
Existence of Simple Tours through Imprecise Points

Maarten Löffler



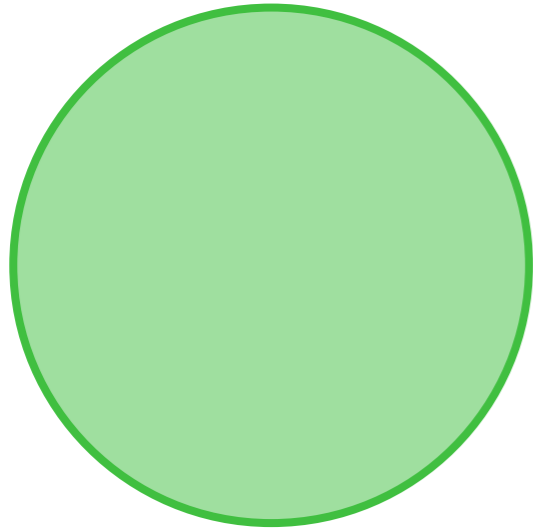
Center for Geometry, Imaging
and Virtual Environments

Utrecht University

Overview

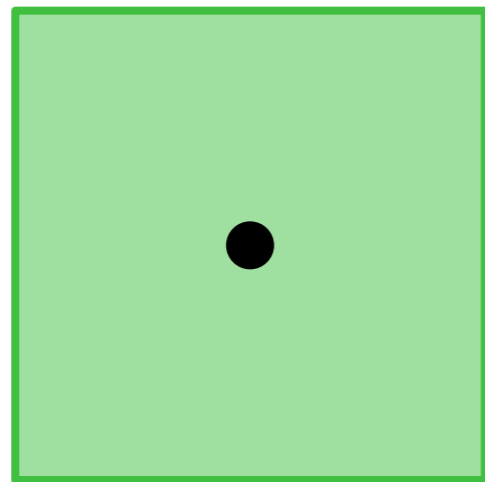
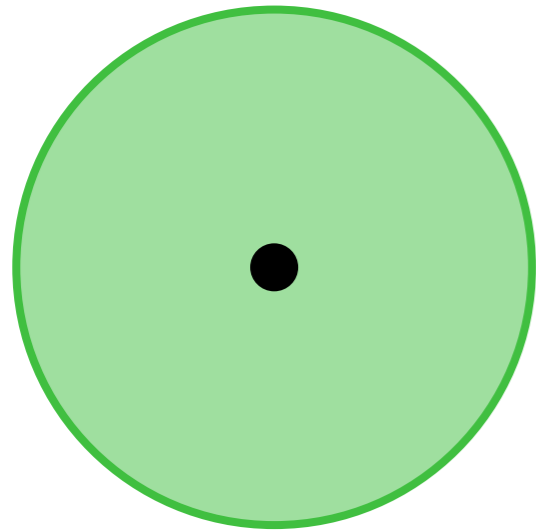
- Imprecise points
- Imprecise simple polygons
- NP-hardness proof
 - Planar 3-SAT
 - Variables as scissors
 - Clauses
 - Further details
- Other results and consequences

Imprecise Points



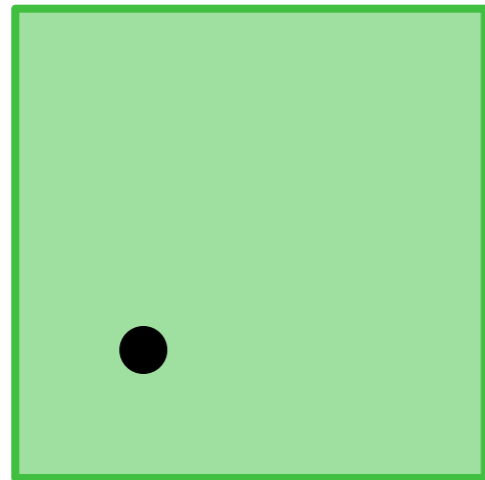
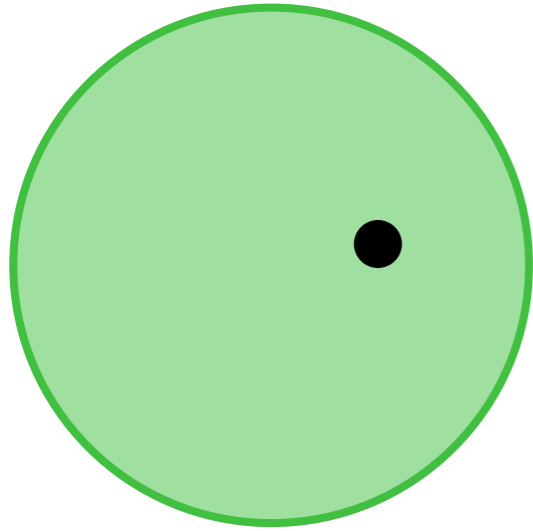
- Unknown location
- Known region of possible locations
- Regions are simple geometric objects
 - Disc
 - Square
 - Rectangle
 - Convex polygon
 - Line segment

Imprecise Points



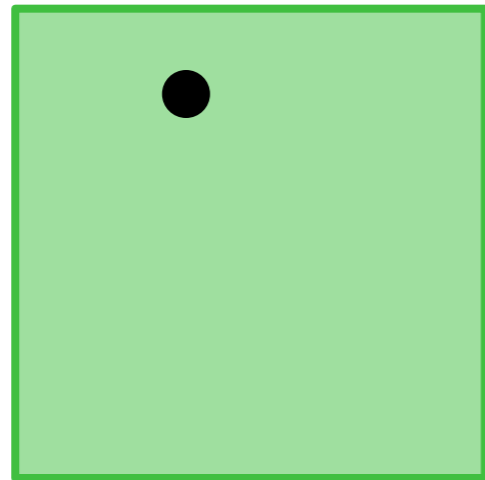
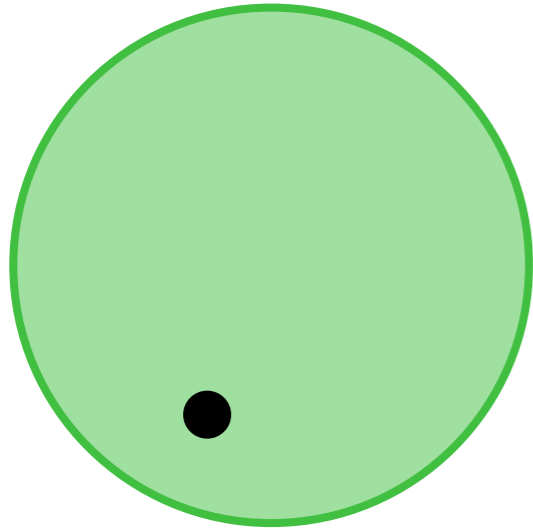
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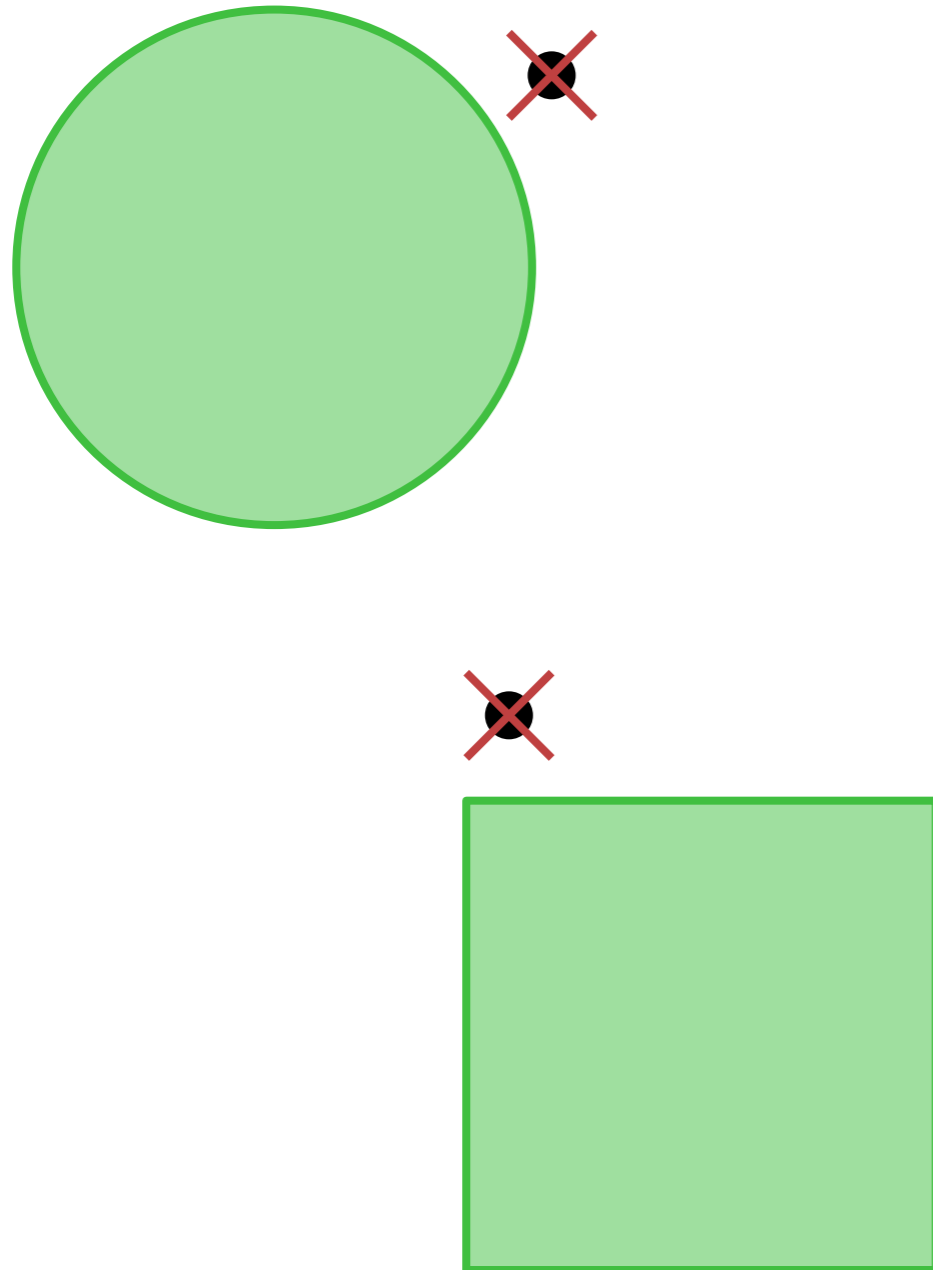
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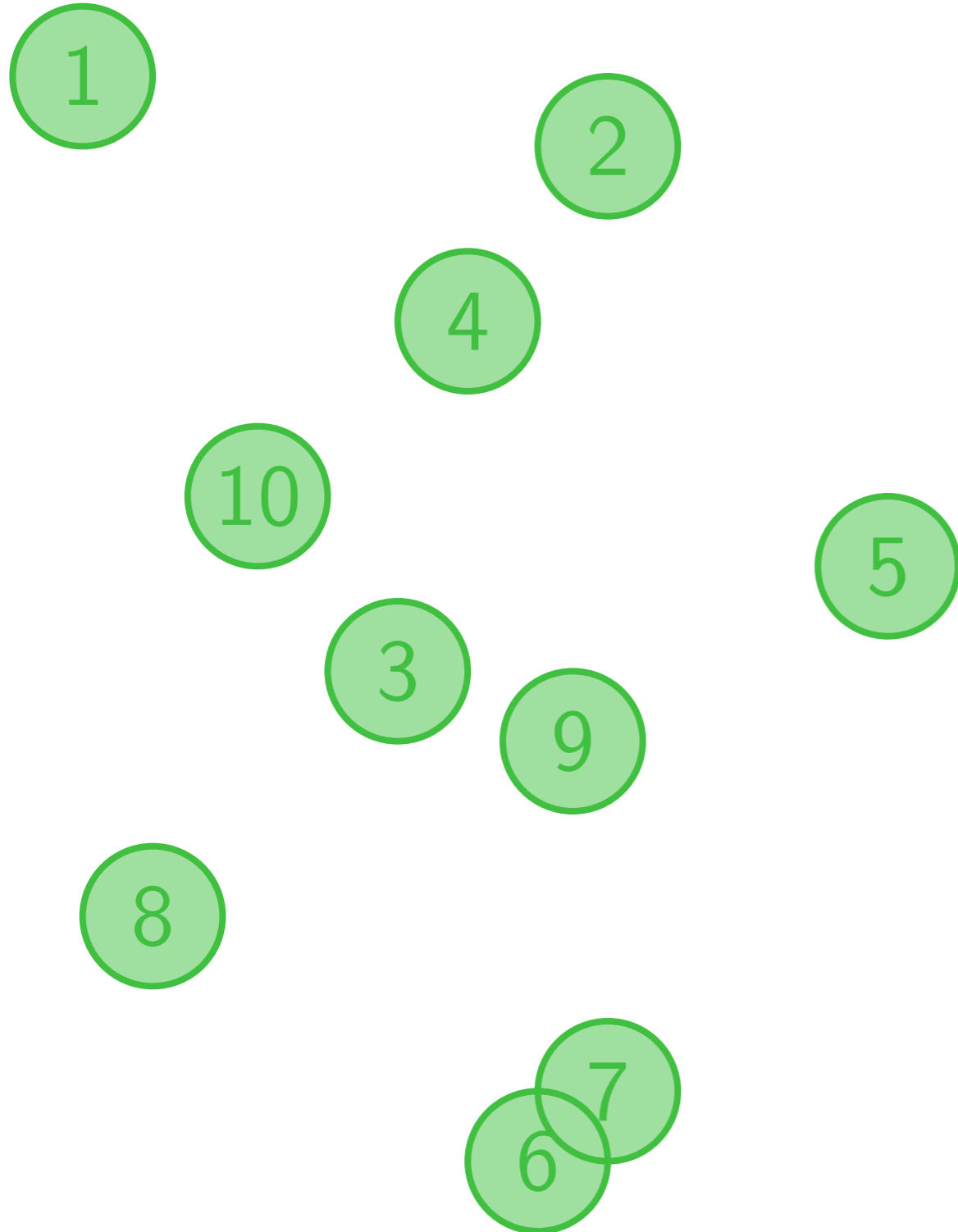
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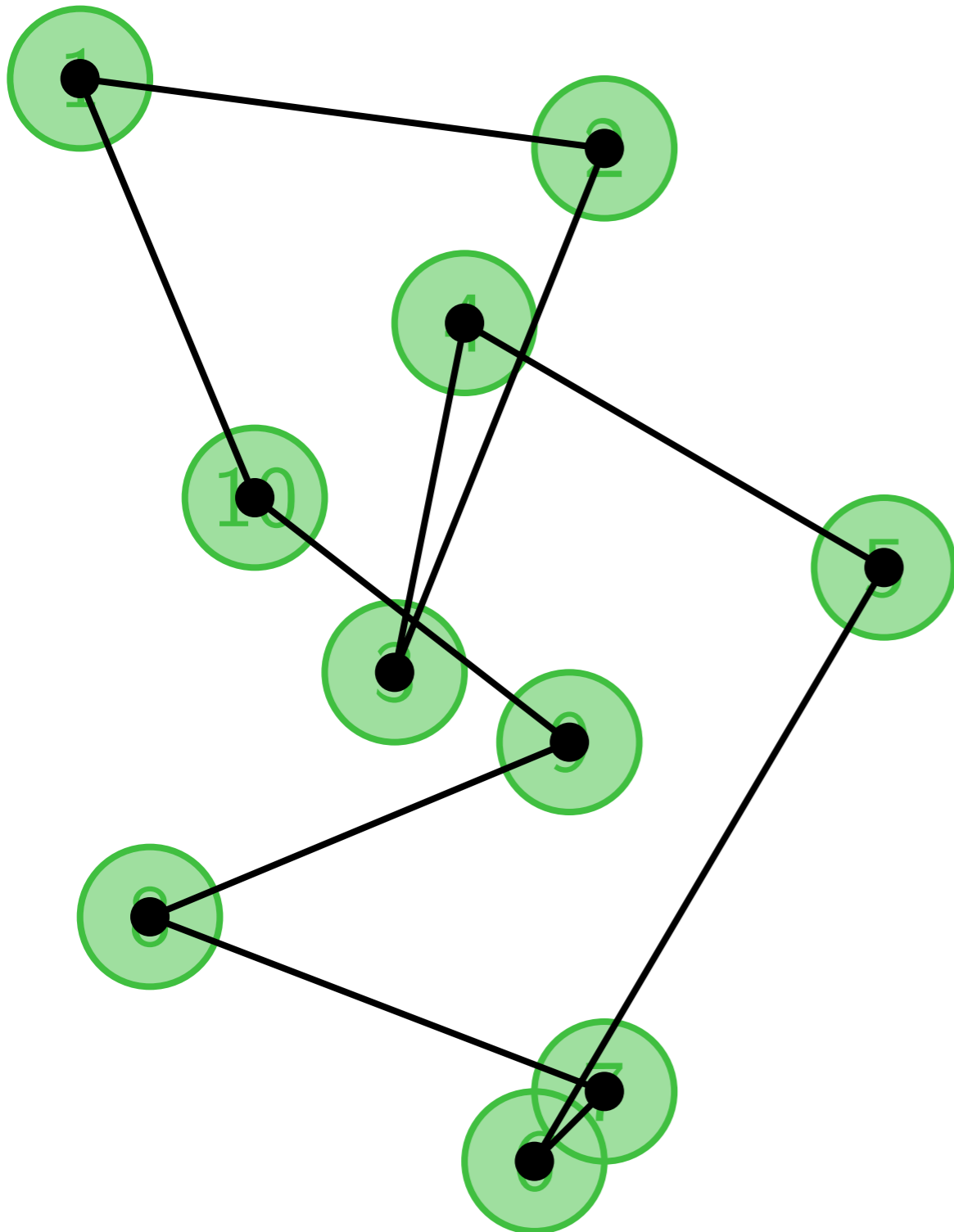
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Imprecise Simple Polygons



