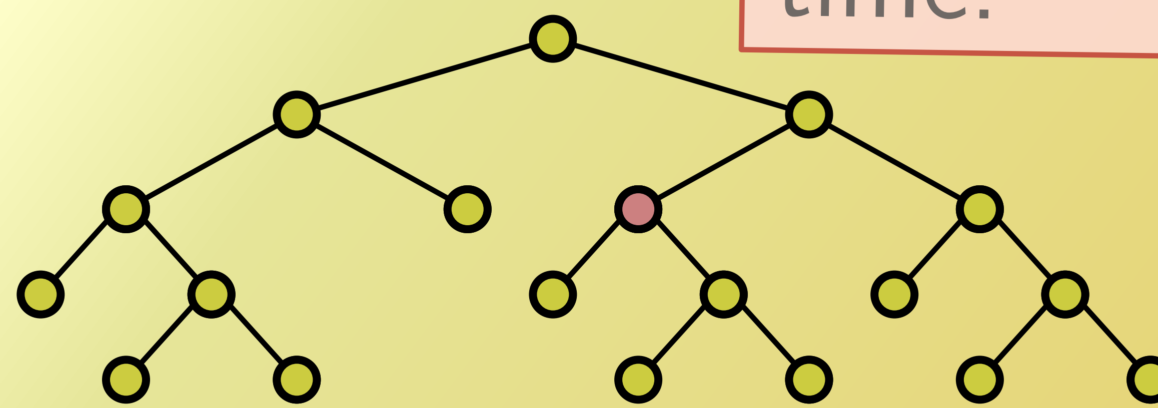

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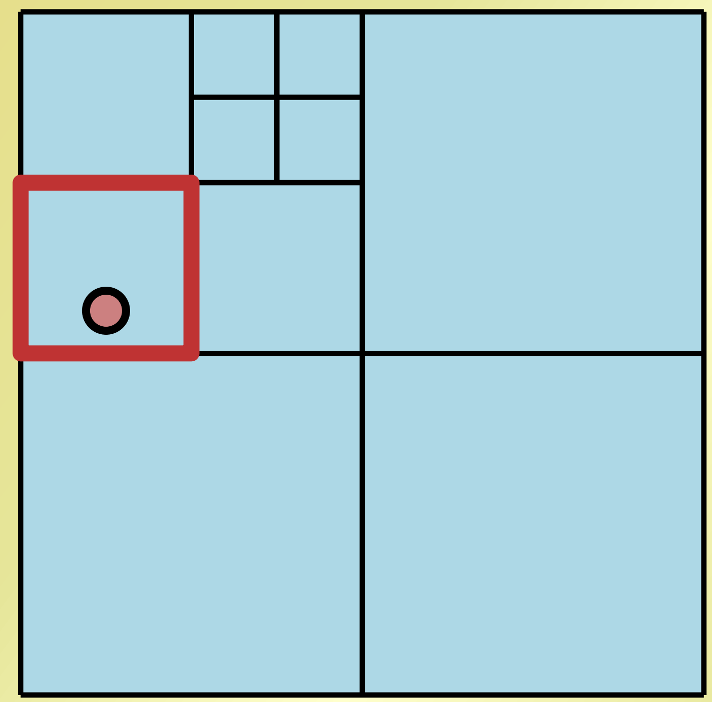

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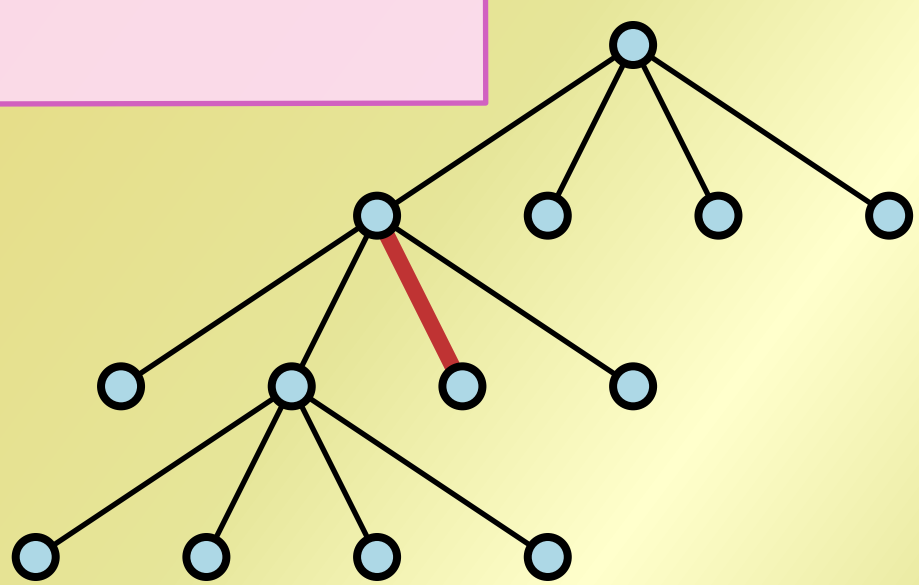

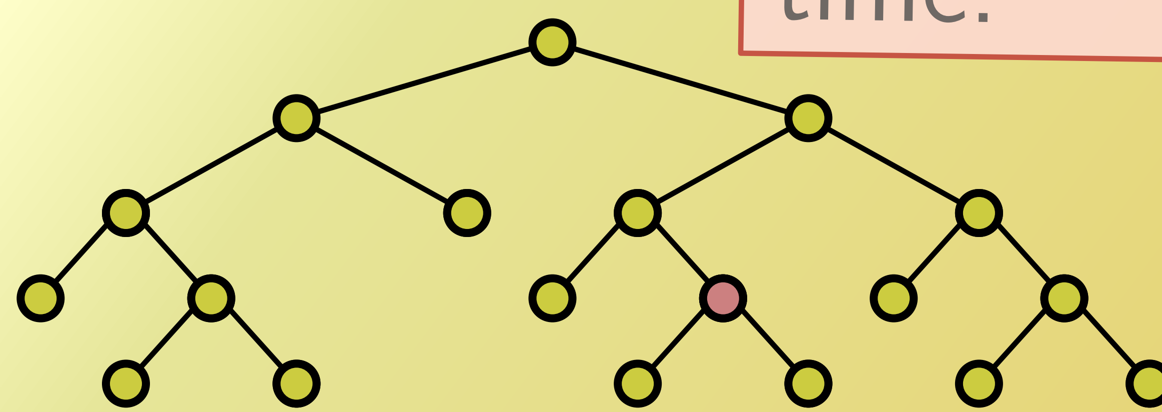

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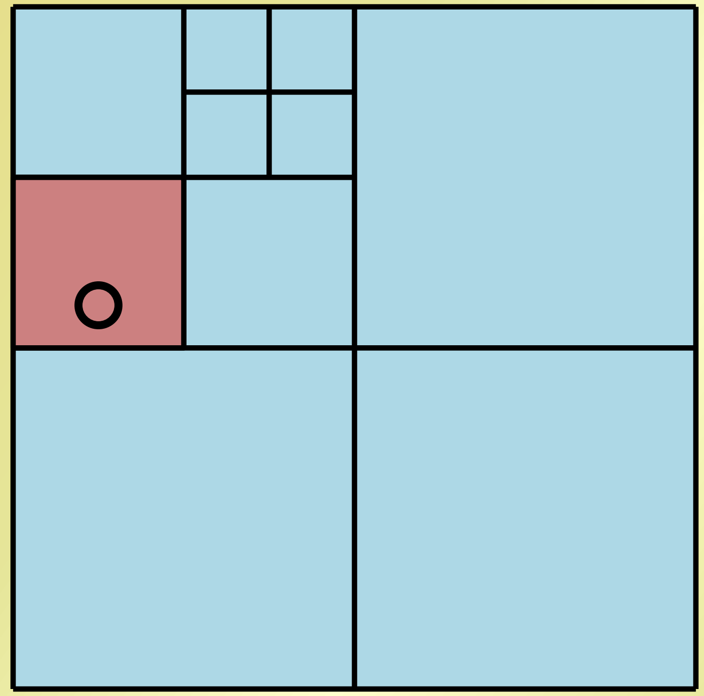

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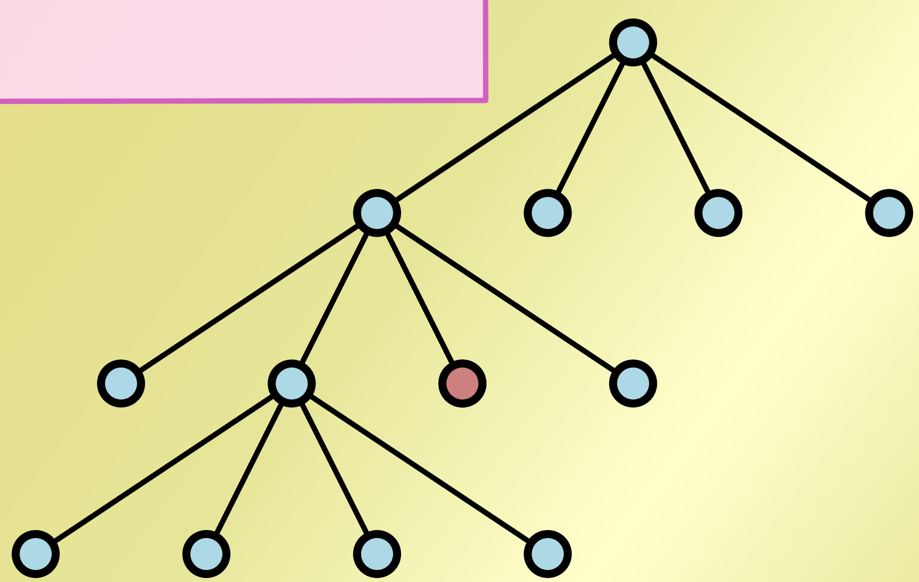

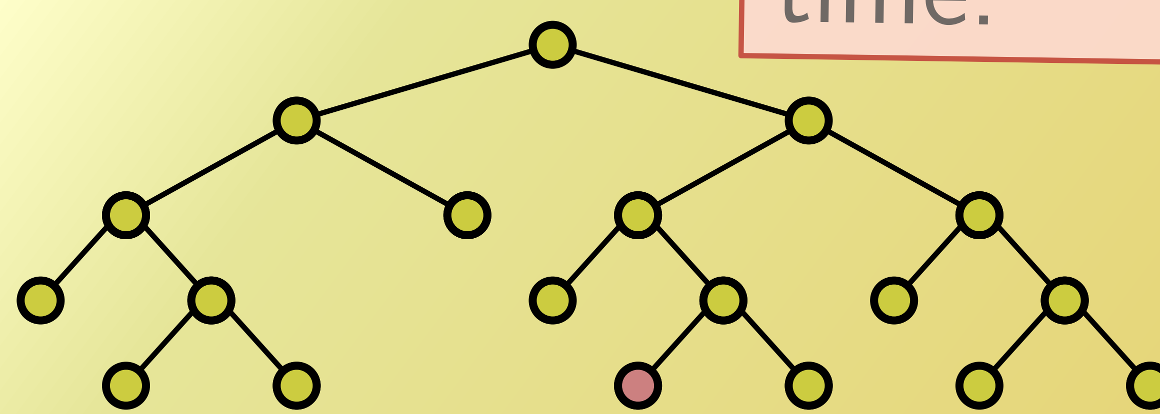

PROBLEM

The number of regions intersecting a quadtree leaf can be linear!

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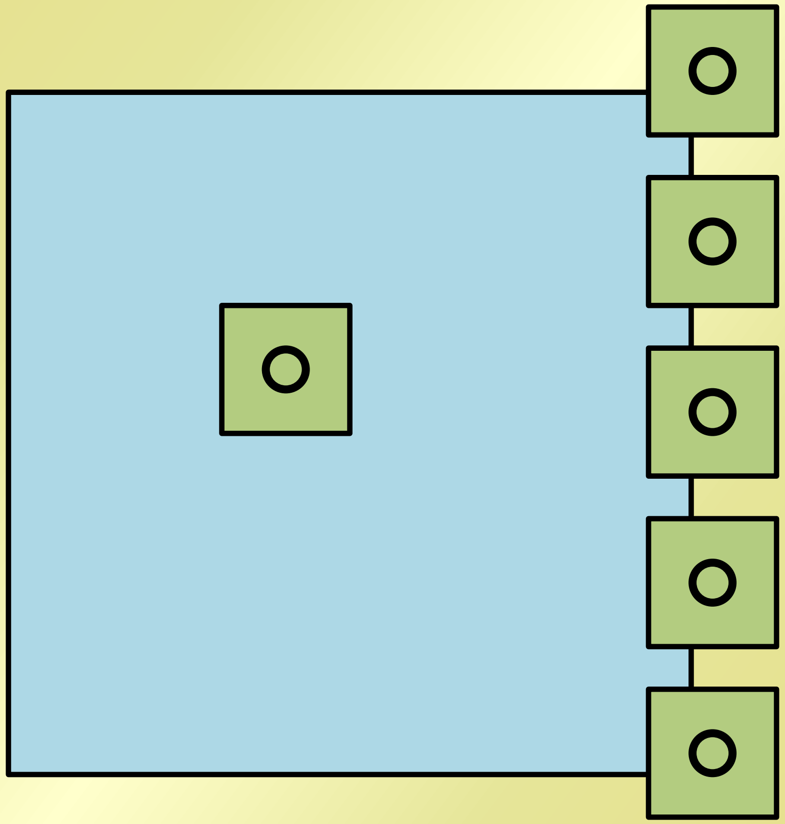

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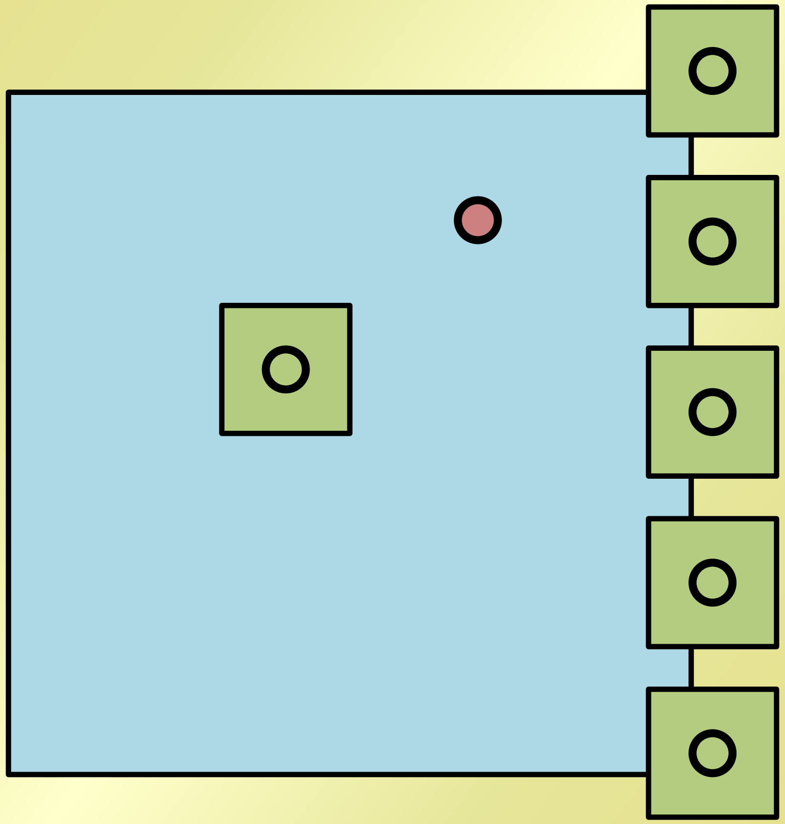

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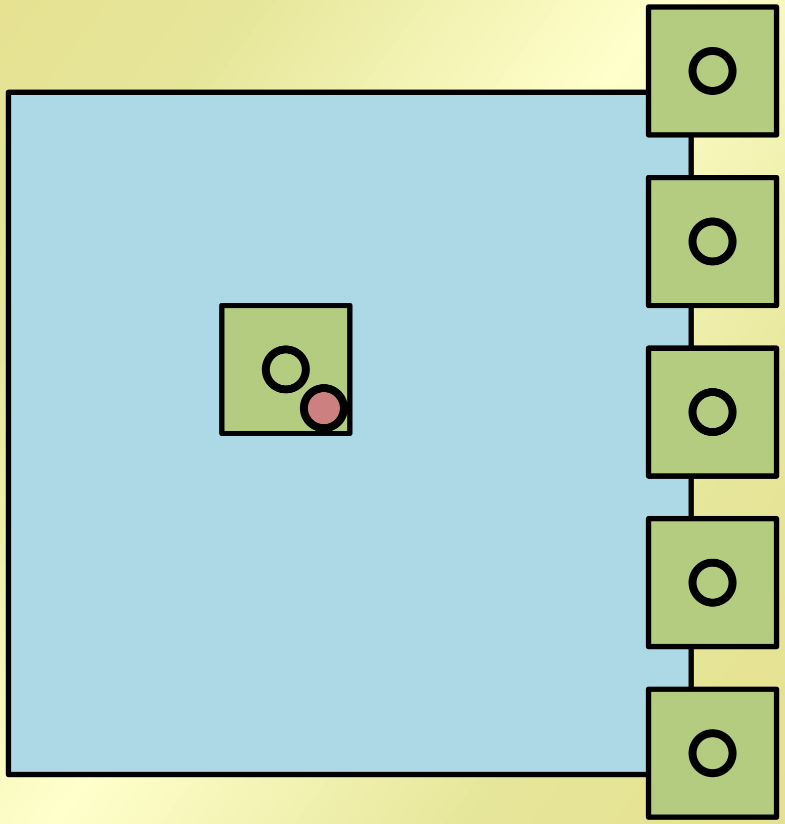

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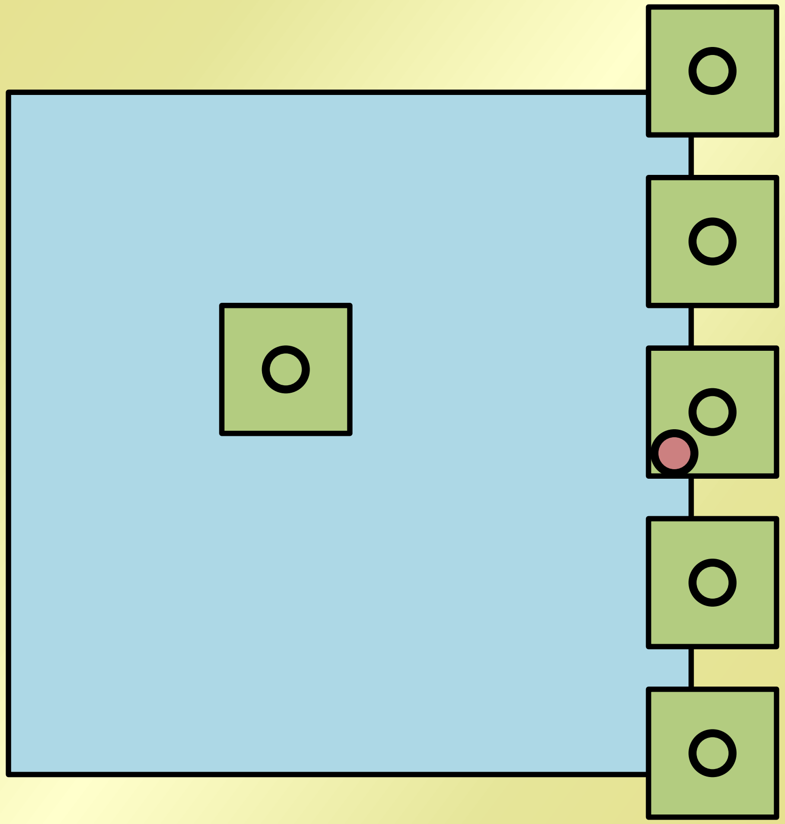

