

Feed Links  
for  
Network Extensions

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Silveira

Maike  
Buchin

Maarten  
Löffler

Boris  
Aronov

Tom  
de Jong

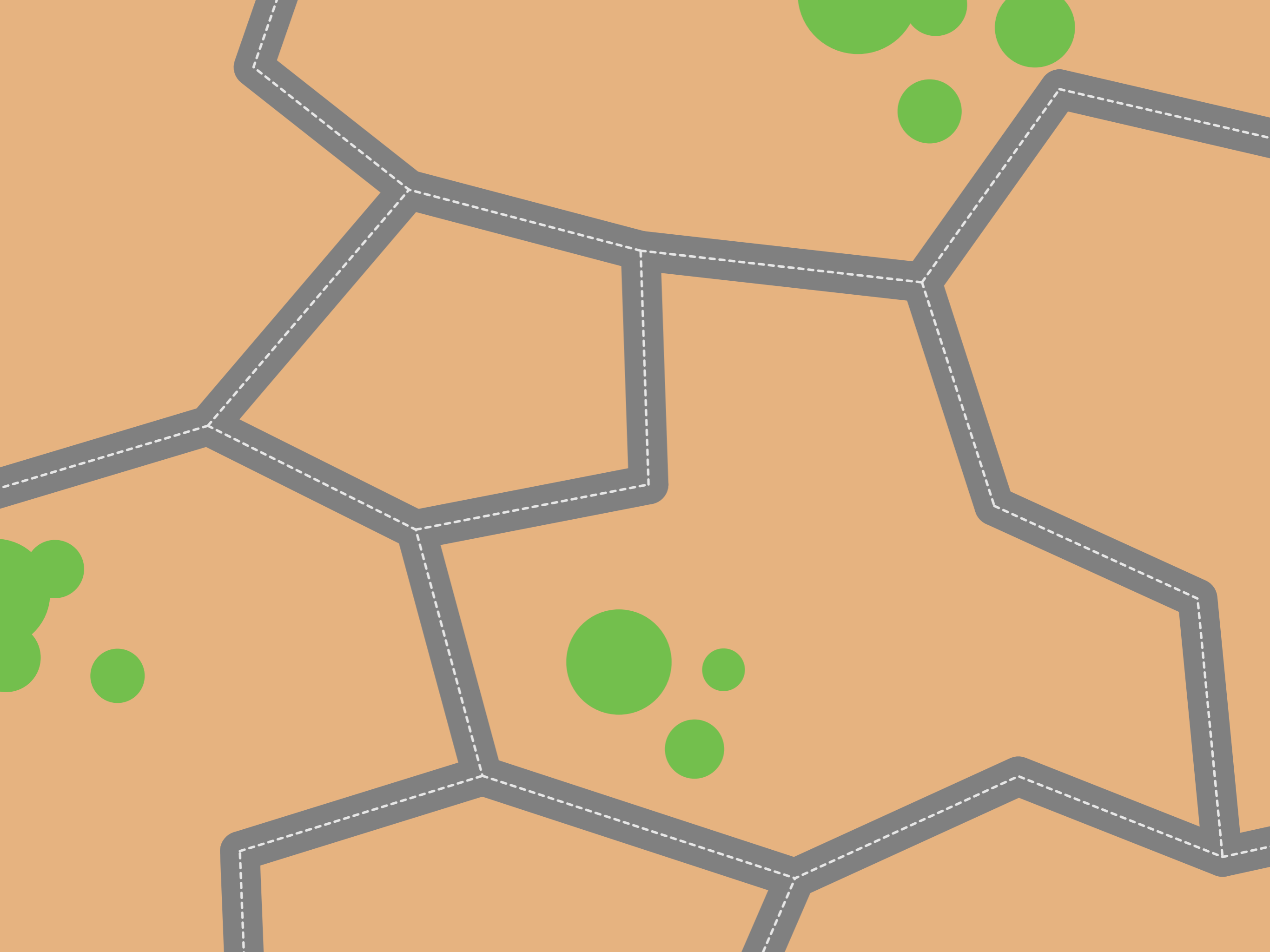
Jun  
Luo

Bettina  
Speckmann

Bart  
Jansen

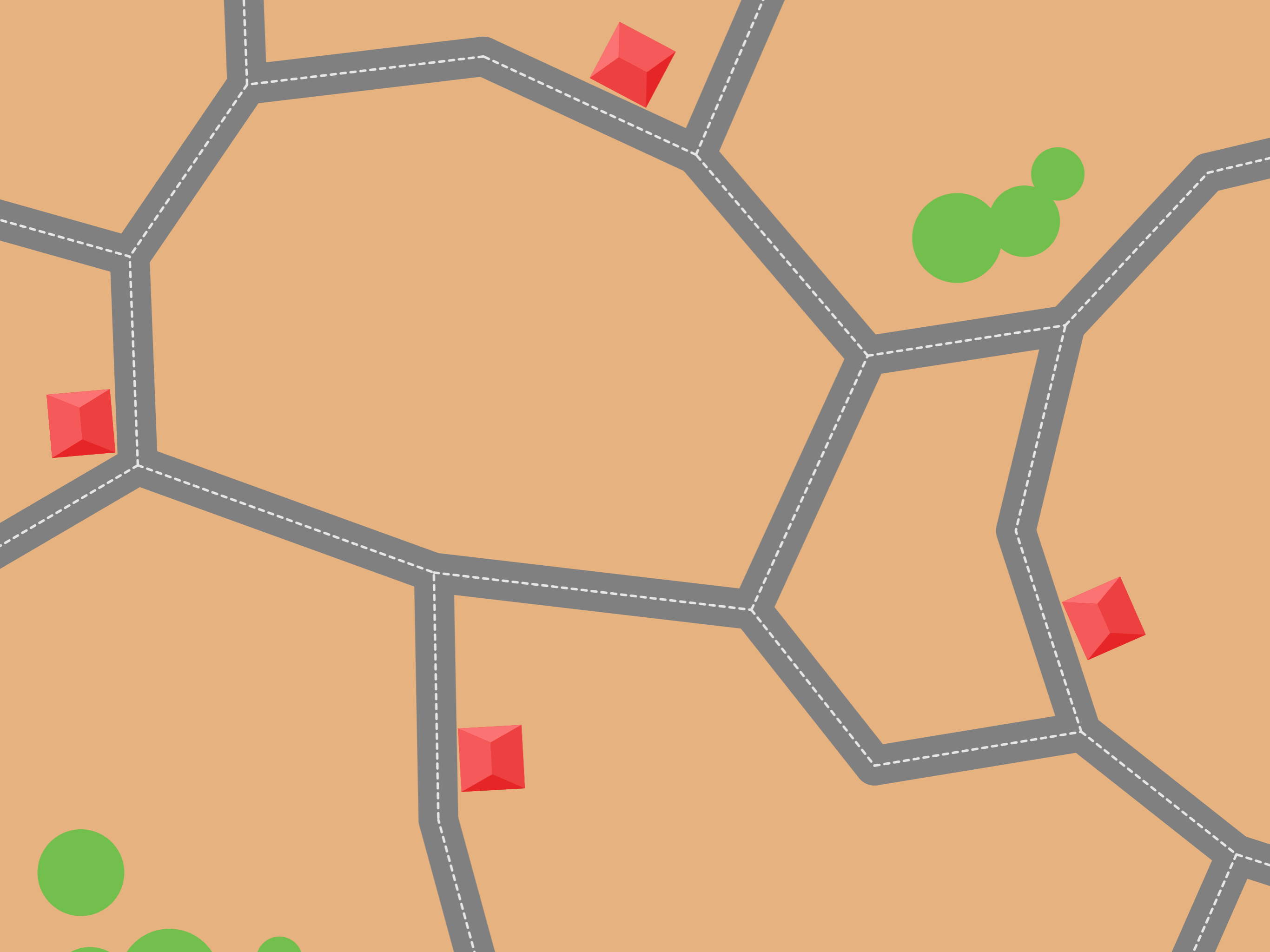
Kevin  
Buchin

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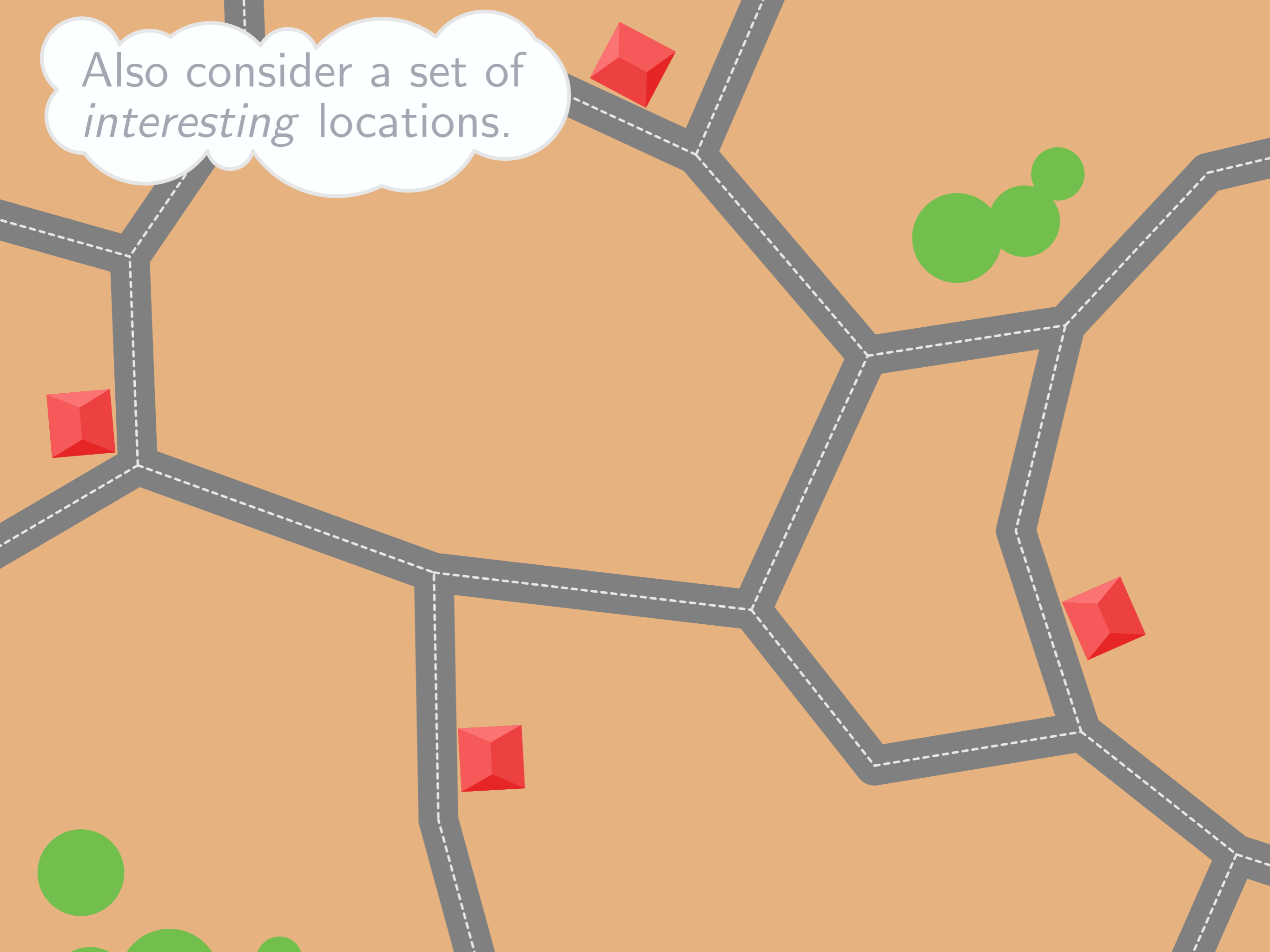


The image features a network of roads on a brown background. The roads are represented by solid grey lines with a dashed white line running parallel to them. The network consists of several interconnected paths. Scattered throughout the scene are several green circles of varying sizes, representing trees. A central white cloud-like bubble contains the text "Consider a network of roads." in a grey, sans-serif font.

Consider a network of roads.



Also consider a set of *interesting* locations.



A network diagram on a tan background. A grey path, consisting of solid and dashed lines, winds through the scene. Four red 3D cubes are placed at various points along this path. In the upper right, three green circles of varying sizes are clustered together. In the lower left, several more green circles are partially visible. Two white, cloud-like callout boxes with grey outlines contain text.

Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.

A network diagram on a brown background. It features a complex network of grey lines representing paths, with a specific path highlighted in yellow. There are four red 3D cube-like shapes representing interesting locations: one at the top, one on the left, one at the bottom, and one on the right. There are also several green circles of varying sizes scattered across the network. Two white, cloud-like text boxes are present. The first box at the top left contains the text "Also consider a set of interesting locations." The second box in the center contains the text "In many applications, the network distance between locations is important."

Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.

A network diagram on a brown background. The network consists of dark grey lines with dashed white outlines, forming a complex path. A specific path is highlighted with a thick yellow border. There are four red 3D cube markers: one at the top, one on the left, one at the bottom, and one on the right. There are also several green circles of varying sizes scattered throughout the network. Two white, cloud-like text boxes are present, one at the top left and one in the center.

Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.



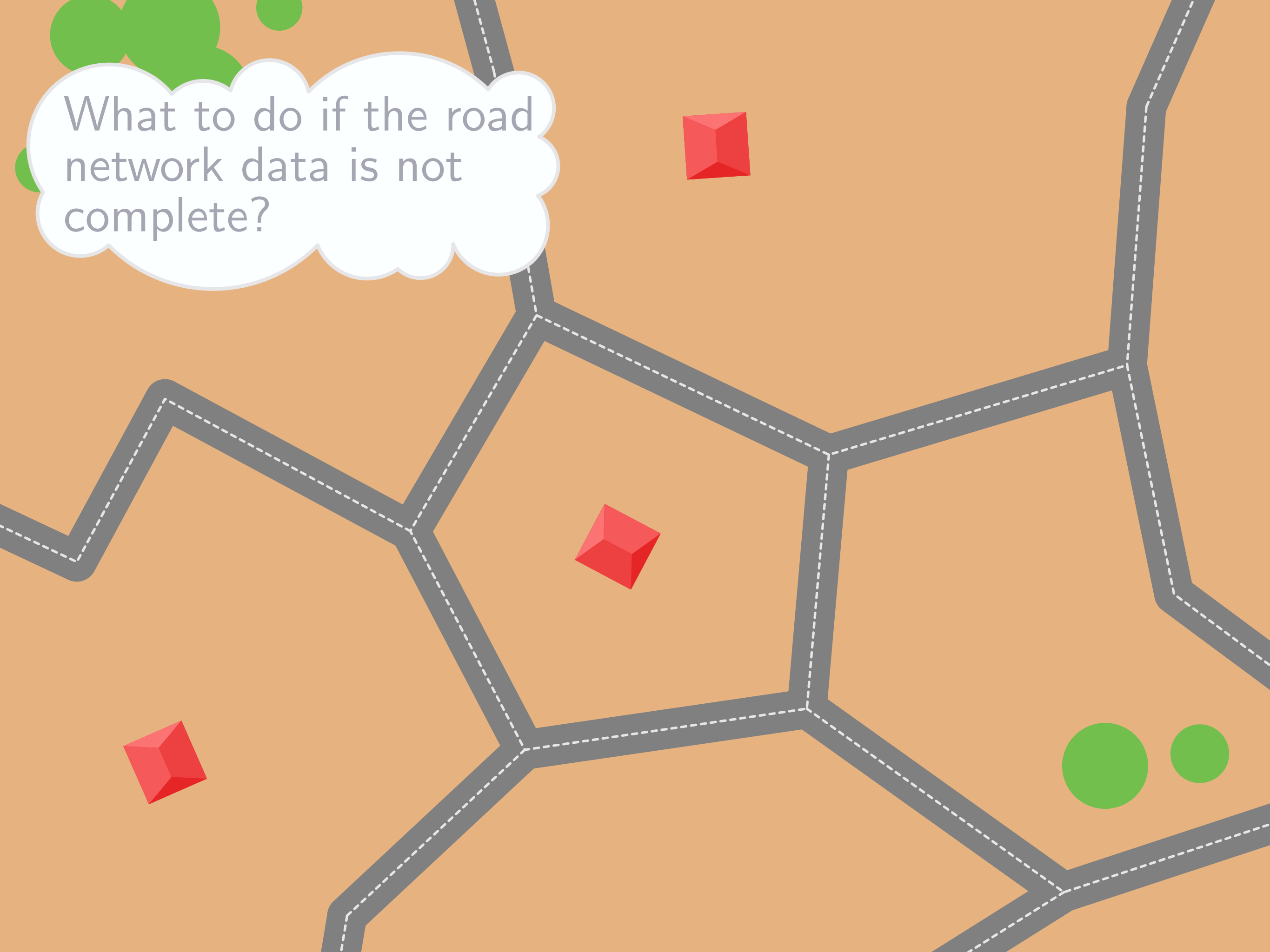
A network diagram on a brown background. The network consists of dark grey lines representing edges, with dashed white lines inside them. A specific path is highlighted in yellow. There are four red 3D cube markers: one at the top, one on the left, one at the bottom, and one on the right. There are also several green circles of varying sizes scattered throughout the network.

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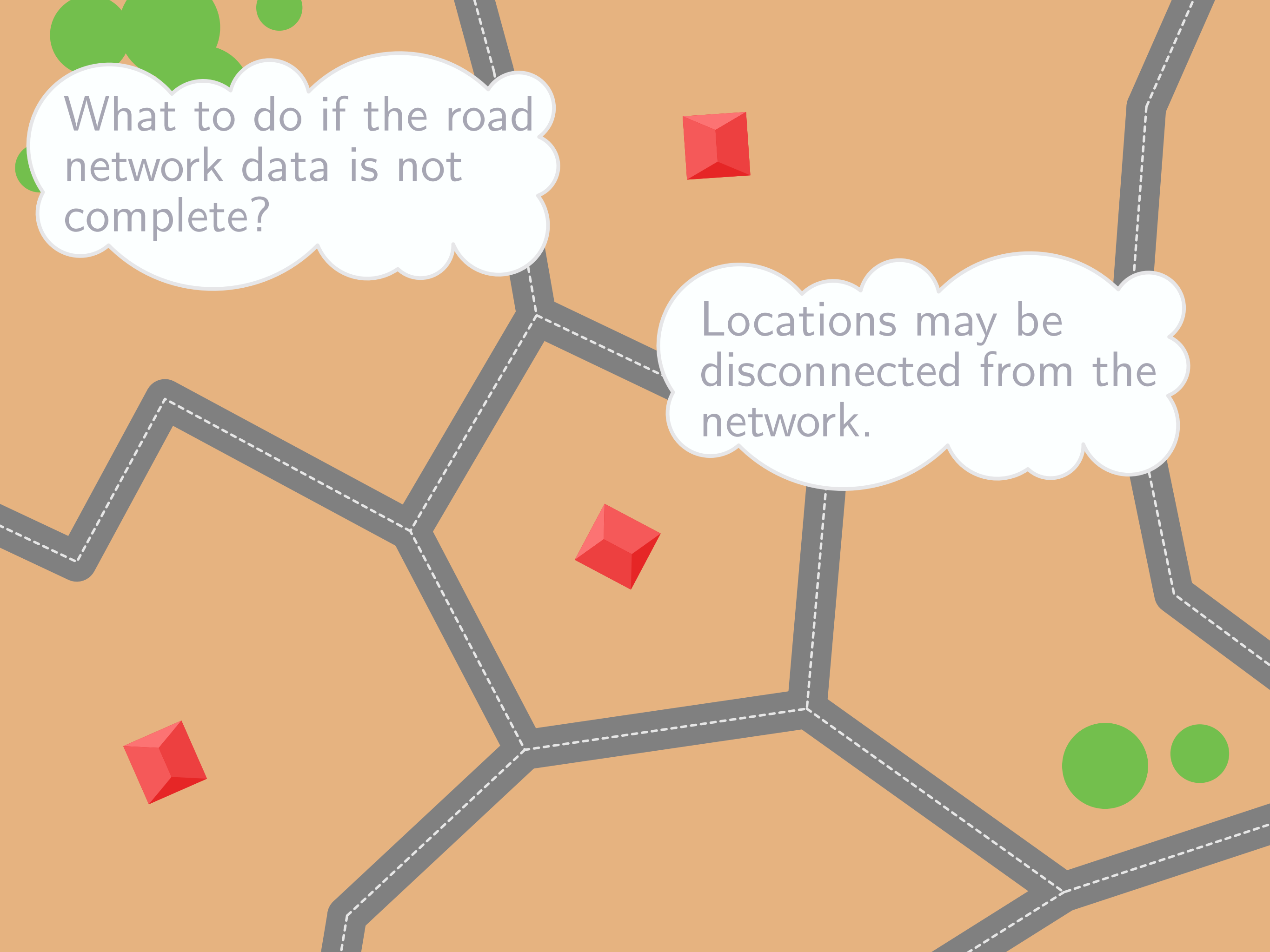
These are well known and easy to compute.





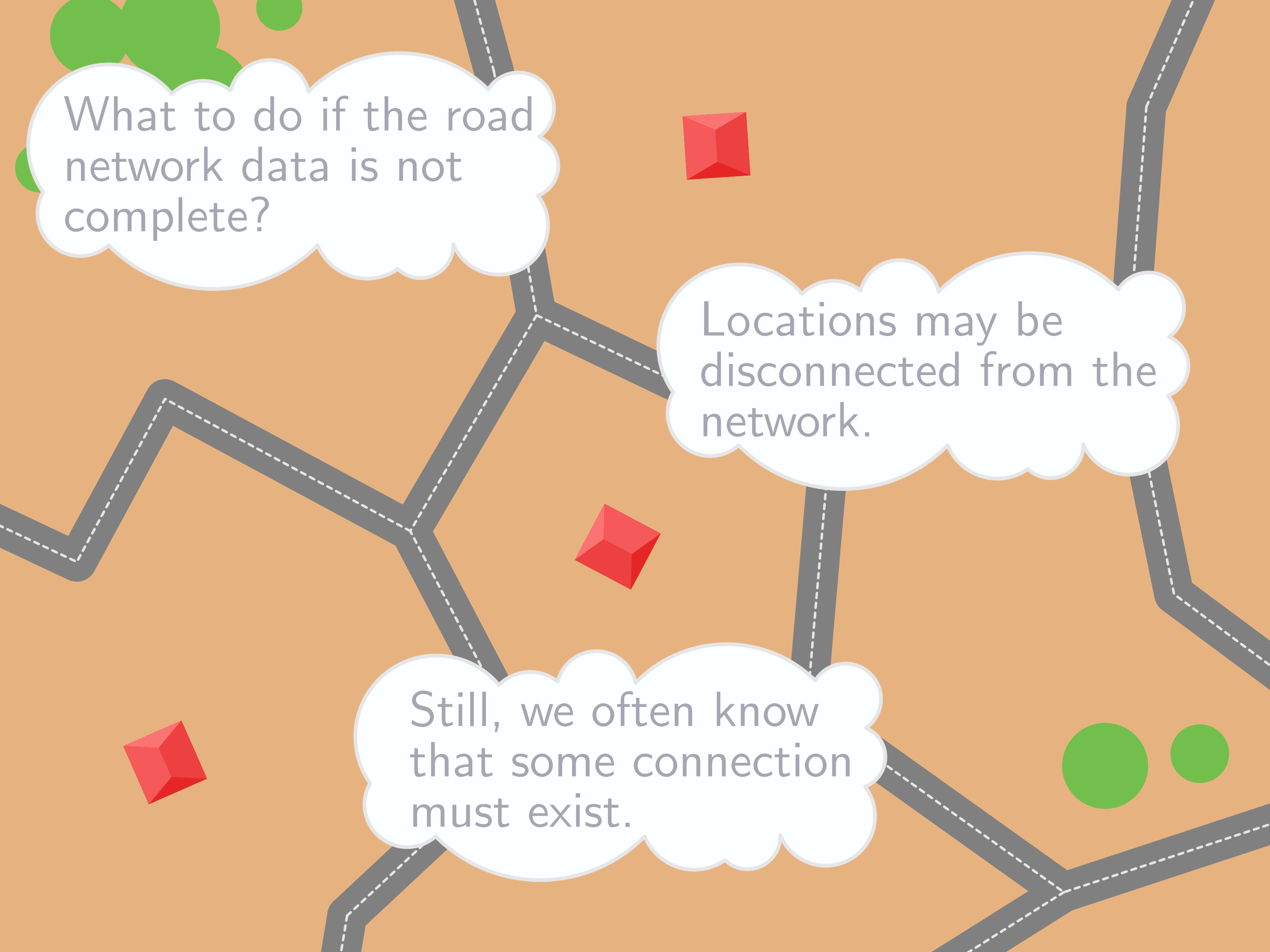
What to do if the road network data is not complete?