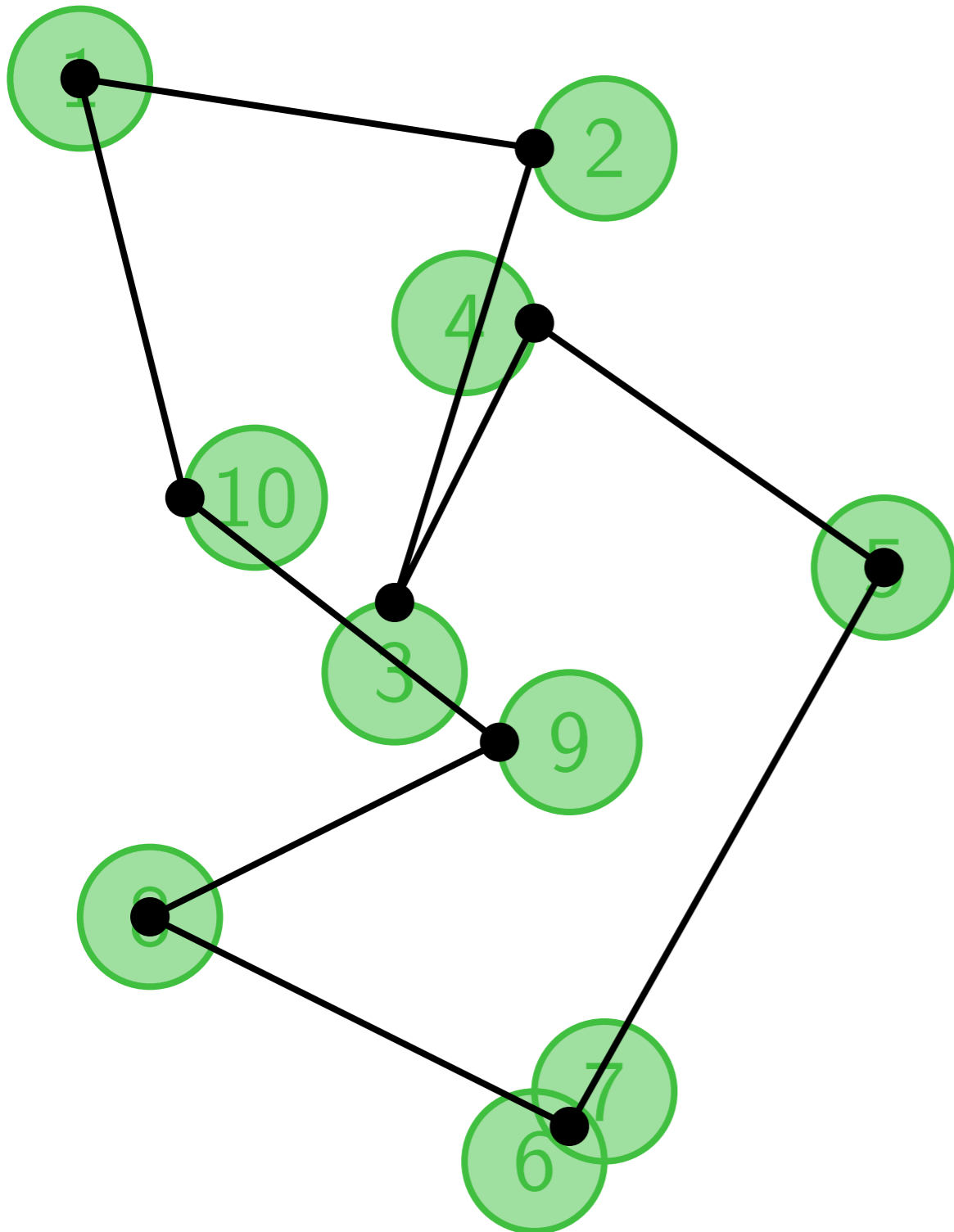
- Sequence of imprecise points
- Place one vertex in each region
- The result should be a simple polygon
- This problem is NP-hard

Imprecise Simple Polygons



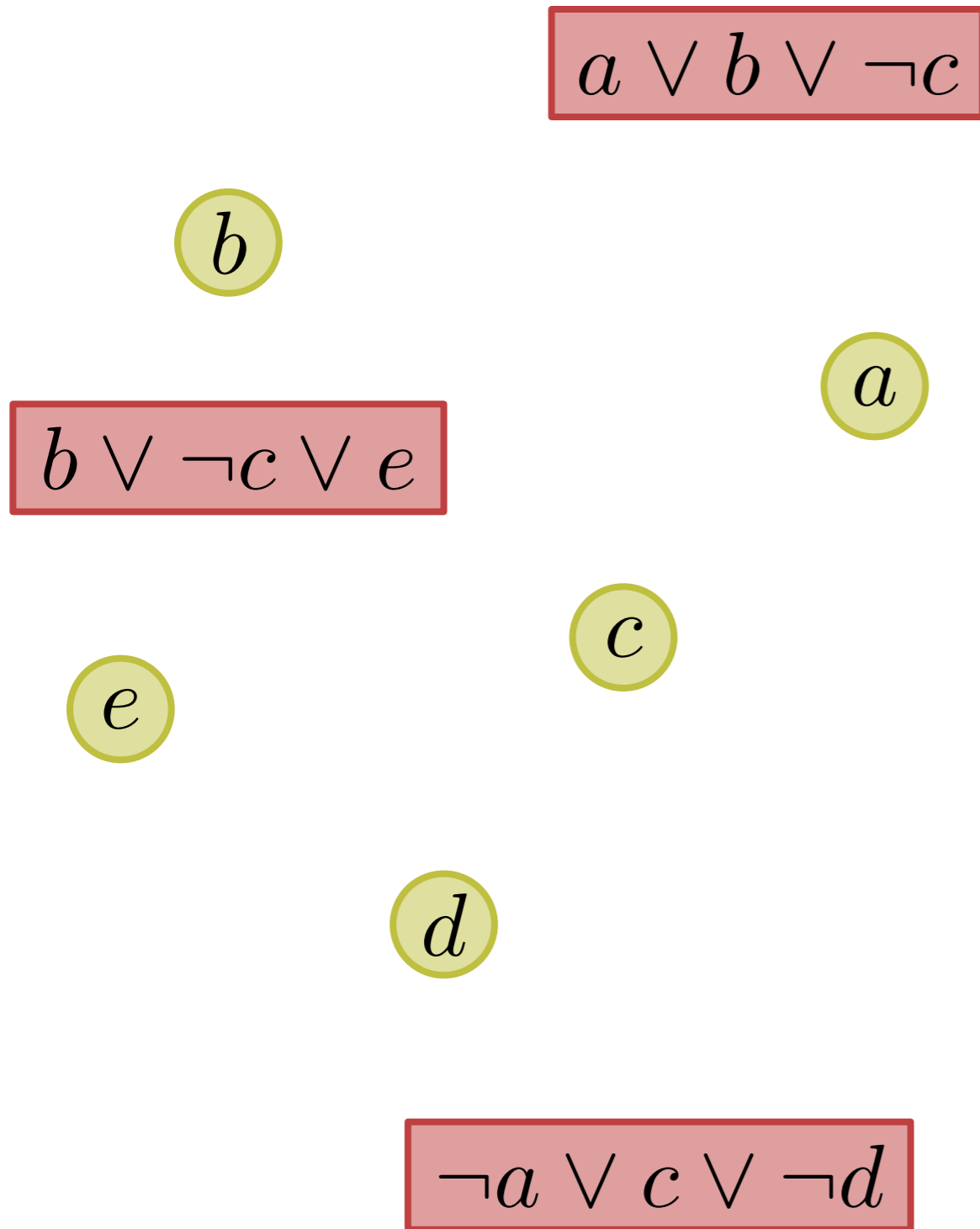
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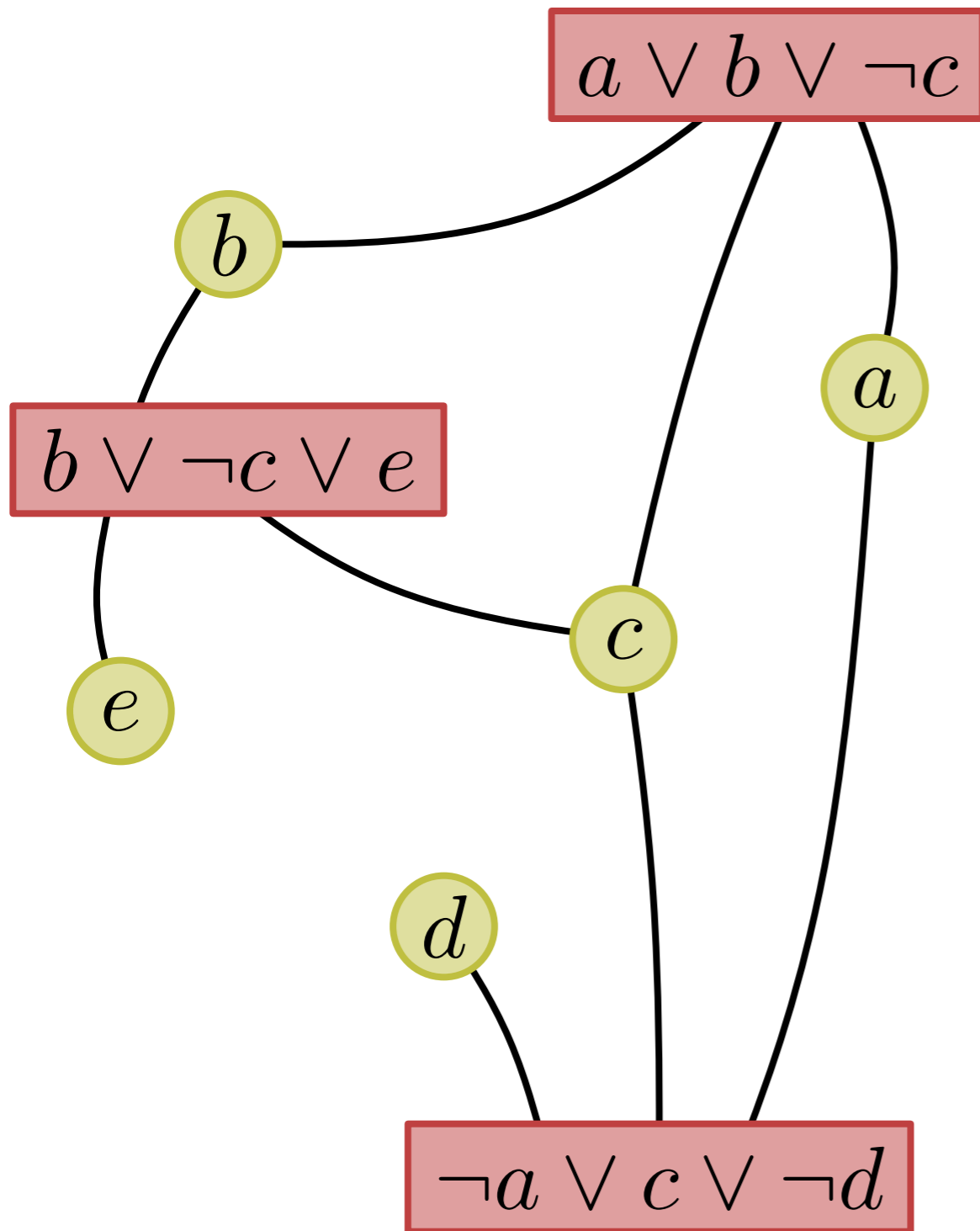
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Planar 3-SAT



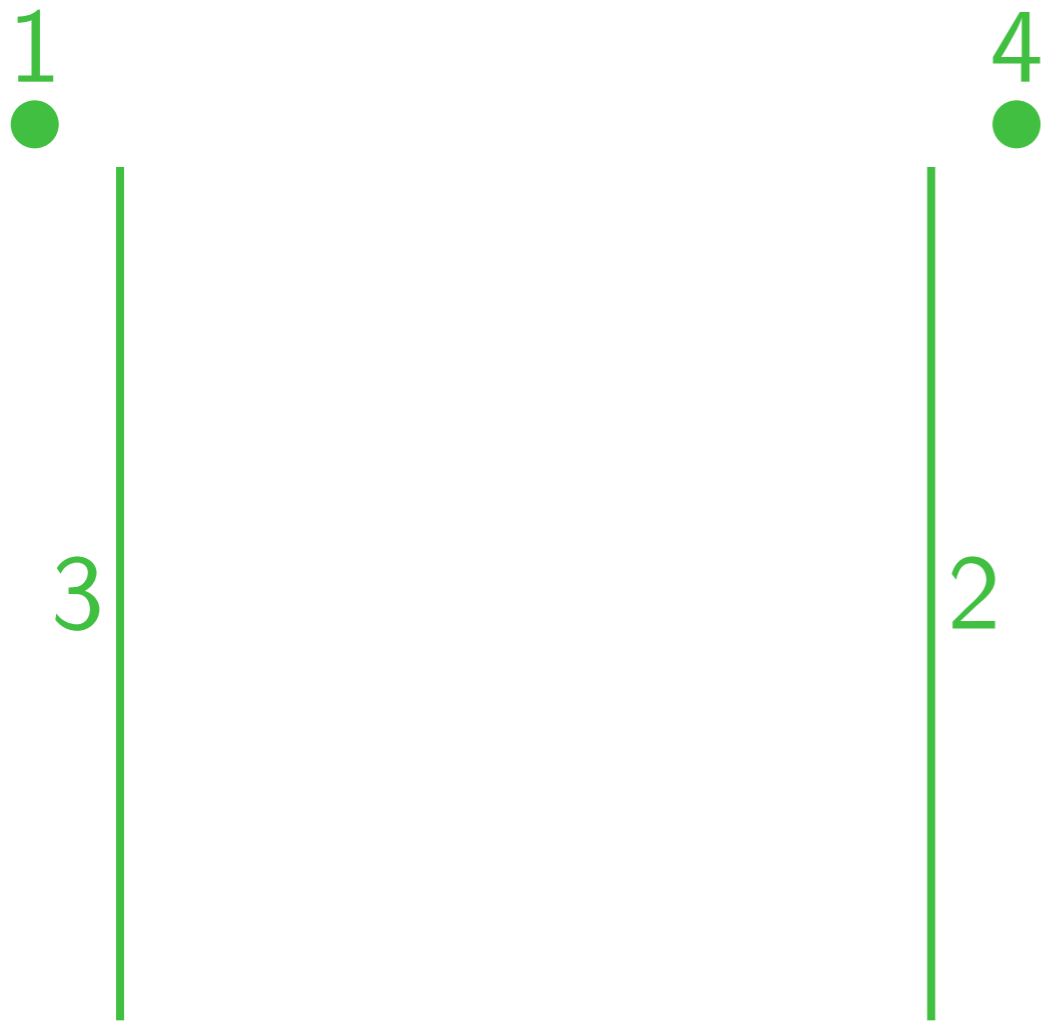
- Boolean satisfiability problem
- At most three variables per clause
- Variable-clause graph must be planar
- Known to be NP-hard [Lichtenstein 1982]