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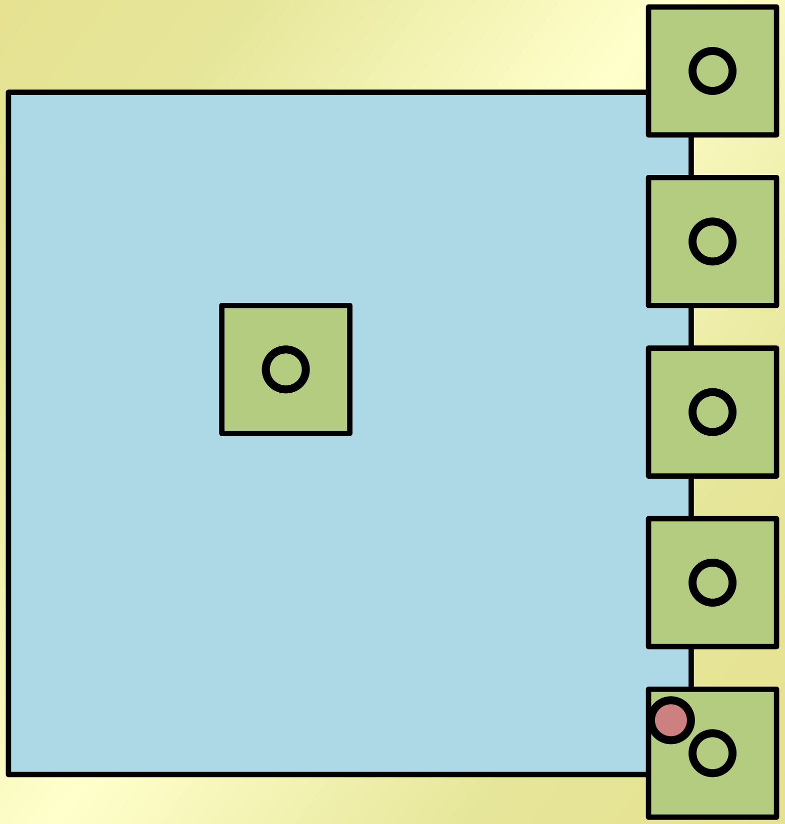

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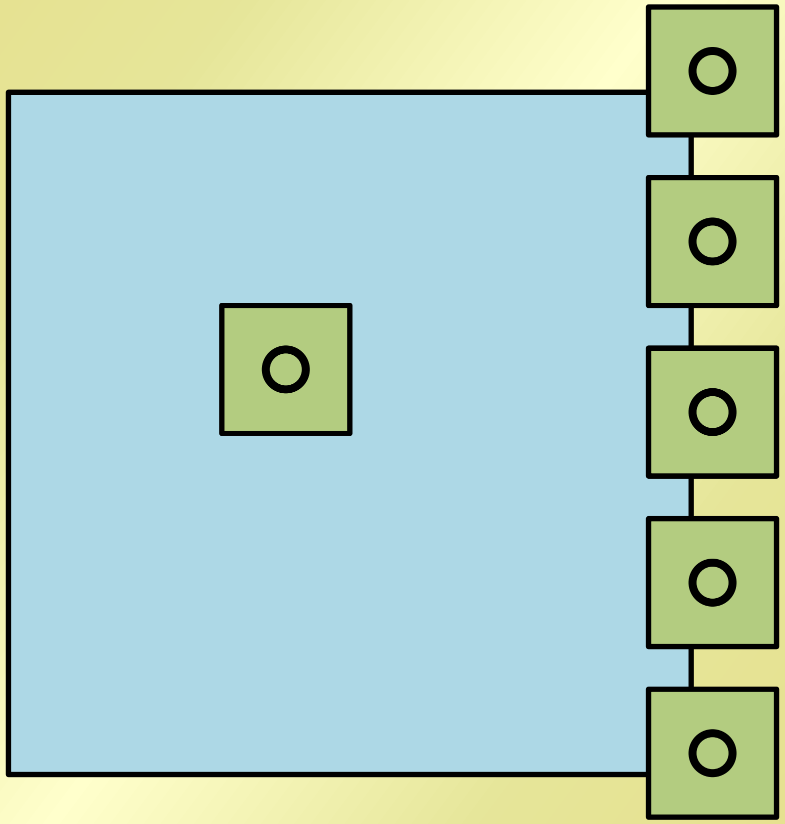
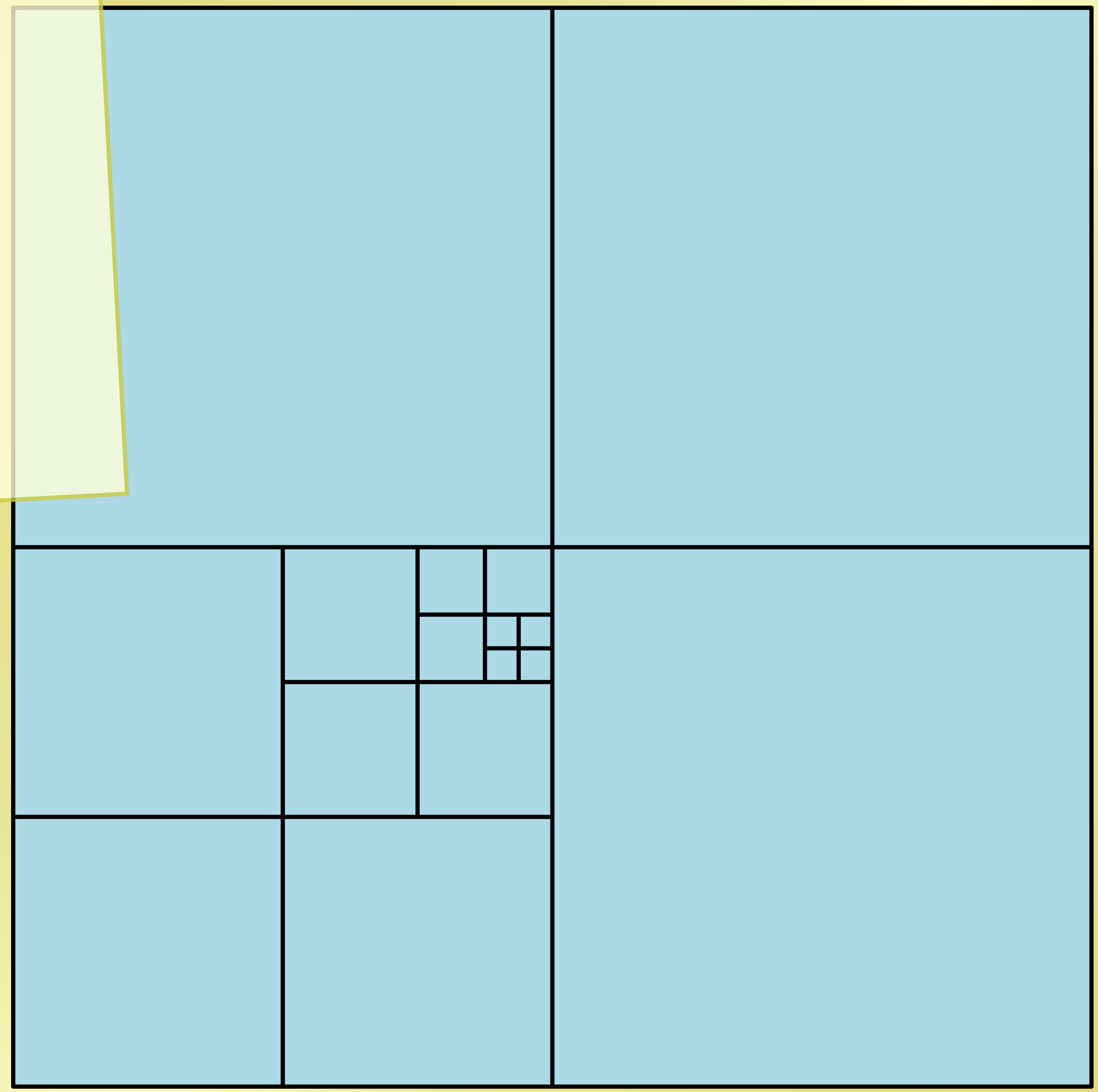


IDEA

Maintain a *balanced* quadtree.

Consider a quadtree again.

Consider a quadtree again.



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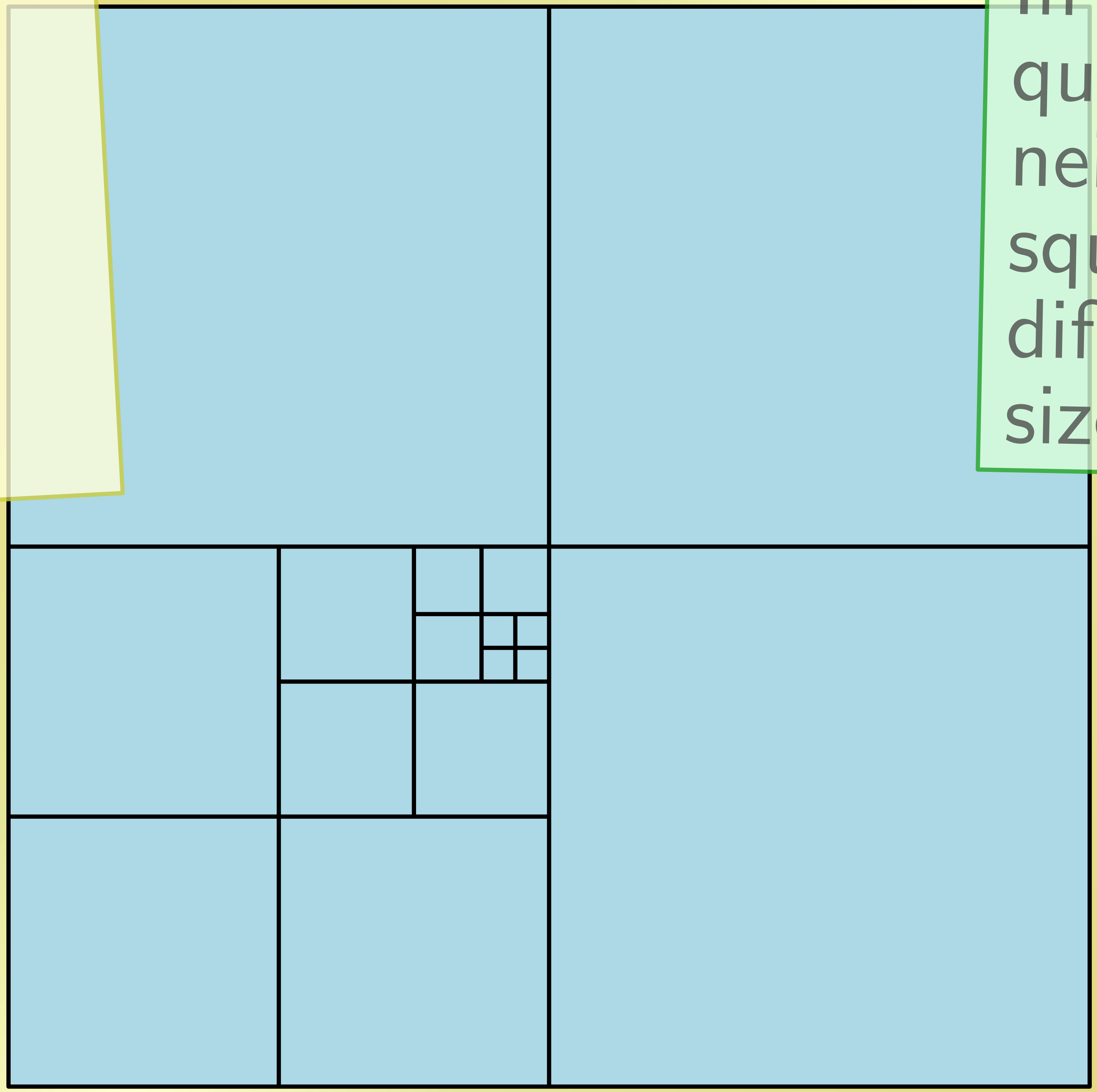
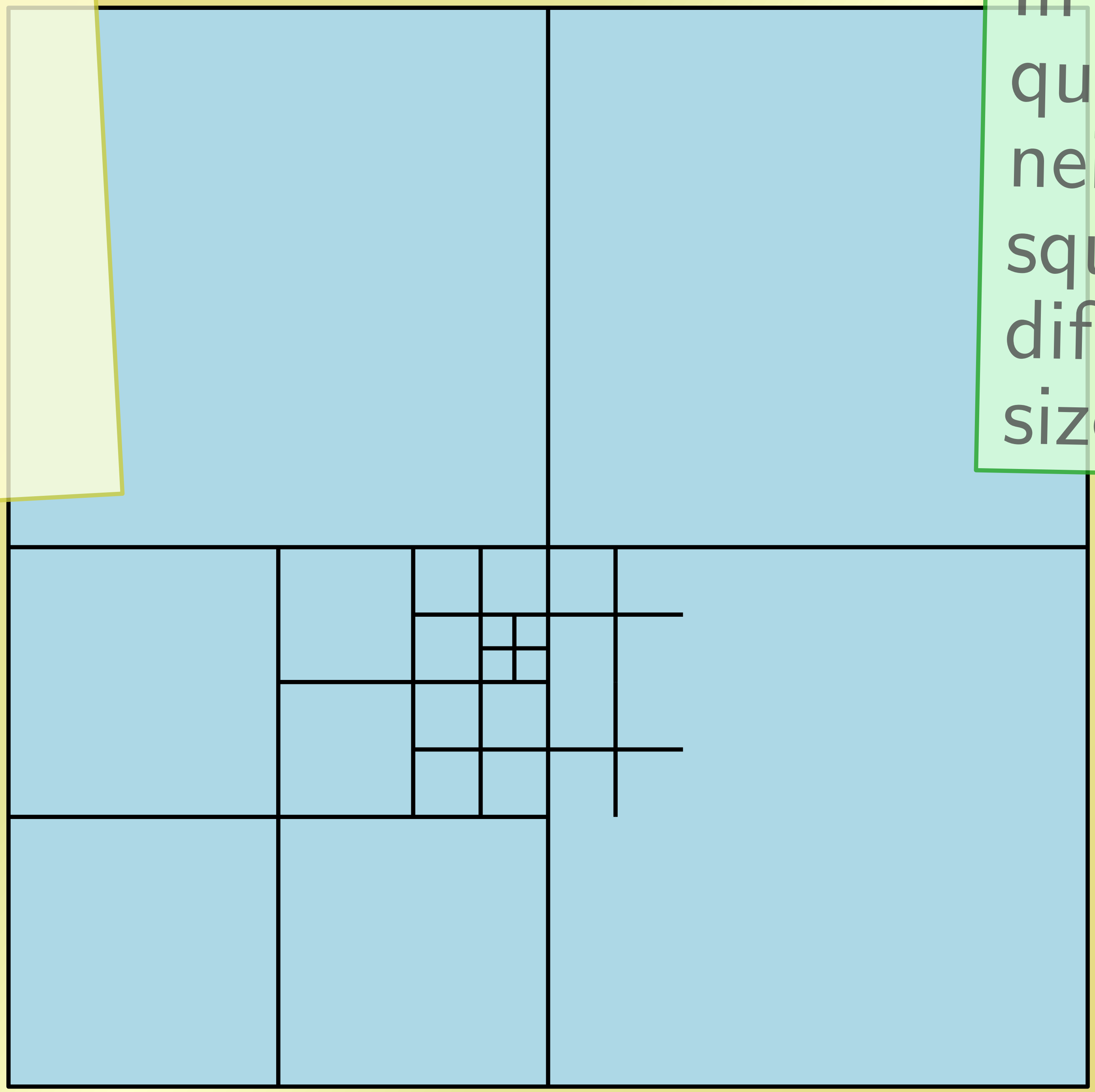


In a *balanced* quadtree, neighbouring squares don't differ much in size.

Consider a quadtree again.



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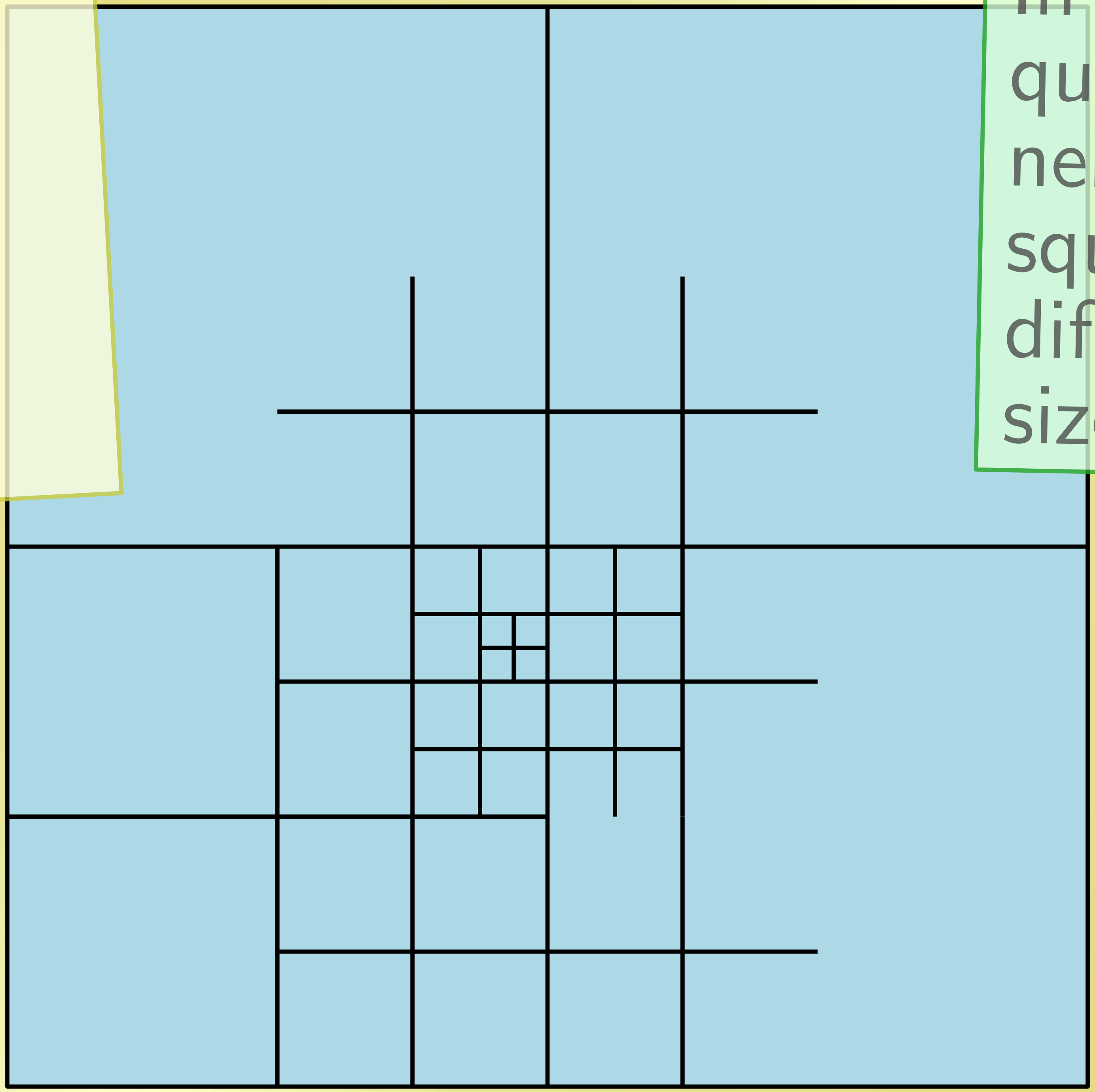