The image shows a network of grey roads on a tan background. The roads are represented by solid grey lines with a dashed white line running parallel to them. There are three red 3D cubes scattered across the map: one in the upper right, one in the center, and one in the lower left. In the bottom right corner, there are two green circles of different sizes. In the top left corner, there are several green circles of various sizes, some overlapping a white cloud-like shape.



What to do if the road network data is not complete?

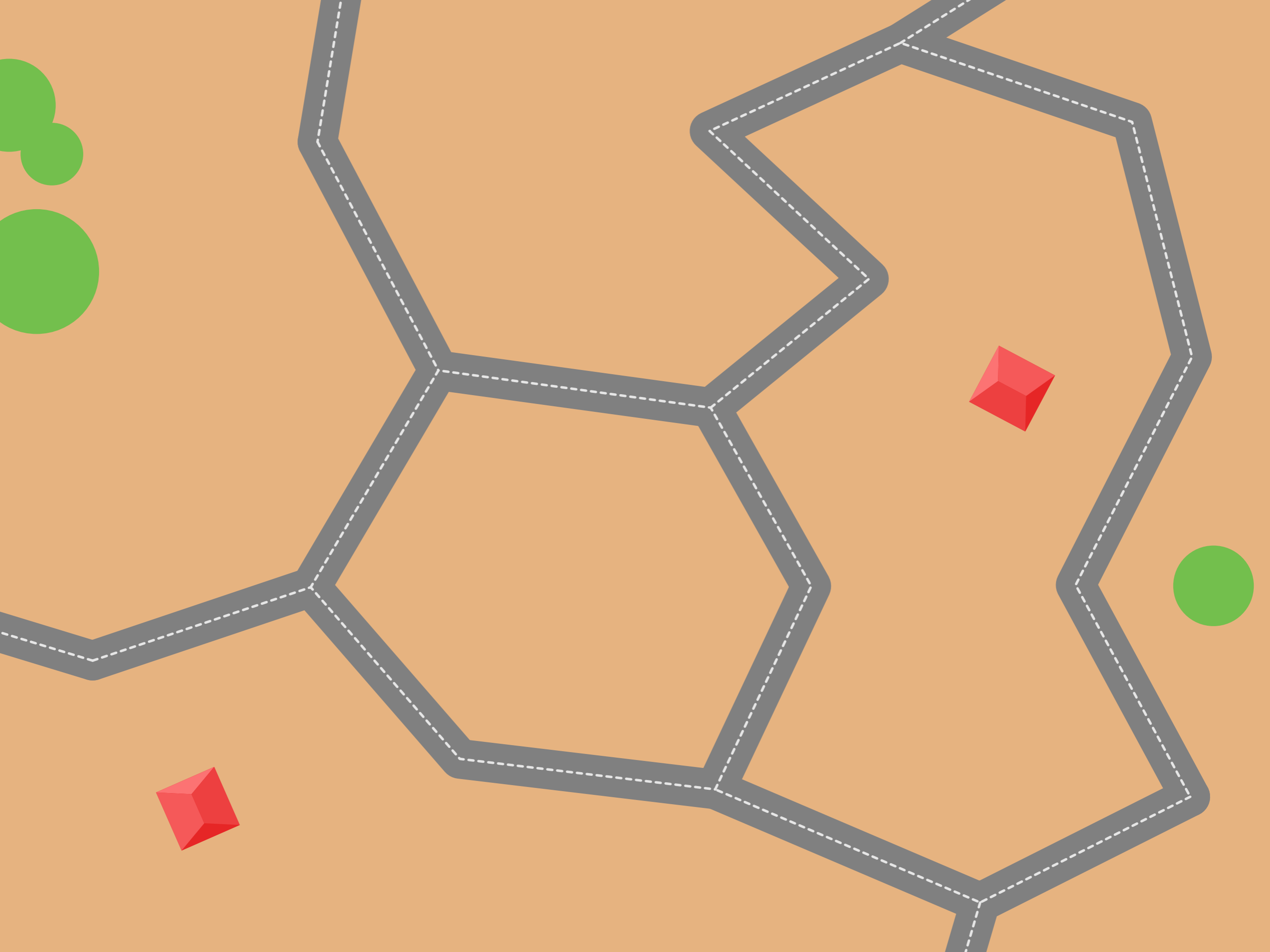
Locations may be disconnected from the network.



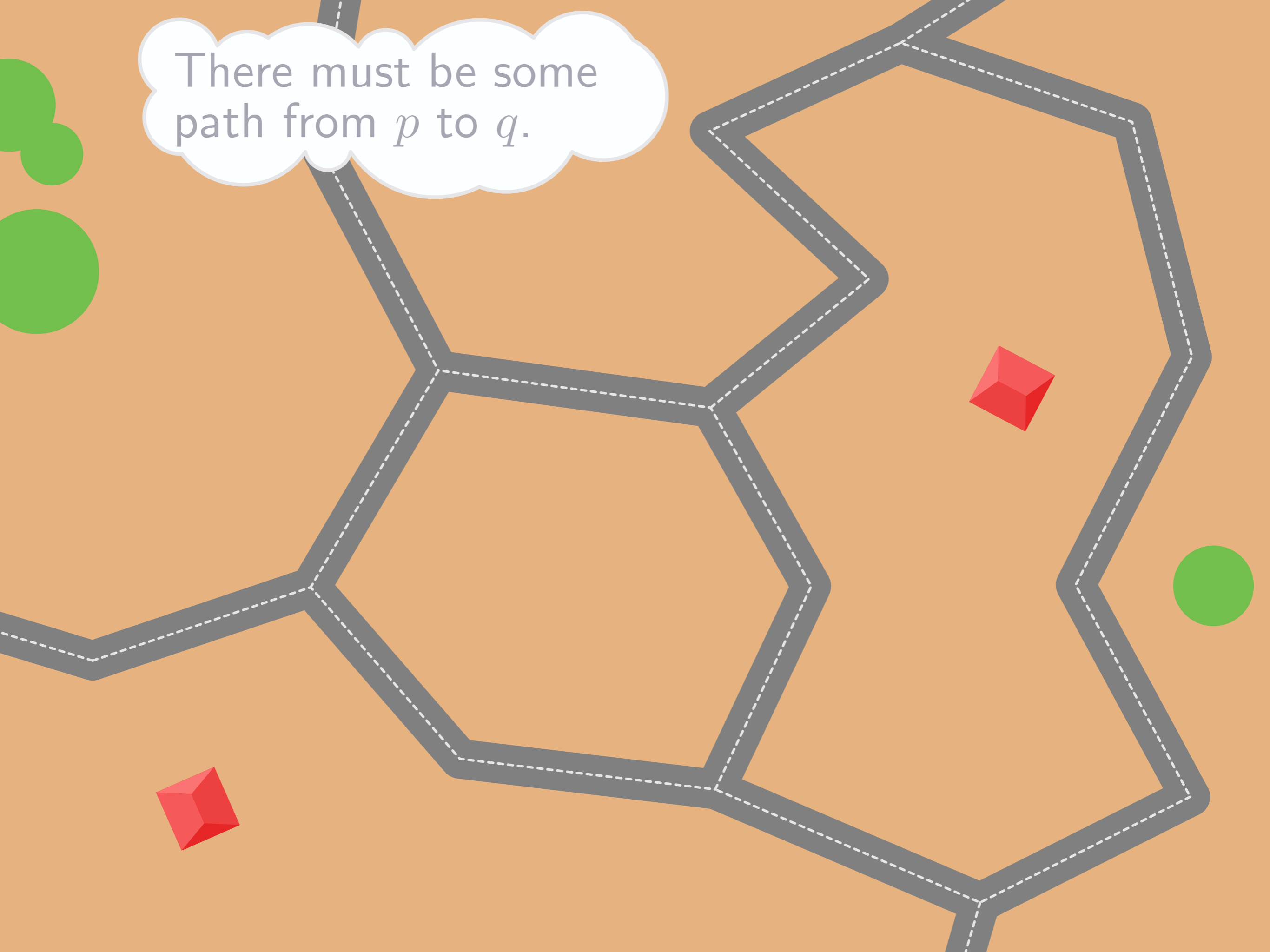
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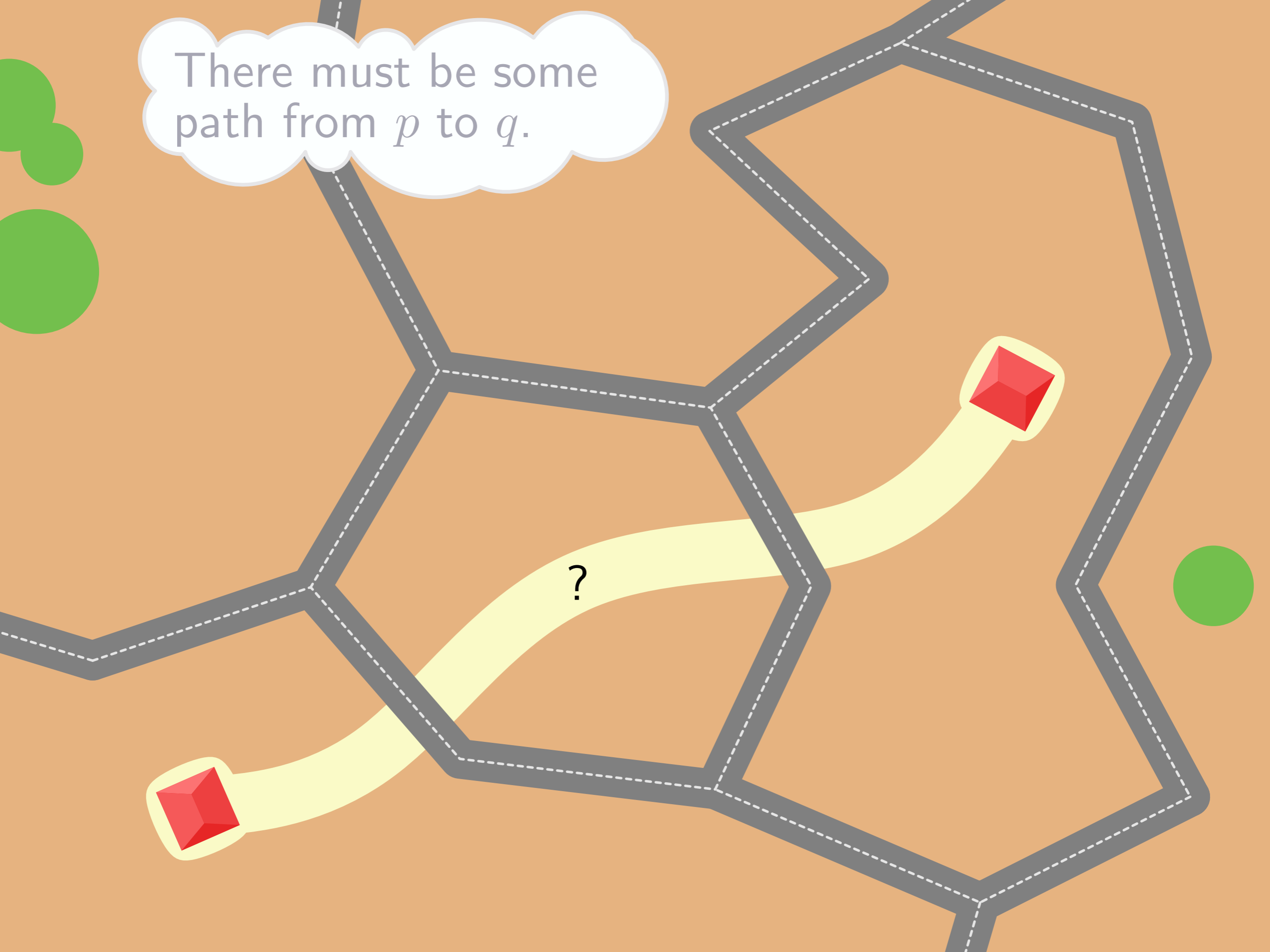
Still, we often know that some connection must exist.



There must be some path from  $p$  to  $q$ .

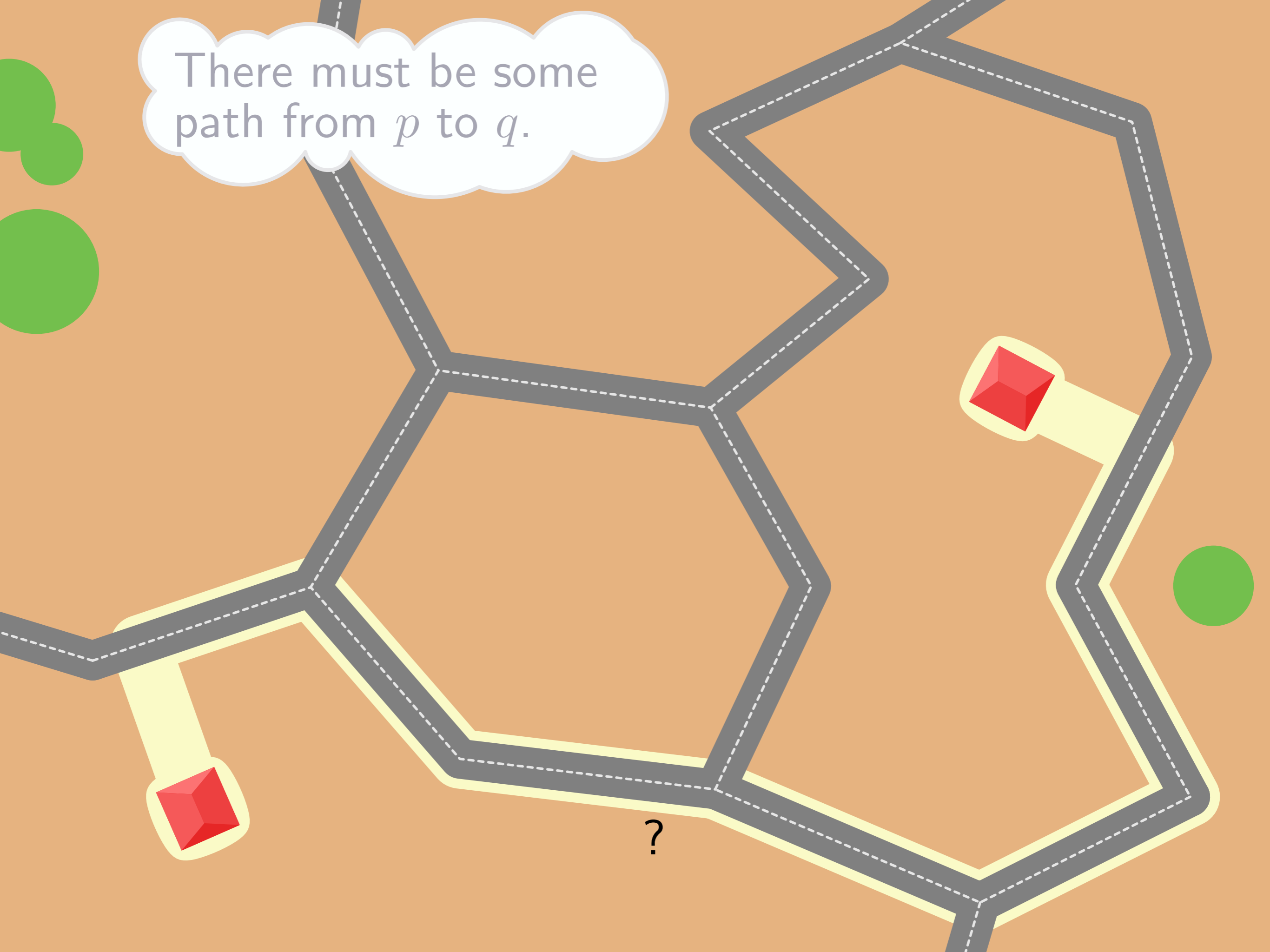


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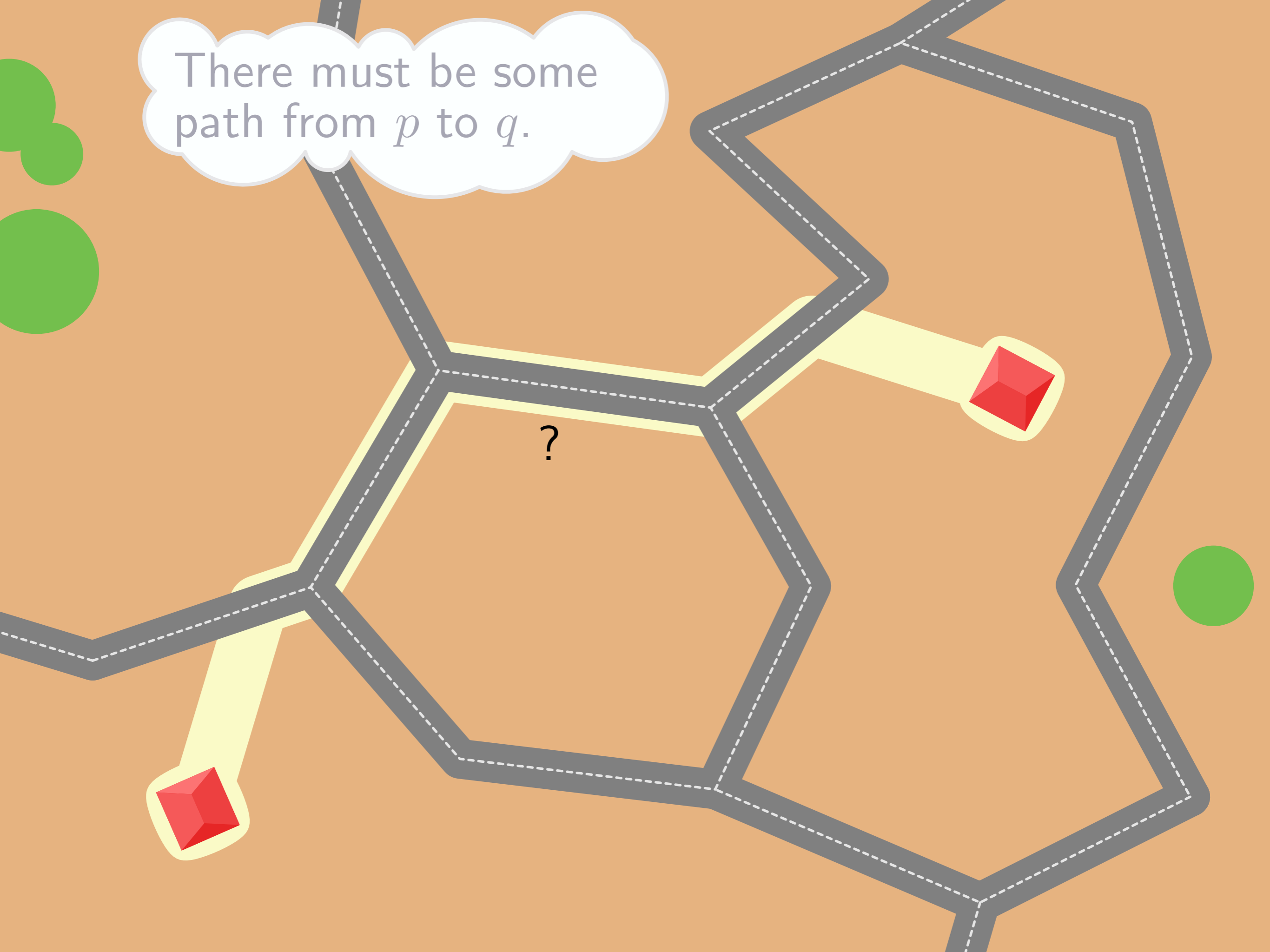


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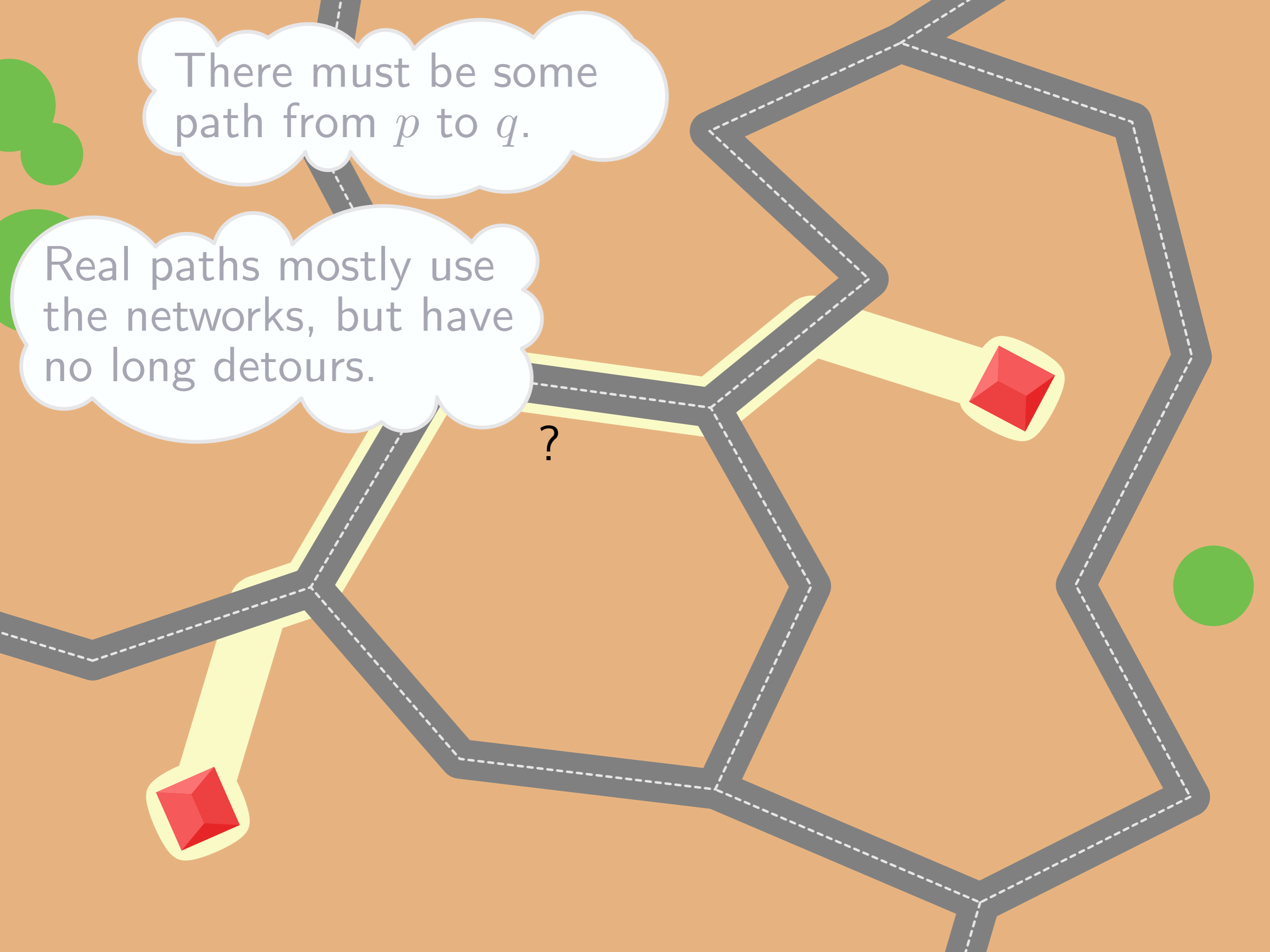
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Real paths mostly use the networks, but have no long detours.