Planar 3-SAT



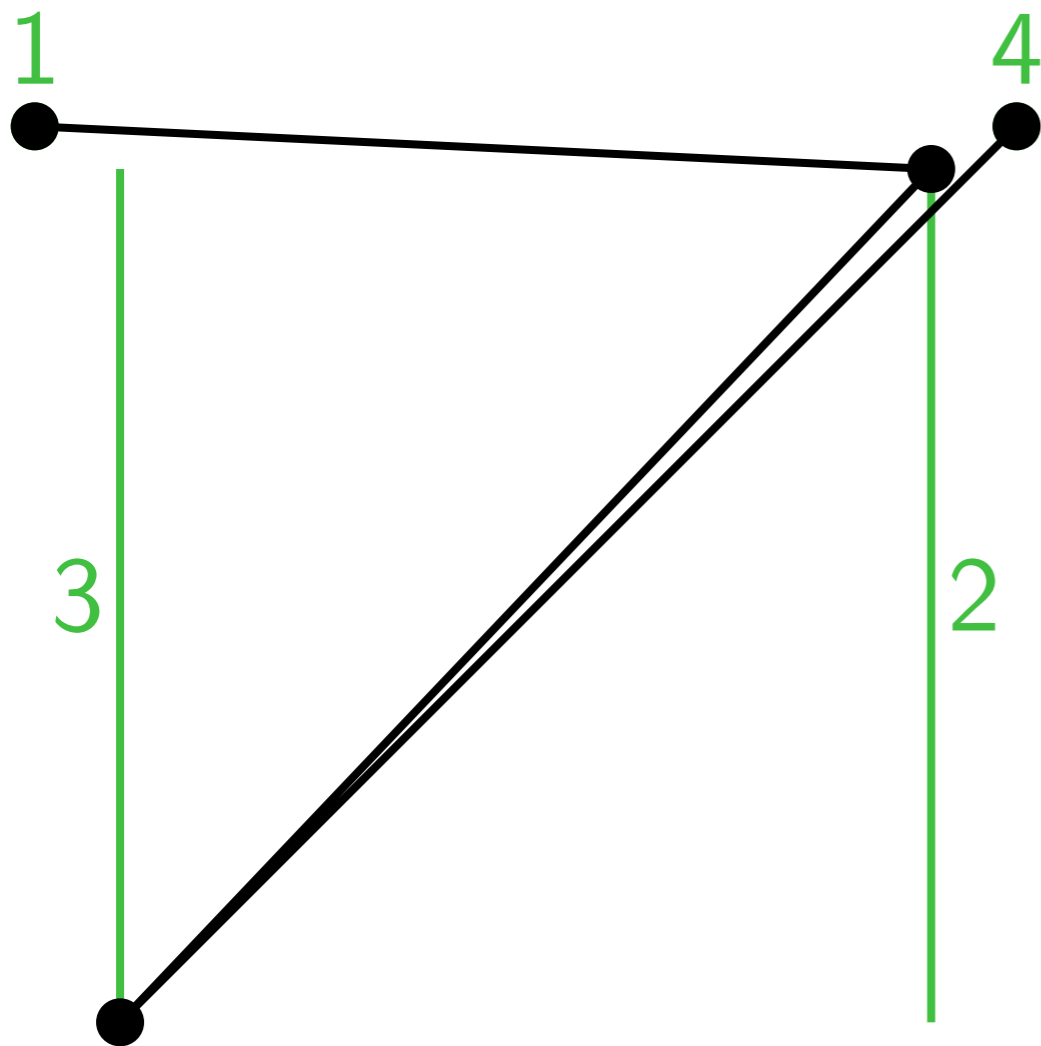
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Scissors Gadget



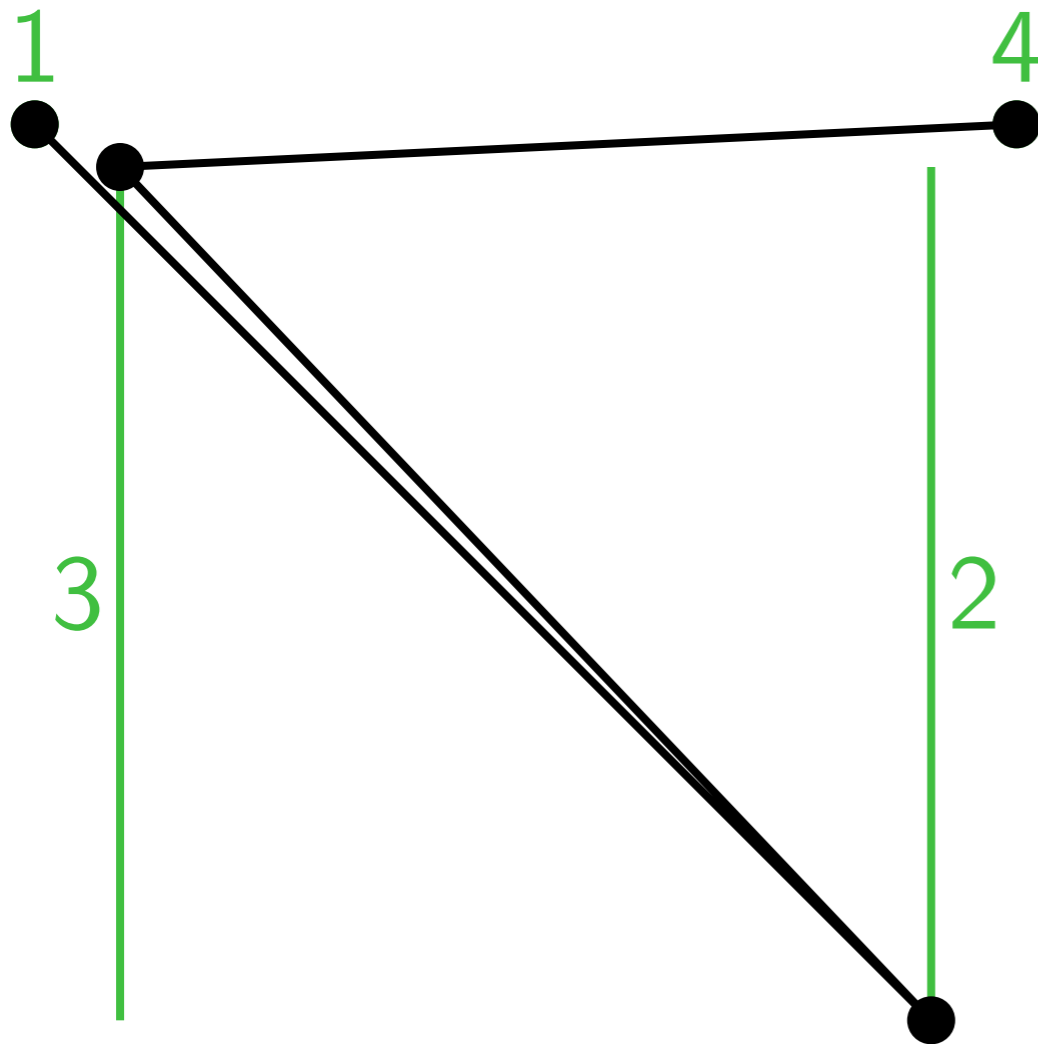
- Configuration of four imprecise points
- Two distinct possible solutions
- Each variable will be represented by a number of scissors
 - Positive slope: *true*
 - Negative slope: *false*

Scissors Gadget



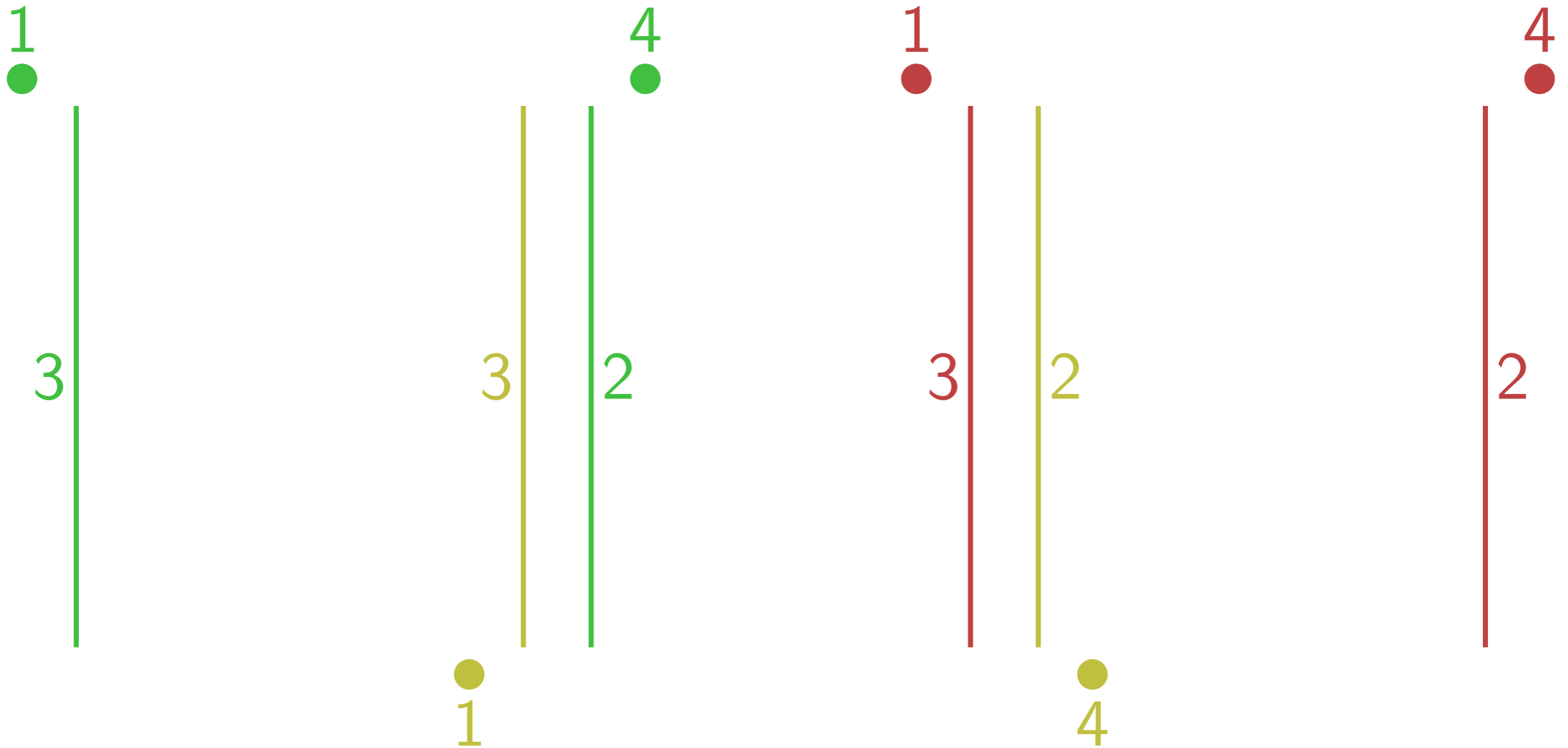
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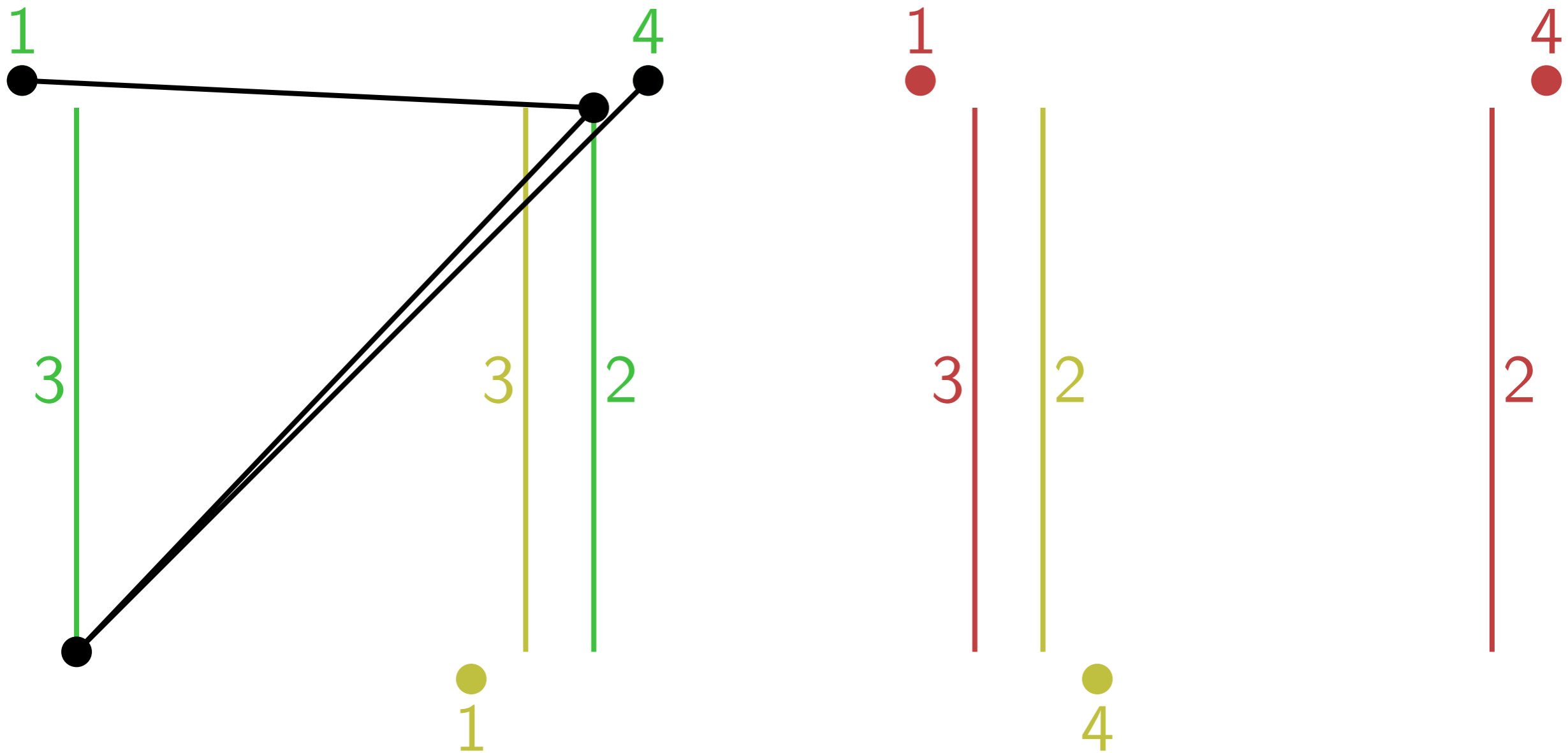