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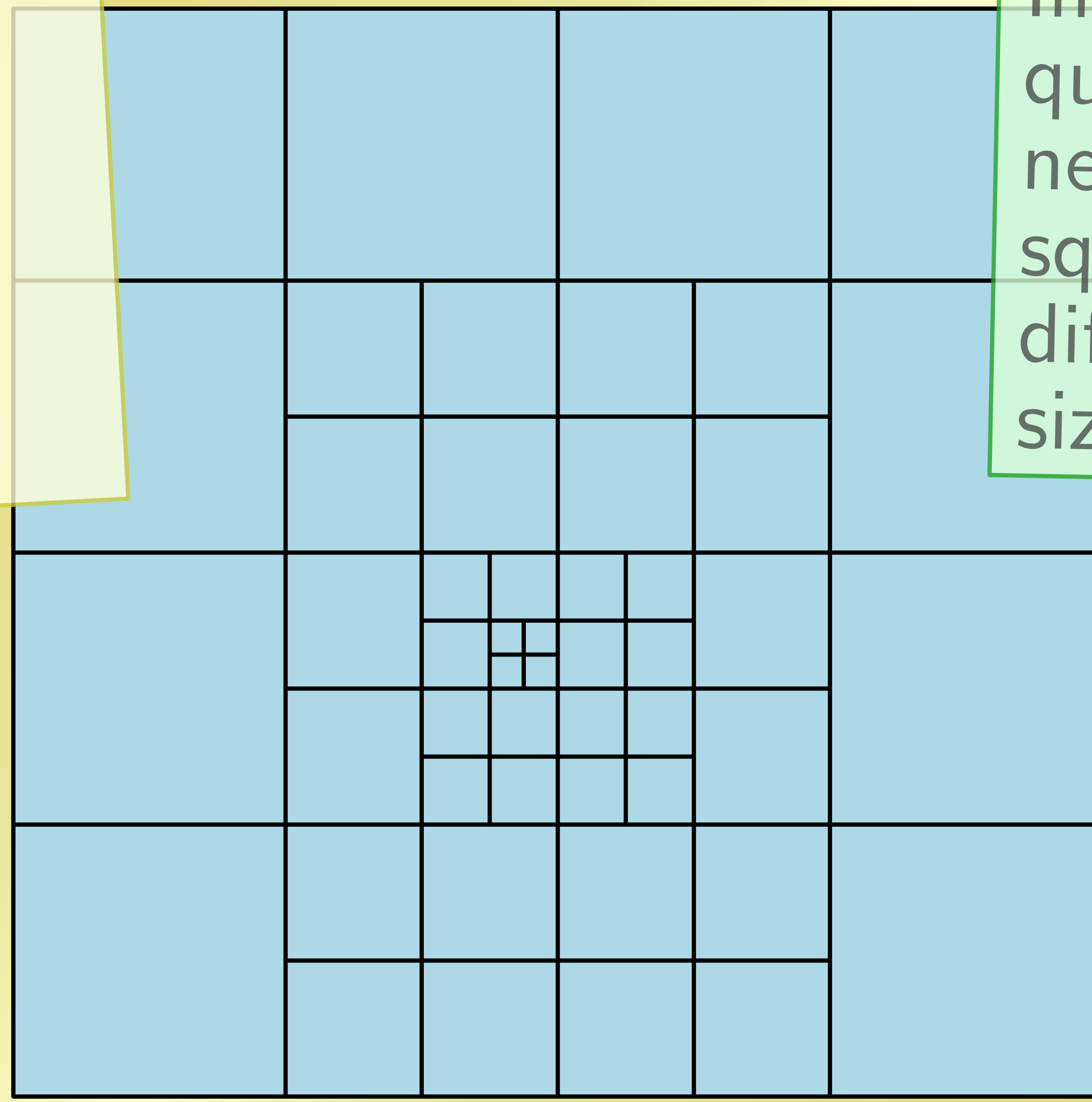
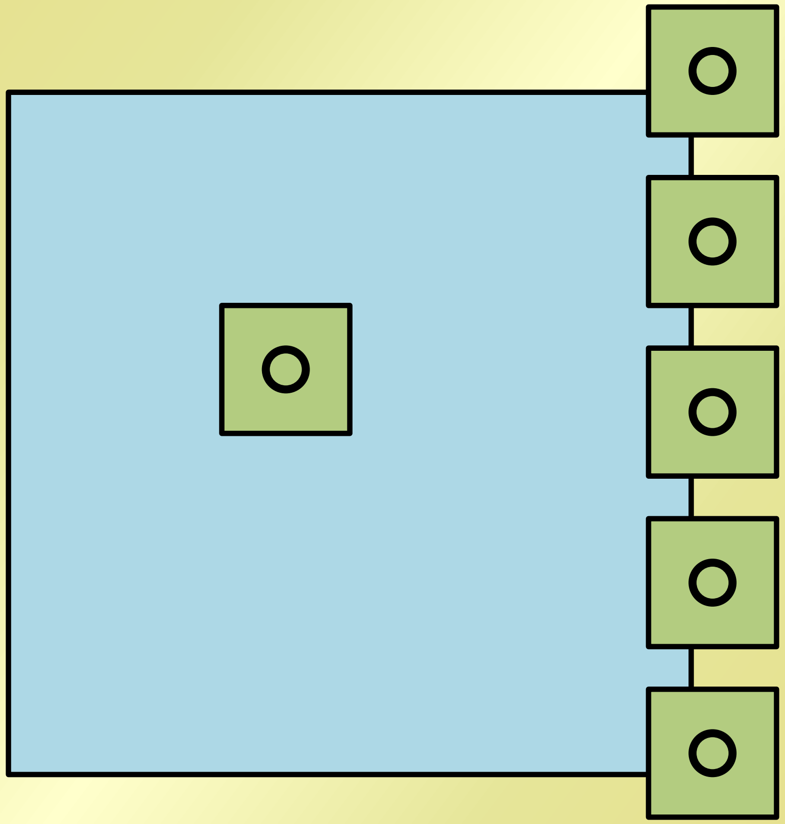


In a *balanced* quadtree, neighbouring squares don't differ much in size.

Consider a quadtree again.



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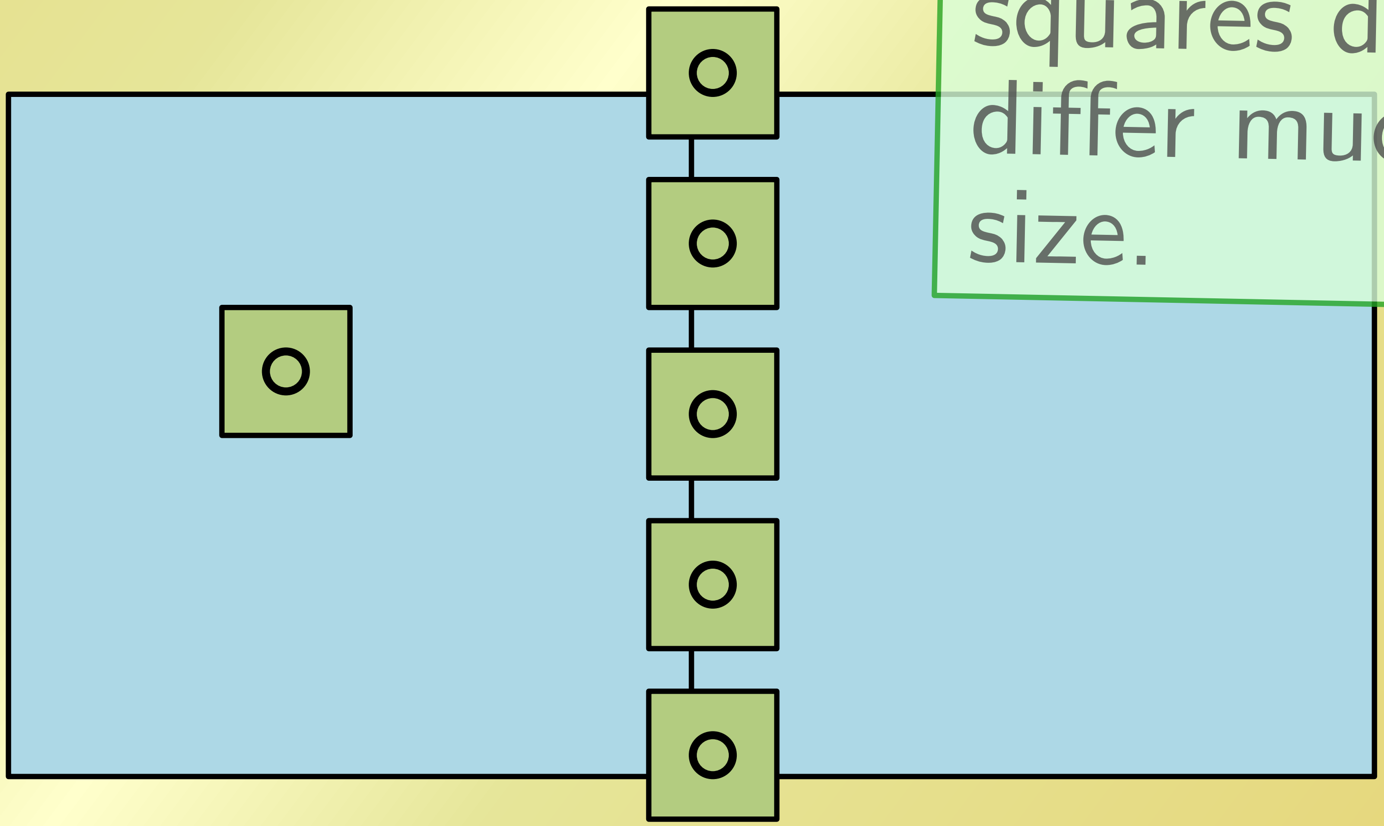

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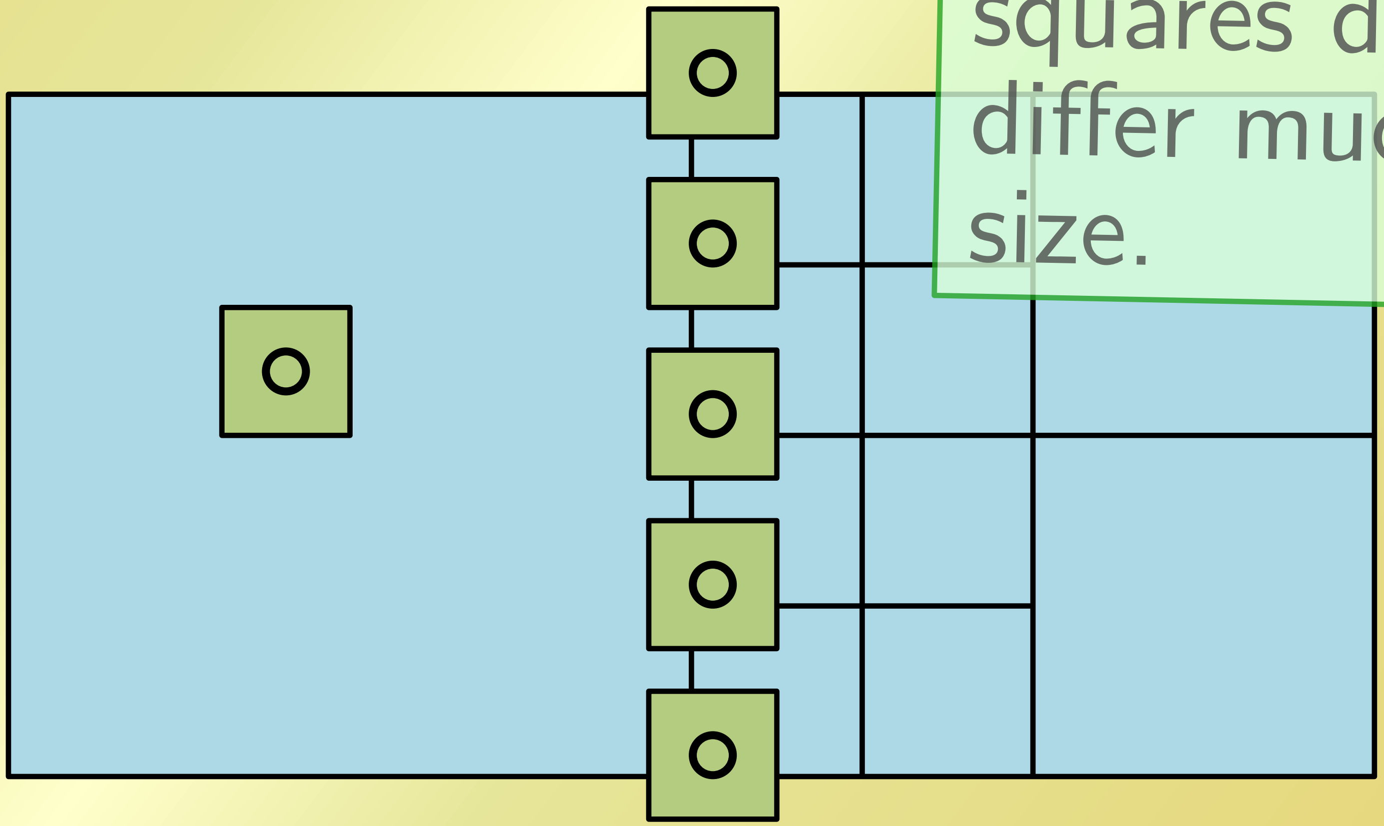

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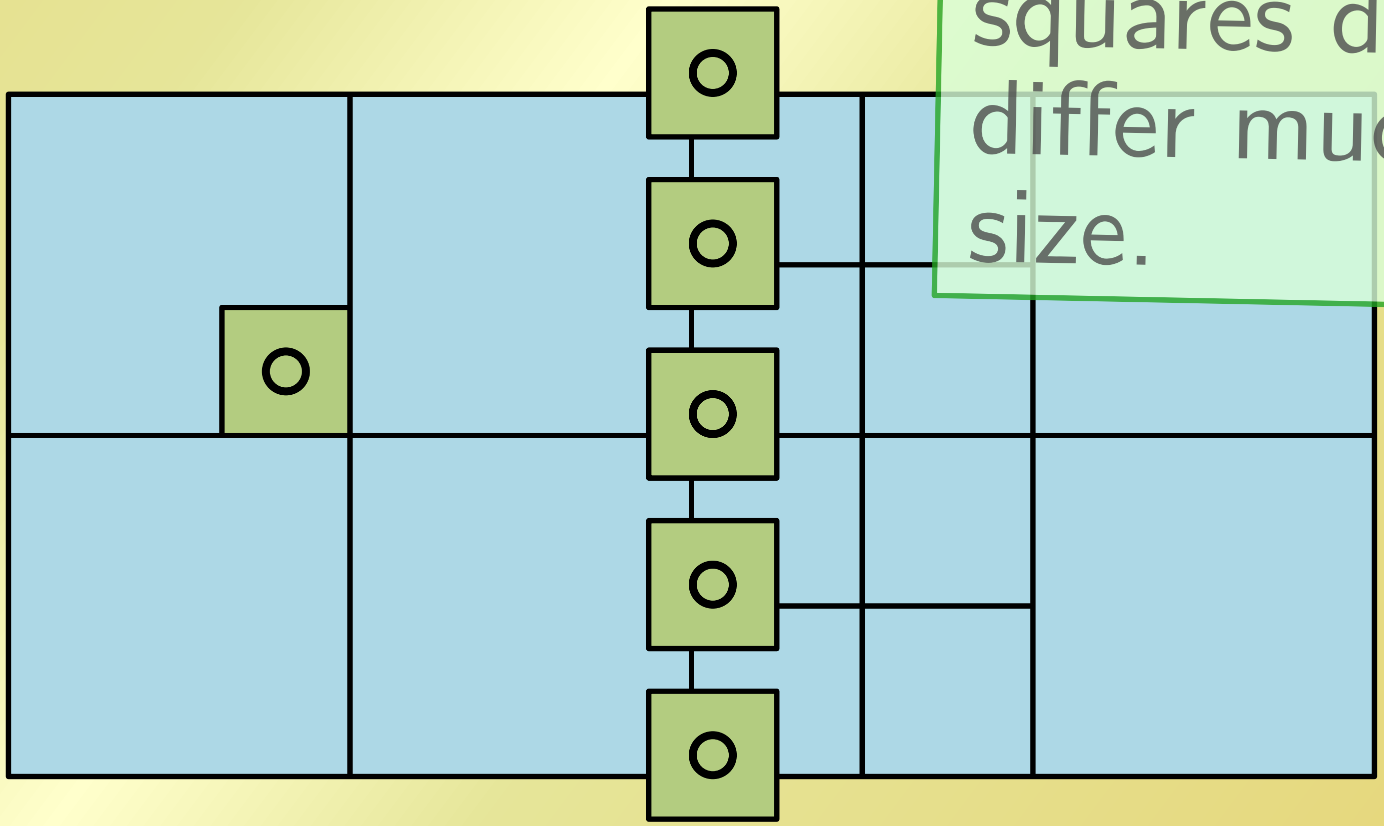

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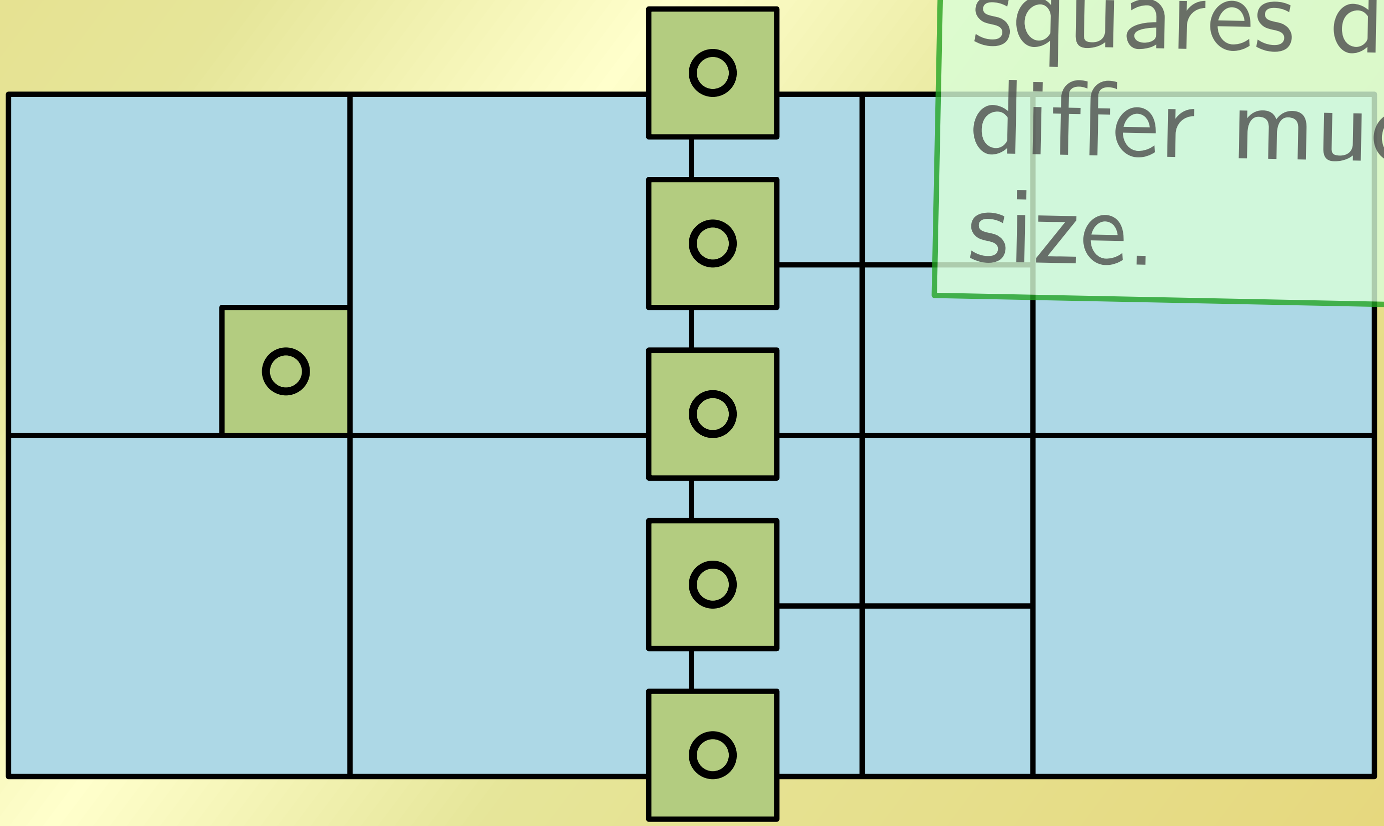

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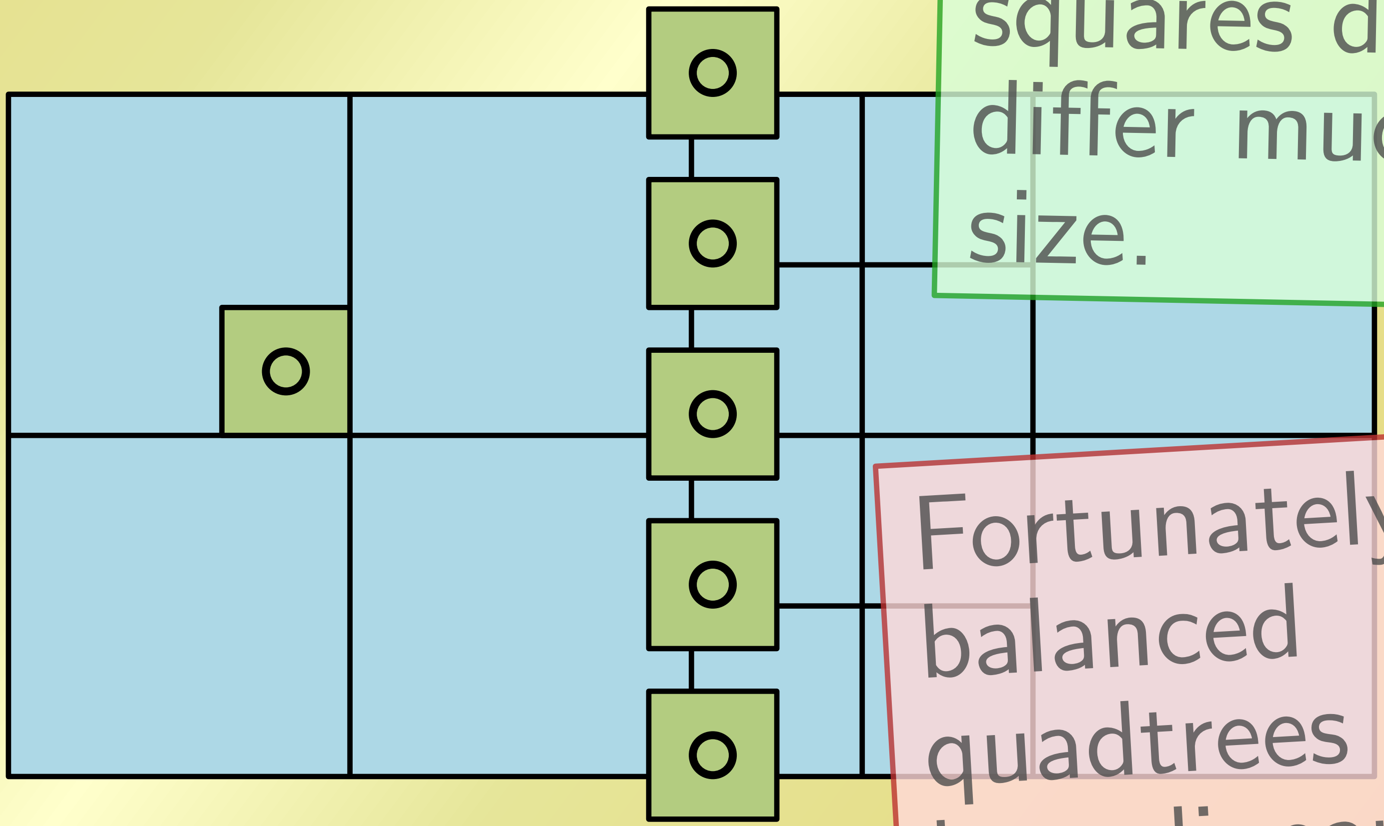

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Fortunately, balanced quadtrees still have linear size.