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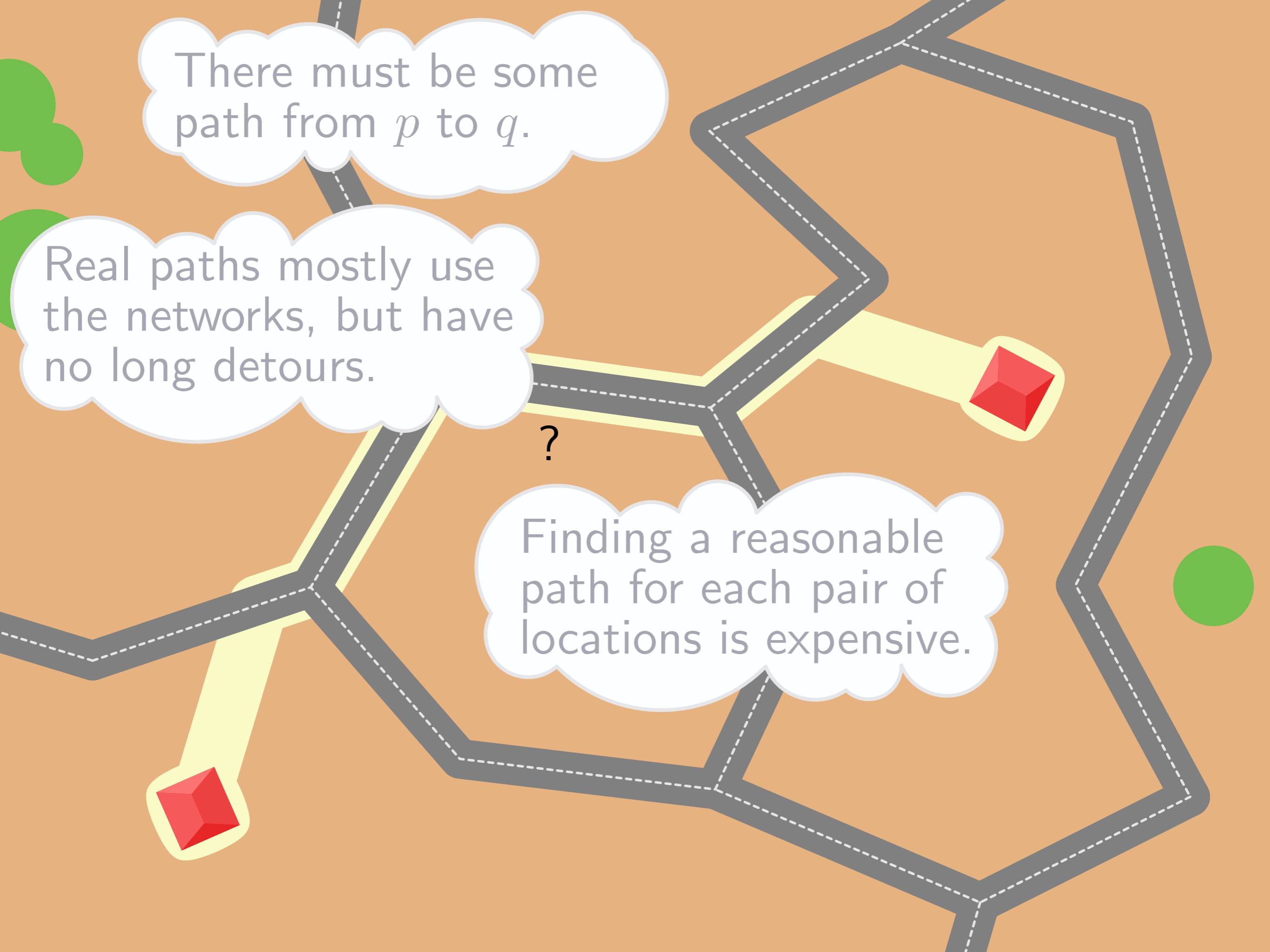


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Real paths mostly use the networks, but have no long detours.

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Finding a reasonable path for each pair of locations is expensive.



A network diagram on a brown background. A grey path with a dashed white line inside starts from the top left and moves towards the bottom right. A yellow path with a red gem at its end branches off from the grey path. A question mark is placed near the junction. Three white callout boxes with grey text are scattered around the network. There are also some green decorative shapes: a cloud-like shape on the top left, a circle on the right, and some foliage on the far left.

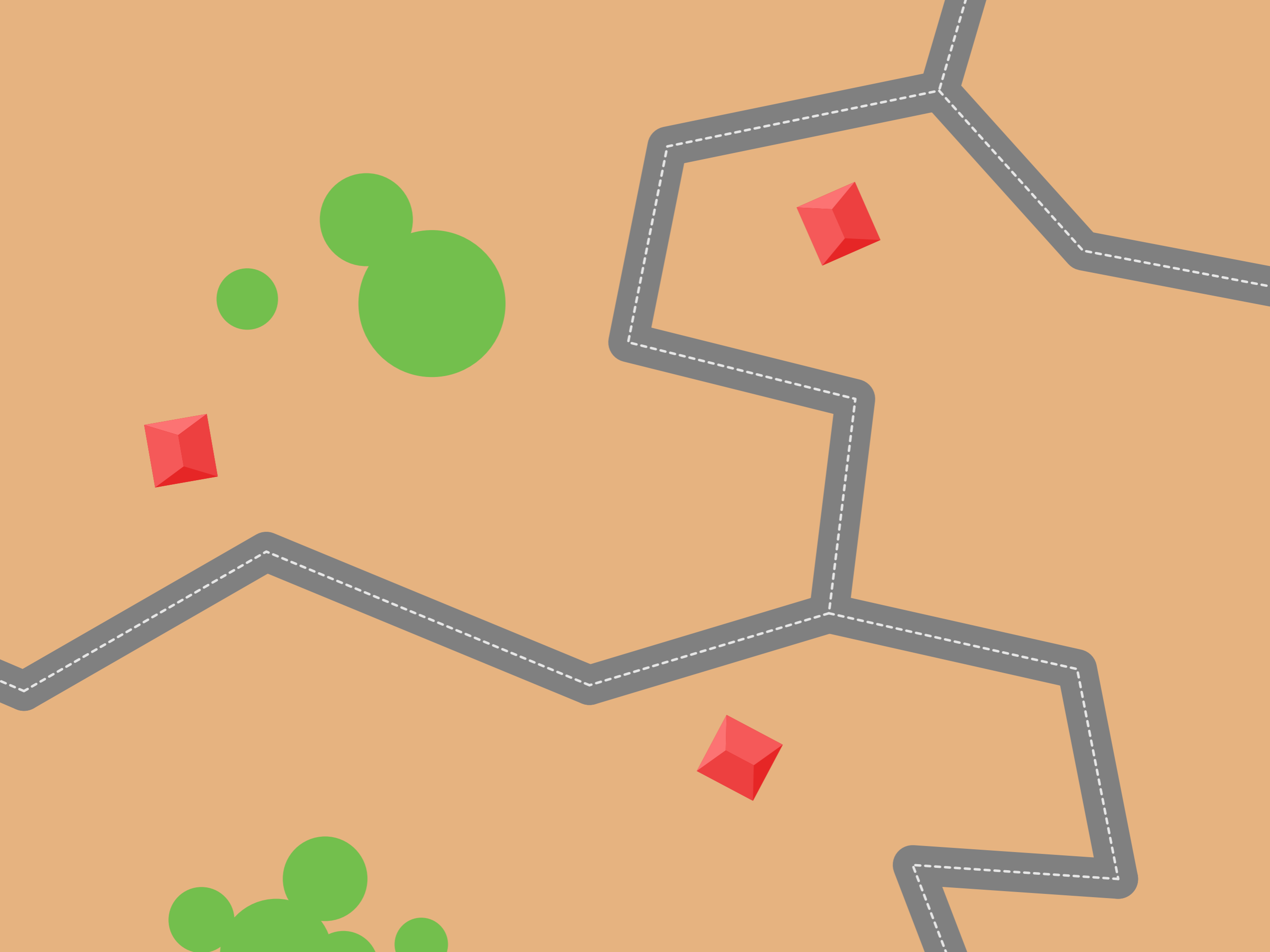
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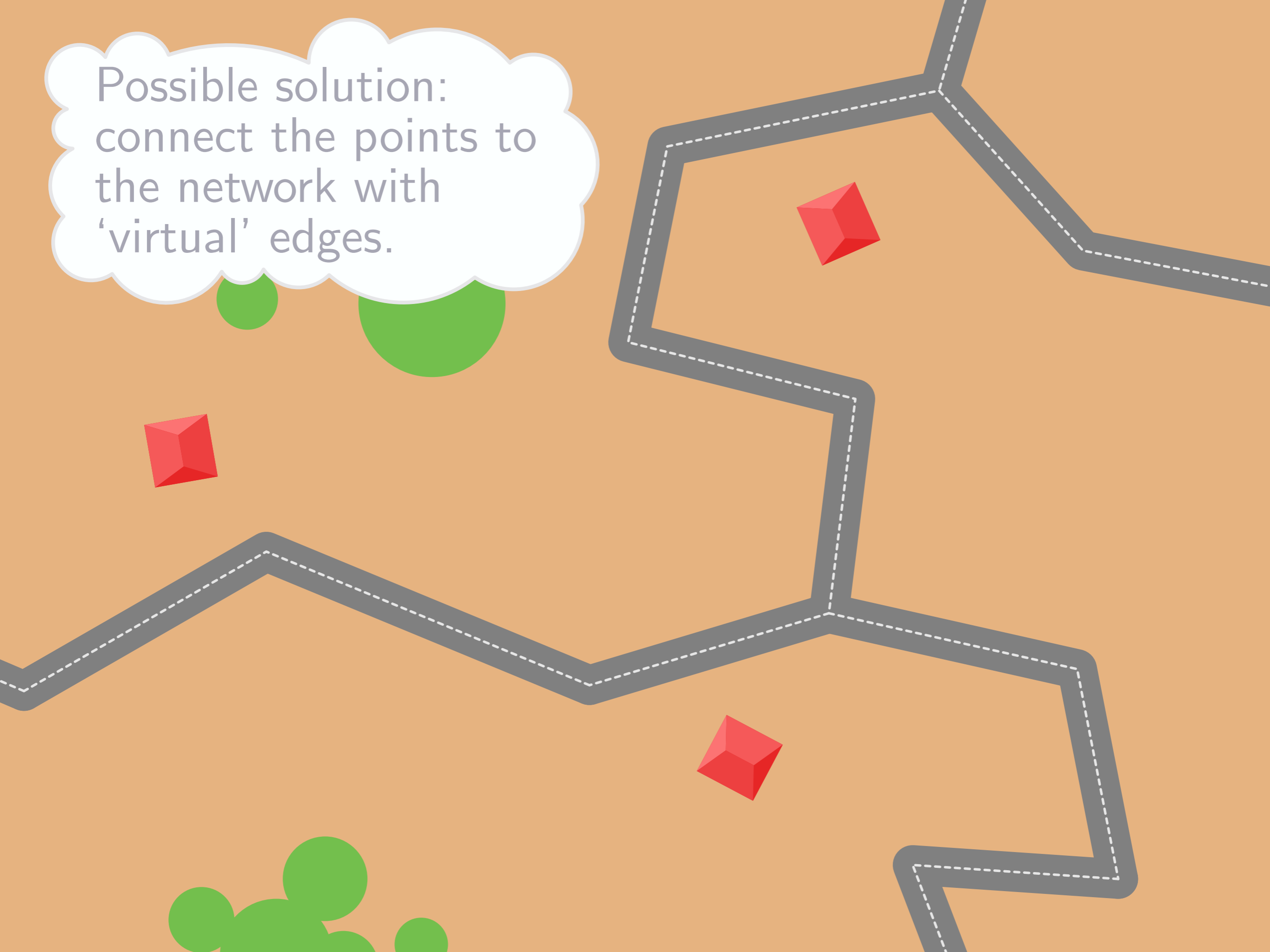
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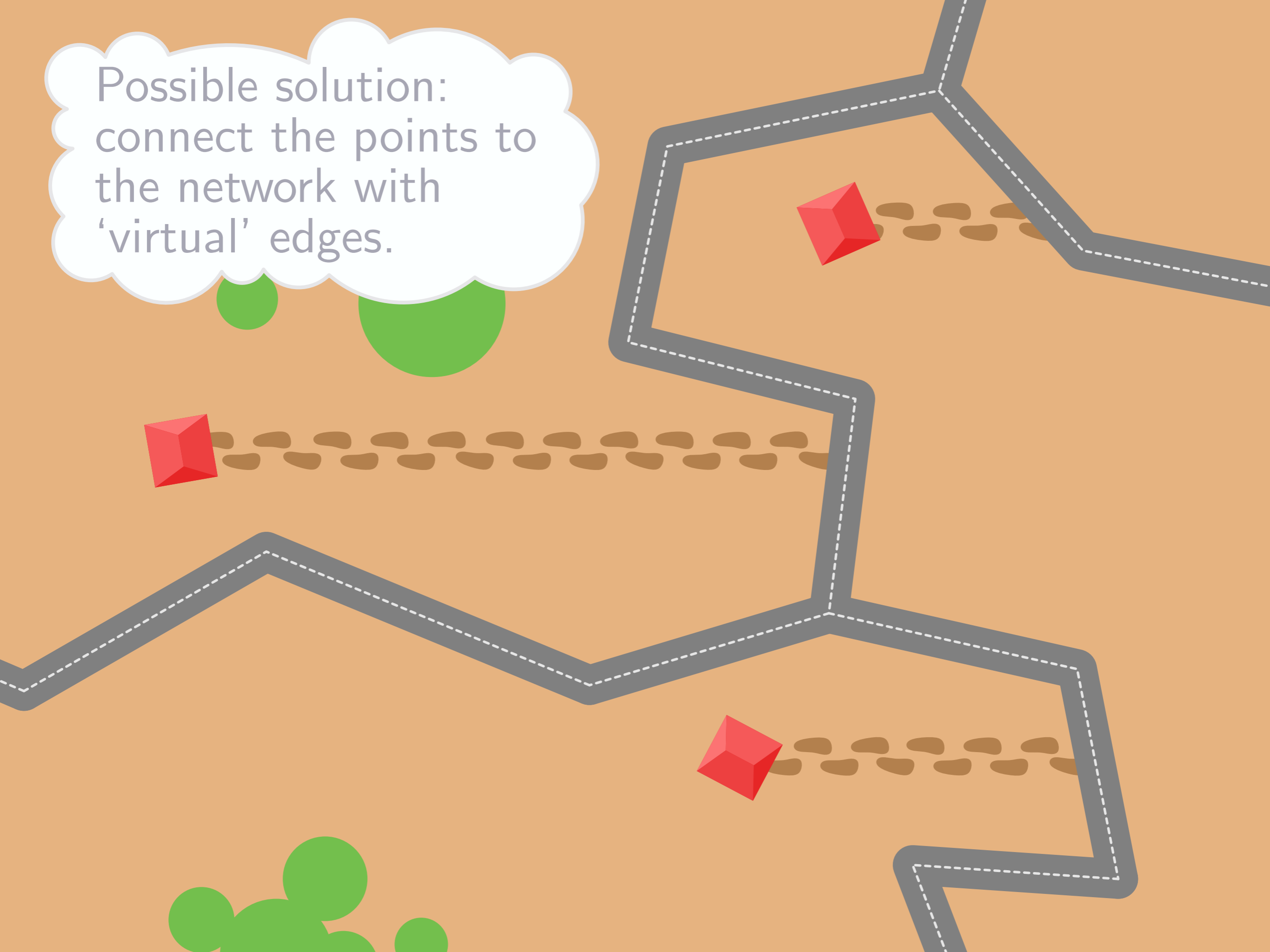
What can we do?



Possible solution:  
connect the points to  
the network with  
'virtual' edges.



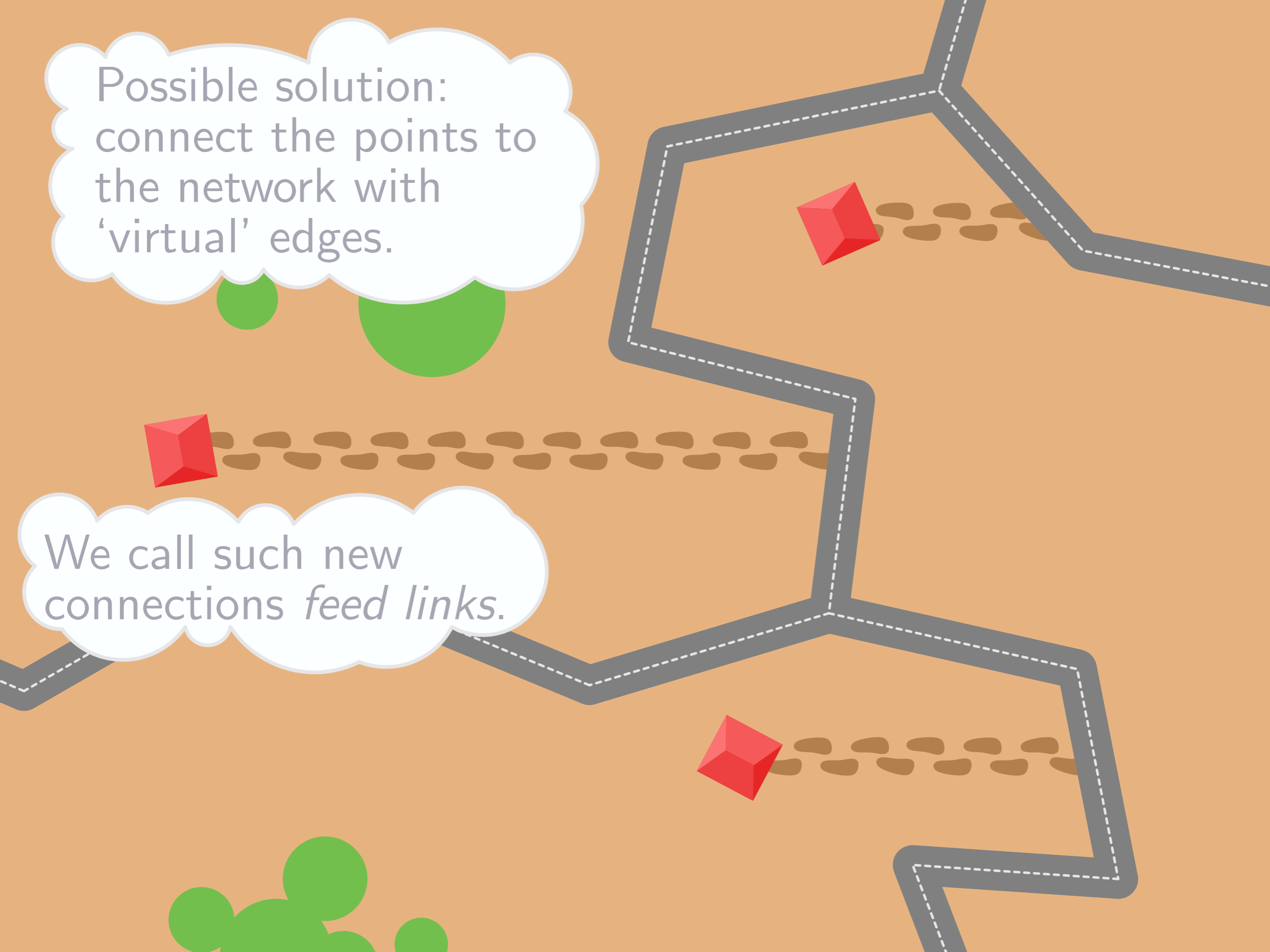
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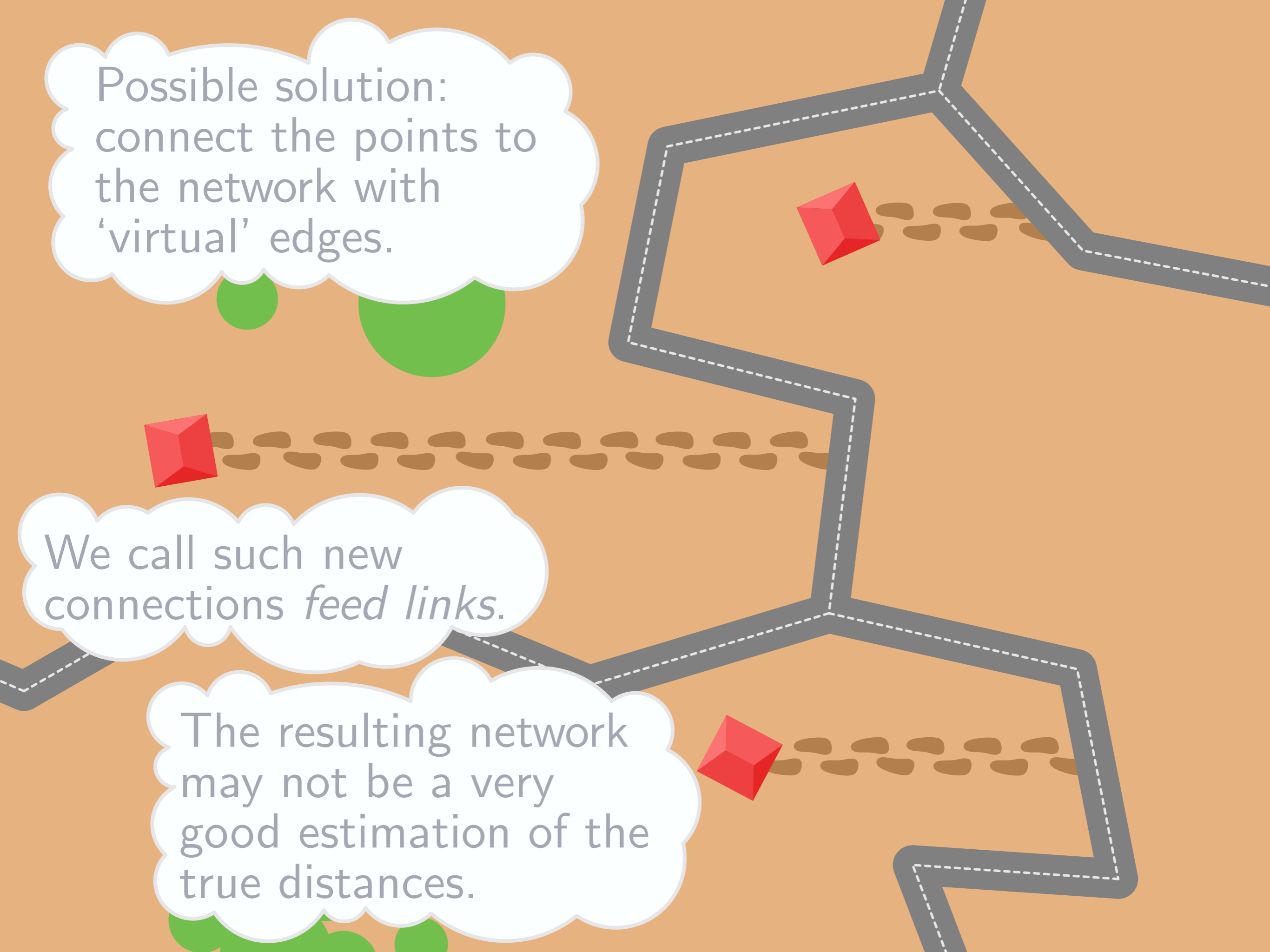
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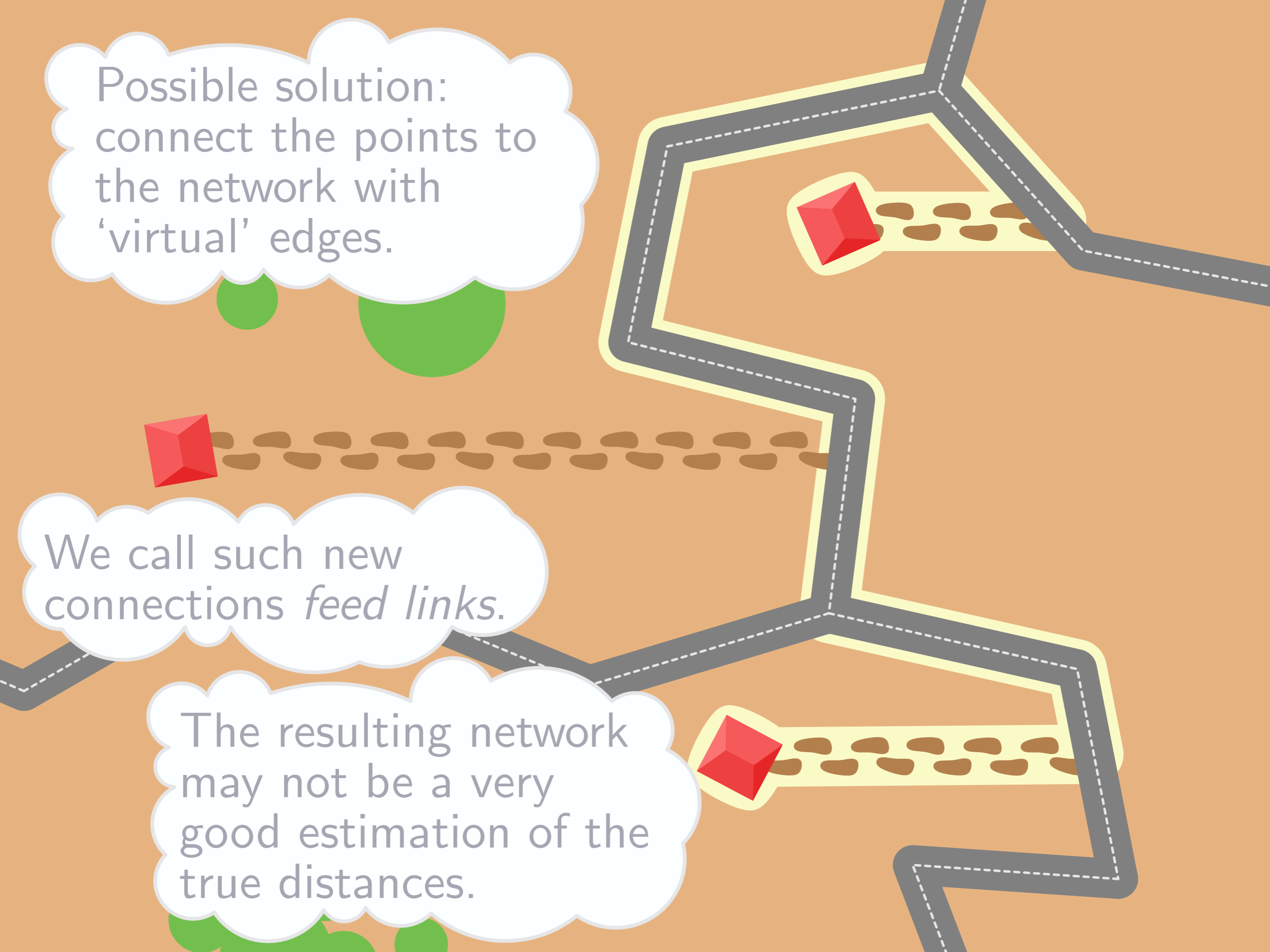
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We may need good connections to many other points.