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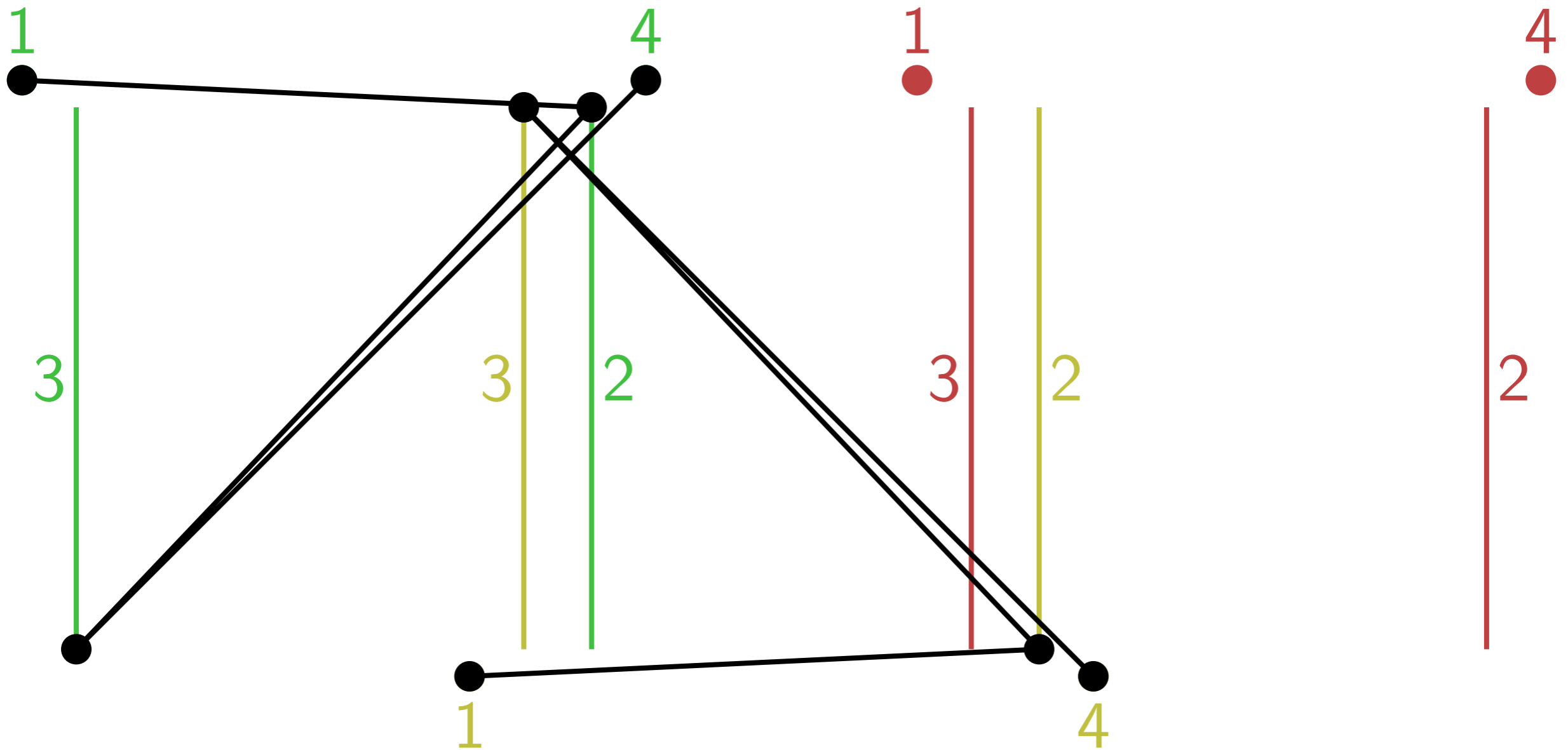
Scissors Chain



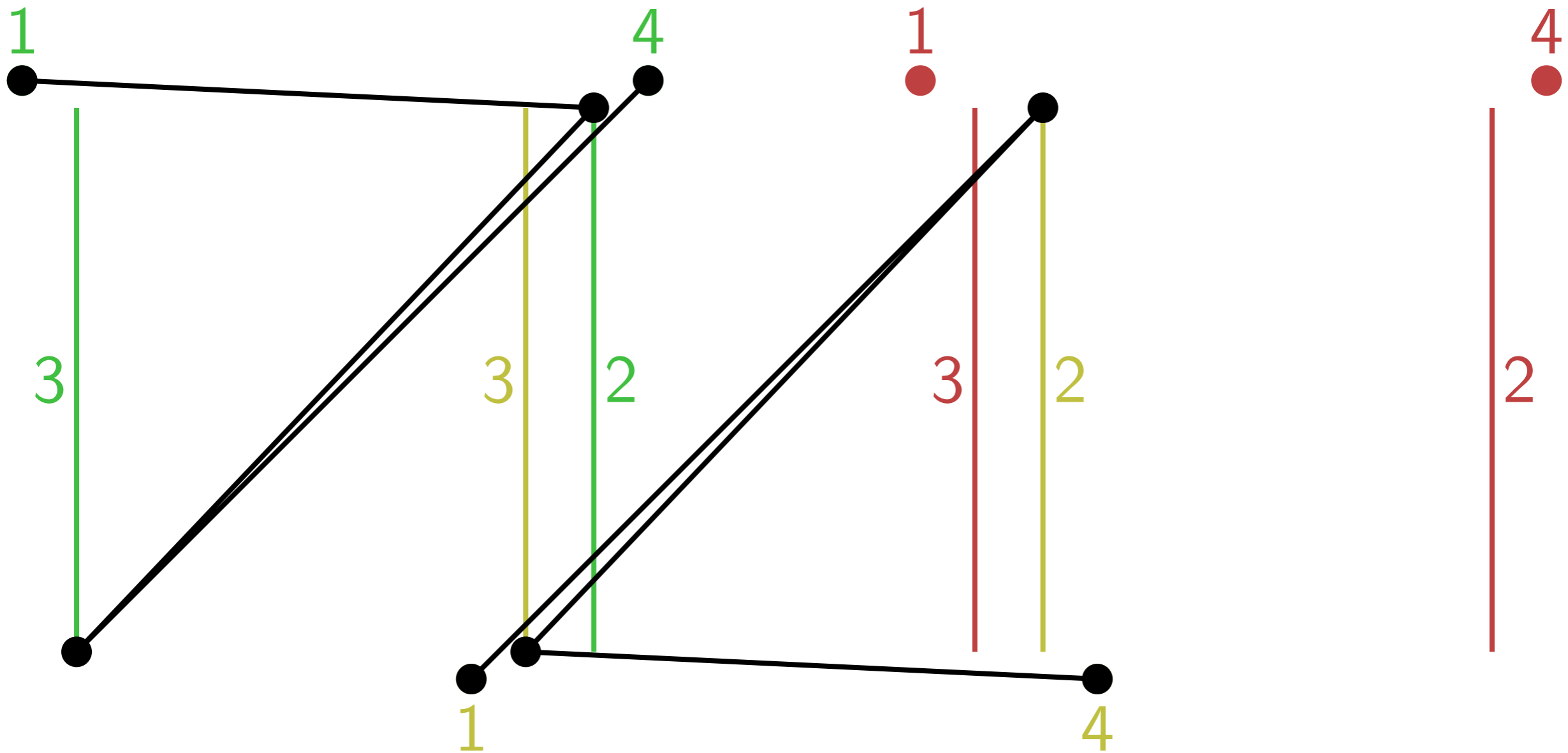
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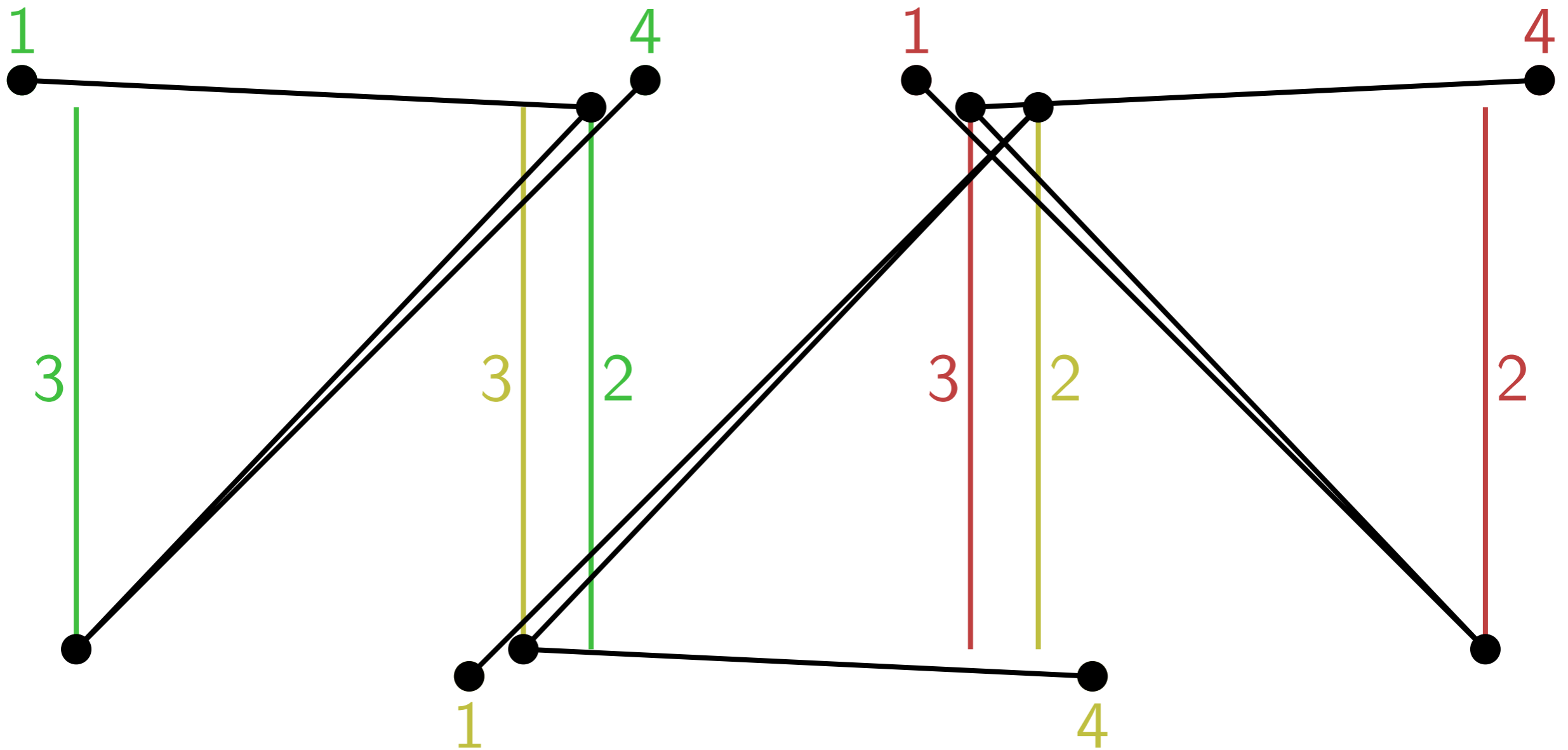
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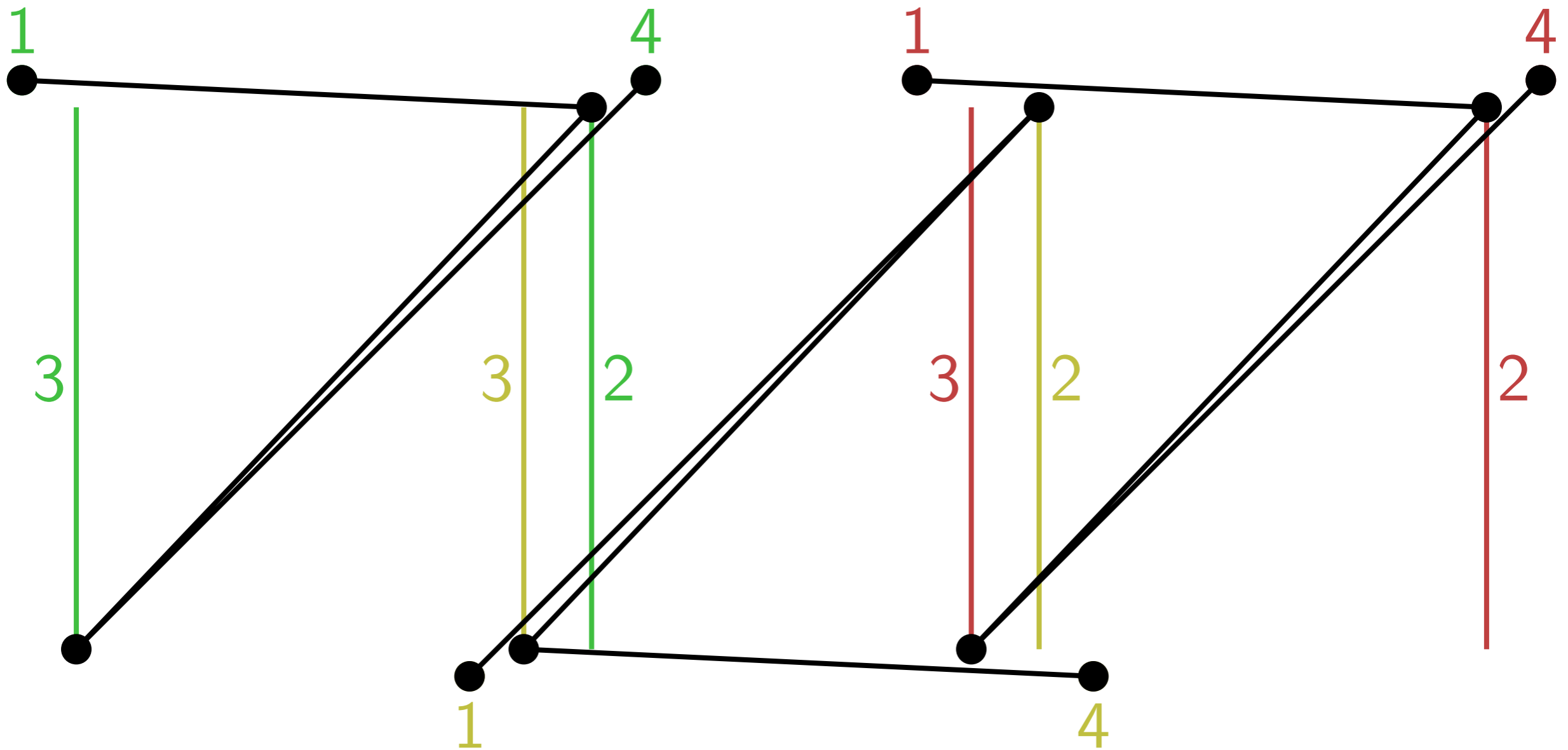
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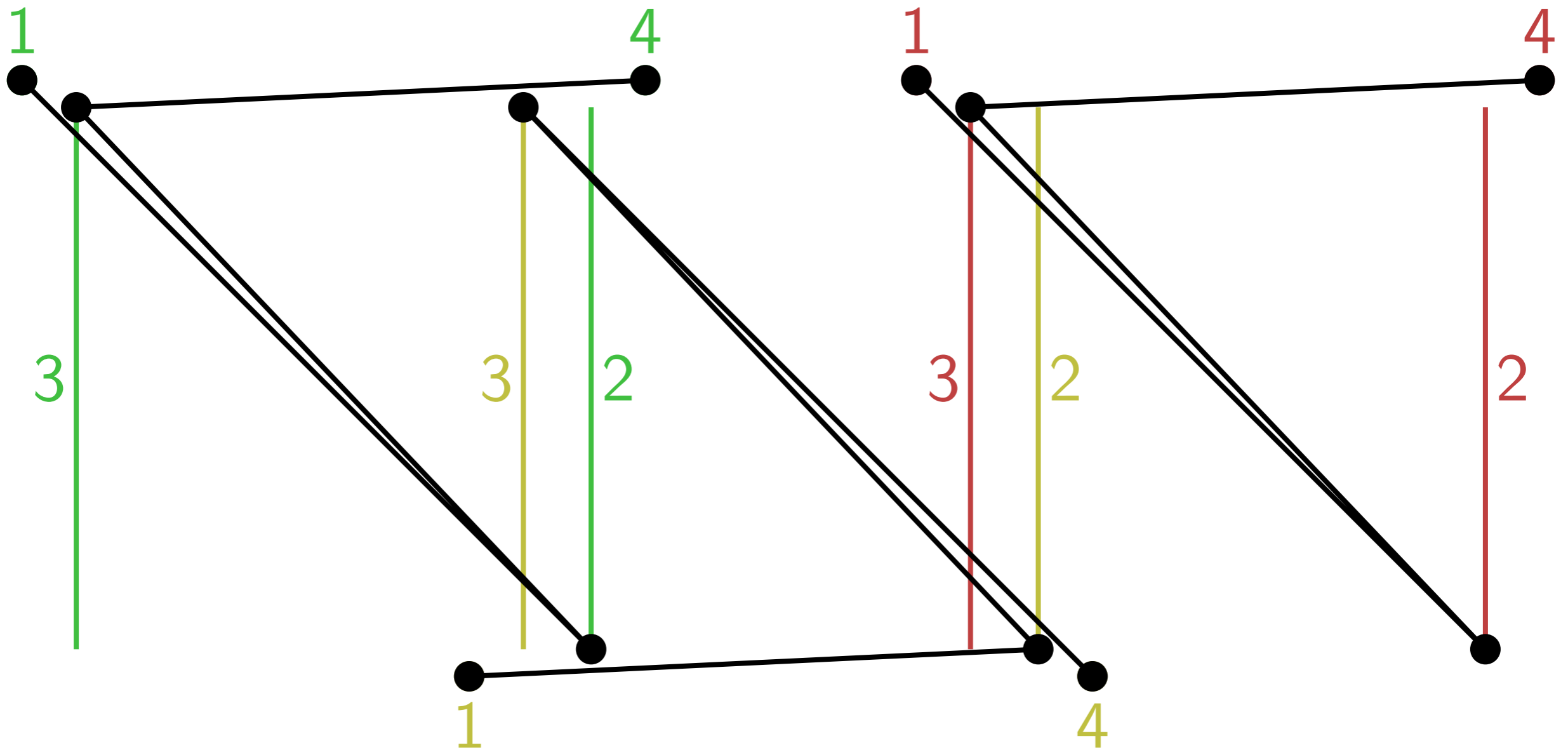
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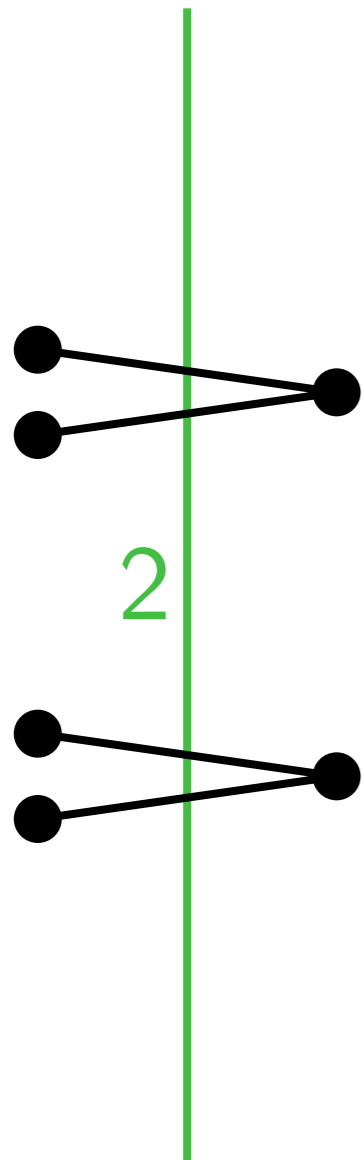
Scissors Chain