PROBLEM

But now we can't change the quadtree locally in $O(1)$ time!

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ADVICE

Don't worry, be happy!

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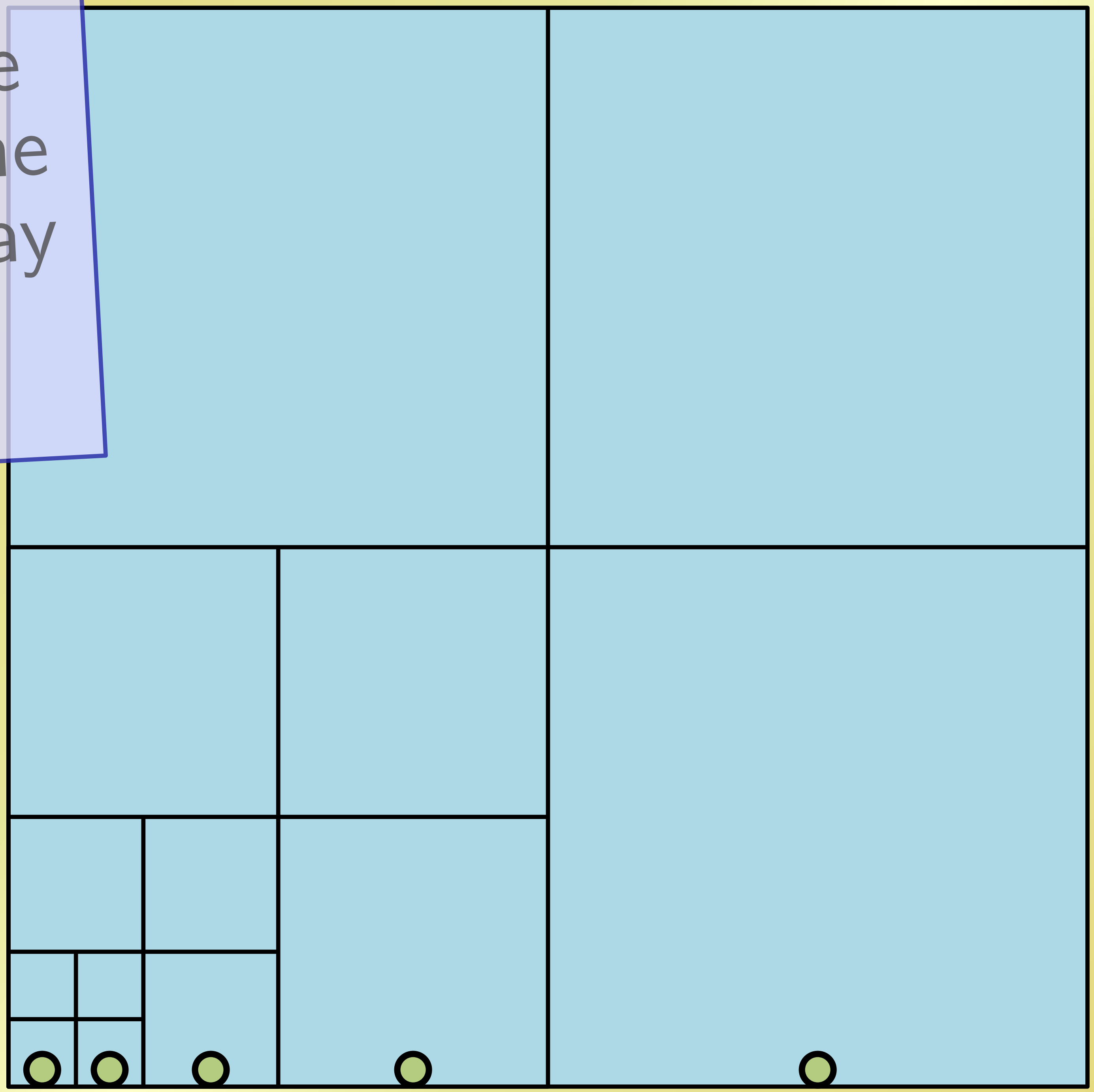

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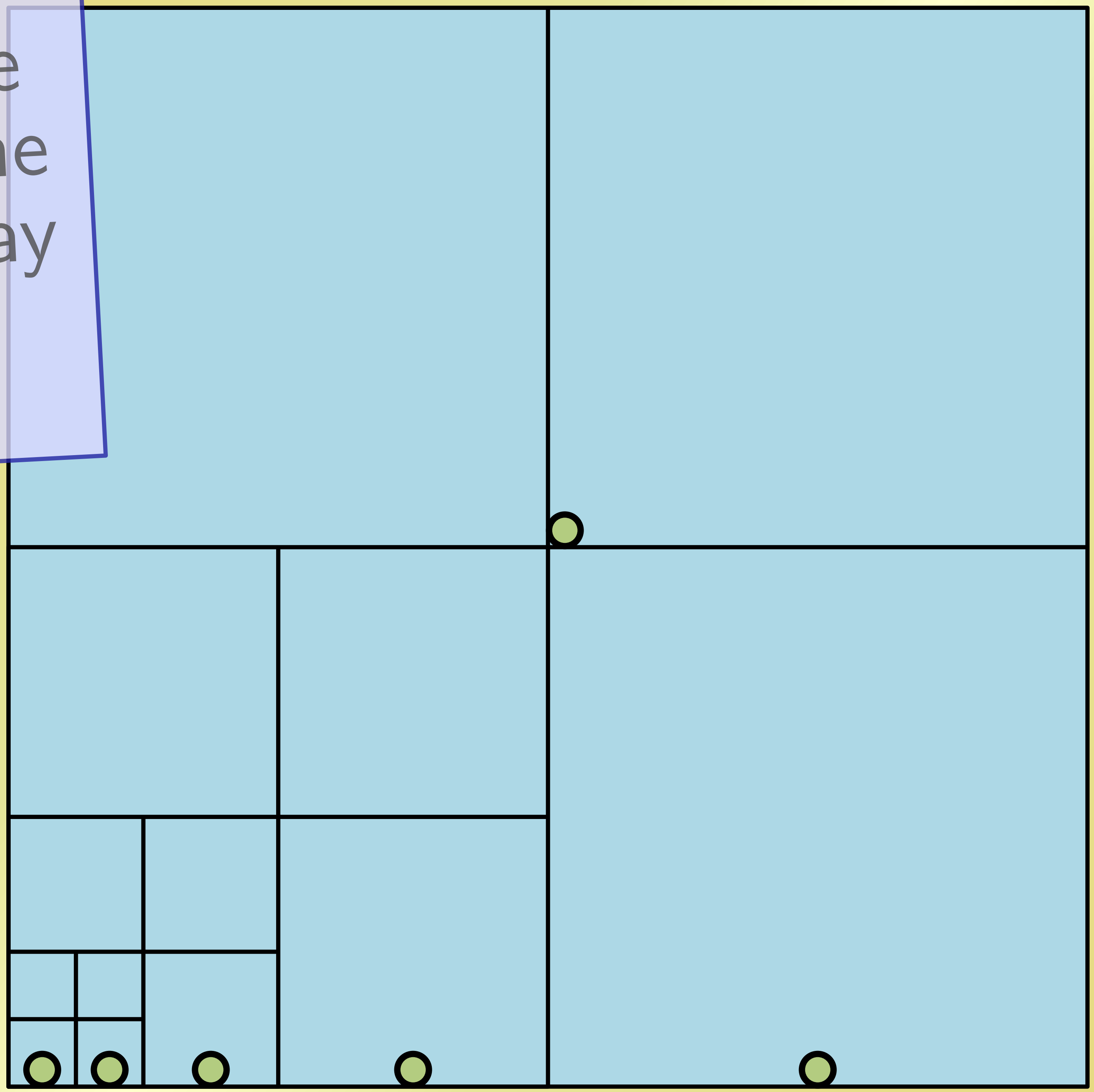

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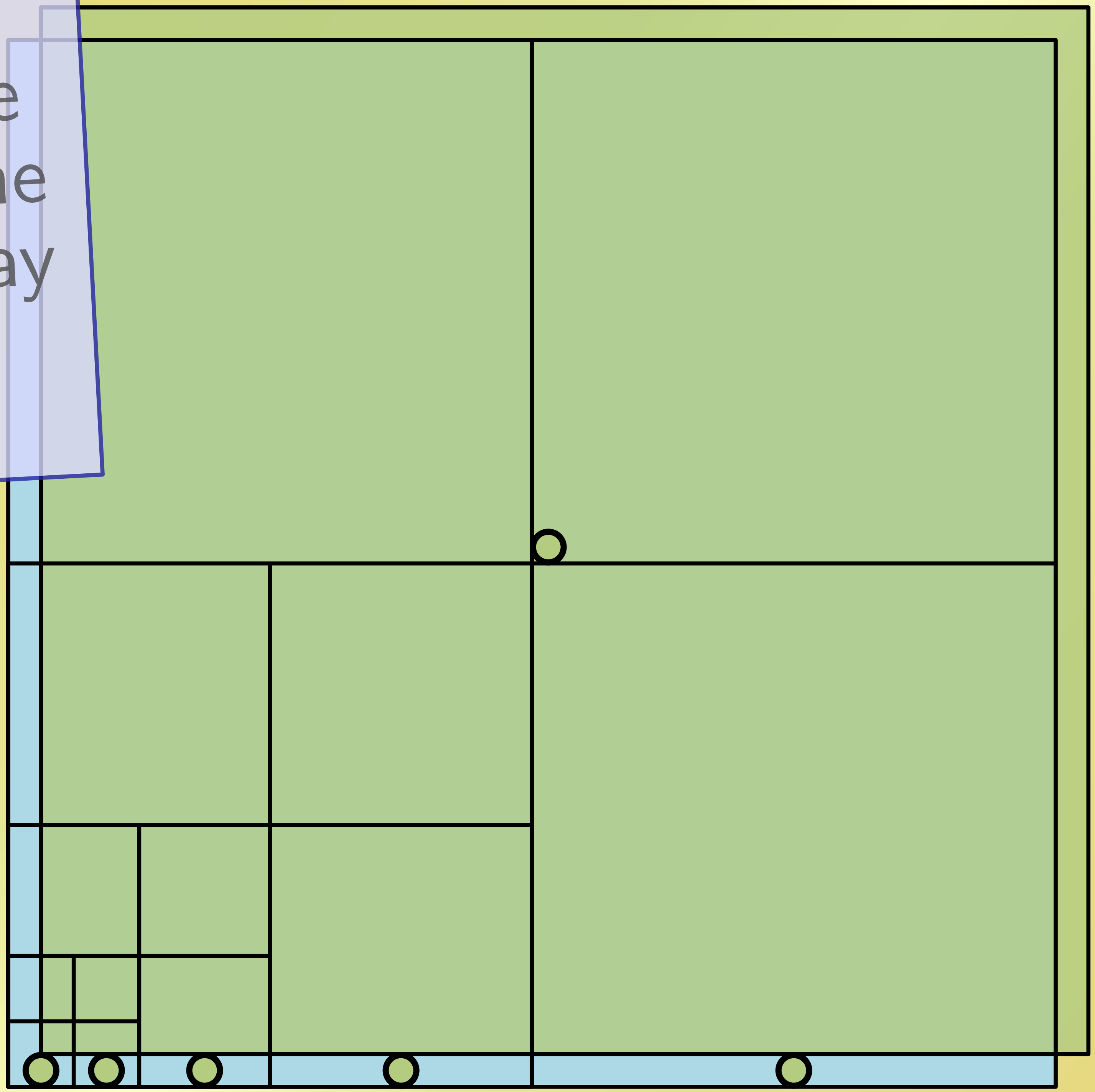

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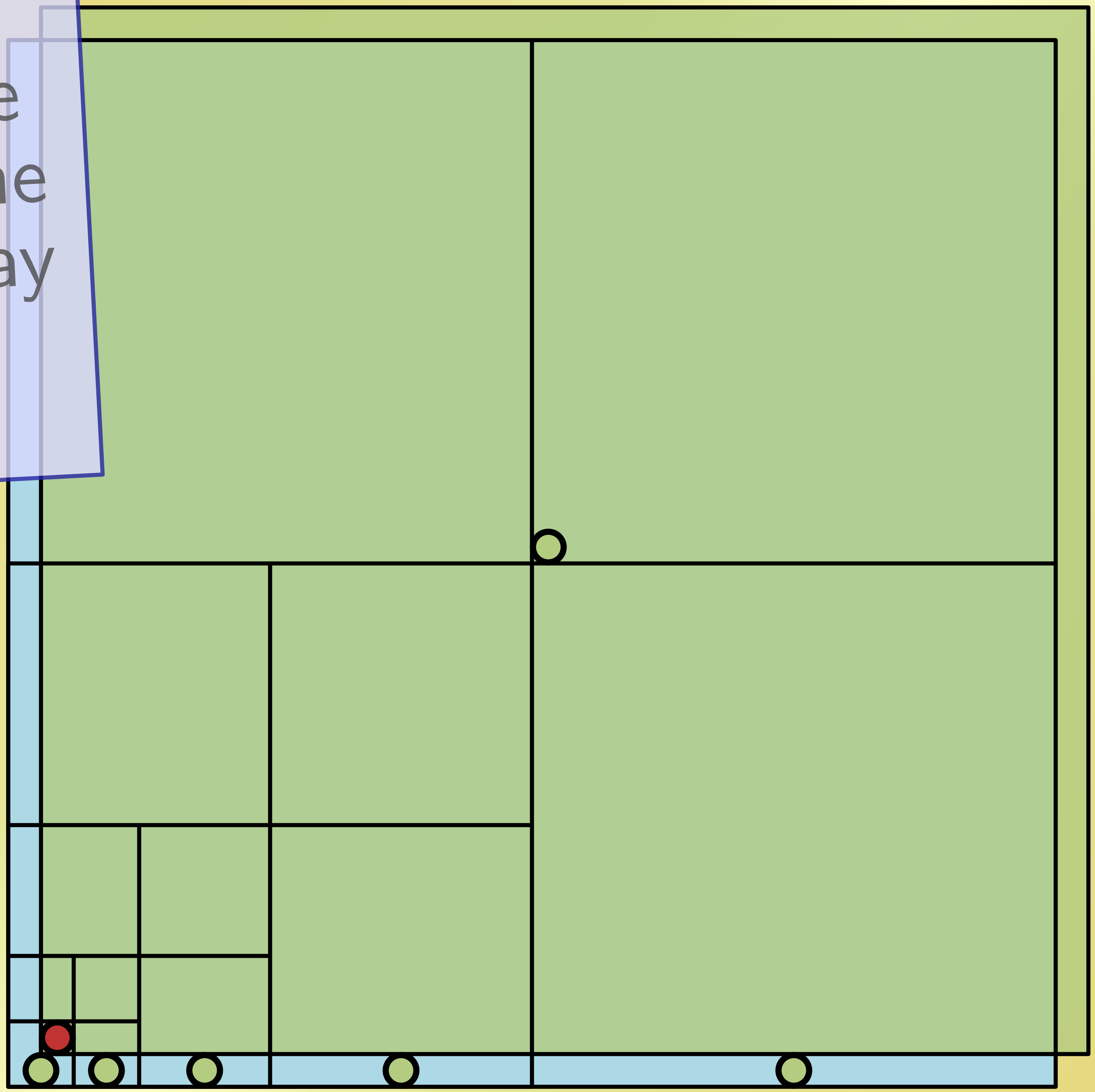

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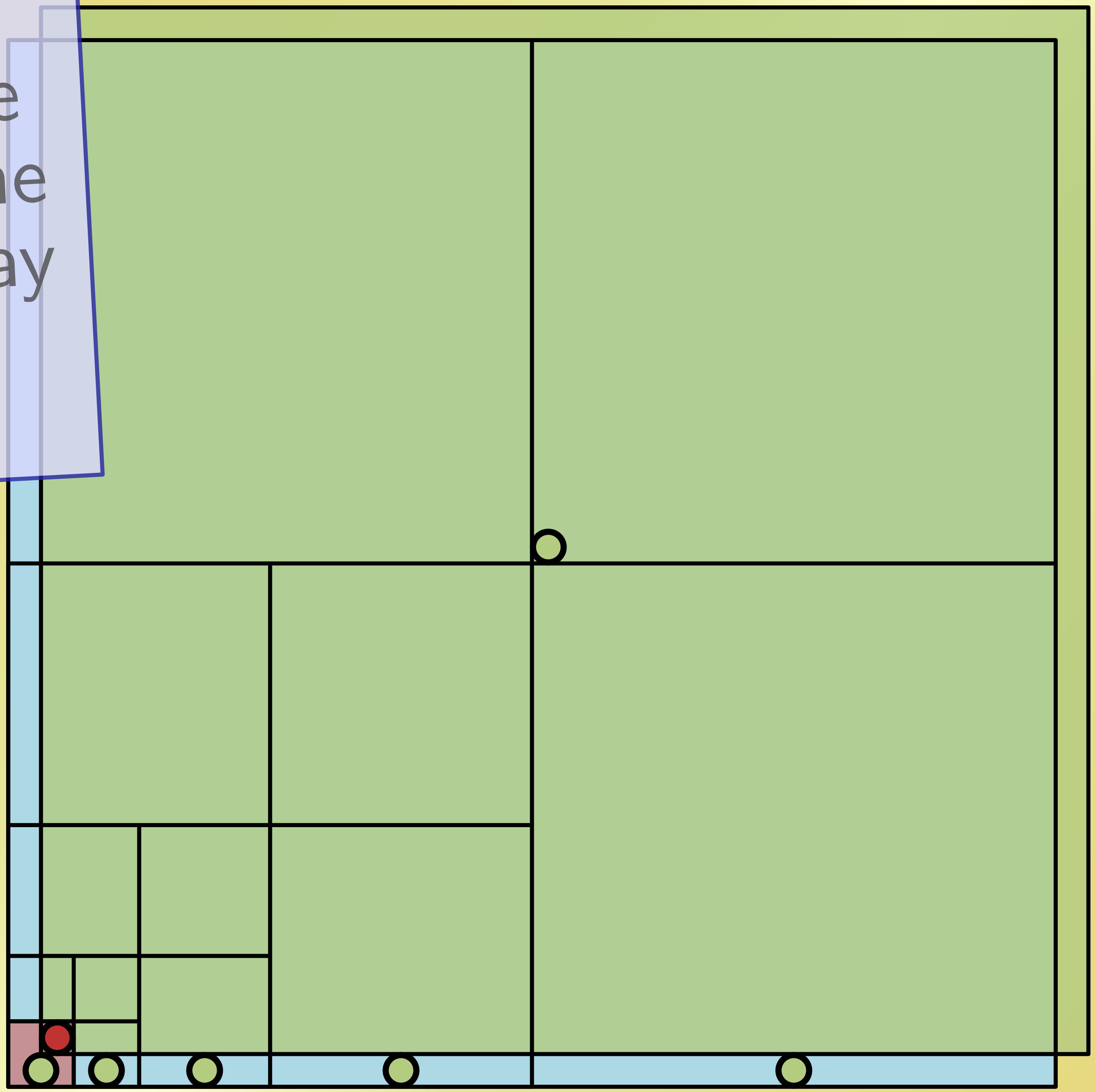