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The diagram shows a network of yellow paths on a brown background. There are five red square nodes: one at the top left, one at the top right, one in the center, one at the bottom left, and one at the bottom right. Grey dashed lines represent connections between these nodes and other points in the network. Green circles are scattered in the top left and bottom right areas. Two white speech bubbles with grey outlines contain text.

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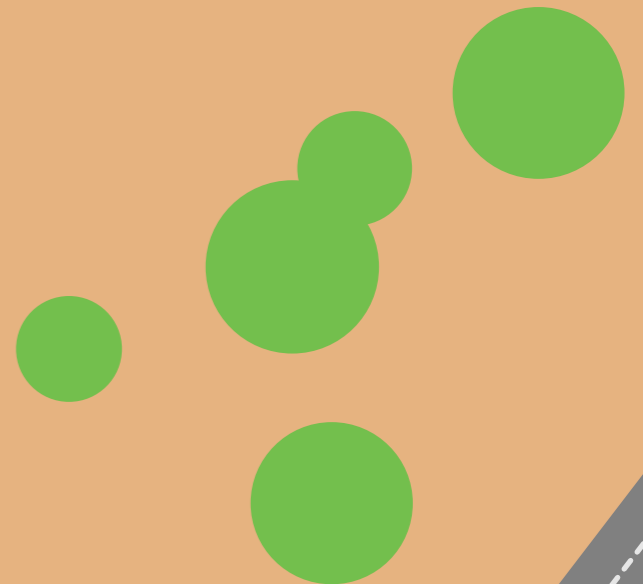
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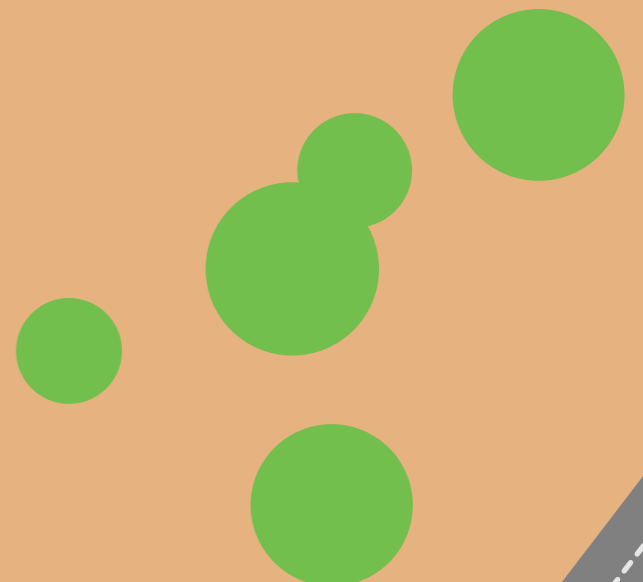


How do we capture the quality of the feed links?



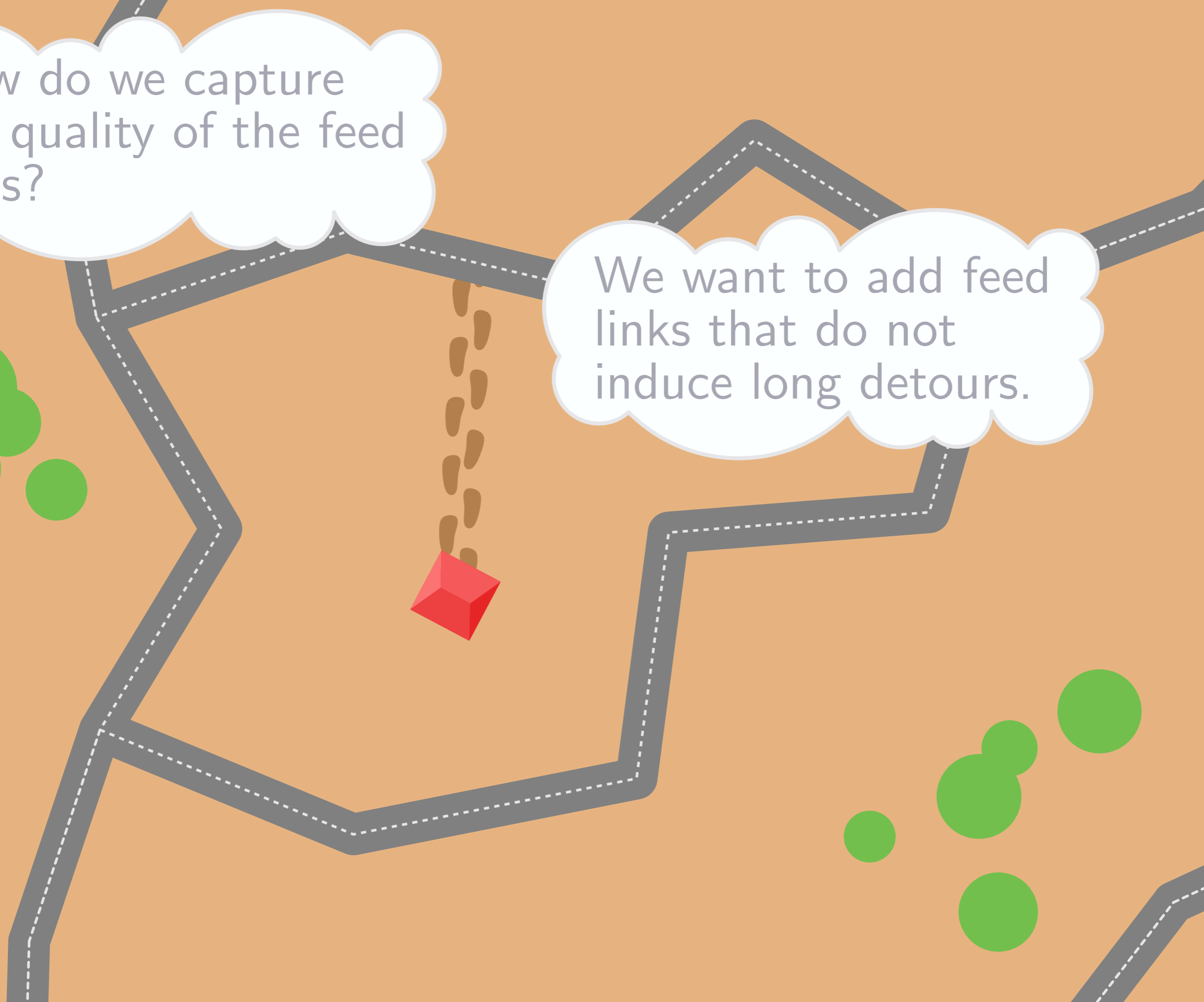
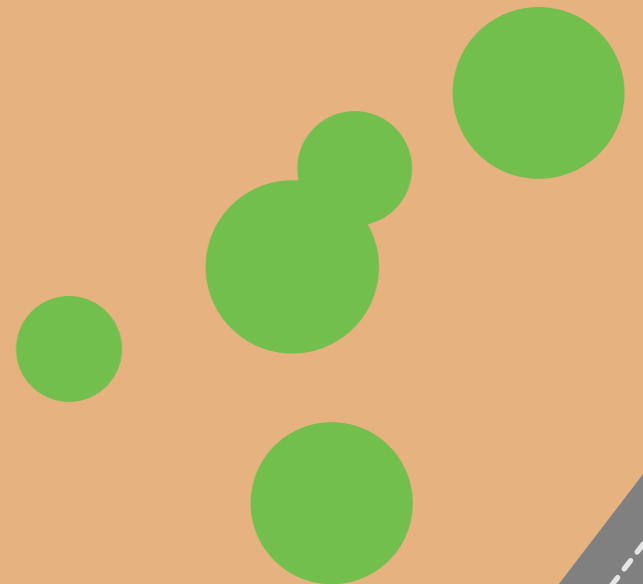
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A network diagram on a brown background. A solid grey path with a dashed white center line starts from the top left and branches out. A red 3D cube is positioned in the center, with a brown path of footprints leading from it upwards towards the network. To the left of the network are three green circles of varying sizes. Three white thought bubbles with grey outlines are connected to the network path.

How do we capture the quality of the feed links?

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The *dilation* between two points  $q$  and  $q$  is:

$$\frac{\text{shortestpath}(p, q)}{\text{distance}(p, q)}$$

A network diagram on a brown background. A solid grey path with a dashed white center line starts from the top left and branches out. A red 3D cube is positioned in the center, with a brown path of footprints leading from it to a junction on the main grey path. To the left of the main path are three green circles of varying sizes. Three white thought bubbles with grey outlines are connected to the network. The first bubble is at the top left, the second is at a junction on the right, and the third is at the bottom right.

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A network diagram on an orange background. A red cube is at the bottom center. A yellow path starts from the cube and goes up, left, and then right, ending at a node on a grey path. The grey path is a network of nodes and edges. A dashed grey line represents the shortest path from the cube to the node. A small white circle with a grey border is on the grey path near the node. There are green bushes on the left side.

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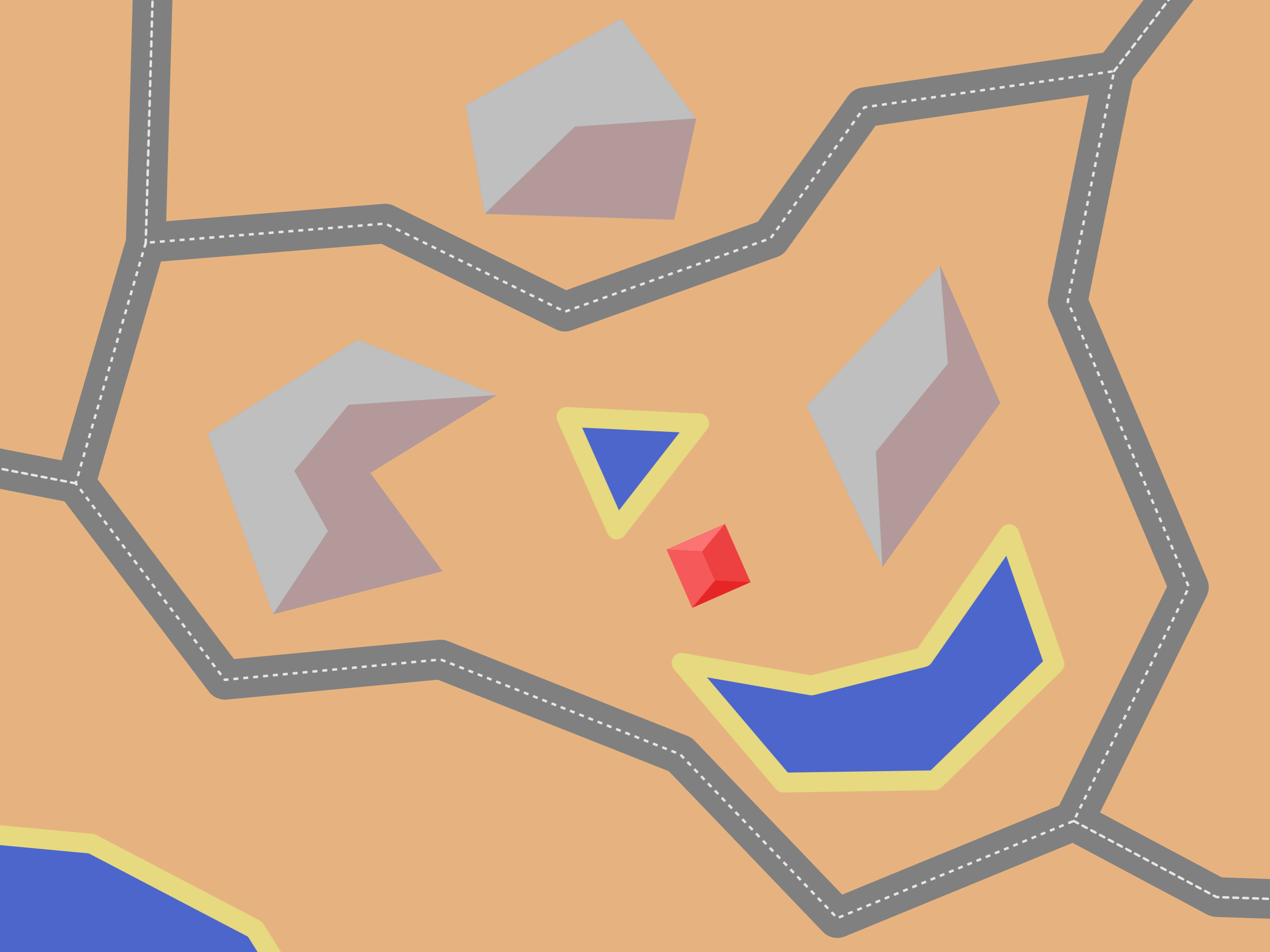
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In real road networks, the dilation is typically around  $1\frac{1}{2}$ .

The *dilation* between two points  $p$  and  $q$  is:

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Usually, not all land  
can be walked on.



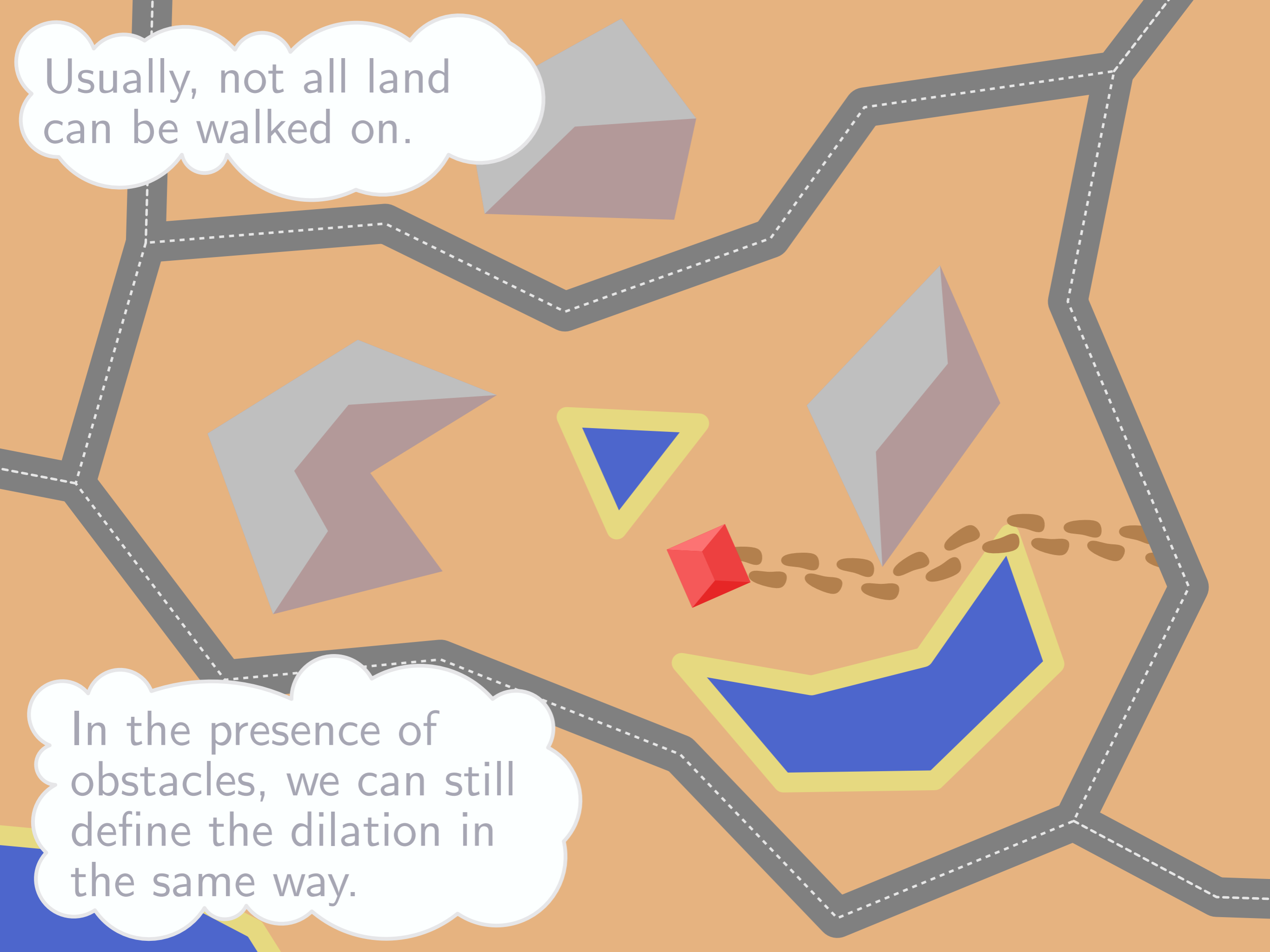


Usually, not all land can be walked on.

In the presence of obstacles, we can still define the dilation in the same way.