Scissors Chain



Clause Gadget

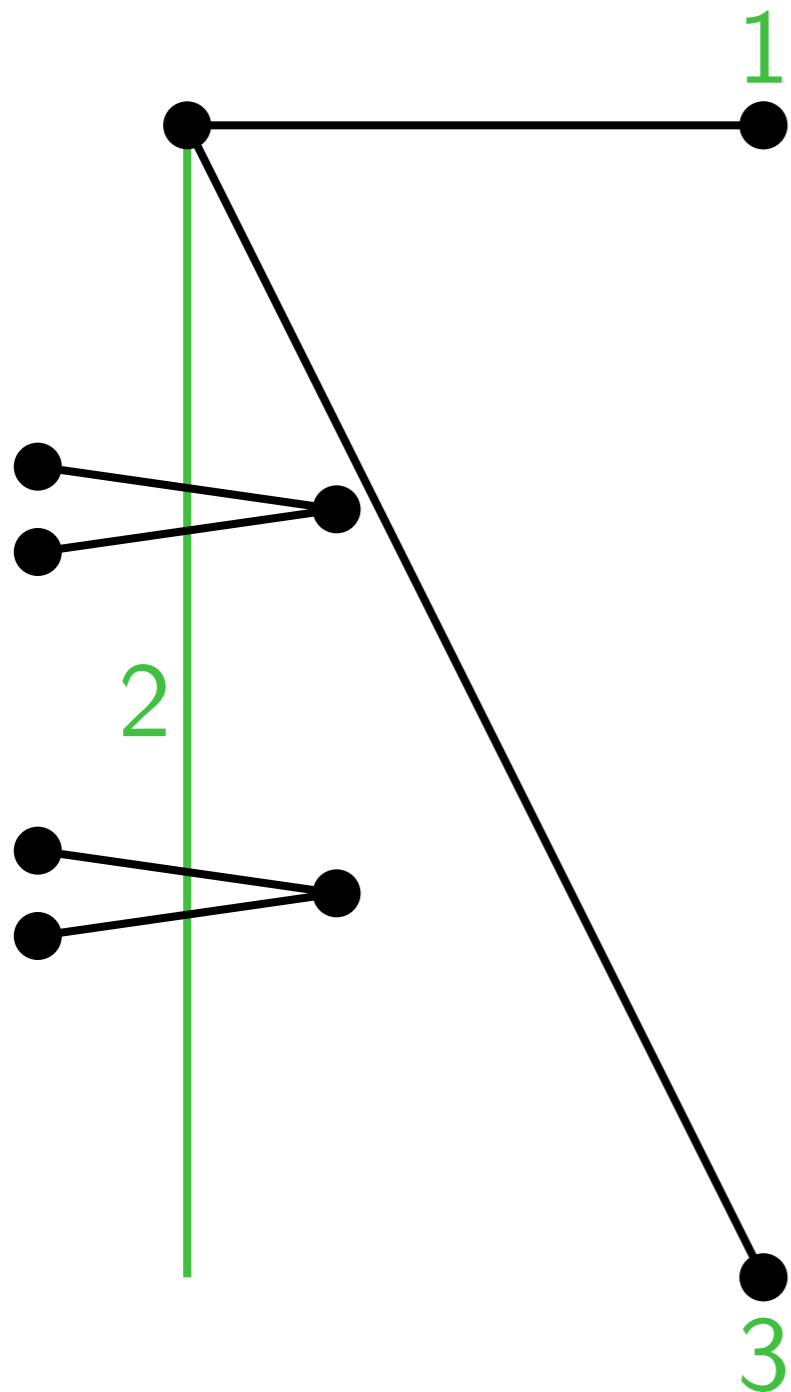


1 ●

- Configuration of three imprecise points
- Two fixed parts of the tour
- Three distinct possible solutions
- Each solution will be connected to one of the clause's variables