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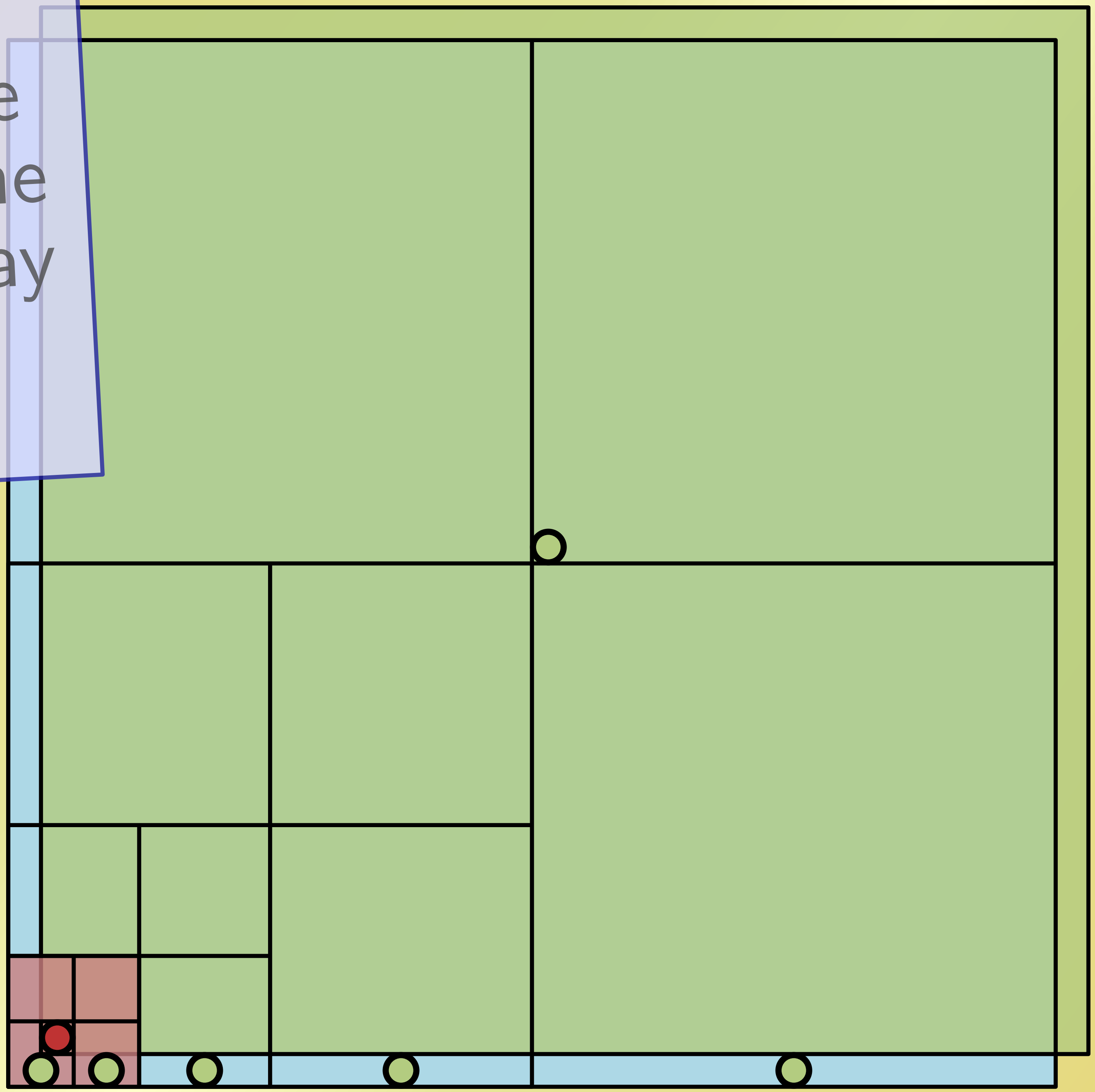

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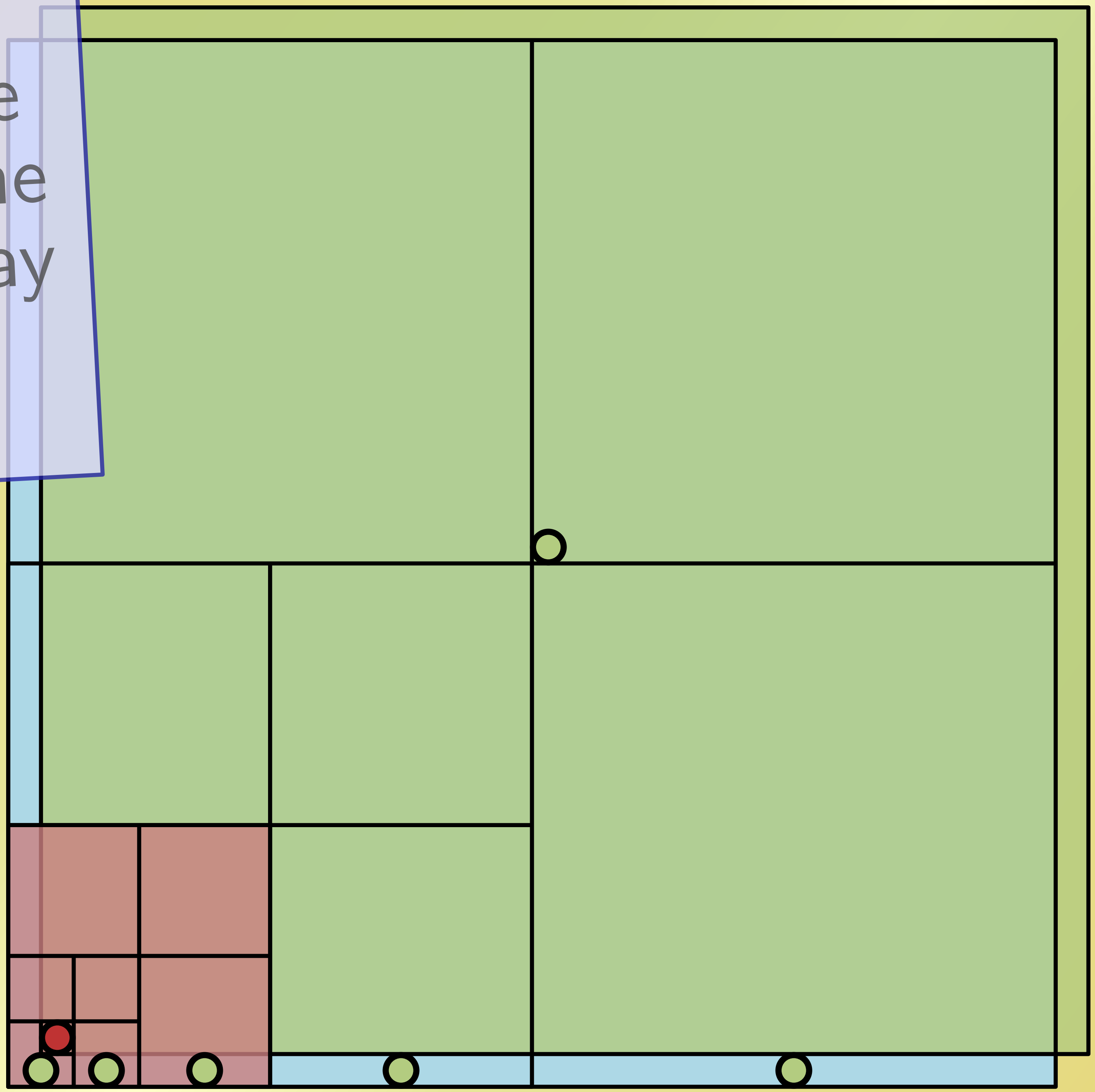

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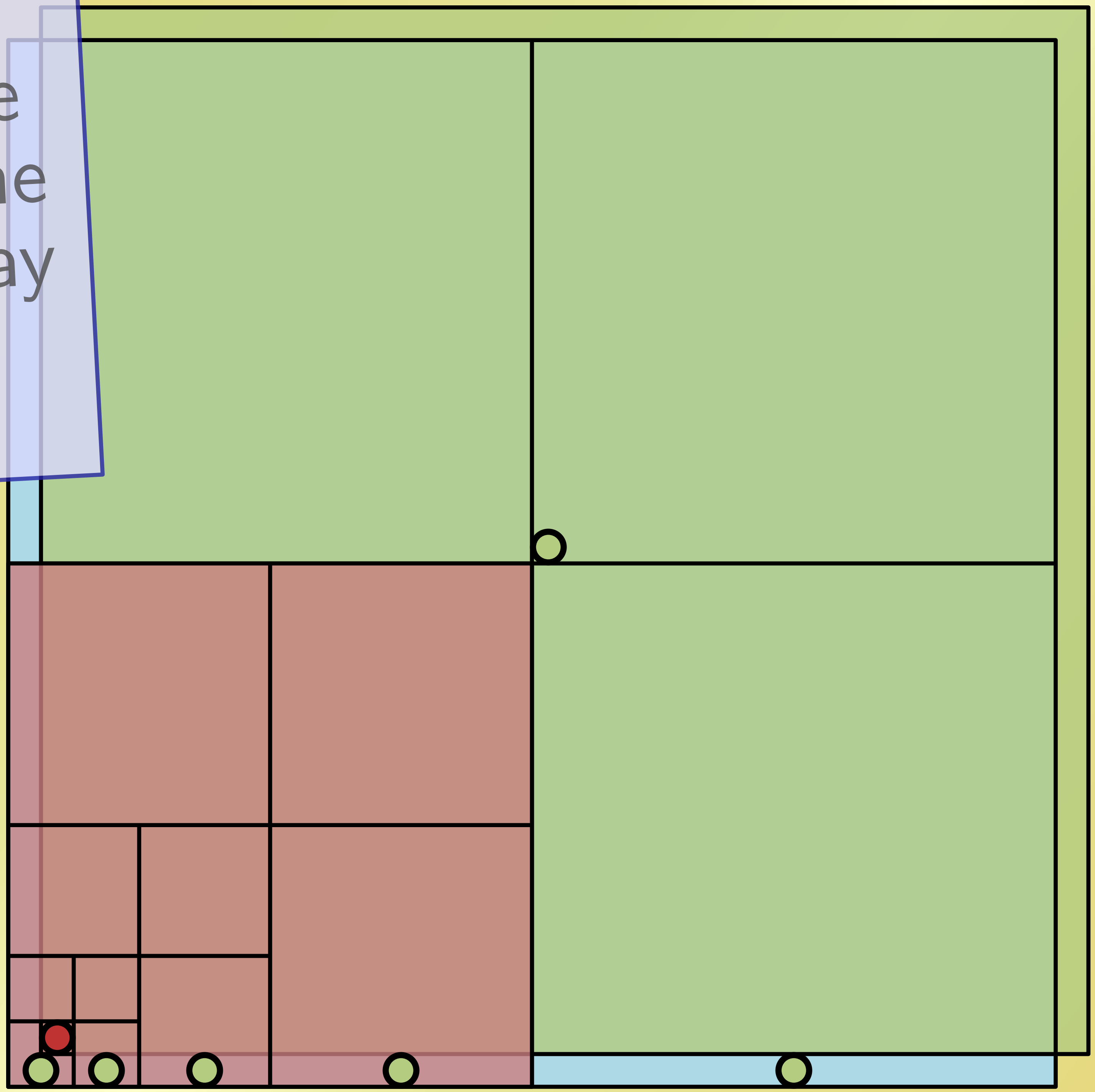

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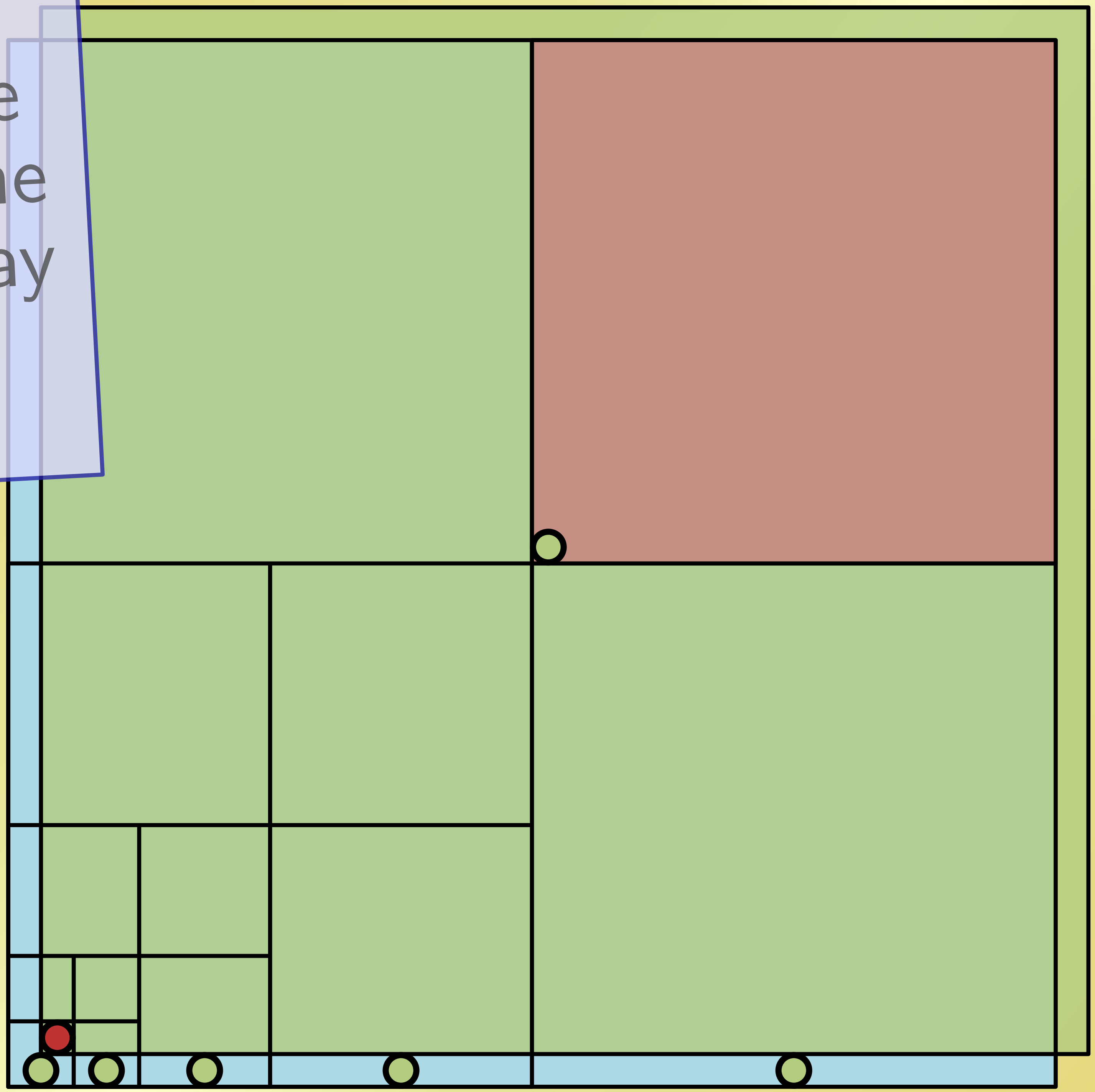

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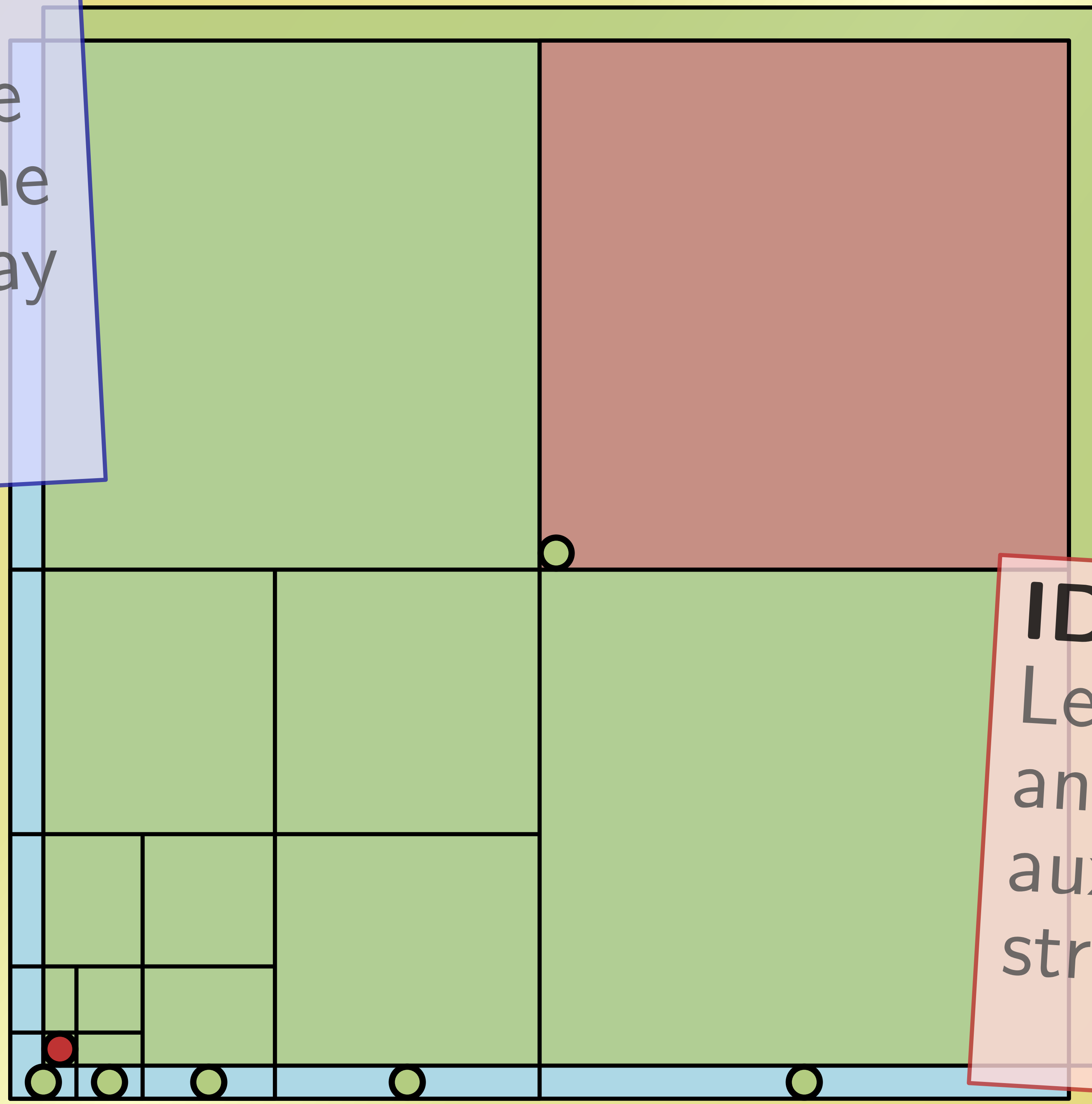

IDEA

Let's add yet another auxliliary structure!