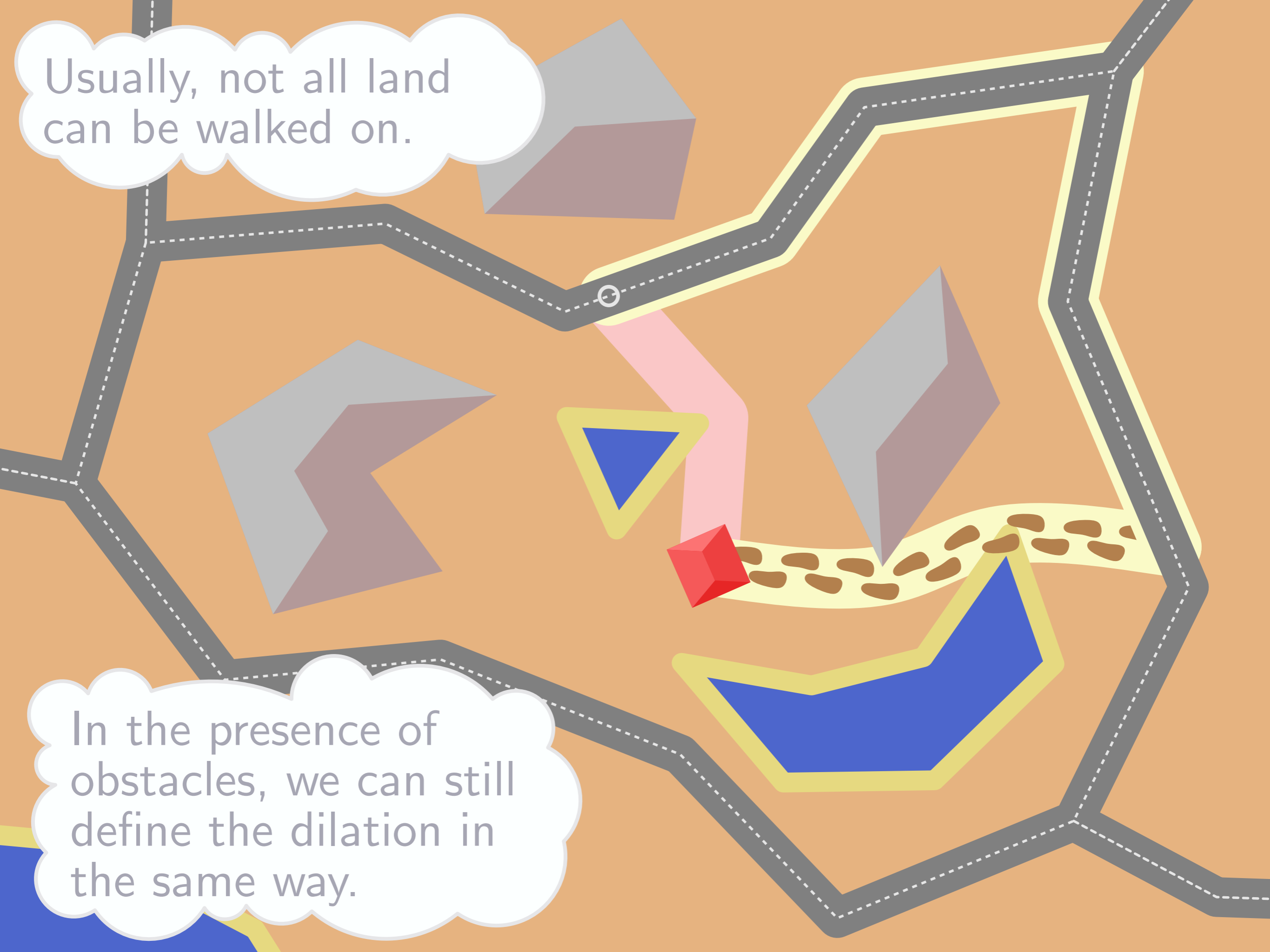
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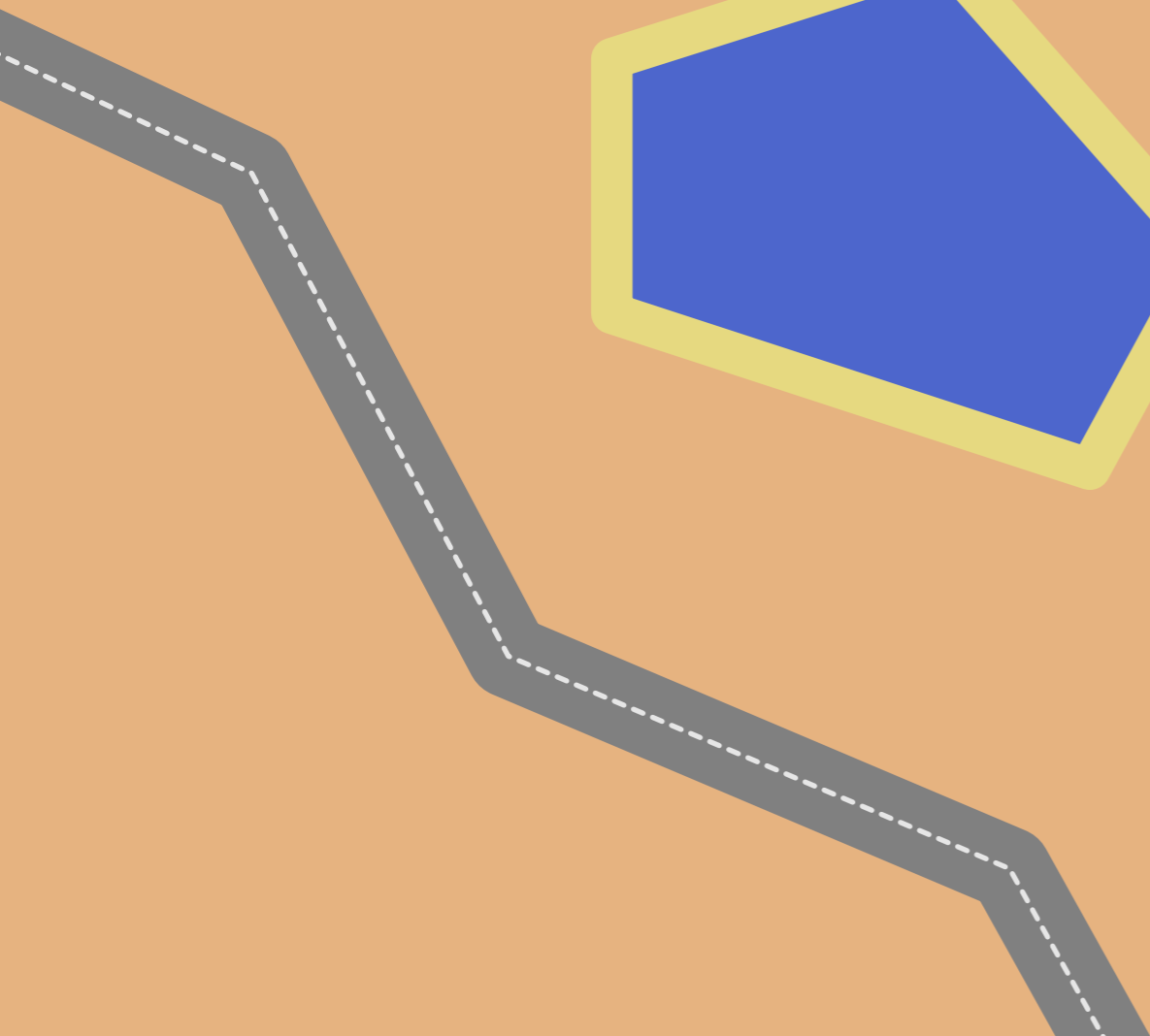
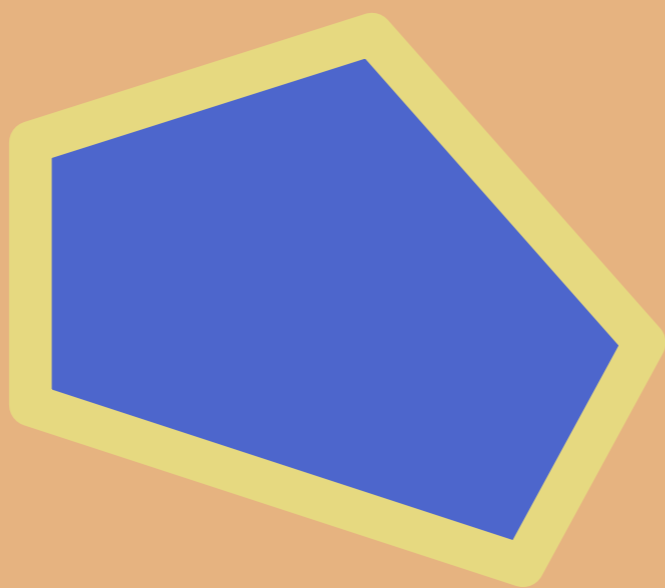
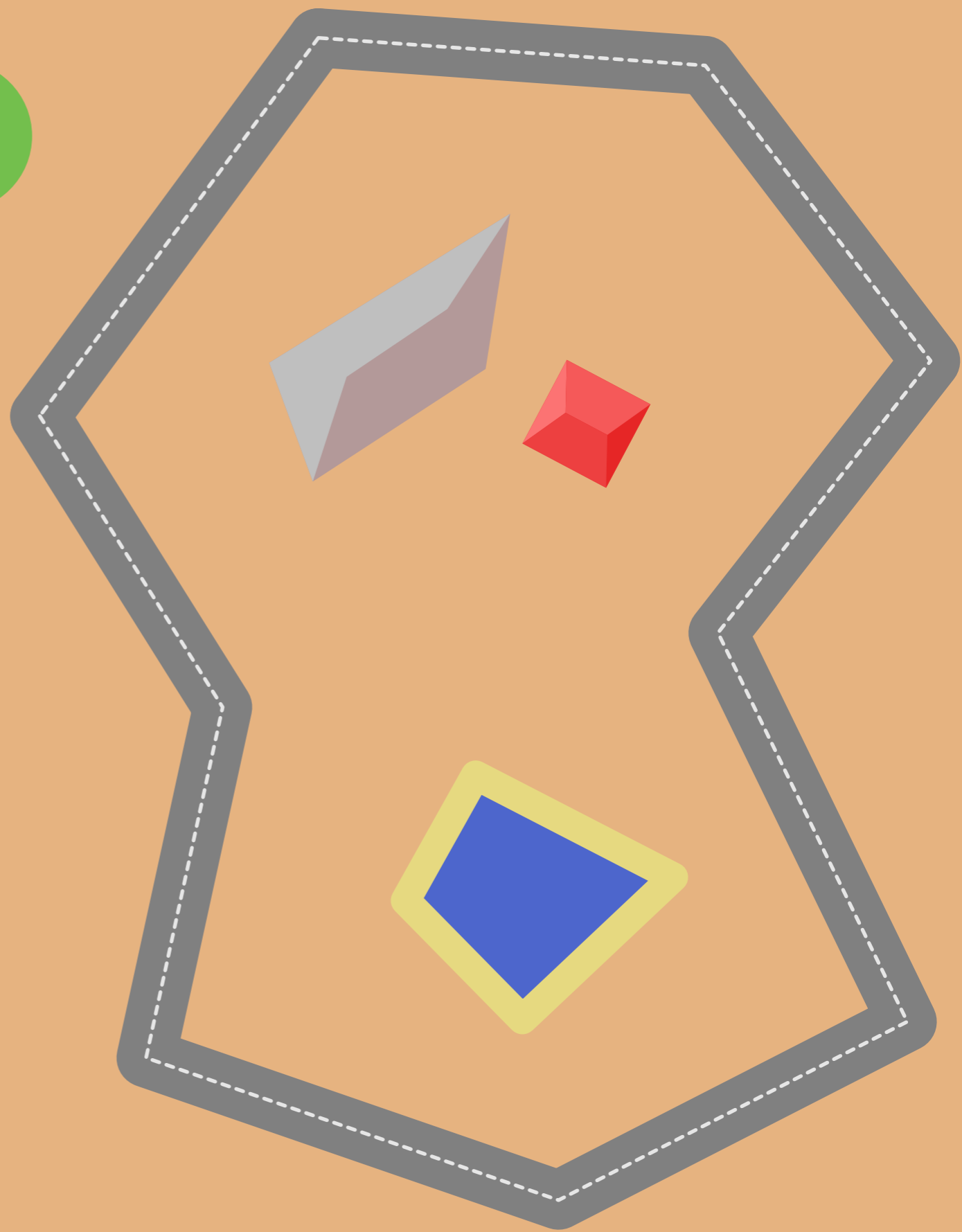
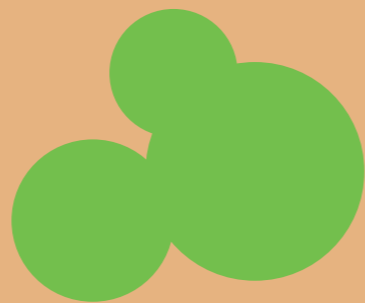
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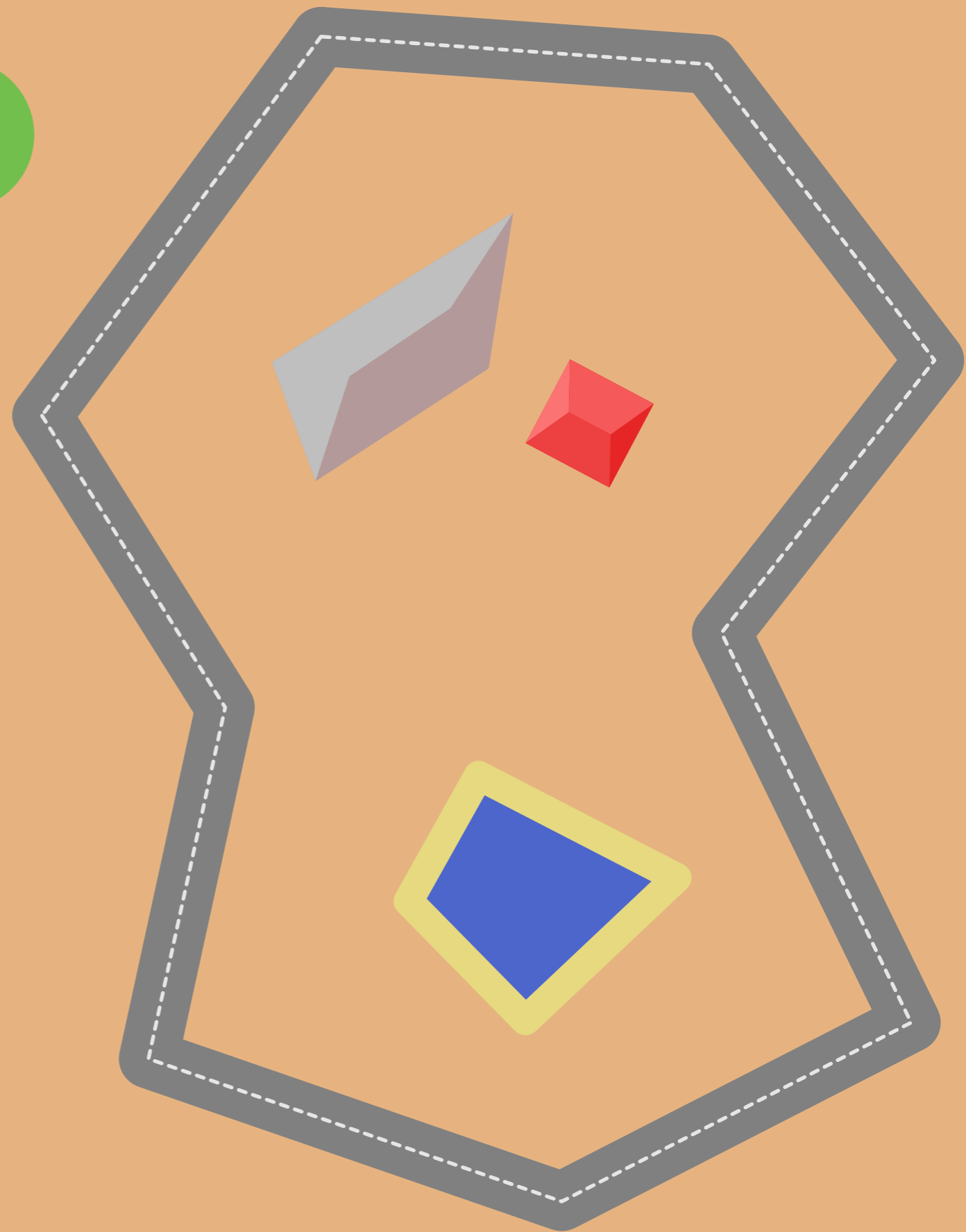




## Problem definition.

Given are a polygon  $P$ ,  
a set of obstacles  $\mathcal{B}$ ,  
and a point  $p$  inside  $P$ .

Add  $k$  feed links from  
 $p$  to  $P$  such that the  
worst dilation between  
 $p$  and any point on  $P$   
is minimal.

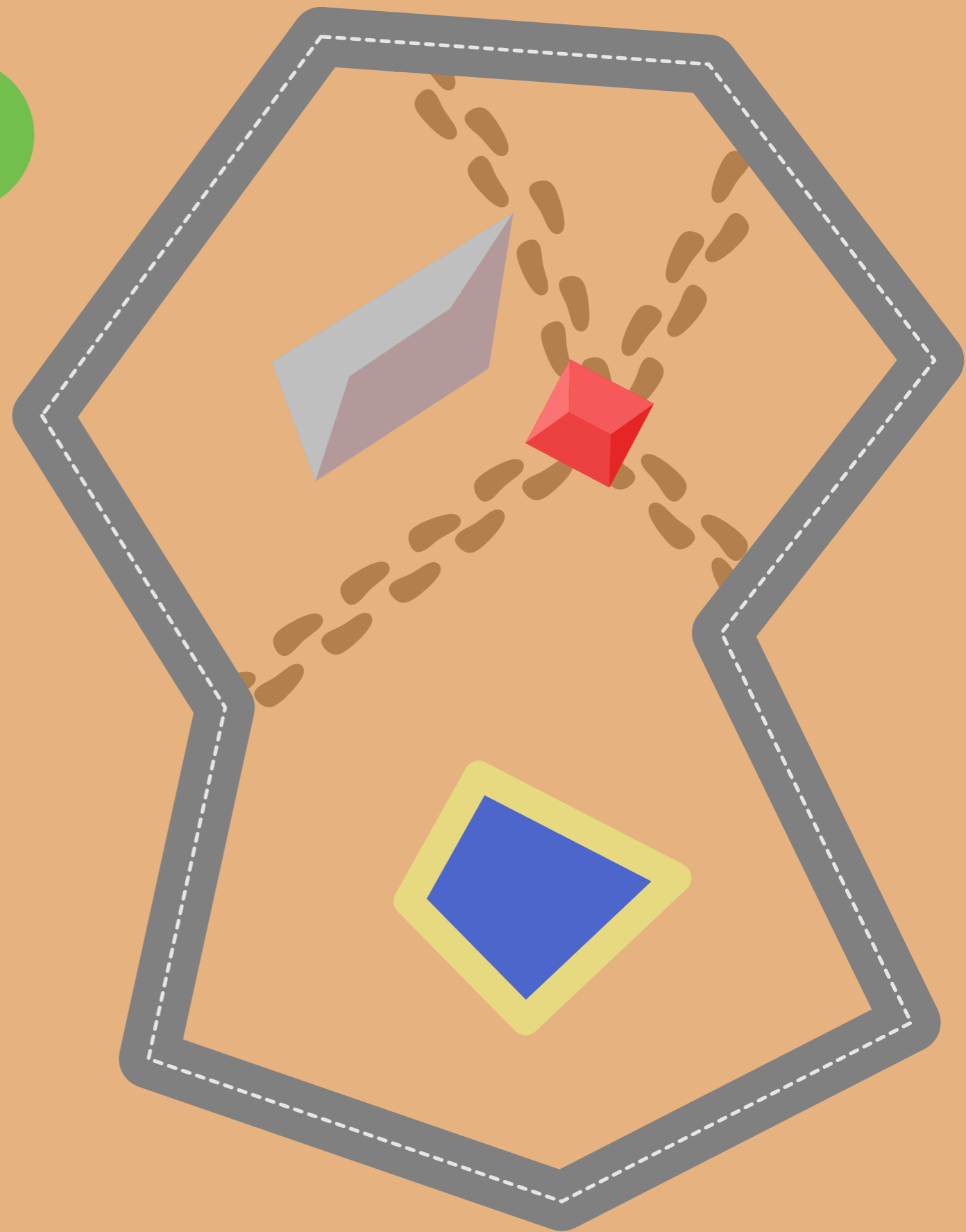




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We cannot solve the problem exactly in reasonable time.





What results do we have?



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Given a set of feed links, we can *compute* the worst dilation.



The background is a stylized landscape with a winding grey road that has a dashed white line on its left side. The landscape features various geometric shapes in shades of grey and brown, suggesting hills or terrain. There are two red 3D cubes: one in the upper left and one in the lower center. In the lower right, there are three green circles of different sizes and a green starburst shape. The overall color palette is warm, with orange and brown tones.

What results do we have?

Given a set of feed links, we can *compute* the worst dilation.

Given a dilation threshold, we can compute  $OPT + 1$  feed links that obtain this.

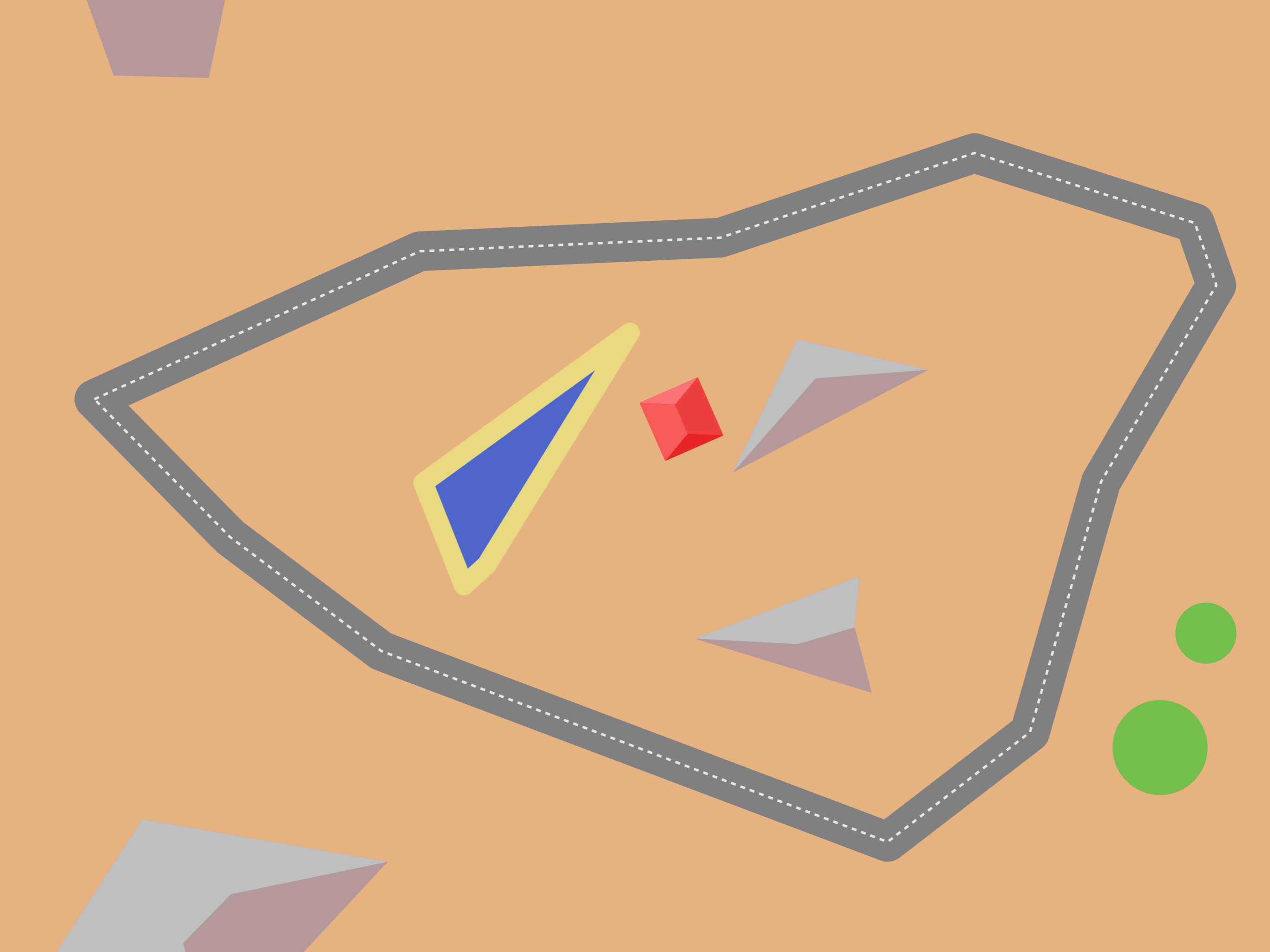
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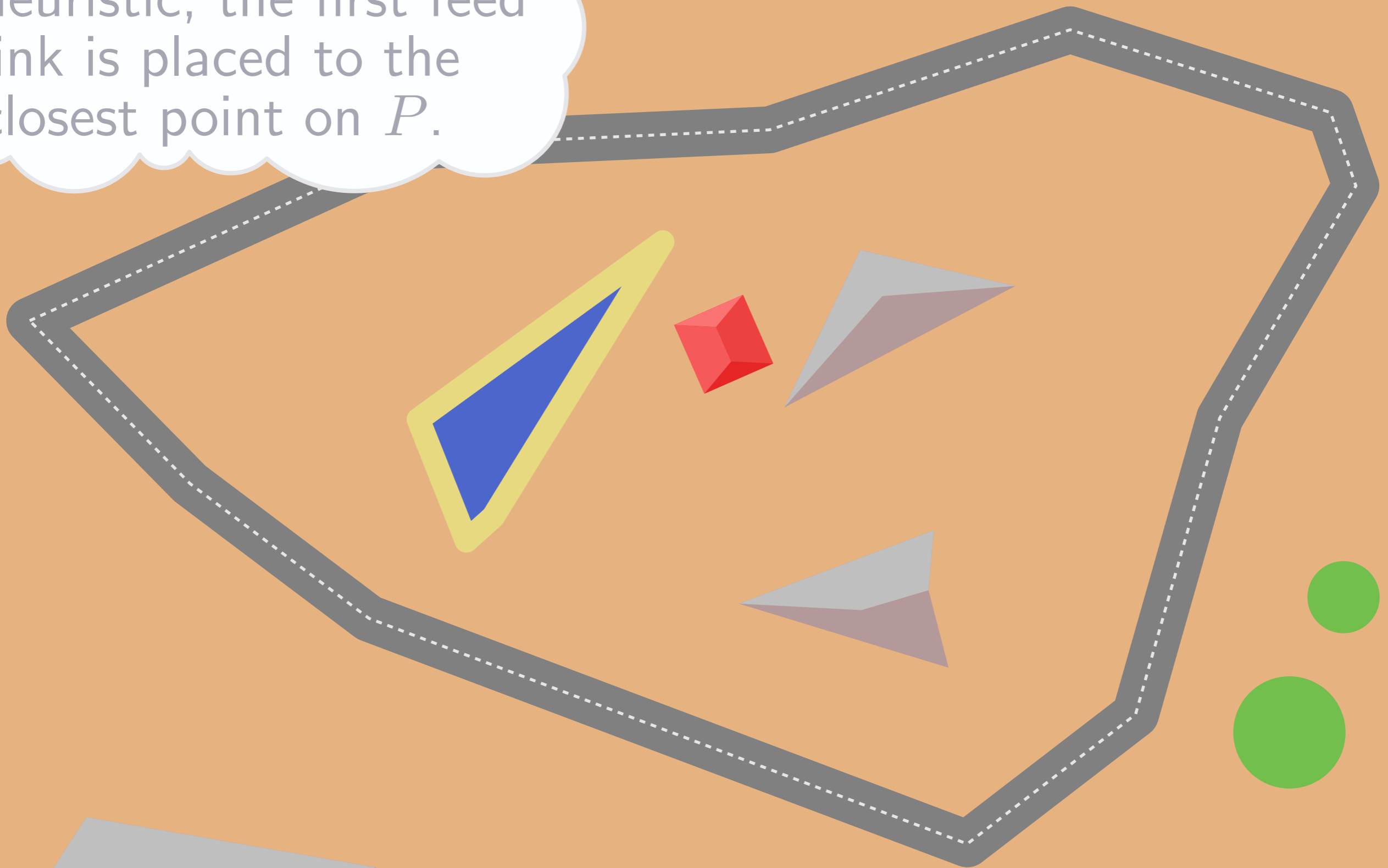
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We implemented two heuristics for placing  $k$  feed links, and analysed the results.