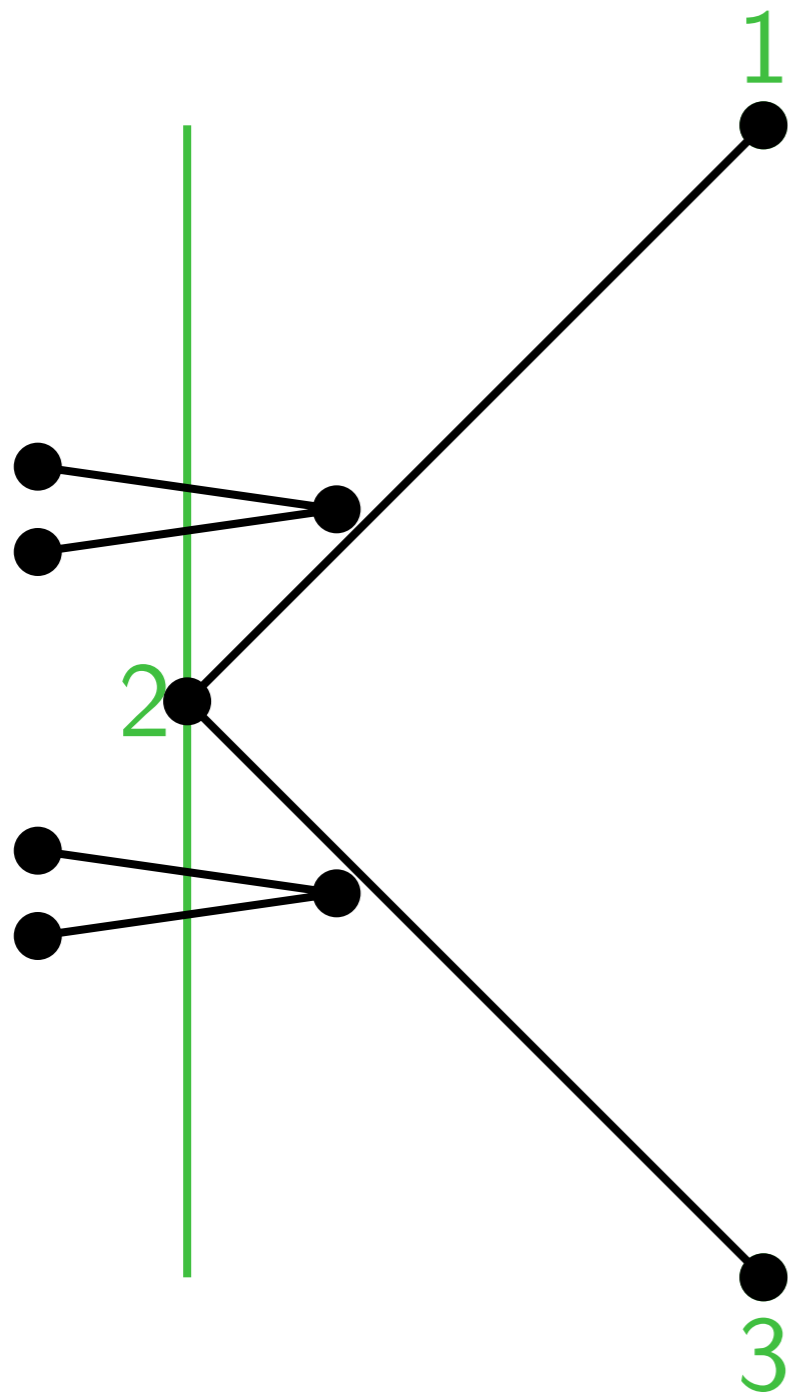
●
3

Clause Gadget



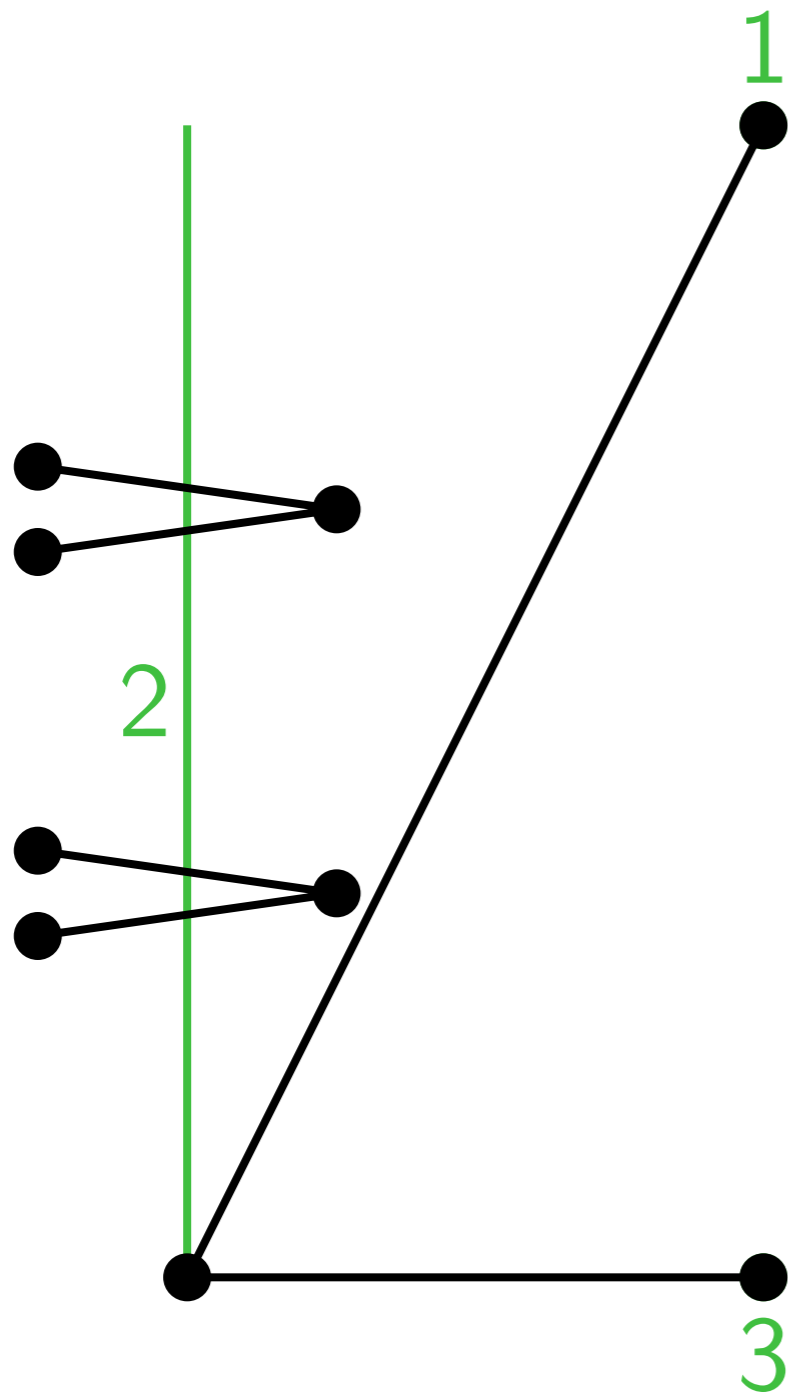
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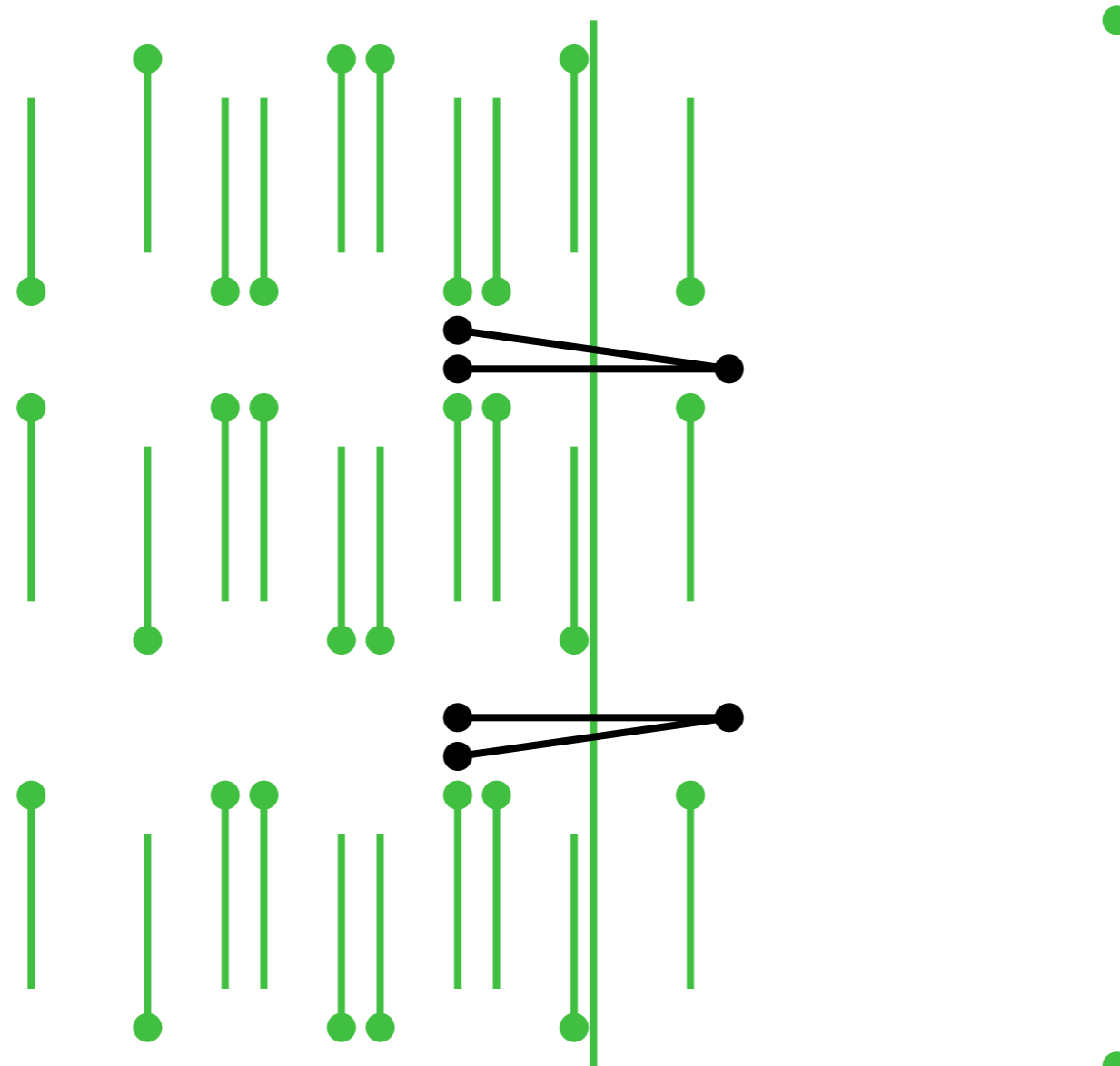
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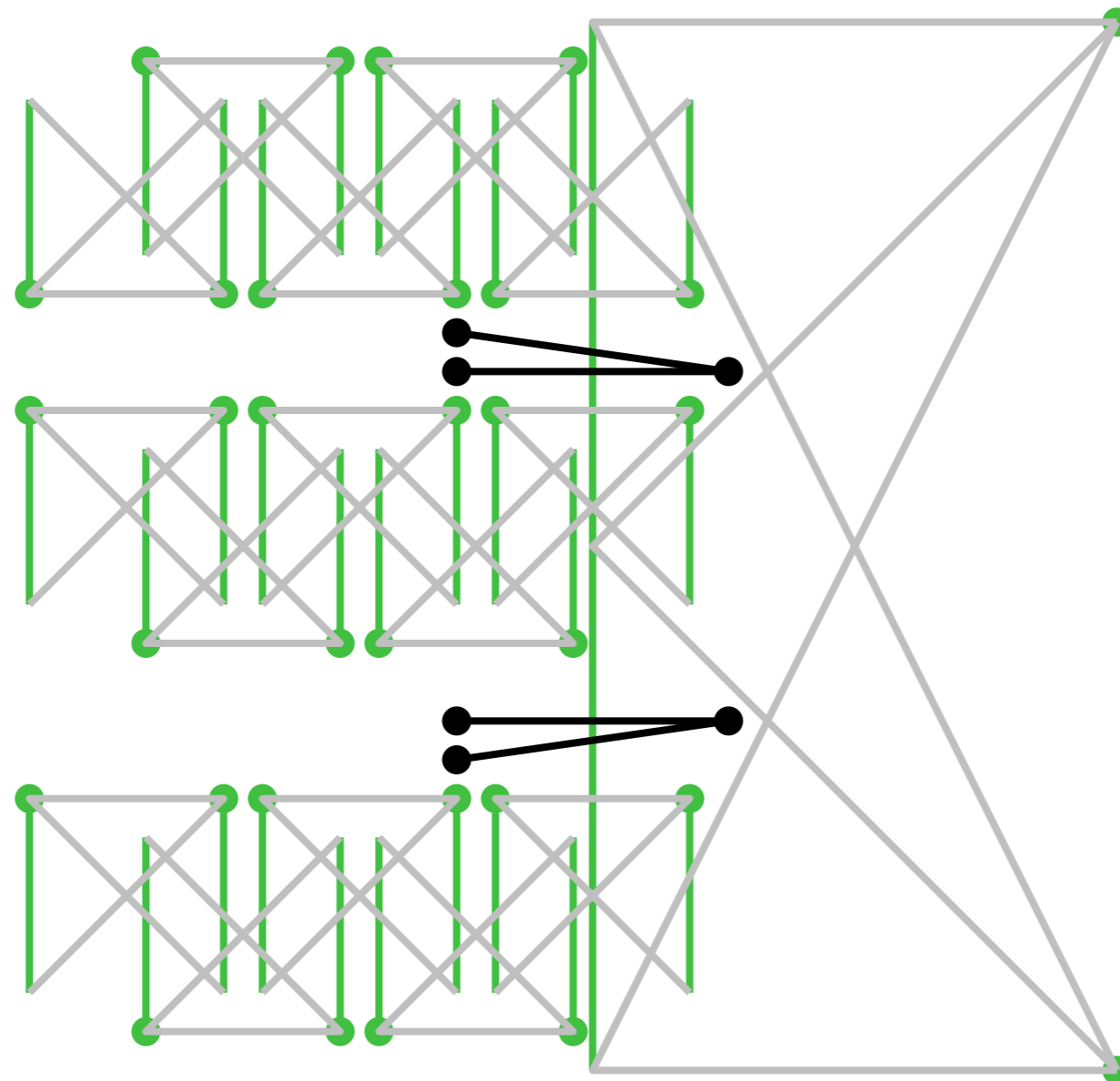


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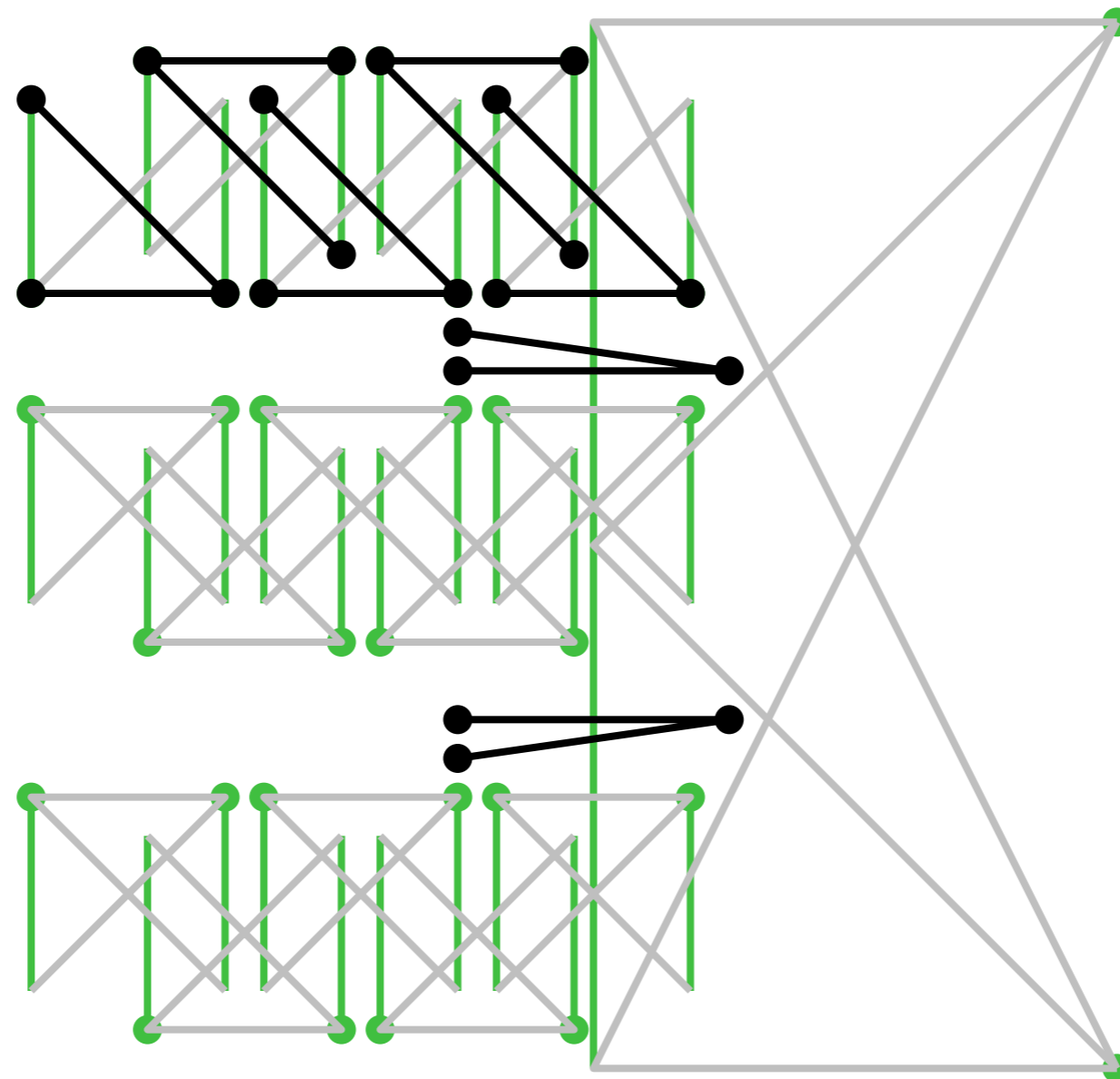
Variables and Clauses