We use a *marked ancestor* data structure.

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Consider a tree, where some nodes are *marked*.

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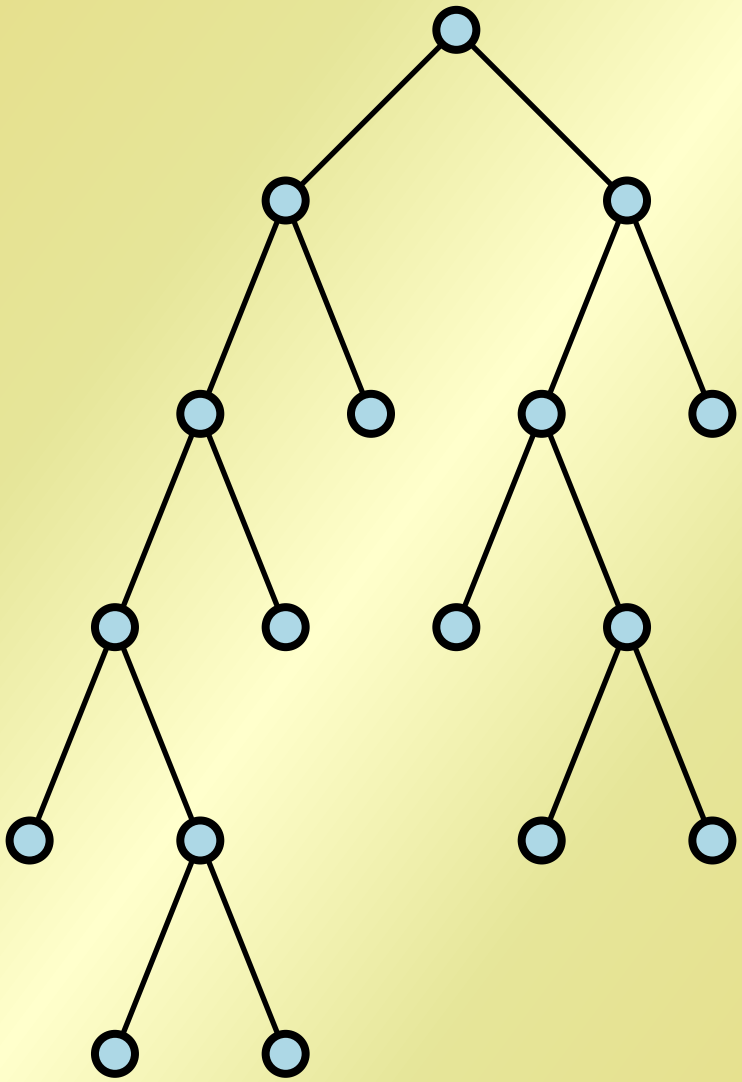

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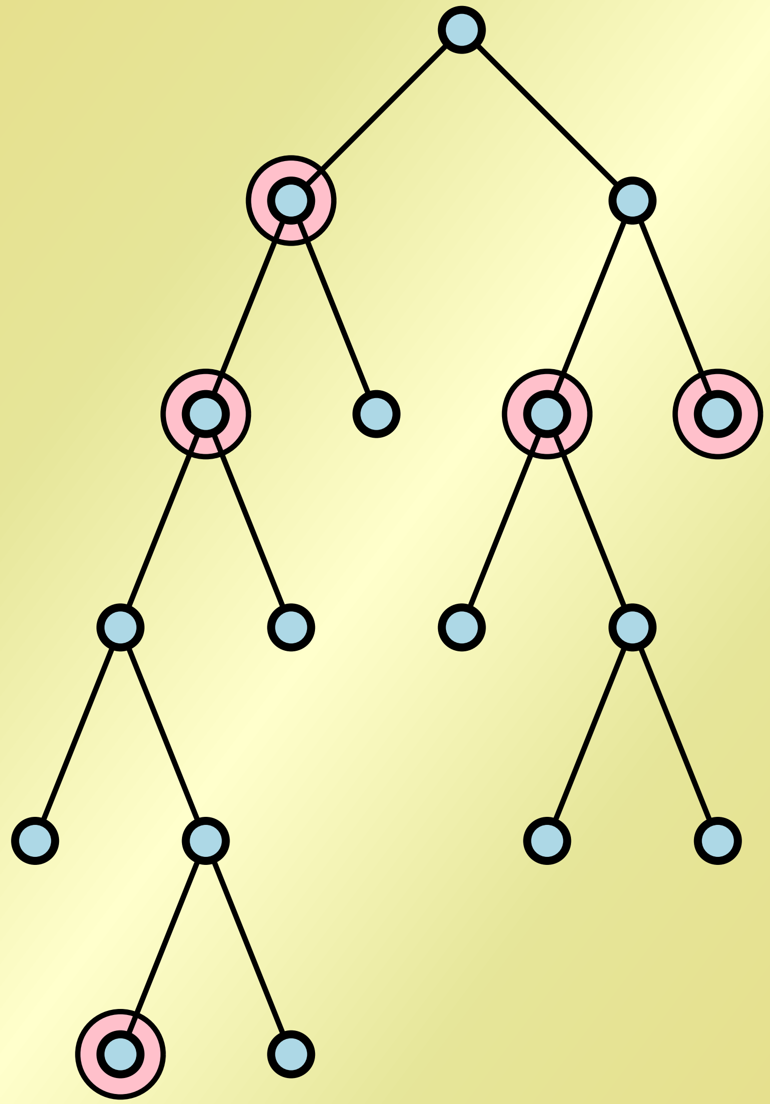

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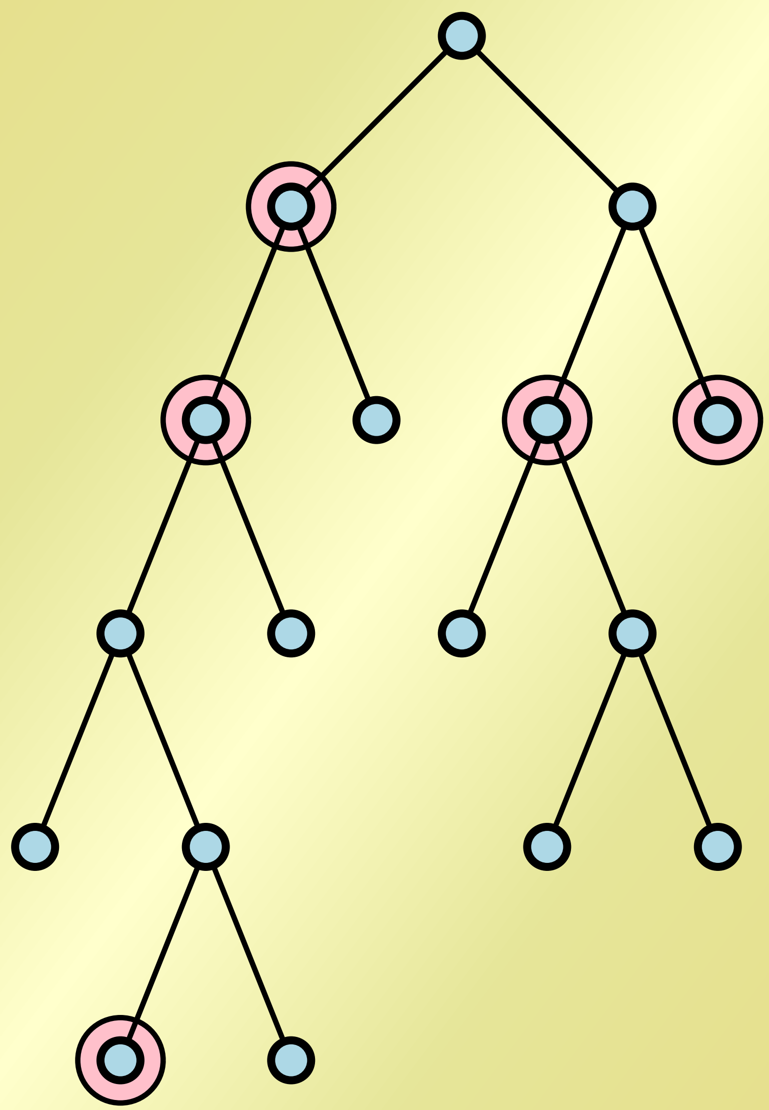

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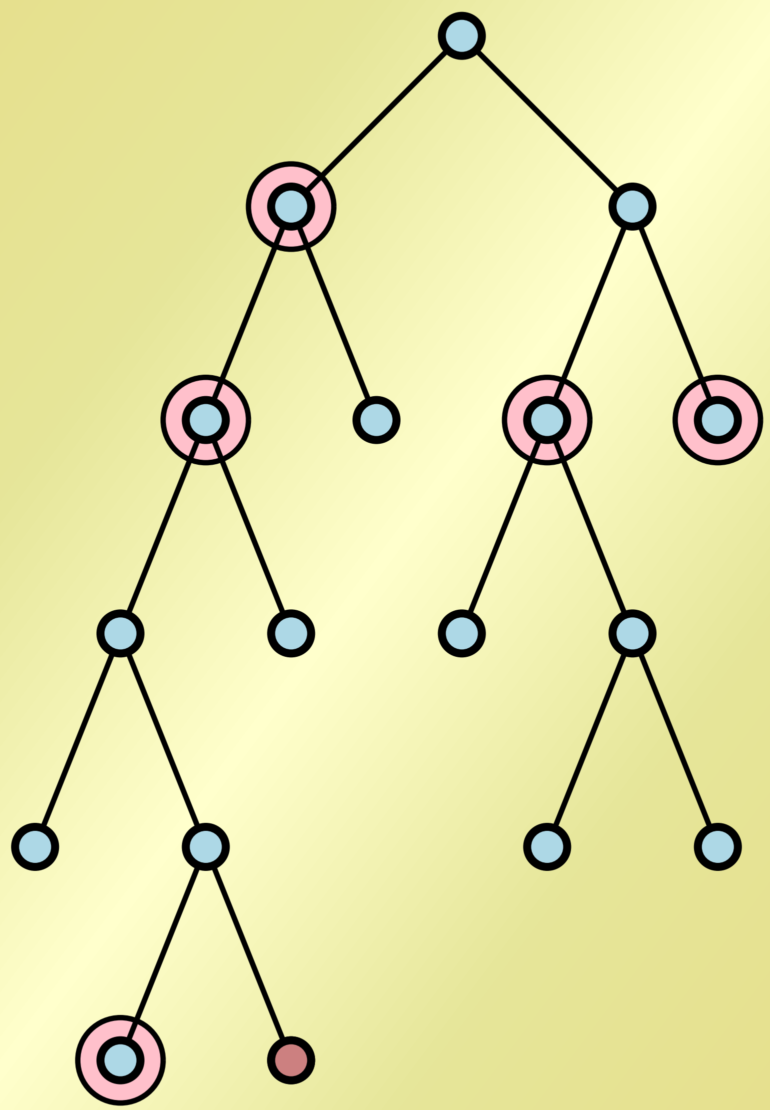

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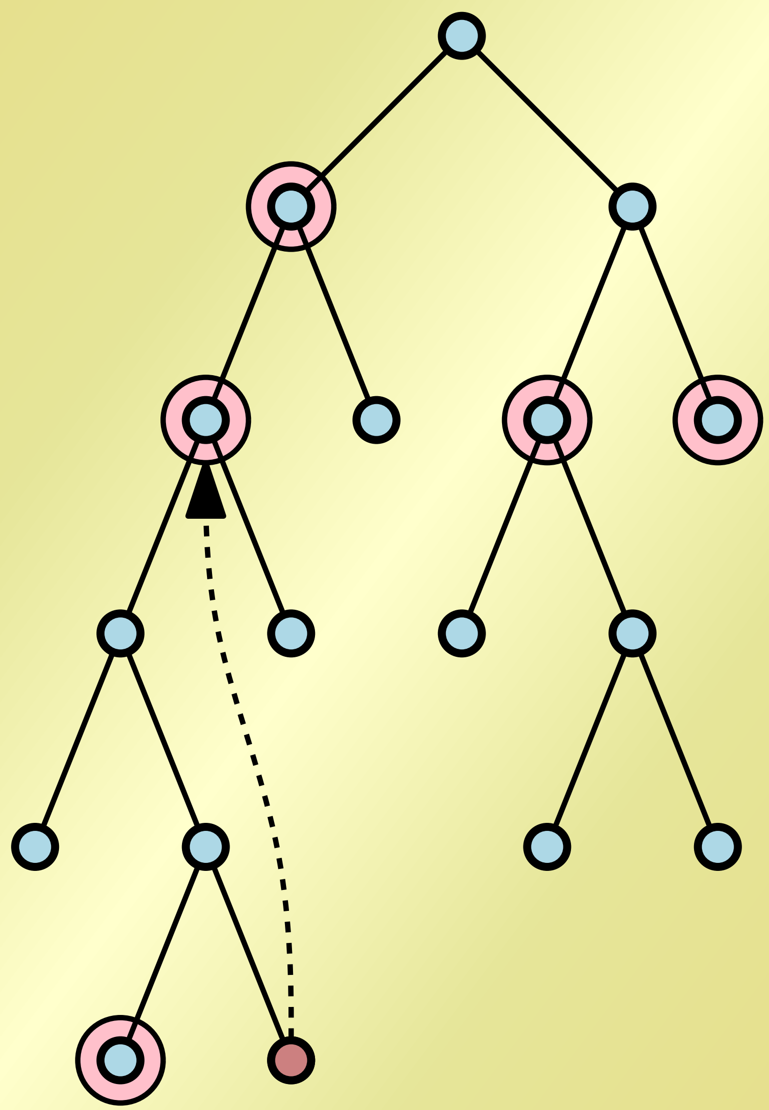

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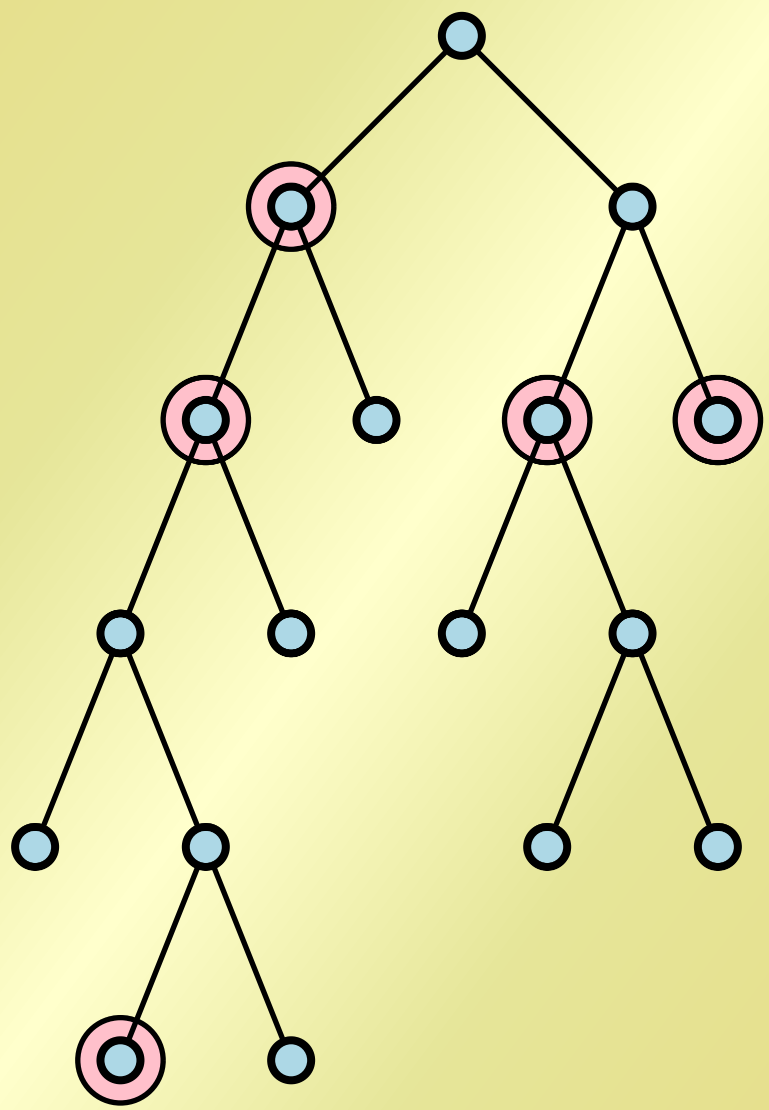

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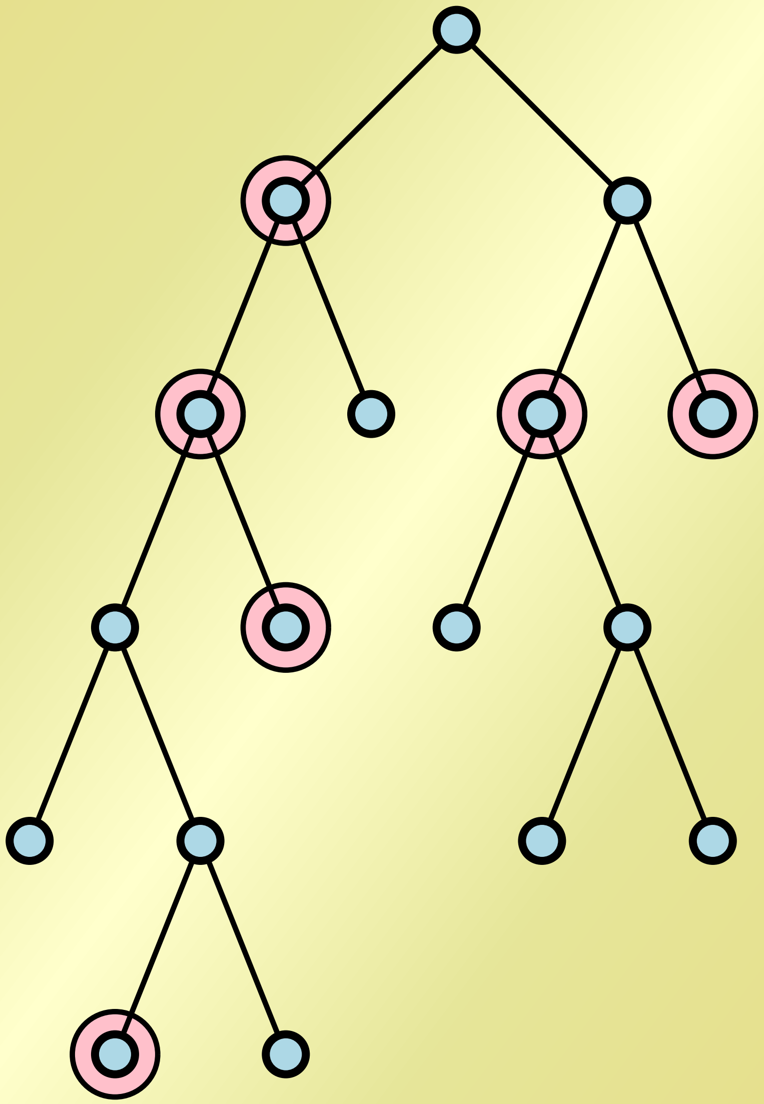