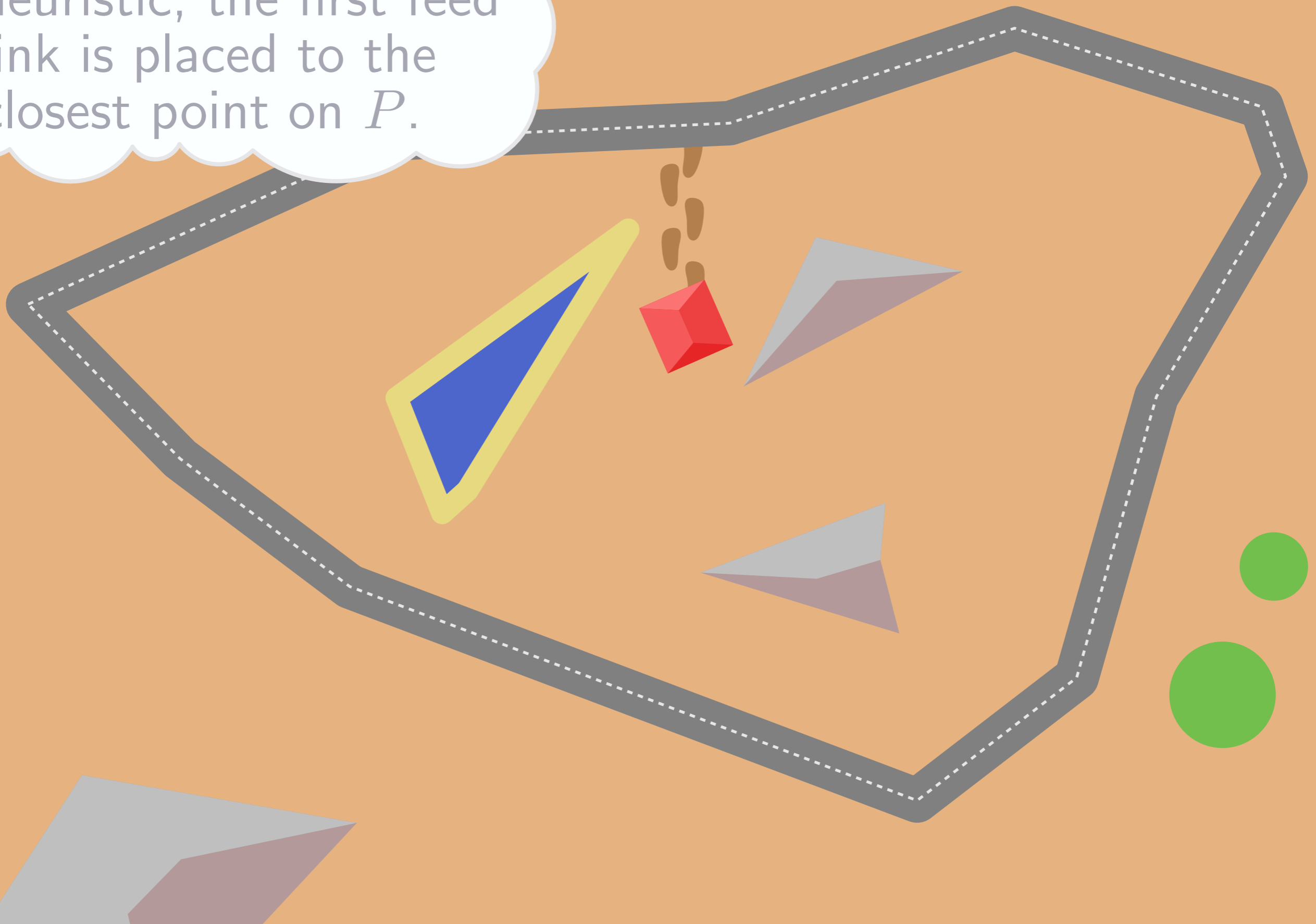




In the *greedy* heuristic, the first feed link is placed to the closest point on  $P$ .

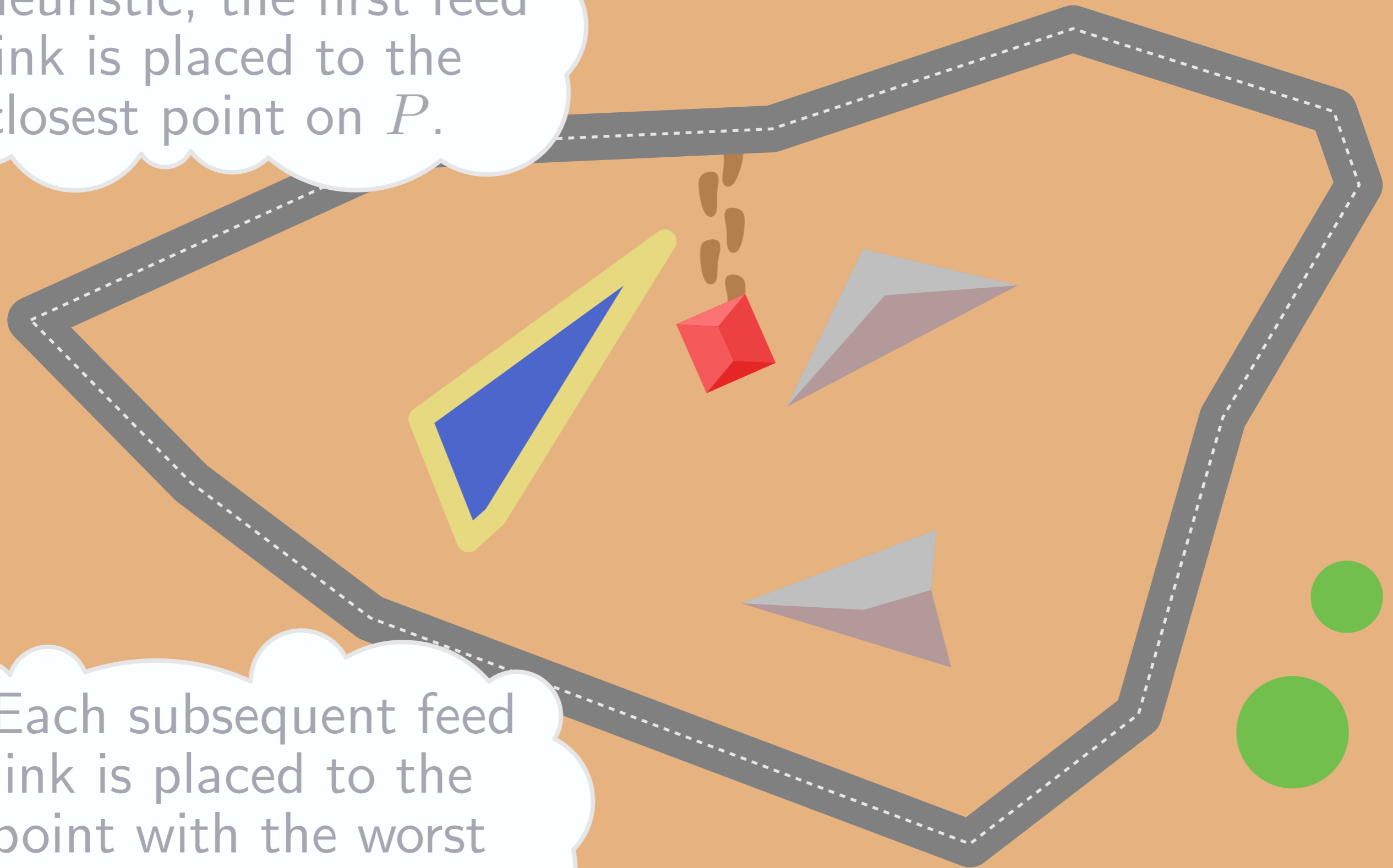


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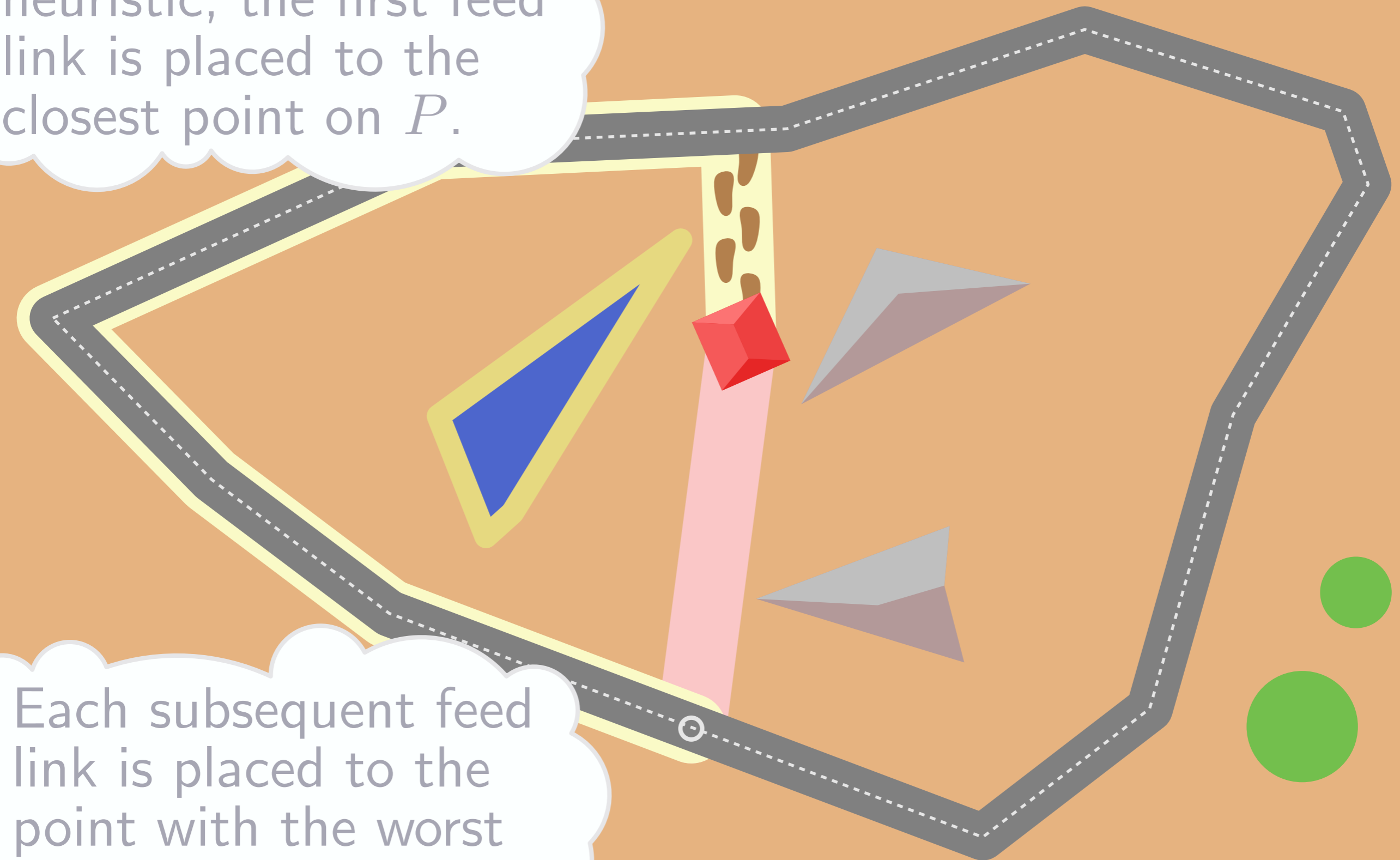