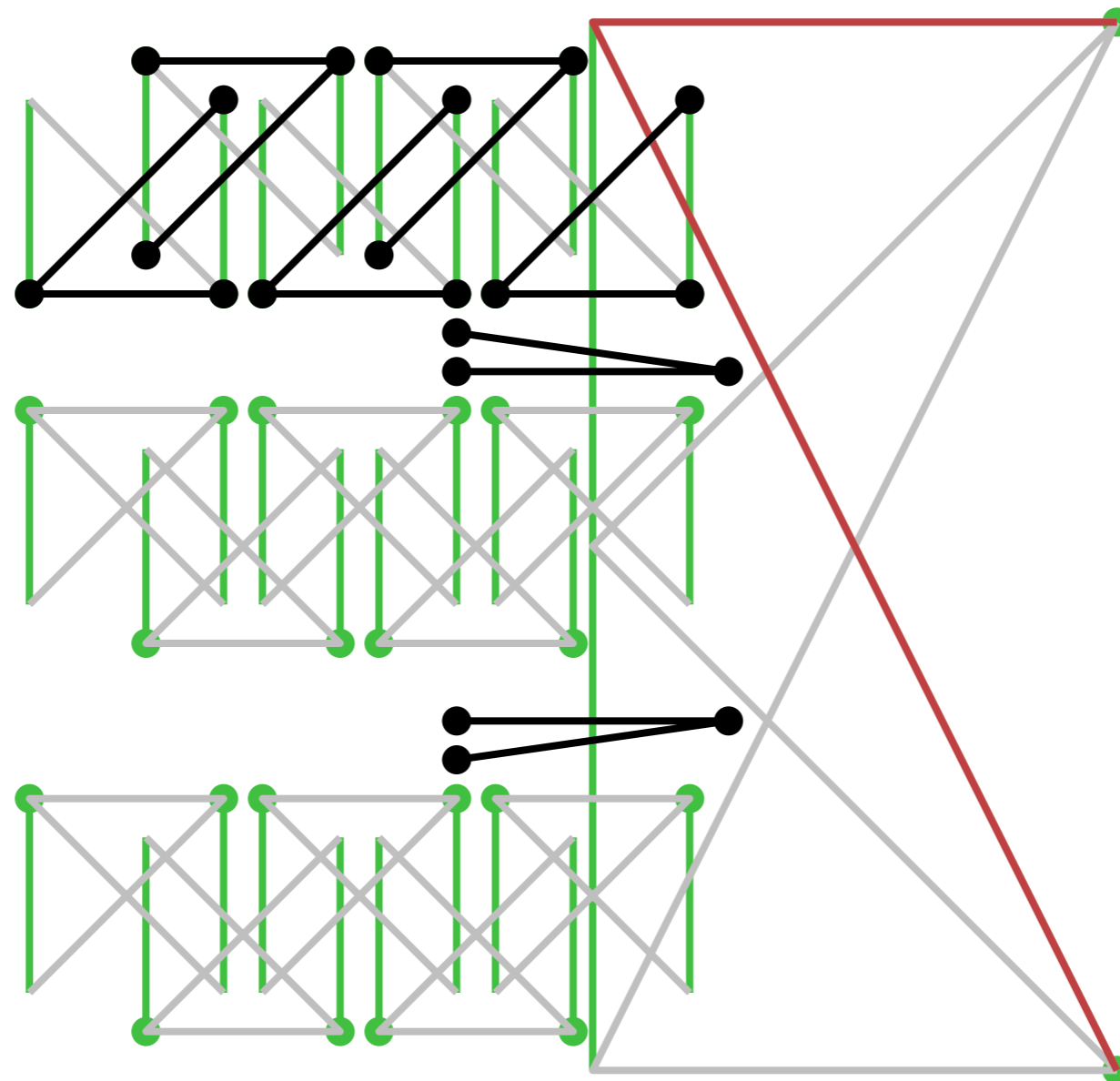
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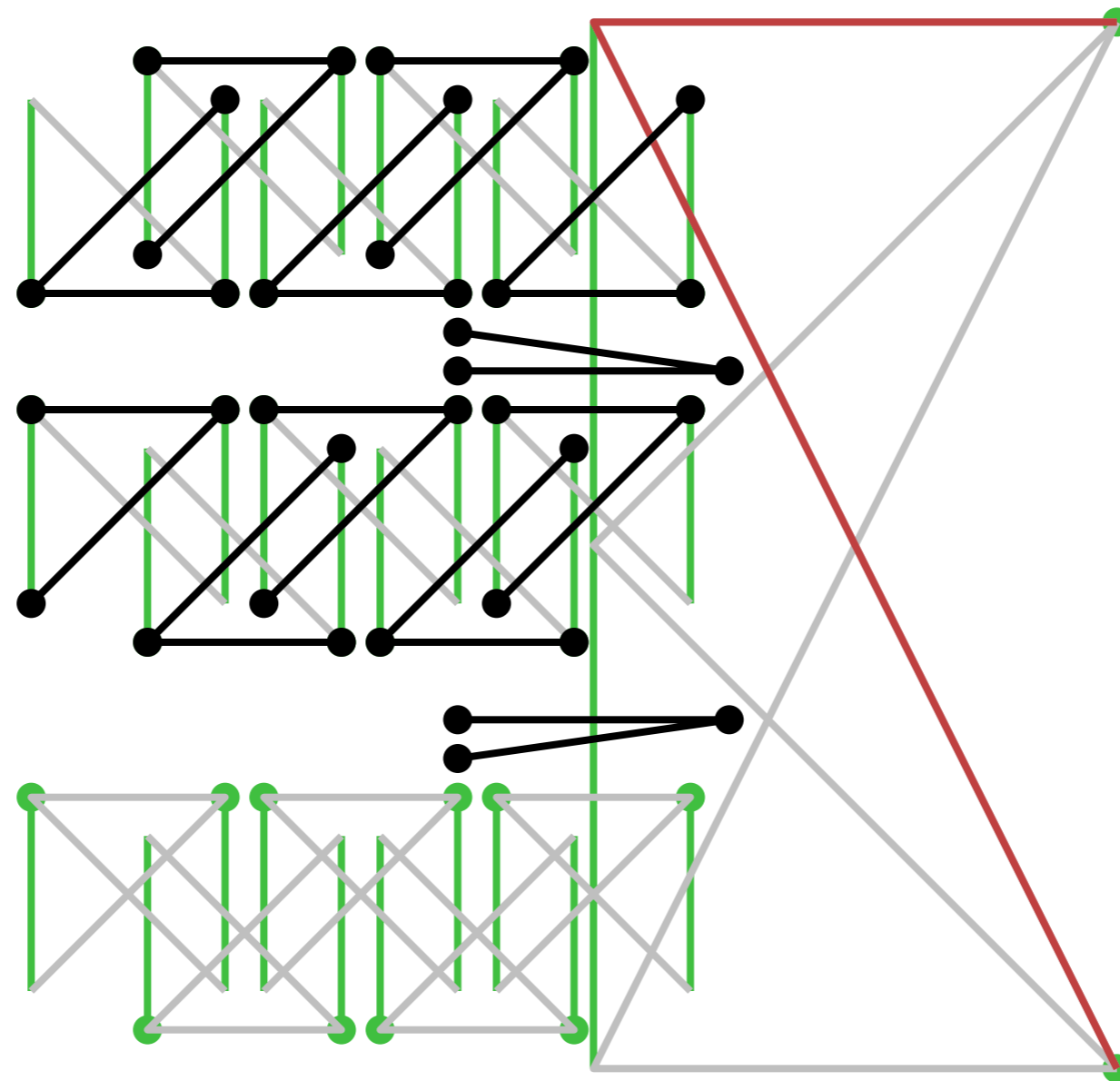
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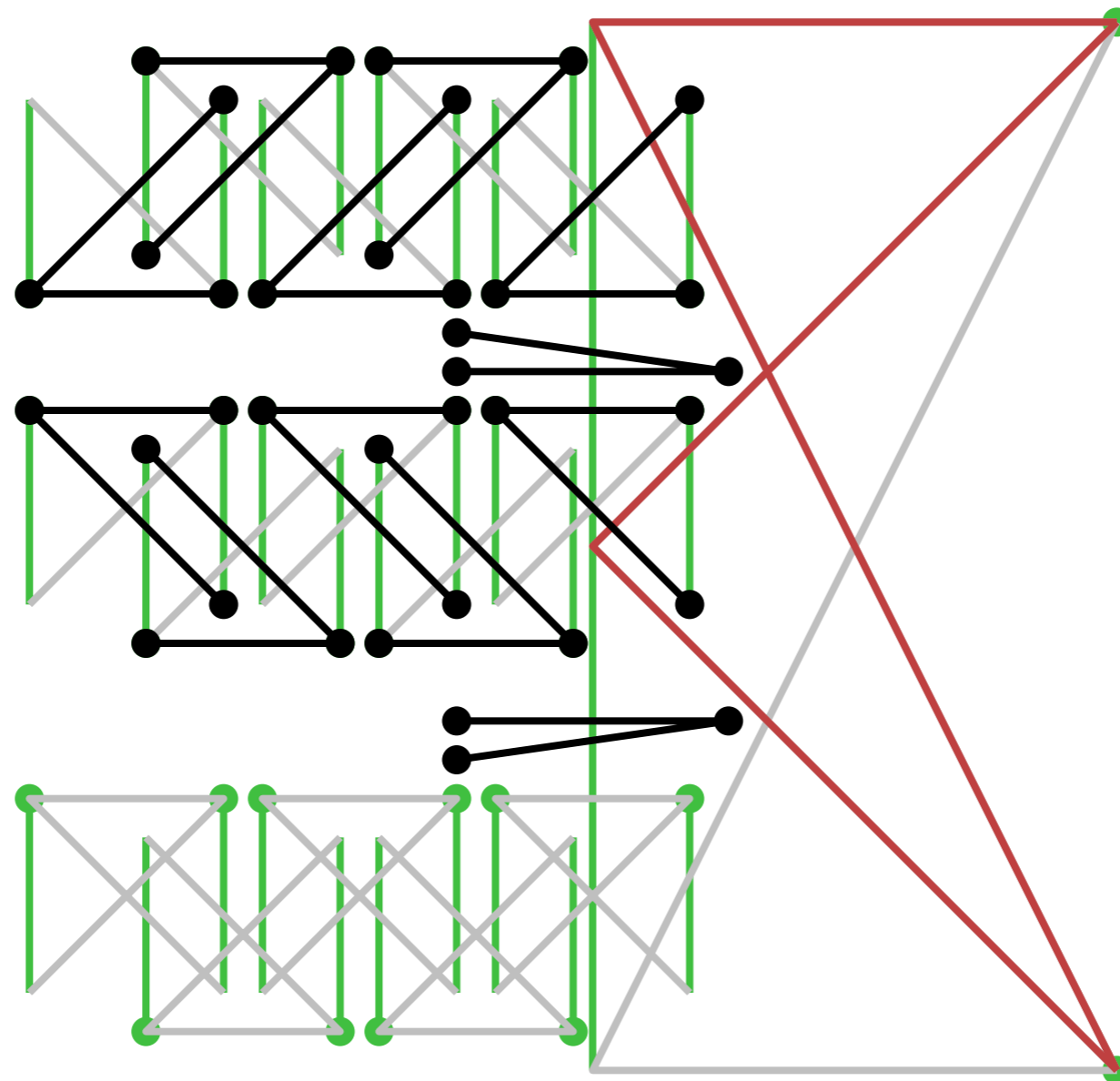
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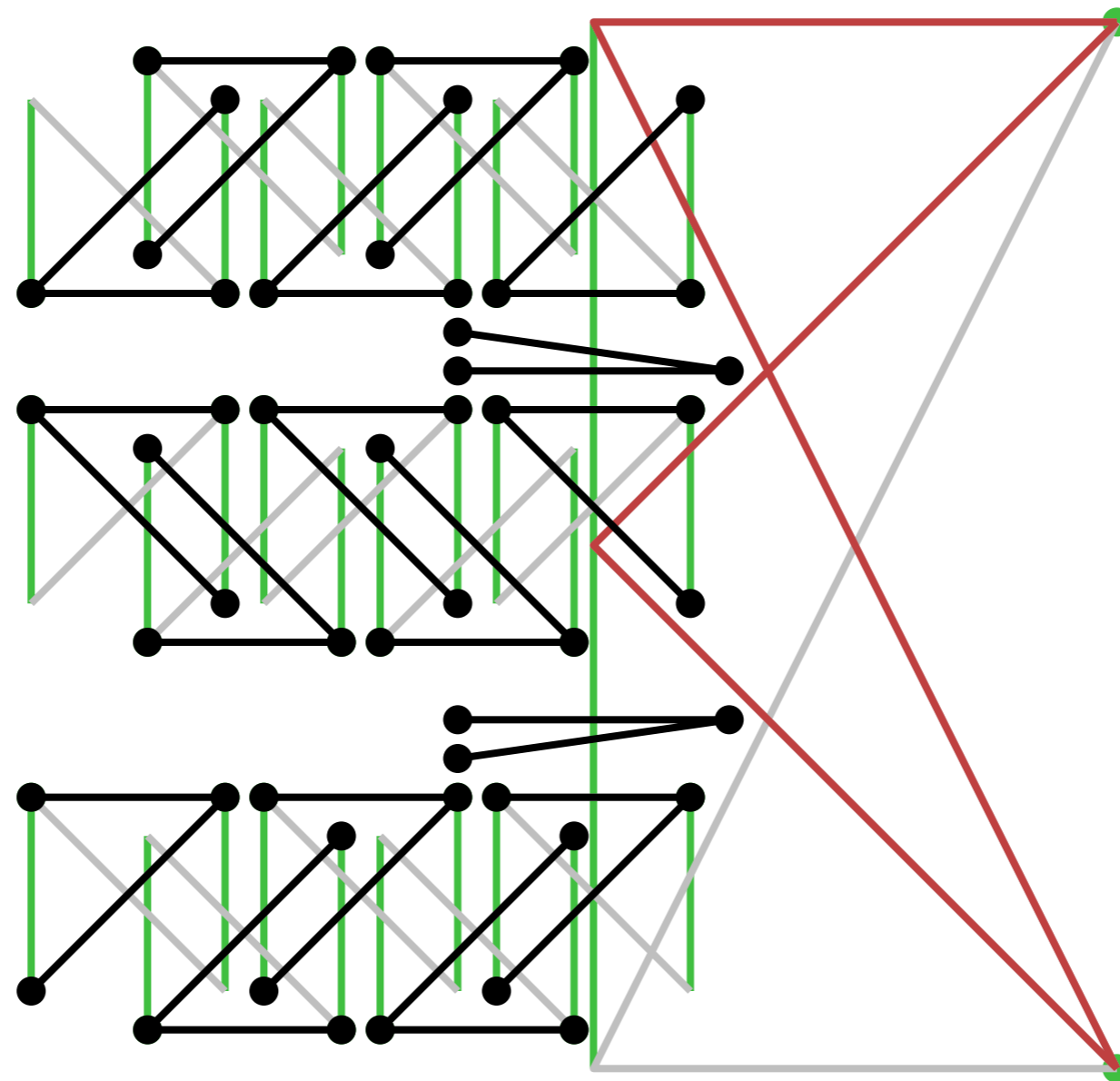
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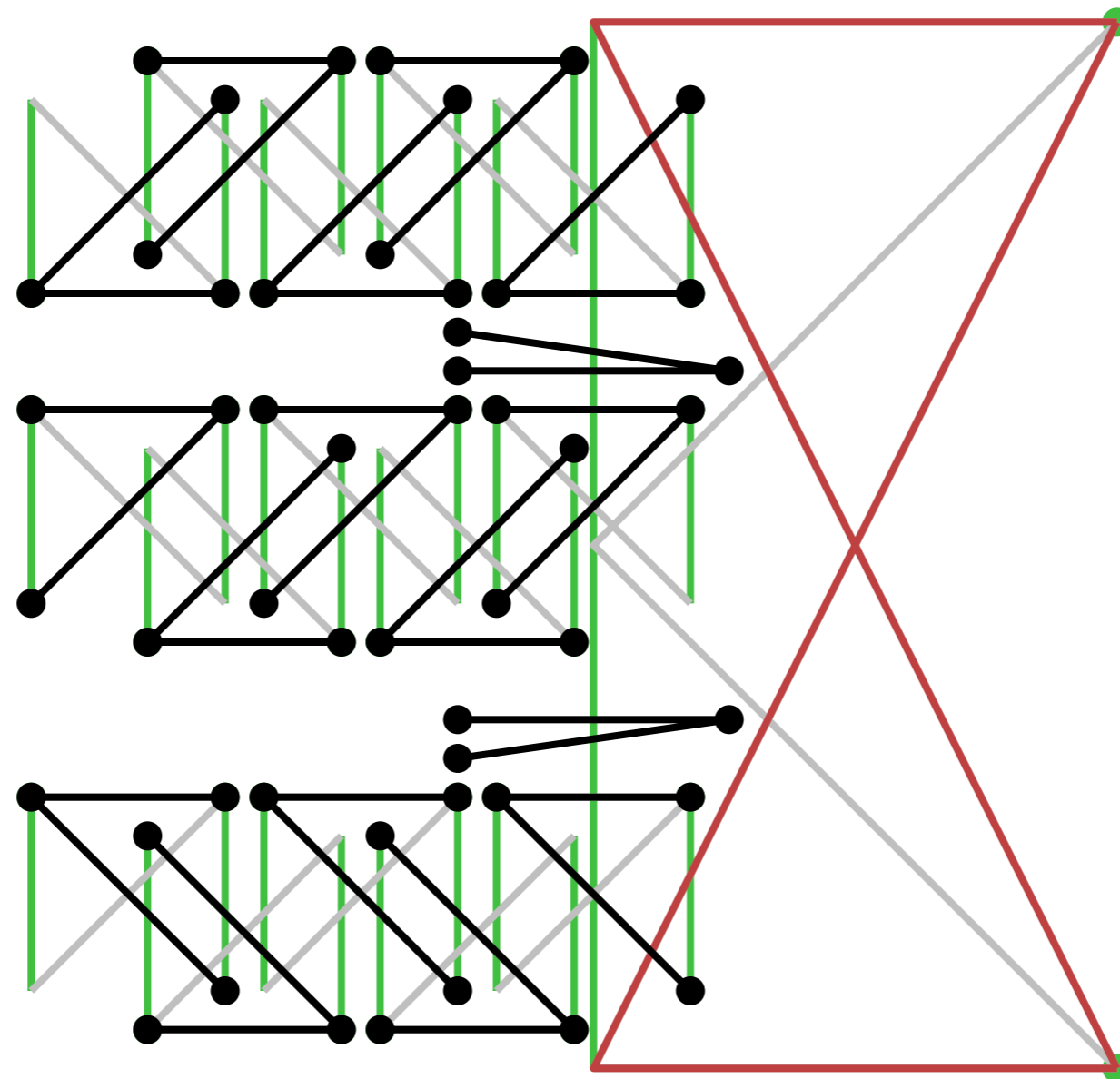
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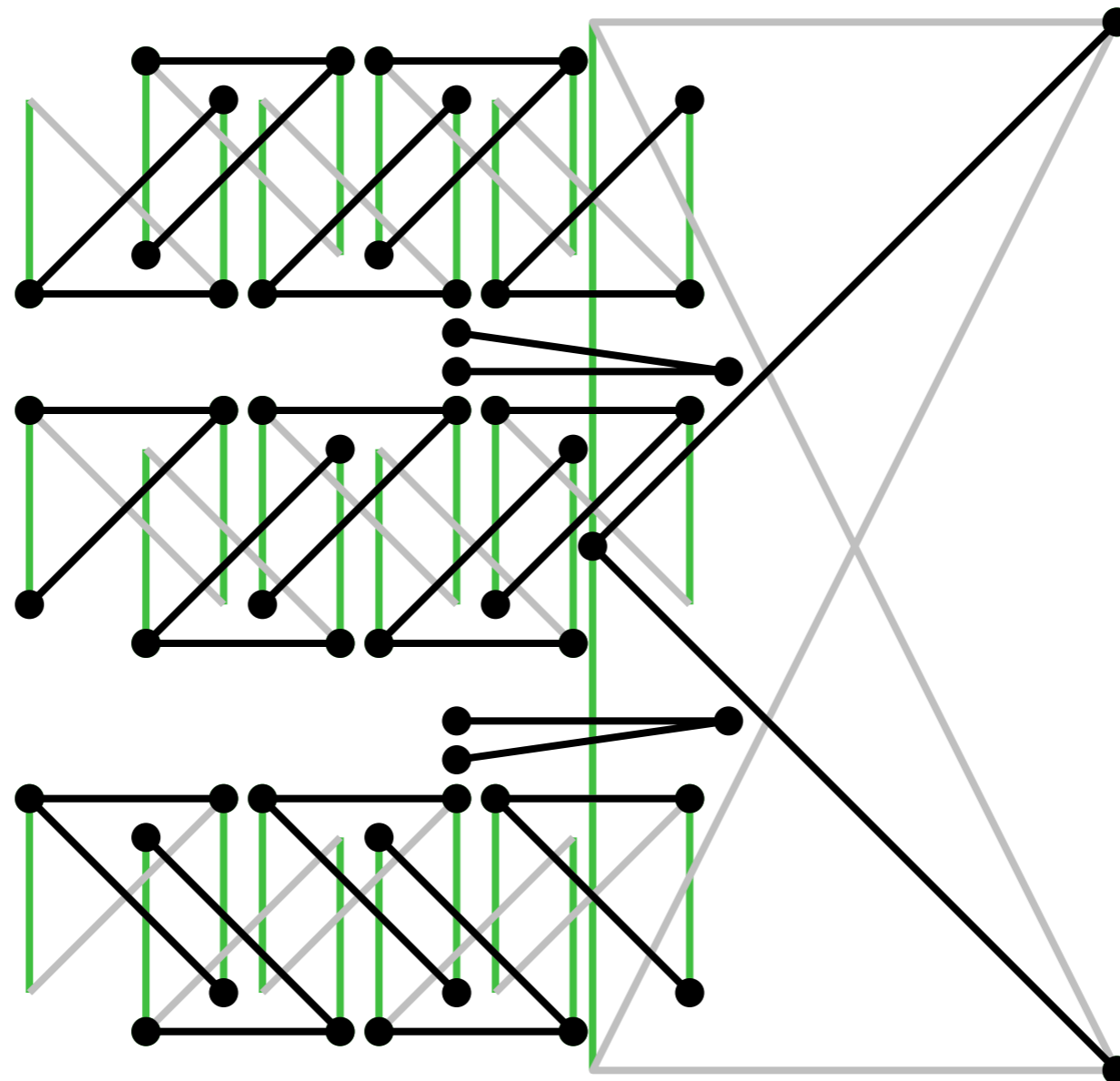
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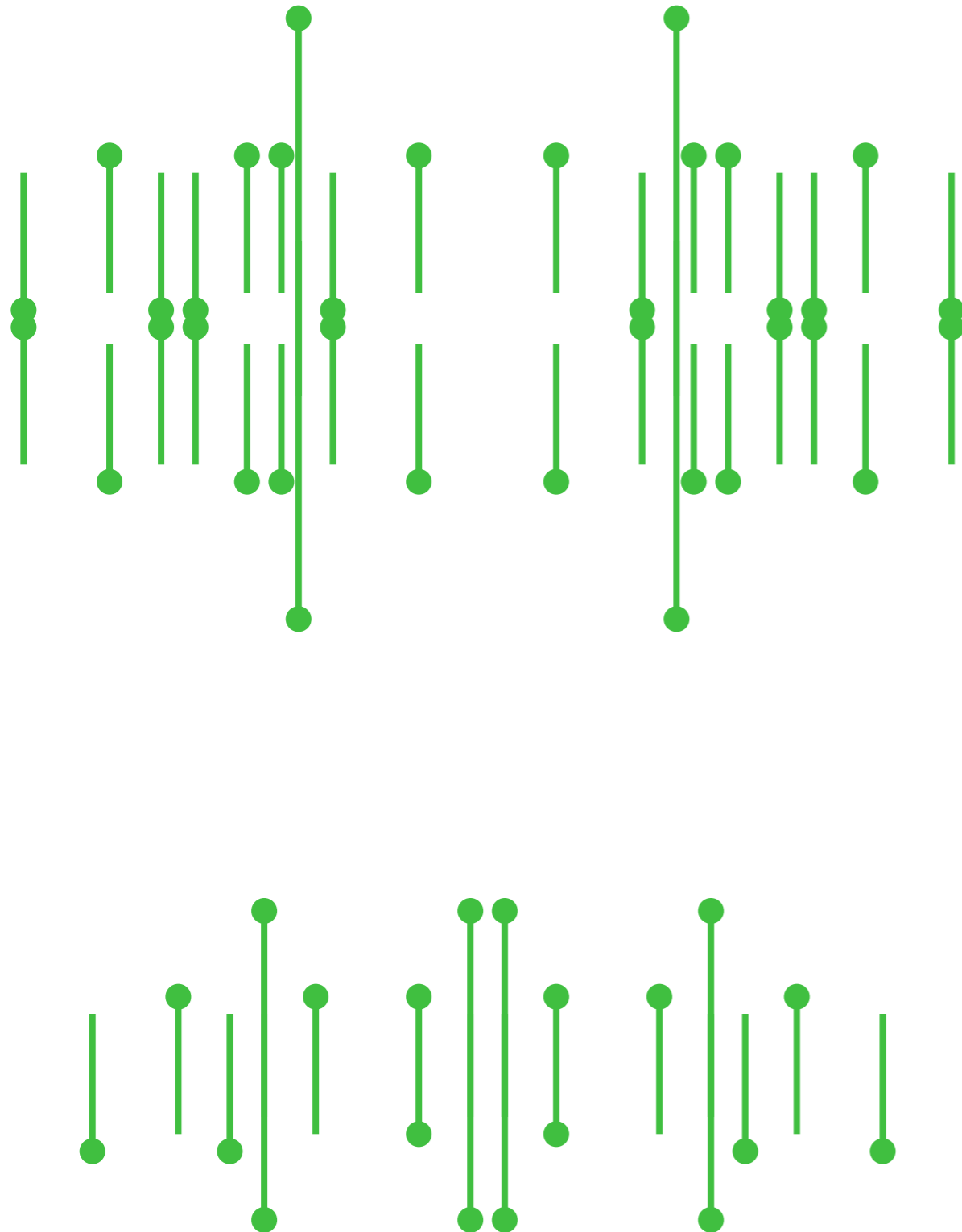
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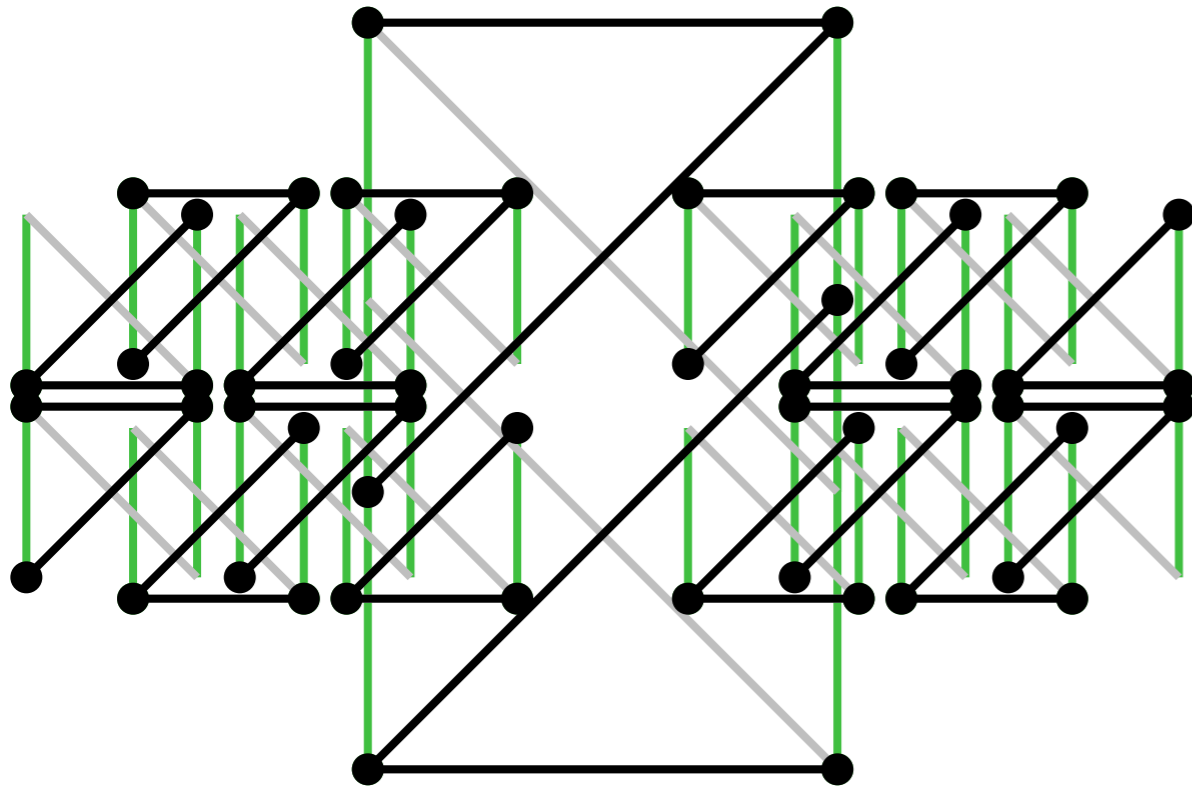