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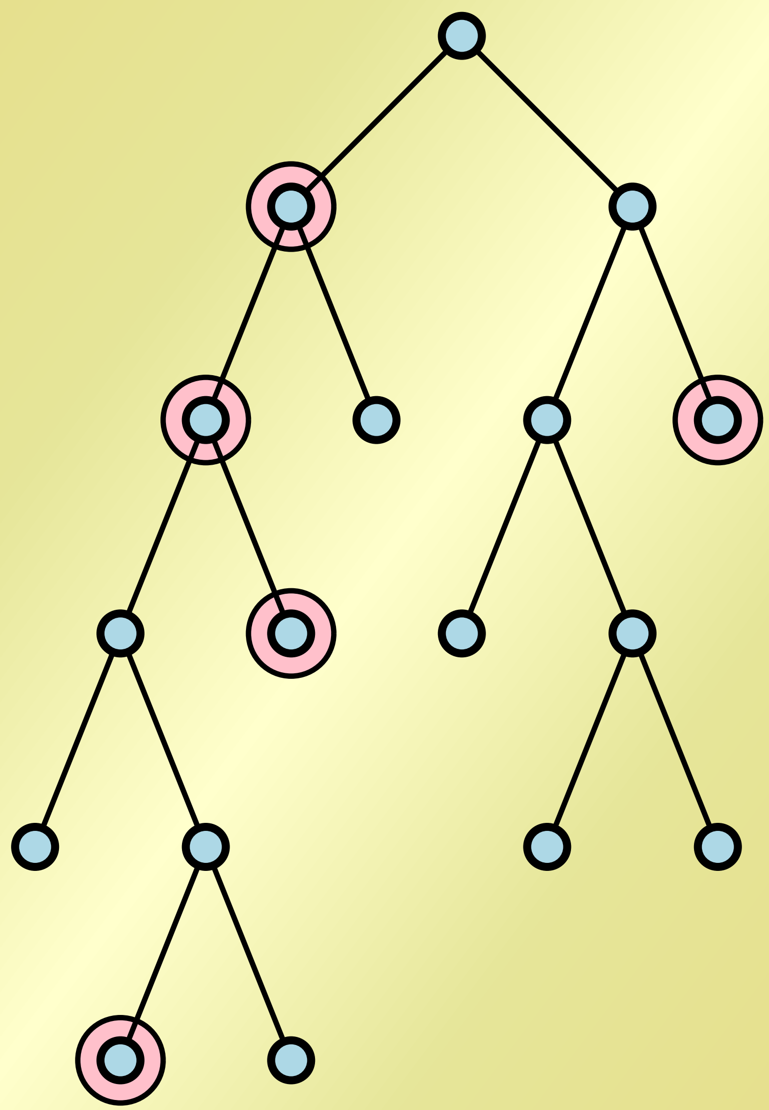

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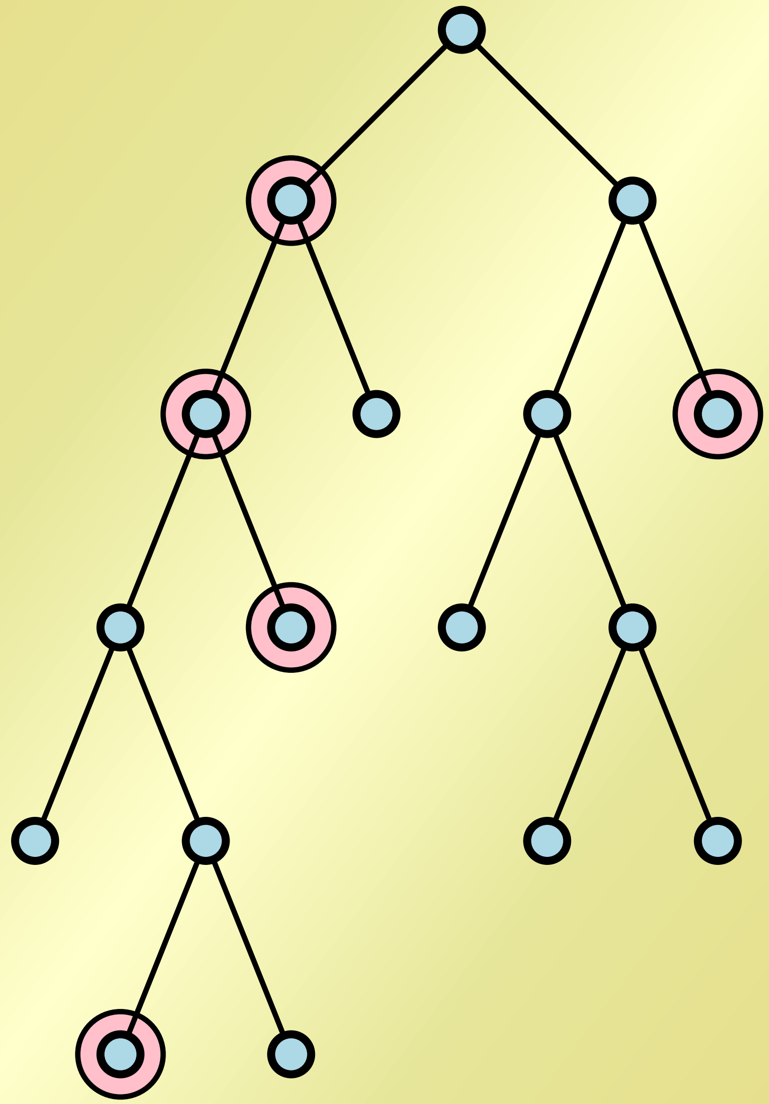

Also, we want to be able to mark and unmark nodes.

$O(\log \log n)$ (un)mark and $O(\frac{\log n}{\log \log n})$ queries is possible.

[Alstrup *et al.*, 1998]

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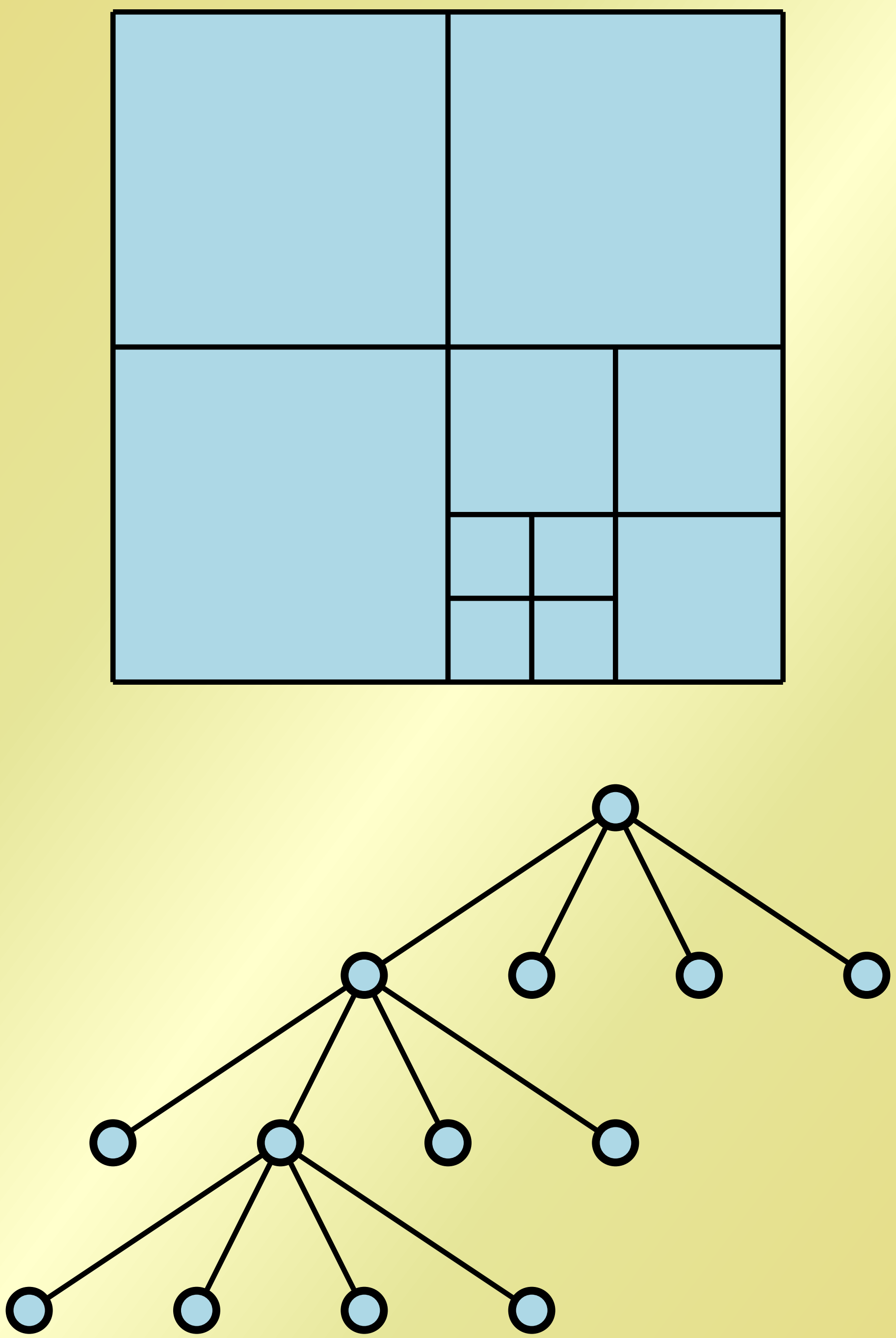

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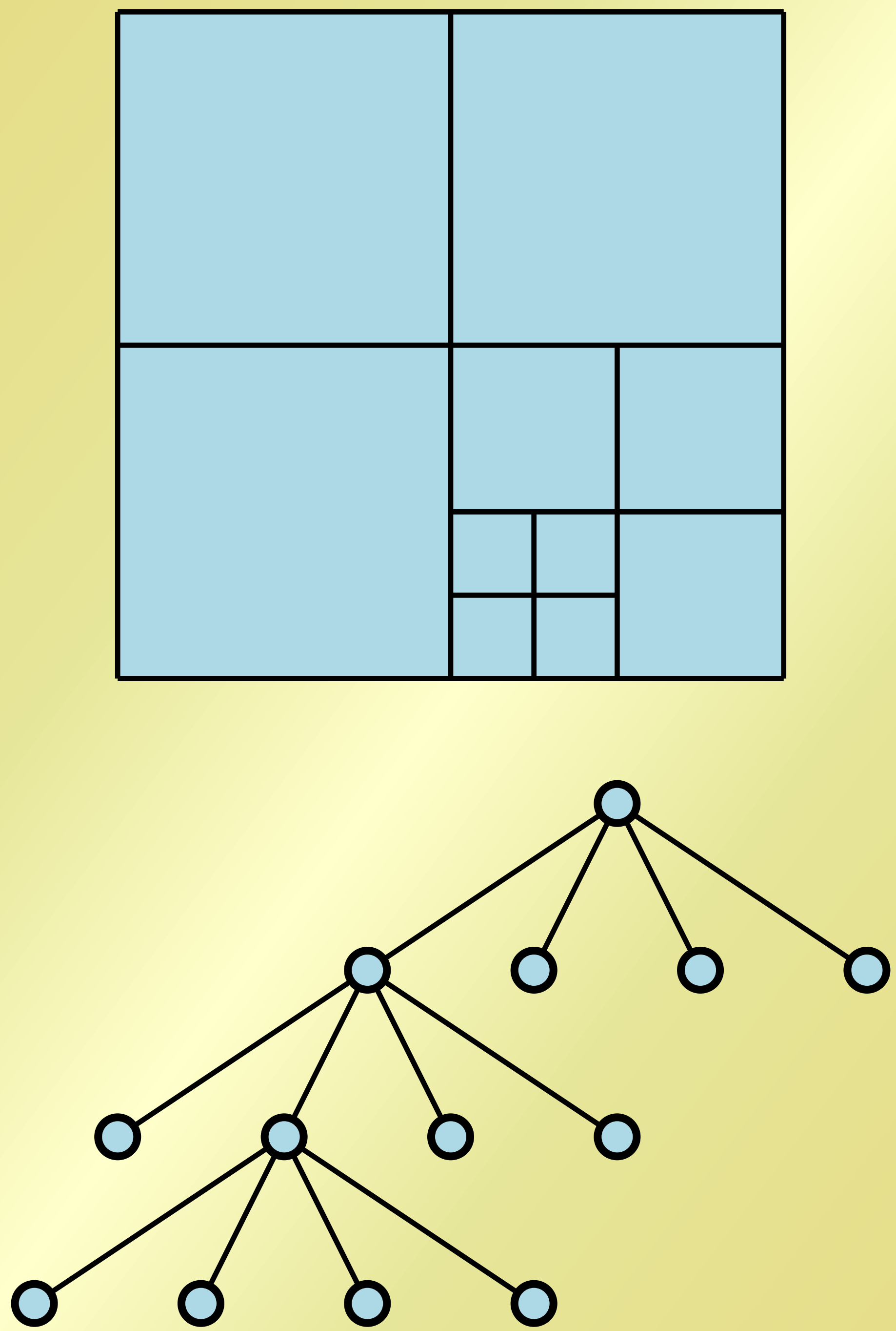

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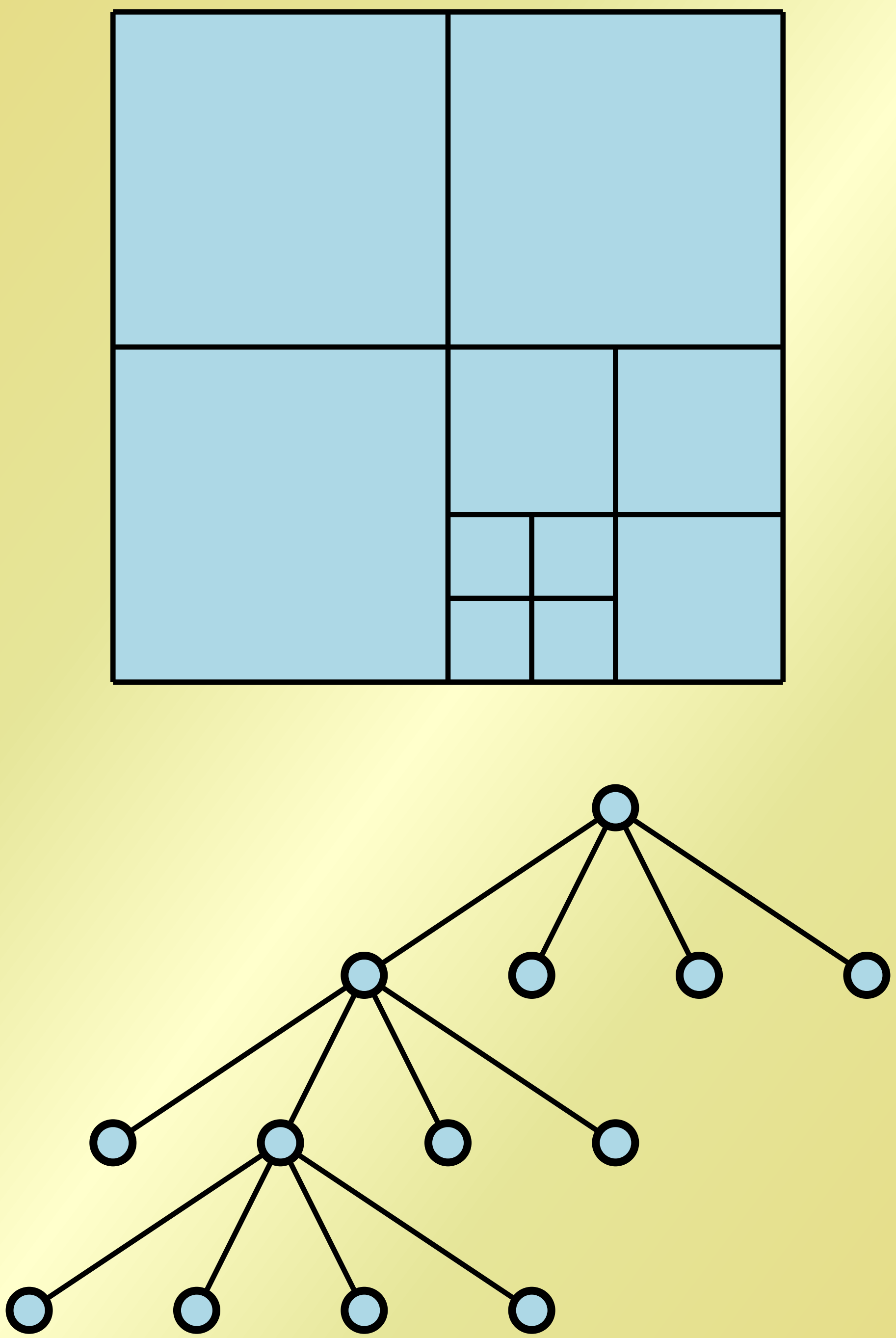

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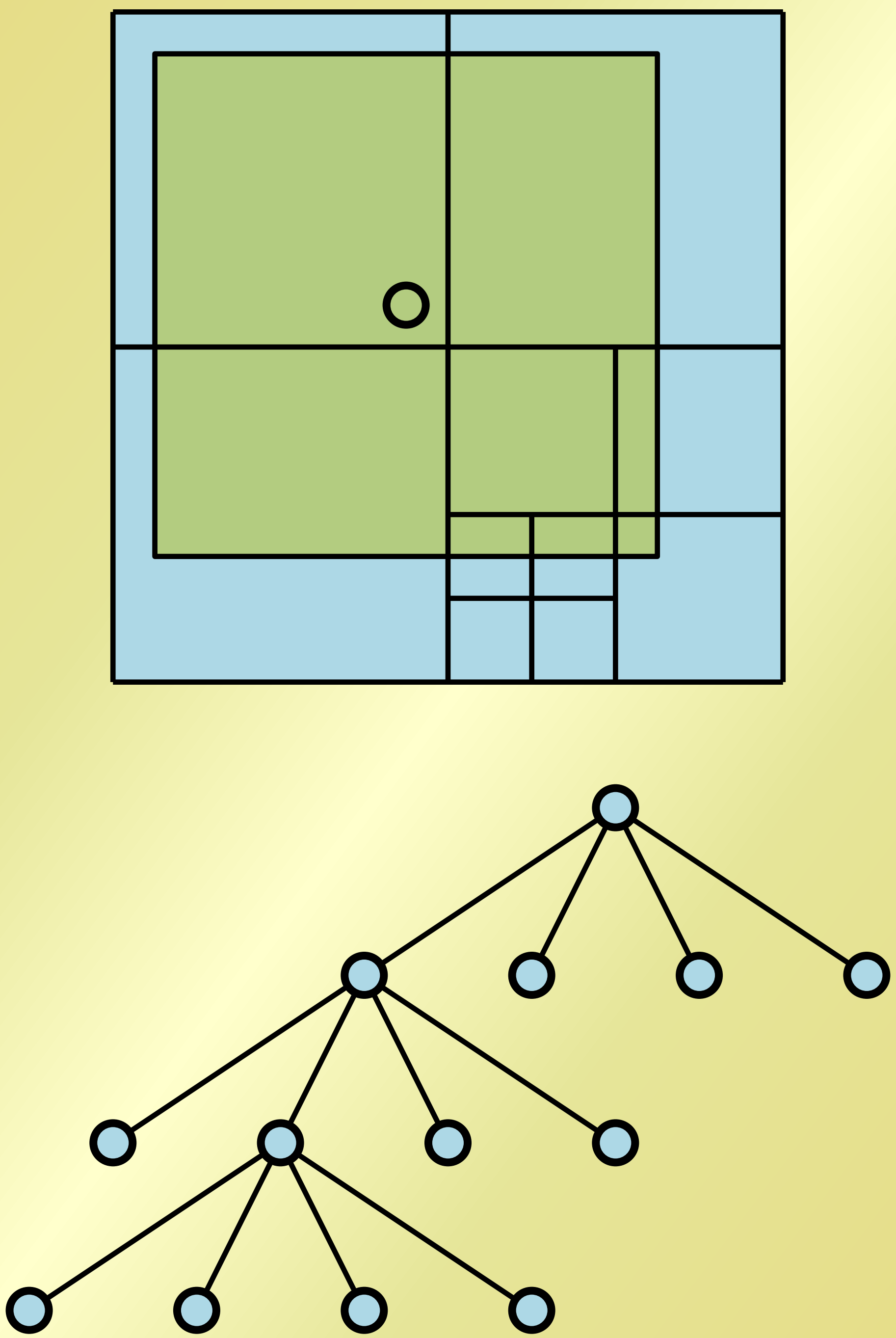