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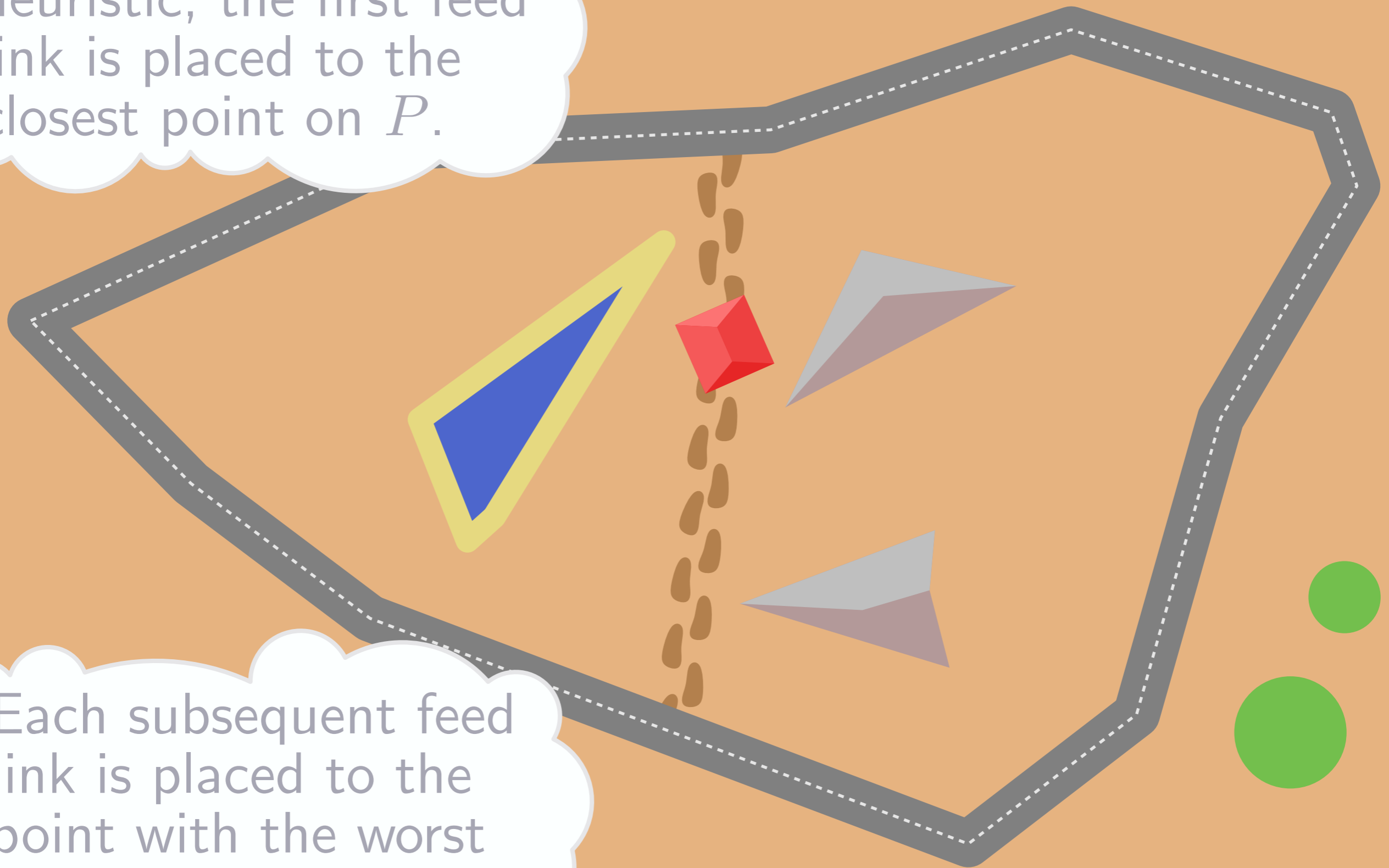
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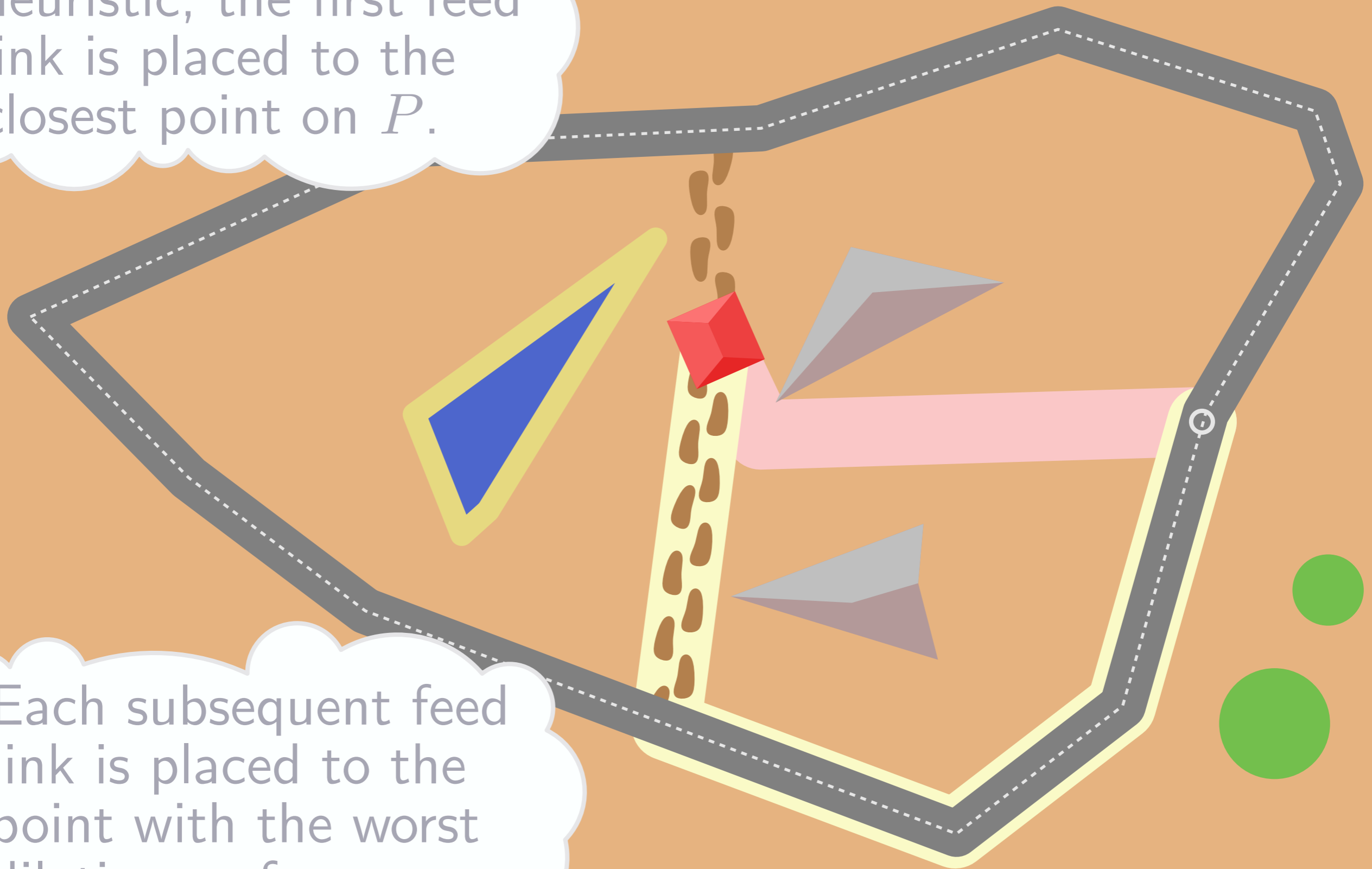
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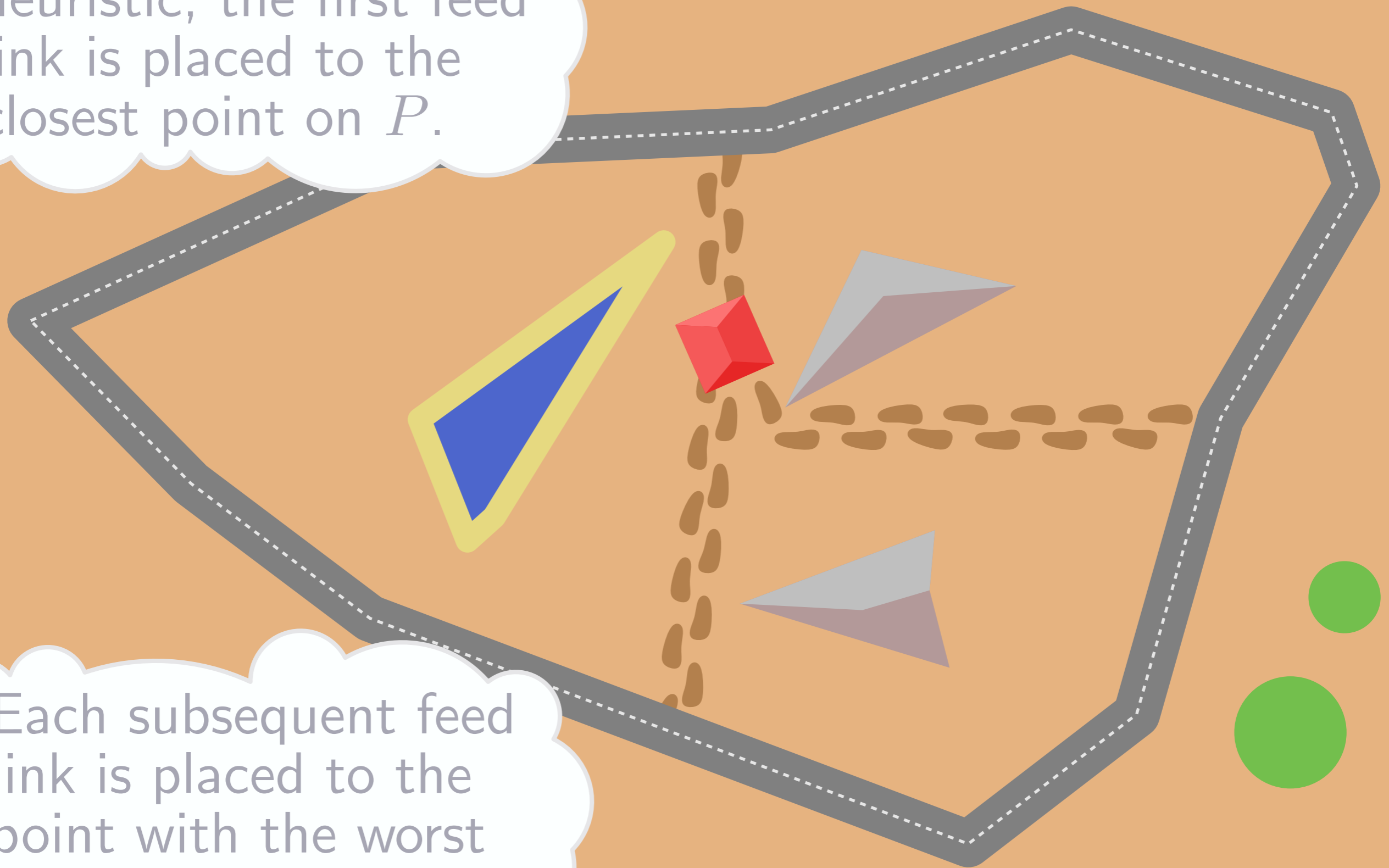
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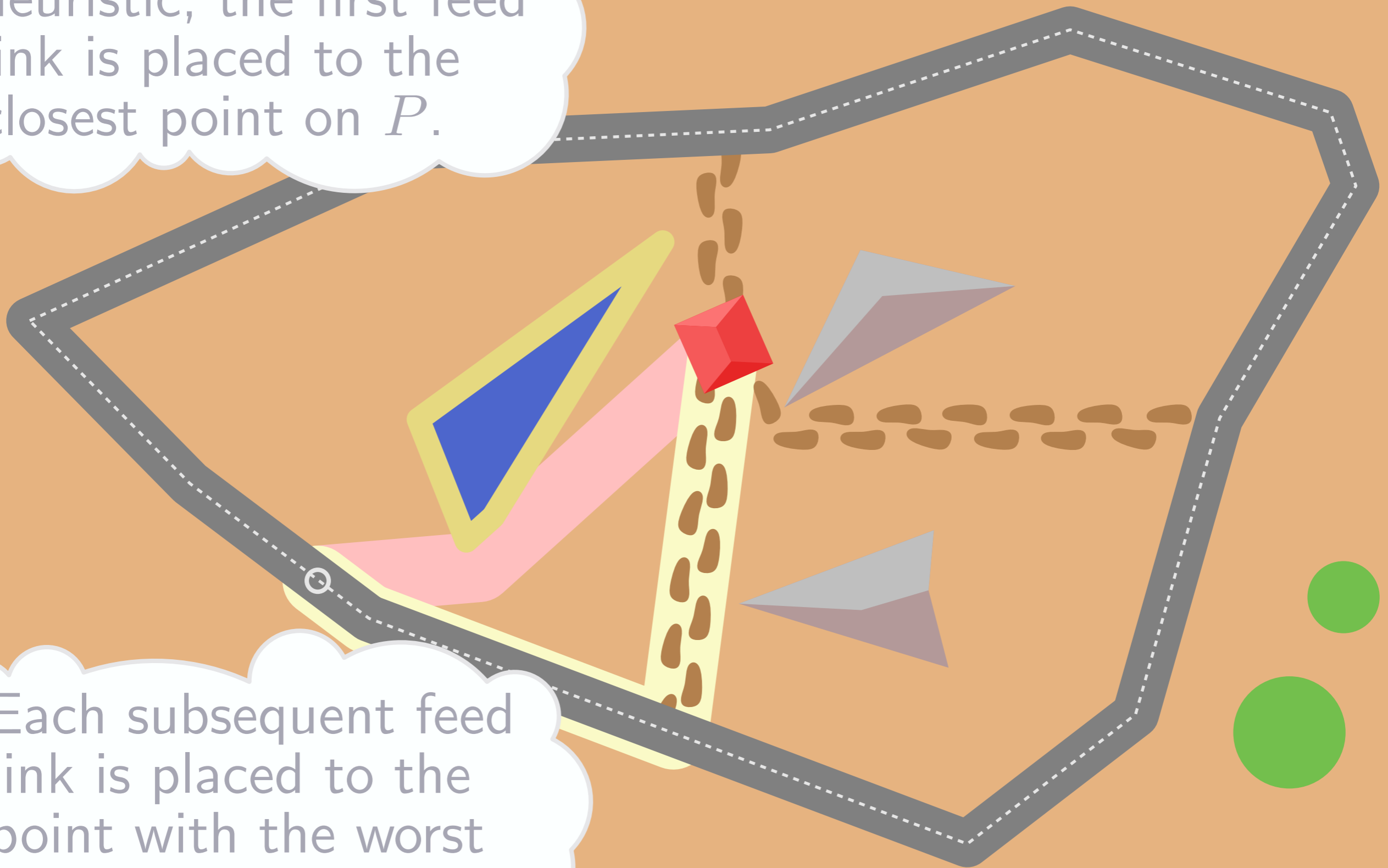
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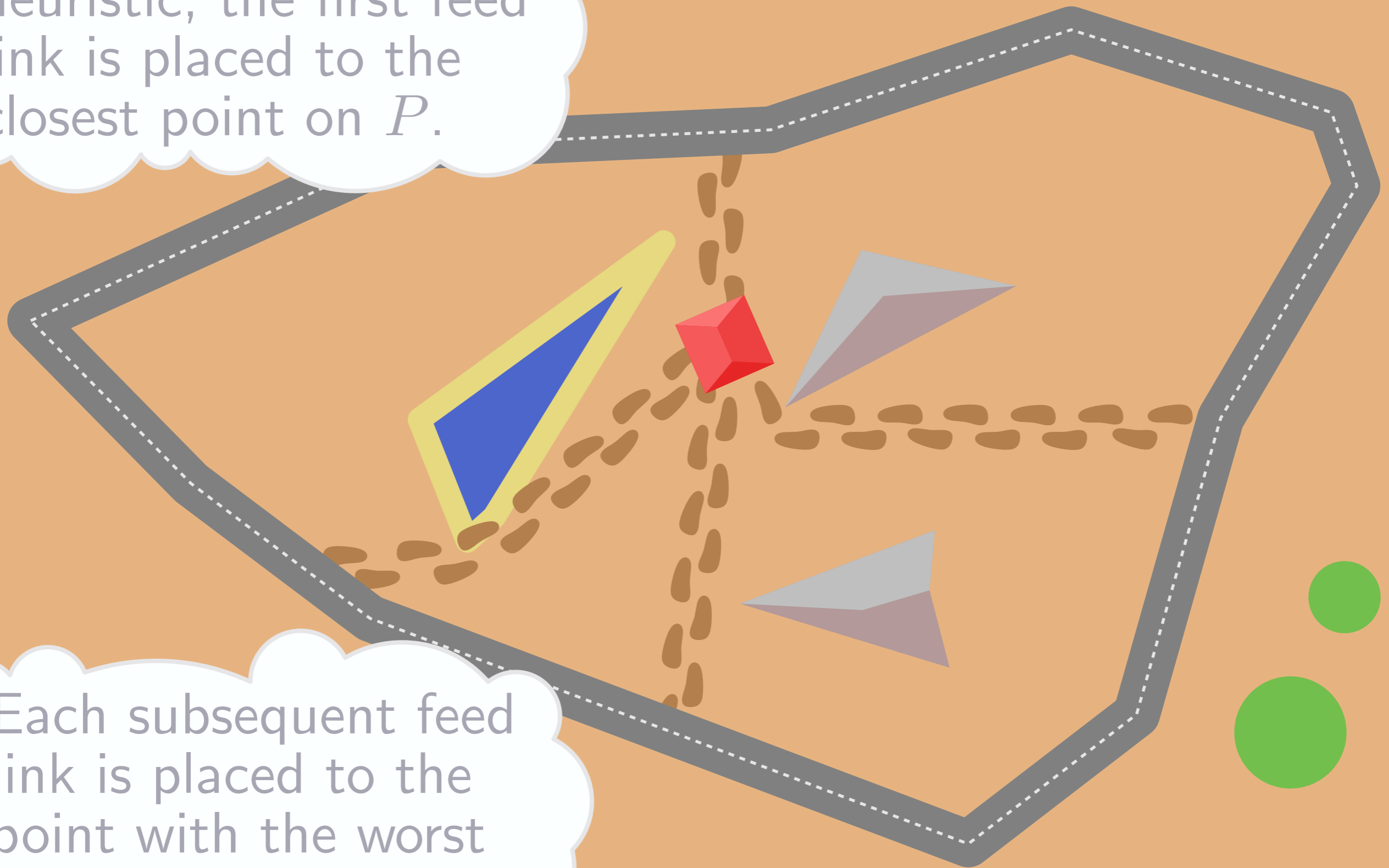
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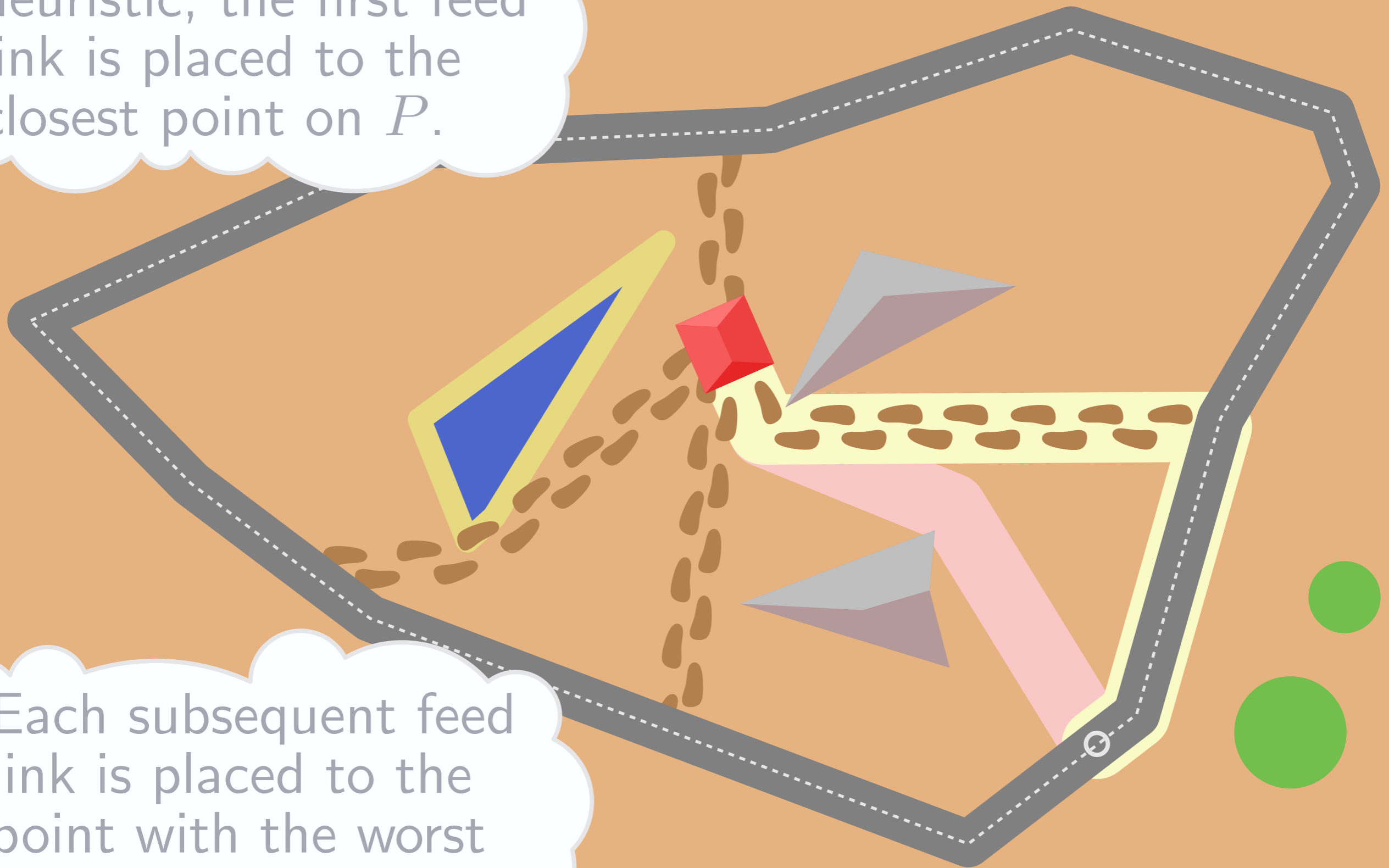
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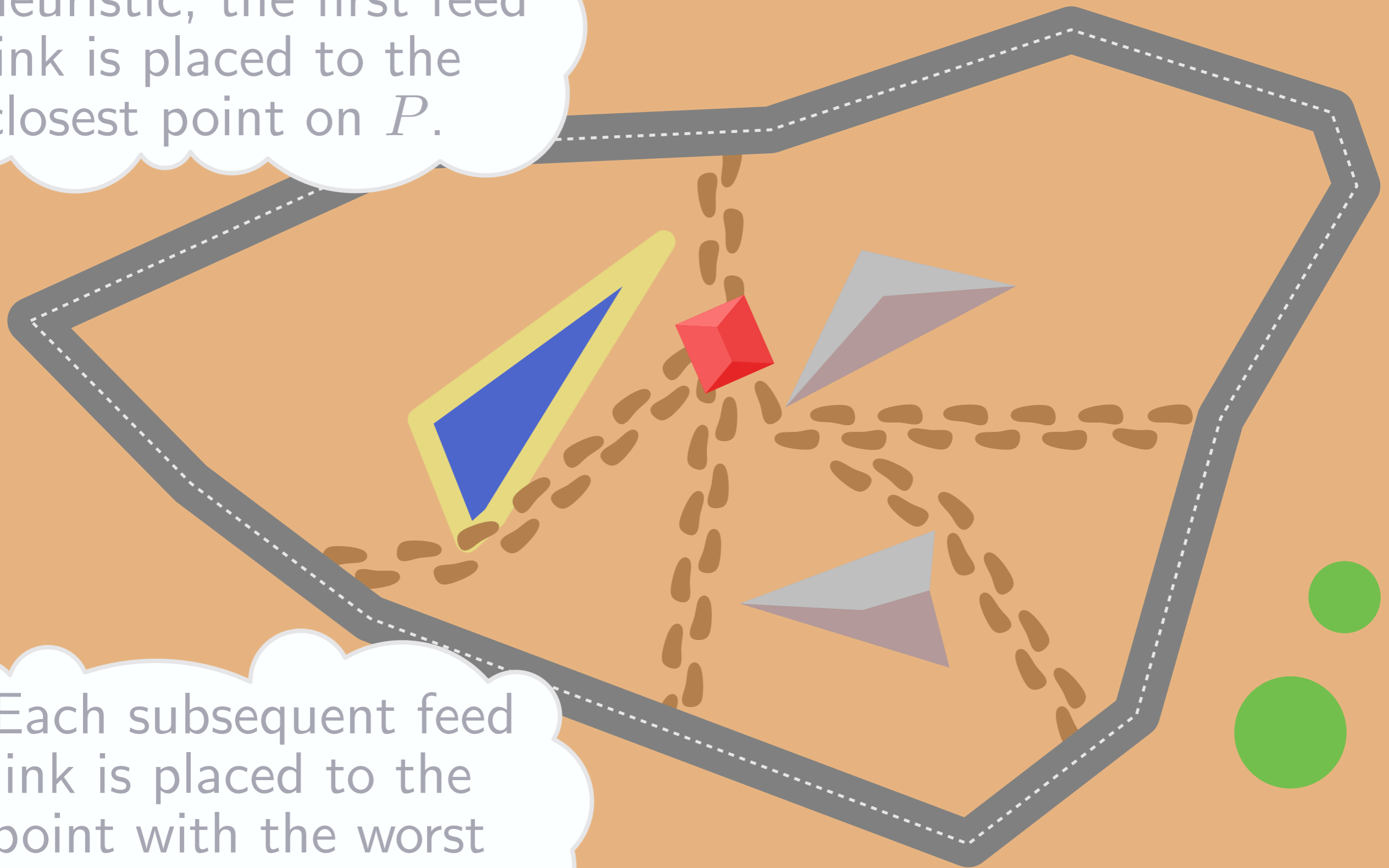
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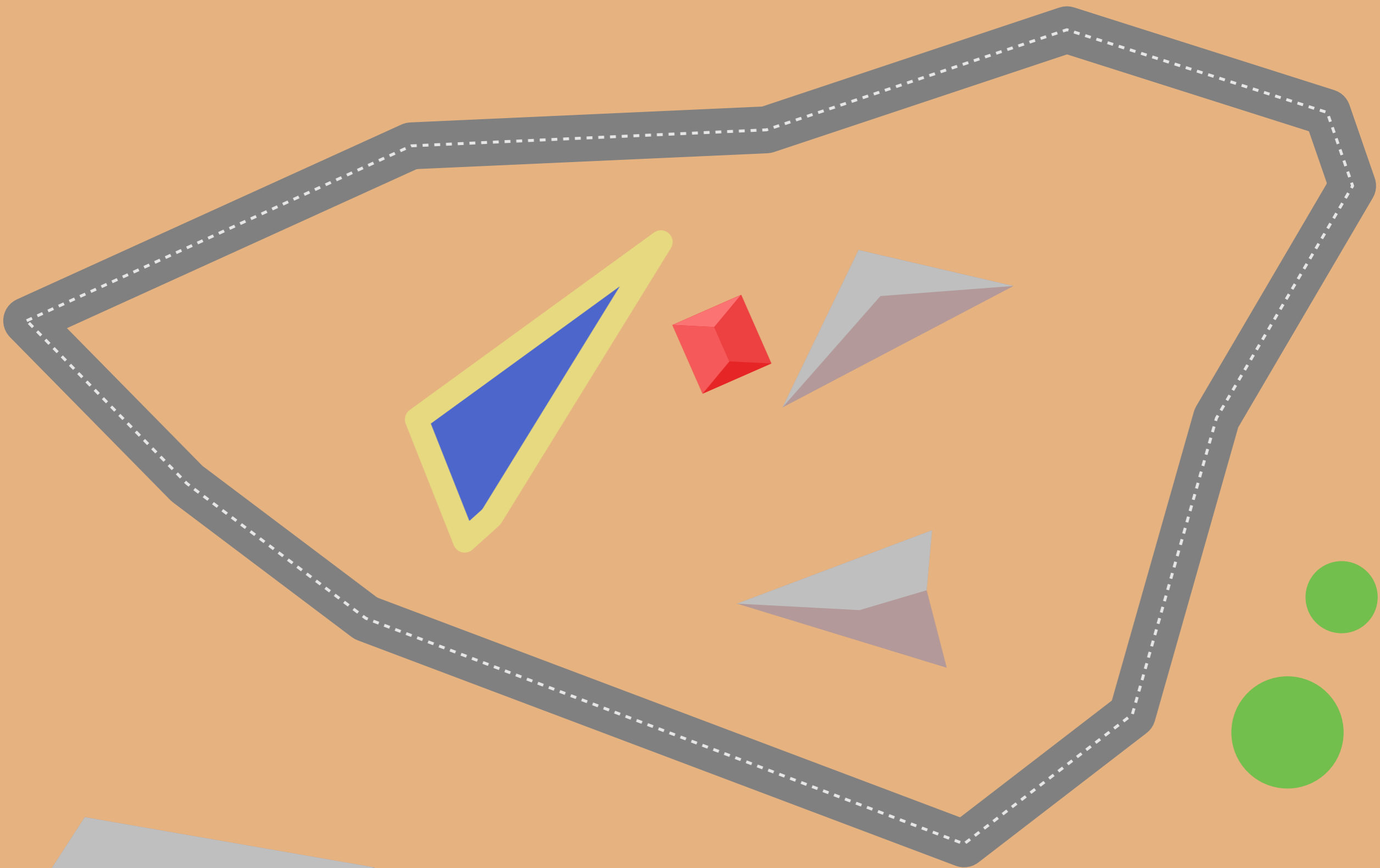
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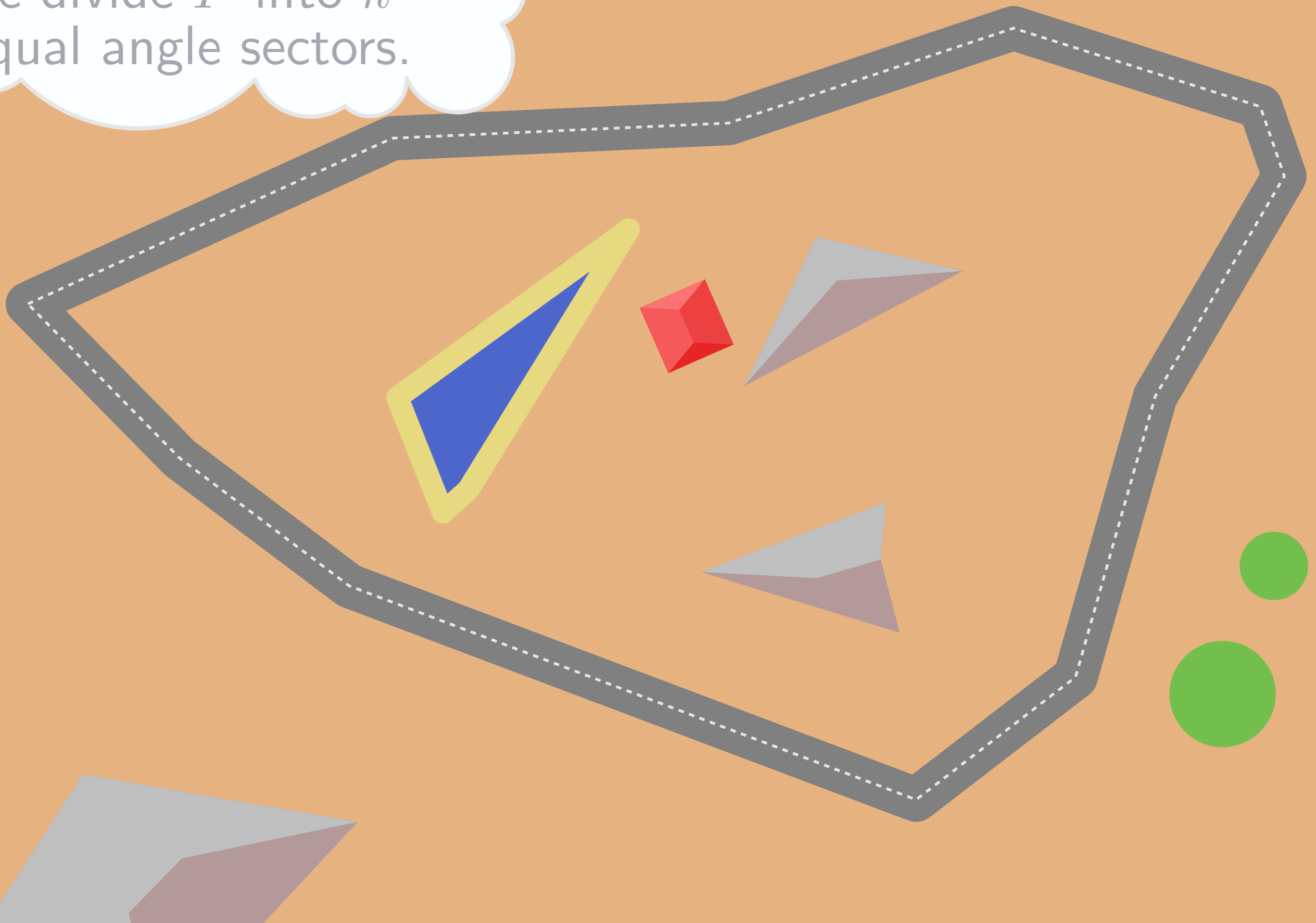
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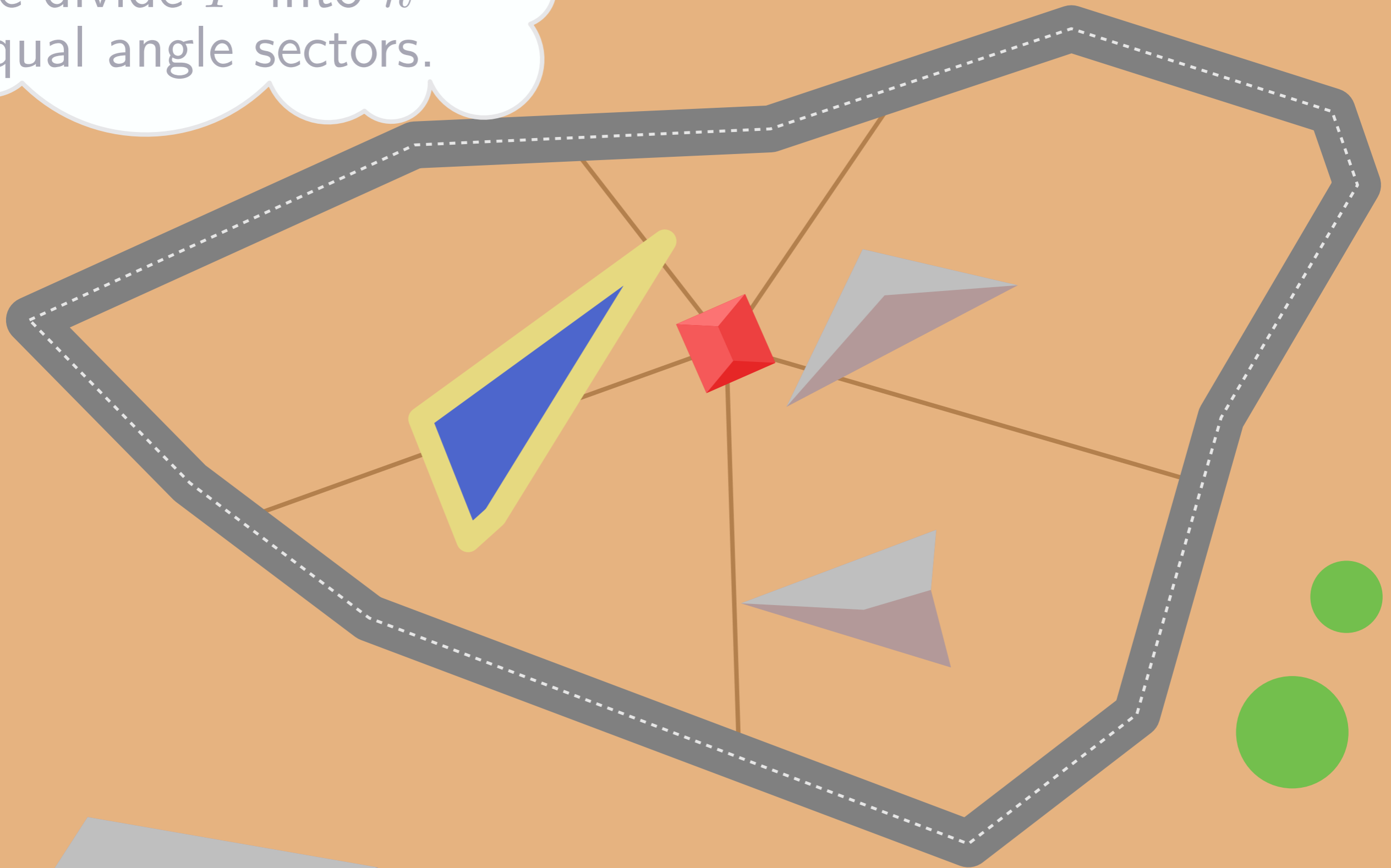