Further Details

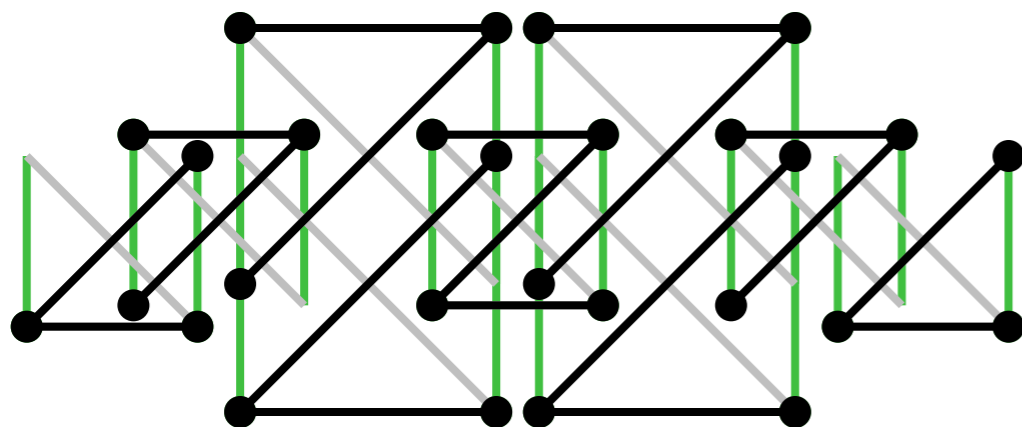


- Splitting variables
 - Variables will occur in many clauses
 - Chains need to move vertically
- Connecting tour parts
 - Many small pieces of tour need to become one big tour
 - Bridges to cross chains

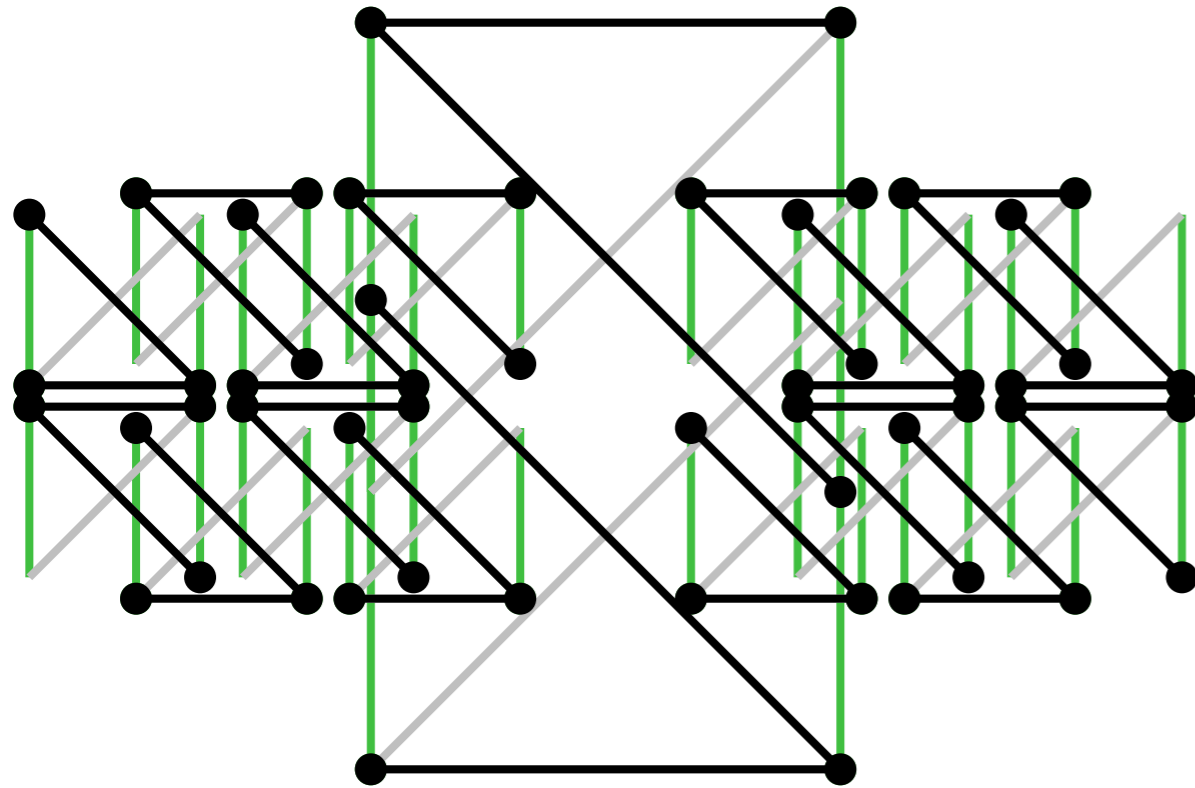
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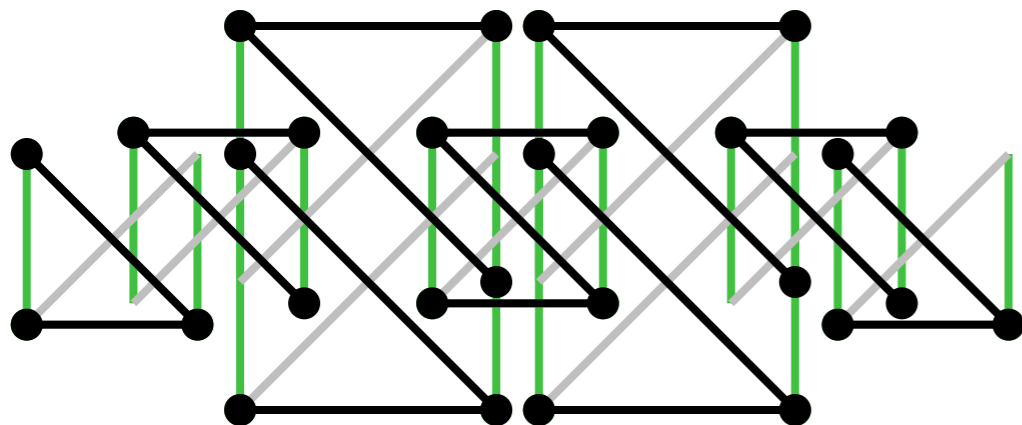
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Conclusions

- Finding a simple polygon with imprecise vertices as vertical line segments is NP-hard
- Other results
 - Imprecision model: any other connected region
 - Finding the shortest simple tour through n regions

Questions?