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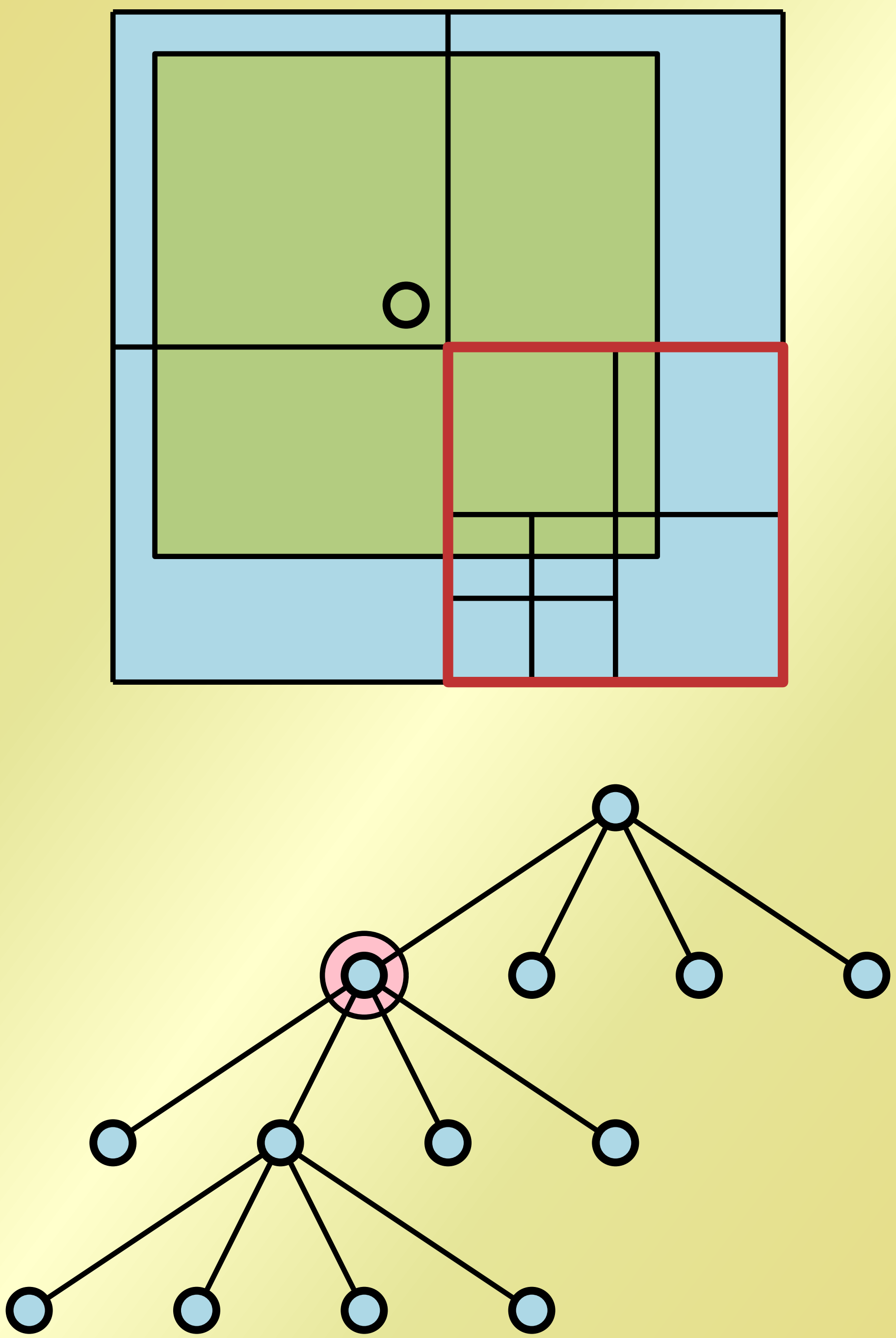

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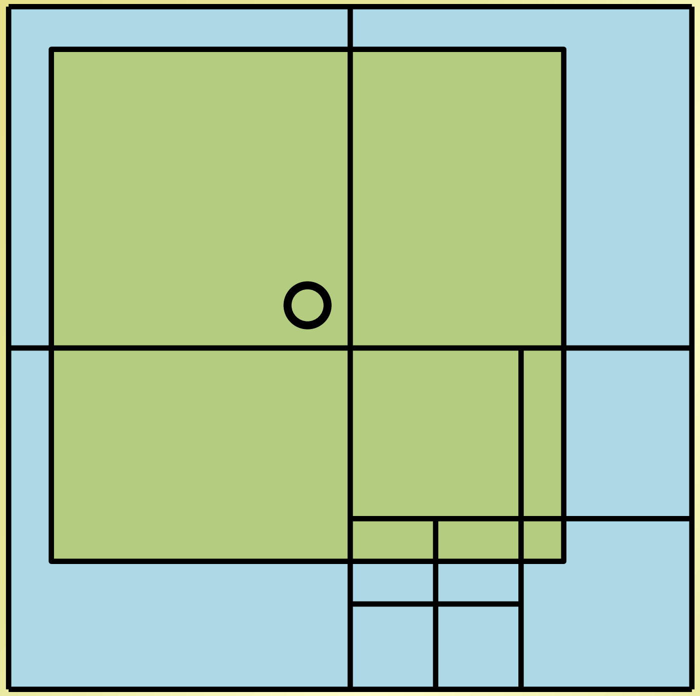

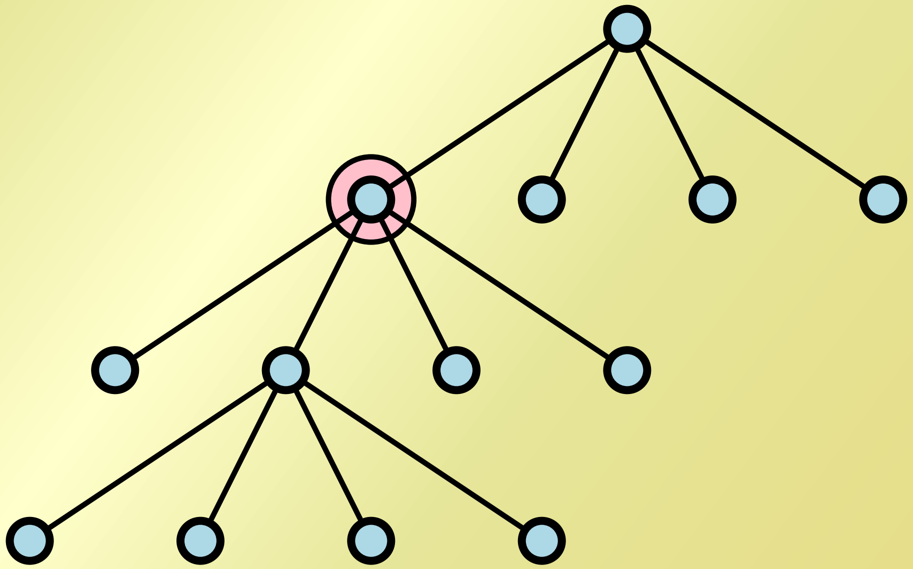

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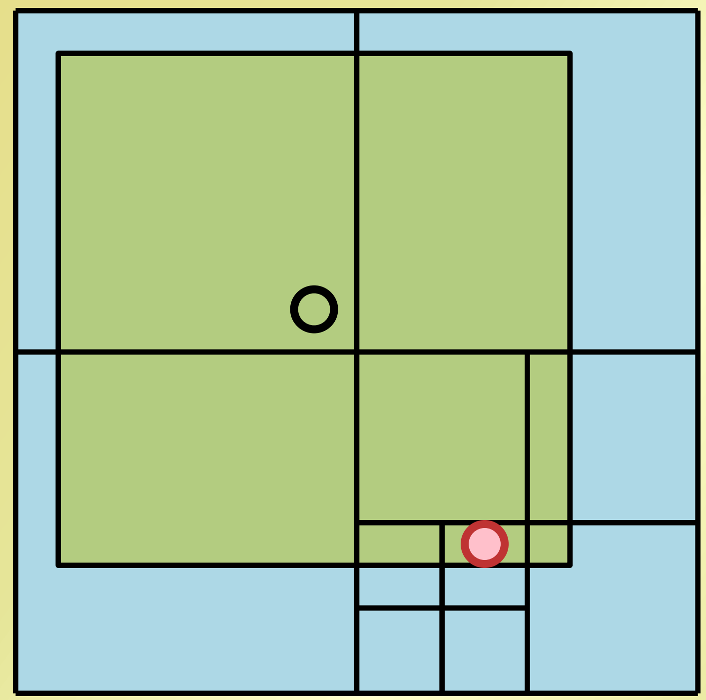

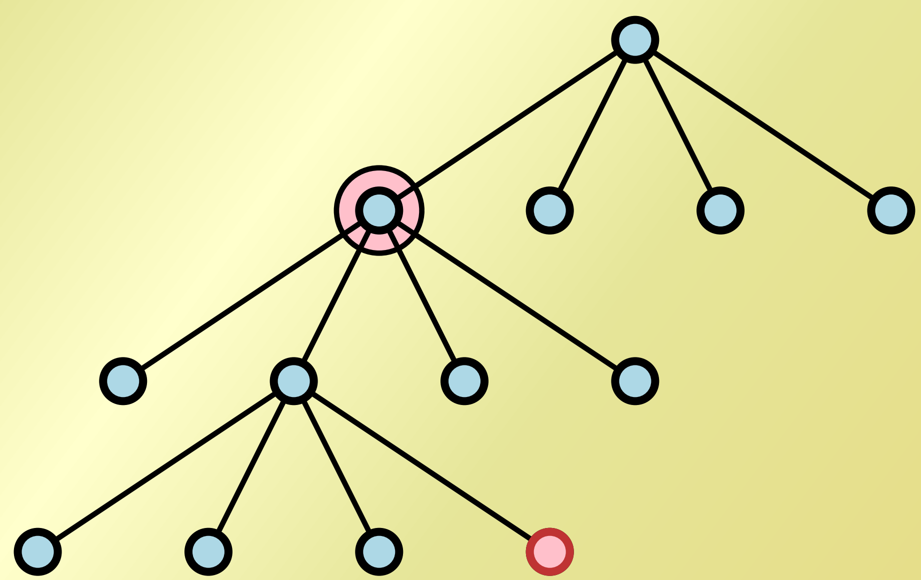

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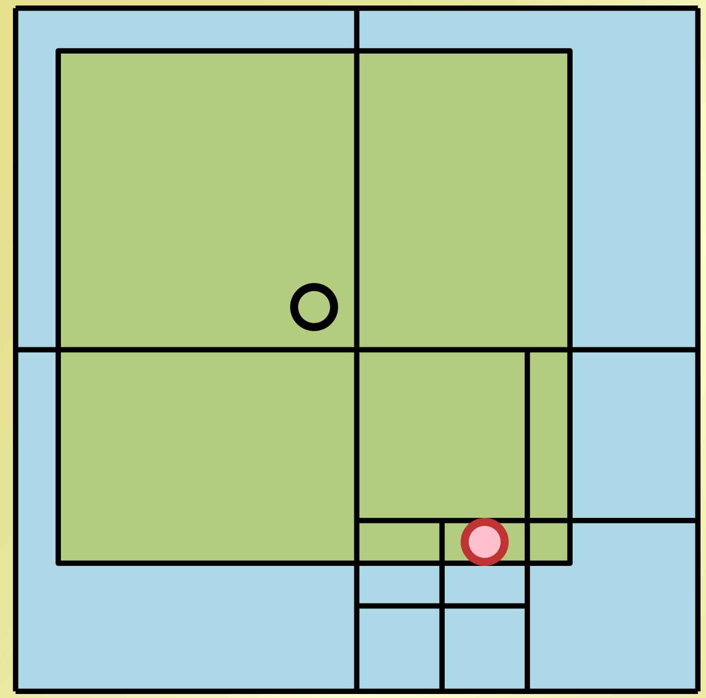

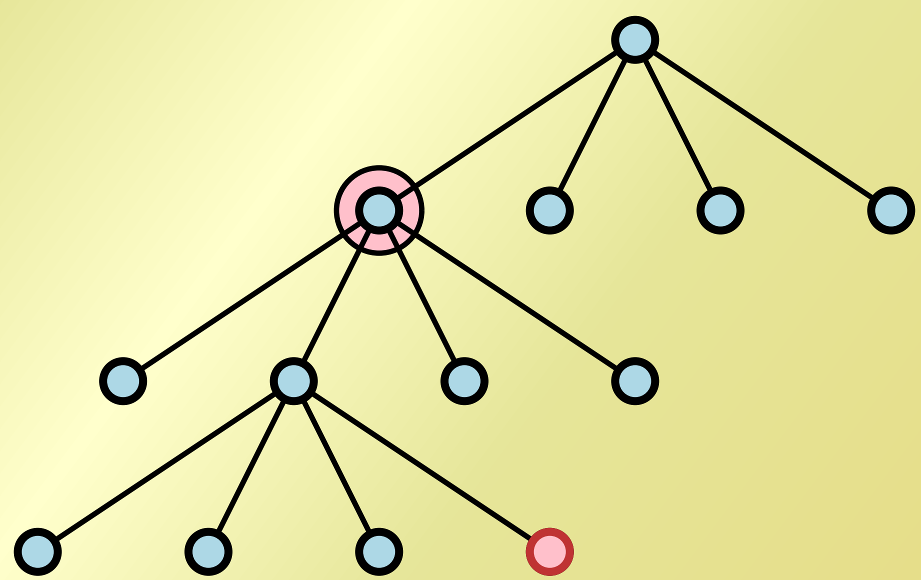

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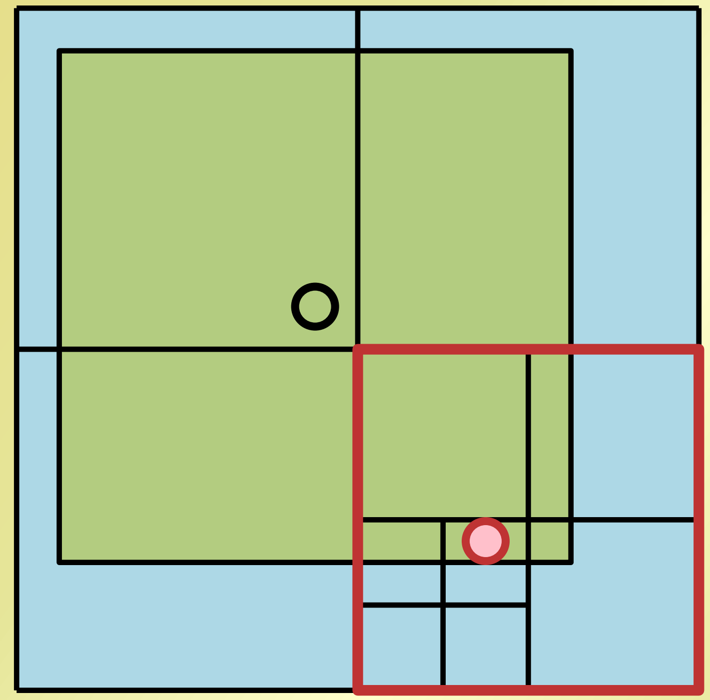

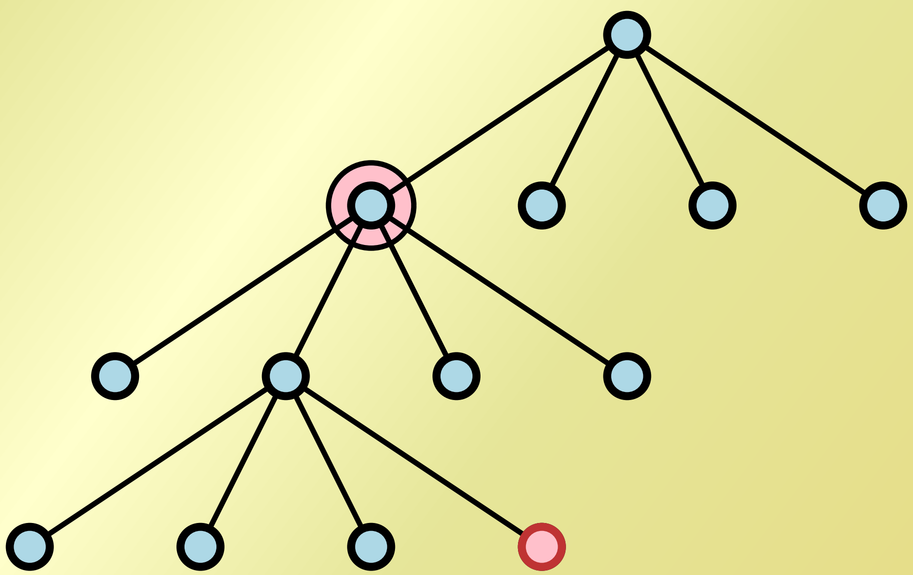

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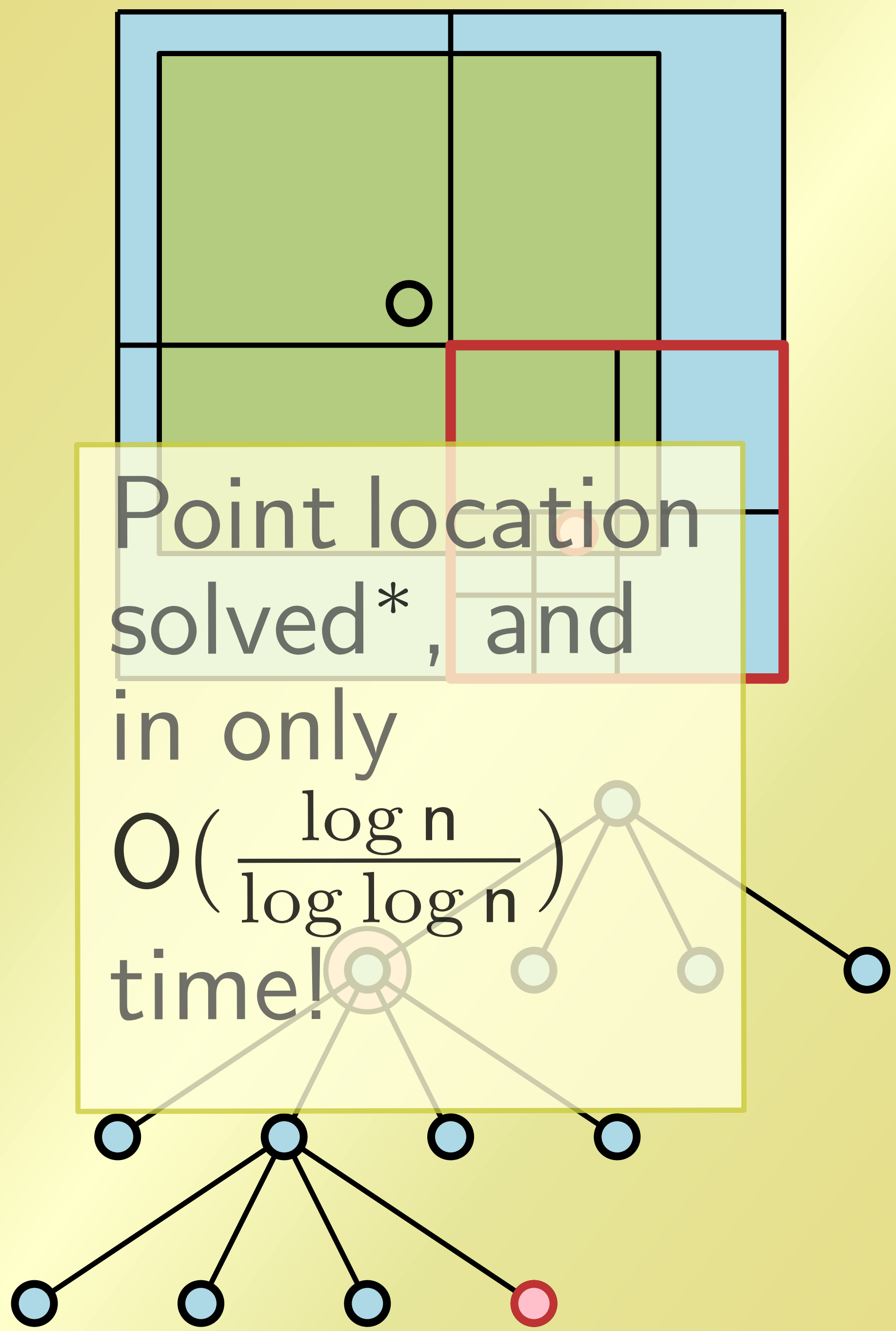

Point location solved*, and in only $O(\frac{\log n}{\log \log n})$ time!

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* CAUTION! Many details have been swept under the rug. Be extremely careful not to trip when walking on the rug.

CONCLUSIONS

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Do realistic input assumptions help?

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