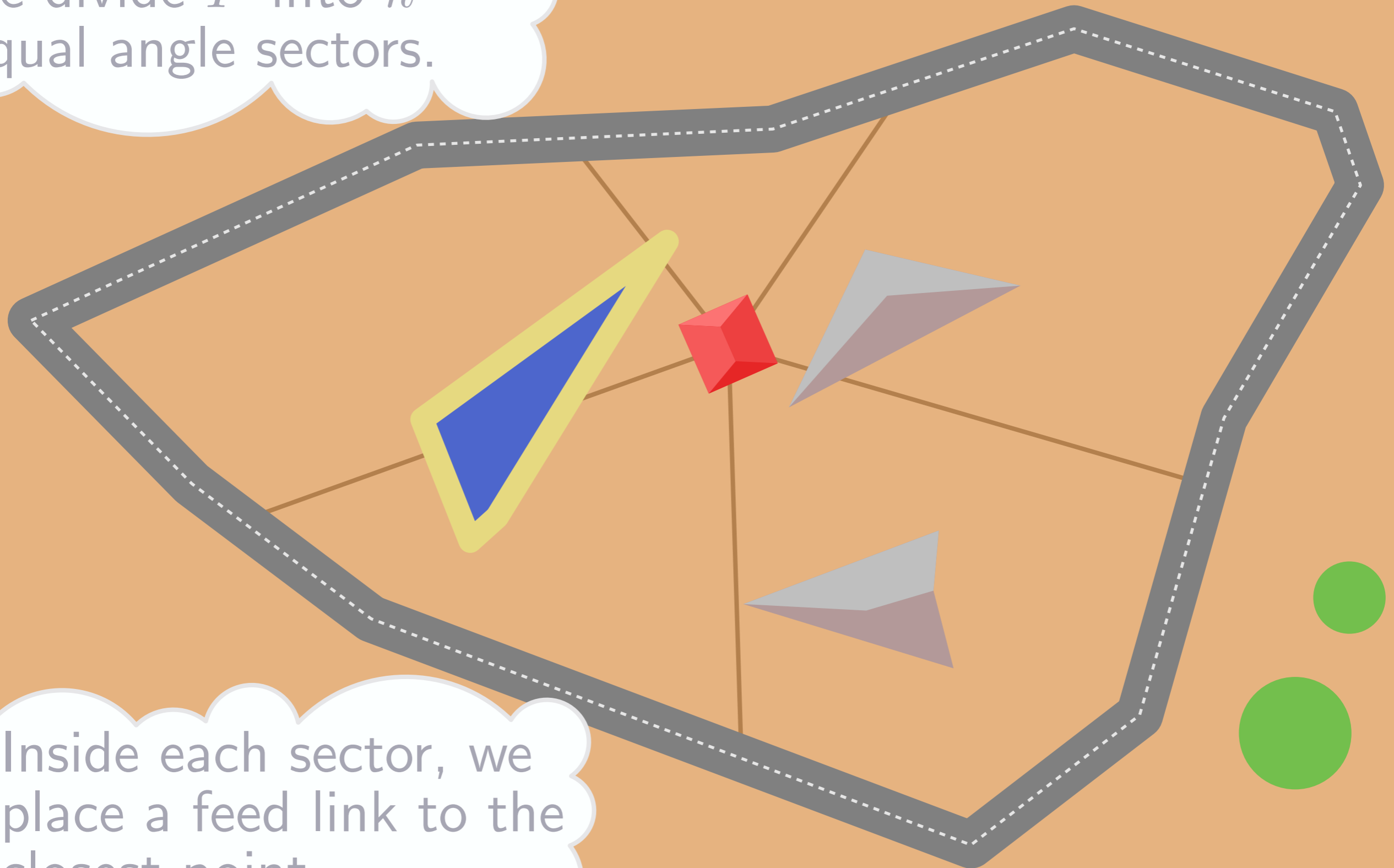
In the *sector* heuristic, we divide  $P$  into  $k$  equal angle sectors.



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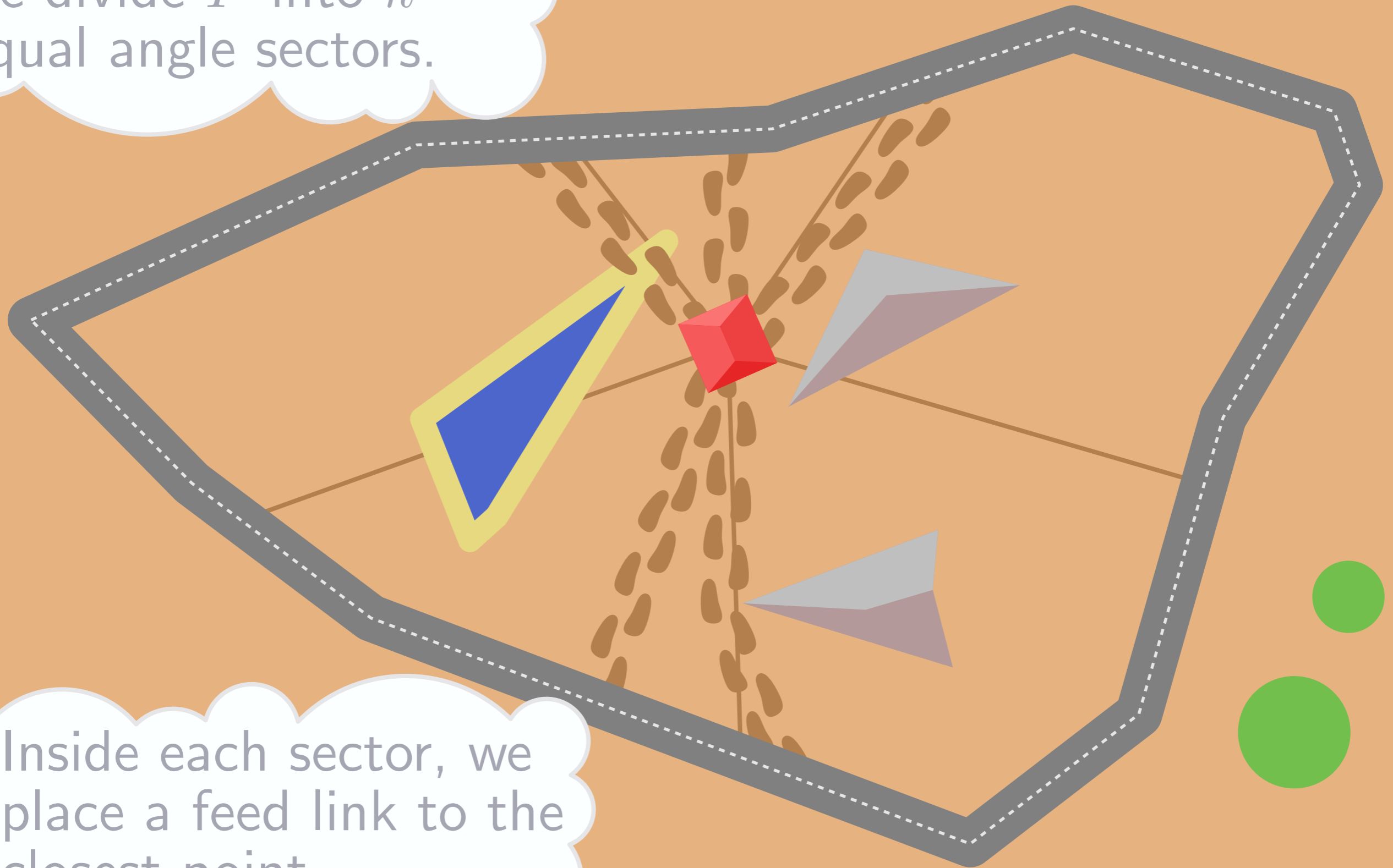
In the *sector* heuristic, we divide  $P$  into  $k$  equal angle sectors.



Inside each sector, we place a feed link to the closest point.

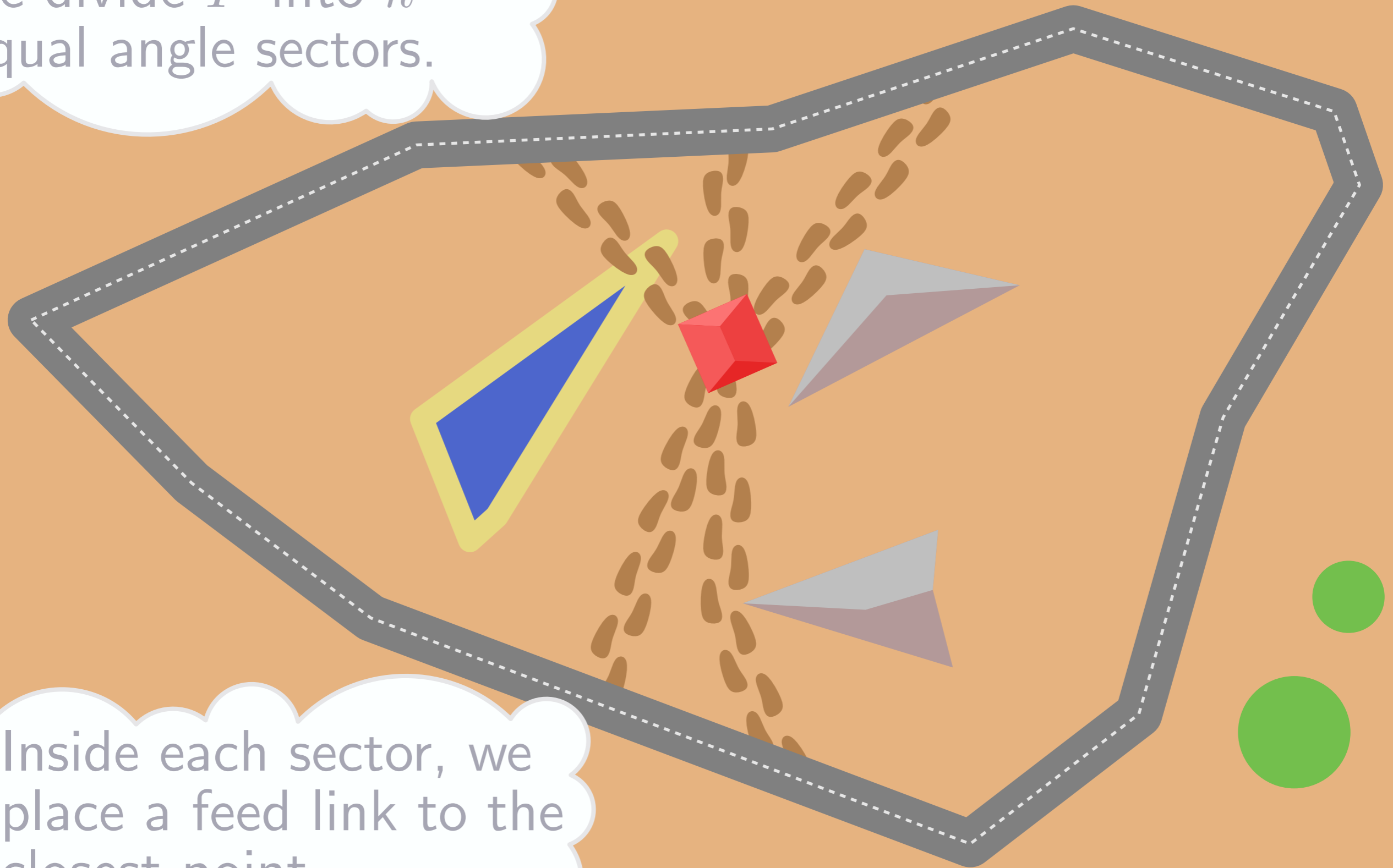


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The image shows a 2D environment with a brown ground. A grey path with a dashed white center line starts from the left and moves towards the right. On the right side, there is a blue area representing water, outlined by a yellow border. A red cube is positioned on the path just before the water. In the bottom-left corner, there are green bushes. In the top-left corner, there is a grey and brown polygon representing a tree. Two white speech bubbles with grey text are overlaid on the scene. The first speech bubble is at the top, and the second is on the left side.

We performed some experiments.

We generated 100 random polygons with two obstacles inside each.

The image shows a stylized maze environment. A grey path with a dashed white line runs through the maze. A red cube is positioned at the end of the path, representing a goal. The maze is composed of various polygons, some of which are obstacles. The background is a mix of orange and blue colors. There are also some green starburst shapes in the bottom left corner.

We performed some experiments.

We generated 100 random polygons with two obstacles inside each.

For each polygon, we placed 1 to 10 feed links with both heuristics.



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We generated 100 random polygons with two obstacles inside each.

For each polygon, we placed 1 to 10 feed links with both heuristics.

We computed the point with the worst dilation in each case, and took the average.

The background features a winding path that starts as a solid grey line with a dashed white center line, then turns into a solid yellow line. A red 3D cube is positioned on the yellow path. The background is a mix of orange and blue colors, with some green starburst shapes on the left and a grey and purple shape in the top left corner.

We performed some experiments.

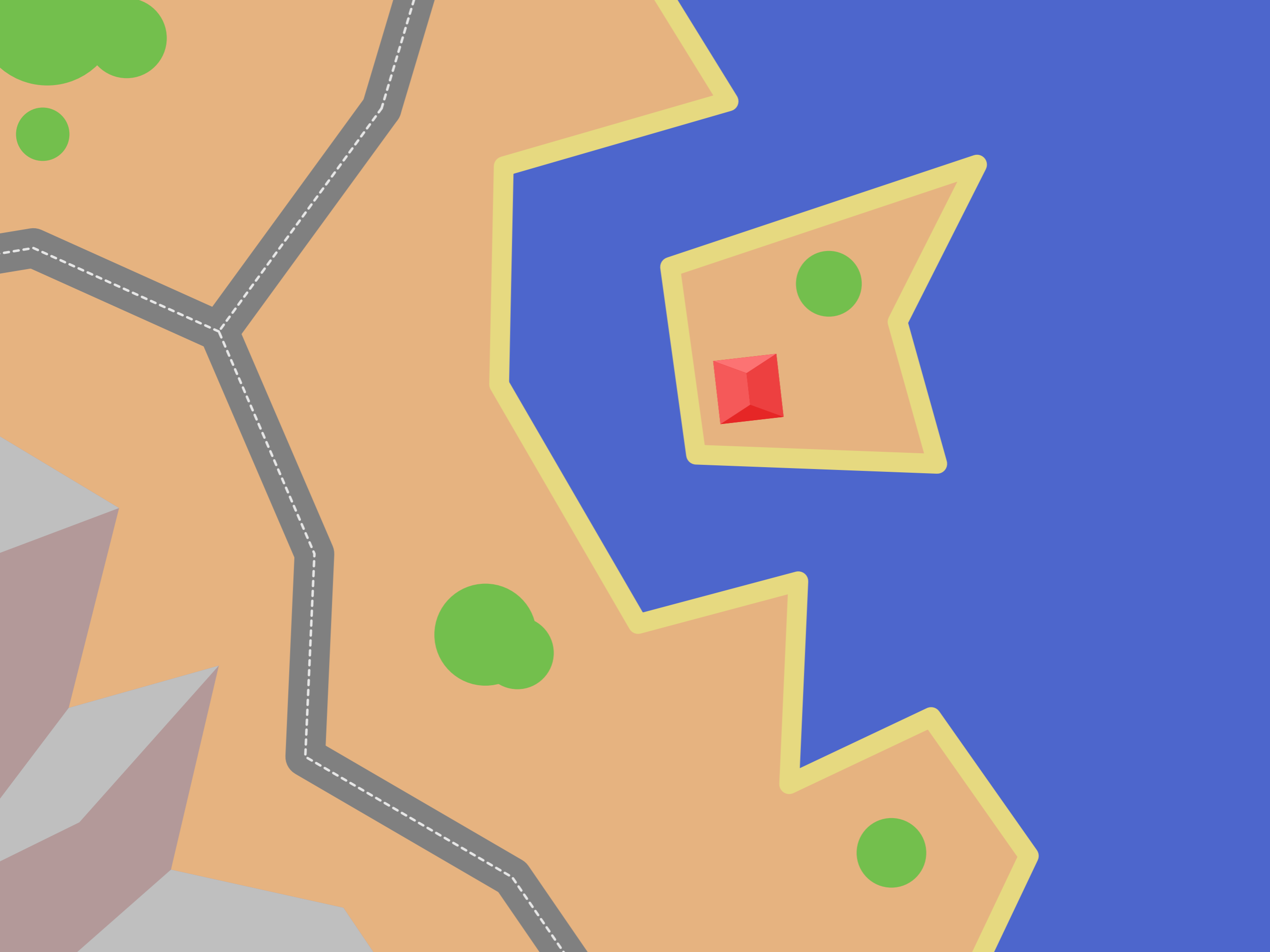
We generated 100 random polygons with two obstacles inside each.

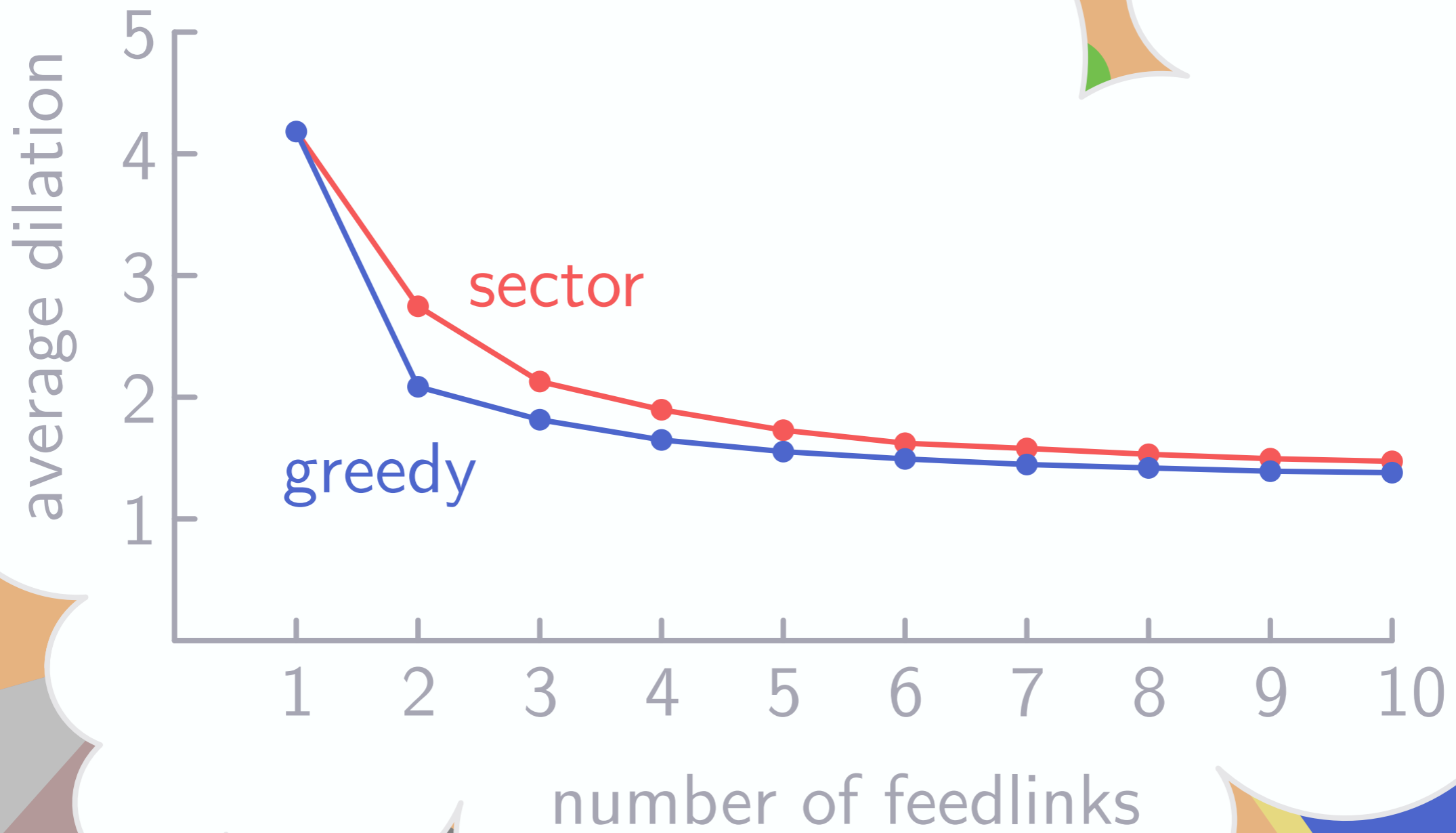
For each polygon, we placed 1 to 10 feed links with both heuristics.

We computed the point with the worst dilation in each case, and took the average.

And the results are...









Time to conclude.





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We tested heuristics to extend a network by several feed links.



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Of the two heuristics, the greedy one appears to perform best.

The background features a stylized landscape with a brown ground area, a blue sky area, and a grey road with a dashed white line. A red square is positioned on the ground, and a green bush is located near the center. The text is contained within white, cloud-like shapes.

Time to conclude.

We tested heuristics to extend a network by several feed links.

Of the two heuristics, the greedy one appears to perform best.

Both heuristics are easy to implement, and produce more realistic networks than ad-hoc methods that do not take dilation into account.







Thank  
you!

Thank  
you!

Rodrigo I.  
Silveira

Maike  
Buchin

Maarten  
Löffler

Boris  
Aronov

Tom  
de Jong

Jun  
Luo

Bettina  
Speckmann

Bart  
Jansen

Kevin  
Buchin

Marc  
van Kreveld

